

# TEX-OWL: a Latex-Style Syntax for authoring OWL 2 ontologies

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# 1 Preliminary Definitions

The grammar presented in this document uses the following two “special” terminal symbols, which affect the process of transforming an input sequence of characters into a sequence of regular (i.e., not “special”) terminal symbols:

- whitespace is a nonempty sequence of space (U+20), horizontal tab (U+9), line feed (U+A), or carriage return (U+D) characters, and
- a comment is a sequence of characters that starts with the % (U+25) character and does not contain the line feed (U+A) or carriage return (U+D) characters.

# 2 General Definitions

$\langle nonNegativeInteger \rangle ::=$  a nonempty finite sequence of digits between 0 and 9

$\langle quotedString \rangle ::=$  a finite sequence of characters in which ‘ (U+22) and \ (U+5C) occur only in pairs of the form \‘ (U+5C, U+22) and \\ (U+5C, U+5C), enclosed in a pair of ‘ (U+22) characters

$\langle languageTag \rangle ::=$  @ (U+40) followed a nonempty sequence of characters matching the langtag production from [2]

$\langle nodeID \rangle ::=$  a finite sequence of characters matching the BLANK\_NODE\_LABEL production of [3]

$\langle fullIRI \rangle ::=$  an IRI as defined in [1], enclosed in a pair of < (U+3C) and > (U+3E) characters

$\langle prefixName \rangle ::=$  a finite sequence of characters matching the as PNAME\_NS production of [3] and not matching any of the keyword terminals of the syntax

$\langle abbreviatedIRI \rangle ::=$  a finite sequence of characters matching the PNAME\_LN production of [3] and not matching any of the keyword terminals of the syntax

$\langle simpleIRI \rangle ::=$  a finite sequence of characters matching the PN\_LOCAL production of [3] and not matching any of the keyword terminals of the syntax

$\langle IRI \rangle ::= \langle fullIRI \rangle \mid \langle abbreviatedIRI \rangle \mid \langle simpleIRI \rangle$

### 3 Including additional input files

Inclusion of additional input files (both local and distributed) allows to organize the ontology in a modular fashion. To specify the input file the following format should be applied:

$\langle includeExternalFile \rangle ::= \backslash input \text{ '}' \langle INPUT\_FILE \rangle \text{ '}'$

$\langle INPUT\_FILE \rangle ::= \langle URL \rangle \mid \langle ABSOLUTE\_FILEPATH \rangle$

Please note that this command may occur anywhere inside the ontology document.

### 4 Ontologies

$\langle ontologyDocument \rangle ::= \langle defaultPrefixDeclaration \rangle? \langle prefixDeclaration \rangle^* \langle Ontology \rangle$

$\langle defaultPrefixDeclaration \rangle ::= \backslash ns \text{ '}' \langle fullIRI \rangle$

$\langle prefixDeclaration \rangle ::= \backslash ns \text{ '}' \langle prefixName \rangle \langle fullIRI \rangle$

$\langle Ontology \rangle ::= \backslash begin \text{ '}' \{ \text{'ontology' '}' \langle ontologyIRIs \rangle (\langle directlyImportsDocument \rangle \mid \langle ontologyAnnotation \rangle)^* \langle axioms \rangle \backslash end \text{ '}' \{ \text{'ontology' '}'$

$\langle ontologyIRIs \rangle ::= [ \text{'['} \langle ontologyIRI \rangle [ \text{' , '}' \langle versionIRI \rangle ] \text{' '}]$

$\langle ontologyIRI \rangle ::= \langle IRI \rangle$

$\langle versionIRI \rangle ::= \langle IRI \rangle$

$\langle directlyImportsDocument \rangle ::= \backslash import \text{ '}' \langle IRI \rangle$

$\langle ontologyAnnotation \rangle ::= \langle Annotation \rangle$

$\langle axioms \rangle ::= \langle Axiom \rangle^*$

Please note that prefixes: *owl:*, *rdf:*, *rdfs:*, *xml:* and *xsd:* are built-in and should not be explicitly declared.

**Example 4.1.** The following code show an example of an ontology document, written with the proposed syntax.

```
% Default namespace
\ ns <http://www.example.com/ontology1#>

\ begin{ontology}[<http://www.example.com/ontology1>]
  \ import <http://www.example.com/ontology2>
```

```

| \a{rdfs:label, "An example"}
|
| % These are comments and they will be skipped
| % during parsing
| % insert axioms below
| Child \cisa owl:Thing
| \end{ontology}

```

## 5 Annotations

### 5.1 Annotations of Ontologies, Axioms, and other Annotations

$\langle \text{Annotation} \rangle ::= \text{'\a' '{' } \langle \text{AnnotationProperty} \rangle \text{' ,' } \langle \text{AnnotationValue} \rangle \text{' }' \langle \text{annotationAnnotations} \rangle$

$\langle \text{annotationAnnotations} \rangle ::= [ \text{'[' } \langle \text{Annotation} \rangle \text{' ,' } \langle \text{Annotation} \rangle \text{' ]' } ]$

$\langle \text{AnnotationValue} \rangle ::= \langle \text{AnonymousIndividual} \rangle \mid \langle \text{IRI} \rangle \mid \langle \text{Literal} \rangle$

### 5.2 Annotation Axioms

$\langle \text{AnnotationAxiom} \rangle ::= \langle \text{AnnotationAssertion} \rangle$   
 $\mid \langle \text{SubAnnotationPropertyOf} \rangle$   
 $\mid \langle \text{AnnotationPropertyDomain} \rangle$   
 $\mid \langle \text{AnnotationPropertyRange} \rangle$

#### 5.2.1 Annotation Assertion

$\langle \text{AnnotationAssertion} \rangle ::= \langle \text{AnnotationSubject} \rangle \text{'\a' '{' } \langle \text{AnnotationProperty} \rangle$   
 $\text{' ,' } \langle \text{AnnotationValue} \rangle \text{' }' \langle \text{axiomAnnotations} \rangle$

$\langle \text{AnnotationSubject} \rangle ::= \langle \text{IRI} \rangle \mid \langle \text{AnonymousIndividual} \rangle$

**Example 5.1.** *The following axiom assigns a human-readable comment to the IRI `a:Person`.*

```

|| a:Person \a{rdfs:label, "Represents the set of all people."}

```

*Since the annotation is assigned to an IRI, it applies to all entities with the given IRI. Thus, if an ontology contains both a class and an individual `a:Person`, the above comment applies to both entities.*

### 5.2.2 Annotation Subproperties

$\langle SubAnnotationPropertyOf \rangle ::= \langle subAnnotationProperty \rangle \text{ '\aisa' } \langle superAnnotationProperty \rangle$   
 $\langle axiomAnnotations \rangle$

$\langle subAnnotationProperty \rangle ::= \langle AnnotationProperty \rangle$

$\langle superAnnotationProperty \rangle ::= \langle AnnotationProperty \rangle$

### 5.2.3 Annotation Property Domain

$\langle AnnotationPropertyDomain \rangle ::= \langle AnnotationProperty \rangle \text{ '\adomain' } \langle IRI \rangle \langle axiomAnnotations \rangle$

### 5.2.4 Annotation Property Range

$\langle AnnotationPropertyRange \rangle ::= \langle AnnotationProperty \rangle \text{ '\arange' } \langle IRI \rangle \langle axiomAnnotations \rangle$

## 6 Entities, Literals, and Anonymous Individuals

### 6.1 Classes

$\langle Class \rangle ::= \langle IRI \rangle$

**Example 6.1.** *Classes  $a:Child$  and  $a:Person$  can be used to represent the set of all children and persons, respectively, in the application domain, and they can be used in an axiom such as the following one:*

*Each child is a person.*

`|| a:Child \cisa a:Person`

### 6.2 Datatype

$\langle Datatype \rangle ::= \langle IRI \rangle$

**Example 6.2.** *The datatype  $xsd:integer$  denotes the set of all integers. It can be used in axioms such as the following one:*

*The range of the  $a:hasAge$  data property is  $xsd:integer$ .*

`|| a:hasAge \drange xsd:integer`

### 6.3 ObjectProperty

$\langle \text{ObjectProperty} \rangle ::= \langle \text{IRI} \rangle$

**Example 6.3.** *The object property `a:parentOf` can be used to represent the parenthood relationship between individuals. It can be used in axioms such as the following one:*

*Peter is a parent of Chris.*

```
|| a:parentOf(a:Peter, a:Chris)
```

### 6.4 DataProperty

$\langle \text{DataProperty} \rangle ::= \langle \text{IRI} \rangle$

**Example 6.4.** *The data property `a:hasName` can be used to associate a name with each person. It can be used in axioms such as the following one:*

*Peter's name is "Peter Griffin".*

```
|| a:hasName(a:Peter, "Peter Griffin")
```

### 6.5 AnnotationProperty

$\langle \text{AnnotationProperty} \rangle ::= \langle \text{IRI} \rangle$

**Example 6.5.** *The comment provided by the following annotation assertion axiom might, for example, be used by an OWL 2 tool to display additional information about the IRI `a:Peter`.*

*This axiom provides a comment for the IRI `a:Peter`.*

```
|| a:Peter \a{rdfs:comment, "The father of the Griffin family  
from Quahog."}
```

### 6.6 Individual

$\langle \text{Individual} \rangle ::= \langle \text{NamedIndividual} \rangle \mid \langle \text{AnonymousIndividual} \rangle$

### 6.6.1 Named Individuals

$\langle NamedIndividual \rangle ::= \langle IRI \rangle$

**Example 6.6.** *The individual  $a:Peter$  can be used to represent a particular person. It can be used in axioms such as the following one:*

*Peter is a person.*

$\| a:Person(a:Peter)$

### 6.6.2 Anonymous Individuals

$\langle AnonymousIndividual \rangle ::= \langle nodeID \rangle$

**Example 6.7.** *Anonymous individuals can be used, for example, to represent objects whose identity is of no relevance, such as the address of a person.*

*Peter lives at some (unknown) address.*

$\| a:livesAt(a:Peter, _:a1)$

*This unknown address is in the city of Quahog and...*

$\| a:city(_:a1, a:Quahog)$

*...in the state of Rhode Island.*

$\| a:state(_:a1, a:RI)$

## 6.7 Literals

$\langle Literal \rangle ::= \langle typedLiteral \rangle | \langle stringLiteralNoLanguage \rangle | \langle stringLiteralWithLanguage \rangle$

$\langle typedLiteral \rangle ::= \langle lexicalForm \rangle \text{ ‘[’ } \langle Datatype \rangle \text{ ‘]’}$

$\langle lexicalForm \rangle ::= \langle quotedString \rangle$

$\langle stringLiteralNoLanguage \rangle ::= \langle quotedString \rangle$

$\langle stringLiteralWithLanguage \rangle ::= \langle quotedString \rangle \text{ ‘[’ } \langle languageTag \rangle \text{ ‘]’}$

**Example 6.8.** *“1” $[xsd:integer]$  is a literal that represents the integer 1.*

**Example 6.9.** “Family Guy” is an abbreviation for “Family Guy@”[*rdf:PlainLiteral*] – a literal with the lexical form “Family Guy@” and the datatype *rdf:PlainLiteral* – which denotes a string “Family Guy” without a language tag.

Furthermore, “Padre de familia”[*@es*] is an abbreviation for the literal “Padre de familia@es”[*rdf:PlainLiteral*], which denotes a pair consisting of the string “Padre de familia” and the language tag *es*.

**Example 6.10.** Even though literals “1”[*xsd:integer*] and “+1”[*xsd:integer*] are interpreted as the integer 1, these two literals are not structurally equivalent because their lexical forms are not identical. Similarly, “1”[*xsd:integer*] and “1”[*xsd:positiveInteger*] are not structurally equivalent because their datatypes are not identical.

## 6.8 Entity Declarations and Typing

$\langle \textit{Declaration} \rangle ::= \langle \textit{Entity} \rangle \langle \textit{axiomAnnotations} \rangle$

$\langle \textit{Entity} \rangle ::= \langle \textit{Class} \rangle \backslash \textit{c}$   
 |  $\langle \textit{Datatype} \rangle \backslash \textit{dt}$   
 |  $\langle \textit{ObjectProperty} \rangle \backslash \textit{o}$   
 |  $\langle \textit>DataProperty} \rangle \backslash \textit{d}$   
 |  $\langle \textit{AnnotationProperty} \rangle \backslash \textit{a}$   
 |  $\langle \textit{NamedIndividual} \rangle \backslash \textit{i}$

**Example 6.11.** The following axioms state that the IRI *a:Person* is used as a class and that the IRI *a:Peter* is used as an individual.

$\parallel \textit{a:Person} \backslash \textit{c}$

$\parallel \textit{a:Peter} \backslash \textit{i}$

## 7 Property Expressions

### 7.1 Object Property Expressions

$\langle \textit{ObjectPropertyExpression} \rangle ::= \langle \textit{ObjectProperty} \rangle$   
 |  $\langle \textit{InverseObjectProperty} \rangle \langle \textit{ObjectProperty} \rangle$

#### 7.1.1 Inverse Object Properties

$\langle \textit{InverseObjectProperty} \rangle ::= \backslash \textit{oinvof} \langle \textit{ObjectProperty} \rangle$

**Example 7.1.** Consider the ontology consisting of the following assertion.

*Peter is Stewie's father.*

```
|| a:fatherOf(a:Peter, a:Stewie)
```

This ontology entails that *a:Stewie* is connected by the following object property expression to *a:Peter*:

```
|| (\oinvof a:fatherOf)
```

## 7.2 Data Property Expressions

$\langle \text{DataPropertyExpression} \rangle ::= \langle \text{DataProperty} \rangle$

## 8 Data Ranges

$\langle \text{DataRange} \rangle ::= \langle \text{Datatype} \rangle$   
 |  $\langle \text{AtomicDataRange} \rangle$   
 |  $\langle '(\langle \text{NonAtomicDataRange} \rangle)'$

### 8.1 Atomic Data Ranges

$\langle \text{AtomicDataRange} \rangle ::= \langle \text{SequenceDataIntersectionOf} \rangle$   
 |  $\langle \text{SequenceDataUnionOf} \rangle$   
 |  $\langle \text{DataOneOf} \rangle$

#### 8.1.1 Sequence Intersection of Data Ranges

$\langle \text{SequenceDataIntersectionOf} \rangle ::= \langle \backslash \text{drandof} \langle \{ \langle \text{DataRange} \rangle ( \langle \text{DataRange} \rangle ) + \} \rangle$

**Example 8.1.** Example 8.4 can be rewritten with the sequence-style notation:

```
|| \drandof{xsd:nonNegativeInteger, xsd:nonPositiveInteger}
```

#### 8.1.2 Sequence Union of Data Ranges

$\langle \text{SequenceDataUnionOf} \rangle ::= \langle \backslash \text{drorof} \langle \{ \langle \text{DataRange} \rangle ( \langle \text{DataRange} \rangle ) + \} \rangle$

**Example 8.2.** Example 8.5 can be rewritten with the sequence-style notation:

```
|| \drorof{xsd:string, xsd:integer}
```

### 8.1.3 Enumeration of Literals

$\langle DataOneOf \rangle ::= \backslash droneof \{ \langle Literal \rangle ( \langle ' \rangle \langle Literal \rangle )^* \}$

**Example 8.3.** *The following data range contains exactly two literals: the string “Peter” and the integer one.*

```
|| \droneof{"Peter", "1"[xsd:integer]}
```

## 8.2 Non-atomic Data Ranges

$\langle NonAtomicDataRange \rangle ::= \langle DataIntersectionOf \rangle$   
|  $\langle DataUnionOf \rangle$   
|  $\langle DataComplementOf \rangle$   
|  $\langle DatatypeRestriction \rangle$

### 8.2.1 Intersection of Data Ranges

$\langle DataIntersectionOf \rangle ::= \langle DataRange \rangle \backslash drand \langle DataRange \rangle$

**Example 8.4.** *The following data range contains exactly the integer 0:*

```
|| (xsd:nonNegativeInteger \drand xsd:nonPositiveInteger)
```

### 8.2.2 Union of Data Ranges

$\langle DataUnionOf \rangle ::= \langle DataRange \rangle \backslash dror \langle DataRange \rangle$

**Example 8.5.** *The following data range contains all strings and all integers:*

```
|| (xsd:string \dror xsd:integer)
```

### 8.2.3 Complement of Data Ranges

$\langle DataComplementOf \rangle ::= \backslash drnot \langle DataRange \rangle$

**Example 8.6.** *The following complement data range contains literals that are not positive integers:*

|| (`\drnot xsd:positiveInteger`)

*In particular, this data range contains the integer zero and all negative integers; however, it also contains all strings (since strings are not positive integers).*

## 8.2.4 Datatype Restrictions

$\langle \text{DatatypeRestriction} \rangle ::= \langle \text{Datatype} \rangle \text{'\drres'} \langle \text{DatatypeRestrictionExpression} \rangle$

$\langle \text{DatatypeRestrictionExpression} \rangle ::= \text{'\{'} \langle \text{constrainingFacet} \rangle \langle \text{restrictionValue} \rangle$   
 $\text{'\{' } \langle \text{constrainingFacet} \rangle \langle \text{restrictionValue} \rangle \text{'\}'}$

$\langle \text{constrainingFacet} \rangle ::= \langle \text{IRI} \rangle$

$\langle \text{restrictionValue} \rangle ::= \langle \text{Literal} \rangle$

**Example 8.7.** *The following data range contains exactly the integers 5, 6, 7, 8, and 9:*

|| (`xsd:integer \drres{xsd:minInclusive "5"[xsd:integer], xsd:maxExclusive "10"[xsd:integer]}`)

## 9 Class Expressions

$\langle \text{ClassExpression} \rangle ::= \langle \text{Class} \rangle$   
 $| \langle \text{AtomicClassExpression} \rangle$   
 $| \text{'(' } \langle \text{NonAtomicClassExpression} \rangle \text{'}'}$

### 9.1 Atomic Class Expression

#### 9.1.1 Propositional Connectives and Enumeration of Individuals

$\langle \text{AtomicClassExpression} \rangle ::= \langle \text{SequenceObjectIntersectionOf} \rangle | \langle \text{SequenceObjectUnionOf} \rangle$   
 $| \langle \text{ObjectOneOf} \rangle$   
 $| \langle \text{ObjectSomeValueFrom} \rangle | \langle \text{ObjectAllValueFrom} \rangle | \langle \text{ObjectHasValue} \rangle$   
 $| \langle \text{ObjectMinCardinality} \rangle | \langle \text{ObjectMaxCardinality} \rangle | \langle \text{ObjectExactCardinality} \rangle$   
 $| \langle \text{DataSomeValueFrom} \rangle | \langle \text{DataAllValueFrom} \rangle | \langle \text{DataHasValue} \rangle |$   
 $| \langle \text{DataMinCardinality} \rangle | \langle \text{DataMaxCardinality} \rangle | \langle \text{DataExactCardinality} \rangle$

#### Sequence Intersection of Class Expressions

$\langle \text{SequenceObjectIntersectionOf} \rangle ::= \text{'\candof'} \text{'\{' } \langle \text{ClassExpression} \rangle \text{'\{' } \langle \text{ClassExpression} \rangle$   
 $\text{'\}' } \text{'\}'}$

**Example 9.1.** *Example 9.17 can be rewritten with the sequence-style notation:*

```
|| \cando{a:Dog, a:CanTalk}
```

### Sequence Union of Class Expressions

$\langle \text{SequenceObjectUnionOf} \rangle ::= \text{'\corof' '{' } \langle \text{ClassExpression} \rangle ( \text{' ,' } \langle \text{ClassExpression} \rangle )_+ \text{'}'}$

**Example 9.2.** *Example 9.18 can be rewritten with the sequence-style notation:*

```
|| \corof{a:Man, a:Woman}
```

### Enumeration of Individuals

$\langle \text{ObjectOneOf} \rangle ::= \text{'\ooneof' '{' } \langle \text{Individual} \rangle ( \text{' ,' } \langle \text{Individual} \rangle )^* \text{'}'}$

**Example 9.3.** *Consider the ontology consisting of the following axioms.*

*The Griffin family consists exactly of Peter, Lois, Stewie, Meg, Chris, and Brian.*

```
|| a:GriffinFamilyMember \ceq \ooneof{a:Peter, a:Lois, a:Stewie, a:Meg, a:Chris, a:Brian}
```

*Quagmire, Peter, Lois, Stewie, Meg, Chris, and Brian are all different from each other.*

```
|| \ialldiff{a:Quagmire, a:Peter, a:Lois, a:Stewie, a:Meg, a:Chris, a:Brian}
```

*The class `a:GriffinFamilyMember` now contains exactly the six explicitly listed individuals. Since we also know that `a:Quagmire` is different from these six individuals, this individual is classified as an instance of the following class expression:*

```
|| (\not a:GriffinFamilyMember)
```

## 9.1.2 Object Property Restrictions

### Existential Quantification

$\langle \text{ObjectSomeValuesFrom} \rangle ::= \backslash \text{oexists} \text{ '}' \langle \text{ObjectPropertyExpression} \rangle \text{ '}' \text{ '}' \langle \text{ClassExpression} \rangle \text{ '}'$

**Example 9.4.** Consider the ontology consisting of the following axioms.

*Peter is Stewie's father.*

`|| a:fatherOf(a:Peter, a:Stewie)`

*Stewie is a man.*

`|| a:Man(a:Stewie)`

The following existential expression contains those individuals that are connected by the `a:fatherOf` property to individuals that are instances of `a:Man`; furthermore, `a:Peter` is classified as its instance:

`|| \oexists{a:fatherOf}{a:Man}`

### Universal Quantification

$\langle \text{ObjectAllValuesFrom} \rangle ::= \backslash \text{forall} \text{ '}' \langle \text{ObjectPropertyExpression} \rangle \text{ '}' \text{ '}' \langle \text{ClassExpression} \rangle \text{ '}'$

**Example 9.5.** Consider the ontology consisting of the following axioms.

*Brian is a pet of Peter.*

`|| a:hasPet(a:Peter, a:Brian)`

*Brian is a dog.*

`|| a:Dog(a:Brian)`

*Peter has at most one pet.*

`|| \o[<=1]{a:hasPet}{a:Peter}`

The following universal expression contains those individuals that are connected through the `a:hasPet` property only with individuals that are instances of `a:Dog` – that is, it contains individuals that have only dogs as pets:

`|| \forallforall{a:hasPet}{a:Dog}`

The ontology axioms clearly state that `a:Peter` is connected by `a:hasPet` only to instances of `a:Dog`: it is impossible to connect `a:Peter` by `a:hasPet` to an individual different from `a:Brian` without making the ontology inconsistent. Therefore, `a:Peter` is classified as an instance of the mentioned class expression.

The last axiom – that is, the one stating that  $a:Peter$  has at most one pet – is critical for the inference from the previous paragraph due to the open-world semantics of OWL 2. Without this axiom, the ontology might not have listed all the individuals to which  $a:Peter$  is connected by  $a:hasPet$ . In such a case  $a:Peter$  would not be classified as an instance of the mentioned class expression.

### Individual Value Restriction

$\langle ObjectHasValue \rangle ::= \backslash\text{ohasvalue} \{ \langle ObjectPropertyExpression \rangle \} \{ \langle Individual \rangle \}$

**Example 9.6.** Consider the ontology consisting of the following axioms.

*Peter is Stewie’s father.*

`|| a:fatherOf(a:Peter, a:Stewie)`

The following has-value class expression contains those individuals that are connected through the  $a:fatherOf$  property with the individual  $a:Stewie$ ; furthermore,  $a:Peter$  is classified as its instance:

`|| \ohasvalue{a:fatherOf}{a:Stewie}`

### 9.1.3 Object Property Cardinality Restrictions

#### Minimum Cardinality

$\langle ObjectMinCardinality \rangle ::= \backslash\text{o}[\geq] \langle nonNegativeInteger \rangle \{ \langle ObjectPropertyExpression \rangle \} [ \{ \langle ClassExpression \rangle \} ]$

**Example 9.7.** Consider the ontology consisting of the following axioms.

*Peter is Stewie’s father.*

`|| a:fatherOf(a:Peter, a:Stewie)`

*Stewie is a man.*

`|| a:Man(a:Stewie)`

*Peter is Chris’s father.*

`|| a:fatherOf(a:Peter, a:Chris)`

*Chris is a man.*

`|| a:Man(a:Chris)`

*Chris and Stewie are different from each other.*

`|| a:Chris \idiff a:Stewie`

*The following minimum cardinality expression contains those individuals that are connected by a:fatherOf to at least two different instances of a:Man:*

`|| \o[>=2]{a:fatherOf}{a:Man}`

*Since a:Stewie and a:Chris are both instances of a:Man and are different from each other, a:Peter is classified as an instance of this class expression. Due to the open-world semantics, the last axiom – the one stating that a:Chris and a:Stewie are different from each other – is necessary for this inference: without this axiom, it is possible that a:Chris and a:Stewie are actually the same individual.*

### Maximum Cardinality

$\langle \text{ObjectMaxCardinality} \rangle ::= \backslash\text{o}[\leq] \langle \text{nonNegativeInteger} \rangle \text{' } \{ \langle \text{ObjectPropertyExpression} \rangle \text{' } [ \text{' } \{ \langle \text{ClassExpression} \rangle \text{' } ]$

**Example 9.8.** *Consider the ontology consisting of the following axioms.*

`|| a:hasPet(a:Peter, a:Brian)`

*Peter has at most one pet.*

`|| \o[<=1]{a:hasPet}{a:Peter}`

*The following maximum cardinality expression contains those individuals that are connected by a:hasPet to at most two individuals:*

`|| \o[<=2]{a:hasPet}`

*Since a:Peter is known to be connected by a:hasPet to at most one individual, it is certainly also connected by a:hasPet to at most two individuals so, consequently, a:Peter is classified as an instance of this class expression. The example ontology explicitly names only a:Brian as being connected by a:hasPet from a:Peter, so one might expect a:Peter to be classified as an instance of the mentioned class expression even without the second axiom. This, however, is not the case due to the open-world semantics. Without the last axiom, it is possible that a:Peter is connected by a:hasPet to other individuals. The second axiom closes the set of individuals that a:Peter is connected to by a:hasPet.*

**Example 9.9.** Consider the ontology consisting of the following axioms.

*Meg is a daughter of Peter.*

`|| a:hasDaughter(a:Peter, a:Meg)`

*Megan is a daughter of Peter.*

`|| a:hasDaughter(a:Peter, a:Megan)`

*Peter has at most one daughter.*

`|| \o[<=1]{a:hasDaughter}{a:Peter}`

One might expect this ontology to be inconsistent: on the one hand, it says that *a:Meg* and *a:Megan* are connected to *a:Peter* by *a:hasDaughter*, but, on the other hand, it says that *a:Peter* is connected by *a:hasDaughter* to at most one individual. This ontology, however, is not inconsistent because the semantics of OWL 2 does not make the unique name assumption – that is, it does not assume distinct individuals to be necessarily different. For example, the ontology does not explicitly say that *a:Meg* and *a:Megan* are different individuals; therefore, since *a:Peter* can be connected by *a:hasDaughter* to at most one distinct individual, *a:Meg* and *a:Megan* must be the same. This example ontology thus entails the following assertion:

`|| a:Meg \ieq a:Megan`

One can axiomatize the unique name assumption in OWL 2 by explicitly stating that all individuals are different from each other. This can be done by adding the following axiom, which makes the example ontology inconsistent.

*Peter, Meg, and Megan are all different from each other.*

`|| \ialldiff{a:Peter, a:Meg, a:Megan}`

### Exact Cardinality

$\langle \text{ObjectExactCardinality} \rangle ::= \backslash \circ [=] \langle \text{nonNegativeInteger} \rangle \text{' } \text{' } \{ \langle \text{ObjectPropertyExpression} \rangle \text{' } \text{' } [ \text{' } \{ \langle \text{ClassExpression} \rangle \text{' } \text{' } ]$

**Example 9.10.** Consider the ontology consisting of the following axioms.

*Brian is a pet of Peter.*

`|| a:hasPet(a:Peter, a:Brian)`

*Brian is a dog.*

`|| a:Dog(a:Brian)`

Each pet of Peter is either Brian or it is not a dog.

```
|| \forall\text{forall}\{a:\text{hasPet}\}\{\(\text{ooneof}\{a:\text{Brian}\} \ \text{cor} \ (\text{not } a:\text{Dog})\}\}(a:\text{Peter})
```

The following exact cardinality expression contains those individuals that are connected by `a:hasPet` to exactly one instance of `a:Dog`; furthermore, `a:Peter` is classified as its instance:

```
|| \o[=1]\{a:\text{hasPet}\}\{a:\text{Dog}\}
```

This is because the first two axioms say that `a:Peter` is connected to `a:Brian` by `a:hasPet` and that `a:Brian` is an instance of `a:Dog`, and the last axiom says that any individual different from `a:Brian` that is connected to `a:Peter` by `a:hasPet` is not an instance of `a:Dog`; hence, `a:Peter` is connected to exactly one instance of `a:Dog` by `a:hasPet`.

#### 9.1.4 Data Property Restrictions

##### Existential Quantification

$\langle \text{DataSomeValuesFrom} \rangle ::= \text{'\dexists'} \text{'\{'} \langle \text{DataPropertyExpression} \rangle \text{'\}' \text{'\{'} \langle \text{DataRange} \rangle \text{'\}'}$

**Example 9.11.** Consider the ontology consisting of the following axioms.

Meg is seventeen years old.

```
|| a:\text{hasAge}(a:\text{Meg}, "17"[\text{xsd}:\text{integer}])
```

The following existential class expression contains all individuals that are connected by `a:hasAge` to an integer strictly less than 20 so; furthermore, `a:Meg` is classified as its instance:

```
|| \dexists\{a:\text{hasAge}\}\{\(\text{xsd}:\text{integer} \ \text{drres}\{\text{xsd}:\text{maxExclusive} "20"[\text{xsd}:\text{integer}]\}\}
```

##### Universal Quantification

$\langle \text{DataAllValuesFrom} \rangle ::= \text{'\forall\text{forall}'} \text{'\{'} \langle \text{DataPropertyExpression} \rangle \text{'\}' \text{'\{'} \langle \text{DataRange} \rangle \text{'\}'}$

**Example 9.12.** Consider the ontology consisting of the following axioms.

The ZIP code of `_:a1` is the integer "02903".

```
|| a:\text{hasZIP}(\_:a1, "02903"[\text{xsd}:\text{integer}])
```

Each object can have at most one ZIP code.

```
|| a:hasZIP \dfunc
```

In United Kingdom and Canada, ZIP codes are strings (i.e., they can contain characters and not just numbers). Hence, one might use the following universal expression to identify those individuals that have only integer ZIP codes (and therefore have non-UK and non-Canadian addresses):

```
|| \dforall{a:hasZIP}{xsd:integer}
```

The anonymous individual  $\_ :a1$  is by the first axiom connected by  $a:hasZIP$  to an integer, and the second axiom ensures that  $\_ :a1$  is not connected by  $a:hasZIP$  to other literals; therefore,  $\_ :a1$  is classified as an instance of the mentioned class expression. The last axiom – the one stating that  $a:hasZIP$  is functional – is critical for the inference from the previous paragraph due to the open-world semantics of OWL 2. Without this axiom, the ontology is not guaranteed to list all literals that  $\_ :a1$  is connected to by  $a:hasZIP$ ; hence, without this axiom  $\_ :a1$  would not be classified as an instance of the mentioned class expression.

### Literal Value Restriction

```
<DataHasValue> ::= '\dhasvalue' '{' <DataPropertyExpression> '}' '{' <Literal> '}'
```

**Example 9.13.** Consider the ontology consisting of the following axioms.

*Meg is seventeen years old.*

```
|| a:hasAge(a:Meg, "17"[xsd:integer])
```

The following has-value expression contains all individuals that are connected by  $a:hasAge$  to the integer 17; furthermore,  $a:Meg$  is classified as its instance:

```
|| \dhasvalue{a:hasAge}{"17"[xsd:integer]}
```

### 9.1.5 Data Property Cardinality Restrictions

#### Minimum Cardinality

```
<DataMinCardinality> ::= '\d[>=]' <nonNegativeInteger> ']' '{' <DataPropertyExpression> '}' [ '{' <DataRange> '}' ]
```

**Example 9.14.** Consider the ontology consisting of the following axioms.

Meg's name is "Meg Griffin".

```
|| a:hasName(a:Meg, "Meg Griffin")
```

Meg's name is "Megan Griffin".

```
|| a:hasName(a:Meg, "Megan Griffin")
```

The following minimum cardinality expression contains those individuals that are connected by `a:hasName` to at least two different literals:

```
|| \d[>=2]{a:hasName}
```

Different string literals are distinct, so "Meg Griffin" and "Megan Griffin" are different; thus, the individual `a:Meg` is classified as an instance of the mentioned class expression.

```
|| \d[>=2]{a:hasName}
```

### Maximum Cardinality

$\langle \text{DataMaxCardinality} \rangle ::= \text{'\d[<=}' \langle \text{nonNegativeInteger} \rangle \text{'}' \text{'{' } \langle \text{DataPropertyExpression} \rangle \text{'}' \text{'}' \text{'{' } \langle \text{DataRange} \rangle \text{'}' \text{'}' ]}$

**Example 9.15.** Consider the ontology consisting of the following axioms.

Each object can have at most one name.

```
|| a:hasName \dfunc
```

The following maximum cardinality expression contains those individuals that are connected by `a:hasName` to at most two different literals:

```
|| \d[<=2]{a:hasName}
```

Since the ontology axiom restricts `a:hasName` to be functional, all individuals in the ontology are instances of this class expression.

### Exact Cardinality

$\langle \text{DataExactCardinality} \rangle ::= \text{'\d[=' } \langle \text{nonNegativeInteger} \rangle \text{'}' \text{'{' } \langle \text{DataPropertyExpression} \rangle \text{'}' \text{'}' \text{'{' } \langle \text{DataRange} \rangle \text{'}' \text{'}' ]}$

**Example 9.16.** Consider the ontology consisting of the following axioms.

Brian's name is "Brian Griffin".

```
|| a:hasName(a:Brian, "Brian Griffin")
```

*Each object can have at most one name.*

$\| a : \text{hasName} \ \backslash \text{dfunc}$

*The following exact cardinality expression contains those individuals that are connected by  $a:\text{hasName}$  to exactly one literal:*

$\| \ \backslash \text{d} [=1] \{ a : \text{hasName} \}$

*Since the ontology axiom restricts  $a:\text{hasName}$  to be functional and  $a:\text{Brian}$  is connected by  $a:\text{hasName}$  to "Brian Griffin", it is classified as an instance of this class expression.*

## 9.2 Non-atomic Class Expression

$\langle \text{NonAtomicClassExpression} \rangle ::= \langle \text{ObjectIntersectionOf} \rangle \mid \langle \text{ObjectUnionOf} \rangle \mid$   
 $\langle \text{ObjectComplementOf} \rangle \mid \langle \text{ObjectHasSelf} \rangle$

### 9.2.1 Propositional Connectives

#### Intersection of Class Expressions

$\langle \text{ObjectIntersectionOf} \rangle ::= \langle \text{ClassExpression} \rangle \ \backslash \text{cand} \ \langle \text{ClassExpression} \rangle$

**Example 9.17.** *Consider the ontology consisting of the following axioms.*

*Brian is a dog.*

$\| a : \text{Dog} ( a : \text{Brian} )$

*Brian can talk.*

$\| a : \text{CanTalk} ( a : \text{Brian} )$

*The following class expression describes all dogs that can talk; furthermore,  $a:\text{Brian}$  is classified as its instance.*

$\| ( a : \text{Dog} \ \backslash \text{cand} \ a : \text{CanTalk} )$

#### Union of Class Expressions

$\langle \text{ObjectUnionOf} \rangle ::= \langle \text{ClassExpression} \rangle \ \backslash \text{cor} \ \langle \text{ClassExpression} \rangle$

**Example 9.18.** *Consider the ontology consisting of the following axioms.*

*Peter is a man.*

$\| a : \text{Man} ( a : \text{Peter} )$

*Lois is a woman.*

`|| a:Woman(a:Lois)`

*The following class expression describes all individuals that are instances of either `a:Man` or `a:Woman`; furthermore, both `a:Peter` and `a:Lois` are classified as its instances:*

`|| (a:Man \cor a:Woman)`

### Complement of Class Expressions

$\langle \text{ObjectComplementOf} \rangle ::= \text{'\cnot'} \langle \text{ClassExpression} \rangle$

**Example 9.19.** *Consider the ontology consisting of the following axioms.*

*Nothing can be both a man and a woman.*

`|| a:Man \cdisjoint a:Woman`

*Lois is a woman.*

`|| a:Woman(a:Lois)`

*The following class expression describes all things that are not instances of `a:Man`:*

`|| (\cnot a:Man)`

*Since `a:Lois` is known to be a woman and nothing can be both a man and a woman, then `a:Lois` is necessarily not a `a:Man`; therefore, `a:Lois` is classified as an instance of this complement class expression.*

**Example 9.20.** *OWL 2 has open-world semantics, so negation in OWL 2 is the same as in classical (first-order) logic. To understand open-world semantics, consider the ontology consisting of the following assertion.*

*Brian is a dog.*

`|| a:Dog(a:Brian)`

*One might expect `a:Brian` to be classified as an instance of the following class expression:*

`|| (\cnot a:Bird)`

*Intuitively, the ontology does not explicitly state that `a:Brian` is an instance of `a:Bird`, so this statement seems to be false. In OWL 2, however, this is not the case: it is true that the ontology does not state that `a:Brian` is an instance*

of  $a:Bird$ ; however, the ontology does not state the opposite either. In other words, this ontology simply does not contain enough information to answer the question whether  $a:Brian$  is an instance of  $a:Bird$  or not: it is perfectly possible that the information to that effect is actually true but it has not been included in the ontology.

The ontology from the previous example (in which  $a:Lois$  has been classified as  $a:Man$ ), however, contains sufficient information to draw the expected conclusion. In particular, we know for sure that  $a:Lois$  is an instance of  $a:Woman$  and that  $a:Man$  and  $a:Woman$  do not share instances. Therefore, any additional information that does not lead to inconsistency cannot lead to a conclusion that  $a:Lois$  is an instance of  $a:Man$ ; furthermore, if one were to explicitly state that  $a:Lois$  is an instance of  $a:Man$ , the ontology would be inconsistent and, by definition, it then entails all possible conclusions.

## 9.2.2 Object Property Restrictions

### Self-Restriction

$\langle ObjectHasSelf \rangle ::= \backslash ohasself' \langle ObjectPropertyExpression \rangle$

**Example 9.21.** Consider the ontology consisting of the following axioms.

*Peter likes Peter.*

`|| a:likes(a:Peter, a:Peter)`

The following self-restriction contains those individuals that like themselves; furthermore,  $a:Peter$  is classified as its instance:

`|| (\backslash ohasself a:likes)`

## 10 Axiom

$\langle Axiom \rangle ::= \langle Declaration \rangle \mid \langle ClassAxiom \rangle \mid \langle ObjectPropertyAxiom \rangle$   
 $\mid \langle DataPropertyAxiom \rangle \mid \langle DatatypeDefinition \rangle \mid \langle HasKey \rangle$   
 $\mid \langle Assertion \rangle \mid \langle AnnotationAxiom \rangle$

$\langle axiomAnnotations \rangle ::= [ \text{'[ ' } \langle Annotation \rangle ( \text{' , ' } \langle Annotation \rangle )^* \text{' ] ' } ]$

### 10.1 Class Expression Axioms

$\langle ClassAxiom \rangle ::= \langle NonSequenceClassAxiom \rangle \mid \langle SequenceClassAxiom \rangle$

### 10.1.1 Non-sequence Class Expression Axioms

$\langle NonSequenceClassAxiom \rangle ::= \langle SubClassOf \rangle \mid \langle EquivalentClasses \rangle \mid \langle DisjointClasses \rangle$   
 $\mid \langle DisjointUnion \rangle$

#### Subclass Axioms

$\langle SubClassOf \rangle ::= \langle subClassExpression \rangle \text{ '\cisa' } \langle superClassExpression \rangle \langle axiomAnnotations \rangle$

$\langle subClassExpression \rangle ::= \langle ClassExpression \rangle$

$\langle superClassExpression \rangle ::= \langle ClassExpression \rangle$

**Example 10.1.** Consider the ontology consisting of the following axioms.

*Each baby is a child.*

$\| a:Baby \text{ \cisa } a:Child$

*Each child is a person.*

$\| a:Child \text{ \cisa } a:Person$

*Stewie is a baby.*

$\| a:Baby(a:Stewie)$

*Since  $a:Stewie$  is an instance of  $a:Baby$ , by the first subclass axiom  $a:Stewie$  is classified as an instance of  $a:Child$  as well. Similarly, by the second subclass axiom  $a:Stewie$  is classified as an instance of  $a:Person$ . This style of reasoning can be applied to any instance of  $a:Baby$  and not just  $a:Stewie$ ; therefore, one can conclude that  $a:Baby$  is a subclass of  $a:Person$ . In other words, this ontology entails the following axiom:*

$\| a:Baby \text{ \cisa } a:Person$

**Example 10.2.** Consider the ontology consisting of the following axioms.

*A person that has a child has either at least one boy or a girl.*

$\| a:PersonWithChild \text{ \cisa } \text{\oexists}\{a:hasChild\}\{(a:Boy \text{ \cor } a:Girl)\}$

*Each boy is a child.*

$\| a:Boy \text{ \cisa } a:Child$

*Each girl is a child.*

```
|| a:Girl \cisa a:Child
```

*If some object has a child, then this object is a parent.*

```
|| \oexists{a:hasChild}{a:Child} \cisa a:Parent
```

The first axiom states that each instance of `a:PersonWithChild` is connected to an individual that is an instance of either `a:Boy` or `a:Girl`. (Because of the open-world semantics of OWL 2, this does not mean that there must be only one such individual or that all such individuals must be instances of either `a:Boy` or `a:Girl`.) Furthermore, each instance of `a:Boy` or `a:Girl` is an instance of `a:Child`. Finally, the last axiom says that all individuals that are connected by `a:hasChild` to an instance of `a:Child` are instances of `a:Parent`. Since this reasoning holds for each instance of `a:PersonWithChild`, each such instance is also an instance of `a:Parent`. In other words, this ontology entails the following axiom:

```
|| a:PersonWithChild \cisa a:Parent
```

### Equivalent Classes

$\langle \text{EquivalentClasses} \rangle ::= \langle \text{ClassExpression} \rangle \backslash \text{ceq} \langle \text{ClassExpression} \rangle \langle \text{axiomAnnotations} \rangle$

**Example 10.3.** Consider the ontology consisting of the following axioms.

*A boy is a male child.*

```
|| a:Boy \ceq (a:Child \cand a:Man)
```

*Chris is a child.*

```
|| a:Child(a:Chris)
```

*Chris is a man.*

```
|| a:Man(a:Chris)
```

*Stewie is a boy.*

```
|| a:Boy(a:Stewie)
```

The first axiom defines the class `a:Boy` as an intersection of the classes `a:Child` and `a:Man`; thus, the instances of `a:Boy` are exactly those instances that are both an instance of `a:Child` and an instance of `a:Man`. Such a definition consists of two directions. The first direction implies that each instance of `a:Child` and `a:Man` is an instance of `a:Boy`; since `a:Chris` satisfies these two conditions,

it is classified as an instance of  $a:Boy$ . The second direction implies that each  $a:Boy$  is an instance of  $a:Child$  and of  $a:Man$ ; thus,  $a:Stewie$  is classified as an instance of  $a:Man$  and of  $a:Boy$ .

**Example 10.4.** Consider the ontology consisting of the following axioms.

*A mongrel owner has a pet that is a mongrel.*

$\| a:MongrelOwner \ \backslash ceq \ \backslash oexists\{a:hasPet\}\{a:Mongrel\}$

*A dog owner has a pet that is a dog.*

$\| a:DogOwner \ \backslash ceq \ \backslash oexists\{a:hasPet\}\{a:Dog\}$

*Functional-Style Syntax – Each mongrel is a dog.*

$\| a:Mongrel \ \backslash isa \ a:Dog$

*Peter is a mongrel owner.*

$\| a:MongrelOwner(a:Peter)$

*By the first axiom, each instance  $x$  of  $a:MongrelOwner$  must be connected via  $a:hasPet$  to an instance of  $a:Mongrel$ ; by the third axiom, this individual is an instance of  $a:Dog$ ; thus, by the second axiom,  $x$  is an instance of  $a:DogOwner$ . In other words, this ontology entails the following axiom:*

$\| a:MongrelOwner(a:DogOwner)$

*By the fourth axiom,  $a:Peter$  is then classified as an instance of  $a:DogOwner$ .*

## Disjoint Classes

$\langle DisjointClasses \rangle := \langle ClassExpression \rangle \ \backslash cdisjoint \ \langle ClassExpression \rangle \ \langle axiomAnnotations \rangle$

**Example 10.5.** Consider the ontology consisting of the following axioms.

*Nothing can be both a boy and a girl.*

$\| a:Boy \ \backslash cdisjoint \ a:Girl$

*Stewie is a boy.*

$\| a:Boy(a:Stewie)$

*The axioms in this ontology imply that  $a:Stewie$  can be classified as an instance of the following class expression:*

$\| (\ \backslash cnot \ a:Girl)$

Furthermore, if the ontology were extended with the following assertion, the ontology would become inconsistent:

$\| a:Girl(a:Stewie)$

### Disjoint Union of Class Expression

$\langle DisjointUnion \rangle ::= \langle Class \rangle \backslash cdisjunction \langle disjointClassExpressions \rangle \langle axiomAnnotations \rangle$

$\langle disjointClassExpressions \rangle ::= \{ \langle ClassExpression \rangle ( \langle ClassExpression \rangle )^* \}$

**Example 10.6.** Consider the ontology consisting of the following axioms.

Each child is either a boy or a girl, each boy is a child, each girl is a child, and nothing can be both a boy and a girl.

$\| a:Child \backslash cdisjunction \{ a:Boy, a:Girl \}$

*Stewie is a child.*

$\| a:Child(a:Stewie)$

*Stewie is not a girl.*

$\| (\backslash cnot a:Girl)(a:Stewie)$

By the first two axioms, *a:Stewie* is either an instance of *a:Boy* or *a:Girl*. The last assertion eliminates the second possibility, so *a:Stewie* is classified as an instance of *a:Boy*.

### 10.1.2 Sequence Class Expression Axioms

$\langle SequenceClassAxiom \rangle ::= \langle SequenceEquivalentClasses \rangle | \langle SequenceDisjointClasses \rangle$

#### Sequence Equivalent Classes

$\langle SequenceEquivalentClasses \rangle ::= \backslash calleq \{ \langle ClassExpression \rangle ( \langle ClassExpression \rangle )^+ \} \langle axiomAnnotations \rangle$

**Example 10.7.** Example 10.3 can be rewritten with the sequence-style notation:

$\| \backslash calleq \{ a:Boy, (a:Child \backslash cand a:Man) \}$

## Sequence Disjoint Classes

$\langle SequenceDisjointClasses \rangle ::= \backslash calldisjoint \{ \langle ClassExpression \rangle ( \text{' , ' } \langle ClassExpression \rangle ) + \text{' } \} \langle axiomAnnotations \rangle$

**Example 10.8.** *Example 10.5 can be rewritten with the sequence-style notation:*

$\| \backslash calldisjoint \{ a : Boy, a : Girl \}$

## 10.2 Object Property Axioms

$\langle ObjectPropertyAxiom \rangle ::= \langle NonSequenceObjectPropertyAxiom \rangle | \langle SequenceObjectPropertyAxiom \rangle$

### 10.2.1 Non-sequence Object Property Axioms

$\langle NonSequenceObjectPropertyAxiom \rangle ::= \langle SubObjectPropertyOf \rangle | \langle EquivalentObjectProperties \rangle$   
 $| \langle DisjointObjectProperties \rangle | \langle InverseObjectProperties \rangle | \langle ObjectPropertyDomain \rangle$   
 $| \langle ObjectPropertyRange \rangle | \langle FunctionalObjectProperty \rangle | \langle InverseFunctionalObjectProperty \rangle$   
 $| \langle ReflexiveObjectProperty \rangle | \langle IrreflexiveObjectProperty \rangle | \langle SymmetricObjectProperty \rangle$   
 $| \langle AsymmetricObjectProperty \rangle | \langle TransitiveObjectProperty \rangle$

### Object Subproperties

$\langle SubObjectPropertyOf \rangle ::= \langle subObjectPropertyExpression \rangle \backslash oisa \langle superObjectPropertyExpression \rangle$   
 $\langle axiomAnnotations \rangle$

$\langle subObjectPropertyExpression \rangle ::= \langle ObjectPropertyExpression \rangle | \langle propertyExpressionChain \rangle$

$\langle propertyExpressionChain \rangle ::= \backslash ochain \{ \langle ObjectPropertyExpression \rangle ( \text{' , ' } \langle ObjectPropertyExpression \rangle ) + \text{' } \}$

$\langle superObjectPropertyExpression \rangle ::= \langle ObjectPropertyExpression \rangle$

**Example 10.9.** *Consider the ontology consisting of the following axioms.*

*Having a dog implies having a pet.*

$\| a : hasDog \backslash oisa a : hasPet$

*Functional-Style Syntax – Brian is a dog of Peter.*

$\| a : hasDog ( a : Peter, a : Brian )$

Since  $a:\text{hasDog}$  is a subproperty of  $a:\text{hasPet}$ , each tuple of individuals connected by the former property expression is also connected by the latter property expression. Therefore, this ontology entails that  $a:\text{Peter}$  is connected to  $a:\text{Brian}$  by  $a:\text{hasPet}$ ; that is, the ontology entails the following assertion:

```
|| a:hasPet(a:Peter, a:Brian)
```

**Example 10.10.** Consider the ontology consisting of the following axioms.

*The sister of someone's mother is that person's aunt.*

```
|| \ochain{a:hasMother, a:hasSister} \oisa a:hasAunt
```

*Lois is the mother of Stewie.*

```
|| a:hasMother(a:Stewie, a:Lois)
```

*Carol is a sister of Lois.*

```
|| a:hasSister(a:Lois, a:Carol)
```

The axioms in this ontology imply that  $a:\text{Stewie}$  is connected by  $a:\text{hasAunt}$  with  $a:\text{Carol}$ ; that is, the ontology entails the following assertion:

```
|| a:hasAunt(a:Stewie, a:Carol)
```

### Equivalent Object Properties

$\langle \text{EquivalentObjectProperties} \rangle ::= \langle \text{ObjectPropertyExpression} \rangle \text{'\oeq'} \langle \text{ObjectPropertyExpression} \rangle$   
 $\langle \text{axiomAnnotations} \rangle$

**Example 10.11.** Consider the ontology consisting of the following axioms.

*Having a brother is the same as having a male sibling.*

```
|| a:hasBrother \oeq a:hasMaleSibling
```

*Stewie is a brother of Chris.*

```
|| a:hasBrother(a:Chris, a:Stewie)
```

*Chris is a male sibling of Stewie.*

```
|| a:hasMaleSibling(a:Stewie, a:Chris)
```

Since  $a:\text{hasBrother}$  and  $a:\text{hasMaleSibling}$  are equivalent properties, this ontology entails that  $a:\text{Chris}$  is connected by  $a:\text{hasMaleSibling}$  with  $a:\text{Stewie}$  – that is, it entails the following assertion:

`|| a:hasMaleSibling(a:Chris, a:Stewie)`

*Furthermore, the ontology also entails that that a:Stewie is connected by a:hasBrother with a:Chris – that is, it entails the following assertion:*

`|| a:hasBrother(a:Stewie, a:Chris)`

### Disjoint Object Properties

$\langle \text{DisjointObjectProperties} \rangle ::= \langle \text{ObjectPropertyExpression} \rangle \text{'\odisjoint'} \langle \text{ObjectPropertyExpression} \rangle$   
 $\langle \text{axiomAnnotations} \rangle$

**Example 10.12.** *Consider the ontology consisting of the following axioms.*

*Fatherhood is disjoint with motherhood.*

`|| a:hasFather \odisjoint a:hasMother`

*Peter is Stewie’s father.*

`|| a:hasFather(a:Stewie, a:Peter)`

*Lois is the mother of Stewie.*

`|| a:hasMother(a:Stewie, a:Lois)`

*In this ontology, the disjointness axiom is satisfied. If, however, one were to add the following assertion, the disjointness axiom would be invalidated and the ontology would become inconsistent:*

`|| a:hasMother(a:Stewie, a:Peter)`

### Inverse Object Properties

$\langle \text{InverseObjectProperties} \rangle ::= \langle \text{ObjectPropertyExpression} \rangle \text{'\oinv'} \langle \text{ObjectPropertyExpression} \rangle$   
 $\langle \text{axiomAnnotations} \rangle$

**Example 10.13.** *Consider the ontology consisting of the following axioms.*

*Having a father is the opposite of being a father of someone.*

`|| a:hasFather \oinv a:fatherOf`

*Peter is Stewie’s father.*

`|| a:hasFather(a:Stewie, a:Peter)`

*Peter is Chris's father.*

`|| a:fatherOf(a:Peter, a:Chris)`

*This ontology entails that a:Peter is connected by a:fatherOf with a:Stewie – that is, it entails the following assertion:*

`|| a:fatherOf(a:Peter, a:Stewie)`

*Furthermore, the ontology also entails that a:Chris is connected by a:hasFather with a:Peter – that is, it entails the following assertion:*

`|| a:hasFather(a:Chris, a:Peter)`

### Object Property Domain

$\langle \text{ObjectPropertyDomain} \rangle ::= \langle \text{ObjectPropertyExpression} \rangle \backslash \text{odomain} \langle \text{ClassExpression} \rangle$   
 $\langle \text{axiomAnnotations} \rangle$

**Example 10.14.** *Consider the ontology consisting of the following axioms.*

*Functional-Style Syntax – Only people can own dogs.*

`|| a:hasDog \odomain a:Person`

*Brian is a dog of Peter.*

`|| a:hasDog(a:Peter, a:Brian)`

*By the first axiom, each individual that has an outgoing a:hasDog connection must be an instance of a:Person. Therefore, a:Peter can be classified as an instance of a:Person; that is, this ontology entails the following assertion:*

`|| a:Person(a:Peter)`

*Domain axioms in OWL 2 have a standard first-order semantics that is somewhat different from the semantics of such axioms in databases and object-oriented systems, where such axioms are interpreted as checks. The domain axiom from the example ontology would in such systems be interpreted as a constraint saying that a:hasDog can point only from individuals that are known to be instances of a:Person; furthermore, since the example ontology does not explicitly state that a:Peter is an instance of a:Person, one might expect the domain constraint to be invalidated. This, however, is not the case in OWL 2: as shown in the previous paragraph, the missing type is inferred from the domain constraint.*

### Object Property Range

$$\langle \text{ObjectPropertyRange} \rangle ::= \langle \text{ObjectPropertyExpression} \rangle \backslash \text{orange} \langle \text{ClassExpression} \rangle \\ \langle \text{axiomAnnotations} \rangle$$

**Example 10.15.** Consider the ontology consisting of the following axioms.

The range of the `a:hasDog` property is the class `a:Dog`.

```
|| a:hasDog \orange a:Dog
```

Brian is a dog of Peter.

```
|| a:hasDog(a:Peter, a:Brian)
```

By the first axiom, each individual that has an outgoing `a:hasDog` connection must be an instance of `a:Person`. Therefore, `a:Peter` can be classified as an instance of `a:Person`; that is, this ontology entails the following assertion:

```
|| a:Dog(a:Brian)
```

Range axioms in OWL 2 have a standard first-order semantics that is somewhat different from the semantics of such axioms in databases and object-oriented systems, where such axioms are interpreted as checks. The range axiom from the example ontology would in such systems be interpreted as a constraint saying that `a:hasDog` can point only to individuals that are known to be instances of `a:Dog`; furthermore, since the example ontology does not explicitly state that `a:Brian` is an instance of `a:Dog`, one might expect the range constraint to be invalidated. This, however, is not the case in OWL 2: as shown in the previous paragraph, the missing type is inferred from the range constraint.

## Functional Object Properties

$$\langle \text{FunctionalObjectProperty} \rangle ::= \langle \text{ObjectPropertyExpression} \rangle \backslash \text{ofunc} \langle \text{axiomAnnotations} \rangle$$

**Example 10.16.** Consider the ontology consisting of the following axioms.

Each object can have at most one father.

```
|| a:hasFather \ofunc
```

Peter is Stewie's father.

```
|| a:hasFather(a:Stewie, a:Peter)
```

Peter Griffin is Stewie's father.

```
|| a:hasFather(a:Stewie, a:Peter_Griffin)
```

By the first axiom, `a:hasFather` can point from `a:Stewie` to at most one distinct individual, so `a:Peter` and `a:Peter_Griffin` must be equal; that is, this ontology entails the following assertion:

```
|| a:Peter \ieq a:Peter_Griffin
```

One might expect the previous ontology to be inconsistent, since the `a:hasFather` property points to two different values for `a:Stewie`. OWL 2, however, does not make the unique name assumption, so `a:Peter` and `a:Peter_Griffin` are not necessarily distinct individuals. If the ontology were extended with the following assertion, then it would indeed become inconsistent:

```
|| a:Peter \idiff a:Peter_Griffin
```

### Inverse-Functional Object Properties

$\langle \text{InverseFunctionalObjectProperty} \rangle ::= \langle \text{ObjectPropertyExpression} \rangle \text{'\oinvfunc'}$   
 $\langle \text{axiomAnnotations} \rangle$

**Example 10.17.** Consider the ontology consisting of the following axioms.

*Each object can have at most one father.*

```
|| a:fatherOf \oinvfunc
```

*Peter is Stewie's father.*

```
|| a:fatherOf(a:Peter, a:Stewie)
```

*Peter Griffin is Stewie's father.*

```
|| a:fatherOf(a:Peter_Griffin, a:Stewie)
```

By the first axiom, at most one distinct individual can point by `a:fatherOf` to `a:Stewie`, so `a:Peter` and `a:Peter_Griffin` must be equal; that is, this ontology entails the following assertion:

```
|| a:Peter \ieq a:Peter_Griffin
```

One might expect the previous ontology to be inconsistent, since there are two individuals that `a:Stewie` is connected to by `a:fatherOf`. OWL 2, however, does not make the unique name assumption, so `a:Peter` and `a:Peter_Griffin` are not necessarily distinct individuals. If the ontology were extended with the following assertion, then it would indeed become inconsistent:

```
|| a:Peter \idiff a:Peter_Griffin
```

### Reflexive Object Properties

$\langle \text{ReflexiveObjectProperty} \rangle ::= \langle \text{ObjectPropertyExpression} \rangle \text{'\oreflex'} \langle \text{axiomAnnotations} \rangle$

**Example 10.18.** Consider the ontology consisting of the following axioms.

*Everybody knows themselves.*

`|| a:knows \oreflex`

*Peter is a person.*

`|| a:Person(a:Peter)`

*By the first axiom, a:Peter must be connected by a:knows to itself; that is, this ontology entails the following assertion:*

`|| a:knows(a:Peter, a:Peter)`

### Irreflexive Object Properties

$\langle \text{IrreflexiveObjectProperty} \rangle ::= \langle \text{ObjectPropertyExpression} \rangle \text{'\oirreflex'} \langle \text{axiomAnnotations} \rangle$

**Example 10.19.** Consider the ontology consisting of the following axioms.

*Nobody can be married to themselves.*

`|| a:marriedTo \oirreflex`

*If this ontology were extended with the following assertion, the irreflexivity axiom would be contradicted and the ontology would become inconsistent:*

`|| a:marriedTo(a:Peter, a:Peter)`

### Symmetric Object Properties

$\langle \text{SymmetricObjectProperty} \rangle ::= \langle \text{ObjectPropertyExpression} \rangle \text{'\osym'} \langle \text{axiomAnnotations} \rangle$

**Example 10.20.** Consider the ontology consisting of the following axioms.

*If x is a friend of y, then y is a friend of x.*

`|| a:friend \osym`

*Brian is a friend of Peter.*

`|| a:friend(a:Peter, a:Brian)`

*Since a:friend is symmetric, a:Peter must be connected by a:friend to a:Brian; that is, this ontology entails the following assertion:*

`|| a:friend(a:Brian, a:Peter)`

### Asymmetric Object Properties

$\langle \text{AsymmetricObjectProperty} \rangle ::= \langle \text{ObjectPropertyExpression} \rangle \text{'\oasym'} \langle \text{axiomAnnotations} \rangle$

**Example 10.21.** *Consider the ontology consisting of the following axioms.*

*If x is a parent of y, then y is not a parent of x.*

`|| a:parentOf \oasym`

*Peter is a parent of Stewie.*

`|| a:parentOf(a:Peter, a:Stewie)`

*If this ontology were extended with the following assertion, the asymmetry axiom would be invalidated and the ontology would become inconsistent:*

`|| a:parentOf(a:Stewie, a:Peter)`

### Transitive Object Properties

$\langle \text{TransitiveObjectProperty} \rangle ::= \langle \text{ObjectPropertyExpression} \rangle \text{'\otrans'} \langle \text{axiomAnnotations} \rangle$

**Example 10.22.** *Consider the ontology consisting of the following axioms.*

*If x is an ancestor of y and y is an ancestor of z, then x is an ancestor of z.*

`|| a:ancestorOf \otrans`

*Carter is an ancestor of Lois.*

`|| a:ancestorOf(a:Carter, a:Lois)`

*Lois is an ancestor of Meg.*

`|| a:ancestorOf(a:Lois, a:Meg)`

*Since a:ancestorOf is transitive, a:Carter must be connected by a:ancestorOf to a:Meg – that is, this ontology entails the following assertion:*

`|| a:ancestorOf(a:Carter, a:Meg)`

## 10.2.2 Sequence Object Property Axioms

$\langle \text{SequenceObjectPropertyAxiom} \rangle ::= \langle \text{SequenceEquivalentObjectProperties} \rangle$   
|  $\langle \text{SequenceDisjointObjectProperties} \rangle$

### Sequence Equivalent Object Properties

$\langle \text{SequenceEquivalentObjectProperties} \rangle ::= \text{'\oalleg' '}' \langle \text{ObjectPropertyExpression} \rangle$   
 $( \text{' ,' } \langle \text{ObjectPropertyExpression} \rangle ) + \text{'}' \langle \text{axiomAnnotations} \rangle$

**Example 10.23.** *Example 10.11 can be rewritten with the sequence-style notation:*

```
|| \oalleg{a:hasBrother, a:hasMaleSibling}
```

### Sequence Disjoint Object Properties

$\langle \text{SequenceDisjointObjectProperties} \rangle ::= \text{'\oalldisjoint' '}' \langle \text{ObjectPropertyExpression} \rangle$   
 $( \text{' ,' } \langle \text{ObjectPropertyExpression} \rangle ) + \text{'}' \langle \text{axiomAnnotations} \rangle$

**Example 10.24.** *Example 10.12 can be rewritten with the sequence-style notation:*

```
|| \oalldisjoint{a:hasFather, a:hasMother}
```

## 10.3 Data Property Axioms

$\langle \text{DataPropertyAxiom} \rangle ::= \langle \text{NonSequenceDataPropertyAxiom} \rangle | \langle \text{SequenceDataPropertyAxiom} \rangle$

### 10.3.1 Non-sequence Object Property Axioms

$\langle \text{NonSequenceDataPropertyAxiom} \rangle ::= \langle \text{SubDataPropertyOf} \rangle | \langle \text{EquivalentDataProperties} \rangle$   
|  $\langle \text{DisjointDataProperties} \rangle$   
|  $\langle \text{DataPropertyDomain} \rangle | \langle \text{DataPropertyRange} \rangle | \langle \text{FunctionalDataProperty} \rangle$

#### Data Subproperties

$\langle \text{SubDataPropertyOf} \rangle ::= \langle \text{subDataPropertyExpression} \rangle \text{'\disa' } \langle \text{superDataPropertyExpression} \rangle$   
 $\langle \text{axiomAnnotations} \rangle$

$\langle \text{subDataPropertyExpression} \rangle := \langle \text{DataPropertyExpression} \rangle$

$\langle \text{superDataPropertyExpression} \rangle := \langle \text{DataPropertyExpression} \rangle$

**Example 10.25.** Consider the ontology consisting of the following axioms.

*A last name of someone is his/her name as well.*

```
|| a:hasLastName \disa a:hasName
```

*Peter's last name is "Griffin".*

```
|| a:hasLastName(a:Peter, "Griffin")
```

*Since a:hasLastName is a subproperty of a:hasName, each individual connected by the former property to a literal is also connected by the latter property to the same literal. Therefore, this ontology entails that a:Peter is connected to "Griffin" through a:hasName; that is, the ontology entails the following assertion:*

```
|| a:hasName(a:Peter, "Griffin")
```

### Equivalent Data Properties

$\langle \text{EquivalentDataProperties} \rangle ::= \langle \text{DataPropertyExpression} \rangle \text{'\deq'} \langle \text{DataPropertyExpression} \rangle$   
 $\langle \text{axiomAnnotations} \rangle$

**Example 10.26.** Consider the ontology consisting of the following axioms.

*a:hasName and a:seLlama (in Spanish) are synonyms.*

```
|| a:hasName \deq a:seLlama
```

*Meg's name is "Meg Griffin".*

```
|| a:hasName(a:Meg, "Meg Griffin")
```

*Meg's name is "Megan Griffin".*

```
|| a:seLlama(a:Meg, "Megan Griffin")
```

*Since a:hasName and a:seLlama are equivalent properties, this ontology entails that a:Meg is connected by a:seLlama with "Meg Griffin" – that is, it entails the following assertion:*

```
|| a:seLlama(a:Meg, "Meg Griffin")
```

*Furthermore, the ontology also entails that a:Meg is also connected by a:hasName with "Megan Griffin" – that is, it entails the following assertion:*

```
|| a:hasName(a:Meg, "Megan Griffin")
```

## Disjoint Data Properties

$\langle \text{DisjointDataProperties} \rangle ::= \langle \text{DataPropertyExpression} \rangle \backslash \text{ddisjoint} \langle \text{DataPropertyExpression} \rangle$   
 $\langle \text{axiomAnnotations} \rangle$

**Example 10.27.** Consider the ontology consisting of the following axioms.

*Someone's name must be different from his address.*

```
|| a:hasName \ddisjoint a:hasAddress
```

*Peter's name is "Peter Griffin".*

```
|| a:hasName(a:Peter, "Peter Griffin")
```

*Peter's address is Quahog, Rhode Island.*

```
|| a:hasAddress(a:Peter, "Quahog, Rhode Island")
```

*In this ontology, the disjointness axiom is satisfied. If, however, one were to add the following assertion, the disjointness axiom would be invalidated and the ontology would become inconsistent:*

```
|| a:hasAddress(a:Peter, "Peter Griffin")
```

## Data Property Domain

$\langle \text{DataPropertyDomain} \rangle ::= \langle \text{DataPropertyExpression} \rangle \backslash \text{ddomain} \langle \text{ClassExpression} \rangle$   
 $\langle \text{axiomAnnotations} \rangle$

**Example 10.28.** Consider the ontology consisting of the following axioms.

*Only people can have names.*

```
|| a:hasName \ddomain a:Person
```

*Peter's name is "Peter Griffin".*

```
|| a:hasName(a:Peter, "Peter Griffin")
```

*By the first axiom, each individual that has an outgoing a:hasName connection must be an instance of a:Person. Therefore, a:Peter can be classified as an instance of a:Person – that is, this ontology entails the following assertion:*

```
|| a:Person(a:Peter)
```

Domain axioms in OWL 2 have a standard first-order semantics that is somewhat different from the semantics of such axioms in databases and object-oriented systems, where such axioms are interpreted as checks. Thus, the domain axiom from the example ontology would in such systems be interpreted as a constraint saying that `a:hasName` can point only from individuals that are known to be instances of `a:Person`; furthermore, since the example ontology does not explicitly state that `a:Peter` is an instance of `a:Person`, one might expect the domain constraint to be invalidated. This, however, is not the case in OWL 2: as shown in the previous paragraph, the missing type is inferred from the domain constraint.

### Data Property Range

$\langle \text{DataPropertyRange} \rangle ::= \langle \text{DataPropertyExpression} \rangle \text{'\drange'} \langle \text{DataRange} \rangle \langle \text{axiomAnnotations} \rangle$

**Example 10.29.** Consider the ontology consisting of the following axioms.

The range of the `a:hasName` property is `xsd:string`.

```
|| a:hasName \drange xsd:string
```

Peter's name is "Peter Griffin".

```
|| a:hasName(a:Peter, "Peter Griffin")
```

By the first axiom, each literal that has an incoming `a:hasName` link must be in `xsd:string`. In the example ontology, this axiom is satisfied. If, however, the ontology were extended with the following assertion, then the range axiom would imply that the literal `"42"[xsd:integer]` is in `xsd:string`, which is a contradiction and the ontology would become inconsistent:

```
|| a:hasName(a:Peter, "42"[xsd:integer])
```

### Functional Data Properties

$\langle \text{FunctionalDataProperty} \rangle ::= \langle \text{DataPropertyExpression} \rangle \text{'\dfunc'} \langle \text{axiomAnnotations} \rangle$

**Example 10.30.** Consider the ontology consisting of the following axioms.

Each object can have at most one age.

```
|| a:hasAge \dfunc
```

*Meg is seventeen years old.*

```
|| a:hasAge(a:Meg, "17"[xsd:integer])
```

By the first axiom, *a:hasAge* can point from *a:Meg* to at most one distinct literal. In this example ontology, this axiom is satisfied. If, however, the ontology were extended with the following assertion, the semantics of functionality axioms would imply that “15”[xsd:integer] is equal to “17”[xsd:integer], which is a contradiction and the ontology would become inconsistent:

```
|| a:hasAge(a:Meg, "15"[xsd:integer])
```

### 10.3.2 Sequence Object Property Axioms

$$\langle \text{SequenceDataPropertyAxiom} \rangle ::= \langle \text{SequenceEquivalentDataProperties} \rangle \mid \langle \text{SequenceDisjointDataProperties} \rangle$$

#### Equivalent Data Properties

$$\langle \text{SequenceEquivalentDataProperties} \rangle ::= \backslash \text{dalleq} \{ \{ \langle \text{DataPropertyExpression} \rangle ( \text{' , ' } \langle \text{DataPropertyExpression} \rangle ) + \text{' } \} \langle \text{axiomAnnotations} \rangle \}$$

**Example 10.31.** *Example 10.26 can be rewritten with the sequence-style notation:*

```
|| \dalleq{a:hasName, a:seLlama}
```

#### Disjoint Data Properties

$$\langle \text{SequenceDisjointDataProperties} \rangle ::= \backslash \text{dalldisjoint} \{ \{ \langle \text{DataPropertyExpression} \rangle ( \text{' , ' } \langle \text{DataPropertyExpression} \rangle ) + \text{' } \} \langle \text{axiomAnnotations} \rangle \}$$

**Example 10.32.** *Example 10.27 can be rewritten with the sequence-style notation:*

```
|| \dalldisjoint{a:hasName, a:hasAddress}
```

## 10.4 Datatype Definitions

$$\langle \text{DatatypeDefinition} \rangle ::= \langle \text{Datatype} \rangle \backslash \text{dtdef} \langle \text{DataRange} \rangle \langle \text{axiomAnnotations} \rangle$$

**Example 10.33.** Consider the ontology consisting of the following axioms.

*a:SSN is a datatype.*

```
|| a:SSN \d
```

*A social security number is a string that matches the given regular expression.*

```
|| a:SSN \dtdef (xsd:string \drres{xsd:pattern
    "[0-9]{3}-[0-9]{2}-[0-9]{4}"})
```

*The range of the a:hasSSN property is a:SSN.*

```
|| a:hasSSN \drange a:SSN
```

The second axiom defines *a:SSN* as an abbreviation for a datatype restriction on *xsd:string*. In order to satisfy the typing restrictions the first axiom explicitly declares *a:SSN* to be a datatype. The datatype *a:SSN* can be used just like any other datatype; for example, it is used in the third axiom to define the range of the *a:hasSSN* property. The only restriction is that *a:SSN* supports no facets and therefore cannot be used in datatype restrictions, and that there can be no literals of datatype *a:SSN*.

## 10.5 Keys

$\langle \text{HasKey} \rangle ::= \langle \text{ClassExpression} \rangle \text{'key'} \langle \text{HasKeyExpression} \rangle \langle \text{axiomAnnotations} \rangle$

$\langle \text{HasKeyExpression} \rangle ::= \text{'\{'} [ \langle \text{ObjectPropertyExpression} \rangle ( \text{'\,'} \langle \text{ObjectPropertyExpression} \rangle )^* ] \text{'\}' } \text{'\{' } [ \langle \text{DataPropertyExpression} \rangle ( \text{'\,'} \langle \text{DataPropertyExpression} \rangle )^* ] \text{'\}' }$

**Example 10.34.** Consider the ontology consisting of the following axioms.

*Each object is uniquely identified by its social security number.*

```
|| owl:Thing \key {\a:hasSSN}
```

*Peter's social security number is "123-45-6789".*

```
|| a:hasSSN(a:Peter, "123-45-6789")
```

*Peter Griffin's social security number is "123-45-6789".*

```
|| a:hasSSN(a:Peter_Griffin, "123-45-6789")
```

The first axiom makes *a:hasSSN* the key for instances of the *owl:Thing* class; thus, only one individual can have a particular value for *a:hasSSN*. Since the values of *a:hasSSN* are the same for the individuals *a:Peter* and *a:Peter\_Griffin*, these two individuals are equal – that is, this ontology entails the following assertion:

$\| a:Peter \ \backslash ieq \ a:Peter\_Griffin$

*One might expect the previous ontology to be inconsistent, since the  $a:hasSSN$  has the same value for two individuals  $a:Peter$  and  $a:Peter\_Griffin$ . However, OWL 2 does not make the unique name assumption, so  $a:Peter$  and  $a:Peter\_Griffin$  are not necessarily distinct individuals. If the ontology were extended with the following assertion, then it would indeed become inconsistent:*

$\| a:Peter \ \backslash idiff \ a:Peter\_Griffin$

## 10.6 Assertion

$\langle Assertion \rangle ::= \langle NonSequenceAssertion \rangle \mid \langle SequenceAssertion \rangle$

$\langle sourceIndividual \rangle ::= \langle Individual \rangle$

$\langle targetIndividual \rangle ::= \langle Individual \rangle$

$\langle targetValue \rangle ::= \langle Literal \rangle$

### 10.6.1 Non-sequence Assertion

$\langle NonSequenceAssertion \rangle ::= \langle SameIndividual \rangle \mid \langle DifferentIndividuals \rangle \mid \langle ClassAssertion \rangle$   
 $\mid \langle ObjectPropertyAssertion \rangle \mid \langle NegativeObjectPropertyAssertion \rangle \mid$   
 $\mid \langle DataPropertyAssertion \rangle \mid \langle NegativeDataPropertyAssertion \rangle$

### Individual Equality

$\langle SameIndividual \rangle ::= \langle Individual \rangle \ \backslash ieq \ \langle Individual \rangle \ \langle axiomAnnotations \rangle$

**Example 10.35.** *Consider the ontology consisting of the following assertion.*

Functional-Style Syntax – *Meg and Megan are the same objects.*

$\| a:Meg \ \backslash ieq \ a:Megan$

*Meg has a brother Stewie.*

$\| a:hasBrother(a:Meg, a:Stewie)$

*Since  $a:Meg$  and  $a:Megan$  are equal, one individual can always be replaced with the other one. Therefore, this ontology entails that  $a:Megan$  is connected by  $a:hasBrother$  with  $a:Stewie$  – that is, the ontology entails the following assertion:*

$\| a:hasBrother(a:Megan, a:Stewie)$

### Individual Inequality

$\langle \text{DifferentIndividuals} \rangle ::= \langle \text{Individual} \rangle \text{'\textbackslash idiff'} \langle \text{Individual} \rangle \langle \text{axiomAnnotations} \rangle$

### Class Assertions

$\langle \text{ClassAssertion} \rangle ::= \langle \text{ClassExpression} \rangle \text{'('} \langle \text{Individual} \rangle \text{'\textbackslash ')'} \langle \text{axiomAnnotations} \rangle$

**Example 10.36.** Consider the ontology consisting of the following assertion.

*Brian is a dog.*

`|| a:Dog(a:Brian)`

*Each dog is a mammal.*

`|| a:Dog \cisa a:Mammal`

The first axiom states that *a:Brian* is an instance of the class *a:Dog*. By the second axiom, each instance of *a:Dog* is an instance of *a:Mammal*. Therefore, this ontology entails that *a:Brian* is an instance of *a:Mammal* – that is, the ontology entails the following assertion:

`|| a:Mammal(a:Brian)`

### Positive Object Property Assertions

$\langle \text{ObjectPropertyAssertion} \rangle ::= \langle \text{ObjectPropertyExpression} \rangle \text{'('} \langle \text{sourceIndividual} \rangle \text{'\textbackslash ','} \langle \text{targetIndividual} \rangle \text{'\textbackslash ')'} \langle \text{axiomAnnotations} \rangle$

**Example 10.37.** Consider the ontology consisting of the following assertion.

*Brian is a dog of Peter.*

`|| a:hasDog(a:Peter, a:Brian)`

*Objects that have a dog are dog owners.*

`|| \oeexists{a:hasDog}{owl:Thing} \cisa a:DogOwner`

The first axiom states that *a:Peter* is connected by *a:hasDog* to *a:Brian*. By the second axiom, each individual connected by *a:hasDog* to an individual is an instance of *a:DogOwner*. Therefore, this ontology entails that *a:Peter* is an instance of *a:DogOwner* – that is, the ontology entails the following assertion:

`|| a:DogOwner(a:Peter)`

### Negative Object Property Assertions

$\langle \text{NegativeObjectPropertyAssertion} \rangle ::= '!' \langle \text{ObjectPropertyExpression} \rangle '(\langle \text{sourceIndividual} \rangle$   
 $'\langle \text{targetIndividual} \rangle)'$   $\langle \text{axiomAnnotations} \rangle$

**Example 10.38.** Consider the ontology consisting of the following assertion.

*Meg is not a son of Peter.*

```
|| !a:hasSon(a:Peter, a:Meg)
```

The ontology would become inconsistent if it were extended with the following assertion:

```
|| a:hasSon(a:Peter, a:Meg)
```

### Positive Data Property Assertions

$\langle \text{DataPropertyAssertion} \rangle ::= \langle \text{DataPropertyExpression} \rangle '(\langle \text{sourceIndividual} \rangle ' \langle$   
 $\text{targetValue} \rangle)'$   $\langle \text{axiomAnnotations} \rangle$

**Example 10.39.** Consider the ontology consisting of the following axiom.

Functional-Style Syntax – *Meg is seventeen years old.*

```
|| a:hasAge(a:Meg, "17"^^xsd:integer)
```

Objects that are older than 13 and younger than 19 (both inclusive) are teenagers.

```
|| \exists a:hasAge { (xsd:integer \drres {xsd:minInclusive  
"13"[xsd:integer], xsd:maxInclusive "19"[xsd:integer]}) }  
|cisa a:Teenager
```

The first axiom states that *a:Meg* is connected by *a:hasAge* to the literal “17” $[xsd:integer]$ . By the second axiom, each individual connected by *a:hasAge* to an integer between 13 and 19 is an instance of *a:Teenager*. Therefore, this ontology entails that *a:Meg* is an instance of *a:Teenager* – that is, the ontology entails the following assertion

```
|| a:Teenager(a:Meg)
```

### Negative Data Property Assertions

$\langle \text{NegativeDataPropertyAssertion} \rangle ::= '!' \langle \text{DataPropertyExpression} \rangle '(\langle \text{sourceIndividual} \rangle$   
 $'\langle \text{targetValue} \rangle)'$   $\langle \text{axiomAnnotations} \rangle$

**Example 10.40.** Consider the ontology consisting of the following axiom.

*Meg is not five years old.*

`|| !a:hasAge(a: Meg, "5"[xsd:integer])`

The ontology would become inconsistent if it were extended with the following assertion:

`|| a:hasAge(a: Meg, "5"[xsd:integer])`

### 10.6.2 Sequence Assertion

$\langle \text{SequenceAssertion} \rangle ::= \langle \text{SequenceSameIndividual} \rangle \mid \langle \text{SequenceDifferentIndividuals} \rangle$

#### Sequence Individual Equality

$\langle \text{SequenceSameIndividual} \rangle ::= \backslash \text{ialleq} \{ \langle \text{Individual} \rangle ( \langle \text{Individual} \rangle ) + \}$   
 $\langle \text{axiomAnnotations} \rangle$

**Example 10.41.** Example 10.35 can be rewritten with the sequence-style notation:

`|| \ialleq{a: Meg, a: Megan}`

#### Individual Inequality

$\langle \text{SequenceDifferentIndividuals} \rangle ::= \backslash \text{ialldiff} \{ \langle \text{Individual} \rangle ( \langle \text{Individual} \rangle ) + \}$   
 $\langle \text{axiomAnnotations} \rangle$

**Example 10.42.** Consider the ontology consisting of the following axioms.

*Peter is Meg's father.*

`|| a:fatherOf(a: Peter, a: Meg)`

*Peter is Chris's father.*

`|| a:fatherOf(a: Peter, a: Chris)`

*Peter is Stewie's father.*

`|| a:fatherOf(a: Peter, a: Stewie)`

*Peter, Meg, Chris, and Stewie are all different from each other.*

```
|| \invalidiff{a:Peter, a:Meg, a:Chris, a:Stewie}
```

*The last axiom in this example ontology axiomatizes the unique name assumption (but only for the four names in the axiom). If the ontology were extended with the following axiom stating that `a:fatherOf` is functional, then this axiom would imply that `a:Meg`, `a:Chris`, and `a:Stewie` are all equal, thus invalidating the unique name assumption and making the ontology inconsistent.*

```
|| a:fatherOf \ofunc
```

## 11 Sample Ontology

In this section a sample ontology is outlined.

```
% define base namespace for this ontology
\Ns <http://basenamespace.owl#>

% define additional custom namespaces
% to refer to concept/property/object defined in a given
  namespace use a prefix notation ns:name, e.g., owl:Thing,
  owl:Nothing
\Ns ns1: <http://www.namespace1.com/ns1#>
\Ns ns2: <http://www.namespace2.com/ns2#>

\begin{ontology}
% import ontologies
\import <http://www.firstontology.org/first.owl>
\import <http://www.firstontology.org/second.owl>

% now you can specify the axioms of the ontology
Person \cisa owl:Thing
Person \a{rdfs:label, "Person"}
Person \a{rdfs:comment, "This is a class for representing
  people"}
Person \cisa \candof{\d[>=1]{hasName}, \d[=1]{hasSurname},
  \dexists{hasSex}{Sex}, \d[=1]{hasAge}}

% if some already existing ontologies are required to be
  reused without cutting-and-pasting them into the current
  ontology, the input command can be used
\input{/input.txt}

% hasName, hasSurname, and hasAge are datatype properties
hasName \ddomain Person
hasName \drange xsd:string
hasName \a{rdfs:comment, "Person's name"}
hasSurname \ddomain Person
```

```

hasSurname \drange xsd:string
hasSurname \a{rdfs:comment, "Person's surname"}
hasAge \ddomain Person
hasAge \drange xsd:integer

% let us define another datatype property by enumerating
  possible values
hasHairColor \ddomain Person
hasHairColor \drange \drangeof{"Blonde", "Brown", "Red"}

% if we have another property hasLastName, we can define it
  as equal to the property hasSurname
hasLastName \deq hasSurname

% sex can be defined as a concept containing two objects,
  male and female
Sex \ceq \ooneof{M, F}

M \a{rdfs:label, "Male"}
F \a{rdfs:label, "Female"}

Father \cisa Person
Child \cisa (Person \cand \oexists{hasFather}{Father})

% if we want to reuse some concept defined in the other
  namespace
Father \cisa ns1:Father
ns1:isFatherOf \oinv hasFather

% property has father is functional
hasFather \ofunc

% let's populate the ontology with some individual data
Person(john)

% to express the fact that john and johny both refer to the
  same individual one can use the object equality construct
john \ieq johny

hasName(john, "John")
hasSurname(john, "Wild")
hasAge(john, "35"[xsd:integer])
hasSex(john, M)

Child(katty)
% to express the fact that john is different from katty one
  can use the object difference construct
katty \idiff john
hasName(katty, "Katty")
hasName(katty, "Wild")

```

```
hasSex(katty, F)
hasAge(katty, "3"[xsd:integer])

% John is Katty's father
hasFather(katty, john)

\end{ontology}
```

## References

- [1] M. Duerst and M. Suignard. RFC 3987: Internationalized Resource Identifiers (IRIs). RFC 3987 (Proposed Standard), see <http://www.ietf.org/rfc/rfc3987.txt>, January 2005.
- [2] A. Phillips and M. Davis. BCP 47 – Tags for Identifying Languages. BCP 47 Standard, see <http://www.rfc-editor.org/rfc/bcp/bcp47.txt>, September 2006.
- [3] Eric Prud'hommeaux and Andy Seaborne. Sparql query language for rdf. Latest version available as <http://www.w3.org/TR/rdf-sparql-query/>, January 2008.