UNIT- III

In a Power system both real and reactive Power demands are never steady and they continually change with raising or falling trend (load). Steam input to turbo generators or water input to hydro generators are must, therefore be continuously regulated to match active Power demand failing which the machine speed will vary with consequent change in frequency and it may be highly undesirable.

The maximum permisible change in frequency is "±2%."

Also the excitation of generators must be continuously regulated to match the reactive Power demand with reactive Power generation otherwise Voltage at Various system buses may go beyond the prescribed limits. The maximum permisible change in Voltage is "±5%."

In modern large interconnected systems manual regulation is not feasable and therefore Automatic generation and Voltage regulation equipment is installed on each generator. The controllers are set for a particular operating condition and they take care of small changes in load demand without exceeding the limits of frequency and Voltage.

As change in load demand becomes large, the controllers must be reset either manually or automatically

Necessity of maintaining Frequency constant:

All AC motors should require constant frequency supply so as to maintain frequency

Constant.

In continuous process industry, it effects the operation of process itself.

For synchronous operation of various units in power system network, it is necessary to maintain frequency constant.

Frequency effects the amount of power transmitted through interconnecting lines.

Electrical clocks will loose or gain time if they are driven by synchronous motors and accuracy of clocks depends on frequency and also the integral of frequency error i.e., loose or gain of time by electrical clocks.

Generator Controllers (P-f and Q-V Controllers):

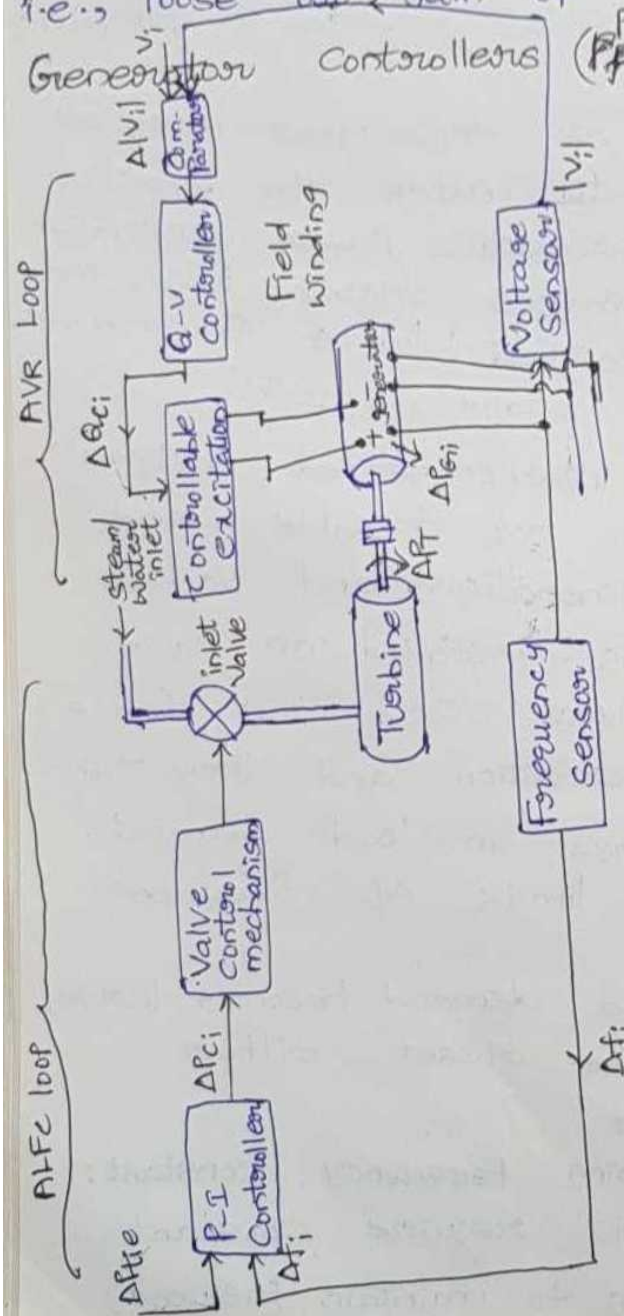


Fig. Schematic diagram of P-f controller and Q-V controller

The active Power "P" mainly depends on internal angle "δ" and is independent of the bus Voltage magnitude (|V|). The bus Voltage is dependent on machine excitation and hence on reactive Power (Q) and is independendant of machine angle "δ"

change in machine angle "δ" is caused by momentarily change in generator speed and hence the frequency. Therefore the load frequency and excitation Voltage controls are not interactive for small changes and can be modeled and analyzed independently.

The objective of P_f control mechanism is to exert control of frequency and simultaneously exchange of real Power flows via interconnecting lines.

In this control, frequency sensor senses change in frequency and gives signal Δf_i, the P_f controller senses the change in frequency signal Δf_i, and the increments in tie line real Powers (ΔP_{TIE}) which will indirectly provide information about incremental state error (ΔS_i). This sensor signals (Δf_i and ΔP_{TIE}) are amplified, mixed and transformed into real Power control signal as input signal and Provides output signal which will change the position of inlet Valve of Prime mover. As a result there will be change in Primemover output and hence change in real Power Position (ΔP_{G1})

This entire P_f controller can be yielded by Automatic Load Frequency Control (ALFC) loop.

The objective of MVAR-Voltage or Q-V control mechanism is to exert control of the Voltage state |V|. A Voltage sensor senses the terminal Voltage and converts it into an equivalent Proportionate DC Voltage. This Proportional

DC Voltage is compared with a reference Voltage V_{iref} by means of comparator. The output obtained from the comparator is error signal $\Delta|V_i|$ and is given as input to Q-V controller which transforms it to reactive power signal command ΔQ_{ci} and is fed to controllable excitation source. This results in a change in rotor field current, which in turn modifies the generator terminal Voltage. This entire Q-V control can be yielded by an automatic Voltage regulator loop (AVR)

In addition to Voltage regulators at generator buses, equipment is used to control Voltage magnitude at other selected buses. Tap-changing transformers, switched capacitor banks and static VAR systems can be automatically regulated for rapid Voltage control.

P-f ~~Vs~~ control Versus Q-V control:

Any static change in real bus power ΔP_i will affect only the bus Voltage phase angles δ_i (since $P \propto \delta$), but will leave the bus Voltage magnitudes almost unaffected.

Static change in reactive bus power ΔQ_i affects essentially only the bus Voltage magnitudes (since $Q \propto V^2$), but leave bus Voltage phase angles almost unchanged.

Static change in reactive bus power at a particular bus affects the magnitude of that bus Voltage most strongly, but in less degree magnitudes of bus Voltages at remote buses.

Speed - Governing system:

The speed governor is main primary tool for the LFC, whether machine is used alone to feed a smaller system (or) whether it is a part of most elaborate ~~m~~ arrangement. A schematic arrangement of the main features of a speed-governing system ~~of~~ ~~kind~~ used on steam turbines to control output of generators to maintain constant frequency is as shown in figure

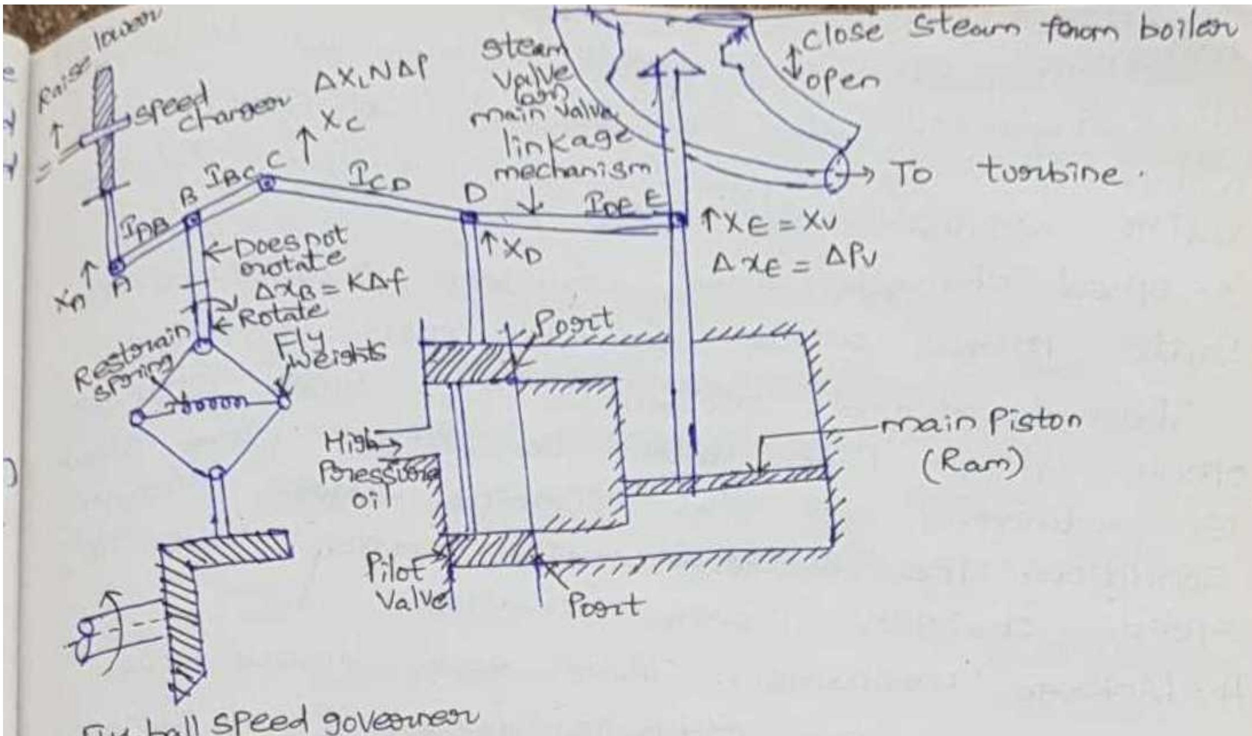


Fig. Speed Governor system

Its main components are as follows.

1. Flyball speed Governor
2. Hydraulic Amplifier
3. Speed Changer
4. Linkage mechanism.

1. Flyball speed Governor:

It is purely a mechanical speed sensitive device coupled directly to and builds directly on prime movers to adjust the control valve opening via linkage mechanism.

It senses a speed deviation (or) a power change command and converts it into appropriate valve action. Hence this is treated as "Heart" of system which controls the change in speed $\left(N_S = \frac{120f}{P}\right)$

As the speed increases, the flyball moves outwards and the point "B" on linkage mechanism moves upwards. The reverse will happen if the speed decreases.

2. Hydraulic Amplifier: It consists of main piston and pilot valve. With this arrangement, hydraulic amplification is obtained by converting the

movement of low power level pilot valve into the movement of high power level piston valve which is necessary to open (or) close the steam valve against the valve

3. Speed changer: Speed changer provides a steady state power output setting for turbine.

The downward movement of speed changer opens upper pilot valve so that more steam is admitted to the turbine under steady condition. The reverse will happen when the speed changer moves upwards.

4. Linkage mechanism: These are linked for transforming the fly balls movement to turbine valve through hydraulic amplifier

"ABC" and "CDE" are rigid links pivoted at "B" and "D" respectively.

The mechanism provides a movement to the control valve in proportion to change in speed.

Link "DE" provides a feedback from the steam valve movement.

Speed Governor model (Transfer function (or) mathematic formulation):

Step I: Consider steady state condition by assuming

1. The linkage mechanism is stationary
2. Pilot valve is closed.
3. Steam valve opened by a definite magnitude.
4. Turbine running at constant speed with turbine power output balancing the generator load.

Let F° = supply frequency

P_G^{1} = Generator output (turbine output neglecting generator losses)

X_E° = steam valve setting

step II:

Let Point "A" of speed changer lower down by an amount ΔX_A. As a result increase in power is considered as ΔP_c.

$\therefore \Delta X_A \propto \Delta P_c$

$$\Delta X_A = K_2 . \Delta P_c \rightarrow ①$$

The increase in frequency Δf causes the link Point "B" to move downwards with a small distance i.e $\Delta X_B \propto \Delta f$

$$\Delta X_B = K_1 . \Delta f \rightarrow ②$$

step III:

Factors contribute to the movement of "C" are ΔX_C and ΔX_B.

$$\therefore \Delta X_C = \Delta X_B - \Delta X_A$$
$$= K_1 . \Delta f - K_2 \Delta P_c \rightarrow ③$$

where K_1 and K_2 are positive constants

step IV:

At Point D, the movement of "D" is contributed by movement of "C" and "E"

Since "C" and "E" move downwards then "D" moves upwards.

$$\therefore \Delta X_D = K_3 \Delta X_C + K_4 \Delta X_E \rightarrow ④$$

Where K_3 and K_4 are positive constants.

step V:

At Point "E" assuming that oil flow into the hydraulic cylinder proportional to position ΔX_E.

$$\therefore \Delta X_E = K_5 \int_0^t -(\Delta X_D) dt \rightarrow ⑤$$

Where K_5 is positive constant.

step VI:

Taking laplace transforms of eqns ③, ④ and ⑤, we get

$$\text{eq ③} \Rightarrow \Delta X_C(s) = K_1 . \Delta f(s) - K_2 \Delta P_c(s) \rightarrow ⑥$$

$$④ \Rightarrow \Delta X_D(s) = K_3 \Delta X_C(s) + K_4 \Delta X_E(s) \rightarrow ⑦$$

$$⑤ \Rightarrow \Delta X_E(s) = \cancel{K_5 \int_0^t - \Delta X_D} \quad - \frac{K_5}{s} \Delta X_D(s) \rightarrow ⑧$$

substitute eq ⑦ in eq ⑧, we get

$$⑧ \Rightarrow \Delta X_E(s) = -\frac{K_5}{s} \cdot [K_3 \Delta X_c(s) + K_4 \cdot \Delta X_E(s)] \rightarrow ⑨$$

substitute eq ⑥ in eq ⑨, we get

$$\Delta X_E(s) = -\frac{K_5}{s} [K_3 (K_1 \Delta f(s) - K_2 \Delta P_c(s)) + K_4 \Delta X_E(s)]$$

$$= -\frac{K_5}{s} (K_3 K_1 \Delta f(s) - K_3 K_2 \Delta P_c(s) + K_4 \Delta X_E(s)]$$

$$= \frac{-K_1 K_3 K_5 \Delta f(s) + K_2 K_3 K_5 \Delta P_c(s) - K_4 K_5 \Delta X_E(s)}{s}$$

$$= \left[\frac{-K_5 K_3}{s} (K_1 \Delta F(s) - K_2 \Delta P_c(s)) - \frac{K_4 K_5}{s} \Delta X_E(s)\right]$$

$$\Rightarrow \Delta X_E(s) + \frac{K_4 K_5}{s} \Delta X_E(s) = -\frac{K_5 K_3}{s} [K_1 \Delta F(s) - K_2 \Delta P_c(s)]$$

$$\Rightarrow \left(1 + \frac{K_4 K_5}{s}\right) \Delta X_E(s) = -\frac{K_5 K_3}{s} [K_1 \Delta F(s) - K_2 \Delta P_c(s)]$$

$$\Rightarrow \Delta X_E(s) = \frac{-\frac{K_5 K_3}{s} [K_1 \Delta F(s) - K_2 \Delta P_c(s)]}{1 + \frac{K_4 K_5}{s}}$$

$$= \frac{-K_5 K_3 [K_1 \Delta F(s) - K_2 \Delta P_c(s)]}{s + K_4 K_5}$$

$$= \frac{-K_3 [K_1 \Delta F(s) - K_2 \Delta P_c(s)]}{\frac{s}{K_5} + K_4}$$

$$= \frac{-K_1 K_3 \Delta F(s) + K_2 K_3 \Delta P_c(s)}{\frac{s}{K_5} + K_4}$$

$$= \frac{K_2 K_3 \Delta P_c(s) - K_1 K_3 \Delta F(s)}{\frac{s}{K_5} + K_4}$$

$$\Delta X_E(s) = \frac{K_2 K_3 \left[\Delta P_c(s) - \frac{K_1}{K_2} \Delta F(s)\right]}{\frac{s}{K_5} + K_4} \rightarrow ⑩$$

Divide eq ⑩ both numerator and denominator with K_4, we get

$$\Delta X_E(s) = \frac{\frac{K_2K_3}{K_4}\left(\Delta P_c(s) - \frac{K_1}{K_2}\Delta F(s)\right)}{\frac{\frac{s}{K_5}+K_4}{K_4}}$$

$$= \frac{\frac{K_2K_3}{K_4}\left[\Delta P_c(s) - \frac{K_1}{K_2}\Delta F(s)\right]}{1+\frac{s}{K_4K_5}}$$

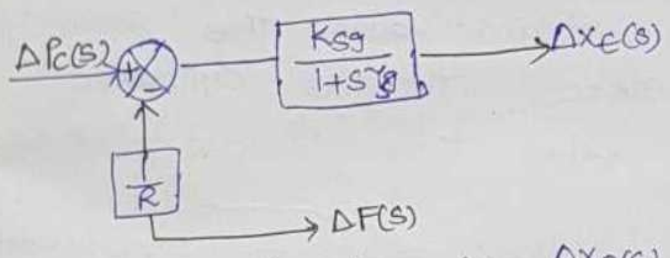
$$\therefore \Delta X_E(s) = \frac{K_{sg}}{1+sT_{sg}}\left[\Delta P_c(s) - \frac{1}{R}\Delta F(s)\right] \rightarrow ⑪$$

where $R = \frac{K_2}{K_1}$ = speed regulation of governor.

$K_{sg} = \frac{K_2K_3}{K_4}$ = Gain of speed governor.

$T_{sg} = \frac{1}{K_4K_5}$ = Time constant of speed governor:

Step VII: Block diagram of speed Governor:



Transfer function is $\frac{\Delta X_E(s)}{\Delta P_c(s)} = \frac{K_{sg}}{1+sT_{sg}}$

Turbine model:

Turbine model is a relation between changes in output power of steam turbine to changes in its steam valve opening $[\Delta X_E(s)]$

There are two types of turbine models:

1. Non reheat type steam turbine.
2. Reheat type steam turbine.

$$G_T(s) = \frac{\Delta P_T(s)}{\Delta X_E(s)} = \frac{\Delta P_G(s)}{\Delta X_E(s)} \rightarrow ① \quad (\because \Delta P_T = \Delta P_G)$$

1. Non Reheat type steam turbine:

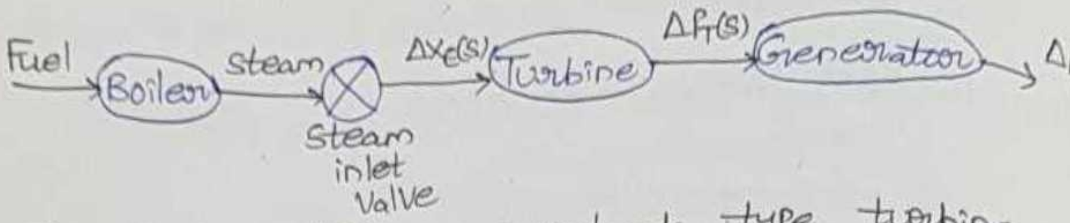


Fig. single ~~stage~~ non reheat type turbine

The above figure shows a single stage non reheat type steam turbine. In this m turbine can be characterised by single gain constant (K_T) and a single time constant ~~τ_{st}~~ (T_t) as

$$G_T(S) = \frac{\Delta P_T(S)}{\Delta X_E(S)} = \frac{\Delta P_G(S)}{\Delta X_E(S)} = \frac{K_t}{1+S\tau_t} \rightarrow ②$$

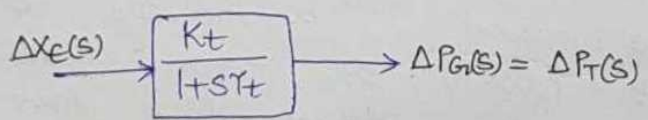
The time constant τ_t lies in the range of 0.2 to 2 sec

From eq ②, $\frac{\Delta P_G(S)}{\Delta X_E(S)} = \frac{K_t}{1+S\tau_t}$

$$\Rightarrow \Delta P_G(S) = \frac{K_t}{1+S\tau_t} \cdot \Delta X_E(S) \rightarrow ③$$

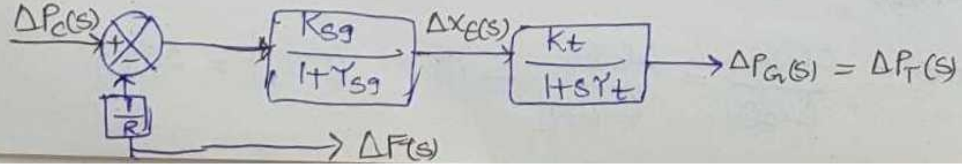
On opening the steam valve, the steam flow ~~stage~~ will not reach turbine cylinder instantaneously. The delay time is 0.2 to 2 sec in steam pipe

From eq ③, the block diagram represents for non reheat type steam turbine model is a shown in figure.



Step II:

Below figure represents block diagram of linearised model of a non reheat type controller including the speed governor mechanism



From the figure, the combined transfer function of turbine and speed governor mechanism will be

$$\text{Transfer Function} = \frac{K_{sg} K_t}{(1+sT_{sg})(1+sT_t)} \rightarrow (4)$$

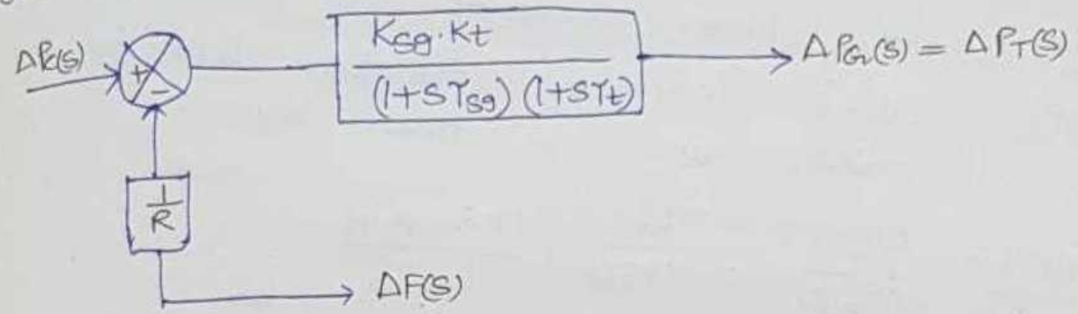
From eq (3) and eq (4),

$$\Delta P_G(s) = \frac{K_{sg} \cdot K_t}{(1+sT_{sg})(1+sT_t)} \cdot \left[\Delta P_C(s) - \frac{1}{R}\Delta F(s)\right] \rightarrow (5)$$

In general the turbine response is low with the response time of several seconds.

$\therefore \; K_{sg} \cdot K_t \simeq 1$

The block diagram of simplified turbine governor system is



2. Reheat type steam Turbine:

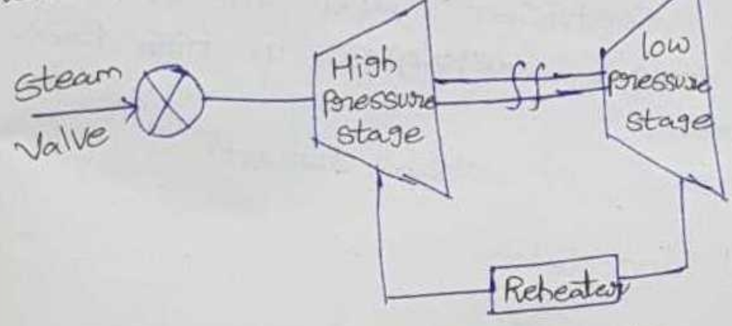


Fig. A two reheat type steam turbine model.

model generating units have reheat type steam turbines as prime movers for higher thermal efficiency.

In this turbine, steam at high pressure and low temperature is withdrawn from the turbine at an intermediate stage.

It is returned to the boiler for resuperheating and then re-introduced into

turbine at low Pressure and high temperature.
This increases the overall efficiency

Two factors influence the dynamic response of reheat type steam turbine.

Entrained steam between the inlet steam Valve and first state of turbine.

The storage action in reheater which causes the output of low Pressure stage to lag behind that of high Pressure stage.

therefore turbine transfer function is characterized by two time constants.

'τ_t' is time constant of the turbine.

'τ_r' is the time constant of reheater.

∴ The turbine transfer function will be of second order and is given by

$$G_T(s) = \frac{\Delta P_G(s)}{\Delta X_E(s)} = \frac{K_t}{1+s\tau_t} \cdot \frac{1+s \cdot K_r \tau_r}{1+s\tau_r}$$

where $\tau_r = 10$ seconds

K_r = Reheat Coefficient and is equal to the Proportion of torque developed in high Pressure section of turbine.

(∴ $K_r = 1$ fraction of steam reheat)

Generator Load model:

Step I:

The generator load model gives the relation between change in frequency (Δf) as a result of change in generation (ΔP_G) when the load changes by a small amount (ΔP_D), when neglecting change in generator loss.

Therefore $\Delta P_G = \Delta P_T$

where ΔP_T = change in turbine Power output

∴ Net surplus Power at busbar = $\Delta P_G - \Delta P_D$

This surplus power can be observed by the system in two ways

i.e $\Delta P_G - \Delta P_D = \frac{dW_{KE}}{dt} + \frac{dP_D}{df}\Delta f \rightarrow (1)$

step II (condition-1):

By increasing the stored kinetic energy of generator rotor at a rate is $\frac{dW_{KE}}{dt}$.

Let W_{KE}° = stored kinetic energy before disturbance at normal speed and frequency "f_0"

W_{KE} = Kinetic energy after disturbance occur when the frequency is changed from "f_0" to "$f_0+\Delta f$"

since kinetic energy is proportional to square of speed and frequency is proportional to speed.

since $K.E \propto N^2$

As we know that, $N_S = \frac{120f}{P}$, as P is constant

$$N_S \propto f$$

$$\therefore K.E \propto N^2 \propto f^2$$

$$\frac{W_{KE}}{W_{KE}^{\circ}} = \left[\frac{f_0^{\circ}+\Delta f}{f_0^{\circ}}\right]^2 \rightarrow (2)$$

$$\therefore W_{KE} = W_{KE}^{\circ}\left[1+\frac{2\Delta f}{f_0} + \text{Higher order terms}\right] \rightarrow (3)$$

since $\frac{\Delta f}{f_0}$ is very small and it is neglected

$$\therefore W_{KE} = W_{KE}^{\circ}\left[1+\frac{2\Delta f}{f_0}\right]$$

By differentiating above equation with respect to 't', we get

$$\frac{dW_{KE}}{dt} = \frac{d}{dt}\left[W_{KE}^{\circ}\left(1+\frac{2\Delta f}{f_0}\right)\right]$$

$$= \frac{2W_{KE}^{\circ}}{f_0}\cdot\frac{d}{dt}\Delta f \rightarrow (4)$$

From fundamentals,

$$W_{KE}^{\circ} = H \times P_r (MW\text{-}s \text{ (or) } MW\text{-}J) \rightarrow ⑤$$

H = Inertia constant of generator in MW-J/MVA

P_r = Rating of turbo generator in MVA.

Substitute eq ⑤ in eq ④, we get

$$\frac{dW_{KE}}{dt} = \frac{2H \times P_r}{f_0} \cdot \frac{d}{dt}\Delta f \rightarrow ⑥$$

~~Substitute eq ⑤ in eq 4~~

Step III (Condition 2):

The load on motors increases with increase in speed. The load on system being mostly motor load, hence some portion of surplus power is absorbed by motor load.

Rate of change of load w.r.to frequency can be regarded as constant for small changes in frequency.

$$\therefore \frac{dP_D}{df} \cdot \cancel{\Delta f} = B\Delta f \rightarrow ⑦$$

Step IV:

$\therefore$ Net surplus power, $\Delta P_G - \Delta P_D = \frac{dW_{KE}}{dt} + \frac{dP_D}{df} \cdot \Delta f$

Substitute eq ⑥ and eq ⑦ in above equation, we get

$$\therefore \Delta P_G - \Delta P_D = \frac{2HP_r}{f_0} \cdot \frac{d}{dt}\Delta f + B\Delta f \rightarrow ⑧$$

Divide eq ⑧ with P_r, we get

$$\Delta P_G(p.u) - \Delta P_D(p.u) = \frac{2H}{f_0} \cdot \frac{d}{dt}\Delta f + B_{p.u}\Delta f \rightarrow ⑨$$

Taking laplace transform for eq ⑨ on both sides, we get

$$\Delta P_G(s) - \Delta P_D(s) = \frac{2H}{f_0} \cdot s \cdot \Delta f(s) + B\Delta f(s).$$

$$\Delta P_G(s) - \Delta P_D(s) = \Delta f(s)\left[\frac{2H}{f_0} \cdot s + B\right]$$

$$\therefore \Delta f(s) = \frac{\Delta P_G(s) - \Delta P_D(s)}{\frac{2H}{f_0} \cdot s + B}$$

$$= \frac{1}{B}\left[\frac{\Delta P_G(s) - \Delta P_D(s)}{\frac{2H}{B f_0} \cdot s + 1}\right]$$

$$\Delta F(s) = \frac{K_{ps}}{1 + s\tau_{ps}} \left[\Delta P_G(s) - \Delta P_D(s)\right] \rightarrow (10)$$

compare the terms

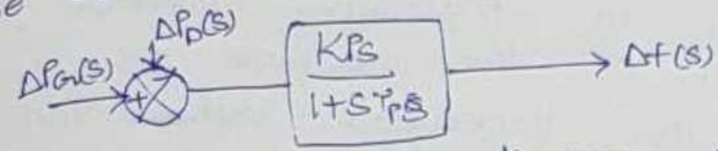
where

$\tau_{ps} = \frac{2H}{B_0}$ = Power system time constant

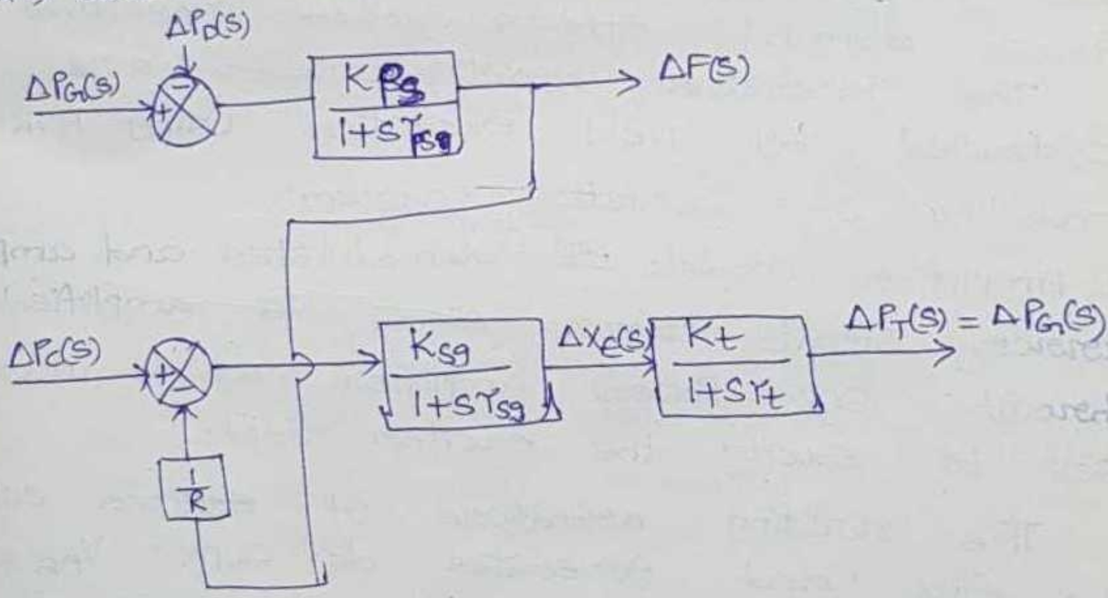
$K_{ps} = \frac{1}{B}$ = Power system gain

step V:

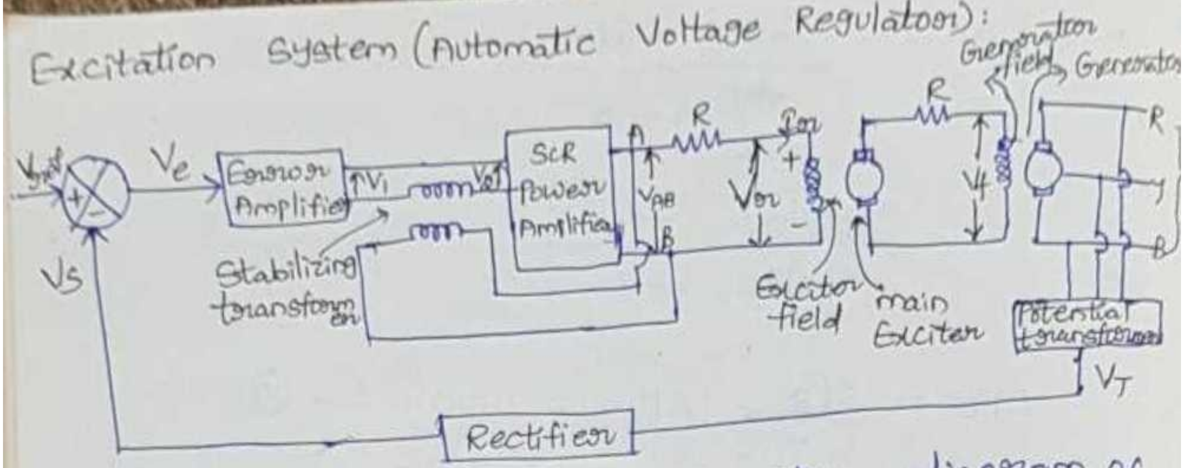
Block diagram of generator load model can be drawn as



The overall block diagram of an isolated power system is obtained by combining the individual block diagrams of speed governor model, turbine model and generator load model.



Excitation System (Automatic Voltage Regulator):



Above figure shows schematic diagram of an automatic Voltage regulator

The terminal Voltage of synchronous machine is controlled by either generator field rheostat (or) exciter field rheostat.

The main objective of an excitation system is to control the field current of synchronous machine in order to maintain constant terminal Voltage (V_t) of the machine.

To regulate the generator Voltage for changes in load automatic Voltage regulators (AVR) are used.

A change in real Power demand effects the frequency, whereas change in reactive Power demand effects Voltage magnitude.

The generator reactive Power are controlled by field excitation using AVR.

modelling of Excitation system:

(1) Amplifier model: It demodulates and amplifies error signal. Error signal is amplified through SCR Power amplifier which inturn Used to excite the exciter field.

The rotating armature of exciter cuts the flux and generates dc supply V_{AB} which is used to excite the Generator field.

The amplifier is represented by gain "K_A" and time constant "T_A"

$$\therefore \text{Transfer function} = \frac{V_r(S)}{V_e(S)} = \frac{K_A}{1+ST_A}$$

where "K_A" ranges from 10 to 400.
"τ_A" ranges from 0.02 to 0.1 sec.

2. Exciter model: The model of modern exciter is a linearised model which takes into account major time constant and ignores the saturation.

$\therefore$ Transfer function $\frac{V_F(s)}{V_R(s)} = \frac{K_E}{1+s\tau_E}$

where 'K_E' = gain of exciter
τ_E = time constant of exciter

3. Generator model: Generator field is excited by the main exciter voltage (V_{AB}). under no load condition, it produces a voltage proportional to field current.
No load transfer Function = $\frac{V_T(s)}{V_F(s)} = \frac{K_G}{1+s\tau_G}$

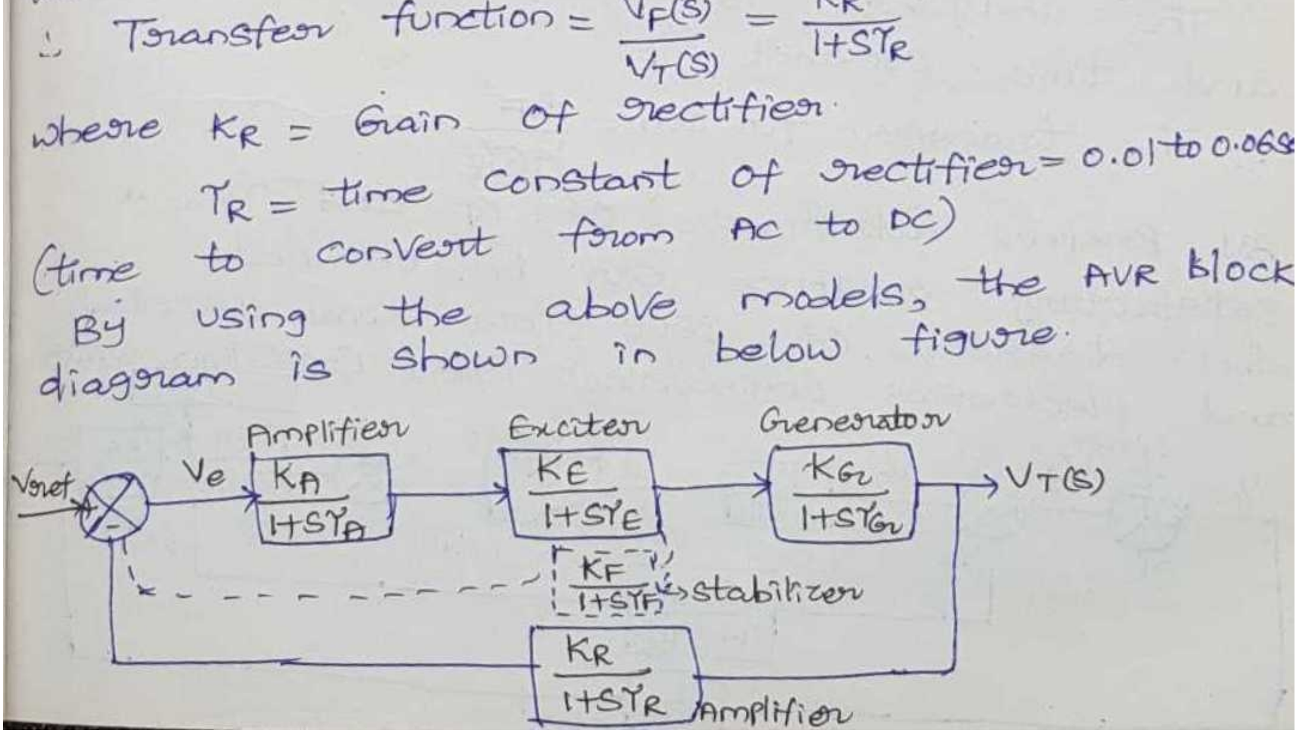
4. Potential Transformer: The voltage magnitude is sensed through a potential transformer and gives terminal voltage (V_T).

5. sensor model (Potential transformer along with rectifier):
The voltage at output of synchronous machine is sensed through potential transformer and it is rectified by bridge rectifier. The sensor is modeled by a simple first order transfer function.

$\therefore$ Transfer function = $\frac{V_F(s)}{V_T(s)} = \frac{K_R}{1+s\tau_R}$

where K_R = Gain of rectifier
τ_R = time constant of rectifier = 0.01 to 0.06s
(time to convert from AC to DC)

By using the above models, the AVR block diagram is shown in below figure.



Condition I:
Open loop transfer function is given by $G(s)H(s)$

i.e. $$G(s)H(s) = \frac{K_A K_E K_G K_R}{(1+sT_A)(1+sT_E)(1+sT_G)(1+sT_R)}$$

where $$H(s) = \frac{K_R}{1+sT_R}$$

Condition II:
Closed loop transfer function is given by

$$\frac{V_T(s)}{V_{ref}(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$= \frac{\frac{K_A K_E K_G}{(1+sT_A)(1+sT_E)(1+sT_G)}}{1+\frac{K_A K_E K_G}{(1+sT_A)(1+sT_E)(1+sT_G)}\cdot\frac{K_R}{1+sT_R}}$$

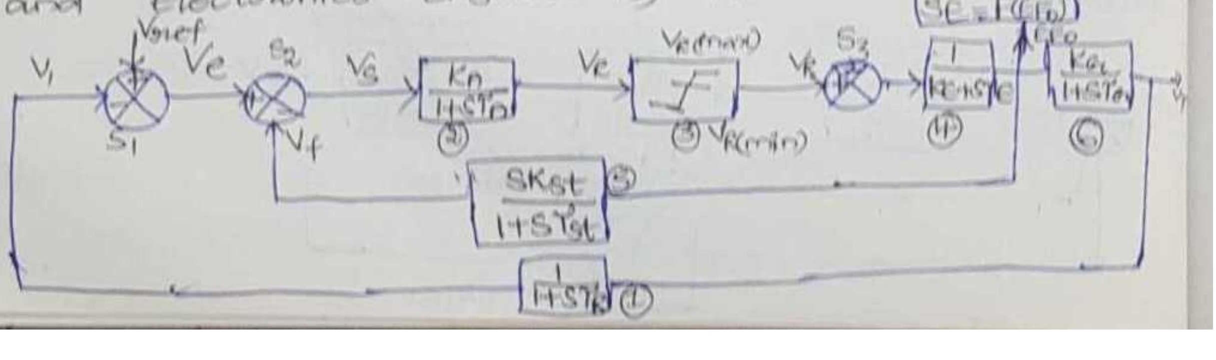
$$= \frac{K_A K_E K_G (1+sT_R)}{(1+sT_A)(1+sT_E)(1+sT_G)(1+sT_R) + K_A K_E K_G K_R}$$

Condition III:
The system has a chance of excessive overshoot and stability problem. In order to get steady response, stabilizer was introduced to increase stability.
The stabilizer is represented by a gain K_F and time constant T_F.

$\therefore$ The transfer function $= \frac{K_F}{1+sT_F}$

By proper adjustment of K_F and T_F, a satisfactory response can be obtained.

Block diagram of IEEE (International Electrical and Electronics Engineering) Type I Excitation system:



In the previous excitation system, saturation dead bands and other non linear effects are not included.

Type I:

This model proposed by IEEE sub committee on excitation system includes the effect of saturation and dead bands (limits)

Consider the block diagram as shown in figure. The terminal voltage "V_t" is measured by potential transformer and it is rectified and filtered.

"τ_R" is the time constant representing the regulation input filtering

The first summing point "S_1" compares the reference voltage (V_{ref}) with the output of filter (V_1) to determine the voltage error (V_e).

The second summing point 'S_2' compares the voltage error (V_e) with the excitation damping loop signal (V_f)

The amplifier transfer function is represented as a gain (K_A) and time constant (τ_A)

Following these, the maximum and minimum limits are imposed. So that ~~are~~ V_R must be in between $V_{R(max)}$ and $V_{R(min)}$

The next summing point 'S_3' subtract a signal which represents the saturation function $S_E = f(E_{FD})$

The resultant is applied to the exciter transferrer i.e $\frac{1}{K_E + S\tau_E}$

For the exciter output E_{FD} to the second summing point S_2, damping is provided by feedback

$\therefore$ Transfer function = $\dfrac{SK_{st}}{1+ST_{st}}$

Step II:

At Block I:

$$V_1 = V_T\left(\frac{1}{1+ST_R}\right)$$

$$\Rightarrow V_T = V_1 + V_1 ST_R$$

$$V_T - V_1 = V_1 ST_R \rightarrow ①$$

At summing point I:

$$V_e = V_{ref} - V_1 \rightarrow ②$$

At summing point II:

$$V_s = V_e - V_f \rightarrow ③$$

At Block II and III:

$$V_R = V_s\left(\frac{K_A}{1+ST_A}\right) \rightarrow ④$$

$$\Rightarrow V_R + V_R \cdot ST_A = V_s \cdot K_A$$

Substituting eq ③ in above equation, we get

$$\Rightarrow V_R + V_R ST_A = (V_e - V_f)K_A$$

$$\Rightarrow V_R + V_R ST_A = V_e K_A - V_f K_A$$

$$\Rightarrow V_R ST_A = -V_R + V_e K_A + V_f K_A$$

$$SV_R = \frac{-V_R + V_e K_A + V_f K_A}{T_A}$$

$$SV_R = \frac{1}{T_A}\left[-V_R + (V_e \pm V_f) K_A\right] \rightarrow ⑤$$

$$SV_R = F_R$$

$$\text{i.e } F_R = \frac{-V_R + (V_e - V_f) K_A}{T_A}$$

Now limits are exist, we take

If $V_R = V_{R(max)}$, set $SV_R = 0$

If $V_R = V_{R(min)}$, set $SV_R = 0$

If the two conditions are not satisfied then set $SV_R = F_R$.

$$\therefore F_R = \frac{1}{T_A}\left[-V_R + K_A(V_e - V_f)\right] \rightarrow ⑥$$

Block IV:

At

$$E_{FD} = (V_R - S_E \cdot E_{FD})\left(\frac{1}{K_E + sT_E}\right)$$

$$E_{FD}K_E + E_{FD}sT_E = V_R - S_E E_{FD}$$

$$E_{FD}K_E + E_{FD}sT_E - V_R + S_E E_{FD} = 0 \Rightarrow E_{FD}sT_E = V_R - S_E E_{FD} - K_E E_{FD}$$

$$\Rightarrow E_{FD}(K_E + sT_E + S_E) - V_R = 0 \qquad E_{FD} \cdot sT_E = V_R - E_{FD}(S_E + K_E)$$

$$\Rightarrow E_{FD} = \frac{V_R}{K_E + sT_E + S_E} \qquad E_{FD} = \frac{1}{sT_E}\left[V_R - E_{FD}(S_E + K_E)\right]$$

$$\Rightarrow E_{FD} = \frac{1}{sT_E}\left(\frac{V_R}{\frac{K_E}{sT_E} + 1 + \frac{S_E}{sT_E}}\right)$$

Let $S_E + K_E = a$ then $E_{FD} = \frac{1}{sT_E}\left[V_R - E_{FD}\,a\right] \rightarrow (7)$

$$\Rightarrow V_R - E_{FD} \cdot a = E_{FD} \cdot sT_E$$

$$V_R = E_{FD} \cdot sT_E + E_{FD}\,a$$

$$V_R = E_{FD}(sT_E + a)$$

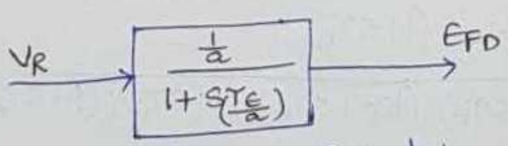
$$\frac{E_{FD}}{V_R} = \frac{1}{sT_E + a}$$

Divide both numerator and denominator with "a"

$$\frac{E_{FD}}{V_R} = \frac{\cancel{a}}{\cancel{(sT_E + a)a}} \quad \frac{\frac{1}{a}}{\frac{sT_E}{a} + 1}$$

$$= \frac{\cancel{a}}{\cancel{a sT_E + a^2}} \quad \cancel{\frac{1}{a}} \quad \frac{\frac{1}{a}}{1 + s\left(\frac{T_E}{a}\right)}$$

$$\Rightarrow E_{FD} = V_R \cdot \frac{\frac{1}{a}}{1 + s\left(\frac{T_E}{a}\right)} \rightarrow (8)$$



Under steady state condition, $V_R - E_{FD}(S_E + K_E) = 0$

$$\therefore E_{FD(min)} \le E_{FD} \le E_{FD(max)}$$

At Block V:

$$V_f = E_{FD}\left[\frac{SK_{st}}{1+S\tau_{st}}\right]$$

Divide both numerator and denominator with SK_{st}, we get

$$V_f = E_{FD}\left[\frac{\frac{\cancel{S}\cancel{K_{st}}}{S\cancel{K_{st}}}}{\frac{1}{SK_{st}}+\frac{\cancel{S}\tau_{st}}{\cancel{S}K_{st}}}\right]$$

$$= E_{FD}\left[\frac{1}{\frac{1}{SK_{st}}+\frac{\tau_{st}}{K_{st}}}\right] \rightarrow (9)$$

At Block VI:

$$V_T = E_{FD}\left(\frac{K_G}{1+S\tau_G}\right)$$

$$V_T + V_T S\tau_G = E_{FD} K_G \rightarrow (10)$$

Step III:

The transfer function of system,

$$\frac{V_T(S)}{V_{ref}(S)} = \frac{G(S)}{1+G(S)H(S)}$$

where $$G(S) = \frac{K_A K_G}{(1+S\tau_A)(K_E+S\tau_E)(1+S\tau_G)}$$

$$H(S) = \frac{1}{1+S\tau_R}$$

$$\frac{V_T(S)}{V_{ref}(S)} = \frac{\frac{K_A K_G}{(1+S\tau_A)(K_E+S\tau_E)(1+S\tau_G)}}{1+\frac{K_A K_G}{(1+S\tau_A)(K_E+S\tau_E)(1+S\tau_G)}\cdot\frac{1}{1+S\tau_R}}$$

$$= \frac{K_A K_G (1+S\tau_R)}{(1+S\tau_A)(K_E+S\tau_E)(1+S\tau_G)(1+S\tau_R)}$$