

EE 281

CIRCUITS

DR. SWIFT

EE 281A SCHEDULE SPRING 2011

DATE	SECTION	TOPIC
Mon Jan 10	1.1	Introduction
	1.2	Circuits, Current and Voltage
Wed Jan 12	1.3	Power and Energy
	1.4	Kirchhoff's Current Law
Fri Jan 14	1.5	Kirchhoff's Voltage Law
	1.6	Circuit Elements
	1.7	Intro to Circuits
		Homework #1 due
Wed Jan 19	2.1	Resistance in Series and Parallel
	2.2	Networks with Series and Parallel Resistors
		Homework #2 due
Fri Jan 21	2.3	Voltage Divider and Current Divider
	2.4	Node Voltage Analysis
		Homework #3 due
Mon Jan 24	2.4	Node Voltage Analysis
		Homework #4 due
Wed Jan 26	2.5	Mesh Current Analysis
		Homework #5 due
Fri Jan 28	2.5	Mesh Current Analysis
Mon Jan 31		Review
		Homework #6 due
Wed Feb 2	TEST 1	
	Chapter 1	ALL SECTIONS
	Chapter 2	Sections 2.1, 2.2, 2.3, 2.4 and 2.5

DATE	SECTION	TOPIC
Fri Feb 4	2.6	Thevenin Circuit
Mon Feb 7	2.6	Thevenin and Norton Maximum Power Transfer Source Transformation
Wed Feb 9	2.7	Superposition Homework #7 due
Fri Feb 11	3.1 3.2	Capacitance Capacitance in Series and Parallel Homework #8 due
Mon Feb 14	3.4 3.5	Inductance Inductance in Series and Parallel
Wed Feb 16	3.7	Mutual Inductance Homework #9 due
Fri Feb 18	4.1	First-Order <i>RC</i> Circuits Homework #10 due
Mon Feb 21	4.2	<i>DC</i> Steady State
Wed Feb 23		Review Homework #11 due
Fri Feb 25	TEST 2 Chapter 2 Chapter 3 Chapter 4	Sections 2.6 and 2.7 Sections 3.1, 3.2, 3.4, 3.5, and 3.7 Sections 4.1 and 4.2

DATE	SECTION	TOPIC
Mon Feb 28	4.3	<i>RL</i> Circuits
	4.4	<i>RC</i> and <i>RL</i> with General Sources
Wed Mar 2	4.5	Second Order Circuits Homework #12 due
Fri Mar 4	4.5	Second Order Circuits
Mon Mar 7	5.1	Sinusoidal Currents and Voltages
	5.2	Phasors Homework #13 due
Wed Mar 9	5.3	Complex Impedance Homework #14 due
Mon Mar 14	5.4	Circuits with Phasors and Complex Impedance
Wed Mar 16	5.5	Power in AC Circuits Homework #15 due
Fri Mar 18		Review Homework #16 due
Mon Mar 21	TEST 3 Chapter 4 Chapter 5	Sections 4.3, 4.4, 4.5 Sections 5.1, 5.2, 5.3, 5.4, 5.5
Wed Mar 23	5.6	Thevenin and Norton Equivalent Circuits
Fri Mar 25	5.6	Thevenin and Norton Maximum Power Transfer
Mon Apr 4	5.7	Balanced 3 Phase
Wed Apr 6	5.7	Balanced 3 Phase Homework #17 due

DATE	SECTION	TOPIC
Fri Apr 8	15.5	Ideal Transformers Homework #18 due
Mon Apr 11	15.5	Ideal Transformers
Wed Apr 13	6.1	Transfer Function Homework #19 due
Fri Apr 15	6.2 6.5	Low pass Filter High pass Filter
Mon Apr 18		Review Homework #20 due
Wed Apr 20	TEST 4 Chapter 5 Chapter 6 Chapter 15	Sections 5.6, 5.7 Sections 6.1, 6.2, 6.5 Section 15.5
Fri Apr 22		Diodes
Mon Apr 25		Diode Circuits
Wed Apr 27		Review for Final Exam Homework #21 due
Fri Apr 29		Review for Final Exam
Wed May 4		FINAL EXAMINATION 4:00 PM - 6:00 PM

EE 281A
SYLLABUS
Spring 2011

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INSTRUCTOR: Dr. Theresa Swift

TEXT: ELECTRICAL ENGINEERING: Principles and Application, 5th edition,
by Allen R. Hambley (Prentice Hall)

PREREQUISITE: Math 204 or Math 229 and Physics 24

OFFICE: 217 EECH
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Email: thswift@mst.edu
Web Site: <http://blackboard.mst.edu>
Office Hours: 1:30-3:00 pm M/W/F and by appointment

GRADING SCALE:	Activity	Points
	Hour Exams	400
not from textbook	Homework	100
	Final Exam	200
	Total Points	700

Letter grades will be assigned by the following scale unless the overall class performance indicates that the scale is too high, in which case the scale may be lowered. However, an "F" grade will be assigned if the final homework percentage is less than 50% or the final exam percentage is less than 50%, regardless of the overall percentage.

A	90.00 - 100.00 %
B	80.00 - 90.00 %
C	70.00 - 80.00 %
D	60.00 - 70.00 %
F	0.00 - 60.00 %

HOMEWORK: Complete the problems and turn them in at the *beginning* of the period on the day that they are due. Homework that is not turned in at the *beginning* of class will not be accepted. If you will be absent, turn the homework in to the instructor early or have someone turn it in for you in class. You may also e-mail the solutions to the instructor. If you cannot scan the solutions, e-mail the answers and then turn in the solutions when you return. Write your name and assignment number on the upper part of the pages that you turn in. Staple all sheets together in the upper left-hand corner. Be neat and show all work. Draw any needed diagrams on your paper. Place answer on the answer space or draw a box around your final answer. Be sure to include units on all final answers. Problems that are not in the book will usually be assigned. You may use both sides of the paper. You may want to keep your solutions to the problems and compare them with the posted solutions. At times the homework may not be returned before the test over the material. Some class time may be used to answer questions about the problems. Each set will be worth 20 points and the lowest *three* sets will not be used in determining the final grade. Students with a homework score of less than **50%** will receive a grade of 'F' for the course. Solutions will be posted on Blackboard.

EXAMS: Four exams worth 100 points each will be given. Late makeup exams will not be given. Early makeup exams will be given before the scheduled exam. You must notify me if you plan to take the makeup exam. If you miss the exam and the makeup exam, your test score will be your average score on the final exam. This will not be done more than one time. If your score on the final exam is higher than your low score on a regular exam, you may replace the low score with the final exam score.

EXAM DATES: The regular exam dates are as follows:

Wednesday, February 2, 2011

Friday, February 25, 2011

Monday, March 21, 2011

Wednesday, April 20, 2011

FINAL EXAM: A final exam that covers all material studied during the semester will be given on

Wednesday, May 4, 2011

4:00 PM - 6:00 PM

You will be required to have a score of at least **50%** on the final exam to pass the course.

ATTENDANCE: You will be expected to be in class every day. Students with excessive absences usually do not do well in the course and I reserve the right to drop students for poor attendance.

HONESTY POLICY: It will be assumed that all students will do their own work. Any copying from another student or using books or notes during a quiz or exam will result in a minimum of a grade of 'F' recorded for the assignment. The academic regulations for Missouri University of Science and Technology are available at <http://registrar.mst.edu/academicregs/index.html>. Calculators will not be cleared before tests and quizzes, so you may bring any helpful material to class on your calculator. If any problem with the class can't be worked out with your instructor, contact the Undergraduate Coordinator or Department Chairman to get assistance in working them out.

SPECIAL SERVICES: If you need special services (extra time) for tests in the course, notify the instructor.

EMERGENCY SAFETY EXITS: Egress maps for all areas of instruction on campus are available on the web at: <http://registrar.mst.edu/links/egress.html>. Students are encouraged to review this site and be aware of the emergency exit signs near their classrooms.

UPDATE: This syllabus was updated as of January 4, 2011.



Exam Practice: See other PDF
"EE281 Practice.pdf"

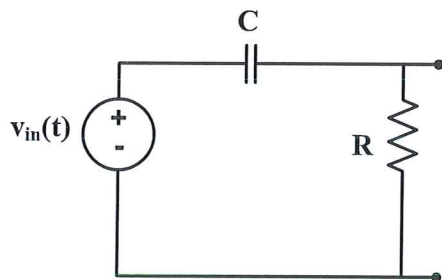
Homework

EE 281 A

Homework #20 -- 20 points

Due: Monday, April 18, 2011

1. For the following highpass filter:

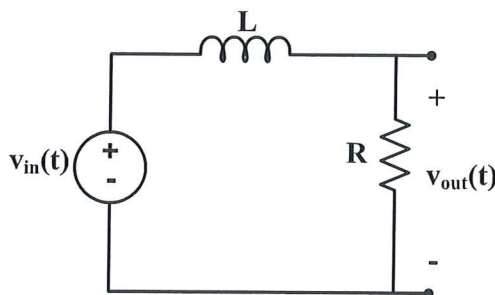


a) $H(j\omega) = \frac{j\omega RC}{1 + j\omega RC}$ ✓

b) $\omega_{\text{cutoff}} = \frac{1}{RC}$ $1000 \text{ rad/sec} = \frac{1}{500C}$
 $C = 2 \mu\text{F}$ show all work (-1)
 c) on back

- Find the transfer function $H(j\omega)$, in terms of ω , R , and C . (3 pts)
- If $R=500 \Omega$, find the value of C needed for a $\frac{1}{2}$ -power frequency of 1000 rad/sec . (3 pts)
- If $v_{\text{in}} = 50\cos(50t) + 50\cos(4000t) \text{ V}$, find $v_{\text{out}}(t)$ for the values of R and C from the previous part. (5 pts)

2. For the following filter, determine:



a) $H(j\omega) = \frac{1}{1 + j\omega RL}$ -3
 b) $\omega_{\text{cutoff}} = \frac{R}{L} = \frac{2000}{.001 \times 10^{-3}} = 2 \times 10^9 \text{ rad/sec}$
 show all work (-1)

- The transfer function $H(j\omega)$, in terms of ω , R , and L . (4 pts)
- The $\frac{1}{2}$ -power frequency (in rad/sec) if $R=2000 \Omega$ and $L=0.001 \text{ mH}$. (3 pts)
- Is this a lowpass or highpass filter? (2 pts)

low pass filter ✓

15/20

$$\frac{V_{out}}{V_{in}} = \frac{j\omega RC}{1 + j\omega RC}$$

$$V_{in} = \underbrace{50 \cos 50t}_a + \underbrace{50 \cos(4000t)}_b$$

a) $\omega = 50 \text{ rad/sec}$ $V_{in_1} = 50 \angle 0^\circ$ $V_{out} = H(j\omega) V_{in}$

$$V_{out_1} = 50 \angle 0^\circ \times \left(\frac{j(50)(500)(2 \times 10^{-6})}{1 + j(50)(500)(2 \times 10^{-6})} \right) \Rightarrow 2.50 \angle 87.1^\circ$$

b)

$$V_{out_2} = 4000 \angle 0^\circ \times \left(\frac{j(4000)(500)(2 \times 10^{-6})}{1 + j(4000)(500)(2 \times 10^{-6})} \right) \Rightarrow 48.5 \angle 14.0^\circ$$

c)

$$V_{out}(t) = 2.5 \cos(50t + 87.1^\circ) + 48.5 \cos(4000t + 14^\circ)$$

✓ (-1/2)

EE281 Homework #21

1. (a) Reverse biased

$$I = I_0 = -0.01 \text{ mA}$$

$$-V_s + V + RI = 0$$

$$24 + V + 1000(-0.01 \times 10^{-3}) = 0$$

$$V = -24 + 0.01 = -23.99 \text{ V}$$

← Note $V > V_{br}$ so not in breakdown. Assumption that $I = I_0$ is correct.

(b) $-V_s + V + RI = 0$

$$-V_s + 0.7 + 1000(1.05 \times 10^{-3}) = 0 \quad \leftarrow \text{Forward biased} \rightarrow V = 0.7 \text{ V.}$$

$$V_s = 0.7 + 1.05 = 1.75 \text{ V}$$

2. (a)

$$\frac{V_0}{1000} + \frac{V_0 - V_s}{1000} - I = 0$$

$$V_s = 12 \text{ V assume not in breakdown, so } I = I_0 = -0.01 \text{ mA}$$

$$\frac{1}{500} V_0 = I + \frac{V_s}{1000}$$

$$V_0 = 500I + \frac{V_s}{2} = 500(-0.01 \times 10^{-3}) + \frac{12}{2} = 6.005 \text{ V}$$

$$V_{diode} = -V_0 = 6.005 \text{ V} > V_{br} \text{ so assumption was correct.}$$

(b)

$$V_0 = 500I + \frac{V_s}{2}$$

$$\text{Assume breakdown so } V_0 = -9.1 \text{ V} \rightarrow V_0 = 9.1 \text{ V.}$$

check current to see if assumption is correct.

$$9.1 = 500I + \frac{42}{2}$$

$$I = \frac{9.1 - 21}{500} = -0.0238 \text{ A} = -23.8 \text{ mA} < I_0 \text{ so}$$

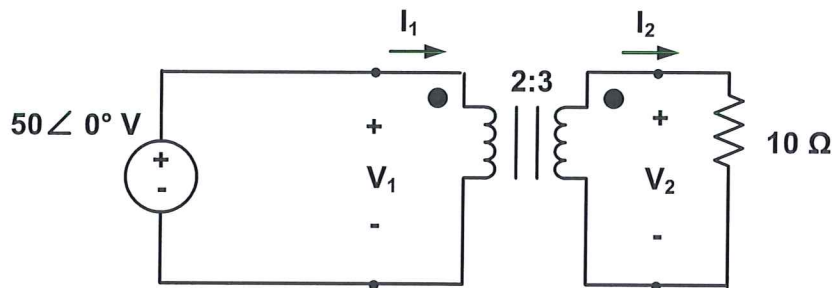
assumption is correct.

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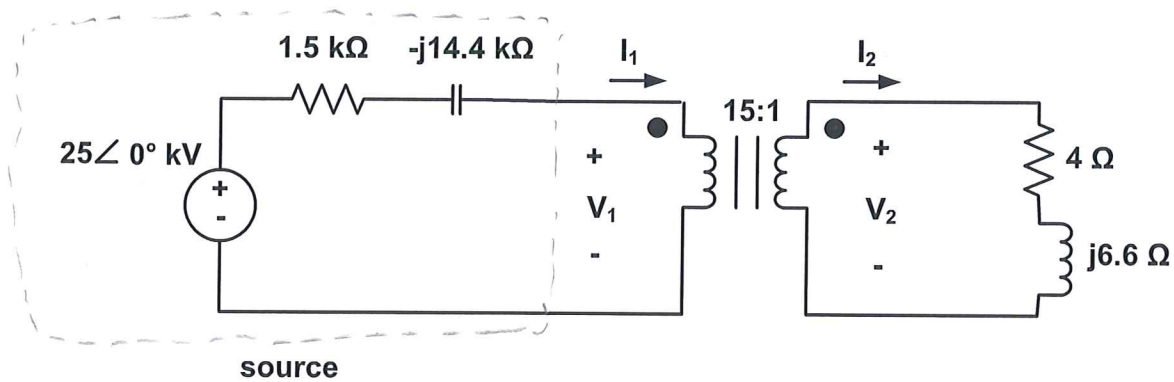
Homework #19 -- 20 points

Due: Wednesday, April 13, 2011

1. Find the effective turns ratio in of ideal transformer needed to step down a $120\text{-V}_{\text{rms}}$ voltage to 50 V_{rms} . (3 pts)
2. Find the effective turns ratio of an ideal transformer needed to step up a 2-A_{rms} current to 10 A_{rms} . (3 pts)
3. Find the average power delivered to the $10\text{-}\Omega$ load. (4 pts)



4. Use impedance transformations (reflected impedances) to find the impedance seen by the source in the following circuit. Then find the phasors V_1 , V_2 , I_1 , and I_2 (10 pts)

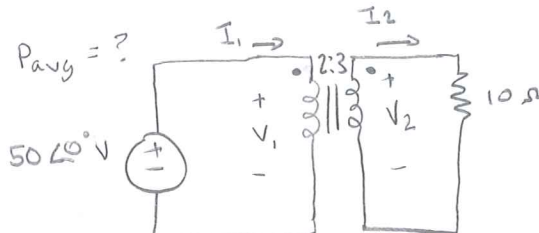


Work on back side →

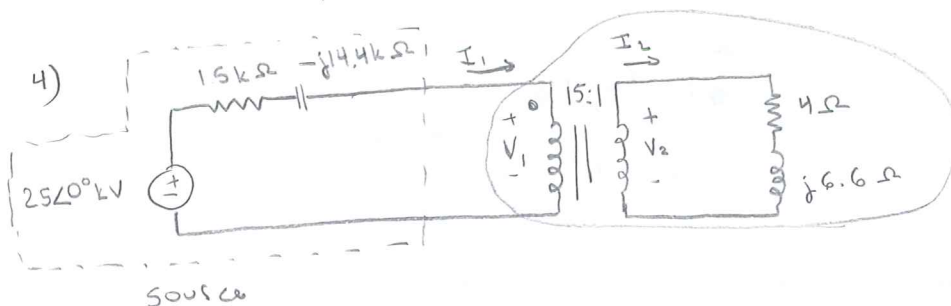
20/20

1) turns ratio $120V_{rms} \rightarrow 50V_{rms}$ $V_2 = \frac{N_2}{N_1} V_1$ $\frac{50}{120} = \frac{N_2}{N_1}$ $N_1:N_2 = 12:5$ ✓

2) $2A_{rms} \rightarrow 10A_{rms}$ $\frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{10A_{rms}}{2A_{rms}}$ $N_1:N_2 = 5:1$ ✓

3) $P_{avg} = ?$ $N_1:N_2 = 2:3$

 $V_1 = 50\angle 0^\circ V$ $V_2 = V_1 \frac{N_2}{N_1}$ $V_2 = 75\angle 0^\circ$
 $I_2 = \frac{V_2}{10} = 7.5A_{rms}$

$P_{avg} = \frac{V_{rms}^2}{R} = \frac{75^2}{10} = 562.5 W$ ✓



$Z_L' = \left(\frac{N_1}{N_2}\right)^2 Z_L$

$Z_L = 4 + j6.6 \Omega$

$Z_L = 7.718 \angle 58.8^\circ$

$Z_L' = 1736 \angle 58.8^\circ \Omega$ ✓

$I_1 = \frac{\text{source}}{\text{imp.}} = \frac{25\angle 0^\circ}{(15 - j14.4) + (1.736 \angle 58.8^\circ)}$

$I_1 = 1.90 \angle 79.5^\circ A_{rms}$ ✓

$I_2 = \frac{N_1}{N_2} I_1 = 15 I_1 \Rightarrow I_2 = 28.5 \angle 79.5^\circ A_{rms}$ ✓

$V_1 = I_1 Z_L' = 1.9 \angle 79.5^\circ \cdot 1736 \angle 58.8^\circ =$

$V_2 = I_2 Z_L$
 $= 28.5 \angle 79.5^\circ \cdot 7.718 \angle 58.8^\circ$

$V_2 = 220 \angle 138^\circ V_{rms}$ ✓

$V_1 = \frac{N_1}{N_2} V_2$

$V_1 = 15 V_2$

$V_1 = 3.30 \angle 138^\circ kV_{rms}$ ✓

10/20

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$$Z_{\Delta} = \frac{50(28.3j)}{50 + (28.3j)}$$

$$\omega = 2\pi f$$

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Homework #18 -- 20 points

Due: Friday, April 8, 2011

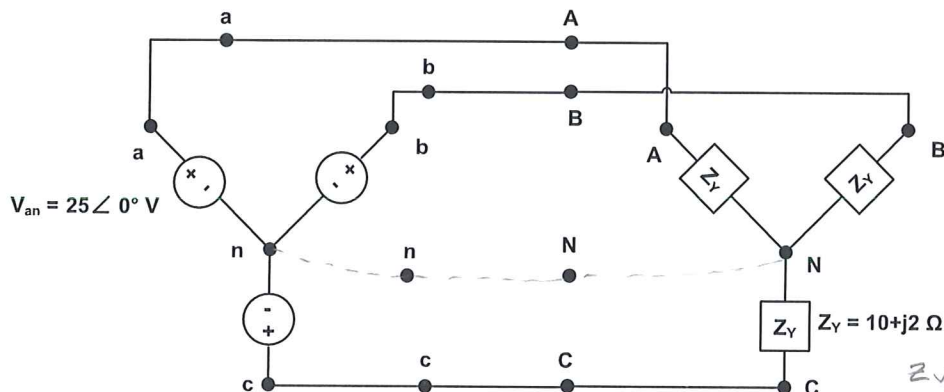
1. Each phase of a Δ -connected load consists of a $50\text{-}\Omega$ resistance in parallel with a 75-mH inductance. Find the impedance of an equivalent Y-connected load. Assume 60-Hz operation. = (4 pts)

$$Z_{\Delta} = 3Z_Y$$

$$Z_Y = 8.21 \angle 60.5^\circ$$

units (-1/2)

2. For the following balanced three-phase system, assume a positive phase sequence and find:



$$Z_Y = 41.67 \angle 33.5^\circ$$

- The line-to-neutral voltages, V_{BN} and V_{CN} , at the load. (3 pts)
- The line-to-line voltages, V_{ab} , V_{bc} , and V_{ca} . (5pts)
- The line currents, I_{aA} , I_{bB} , and I_{cC} . (5 pts)
- If, instead, the system had a negative phase sequence, find V_{BN} and V_{CN} . (3pts)

a) $V_{an} = 25 \angle 0^\circ \text{ Vrms}$
 $V_{BN} = 25 \angle -120^\circ \text{ Vrms}$
 $V_{CN} = 25 \angle 120^\circ \text{ Vrms}$

$$V_{ab} = V_{an} - V_{bn} = 43.3 \angle 30^\circ$$

$$V_{bc} = 43.3 \angle -90^\circ$$

$$V_{ca} = 43.3 \angle 150^\circ$$

units (-1)

c) $I_{aA} = \frac{V_{an}}{Z_Y} = \frac{25 \angle 0^\circ}{10 + j2} = 2.45 \angle -11.3^\circ \text{ Arms}$

$$I_{bB} = 2.45 \angle -131^\circ \text{ Arms}$$

$$I_{cC} = 2.45 \angle 109^\circ \text{ Arms}$$

1) Negative Phase sequence Switches BN & CN

$$V_{BN,2} = 25 \angle 120^\circ \text{ Vrms}$$

$$V_{CN,2} = 25 \angle -120^\circ \text{ Vrms}$$

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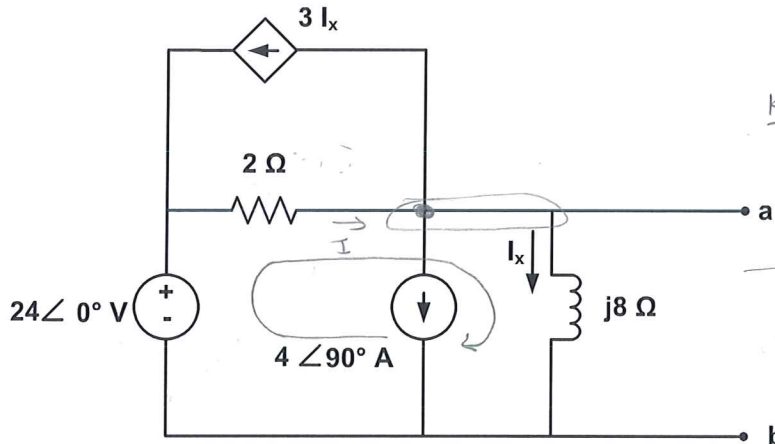
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Homework #17 -- 20 points

Due: Wednesday, April 6, 2011c

1. Find the Thévenin equivalent of the following circuit. (8 pts)

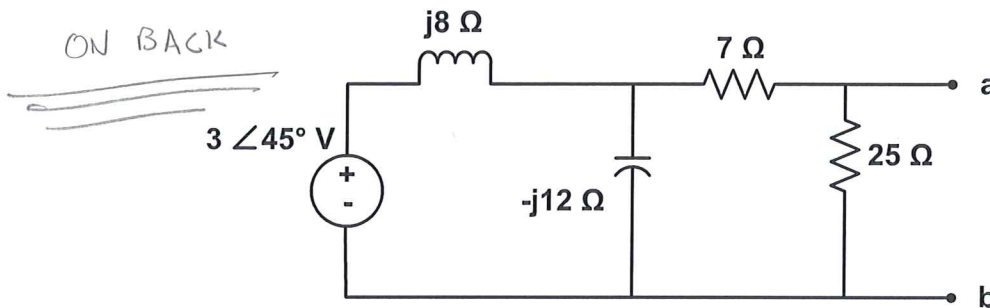


KVL
 $-24\angle 0^\circ + 2I + 8jI_x = 0$
 $2I + 8jI_x = 24 \quad (1)$

KCL
 $-I - 4\angle 90^\circ + I_x + 3I_x = 0$
 $4I_x - I = -j4 \quad (2)$

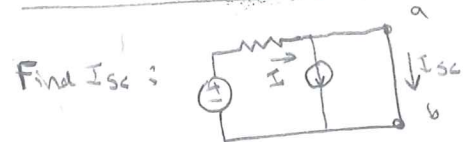
$8I_x + 2I = 24$
 $8jI_x - 2I = -j8$
 $(8+8j)I_x = 24 - j8$
 $I_x = \frac{(24 - j8)}{(8+8j)} = 1 - j2$

2. Find the Thévenin and Norton equivalents of the following circuit. (8 pts)



$V_{oc} = I_x (8j) \quad \checkmark$

$V_{oc} = j8 + 16 \text{ V} \quad \checkmark$



$-24\angle 0^\circ + 2I = 0$

$I = 12 \text{ A} \quad \checkmark$

$I_{sc} = I - 4\angle 90^\circ$

$I_{sc} = 12 - j4 \text{ A}$

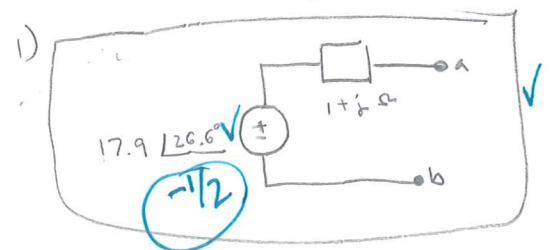
3. What load should be attached between the terminals a and b in the circuit in problem 2 to maximize the real power delivered to the load? (4 pts)

Load should be

$Z_L = Z_s^*$, complex conjugate

$15.6 \angle 36.8^\circ \rightarrow 15.6 \angle -36.8^\circ \quad \checkmark$
 $-j$

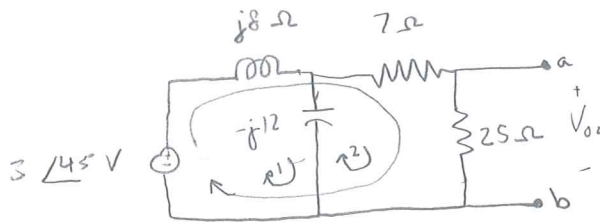
$Z_{th} = \frac{V_{oc}}{I_{sc}} = \frac{j8 + 16}{12 - j4} = 1 + j \Omega \quad \checkmark$



17/20

#2

$$V = IR$$



KVL $-3\angle 45^\circ +$

M1 $-3\angle 45^\circ + I_1 j8 - j12(I_1 - I_2) = 0$
 $I_1(-4j) + I_2(12j)$

$$I_1 = .641 \angle 77.6^\circ$$

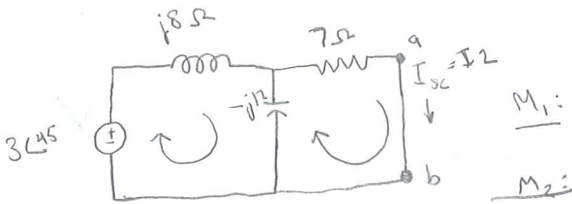
$$I_2 = .225 \angle 8.13^\circ$$

M2 $7I_2 + 25I_2 + (I_2 - I_1)j12 = 0$
 $I_1(12j) + I_2(32 - 12j)$

$$V_{oc} = 25\Omega(I_2) = \underline{5.625 \angle 8.13^\circ}$$

$$Z_{Th} = \frac{V_{oc}}{I_{Th}} = \frac{5.63 \angle 8.13^\circ}{.36 \angle -28.7^\circ} = 15.6 \angle 36.8^\circ$$

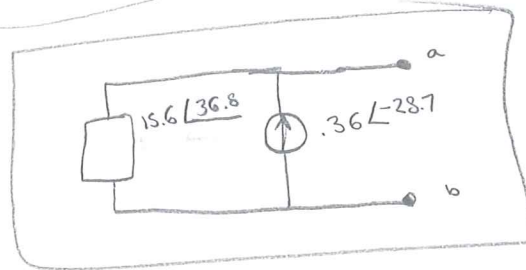
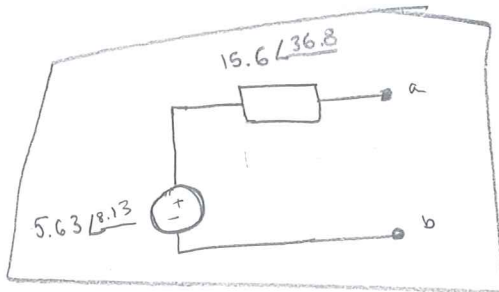
I_{sc}



$$I_1(-4j) + I_2(12j) = 3\angle 45^\circ$$

$$7I_2 + -j12(I_2 - I_1) = 0$$

$$I_2 = 0.36 \angle -28.7^\circ$$



units $\ominus$
 labels $\ominus$

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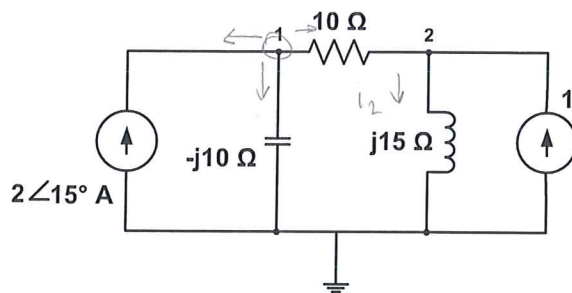
EE 281 A

Homework #16 -- 20 points

Due: Friday, March 18, 2011

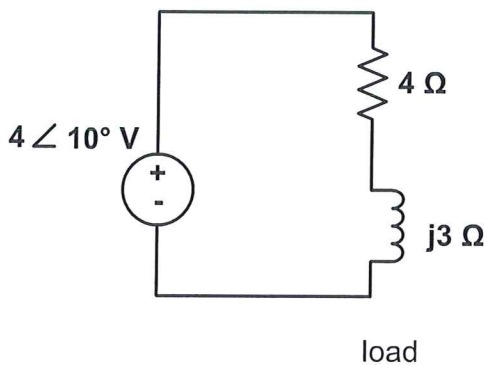
$$\frac{1}{j} = -j$$

1. Set up the node voltage equations in matrix form for the following circuit. (5 pts) $V = 12$



$$\begin{aligned} N_1] \quad & -2\angle 15^\circ + \frac{V_1}{-j10} + \frac{V_1 - V_2}{10} = 0 \\ & -2\angle 15^\circ + V_1 \left(\frac{1}{-j10} + \frac{1}{10} \right) - V_2 \frac{1}{10} = 0 \\ N_2] \quad & -1\angle 35^\circ + \frac{V_2}{j15} + \frac{V_2 - V_1}{10} = 0 \\ & -1\angle 35^\circ + V_2 \left(\frac{1}{j15} + \frac{1}{10} \right) - V_1 \left(\frac{1}{10} \right) = 0 \end{aligned}$$

2. Given the following circuit, find:



$$-\frac{1}{j10} + \frac{1}{10} = \frac{j}{10} + \frac{1}{10}$$

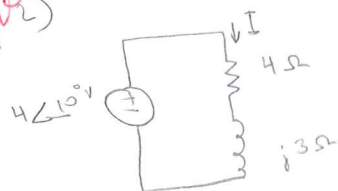
$$\frac{1}{j15} + \frac{1}{10} = -j\left(\frac{1}{15}\right) + \frac{1}{10}$$

$$\begin{bmatrix} \frac{j}{10} + \frac{1}{10} & -\frac{1}{10} \\ -\frac{1}{10} & -j\frac{1}{15} + \frac{1}{10} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2\angle 15^\circ \\ 1\angle 35^\circ \end{bmatrix}$$

- The power factor of the load (including whether it is leading or lagging). (3 pts)
- The average power absorbed by the load. (3 pts)
- The reactive power absorbed by the load. (3 pts)
- The apparent power absorbed by the load. (3 pts)
- Sketch the power triangle for the load. (3 pts)

on back

14/20



$$Z = 4 + j3 \Omega = 5 \angle 36.87^\circ \Omega$$

$$I = \frac{V}{Z} = \frac{4 \angle 10^\circ}{4 + j3} = 0.8 \angle -26.87^\circ \text{ A}$$

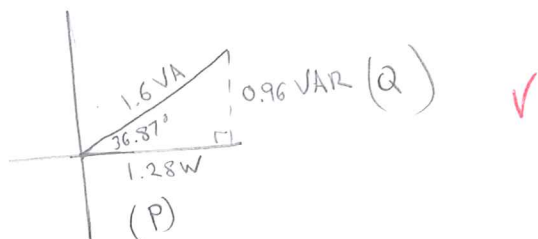
a) $P_{\text{factor}} = \frac{1}{2} V_m I_m \cos \theta = \frac{1}{2} (4)(.8) \cos(36.87) = \boxed{1.28 \text{ W}}$ ✓

b) $Q = \frac{1}{2} V_m I_m \sin \theta = \frac{1}{2} (4)(.8) \sin 36.87 = \boxed{0.96 \text{ VAR}}$ ✓

c) $AP = \frac{P}{\text{PF}} = \frac{1.28}{\cos(36.87)} = \boxed{1.6 \text{ VA}}$ ✓

d) $\text{PF} = \cos \theta = \boxed{0.8 \text{ lagging}}$ ✓

e)



David Maloney

inductor $Z_L = j\omega L$

EE281 A

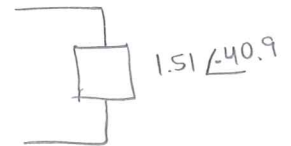
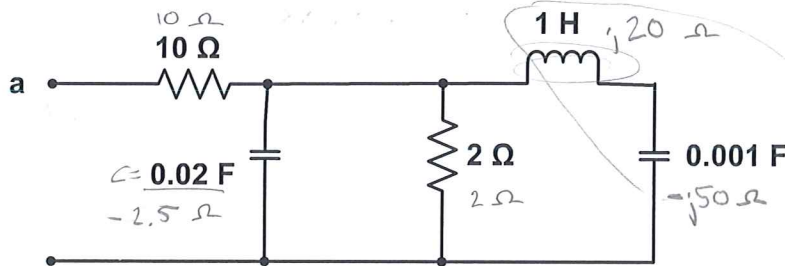
Homework #15 -- 20 points

Due: Wednesday, March 16, 2011

$\frac{1}{j} = -j$

capacitor $Z = \frac{1}{Cj\omega} = \frac{1}{.02j} \Omega$

1. Write the following as a single impedance. Assume that $\omega = 20$ rad/sec. (6 pts)



work is on back

1. Evaluate each of the following. Write your answers in rectangular and in polar form. (8 pts)

✓ a. $\frac{(4\angle 45^\circ)(2e^{j60^\circ})}{1-j} = \frac{(4\angle 45^\circ)(2\angle 60^\circ)}{(1\angle -45^\circ)}$

$8\angle 150$

$-6.93 + 4j$

-2

✓ b. $\frac{2\angle 45^\circ + 5\angle 120^\circ}{(10\angle 30^\circ)(1\angle -45^\circ)}$

$0.584\angle 115.7$

$-0.254 + .527j$

✓

✓ c. $\frac{10+j10}{2e^{j\pi}} + e^{j30^\circ} = \frac{\sqrt{200}\angle 45^\circ}{2\angle 180^\circ} + 1\angle 30^\circ$

$6.11\angle -133$

$-4.134 - 4.5j$

✓

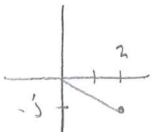
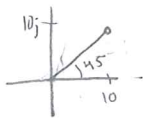
✓ d. $e^{j45^\circ} + 2 - j$

$2.73\angle -5.74$

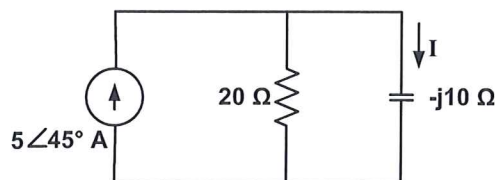
$2.72 - .273j$

✓

$= 1\angle 45^\circ + 2\angle -90^\circ$



- ✓ 2. Find the steady-state current $i(t)$ in the following circuit if $\omega = 100$ rad/sec. Hint: Find the phasor current I first then convert it back to a cosine. (6 pts)



$I = \frac{-j10}{20-j10} (5\angle 45^\circ)$

✓

$= \frac{(10\angle -90^\circ)}{(22.36\angle 26.57^\circ)} (5\angle 45^\circ)$

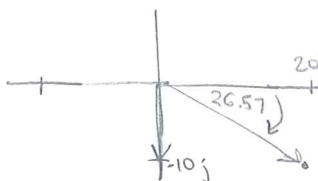
$= 2.236\angle -71.6$

-1

$= 2.24 (\cos(-71.6) + j\sin(-71.6))$

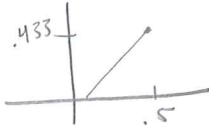
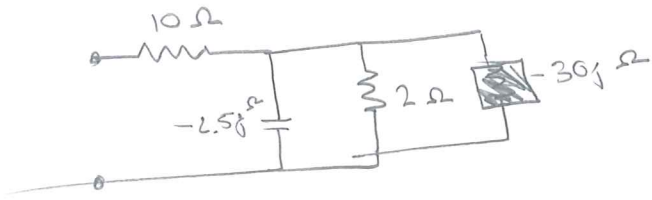
$i(t) = 2.24 \cos(100t - 71.6)$

unit
-1/2



15/20

#1



$$= .6614 \angle 40.9$$

$$\frac{1}{\frac{1}{-2.5j} + \frac{1}{2} + \frac{1}{-30j}}$$

$$\frac{1}{j} = -j$$

$$= (0.4j + \frac{1}{2} + .0333j)^{-1}$$

$$= \frac{1}{(\frac{1}{2} + 0.433j)}$$

$$= \frac{1}{.6614 \angle 40.9}$$

$$= 1.51 \angle -40.9$$

-1 unit -1/2

David Malawey

EE281 A

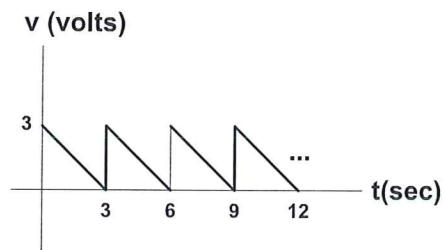
Homework #14 -- 20 points

Monday March 14
Due: Wednesday, March 9, 2011

1. Find the RMS value of each of the following signals. (4 pts each)

a. $i(t) = 4 \sin(25t - 45^\circ) \text{ A}$

b.

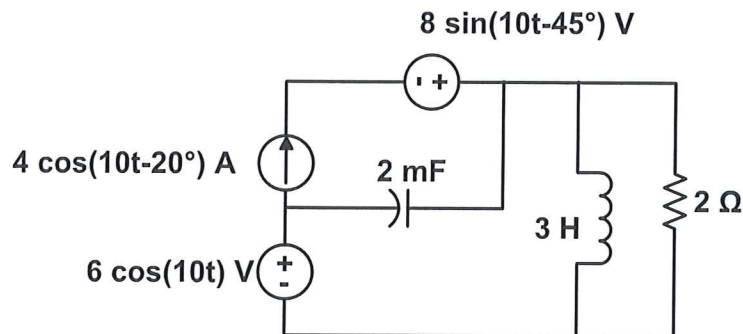


2. Write each of the following signals as phasors. (3 pts each)

a. $v(t) = 11 \cos(10t + 55^\circ) \text{ V}$

b. $i(t) = 9.1 \sin(8t + 10^\circ) \text{ A}$

3. Redraw the following circuit in the frequency domain (in terms of phasors and impedances). (6 pts)



David Maloney
Homework 14

~~For top of page~~

1) $i(t) = 4 \sin(25t - 45^\circ) \text{ A}$

a) $I_{\text{RMS}} = \frac{4}{\sqrt{2}} \text{ Arms}$ ✓ ✓

b) amplitude = 3 $V_{\text{rms}} = \frac{16}{\sqrt{3}}$
- sawtooth -
 $V = (3 - t)$ repeated for each period

$V_{\text{rms}} = \sqrt{3} V_{\text{rms}}$ ✓
(more work on back)

2) Write as phasor

a) $11 \cos(10t + 55^\circ) \text{ V}$

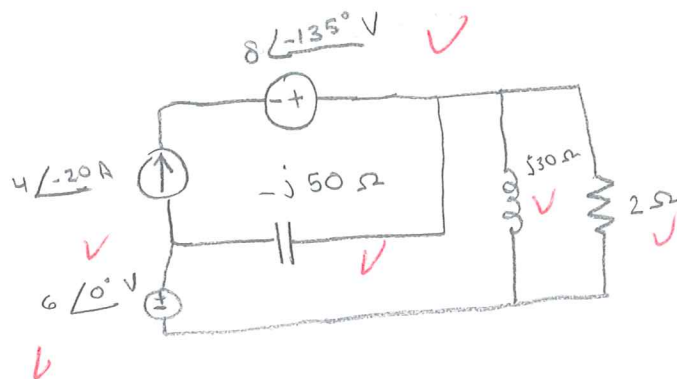
$V_m = 11 \quad \omega = 10 \quad \theta = 55^\circ$

$\underline{V} = 11 \angle 55^\circ \text{ V}$ ✓

b) $i(t) = 9.1 \sin(8t + 10^\circ) \text{ A}$
 $= 9.1 \cos(8t + 80^\circ) \text{ A}$

$\underline{i} = 9.1 \angle 80^\circ \text{ A}$

3)



$\frac{1}{j\omega C} = -j \frac{1}{(2 \times 10^{-3})(10)} = -j50$

$j\omega L = j(10)(3) = j30$

20/20

$$V_{\text{rms}} = \sqrt{\frac{1}{3} \int_0^3 (3-t)^2 dt} = \sqrt{\frac{1}{3} \left[-\frac{1}{3} (3-t)^3 \right]_0^3} = \sqrt{-\frac{1}{9} (3^3 - 0^3)} \\ = \sqrt{-\frac{1}{9} (-27)} = \sqrt{\frac{27}{9}} = \sqrt{3} \text{ V}$$

David Maloney

#13 HW

$$1. a) -i_s + \frac{v}{6} + \frac{1}{L} \int_{-\infty}^t v dx + \frac{1}{13} \frac{dv}{dt} = 0 \quad i_s = 250e^{-2t} A$$

$$\frac{d^2v}{dt^2} + \frac{13}{6} \frac{dv}{dt} + 13v = -500e^{-2t}$$

$$\frac{di_s}{dt} = -500e^{-2t} A$$

$$b) \begin{cases} i(0^+) = 0 \\ v(0^+) = 0 \end{cases}$$

$$\frac{d^2v}{dt^2} + \frac{13}{6} \frac{dv}{dt} + 13v = 0$$

$$s^2 + \frac{13}{6}s + 13 = 0$$

$$s = \frac{-\frac{13}{6} \pm \sqrt{\left(\frac{13}{6}\right)^2 - 4(13)}}{2}$$

c)

$$v_h(t) = k_1 e^{-1.08t} \cos(3.45t) + k_2 e^{-1.08t} \sin(3.45t)$$

d)

$$v_p(t) = Ae^{-2t}$$

$$v_p'(t) = -2Ae^{-2t}$$

$$v_p''(t) = 4Ae^{-2t}$$

$$4A + \frac{13}{6}(-2A) + 13A = -500$$

$$A = \frac{-500}{\left(4 - \frac{13}{3} + 13\right)} = -39.46$$

$$v_p(t) = -39.467e^{-2t}$$

$$e) v(t) = k_1 e^{-1.08t} \cos(3.45t) + k_2 e^{-1.08t} \sin(3.45t) - 39.47e^{-2t}$$

$$v(0^+) = 0 = k_1 - 39.47 \Rightarrow k_1 = 39.47$$

$$i(0) = 0 \Rightarrow \frac{dv}{dt} = 0$$

$$v'(t) = k_1 e^{-1.08t} (-3.45 \sin(3.45t)) - 1.08 k_1 e^{-1.08t} \cos(3.45t) + k_2 e^{-1.08t} (3.45 \cos(3.45t)) - 1.08 k_2 e^{-1.08t} \sin(3.45t) + 78.94 e^{-2t}$$

$$v'(0) = -1.08 k_1 + 3.45 k_2 + 78.94 \Rightarrow k_2 = \frac{1.08 \times 39.47 - 78.94}{3.45} = -10.56$$

$$v(t) = 39.47e^{-1.08t} \cos(3.45t) - 10.56e^{-1.08t} \sin(3.45t) - 39.47e^{-2t} \quad \checkmark$$

f) underdamped

EE281 Hwk 13 Solutions, 2010

1. a) $-i_s + \frac{V}{6} + \frac{1}{L} \int_{-\infty}^t v d\lambda + \frac{1}{13} \frac{dv}{dt} = 0$

$i_s = 250e^{-2t} A$
 $\frac{di_s}{dt} = -500e^{-2t} A$

$$\frac{d^2v}{dt^2} + \frac{13}{6} \frac{dv}{dt} + 13v = -500e^{-2t}$$

b) $i(0^+) = 0$
 $v(0^+) = 0$

c) $\frac{d^2v}{dt^2} + \frac{13}{6} \frac{dv}{dt} + 13v = 0$

$A^2 + \frac{13}{6}A + 13 = 0$

$\Delta = \frac{-\frac{13}{6} \pm \sqrt{(\frac{13}{6})^2 - 4(13)}}{2} = -1.08 \pm j 3.44$

$$v_h(t) = K_1 e^{-1.08t} \cos(3.44t) + K_2 e^{-1.08t} \sin(3.44t)$$

d) $v_p(t) = Ae^{-2t}$
 $v_p' = -2Ae^{-2t}$
 $v_p'' = 4Ae^{-2t}$

$4Ae^{-2t} + \frac{13}{6}(-2Ae^{-2t}) + 13(Ae^{-2t}) = -500e^{-2t}$

$4A - \frac{13}{3}A + 13A = -500$

$A = -500 / (4 - \frac{13}{3} + 13) = -39.47$

$$v_p(t) = -39.47 e^{-2t}$$

e) $v(t) = K_1 e^{-1.08t} \cos(3.44t) + K_2 e^{-1.08t} \sin(3.44t) - 39.47 e^{-2t}$
 $v(0^+) = 0 = K_1 - 39.47$

$K_1 = 39.47$

$i(0) = 0 \rightarrow \frac{dv}{dt} = 0$

$v'(t) = K_1 e^{-1.08t} (-3.44 \sin(3.44t)) - 1.08 K_1 e^{-1.08t} \cos(3.44t) + K_2 e^{-1.08t} (3.44 \cos(3.44t)) - 1.08 K_2 e^{-1.08t} \sin(3.44t) + 78.94 e^{-2t}$

$v'(0) = 0 = -1.08 K_1 + 3.44 K_2 + 78.94$

$K_2 = (1.08(39.47) - 78.94) / 3.44 = -10.56$

$$v(t) = 39.47 e^{-1.08t} \cos(3.44t) - 10.56 e^{-1.08t} \sin(3.44t) - 39.47 e^{-2t} V$$

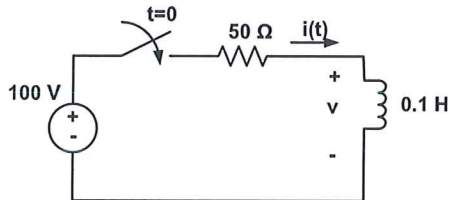
f) underdamped

EE281 A

Homework #12 -- 20 points

Due: Wednesday, March 2, 2011

1. Consider the circuit shown below.



b)

$$\frac{di}{dt} + \frac{R}{L}i = \frac{V_s}{L}$$

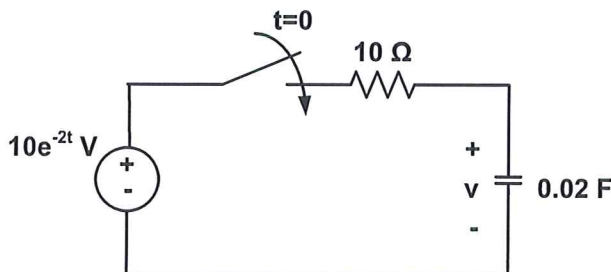
homogeneous $\frac{di}{dt} + \frac{50}{0.1}i = 0$

$$i_c(t) = k_1 e^{-500t}$$

Steady state $i_p(t) = k_2 = \frac{100}{50} = 2A$

- a. What is the initial current in the loop, $i(0^+)$? (1 pt) 0A ✓
- b. Find the current, $i(t)$, through the inductor for $t > 0$. (6 pts) $i(t) = 2 - 2e^{-500t} A$ ✓
- c. Find the voltage, $v(t)$, across the inductor for $t > 0$. (3 pts) $v(t) = 100e^{-500t} V$ ✓
- d. What is the maximum value of the current through the inductor? (1 pt) 2A ✓
- e. How long does it take for the current to reach 85% of its maximum? (2 pts) $t = .00379 s$ ✓

2. Consider the circuit shown below. Find the voltage, $v(t)$, across the capacitor for $t > 0$ if the initial voltage is $v(0^+) = 2V$. (7 pts)



$$v(t): \frac{dv}{dt} + \frac{1}{RC}(v - v_s) = 0$$

$$\frac{dv}{dt} + \frac{1}{0.2}(v - v_s) = 0 \Rightarrow \frac{dv}{dt} + 5v = 50e^{-5t}$$

homogeneous $v_c(t) = k_1 e^{-5t}$

$$v(t) = -2k_1 e^{-2t} + 5(k_2 e^{-2t}) = 50e^{-2t}$$

$$3k_2 e^{-2t} = 50e^{-2t}$$

$$k_2 = 50/3 \quad \checkmark$$

$$v(t) = -\frac{50}{3}e^{-5t} + \frac{50}{3}e^{-2t} V$$

$$v_p(t) = \frac{dv}{dt} = .2k_2 e^{-.2t}$$

$$v(t) = k_1 e^{-.5t} + \frac{50}{3}e^{-.2t}$$

$$v(0) = k_1 + 50/3 = 0$$

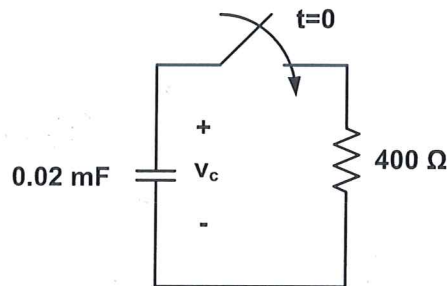
$$k_1 = -50/3 \quad \times$$

EE281 A

Homework #11 -- 20 points

Due: Wednesday, February 23, 2011

1. If the initial voltage across the capacitor is $v_c(0) = 50\text{V}$, Find the voltage across the capacitor, v_c , as a function of time. Then find the time (in seconds) for the voltage v_c to be reduced to 10 V as the capacitor discharges. (7 pts)



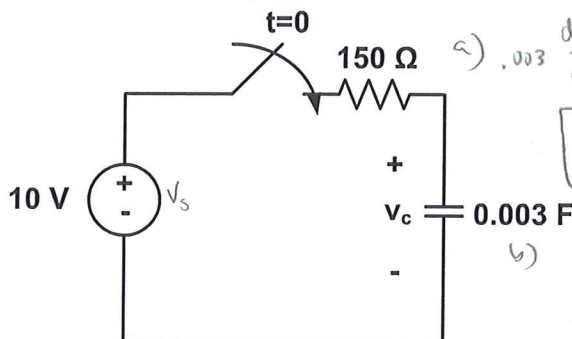
$$V(t) = V_0 e^{-\frac{1}{RC}t} \quad 400 \times .02 \times 10^{-3}$$

$$= 50 e^{-\frac{t}{.008}} \quad V - 1/2$$

$$10 = 50 e^{-\frac{t}{.008}} \quad \text{no}$$

$$t = .0129 \quad s - 1/2$$

2. Consider the RC circuit in which v_c is the capacitor voltage.



$$a) .003 \frac{dv_c}{dt} + \frac{v_c - 10}{150} = 0 \Rightarrow \frac{dv_c}{dt} + \frac{1}{.45} v_c = \frac{10}{.45}$$

$$\frac{dv_c}{dt} + 2.22 v_c = 22.22$$

$$b) \frac{dv_c}{dt} + 2.22 v_c = 0$$

$$s + 2.22 = 0$$

$$v_c = k_1 e^{-2.22t}$$

$$V_p = k_2$$

$$0 + 2.22 k_2 = 22.22$$

$$k_2 = 10$$

$$v_c = k_1 e^{-2.22t} + 10$$

$$v_c(0^+) = 2 = k_1 e^0 + 10$$

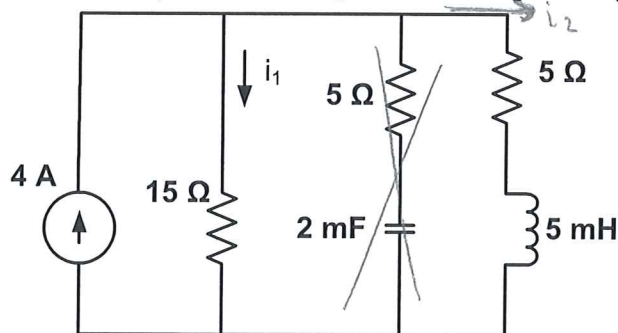
$$k_1 = -8$$

- a. Write the differential equation in standard form used to find $v_c(t)$ for $t > 0$. (3 pts)

- b. If $v(0^+) = 2\text{V}$, find the solution of the differential equation (5 pts)

$$V_c(t) = 10 - 8e^{-2.22t} \text{ V}$$

3. Solve the steady-state current i_1 in the following circuit. (5 pts)



$$4 = i_1 + i_2 \Rightarrow 4 = 4i_1$$

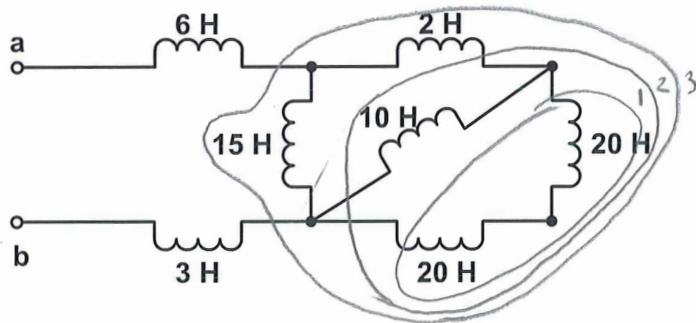
$$15i_1 = 5i_2$$

$$i_2 = 3i_1$$

$$i_1 = 1\text{A}$$

Did Not turn in on time

1. Simplify the following to a single equivalent component. (6 pts)



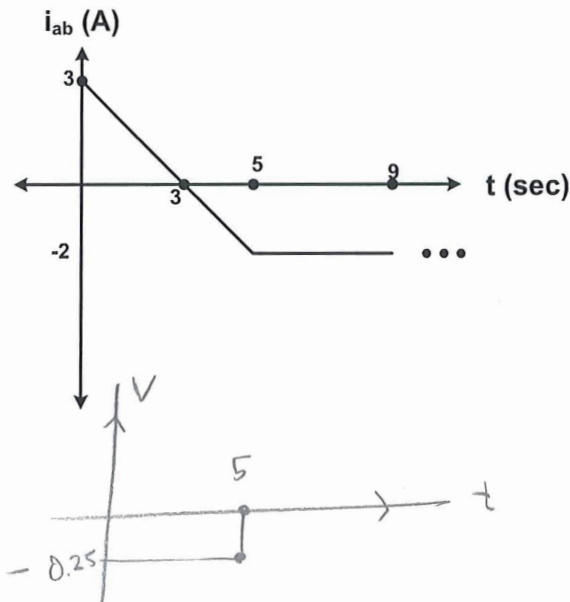
$$\lambda_1 = 20 + 20 = 40$$

$$\lambda_2 = \left(\frac{1}{40} + \frac{1}{10}\right)^{-1} = 8$$

$$\lambda_3 = \left(\frac{1}{15} + \frac{1}{2+20}\right)^{-1} = \frac{30}{5} = 6$$

$$\lambda_4 = 6 + 6 + 3 = 15 \text{ H}$$

2. If the voltage across a 4-mH inductor is $v(t) = 8\cos(2t)$ V, find the current through the inductor as a function of t if the initial current at $t=0$ is 0.25 A. (4 pts) ✓ Bank
3. If the current through a 2- μ H inductor is $i(t) = 4e^{-2t}$ A, find the voltage across the inductor as a function of t . (4 pts) Bank
4. If the current through a 0.25-H inductor is given by the graph below, find the piecewise function that describes the voltage across the inductor and sketch the graph of the voltage. (6 pts)



$$\begin{cases} i = 3 - t & 0 < t < 5 \\ i = -2 & t \geq 5 \end{cases}$$

$$v = L \frac{di}{dt} = \begin{cases} 0.25(-1) & 0 < t < 5 \\ 0 & t \geq 5 \end{cases}$$

2) $v(t) = 8\cos(2t)$ V $i_0 = 0.25$ A

$$i(t) = i_0 + \int_0^t v d\lambda = 0.25 + \left[-16\sin\lambda\right]_0^t = 0.25 - 16\sin t - 0$$

$$i(t) = 0.25 - 16\sin t$$

3) $i(t) = 4te^{-2t}$ A $L = 2\mu\text{H} = 2 \times 10^{-6}$ H

$$v(t) = L \frac{di}{dt} = 2 \times 10^{-6} (4e^{-2t} - 2(4t)e^{-2t})$$

$$v(t) = 8 \times 10^{-6} e^{-2t} (1 - 8t) \text{ V}$$

David Maloney

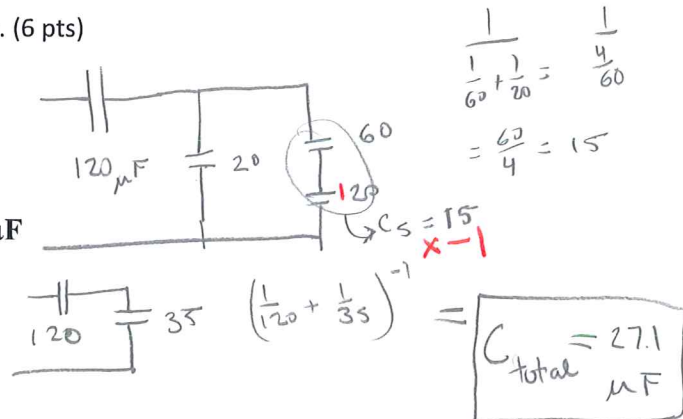
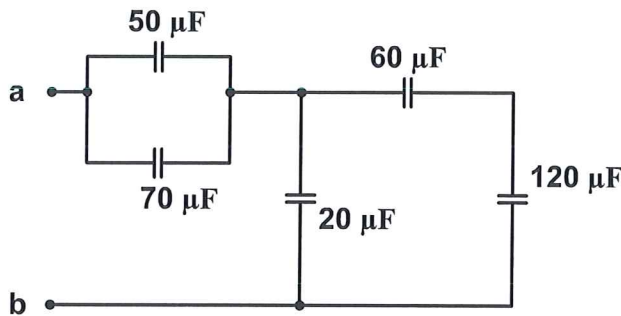
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EE281 A

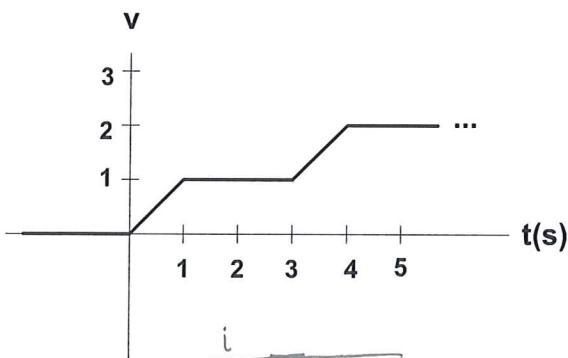
Homework #9 -- 20 points

Due: Wednesday, February 16, 2011

1. Simplify the following to a single equivalent component. (6 pts)



2. If the current through a 2-mF capacitor is $i(t) = 6e^{-4t}$ A, find the voltage across the capacitor as a function of t if the initial voltage at $t=0$ is 3 V. (4 pts)
3. If the voltage across a 2-mF capacitor is $v(t) = 5\sin(3t)$ V, find the current through the capacitor as a function of t . (4 pts)
4. If the voltage across a 0.5-F capacitor is given by the graph below, find the piecewise function that describes the current through the capacitor and sketch the graph of the current. (6 pts)



4)

$$v = \begin{cases} t & 0 \leq t < 1 \\ 1 & 1 \leq t < 3 \\ t-2 & 3 \leq t < 4 \\ 2 & 4 \leq t \end{cases}$$

$$i = \begin{cases} 0.5(1) = 0.5 & 0 \leq t < 1 \\ 0.5(0) = 0 & 1 \leq t < 3 \\ 0.5(1) = 0.5 & 3 \leq t < 4 \\ 0.5(0) = 0 & 4 \leq t \end{cases}$$

sketch the graph
-3

$$\begin{aligned} 2) \quad v(t) &= v(t_0) + \frac{1}{C} \int_{t_0}^t i dx \\ &= 3V + \frac{1}{2mF} \int_{t_0}^t 6e^{-4x} dx \\ &= 3 + \frac{1}{2mF} \left[\frac{6}{-4} e^{-4x} \right]_{t_0}^t \\ &= 3 + 500 \left(-\frac{6}{4} e^{-4t} \right) - 500 \left(-\frac{6}{4} e^0 \right) \end{aligned}$$

$$v(t) = 3V - 750e^{-4t} + 750V$$

3)

$$i = C \frac{dv}{dt} = 2 \times 10^{-3} \frac{d}{dt} 5 \sin 3t$$

$$= +.002(15) \cos 3t$$

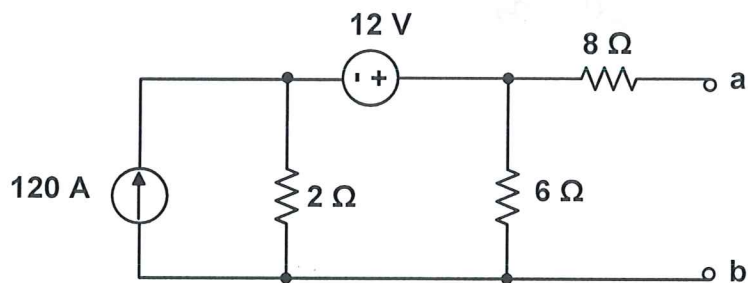
$$i(t) = +.030 \cos 3t \text{ A}$$

EE281 A

Homework #8 -- 20 points

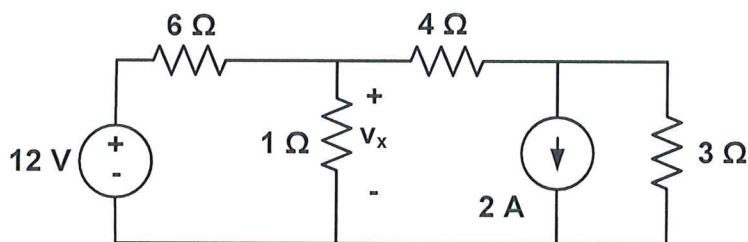
Due: Friday, February 11, 2011

1. Using the source transformation method, determine the Thévenin equivalent circuit for the following circuit. (6 pts)



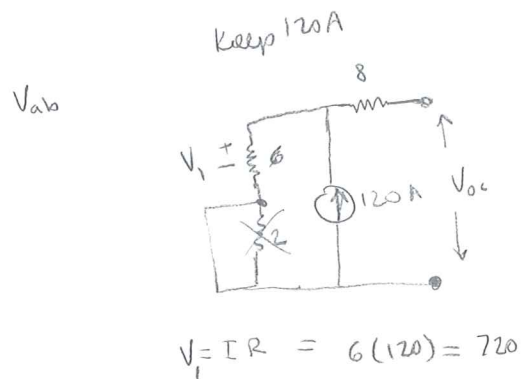
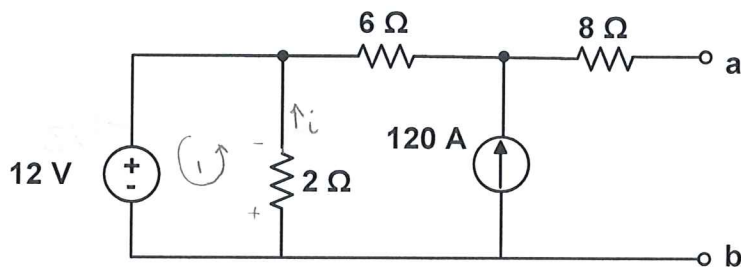
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2. Use superposition to find v_x in the following circuit. (6 pts)



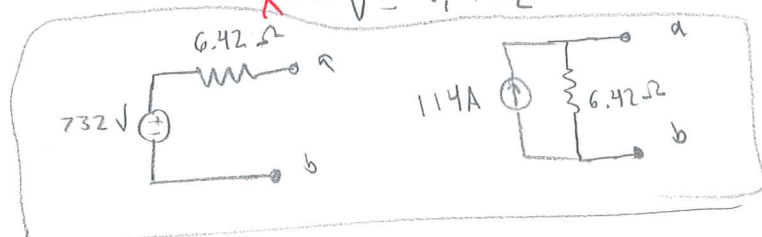
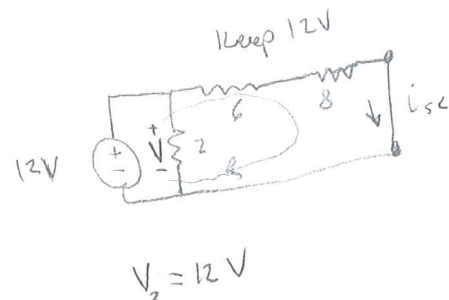
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3. Find the Thévenin equivalent circuit with respect to the terminals a and b . (8 pts)



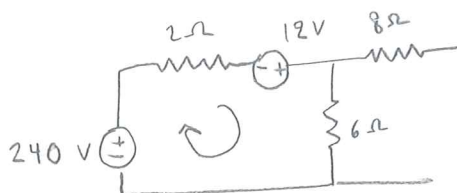
Mesh 1 KVL $12V + 2i_1 = 0$
 $i_1 = -6A$

$V = V_1 + V_2 = 720 + 12 = 732V$

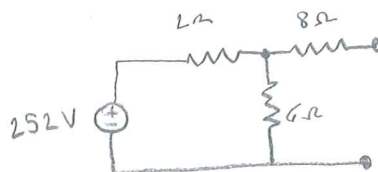


13/20

1)



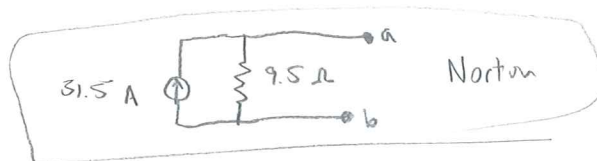
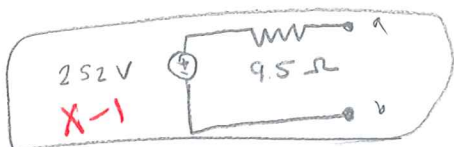
→



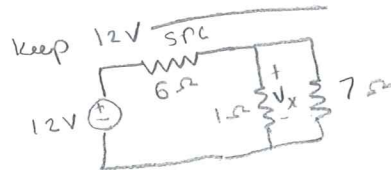
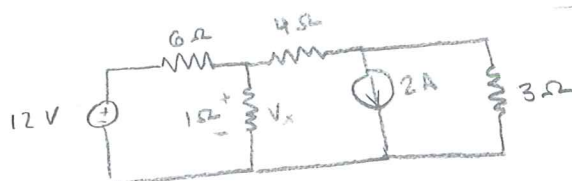
KVL $240V + 2i_{sc} + 12V + 6i_{sc} = 0$

$$i_{sc} = -\frac{252}{8}$$

use source transform
→



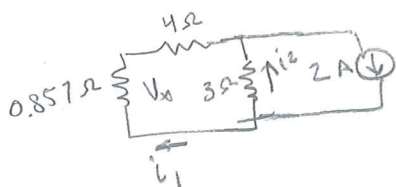
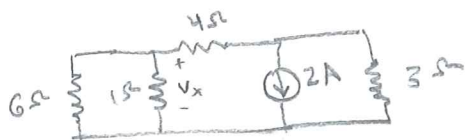
2)



Voltage divider $\frac{1}{1+1} = .5$

$v_x = \frac{.5}{1+1} (12V) = 1.5V$ ✓

keep the 2A



current divider

$$i_1 = \frac{\frac{1}{4.857}}{\frac{1}{4.857} + \frac{1}{3}} (2) = .764A$$

V divider



$$v_x = \frac{.857}{4.857} (.764)(4.857) = 0.655V$$

$V_x = V_1 + V_2 = 2.185V$ ✓

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David Malawey

EE281 A

Homework #7 -- 20 points

Due: Wednesday, February 9, 2011

$$\begin{aligned} ③ \quad 0 &= 3i_3 - 4i_2 \Leftrightarrow i_3 = 4i_2 - 4i_2 \\ i_3 &= 4i_x \\ \downarrow \\ i_x &= (i_3 - i_2) \Rightarrow \frac{i_3}{4} = i_3 - i_2 \end{aligned}$$

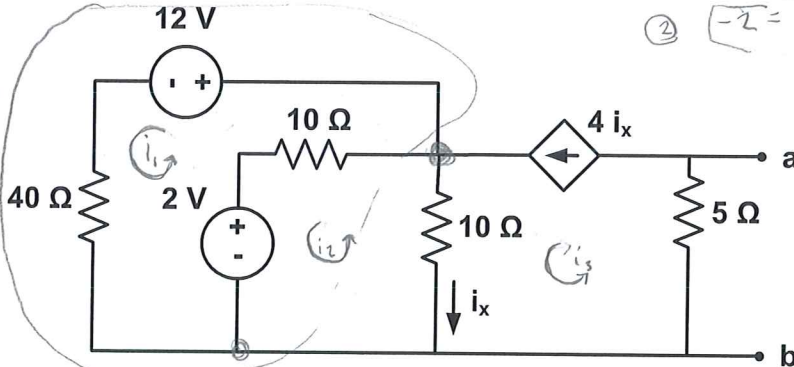
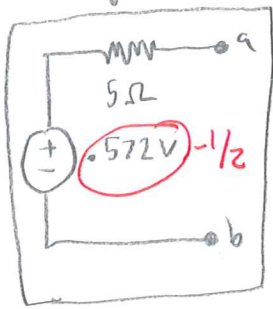
$$0 = -12 + 40i_1 - 2 + 10(i_1 - i_2)$$

$$① \quad -10 = 50i_1 - 10i_2$$

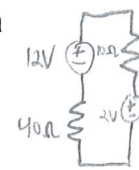
$$0 = 10(i_2 - i_1) + 2 + 10(i_2 - i_3)$$

$$② \quad -2 = 20i_2 - 10i_1 - 10i_3$$

1. Find the Thévenin equivalent for the following circuit. (10 pts)



show work for 5 ohm -4

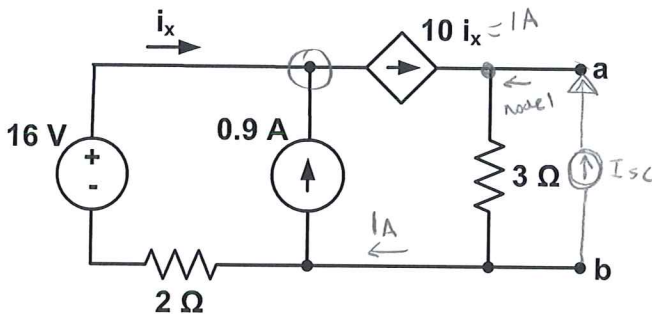


$$\begin{aligned} i_x &= i_3 - i_2 \\ &= -1.143 - (-0.857) \\ i_x &= -0.286 \checkmark \end{aligned}$$

$$i_3 = -1.143$$

$$V_{ab} = -5(-1.143) = 5.72$$

2. Find the Thévenin and Norton equivalents of the following circuit. (10 pts)



$$10i_x - i_x - 9A = 0$$

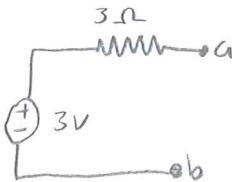
$$i_x = 0.1A \checkmark$$

$$I_{sc} = I_{ab} = -1A$$

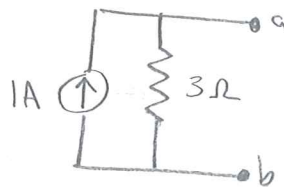
show all work

$$V_{oc} = 10i_x(3\Omega) = 3V \checkmark$$

show work for 3 ohm -1

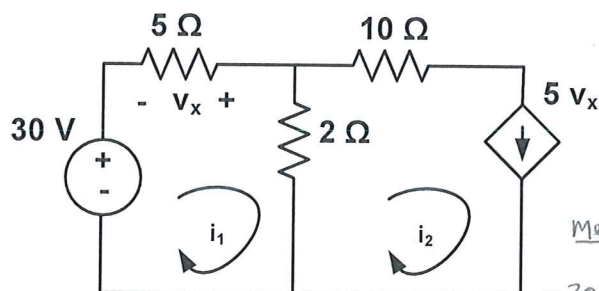


Thévenin



Norton

1. Solve for the mesh currents i_1 and i_2 in the following circuit. (8 pts)



Mesh 2)

$$i_2 = 5v_x \quad v_x = 5i_1$$

$$i_2 = 5(5i_1) = 25i_1 \quad \checkmark$$

Mesh 1)

$$-30 + 5i_1 + 2(i_1 - i_2) = 0$$

$$-3i_1 - 2(-25i_1) = 30$$

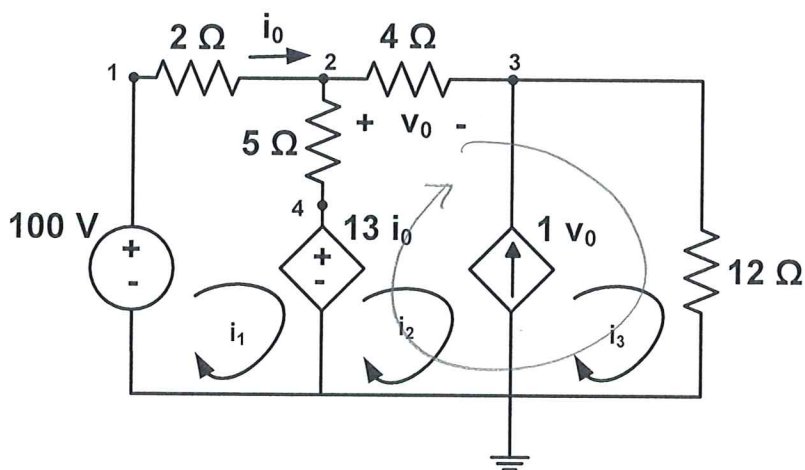
$$i_1 = 30/47$$

$$i_2 = -15.98$$

$$i_1 = \frac{30}{47} \text{ A} = 0.638 \text{ A}$$

$$-43i_1 = 30$$

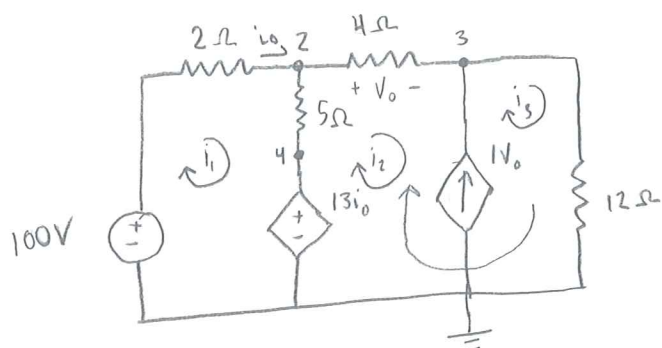
2. Set up the mesh current equations to find i_1 , i_2 , and i_3 for the following circuit in matrix form. You must eliminate the control variables i_0 and v_0 from your equations. (10 pts)



3. Why is a supermesh required in problem 2 but not in problem 1? (2 pts)

the current source is shared between 2 meshes for prob 2 $\checkmark$

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Supermesh (2,3)

$$\text{KVL} \quad 12 i_3 - 13 i_0 + 5 (i_2 - i_1) + 4 i_2 = 0$$

$$12 i_3 + 9 i_2 - 5 i_1 - 13 i_0 = 0 \quad \checkmark$$

Mesh 1

$$-100 + 2 i_1 + 5 (i_1 - i_2) + 13 i_0 = 0 \quad \checkmark$$

$$0 i_3 - 5 i_2 + 7 i_1 + 13 i_0 = 100$$

branch currents:

$$\left. \begin{array}{l} 1V_0 = i_3 - i_2 \\ V_0 = +4i_2 \end{array} \right\} +4i_2 = i_3 - i_2 \Rightarrow \begin{array}{l} 5i_2 = i_3 \\ i_3 + 5i_2 = 0 \quad \checkmark \end{array}$$

$$i_1 = i_0$$

$$\begin{bmatrix} -18 & 9 & 12 \\ 20 & -5 & 0 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 0 \end{bmatrix}$$

reduces to:

$$\boxed{\begin{bmatrix} -6 & 3 & 4 \\ 4 & -1 & 0 \\ 0 & -5 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 20 \\ 0 \end{bmatrix}} \quad \checkmark$$

David Malawey

EE281 A

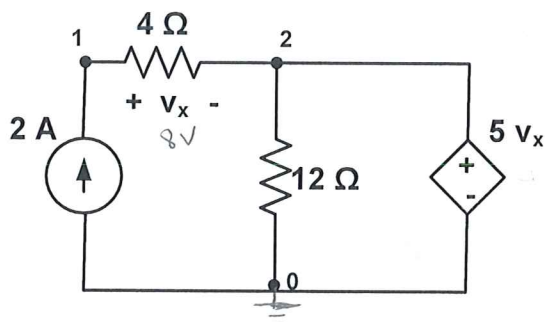
Homework #5 -- 20 points

Due: Wednesday, January 26, 2011

1. A source supplies 4 A to the parallel combination of a $10\text{-}\Omega$ resistance, a $5\text{-}\Omega$ resistance, and an unknown resistance R . The current through the $10\text{-}\Omega$ resistance is 0.2 A . Determine the value of the unknown resistance R . (4 pts)

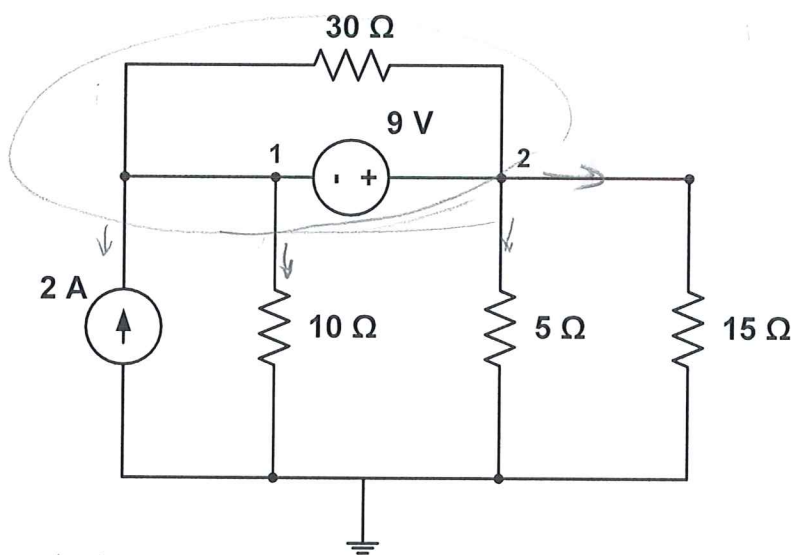
(back)

2. Solve for the node voltages v_1 , and v_2 in the following circuit. (8 pts)



(back)

3. Solve for the node voltage, v_1 and v_2 , in the following circuit. (8 pts)



Supernode (1,2)

$$v_2 - v_1 = 9V \quad (1)$$

$$-2 + \frac{v_1}{10} + \frac{v_2}{5} + \frac{v_2}{15} = 0$$

$$-20 + v_1 + 2v_2 + \frac{2}{3}v_2 = 0 \quad (2)$$

$$v_1 + \frac{8}{3}v_2 = 20$$

$$v_2 - 9 + \frac{8}{3}v_2 = 20 \Rightarrow \frac{11}{3}v_2 = 29 \quad v_2 = \frac{87}{11} \Rightarrow$$

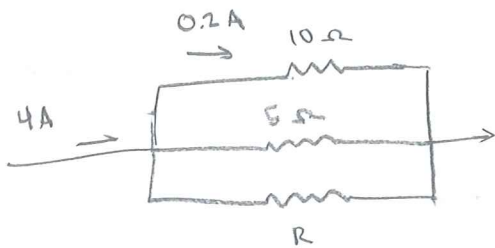
$$v_1 = -1.09V$$

$$v_2 = 7.91V$$



20/20

1)



$$R = .588 \Omega$$

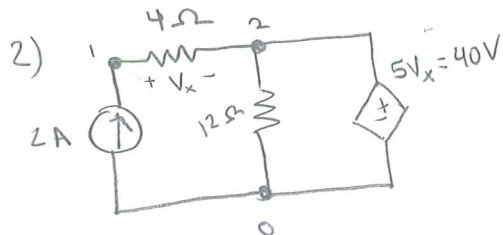
current divider

$$0.2A = \frac{\frac{1}{10}}{\frac{1}{10} + \frac{1}{5} + \frac{1}{R}} (4A)$$

$$\frac{.2}{10} + \frac{.2}{5} + \frac{.2}{R} = \frac{4}{10}$$

$$\frac{.2}{R} = .4 - .02 - .04$$

$$.2 = .34 R \quad \frac{.2}{.34} = R = \frac{1}{1.7}$$



$$V_x = 4(2) = 8V$$

$$V_2 = 40V$$

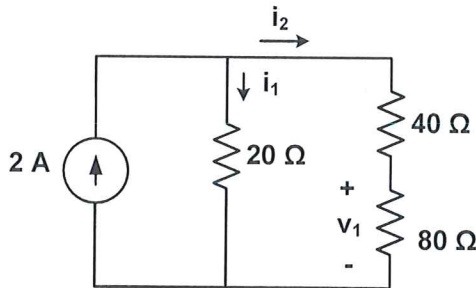
node 1

$$-2A + \frac{V_1 - V_2}{4\Omega} = 0$$

$$V_1 - V_2 = 2A(4\Omega) = 8V$$

$$V_1 = V_2 + 8 = 48V$$

1. Find
- i_1
- ,
- i_2
- , and
- v_1
- in the following circuit. (8 pts)



$$i_1 + i_2 = 2 \text{ A}$$

$$V = IR \quad 120\Omega(i_2) = 20\Omega(i_1)$$

$$i_1 = 6i_2$$

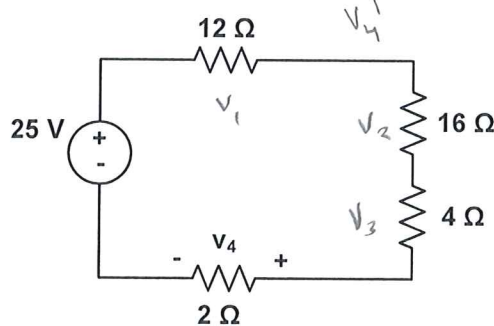
$$7i_2 = 2 \text{ A} \Rightarrow$$

$$i_2 = \frac{2}{7} \text{ A}$$

$$i_1 = \frac{12}{7} \text{ A}$$

$$V_1 = \frac{2}{7} \text{ A}(80\Omega) \Rightarrow V_1 = 22.86 \text{ V}$$

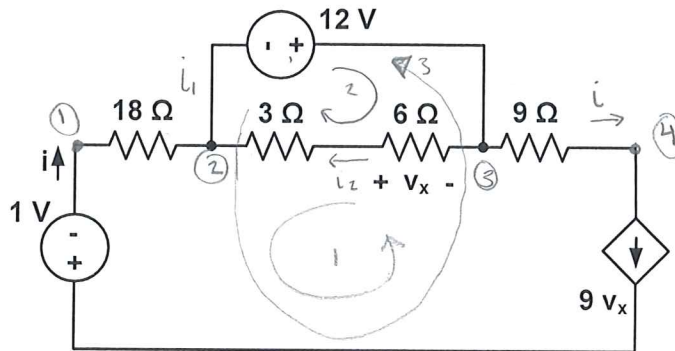
2. Use the voltage divider to find
- v_3
- in the following circuit. (4 pts)



$$v_3 = \frac{4}{12+16+4+2} (25 \text{ V})$$

$$v_3 = 1.47 \text{ V}$$

3. Find the current,
- i
- , in the following circuit. (8 pts)



KVL 1

$$1V - 9V_x + i(9) - 9i_2 + 18i = 0$$

$$1 - 9V_x + 27i = 9i_2 \quad (1)$$

KVL 2

$$1V - 9V_x + 9i - 12 + 18i = 0$$

$$1 - 9V_x + 27i - 12 = 0$$

$$-9V_x + 27i = 11$$

(2)

$$V_x = i_2(6)$$

(3)

$$1 - 54i_2 + 27i = 9i_2$$

$$1 + 27i = 63i_2 \quad i_2 = \frac{1}{63} + \frac{3}{7}i$$

$$i = \frac{-1}{27} + \frac{2}{3}i_2$$

$$-54i_2 + 27\left(\frac{-1}{27} + \frac{2}{3}i_2\right) = 11$$

$$-54i_2 - 1 + 63i_2 = 11$$

$$9i_2 = 12 \quad i_2 = 1.25 \text{ A}$$

- Voltage divider -

$$V_x = \frac{6}{9}(-12) = -8 \text{ V}$$

$$-9V_x + 27i = 11$$

$$+81V + 27i = 11$$

$$i = \frac{11}{27} - \frac{81}{27} = -\frac{70}{27} \text{ A} = -2.59 \text{ A}$$

X -3

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$$V = ir$$

$$i = \frac{V}{r} \quad \frac{1}{r}$$

answer to
p 3)

$$9V_x = \underset{\substack{\uparrow \\ \frac{1}{r}}}{9} (-8V) = -72i$$

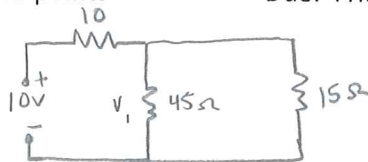
David Maloney

20/20

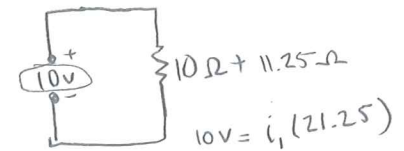
EE281 A

Homework #3 -- 20 points

Due: Friday, January 21, 2011



$$\frac{45(15)}{45+15} = \frac{675}{60} \rightarrow$$



$$10V = i_1(21.25)$$

1. Determine i_1 , i_2 , i_3 , v_1 , and v_2 for the following circuit. (10 pts)

$$\text{Node A: } i_1 = i_2 + i_3$$

$$i_2 = \frac{v_1}{45} \quad i_3 = \frac{v_2}{5} = \frac{v_1}{10}$$

$$10V - i_1(10) - i_3(10) - i_3(5) = 0$$

$$10 = 10i_1 + 15i_3$$

$$v_1 = i_2(45) = i_3(15) \quad i_2 = \frac{i_3}{3}$$

$$.471A = i_3\left(\frac{4}{3}\right) \quad i_3 = .353A$$

$$v_2 = .353(5)$$

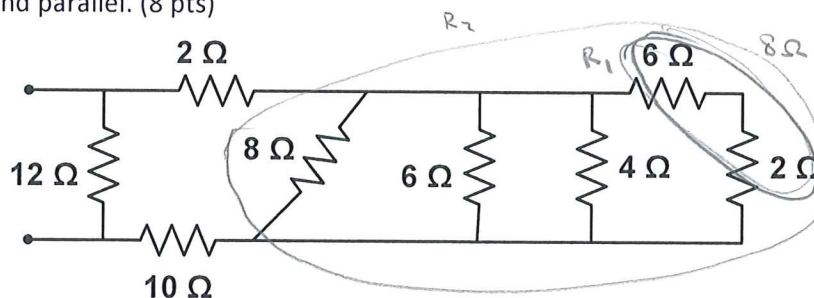
$$v_2 = 1.765V$$

$$v_1 = (.118)(45)$$

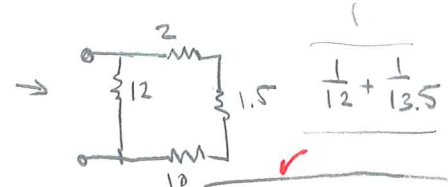
$$v_1 = 5.31V$$

$$i_2 = .118A$$

2. Reduce the network shown to a single equivalent resistance by combining resistances in series and parallel. (8 pts)

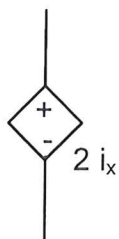


$$\frac{1}{R_2} = \frac{1}{8} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} = \frac{4}{6}$$



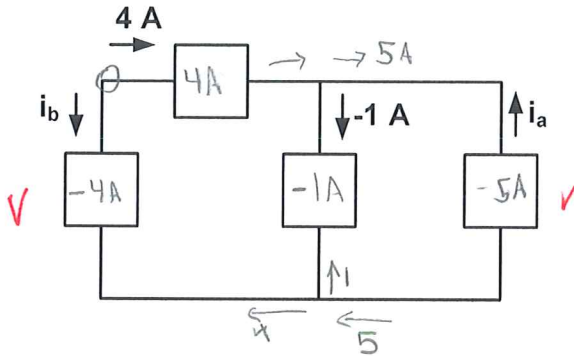
$$R_{eq} = 6.353\Omega$$

3. What are the units of the gain parameter (i.e. the 2) in the dependent source below if i_x is a current (with units in amperes) flowing in another part of the circuit? (2 pts)



$$\Omega \text{ ohms}$$

1. Use KCL to determine the values of each of the unknown currents in the figure shown. (6 pts)



2. The terminals of an electrical device are labeled a and b . If $v_{ab} = 10\text{ V}$, how much energy is exchanged when a charge of 1 C moves through the device from a to b ? Is the energy delivered to the device or taken from the device? (6 pts)

$$V = \frac{J}{C} \quad 10\text{ V} = \frac{J}{1\text{ C}} \quad \boxed{E = 10\text{ J}}$$

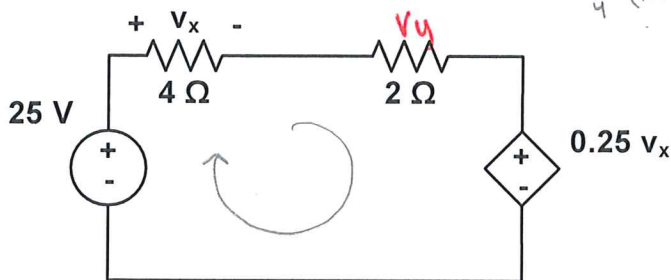
3. Calculate the resistance of a 10-meter section of gold wire which has a diameter of 2 mm. (4 pts)

$$A = \frac{\pi}{4} (.002)^2$$

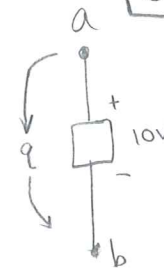
$$\text{resistivity of gold: } 2 \times 10^{-8} \Omega \cdot \text{m}$$

$$\boxed{0.077 \Omega}$$

4. Find v_x in the following circuit. (4 pts)



$$\frac{10\text{ m}}{\frac{\pi}{4} (.002\text{ m})^2} (2.44 \times 10^{-8} \Omega \cdot \text{m})$$



$$V = IR$$

$$25 - v_x$$

$$-0.25 v_x = 0$$

$$\text{KVL } 25\text{ V} - v_x - 2I - .25 v_x = 0$$

$$25 - 2I - 1.25 v_x = 0$$

$$\boxed{v_x = 20\text{ V}}$$

Rework

$$25 - v_x - v_y - .25 v_x = 0$$

$$.75 v_x = 25 - v_y$$

$$.75 v_x = 25 - \frac{v_x}{2}$$

$$1.25 v_x = 25$$

$$v_x = \frac{25}{1.25} = 20\text{ V}$$

$$v = ir$$

$$\frac{v_x}{4} = \frac{v_y}{2}$$

$$v_x = 2v_y$$

EE281 A

Homework #1 -- 20 points

Due: Friday, January 14, 2011

did not turn in because
I was in Guatemala



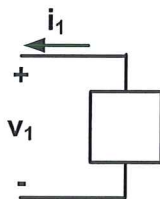
1. A constant current of 5 A flows through a circuit element for 4 seconds. How much net charge passes through the element in that time? (5 pts)

$$5 \text{ A} = \frac{5 \text{ coulombs}}{\text{sec}} (4 \text{ sec}) = \boxed{20 \text{ coulombs}}$$

2. The charge that passes through a circuit element is given by $q(t) = 5 \cos(20t) \text{ C}$. Find the current as a function of time. (5 pts)

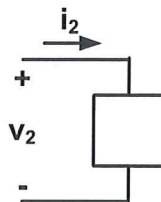
$$i = \frac{dq}{dt} = \frac{d}{dt}(5 \cos 20t) = (-20(5) \sin 20t) = \boxed{-100 \sin 20t \text{ Amperes}}$$

3. Compute the power as a function of time for each of the following elements. Find the energy transferred between $t=0$ and $t=1$ seconds. In each case is energy supplied or absorbed by the element? (10 pts)



$$v_1 = t - 3 \text{ V}$$

$$i_1 = t^2 + 4 \text{ A}$$



$$v_2 = 2e^{-3t} \text{ V}$$

$$i_2 = 4 \text{ A}$$

$$P_1 = v_1 i_1 = (t - 3)(t^2 + 4) = (t^3 - 3t^2 + 4t - 12)(-) \text{ due to current flows into neg side}$$

$$\begin{aligned} \text{energy } W &= \int_0^1 (-t^3 + 3t^2 - 4t + 12) dt = \left[-\frac{t^4}{4} + t^3 - 2t^2 + 12t \right]_0^1 \\ &= -\frac{1}{4} + 1 - 2 + 12 = \boxed{10.75 \text{ J}} \quad \text{-absorbed} \end{aligned}$$

$$P_2 = v_2 i_2 = 2e^{-3t} (4)$$

$$W = \int_0^1 8e^{-3t} dt = -\frac{8}{3} e^{-3t} \Big|_0^1 = \frac{8}{3} - \frac{8}{3} e^{-3} = \boxed{2.53 \text{ J}} \quad \text{-absorbed}$$

Notes

EE 281

Notes 1/10/11

1st homework due Friday

1.1] Overview of EE

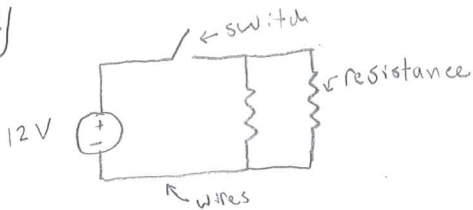
- information
- energy

Mech-tronics: integration of electronic & mechanical systems

EE branches:

1. Power systems
2. computer systems
3. control systems
4. signal processing
5. electronics
6. communication systems
7. electromagnetics
8. photonics

1.2]



← headlight circuit of a car

1. Voltage source = battery
2. wires = insulated copper - confines electrons through single path
3. switch - controls flow of electrons
4. resistances - convert electrical energy to heat/light
tungsten wires in headlight

(i) Current = flow of charge through a cross-section of a circuit

Coulomb = unit of charge (C)

Ampere (A) = current units, C/sec

Voltage: electrical potential between 2 points

fluid flow analogy:

battery = pump
charges: fluid
conductors: frictionless pipes
current: flow rate of fluid

switches: valves

resistance: constriction resulting in turbulence

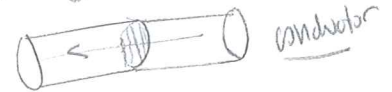
David M.

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q = charge units = coulombs i = current units = amperes, A

$$1 \text{ amp} = \frac{1 \text{ coulomb}}{\text{sec}}$$

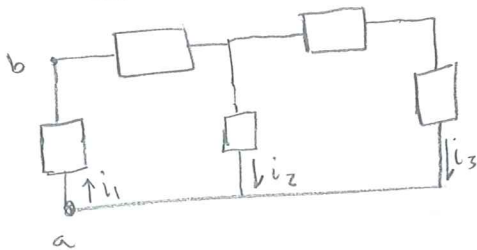
passing thru a x-section



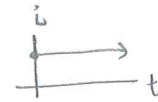
$$i = \frac{dq}{dt} \quad q(t) = \int_{-\infty}^t i d\lambda = q(t_0) + \int_{t_0}^t i d\lambda$$

$i_1 = i_{ab}$ if $i < 0 \rightarrow$ current is flowing in opposite direction

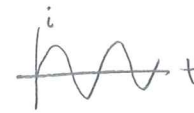
$$i_{ab} = -i_{ba}$$



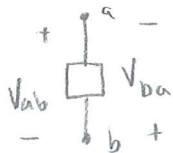
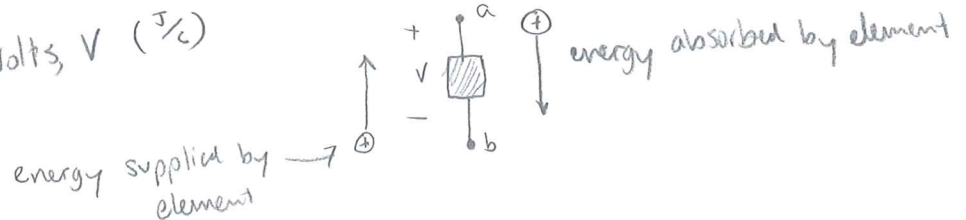
DC (direct)



AC (alternating)

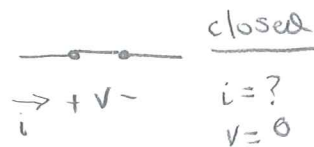


Voltage v units: volts, V ($\frac{J}{C}$)



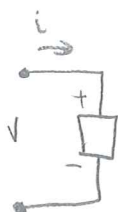
$$V_{ab} = -V_{ba}$$

Switches



Power = P = rate of energy transfer, $P = vi$ $\frac{J}{C} \cdot \frac{C}{s} = \frac{J}{s} = \text{Watts}$

• passive sign convention: if current enters pos. side of voltage, then if P is positive, element is absorbing power. if P neg, element is supplying (we will use)

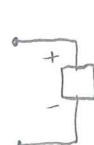


$$V = 20 - 2t \text{ V}$$

$$i = 10 \text{ A}$$

find $P(t)$

$$p = vi = (20 - 2t)(10) = 200 - 20t \text{ W}$$



$$V = 10e^{-2t} \text{ V}$$

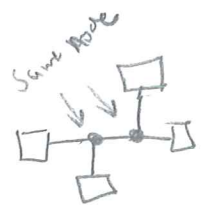
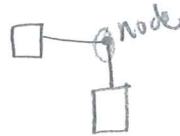
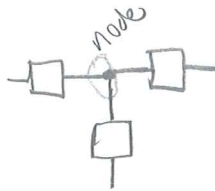
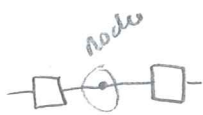
$$i = 5 \text{ A}$$

Notes 1/12 continued

Energy $W = \int_{t_1}^{t_2} p(t) dt$ energy delivered to an element in time interval $t_1 \leq t \leq t_2$

ex $p(t) = 200 - 2t$ $W = \int_0^1 (200 - 20t) dt = 200t - 10t^2 \Big|_0^1 = 200 - 10 = 190 \text{ J}$

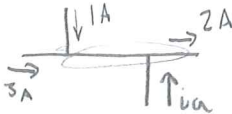
KCL Kirchoff's current law



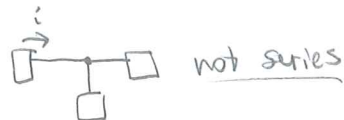
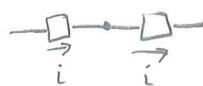
- nodes connected by wires (short circuits) are the same node

KCL: net current leaving a node must be zero (node OR any closed surface)

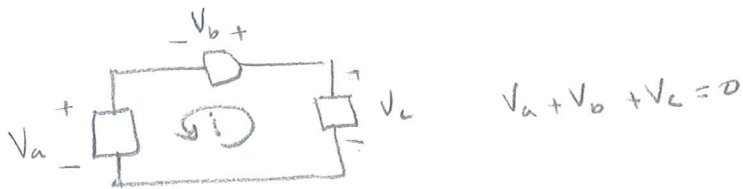
Swift uses convention "current leaving is positive"

ex)  $-3A - 1A + 2A - i_a = 0$ $i_a = -2A$

Series

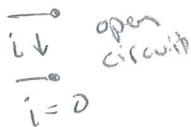
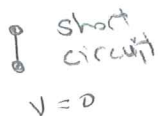


KVL Σ voltage around closed path = 0 (comes from Cons.-E.)



Parallel components - connected between the same pair of nodes

1.6 | conductors: wires connecting components - assumed perfect conductors

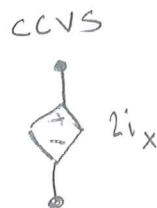
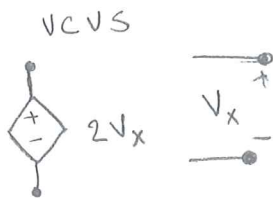


Independent Voltage Source:

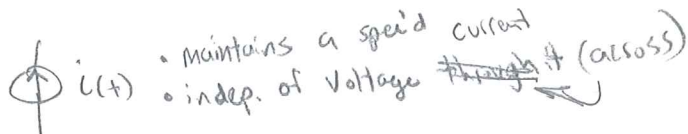
- maintains a specified voltage between the terminals
- independent of i flowing through it (ideally)



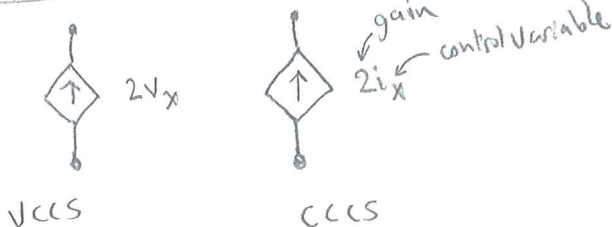
Dependent Voltage Src.



independent current source



dependent current source



$$S = \frac{1}{\Omega} = \frac{A}{V} \text{ (Siemens)}$$

$$\frac{V}{A} = \Omega \text{ (ohms)} \quad \text{or mhos}$$

Notes 1-14-2011 continued:

Resistor:



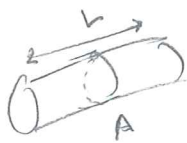
units - ohms Ω

ohm's law $\Rightarrow V = iR$ R is positive

$$R = \frac{V}{i}$$

• conductance $G = \frac{1}{R}$ Siemens (S)

- resistance depends on ~~the~~ material & dimensions of conductor



$$R = \frac{\rho L}{A}$$

ρ = resistivity of mat'l (Ωm)

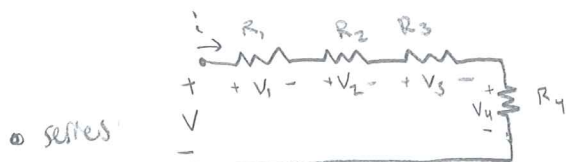
Power = J/s $P = Vi$

for resistors $V = iR$

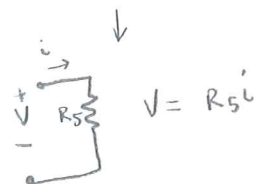
$$P = i^2 R$$

$$P = \frac{V^2}{R}$$

2.1) Parallel & series resistors

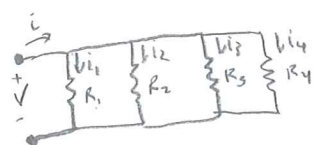


KVL $-V_1 - V_2 - V_3 - V_4 + V = 0$
 $V = V_1 + V_2 + V_3 + V_4 = R_1 i + R_2 i + R_3 i + R_4 i$



$$R_S = R_1 + R_2 + R_3 + R_4$$

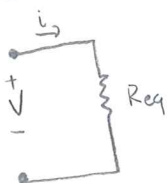
• parallel



$$i = i_1 + i_2 + i_3 + i_4$$

$$i_1 = \frac{V}{R_1}, \quad i_2 = \frac{V}{R_2} \dots$$

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \frac{V}{R_4}$$

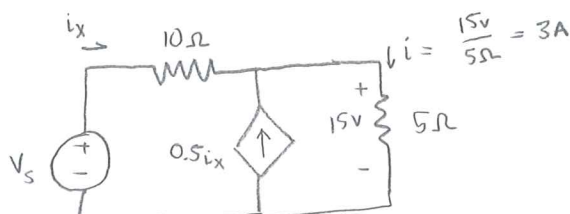


$$R_P = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}}$$

- 2 resistors in parallel $R_P = \frac{R_1 R_2}{R_1 + R_2}$

Ohm's law

KVL
KCL

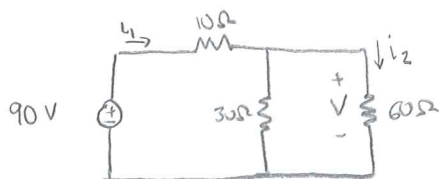


KCL: $-i_x - 0.5i_x + 3 = 0$
 $1.5i_x = 3$
 $i_x = 2A$

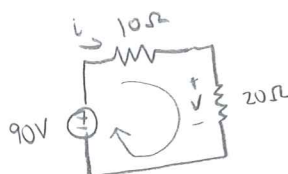
KVL: $-V_s + 10i_x + 15 = 0$ outer loop

$$V_s = 10(2A) + 15$$

$$V_s = 35V$$



$$\frac{30 \cdot 60}{90} = 20\Omega$$



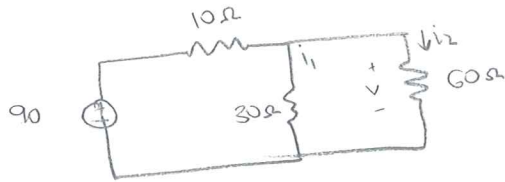
KVL: $-90 + 10i_1 + 20i_1 = 0$
 $30i_1 = 90$
 $i_1 = 3A$

$$V = 20i_1 = 20(3) = 60V$$

$$V = 60i_2 \quad 60 = 60i_2$$

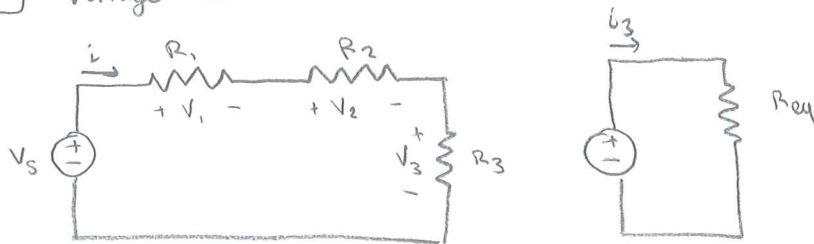
$$i_2 = 1A$$

Notes 1-19 continued



Notes 1-21-2011

2.3 Voltage divider



$$i = \frac{V_s}{R_{eq}} = \frac{V_s}{R_1 + R_2 + R_3}$$

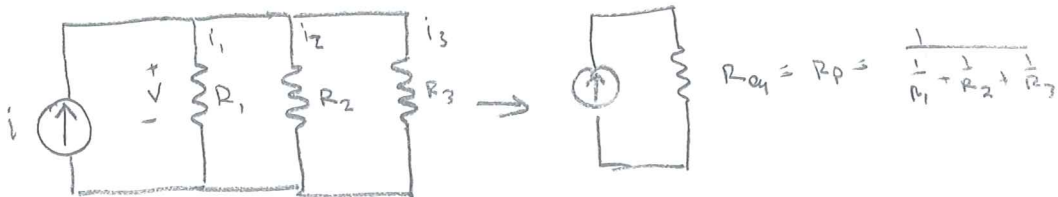
$$V_1 = R_1 \left(\frac{V_s}{R_1 + R_2 + R_3} \right)$$

Voltage divider

$$V_1 = \frac{R_1}{R_1 + R_2 + R_3} V_s$$

ex] done in class, not copied

— Current divider —



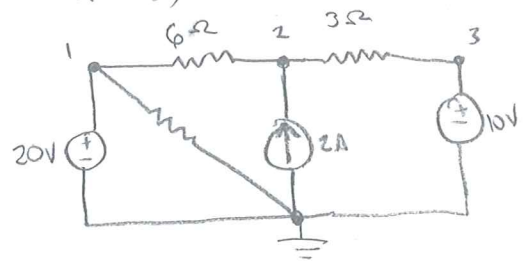
$$i_1 = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} (i_s) \leftarrow \text{current divider}$$

2.4 / node-voltage analysis

① select a node to be the reference node (node 0 or $\perp$)

② number the remaining nodes

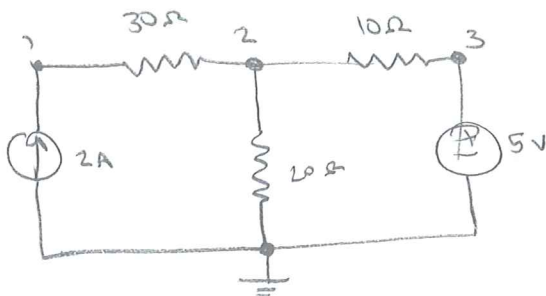
③ Use KCL at each node to get a set of eq's



ex) find V_1, V_2, V_3

node 1 $V_1 = 20V$ $V_3 = 10V \leftarrow$ Node 3 :

node 2: KCL $\frac{V_2 - V_1}{6} - 2 + \frac{V_2 - V_3}{3} = 0$

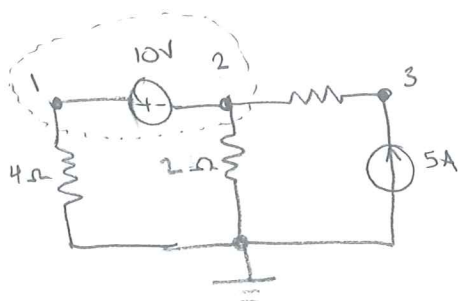


find V 's

node 3 $V_3 = 5V$
node 1 $-2 + \frac{V_1 - V_2}{30} = 0 \Rightarrow \frac{1}{30}V_1 - \frac{1}{30}V_2 = 2$

node 2 $\frac{V_2 - V_1}{30} + \frac{V_2}{20} + \frac{V_2 - V_3}{10} = 0$

$-\frac{1}{30}V_1 + \frac{11}{60}V_2 - \frac{1}{10}V_3 = 0$



• if voltage source is not grounded through a path of voltage sources

$\rightarrow$ supernode

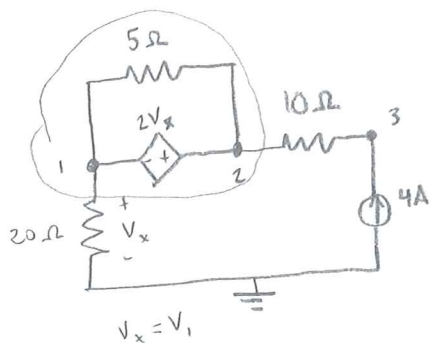
ex) supernode (1,2)

① $V_1 - V_2 = 10V$

KCL: $\frac{V_1}{4} + \frac{V_2}{2} + \frac{V_2 - V_3}{8} = 0$

$\frac{1}{4}V_1 + \dots$ matrix

$$\begin{bmatrix} 1 & -1 & 0 \\ \frac{1}{4} & \frac{5}{8} & -\frac{1}{8} \\ 0 & -\frac{1}{8} & \frac{1}{8} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 5 \end{bmatrix}$$



Supernode (1,2)

$$V_2 - V_1 = 2V_x = 2V_1$$

$$\textcircled{1} -3V_1 + V_2 = 0$$

$$\text{KCL: } \frac{V_1}{20} + \frac{V_2 - V_3}{10} = 0$$

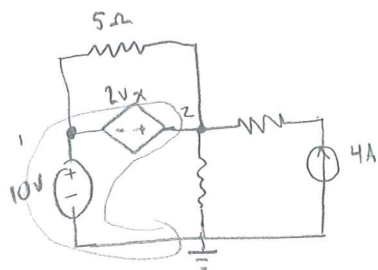
$$\frac{V_1}{20} + \frac{V_2}{10} - \frac{V_3}{10} = 0$$

node 3

$$\text{KCL: } \frac{V_3 - V_2}{10} - 4 = 0$$

$$\frac{V_3}{10} - \frac{V_2}{10} - 4 = 0$$

$$\begin{bmatrix} -3 & 1 & 0 \\ \frac{1}{20} & \frac{1}{10} & -\frac{1}{10} \\ 0 & -\frac{1}{10} & \frac{1}{10} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix}$$



node 1:

$$\textcircled{1} V_1 = 10V$$

node 2

$$V_2 = 2V_x + 10V$$

$$V_2 = 2(V_2 - V_3) + 10V$$

$$\textcircled{2} -V_2 + 2V_3 = 10$$

$$V_x = V_2 - V_3$$

node 3:

$$\frac{V_3 - V_2}{10} - 4 = 0$$

$$\textcircled{3} -\frac{1}{10}V_2 + \frac{1}{10}V_3 = 4$$

path of voltage sources, NOT a supernode

b) (on handout) Supernode (1,2)

$$V_1 - V_2 = 2i_x = 2\left(\frac{V_3}{3}\right)$$

$$\textcircled{1} V_1 - V_2 - \frac{2}{3}V_3 = 0$$

$$\text{KCL: } \frac{V_1}{6} - 2.5 + \frac{V_2 - V_3}{4} = 0$$

$$\textcircled{2} \frac{1}{6}V_1 + \frac{1}{4}V_2 - \frac{1}{4}V_3 = 2.5$$

node 3

$$\text{KCL: } \frac{V_3 - V_2}{4} + \frac{V_3}{3} = 0$$

$$\textcircled{3} -\frac{1}{4}V_2 - \frac{7}{12}V_3 = 0$$

$$\begin{bmatrix} 1 & -1 & -\frac{2}{3} \\ \frac{1}{6} & \frac{1}{4} & -\frac{1}{4} \\ 0 & -\frac{1}{4} & \frac{7}{12} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2.5 \\ 0 \end{bmatrix}$$

d) node 1:

$$V_1 = 100V \textcircled{1}$$

$$\text{node 4: } V_4 = 13i_o = \frac{13}{2}(V_1 - V_2) \textcircled{2}$$

$$\text{node 2: } \frac{V_2 - V_1}{2} + \frac{V_2 - V_4}{5} + \frac{V_2 - V_3}{4} = 0$$

$$\textcircled{3} -\frac{1}{2}V_1 + \frac{19}{20}V_2 - \frac{1}{4}V_3 - \frac{1}{5}V_4 = 0$$

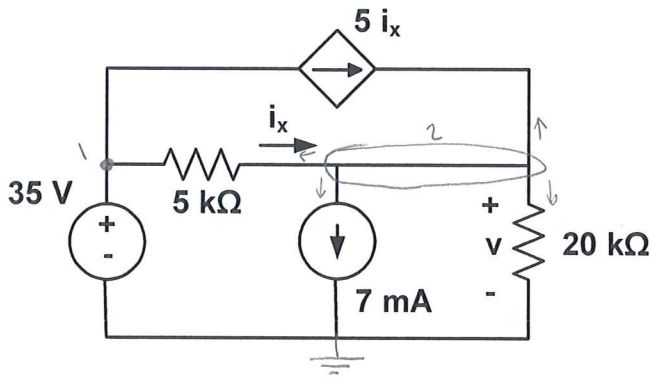
node 3

$$\frac{V_3 - V_2}{4} - 1V_o + \frac{V_3}{12} = 0$$

$$\frac{V_3 - V_2}{4} - (V_2 - V_3) + \frac{V_3}{12} = 0$$

$$\textcircled{4} -\frac{5}{4}V_2 + \frac{4}{3}V_3 = 0$$

a)



Node 1

$$V_1 = 35V$$

Node 2

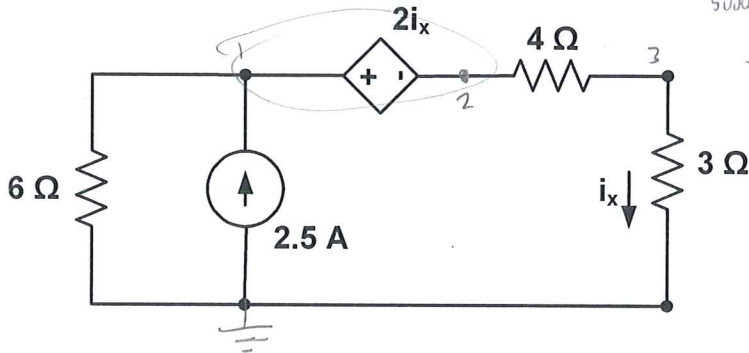
$$\frac{V_2 - V_1}{5000} + 0.007 + \frac{V_2}{20000} - 5i_x = 0$$

$$i_x = \frac{V_1 - V_2}{5000}$$

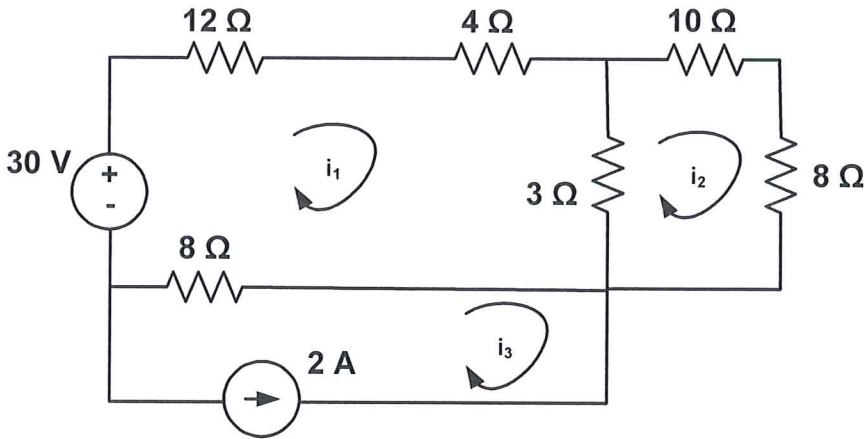
$$\frac{V_2}{5000} - \frac{V_1}{5000} + 0.007 + \frac{V_2}{20000} - \frac{5V_1}{5000} + \frac{5V_2}{5000} = 0$$

$$-\frac{6V_1}{5000} + \left(\frac{6}{5000} + \frac{1}{20000}\right)V_2 = -0.007 \text{ Simplify.}$$

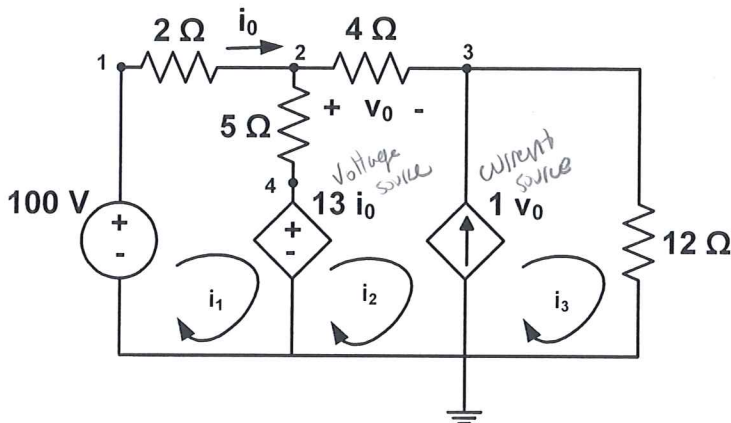
b)



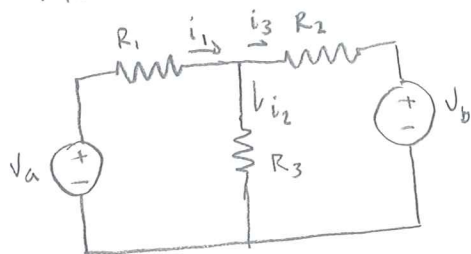
c)



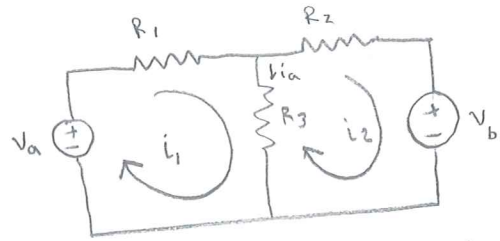
d)



2.5 Mesh-Current Analysis



i_1, i_2, i_3 are branch currents



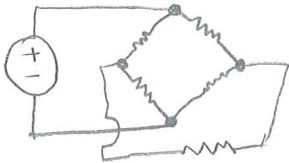
i_1 & i_2 are mesh currents

i_a = branch current = $i_1 - i_2$ through R_3

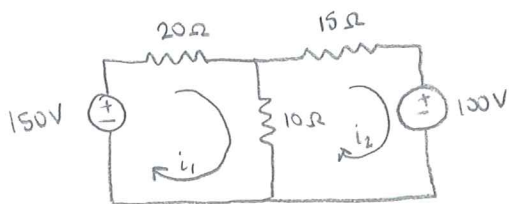
Mesh - loop that does not contain any other loops

• mesh-current analysis only works for planar circuits

• all loops must go in same direction



still planar but it looks like wires cross

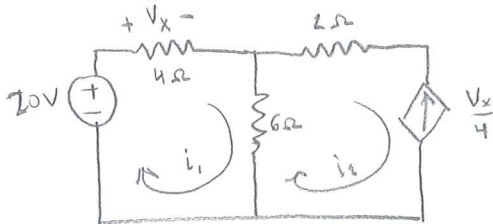


find mesh currents i_1 & i_2

mesh 1: KVL $-150 + 20i_1 + 10(i_1 - i_2) = 0$

$$\textcircled{1} 3i_1 - i_2 = 15$$

Mesh 2 $15i_2 + 100 + 10(i_2 - i_1) = 0$



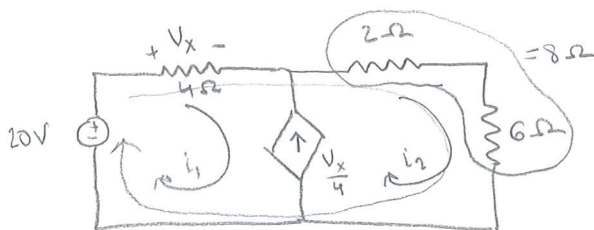
mesh 2 $i_2 = -\frac{V_x}{4}$ $V_x = 4i_1$

$$i_2 = -\frac{4i_1}{4} = -i_1$$

$$\textcircled{1} i_1 + i_2 = 0$$

mesh 1 $\textcircled{2} -20 + 4i_1 + 6(i_1 - i_2) = 0$

} solve



current source shared between 2 meshes

→ supermesh

S.M. (1,2) KVL around combined meshes

$$-20 + 4i_1 + 8i_2 = 0$$

$$\textcircled{1} i_1 + 2i_2 = 5$$

branch current $\frac{V_x}{4} = i_2 - i_1$

$$\frac{4i_1}{4} = i_2 - i_1 \Rightarrow 2i_1 - i_2 = 0$$

Notes 1-26-11

Part c on handout) mesh 3: $i_3 = -2A$ ①

mesh 1 $-30 + 16i_1 + 3(i_1 - i_2) + 8(i_1 - i_3) = 0$

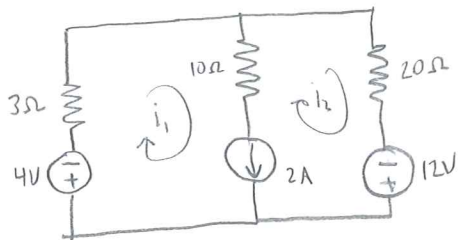
mesh 2 $3(i_2 - i_1) + 18i_2 = 0$ ③ $-3i_1 + 21i_2 = 0$

② $27i_1 - 3i_2 - 8i_3 = 30$

$$\begin{bmatrix} 0 & 0 & 1 \\ 17 & -3 & -8 \\ -3 & 21 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 30 \\ 0 \end{bmatrix}$$

Notes 1-28-11

Mesh current analysis

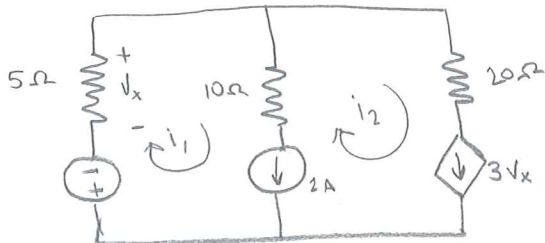


Supermesh (1,2)

KVL: $4 + 5i_1 + i_2 - 12 = 0$

① $5i_1 + 20i_2 = 8$

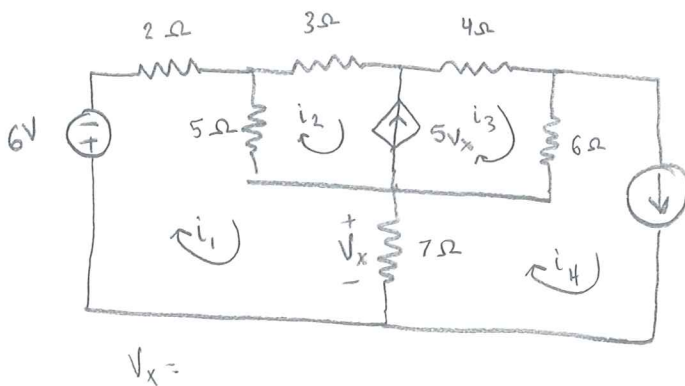
branch current: ② $i_1 - i_2 = 2$



mesh 2: $i_2 = 3V_x = 3(-5i_1)$

$V_x = -5i_1$ ① $15i_1 + i_2 = 0$

"branch" eqn ② $i_1 - i_2 = 2A$



mesh 4: $i_4 = 4A$ ①

Super 2,3

KVL: $3i_2 + 4i_3 + 6(i_3 - i_4) + 5(i_2 - i_1) = 0$

② $-5i_1 + 8i_2 + 10i_3 - 6i_4 = 0$

branch $i_3 - i_2 = 5V_x$ $V_x = 7(i_1 - i_4)$
 $= 5(7i_1 - 7i_4)$

③ $35i_1 + i_2 - i_3 - 35i_4 = 0$

mesh 1

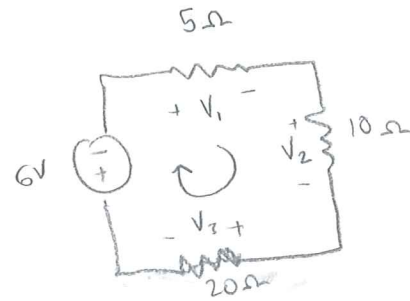
$6 + 2i_1 + 5(i_1 - i_2) + 7(i_1 - i_4) = 0$

④ $14i_1 - 5i_2 - 7i_4 = -6$

1-28-11 continued #2

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ -5 & 8 & 10 & -6 \\ 35 & 1 & -1 & -35 \\ 14 & 5 & 0 & -7 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \\ -6 \end{bmatrix}$$

KVL sign convention



$$6 + V_1 + V_2 + V_3 = 0$$
$$6 + 5i + 10i + 20i = 0$$

★ Test Wed.

1-31-10 Notes

3 formulas given on exam: current divider

voltage divider

Resistance $R = \frac{pL}{k}$

Not given: $P = vi$
 $w = \int_{t_0}^{t_1} p(t) dt$

KVL
KCL

Test: 7 questions

Node Voltage } including supernodes
Mesh Currents }

Power & energy

Equivalent resistance (parallel & series)

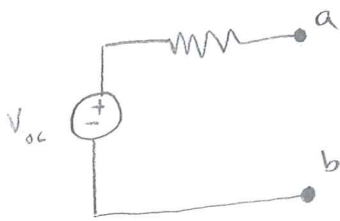
KVL, KCL, Ohm's law

Feb 7 2011

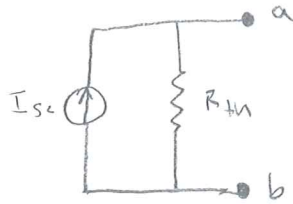
26

Thévenin & Norton Equivalents

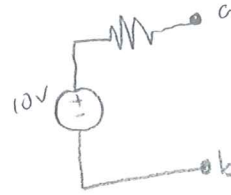
OC: open circuit
SC: short circuit
source transformation



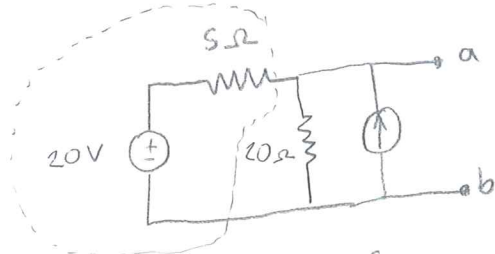
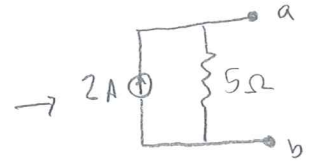
↑ Thévenin eq. ↓



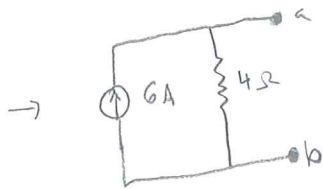
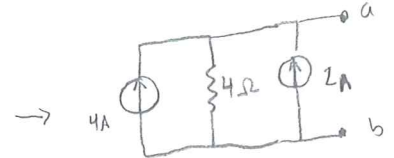
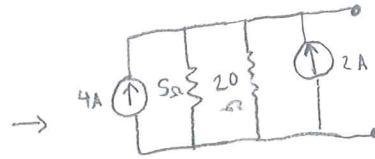
↑ Norton Eq ↓



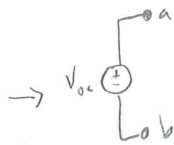
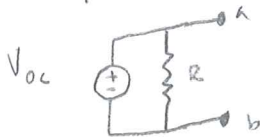
$$I = \frac{10V}{5\Omega} = 2A$$



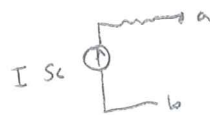
Method 1 - source transf.



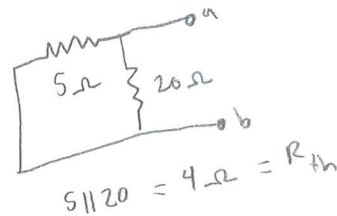
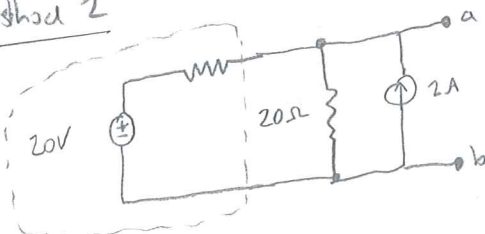
- Exceptions -



• no dependent sources for source transformation



Method 2



$$5 \parallel 20 = 4\Omega = R_{th}$$

Find R_{th}

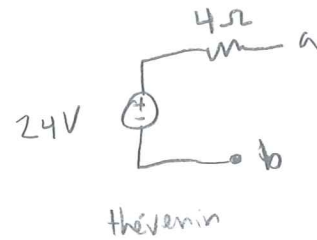
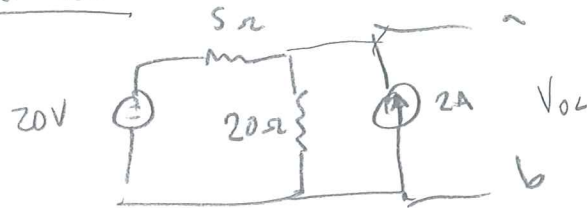
no dep. sres $\Rightarrow$ set all indep. sres = 0 $\rightarrow$ short circuit

use series & parallel combos of resistors

$\phi \rightarrow$ open circuit

#2 2-7-2011

Find V_{oc}

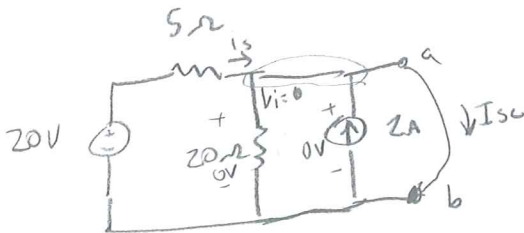


$$V_{oc} = V_2$$

$$V_1 = 20V$$

$$\frac{V_2 - V_1}{5} + \frac{V_2}{20} - 2 = 0$$

$$\dots V_2 \left(\frac{1}{4} \right) = 6 \rightarrow V_2 = 24V \Rightarrow V_{oc} = 24V$$

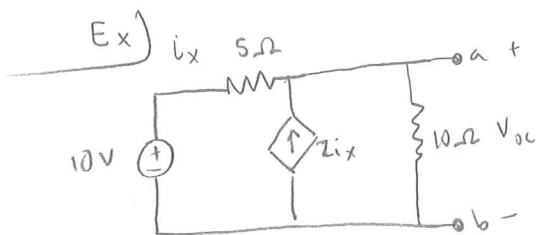


$$\begin{aligned} \text{KCL: } -i_s + 0 + I_{sc} &= 0 \\ \text{KVL: } -20 + 5i_s + 0 &= 0 \end{aligned} \quad \left. \vphantom{\begin{aligned} \text{KCL: } -i_s + 0 + I_{sc} &= 0 \\ \text{KVL: } -20 + 5i_s + 0 &= 0 \end{aligned}} \right\} \begin{aligned} I_{sc} &= 6A \\ i_s &= 4A \end{aligned}$$

Dependent Sources

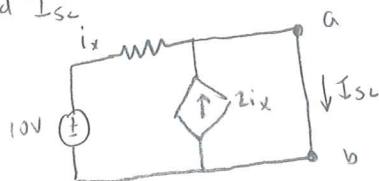
- cannot do a source transformation of a dependent source
- cannot source transform through a control variable
- Cannot find R_{th} by zeroing independent sources

• Can find V_{oc} & I_{sc}



Find V_{oc} KCL: $-i_x - 2i_x + \frac{V_{oc}}{10} = 0$ $V_{oc} - 30i_x = 0$
KVL: $-10 + 5i_x + V_{oc} = 0$ $V_{oc} = 30i_x \Rightarrow 30\left(\frac{2}{7}\right) = \frac{60}{7}V$
 $-35i_x = -10$
 $i_x = \frac{10}{35} = \frac{2}{7}A$

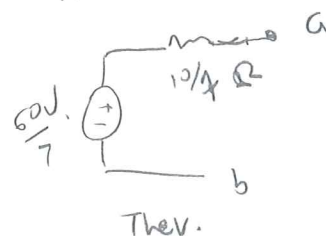
Find I_{sc}



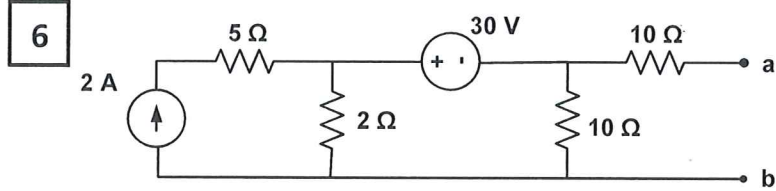
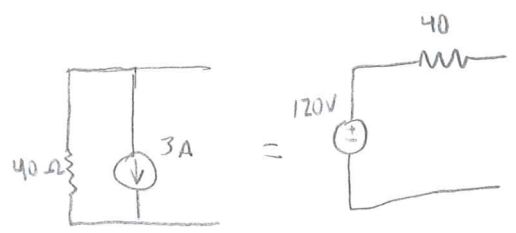
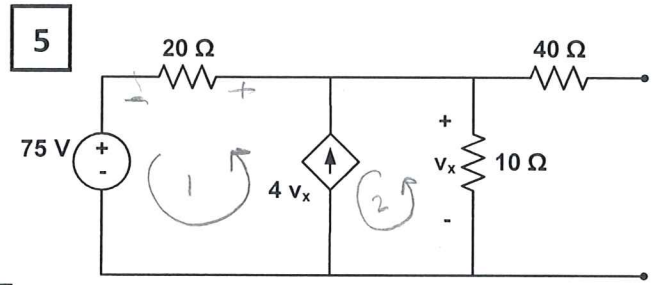
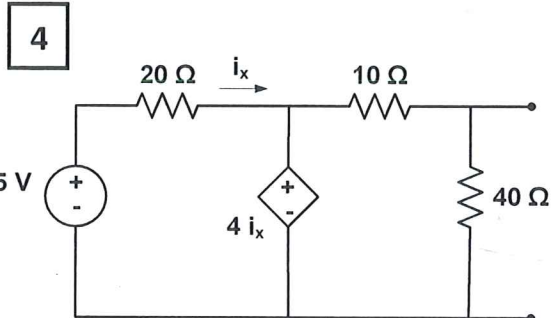
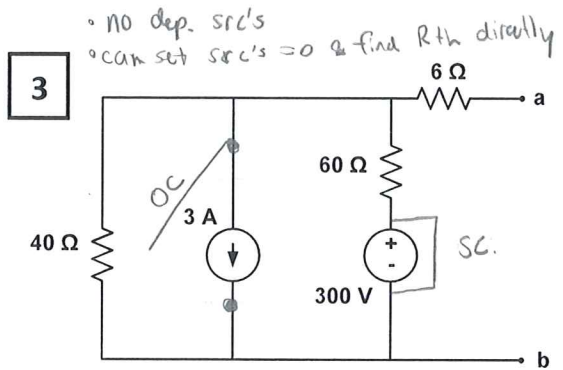
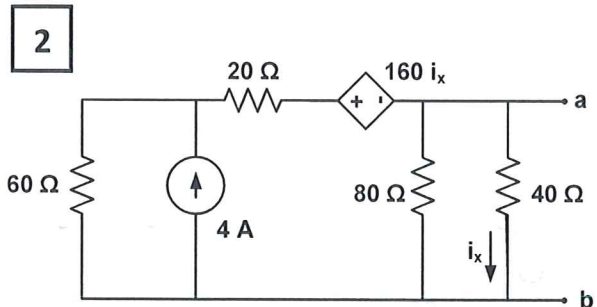
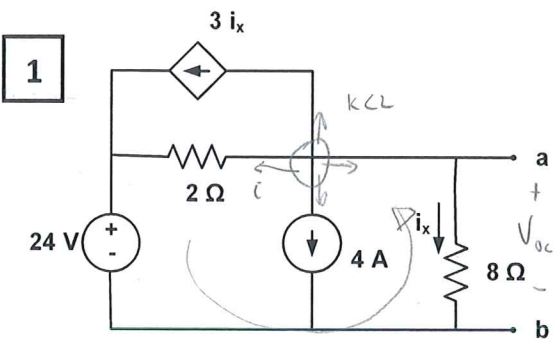
$$\begin{aligned} i_x - 2i_x + I_{sc} &= 0 \\ I_{sc} &= 3i_x \\ I_{sc} &= 6A \end{aligned}$$

$$-10 + 5A$$

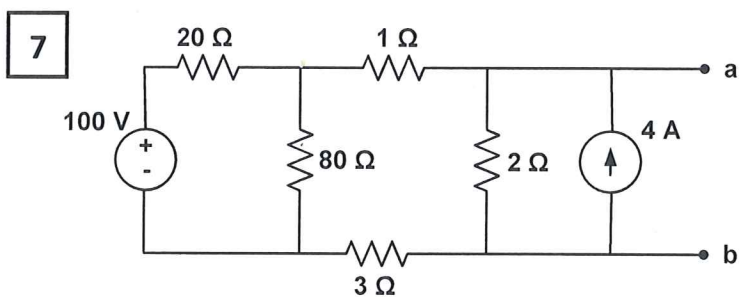
$$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{60/7V}{6A} = \frac{10}{7}\Omega$$

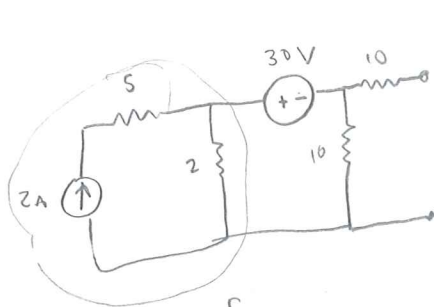


2-9-11

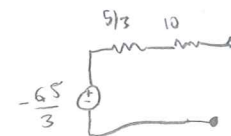
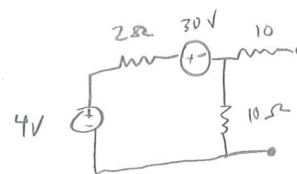
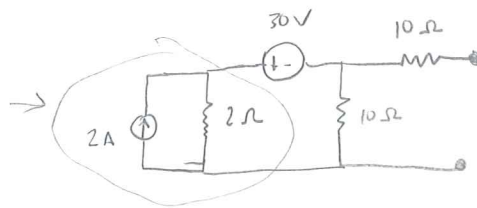


ON Back

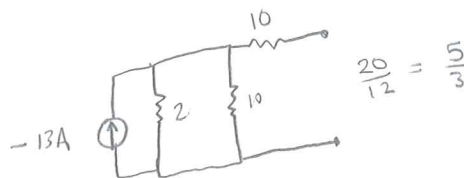
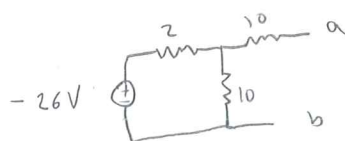




Source transf.

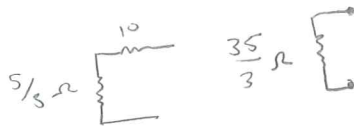
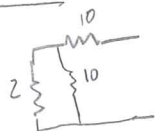


↑
makes Thévenin



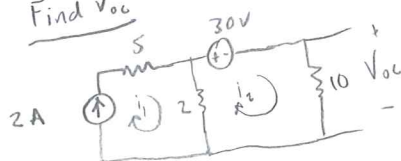
$$\frac{20}{12} = \frac{5}{3}$$

Find R_{th}



$$\frac{35}{3} \Omega$$

Find V_{oc}



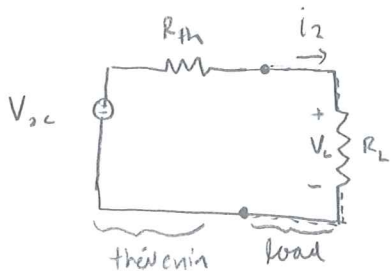
$$i_1 = 2A \quad V_{oc} = 10i_2 = 10 \left(\frac{-13}{6} \right) = -\frac{130}{6} V = -\frac{65}{3} V$$

$$2(i_2 - i_1) + 30 + 10i_2 = 0$$

$$12i_2 = -30 + 4 = -26$$

$$i_2 = -\frac{13}{6}$$

Notes 2-9-11) Maximum Power Transfer



Power delivered to load $P_L = i_L V_L$

$$V_L = \frac{R_L}{R_L + R_{th}} V_{oc}$$

$$P_L = \frac{R_L}{(R_L + R_{th})^2} V_{oc}^2 \leftarrow \text{find } R_L \text{ s.t. this function is a maximum}$$

$$i_L = \frac{V_{oc}}{R_L + R_{th}}$$

$$R_L = R_{th}$$

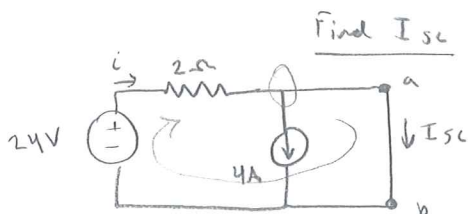
(load matching)

$$P_{L, \max} = \frac{R_{th}}{(2R_{th})^2} V_{oc}^2 = \frac{V_{oc}^2}{4R_{th}}$$

form on test: 1 step source transformation then find R_{th} & ask what should R_L be

ex) #1 on handout Find V_{oc}

$$\begin{aligned} \textcircled{1} \quad & \delta i_x = V_{oc} \quad \underline{\text{KCL}} \quad 4i_x + i = 4 \quad \textcircled{2} \\ \underline{\text{KVL}} \quad & 2i + 24 - V_{oc} = 0 \\ & i = \frac{V_{oc}}{2} - 12 \quad \textcircled{3} \end{aligned} \quad \left. \vphantom{\begin{aligned} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \end{aligned}} \right\} V_{oc} = 8V$$

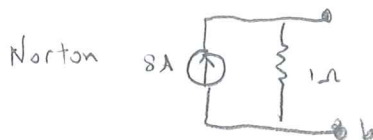
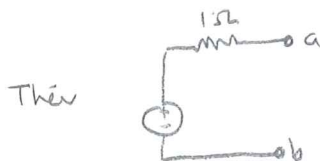


$$\underline{\text{KCL}}: -i + 4 + I_{sc} = 0$$

$$I_{sc} = i - 4 = 12 - 4 = 8A$$

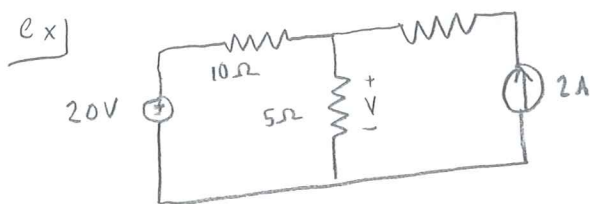
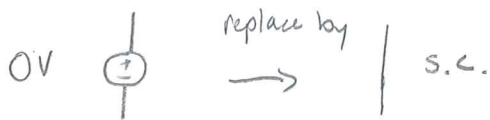
$$\underline{\text{KVL}}: -24 + 2i = 0 \quad i = 12A$$

$$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{8V}{8A} = 1\Omega$$



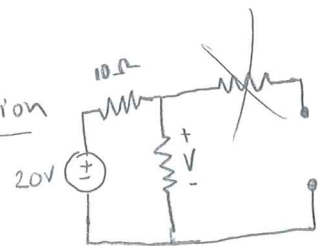
2.7] superposition - total response (voltage or current) is the sum of the responses to the individual (independent) src's

keep 1 src on, turn other indep. src. to zero



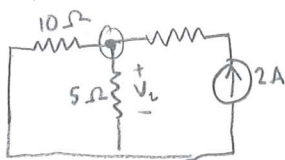
← Find V. using superposition

Keep 20V src.



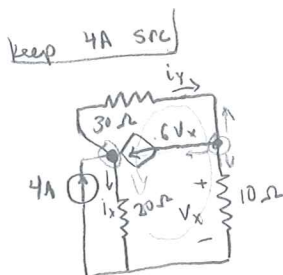
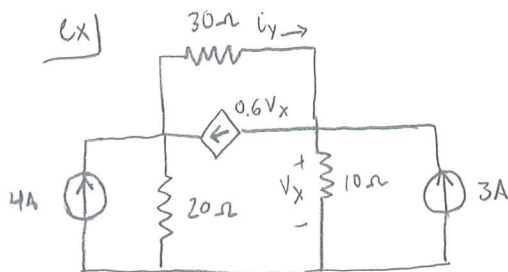
$$V = \frac{5}{5+10} (20) = \frac{20}{3} \text{ V}$$

② keep the 2A source



$$-2A + \frac{V_2}{5} + \frac{V_2}{10} \Rightarrow \frac{3V_2}{10} = 2$$

$$V_2 = \frac{20}{3} \text{ V}$$



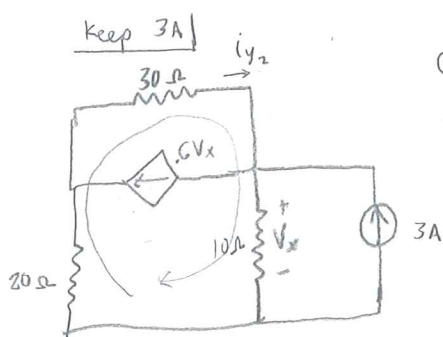
KCL $-i_{y1} + 0.6V_x + \frac{V_x}{10} = 0$

$-i_{y1} + 0.7V_x = 0$ ①

KCL $-4 + i_{y1} - 0.6V_x + i_x = 0$ ②

$30(i_{y1}) + 20i_x - V_x = 0$ ③

$i_{y1} = 2.33 \text{ A}$



① $0.6V_x - i_{y2} - 3 + \frac{V_x}{10} = 0$

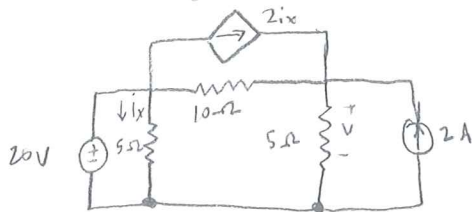
② $-i_x + i_{y2} - 0.6V_x = 0$

③ $20i_x + 30i_{y2} + V_x = 0$

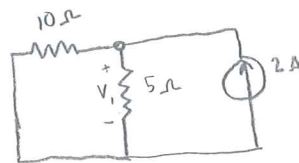
$i_{y2} = 1.375 \text{ A}$

$i_y = i_{y1} + i_{y2} = 2.33 + 1.375 = 3.704 \text{ A}$

Find V using superposition



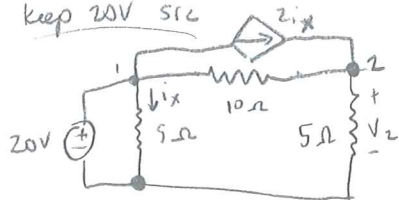
Keep 2A src. $i_x = 0$



$$-2 + \frac{V_1}{5} + \frac{V_1}{10} = 0$$

$$V_1 = \frac{20}{3} V$$

Keep 20V src



Node Voltages

$$V_2 = ? \quad V_1 = 20V$$

$$\frac{V_2 - V_1}{10} - 2i_x + \frac{V_2}{5} = 0$$

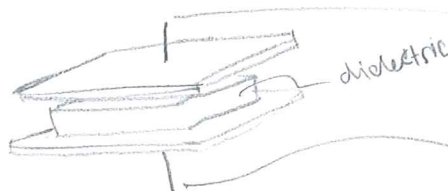
$$V_2 = -\frac{100}{3} V$$

$$V = V_1 + V_2 = \frac{20}{3} + \frac{100}{3} = \frac{120}{3} = 40V$$

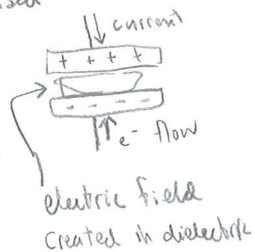
• why get rid of $2i_x$ for 1st part of solving?

13.2] capacitance - Energy is stored in the electric field and can be released back into circuit

parallel plate capacitor



conducting plates



$$q = Cv = \text{charge stored}$$

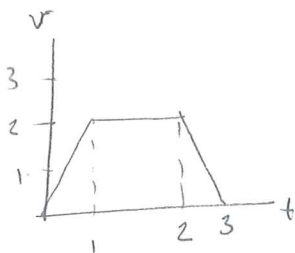
V = voltage

$$C = \text{capacitance, Farads } F = \frac{\text{Coulombs}}{\text{Volt}}$$

$$i = \frac{dq}{dt} = C \frac{dv}{dt} \quad \text{current}$$

$$q(t) = q(t_0) + \int_{t_0}^t i dx \quad \text{charge}$$

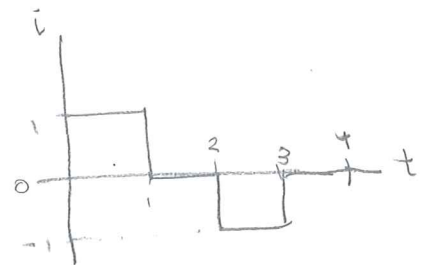
$$v(t) = v(t_0) + \frac{1}{C} \int_{t_0}^t i dx = \frac{1}{C} \int_{-\infty}^t i dx \quad \text{voltage}$$



$$C = 0.5 F$$

find $i(t)$

$$i(t) = \begin{cases} 0.5(2) & 0 < t < 1 \\ 0.5(0) & 1 < t < 2 \\ 0.5(-2) & 2 < t < 3 \\ 0.5(0) & t > 3 \end{cases} = \begin{cases} 1 \\ 0 \\ -1 \\ 0 \end{cases}$$



Notes Feb 14 2011

$$i(t) = 2e^{-4t} \text{ A}$$

$$v(0) = 2 \text{ V}$$

find $v(t)$ for $t > 0$

$$v(t) = v(0) + \frac{1}{0.5} \int_0^t 2e^{-4\lambda} d\lambda$$

$$= \dots 3 - e^{-4t} \text{ V}$$

Stored Energy

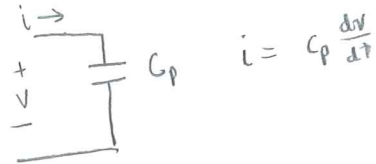
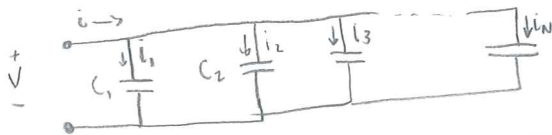
u-substitution

$$W(t) = \int_{t_0}^t p(\lambda) d\lambda = \int_{t_0}^t C \frac{dv}{d\lambda} v d\lambda$$

$$W(t) = \frac{1}{2} C v^2$$

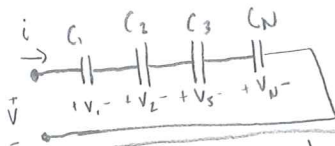
$$q = C v$$

3.2) - Series & parallel capacitors



$$i = i_1 + i_2 + \dots + i_N$$

$$C_p = C_1 + C_2 + C_3 + \dots + C_N$$



$$V = V_1 + V_2 + V_3 + \dots + V_N$$

$$V = \frac{1}{C_s} \int_{-\infty}^t i d\lambda$$

$$C_s = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_N}}$$

ex)

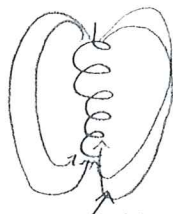
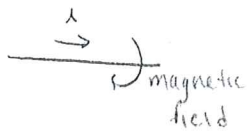
$$= \frac{4(8)}{4+8} = \frac{8}{3} \text{ F}$$

$$= \frac{2(\frac{20}{3})}{2 + \frac{20}{3}} = \dots \frac{20}{13} \text{ F}$$

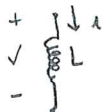
Feb 16

3.4 Inductance

Inductance - formed by coiling a wire around a form.



energy is stored in the magnetic field.



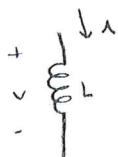
$$v(t) = L \frac{di}{dt}$$

L = inductance
Henries (H)

$$(H = \frac{Vs}{A})$$

$$i(t) = \frac{1}{L} \int_{-\infty}^t v d\lambda = i(t_0) + \frac{1}{L} \int_{t_0}^t v d\lambda$$

stored energy $w(t) = \int_{t_0}^t p(\lambda) d\lambda = \frac{1}{2} L i^2$



$$i(t) = 10 \sin(4t) \text{ A}$$

$$L = 5 \text{ mH}$$

Find $v(t)$

$$v(t) = L \frac{di}{dt} = .005 (40 \cos(4t)) = .2 \cos(4t) \text{ V}$$

$$v(t) = 10e^{-2t} \text{ V}$$

$$L = .1 \text{ H}$$

$$i(0) = 5 \text{ A}$$

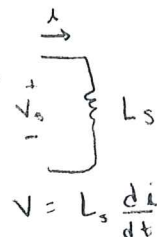
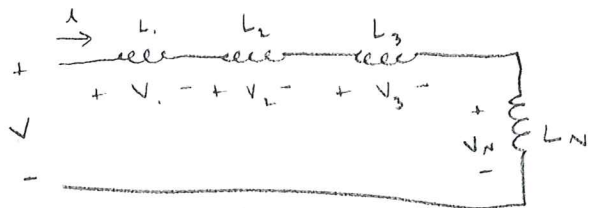
Find $i(t)$ for $t > 0$

$$i(t) = i(0) + \frac{1}{L} \int_0^t v(\lambda) d\lambda$$

$$5 + 10 \int_0^t 10e^{-2\lambda} d\lambda$$

$$5 + 10 (-5e^{-2\lambda})_0^t$$

$$i(t) = 55 - 50e^{-2t} \text{ A}$$



$$v = v_1 + v_2 + v_3 + \dots + v_N$$

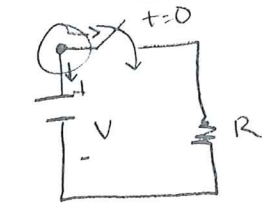
$$L_s \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt} + \dots + L_N \frac{di}{dt}$$

$$L_s = L_1 + L_2 + L_3 + \dots + L_N$$

$$\lambda = \lambda_1 + \lambda_2 + \lambda_3 + \dots + \lambda_N$$

$$\frac{1}{L_s} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots + \frac{1}{L_N}$$

4.1 RC Circuits



Discharging a capacitor

$$\text{KCL: } \frac{V}{R} + C \frac{dV}{dt} = 0$$

$$\frac{dV}{dt} + \frac{1}{RC} V = 0$$

$$V = k e^{st}$$

$$\frac{dV}{dt} = k s e^{st}$$

$$V = k e^{-\frac{1}{RC}t}$$

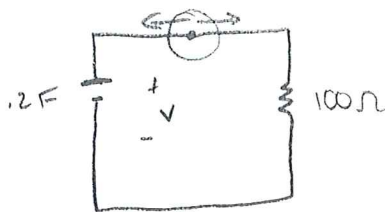
$$V(0^+) = k e^{-\frac{1}{RC}(0)} = V_0$$

$$V(t) = V_0 e^{-\frac{1}{RC}t}$$

$$\cancel{k s e^{st}} + \frac{1}{RC} k e^{st} = 0$$

$$s + \frac{1}{RC} = 0$$

Characteristic eq.



$$V(0^+) = 5V$$

$$C \frac{dV}{dt} + \frac{V}{R} = 0$$

$$0.2 \frac{dV}{dt} + \frac{V}{100} = 0$$

$$\frac{dV}{dt} + \frac{1}{20} V = 0$$

$$s + \frac{1}{20} = 0$$

$$s = -\frac{1}{20}$$

$$V(t) = k e^{-\frac{1}{20}t}$$

$$V(0^+) = k e^{-\frac{1}{20}(0)} = 5$$

$$k = 5$$

$$V(t) = 5 e^{-\frac{1}{20}t} \checkmark$$

Feb 18]

$$\text{Time constant } \tau = RC$$

$$\frac{dV}{dt} + \frac{1}{\tau} V = 0$$

$$V(t) = V_0 e^{-t/\tau}$$

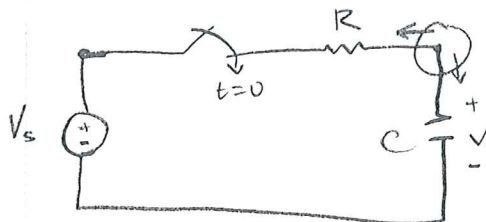
$$\tau = RC = 0.2(100) = 20 \text{ sec}$$

5 time constants for the voltage to be negligible

$$5\tau = 5(20) = 100 \text{ sec}$$

$$V(100 \text{ sec}) = 5 e^{-\frac{100}{20}} = 5 e^{-5} = 0.0337 V$$

Charge a Capacitor with a DC Source



$$\text{KCL: } C \frac{dV}{dt} + \frac{V - V_s}{R} = 0$$

$$\frac{dV}{dt} + \frac{1}{RC} V = \frac{1}{RC} V_s$$

Solve homogeneous (source-free)

Find particular solution (steady-state)

$$\frac{dV}{dt} + \frac{1}{RC} V = 0 \rightarrow V_c(t) = k e^{-\frac{1}{RC}t}$$

$$V_s = \# (\text{constant})$$

$$V_p(t) = k_2$$

$$k_2 = V_s$$

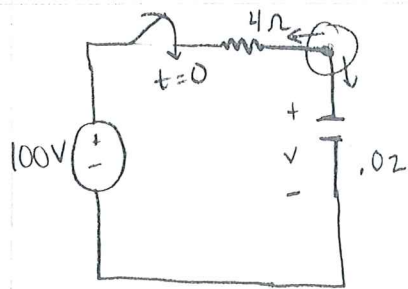
$$0 + \frac{1}{RC} k_2 = \frac{1}{RC} V_s$$

$$V(t) = V_c + V_p$$

$$= k_1 e^{-\frac{1}{RC}t} + V_s$$

then find

k, (initial conditions)



$$V(0^+) = 2V$$

Find $V(t)$ for $t > 0$

$$C \frac{dV}{dt} + \frac{V}{R} = 0$$

$$.02 \frac{dV}{dt} + \frac{V-100}{4} = 0$$

$$\frac{dV}{dt} + 12.5 V = 1250$$

$$\frac{dV}{dt} + 12.5 V = 0$$

$$s + 12.5 = 0$$

$$s = -12.5$$

$$V_c = K_1 e^{-12.5t}$$

$$\tau = \frac{1}{12.5} = .08 \text{ sec}$$

particular soln

$$V_p = K_2$$

$$0 + 12.5 K_2 = 1250$$

$$K_2 = 100$$

$$V(t) = K_1 e^{-12.5t} + 100$$

$$V(0) = 2 = K_1 e^0 + 100$$

$$K_1 = -98$$

$$V(t) = 100 - 98 e^{-12.5t} \text{ V}$$

Notes Feb 21)

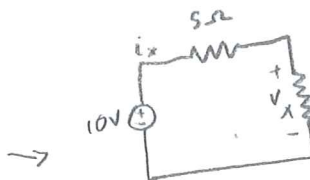
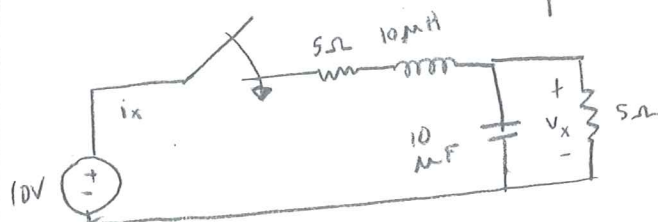
Dc steady-state
↓
constant sources

$$i_c(t) = C \frac{dV}{dt} \rightarrow V = \text{constant} \Rightarrow i_c(t) = 0$$

$$\frac{1}{C} \rightarrow \text{open circuit}$$

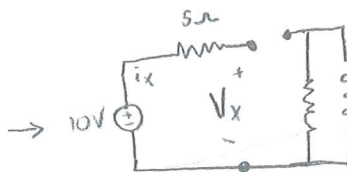
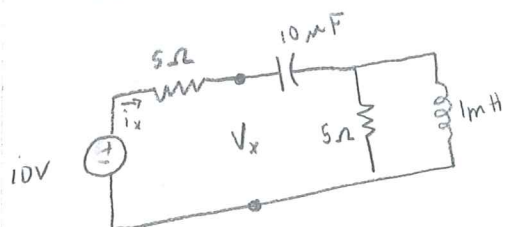
$$V_L(t) = L \frac{di}{dt} \rightarrow i \text{ constant} \rightarrow V_L(t) = 0$$

$$L \rightarrow \text{short circuit}$$



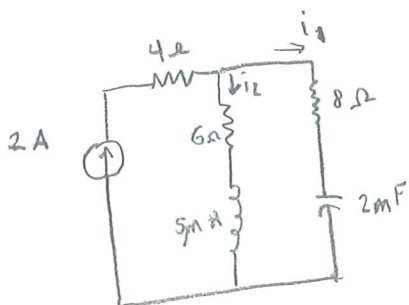
$$V_x = 5V$$

$$i_x = 1A$$



$$i_x = 0$$

$$V_x = 10V$$



$$i_1 = 0$$

$$i_2 = 2A$$

2-21-2010

Exam topics:

- Thevenin & Norton
 - with dependent sources
- Source transformations
- Maximum power transfer
- Superposition

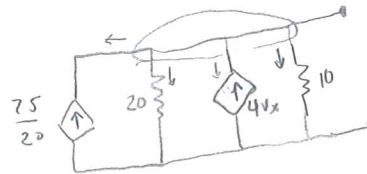
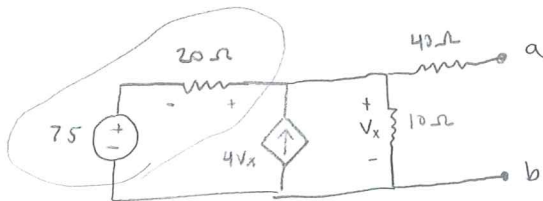
- RC circuits (1st order D.E)
- D.C. steady state

- Parallel/series combos of L & C
- given Voltage or Current, find opposite

$$V(t) + \frac{1}{T} i(t)$$

$i(t)$ & $v(t)$ relationships are given on front page of test

2-23-11 Exam review

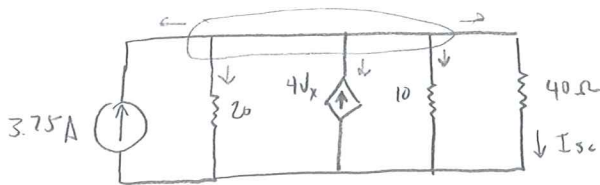


$$-3.75 + \frac{V_{OC}}{20} - 4V_{OC} + \frac{V_{OC}}{10} = 0$$

$$V_{OC} \left(\frac{1}{20} - \frac{40}{20} + \frac{1}{10} \right) = 3.75$$

$$V_{OC} = -0.97$$

find I_{SC}



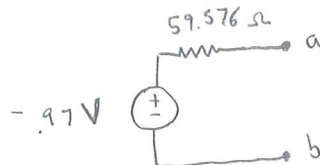
$$I_{SC} = \frac{V_x}{40} \quad \left(\frac{7}{40} - \frac{160}{40} \right) V_x = 3.75$$

$$V_x = -0.97$$

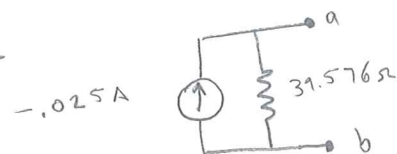
$$I_{SC} = \frac{V_x}{40} = -0.025 A$$

$$R_{Th} = \frac{V_{OC}}{I_{SC}} = 39.576 \Omega$$

Thevenin:

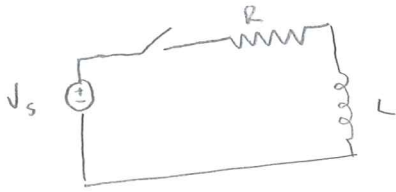


Norton



Feb 28 2011

4.3 RL Circuits



$$i(0) = 0 \text{ A} \quad -V_s + Ri + L \frac{di}{dt} = 0$$

$$\frac{di}{dt} + \left(\frac{R}{L}\right)i = \frac{V_s}{L} \rightarrow \frac{1}{\tau} = \frac{R}{L}$$

$$\tau = \frac{L}{R} \quad (\text{Time constant})$$

$$i_c = k_1 e^{\Delta t} \quad \text{homogeneous solution}$$

$$\frac{di}{dt} + \frac{R}{L}i = 0$$

$$i_c = k_1 e^{-\frac{R}{L}t} = k_c e^{-\frac{t}{\tau}}$$

$$s + \frac{R}{L} = 0$$

$$2 \frac{di}{dt} + 50i - 10 = 0$$

$$\frac{di}{dt} + 25i = 5$$

$$s + 25 = 0 \Rightarrow s = -25$$

$$i_c(t) = k_1 e^{-25t}$$

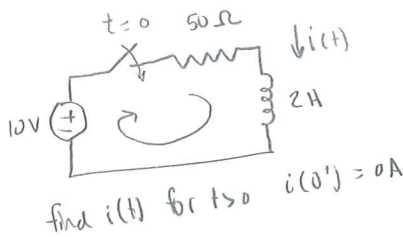
forced solution (steady state soln)

$$i_p(t) = k_2$$

$$\frac{di_p}{dt} = 0$$

$$0 + 25k_2 = 5 \quad k_2 = \frac{1}{5}$$

$$i_p(t) = \frac{1}{5} \text{ A}$$



$$i(t) = k_1 e^{-25t} + \frac{1}{5}$$

$$i(0) = 0 = k_1 e^0 + \frac{1}{5} \Rightarrow k_1 = -\frac{1}{5}$$

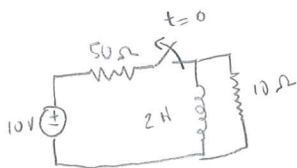
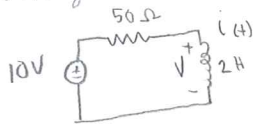
$$i(t) = -\frac{1}{5} e^{-25t} + \frac{1}{5} \text{ A}$$

DC steady state:

$$10 \text{ V} \quad 50 \Omega \quad i_p(t) = \frac{10}{50} = \frac{1}{5} \text{ A}$$

Voltage across inductor $v(t) = L \frac{di}{dt} = 2(5e^{-25t} + 0)$

$$v(t) = 10e^{-25t} \text{ V}$$



circuit has been closed for long time before $t = 0$
prior to $t = 0$ ($t < 0$)



KVL $L \frac{di}{dt} + Ri = 0$

$$2 \frac{di}{dt} + 10i = 0$$

$$\frac{di}{dt} + 5i = 0$$

$$i(t) = k e^{-5t}$$

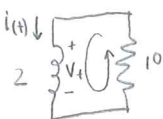
$$i(0^-) = i(0^+) = \frac{1}{5} \text{ A}$$

$$i(0^+) = \frac{1}{5} = k e^0 \quad k = \frac{1}{5}$$

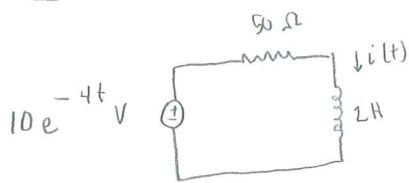
$$i(t) = \frac{1}{5} e^{-5t} \text{ A}$$

$$v(t) = L \frac{di}{dt} = 2(-e^{-5t})$$

$$v(t) = -2e^{-5t} \text{ V}$$



Feb 28 2011



$$-4k_2 e^{-4t} + 25k_2 e^{-4t} = 5e^{-4t}$$

$$21k_2 = 5 \quad k_2 = \frac{5}{21}$$

$$i_p(t) = \frac{5}{21} e^{-4t}$$

$$i(t) = k_1 e^{-25t} + \frac{5}{21} e^{-4t}$$

Given $i(0^+) = 0A$ Find $i(t)$ for $t > 0$

KVL $2 \frac{di}{dt} + 50i - 10e^{-4t} = 0$

$\frac{dw}{dt} + 25i = 5e^{-4t}$

$\downarrow$

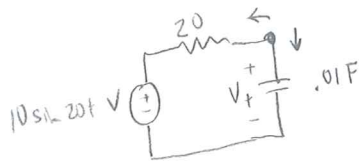
Solve $\frac{di}{dt} + 25i = 0$

$i_c(t) = k_1 e^{-25t}$

$i_p(t) = k_2 e^{-4t}$

$\frac{di_p}{dt} = -4k_2 e^{-4t}$

$$i(0^+) = 0 = k_1 e^0 + \frac{5}{21} e^0 \rightarrow k_1 = -\frac{5}{21}$$



solve homogen. $V_c(t) = k_1 e^{-5t}$

$V(0^+) = 6V$ find $V(t)$ for $t > 0$

$$.01 \frac{dV}{dt} + \frac{V - V_s}{20} = 0 \quad \frac{dw}{dt} + 5V = 50 \sin(20t)$$

$$V_p(t) = A \sin(20t) + B \cos(20t)$$

$$V_p'(t) = 20A \cos(20t) - 20B \sin(20t)$$

$$20A \cos(20t) - 20B \sin(20t) + 5(A \sin(20t) + B \cos(20t)) = 50 \sin 20t$$

$$(20A + 5B) \cos(20t) + 5A - 20B$$

$$V(t) = k_1 e^{-5t} + \left(\frac{10}{17}\right) \sin(20t) - \left(\frac{40}{17}\right) \cos(20t)$$

$$V(0^+) = k_1 - \frac{40}{17} \rightarrow k_1 = \frac{40}{17} \text{ plug in.}$$

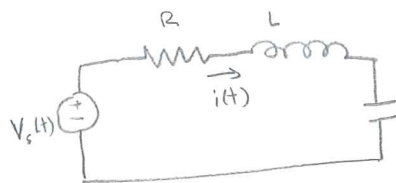
$$f(t) = A \quad V_p = k_2$$

$$f(t) = B e^{at} \quad V_p = k_2 e^{at}$$

$$f(t) = A \sin(bt) \quad V_p = k_2 \cos bt + k_3 \sin bt$$

$$= B \cos(bt)$$

4.2] 2nd order circuits



KVL $-V_s + Ri + L \frac{di}{dt} + \frac{1}{C} \int_{-\infty}^t i d\lambda = 0$

$$-\frac{dV_s}{dt} + R \frac{di}{dt} + L \frac{d^2 i}{dt^2} + \frac{1}{C} i = 0$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{1}{LC} i = \frac{1}{L} \frac{dV_s}{dt}$$

Solve for homogeneous
Solution

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

factor & solve for s Real, distinct roots
 $i(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t}$

Real repeated roots

$$i(t) = k_1 e^{s_1 t} + k_2 t e^{s_1 t}$$

Complex roots

$$i(t) = k_1 e^{s_1 t} \cos(\omega_d t) + k_2 e^{s_1 t} \sin(\omega_d t)$$

$$s = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4\left(\frac{1}{LC}\right)}}{2} = \frac{-\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}}{1}$$

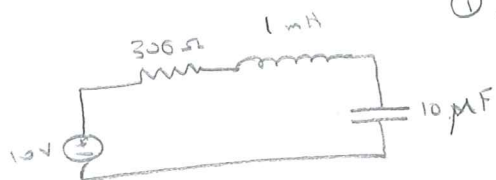
$$\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} \quad \text{damped natural frequency}$$

$$\frac{R}{2L} = \text{damping coefficient}$$

$$s^2 + 2\omega_n \zeta s + \omega_n^2$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

Roots $\left\{ \begin{array}{l} \text{Real distinct} - \text{overdamped} \\ \text{Real repeated} - \text{critically damped} \\ \text{Complex} - \text{underdamped} \end{array} \right.$



①

model diff eq

$$-10 + 300i + 0.001 \frac{di}{dt} + \frac{1}{1 \times 10^{-6}} \int_{-\infty}^t i d\lambda = 0$$

$$\frac{di}{dt} + 300 \frac{di}{dt} + 10^9 i = 0$$

$$s^2 + 3 \times 10^5 s + 10^9 = 0$$

$$s = \frac{3 \times 10^5 \pm \sqrt{(3 \times 10^5)^2 - 4(10^9)}}{2}$$

$$s = -3371, -296629$$

$$i(t) = k_1 e^{-3371t} + k_2 e^{-296600t}$$

② use initial conditions

$$i(0^+) = i_0$$

$$V(0^+) = V_0$$

gives $k_1 = k_2$

$$\text{KVL} = -10 + 300i + 0.001 \frac{di}{dt} + V = 0$$

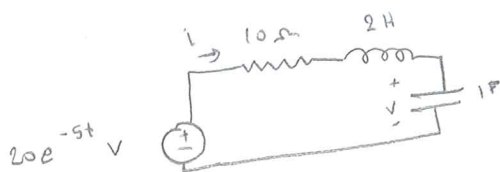
$$-10 + 300i(0) + 0.001 \frac{di}{dt} \Big|_{t=0} + V(0) = 0 \Rightarrow \frac{di}{dt} \Big|_{t=0} = \frac{10-2}{0.001} = 8000$$

take derivative of main form of solution, for 2nd boundary condition

$$\frac{di}{dt} \Big|_{t=0} = -3371 k_1 e^{-3371t} - 296000 k_2 e^{-296000t}$$

$$\frac{di}{dt} \Big|_{t=0} = -3371 k_1 - 296000 k_2 = 8000$$

3-4-2011



Find $i(t)$ if $i(0^+) = 0A$
 $V(0^+) = 4V$

$$KVL = -20e^{-5t} + 10i + 2 \frac{di}{dt} + \frac{1}{1} \int_{-\infty}^t i d\lambda = 0$$

$$100e^{-5t} + 10 \frac{di}{dt} + 2 \frac{d^2i}{dt^2} + i = 0$$

$$\frac{d^2i}{dt^2} + 5 \frac{di}{dt} + \frac{1}{2} i = -50e^{-5t}$$

Solve $\frac{d^2i}{dt^2} + 5 \frac{di}{dt} + \frac{1}{2} i = 0$

$$s^2 + 5s + \frac{1}{2} = 0 \Rightarrow s = \frac{-5 \pm \sqrt{23}}{2} = -4.9, -0.1$$

Find Particular solution

$$i_p(t) = A e^{-5t}$$

$$\frac{di_p}{dt} = -5A e^{-5t}$$

$$\frac{d^2i}{dt^2} = 25A e^{-5t}$$

$$\frac{1}{2}A = -50 \Rightarrow A = -100$$

$$i(t) = i_c + i_p = 100e^{-5t} + k_1 e^{-4.9t} + k_2 e^{-0.1t}$$

$$i(0) = 0 = k_1 + k_2 - 100 \Rightarrow k_1 + k_2 = -100$$

$$0 = -20 + 10 \left(\frac{di}{dt} \right)_{t=0} + 2 \left(\frac{d^2i}{dt^2} \right)_{t=0} + i(0) \Rightarrow 2 \left(\frac{di}{dt} \right)_{t=0} = 16 \Rightarrow \left(\frac{di}{dt} \right)_{t=0} = 8$$

$$\left(\frac{di}{dt} \right)_{t=0} = 8 = -4.9k_1 - 0.1k_2 + 500 \leftarrow \text{comes from}$$

$$-4.9k_1 - 0.1k_2 = -492$$

$$-49k_1 - k_2 = 4920$$

$$k_1 + k_2 = -100$$

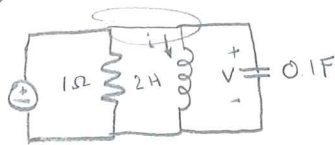
$$-48k_1 = -4800$$

$$i(t) = 100.4 e^{-4.9t} - 0.4 e^{-0.1t} - 100 e^{-5t} \text{ A}$$

March 4 2011

Ex)

$20 \cos(2t)$



Find $v(t)$ for $t > 0$ if $i(0) = 2A$
 $v(0) = 1V$

Skipped notes for this part

1) KCL in terms of v

2) Evaluate at zero, solve for $\frac{dv}{dt}|_{t=0}$

3) derivative of KCL in ① $\Rightarrow$ ODE

4) Source free solution $s^2 + 10s + 5 = 0$, solve for s

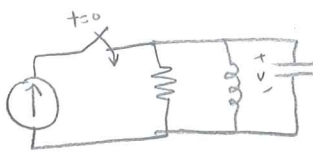
5) write general form $v_c(t) = k_1 e^{-1.5t} + k_2 e^{-0.5t}$

6) Particular: find $v_p(t) = A \sin(2t) + B \cos(2t)$
 $v_p'(t) = \dots$
 $v_p''(t) = \dots$ } $\rightarrow$ plug into ODE, solve for A & B

Hint: ODE is $\frac{d^2 v}{dt^2} + 10 \frac{dv}{dt} + 5v = 400 \sin(2t)$

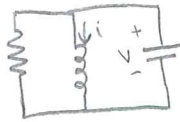
$$v(t) = k_1 e^{-9.5t} + k_2 e^{-0.5t} - 0.918 \sin(2t) + 19.95 \cos(2t)$$

Ex)



Switch has been open
 for long time before being
 closed @ $t=0$

$t < 0$:



$$i(0^-) = 0 \rightarrow i(0^+) = 0$$

$$v(0^-) = 0 \rightarrow v(0^+) = 0$$

Notes 3-7-2011

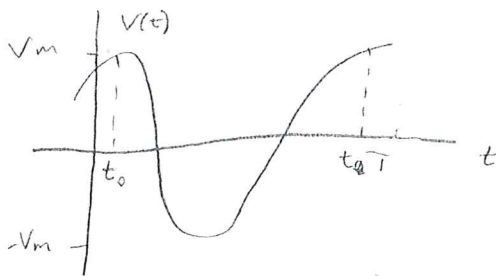
5.1 Sinusoidal Currents + Voltages

$$v(t) = V_m \cos(\omega t + \theta)$$

$$\omega = 2\pi f$$

T = period (sec)

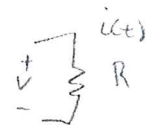
$$1/f = T = 2\pi/\omega$$



V_m = amplitude
 ω = angular frequency ($\frac{\text{rad}}{\text{sec}}$)
 θ = phase angle
 f = frequency (Hertz) (Hz)

RMS - Root-mean-Square

$$p(t) = v i = v^2/R = i^2 R$$



$$v(t) = V_m \cos(\omega t + \theta)$$

find average power P_{avg}

$$P_{avg} = \frac{\int_0^T p(t) dt}{T} = \frac{\int_0^T v^2/R dt}{T}$$

$$= \frac{1}{T} \frac{\int_0^T v^2 dt}{R} = \left(\frac{\sqrt{\frac{1}{T} \int_0^T v^2 dt}}{R} \right)^2 = \frac{V_{rms}^2}{R}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

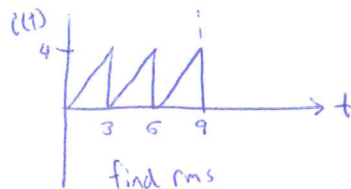
$$I_{rms} = \sqrt{\frac{1}{T} \int_0^T i^2 dt}$$

Notes 3-7 2011 continued. copy page 1st page from JMP.

$$V(t) = 10 \cos(20t - 40^\circ) \text{ V} \rightarrow V_{\text{rms}} = \frac{10}{\sqrt{2}} V_{\text{rms}}$$

$$i(t) = 20 \sin(15t + 30^\circ) \text{ A} \quad I_{\text{rms}} = \frac{20}{\sqrt{2}} A_{\text{rms}}$$

$$P_{\text{avg}} = \frac{V_{\text{rms}}^2}{R} = \frac{I_{\text{rms}}^2 R}{R}$$



$$\begin{aligned} I_{\text{rms}} &= \sqrt{\frac{1}{T} \int_0^T i^2 dt} \\ &= \sqrt{\frac{1}{6} \int_0^3 \left(\frac{4}{3}t\right)^2 dt} \\ &= \sqrt{\frac{1}{6} \int_0^3 \frac{16}{9}t^2 dt} \\ &= \sqrt{\frac{1}{6} \left[\frac{16}{9} \frac{t^3}{3}\right]_0^3} = \sqrt{\frac{16}{3}} \end{aligned}$$

Formulas for: I_{rms} (sinusoid) = $y = a \sin(2\pi ft)$ $RMS = \frac{a}{\sqrt{2}}$
 (Sawtooth) $y = 2a\{ft\} - a$ $RMS = \frac{a}{\sqrt{3}}$

5.2] Phasors If $u(t) = V_m \cos(\omega t + \theta)$ V
 the phasor $\underline{V} = V_m \angle \theta$ V

phasor is a complex # used to represent a sinusoidal voltage or current. $\angle$ angle

Euler's formula

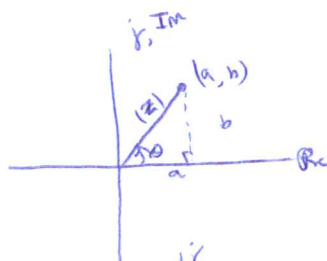
$$e^{j\omega t} = \cos \omega t + j \sin \omega t$$

$$\text{Re}\{e^{j\omega t}\} = \cos \omega t$$

$$|e^{j\omega t}| = 1$$

$$v(t) = V_m \cos(\omega t + \theta) = \text{Re}\{V_m e^{j(\omega t + \theta)}\} = \text{Re}\{V_m e^{j\theta} e^{j\omega t}\} = \text{Re}\{\underline{V} e^{j\omega t}\}$$

$$\underline{V} = V_m \angle \theta = V_m e^{j\theta}$$

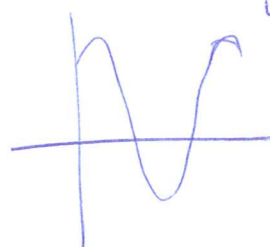
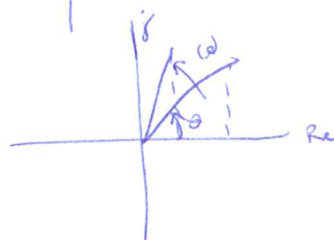


$$a + jb = |z| \angle \theta = |z| e^{j\theta}$$

Polar coordinate

$$|z| = \sqrt{a^2 + b^2}$$

$$\tan \theta = \frac{b}{a}$$



$$u(t) = V_m \cos(\omega t + \theta)$$

5-17-11 cont.

$$v(t) = 20 \cos(10t + 25^\circ) \text{ V}$$

"bold" $\rightarrow \underline{v} = 20 \angle 25^\circ \text{ V}$

$e^{j\omega t}$ gets dropped from everything

must have cos for phaser?

$$i(t) = 10 \sin(50t + 40^\circ) \text{ A} = 10 \cos(50t + 40^\circ - 90^\circ) \text{ A}$$

$$= 10 \cos(50t - 50^\circ) \text{ A}$$

$$\Rightarrow \underline{I} = 10 \angle -50^\circ \text{ A}$$

3-14-11

Exam 3 Monday March 21

Ohm's law

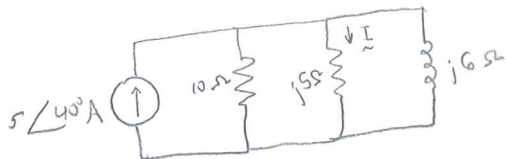
KVL

KCL

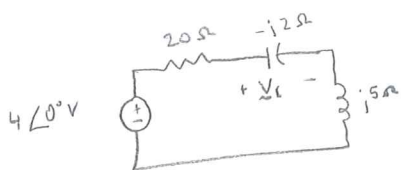
current & voltage dividers

mesh current analysis
node voltage analysis

with phasors



$$\text{Find } \underline{I} = \frac{\frac{1}{j5}}{\frac{1}{10} + \frac{1}{j5} + \frac{1}{j6}} 5 \angle 40^\circ = (0.526 \angle -15.3^\circ)(5 \angle 40^\circ) = 2.63 \angle 24.7^\circ \text{ A}$$

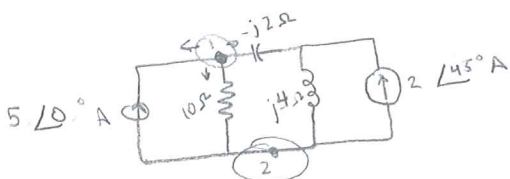


$$\underline{V}_1 = \frac{-j2}{20 - j2 + j5} (4 \angle 0^\circ) = .98 \times 10^{-2} (-98.5^\circ)(4 \angle 0^\circ) = 0.396 \angle -98.5^\circ \text{ V}$$

ON CALCULATOR $-j2 = -1 \cdot (-2)$

TI89 $a \angle b = (a \angle b)$ polar complex mode

* Standard form of phasor has a positive magnitude



$$\underline{n1}: -5 \angle 0^\circ + \frac{V_1}{10} + \frac{V_1 - V_2}{-j2} = 0$$

$$\left(\frac{1}{10} + j\frac{1}{2}\right)V_1 - j\frac{1}{2}V_2 = 5 \angle 0^\circ \quad (1)$$

$$\begin{cases} \frac{1}{j} \cdot \frac{j}{j} = \frac{j}{-1} = -j \\ \frac{1}{j} = -j \end{cases} \text{ special property}$$

$$\underline{n2}: \frac{V_2 - V_1}{-j2} + \frac{V_2}{j4} - 2 \angle 45^\circ = 0 \quad -j\frac{1}{2}V_1 + (j\frac{1}{2} - j\frac{1}{4})V_2 = 2 \angle 45^\circ$$

$$-j\frac{1}{2}V_1 + j\frac{1}{4}V_2 = 2 \angle 45^\circ \quad (2)$$

$$\text{Find } \begin{bmatrix} \frac{1}{10} + j\frac{1}{2} & -j\frac{1}{2} \\ -j\frac{1}{2} & j\frac{1}{4} \end{bmatrix} \begin{bmatrix} \underline{V}_1 \\ \underline{V}_2 \end{bmatrix} = \begin{bmatrix} 5 \angle 0^\circ \\ 2 \angle 45^\circ \end{bmatrix}$$

translated,

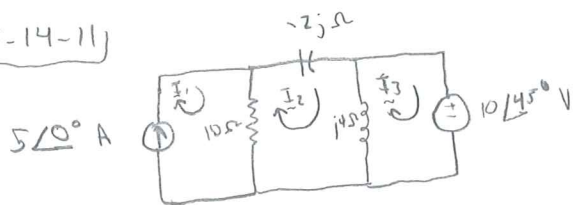
calculator

$$\underline{V}_1 = 16.32 \angle 98.6^\circ \text{ V} \quad \underline{V}_2 = 26.63 \angle 88.3^\circ \text{ V}$$

$$V_1(t) = 16.32 \cos(\omega t + 98.6^\circ)$$

$$V_2(t) = 26.63 \cos(\omega t + 88.3^\circ) \text{ V}$$

3-14-11)



Find mesh current phasors $\underline{I}_1$, $\underline{I}_2$, $\underline{I}_3$

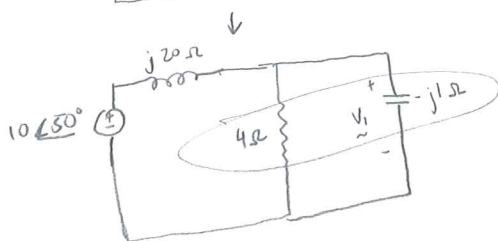
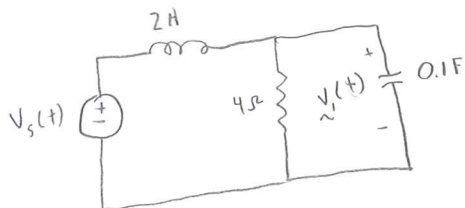
M₁: $\underline{I}_1 = 5\angle 0^\circ$

M₂: $10(\underline{I}_2 - \underline{I}_1) + (-j2)\underline{I}_2 + j4(\underline{I}_2 - \underline{I}_3) = 0$

M₃: $j4(\underline{I}_3 - \underline{I}_2) + 10\angle 45^\circ V = 0$

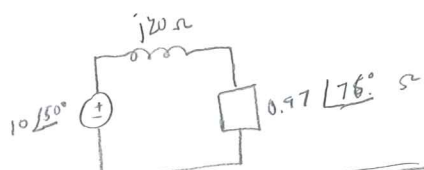
$$\begin{bmatrix} 1 & 0 & 0 \\ -10 & 10-j2 & -j4 \\ 0 & -j4 & j4 \end{bmatrix} \begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \\ \underline{I}_3 \end{bmatrix} = \begin{bmatrix} 5\angle 0^\circ \\ 0 \\ -10\angle 45^\circ \end{bmatrix}$$

$V_s(t) = 10 \cos(10t + 50^\circ) V$ Find steady state voltage



L: $Z = j\omega L = j(10)(2)$
C: $Z = \frac{1}{j\omega C} = -j \frac{1}{10(0.1)} = -j1$

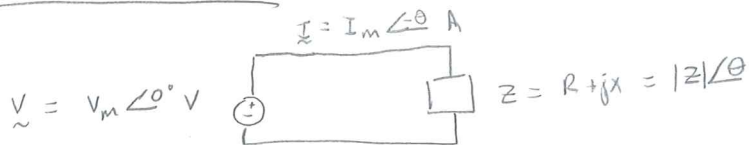
$4\Omega \parallel -j1\Omega = \frac{4(-j1)}{4-j1} \xrightarrow{\frac{Z_1 Z_2}{Z_1 + Z_2}} = 0.97\angle 75.96^\circ \Omega$



$\underline{V}_1 = \frac{0.97\angle -75.96^\circ}{0.97\angle -75.96^\circ + j20} (10\angle 50^\circ) = 0.51\angle -115.3^\circ V$

$V_1(t) = 0.51 \cos(10t - 115.3^\circ) V$

5.5 Power in AC circuits



$$\underline{I} = \frac{\underline{V}}{\underline{Z}}$$

$$v(t) = V_m \cos(\omega t) \text{ V} \quad i(t) = I_m \cos(\omega t - \theta) \text{ A}$$

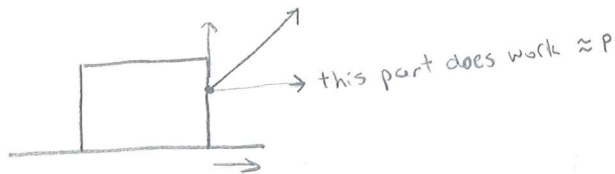
$p(t) = v(t) \cdot i(t) = \text{instantaneous power}$

$$= V_m I_m \cos(\omega t) \cos(\omega t - \theta)$$

$$= V_m I_m \cos \theta \cos^2(\omega t) + V_m I_m \sin \theta \cos(\omega t) \sin(\omega t)$$

$$= \underbrace{\frac{1}{2} V_m I_m \cos \theta (1 + \cos(2\omega t))}_{= P} + \underbrace{\frac{1}{2} V_m I_m \sin \theta \sin(2\omega t)}_{= Q}$$

Analogy: pulling a block



Avg. Power $P = \frac{1}{2} V_m I_m \cos \theta$ (Watts)

Reactive Power $Q = \frac{1}{2} V_m I_m \sin \theta$ (VAR)

(VAR = Volt-ampere reactive)

Apparent power:

$$\frac{P}{\cos \theta} = \frac{1}{2} V_m I_m = \frac{P}{\text{PF}} \text{ (VA)}$$

$$\cos \theta = \text{power factor PF}$$

↑ also designated $|S|$

Complex Power

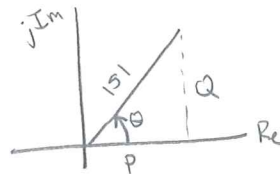
does not represent a real quantity, only a math expression

$$S = P + jQ \text{ (VA)}$$

$$= |S| \angle \theta$$

$$= \frac{1}{2} \underline{V} \underline{I}^* = \underline{V}_{\text{rms}} \underline{I}_{\text{rms}}^*$$

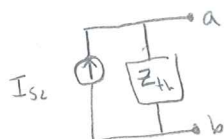
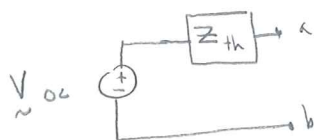
Power Triangle



Exam covers HW's 12-16

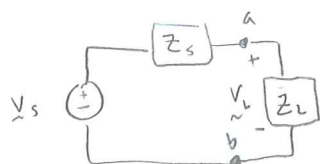
MONDAY

5.6 Thevenin & Norton Equivalents



$$V_{oc} = Z_{th} I_{sc}$$

Max Pwr Transfer



$$S_L = \frac{1}{2} V_L I_L^* = \frac{1}{2} \left(\frac{Z_L V_s}{Z_L + Z_s} \right) \left(\frac{V_s}{Z_L + Z_s} \right)^*$$

Voltage divider Ohm's law

$$Z_L = R_L + jX_L$$

$$Z_s = R_s + jX_s$$

$$S_L = \frac{\frac{1}{2} V_{sm}^2 (R_L + jX_L)}{(R_L + R_s)^2 + (X_L + X_s)^2}$$

$$= \frac{\frac{1}{2} V_{sm}^2 R_L}{(R_L + R_s)^2 + (X_L + X_s)^2} + \frac{\frac{1}{2} V_{sm}^2 jX_L}{(R_L + R_s)^2 + (X_L + X_s)^2}$$

$$= P + jQ$$

Case 1: $Z_s = R_s + jX_s$ is fixed $\rightarrow$ find Z_L

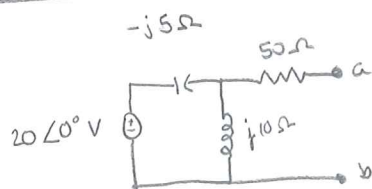
$$\left. \begin{matrix} X_L = -X_s \\ R_L = R_s \end{matrix} \right\} Z_L = Z_s^*$$

* means complex conjugate

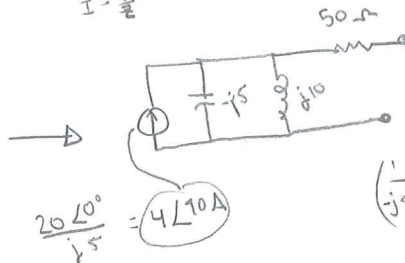
Case 2: $Z_L = R_L + jX_L$ is fixed $\rightarrow$ find Z_s

$$\left. \begin{matrix} X_s = -X_L \\ R_s = 0 \end{matrix} \right\} Z_s = 0 - jX_L$$

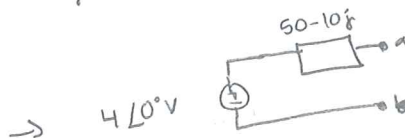
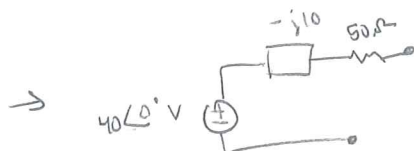
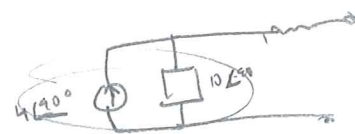
Source Transformations



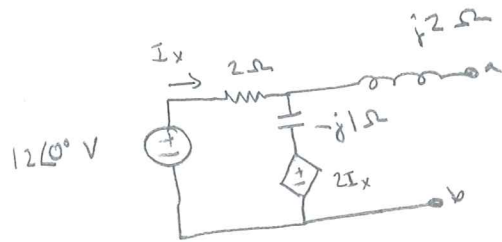
$$I = \frac{V}{Z}$$



$$\left(\frac{1}{-j5} + \frac{1}{j10} \right)^{-1} = \frac{-j10}{1} = 10 \angle 90^\circ$$



3-23-11



Find V_{oc} :

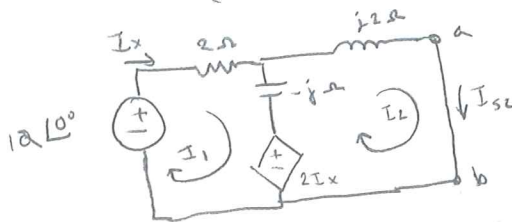
$$\text{KVL: } -12\angle 0^\circ + 2I_x - j1 \cdot I_x + 2I_x = 0$$

$$(4-j)I_x = 12\angle 0^\circ$$

$$I_x = 2.91\angle 14.04^\circ \text{ A}$$

$$V_{oc} = -j1I_x + 2I_x$$

$$(2-j)I_x = (2-j)(2.91\angle 14.04^\circ) = 6.51\angle -12.53^\circ \text{ V}$$



$$\text{M1: } -12\angle 0^\circ + 2I_1 - j1(I_1 - I_2) + 2I_1 = 0$$

$$\text{M2: } j2I_2 - 2I_1 - j1(I_2 - I_1) = 0$$

$$I_2(j) + (-2+j)I_1 = 0$$

$$\dots I_1 = \frac{-jI_2}{-2+j}$$

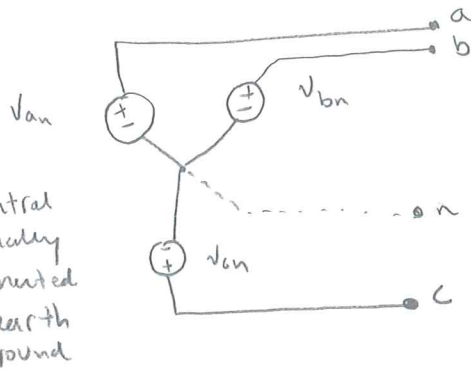
$$I_{sc} = 4.24\angle -98.13^\circ \text{ A}$$

$$Z_{th} = \frac{V_{oc}}{I_{sc}} = \frac{6.51\angle -12.53^\circ}{4.24\angle -98.13^\circ} = 1.53\angle 85.6^\circ$$

Notes 4-4-2011

5.7 Balanced 3 phase

Y-connected



neutral usually connected to earth ground

$$V_{an} = V_Y \cos \theta$$

$$V_{bn} = V_Y \cos(\theta - 120^\circ)$$

$$V_{cn} = V_Y \cos(\theta + 120^\circ)$$

positive phase sequence (abc)

reverse $\pm 120^\circ$ on b & c

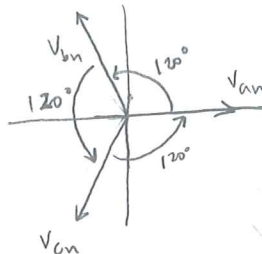
negative phase sequence (acb)

line voltages

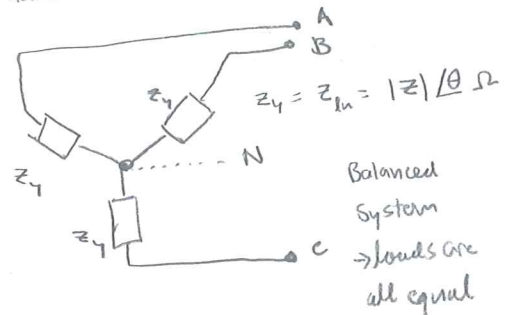
$$V_{an} = V_Y \angle 0^\circ$$

$$V_{bn} = V_Y \angle -120^\circ$$

$$V_{cn} = V_Y \angle 120^\circ$$



3-Phase Y-connected load



Balanced System
→ loads are all equal

wires a-A n-N neutral
b-B
c-C
if balanced, $I_{nN} = 0$

corresponds to source drawing and load drawings connected together

"ln" is "line to neutral"

line currents

$$I_{aA} = I_L \angle 0$$

$$I_{bB} = I_L \angle \theta - 120^\circ$$

$$I_{cC} = I_L \angle \theta - 240^\circ$$

V_Y = phase voltage = magnitude of the line-to-neutral voltage

V_{AN}, V_{BN}, V_{CN} = line-to-neutral voltages on load side

V_{an}, V_{bn}, V_{cn} = line-to-neutral voltages on source side

I_{aA}, I_{bB}, I_{cC} = line currents (phase currents)

$I_L = |I_{aA}|$ line current (magnitude)

line to line voltages

$$V_{AB} = V_L \angle 0^\circ$$

$$V_{BC} = V_L \angle \theta - 120^\circ$$

$$V_{CA} = V_L \angle \theta - 240^\circ$$

$$V_{AB} = V_{AN} - V_{BN} = V_Y \angle 0^\circ - V_Y \angle -120^\circ$$

$$\text{can do } \rightarrow = \sqrt{3} V_Y \angle 30^\circ$$

$$\text{this because it is } = \sqrt{3} \angle 30^\circ V_{AN}$$

balanced (120°)

V_L denotes line voltage or line-to-line

Notes 4-4-2011, continued

instantaneous power

$$p(t) = V_{an} i_{aA} + V_{bn} i_{bB} + V_{cn} i_{cC}$$

$$= V_Y \cos(\omega t) I_L \cos(\omega t - \theta) + V_Y \cos(\omega t - 120^\circ) I_L \cos(\omega t - \theta - 120^\circ) + V_Y \cos(\omega t - 240^\circ) I_L \cos(\omega t - \theta - 240^\circ)$$

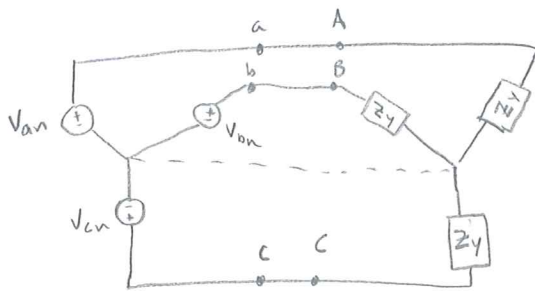
$$\cos x \cos y = \frac{1}{2} \cos(x-y) + \frac{1}{2} \cos(x+y)$$

$$P(t) = \frac{3V_Y I_L}{2} \cos \theta = 3V_{Yrms} I_{Lrms} \cos \theta = \text{constant!} = P(\text{average power})$$

$$Q = \frac{3V_Y I_L}{2} \sin \theta = 3V_{Yrms} I_{Lrms} \sin \theta = \text{Var}$$

Notes 4-6-11

Exam 4 Wed Apr 20



Given:

$$V_{ab} = (\sqrt{3} \angle 30^\circ) V_{an}$$

balanced 3-phase pos. sequence Y connected
60 Hz source

line to line voltage is 1000 Vrms

Z_Y is a 100Ω resistor in series w/ a .2 H inductor

find: line-to-line voltage phasors line-to-neutral voltage phasors line current phasors P, Q,

$$V_L = 1000 \text{ Vrms}$$

$$Z_Y = \frac{100 \Omega \parallel 0.2 \text{ H}}{\omega = 2\pi f} \rightarrow j\omega L = j(2\pi(60))(0.2) = j75.4 \Omega$$

$$V_L = 1 V_{ab}$$

$$V_{an} = V_Y \angle 0^\circ \text{ Vrms}$$

$$Z_Y = 100 + j75.4 \Omega$$

$$V_{ab} = \sqrt{3} \angle 30^\circ V_{an} \Rightarrow 1000 \angle 30^\circ = \sqrt{3} \angle 30^\circ V_Y \angle 0^\circ \Rightarrow V_Y = \frac{1000}{\sqrt{3}}$$

$$V_{an} = V_Y \angle 0^\circ = \frac{1000}{\sqrt{3}} \angle 0^\circ$$

$$V_{bn} = \frac{1000}{\sqrt{3}} \angle -120^\circ \text{ Vrms}$$

$$V_{cn} = \frac{1000}{\sqrt{3}} \angle 120^\circ \text{ Vrms}$$

$$V_{ab} = 1000 \angle 30^\circ \text{ Vrms}$$

$$V_{bc} = 1000 \angle -90^\circ \text{ Vrms}$$

$$V_{ca} = 1000 \angle 150^\circ \text{ Vrms}$$

line-to-neutral voltages

$$I_{aA} = \frac{V_{an}}{Z_Y} = \frac{1000/\sqrt{3} \angle 0^\circ}{100 + j75.4} = 4.6 \angle -37.0^\circ \text{ Arms}$$

$$I_{bB} = 4.6 \angle -157^\circ \text{ Arms}$$

$$I_{cC} = 4.6 \angle 83.0^\circ \text{ Arms}$$

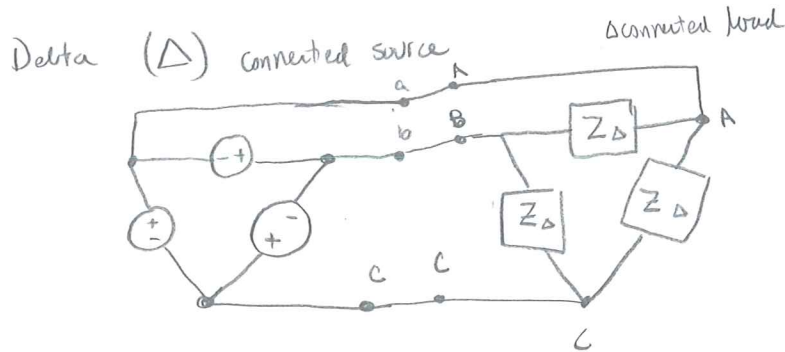
Notes 4-6-11) continued.

$$p(t) = \frac{3V_Y I_L}{2} \cos \theta = 3V_{rms} I_{rms} \cos \theta$$

$$I_L = |I_{aA}| = 4.6$$

$$Z_Y = 100 + j75.4 = 125 \angle 37.0^\circ \Omega$$

$$p(t) = 3 \frac{1000}{\sqrt{3}} (4.6) \cos(37.0) = \boxed{6363 \text{ W}}$$



$$\boxed{Z_\Delta = 3Z_Y}$$

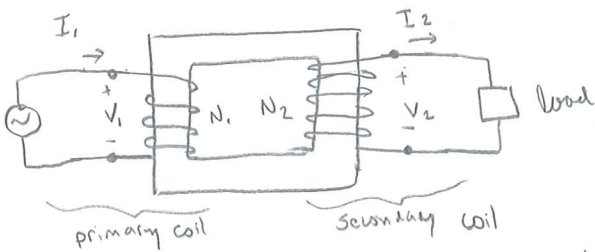
$$I_{AB} = \frac{V_{AB}}{Z_\Delta} \leftarrow \text{phase current}$$

$$\begin{aligned} I_{aA} &= I_{AB} + I_{AC} \\ &= I_{AB} - I_{CA} \\ &= \boxed{\sqrt{3} \angle -30^\circ I_{AB}} \end{aligned}$$

line current

Notes 4-8-11

15.5 Ideal Transformer



N = number of turns in coil

can step-up or step-down currents or voltages

can block D.C. voltage from a signal

$$P_{loss} = R_{line} I_{rms}^2 \leftarrow \text{reduce current to reduce line losses}$$

power loss in transmission line

$$P_2(t) = v_2(t) i_2(t)$$

$$= \frac{N_2}{N_1} v_1(t) \cdot \frac{N_1}{N_2} i_1(t) =$$

$$\boxed{P_2(t) = P_1(t)}$$

$\hookrightarrow$ ideal transformer neither generates nor absorbs power

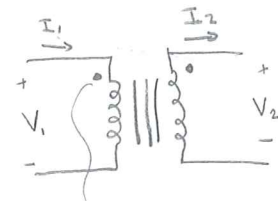
Voltage ratio:

$$\boxed{V_1 = \frac{N_1}{N_2} V_2}$$

Current ratio:

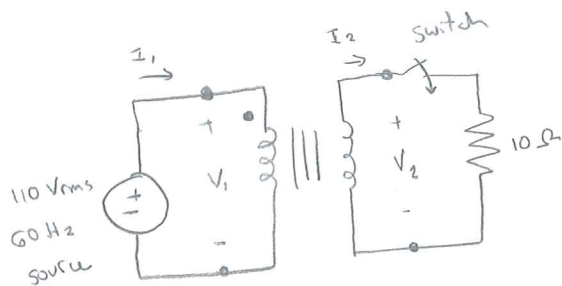
$$\boxed{I_2 = \frac{N_1}{N_2} I_1}$$

$N_1:N_2$ effective turns ratio



polarity marks, (+ goes with dot)

Notes 4-8-11) continued



Switch is open

$$\begin{aligned} I_2 &= 0 \\ I_1 &= 0 \\ V_1 &= 110 \angle 0^\circ \text{ Vrms} \end{aligned}$$

$$V_2 = \frac{N_2}{N_1} V_1 = \frac{1}{5} (110 \angle 0^\circ) = 22 \angle 0^\circ \text{ Vrms}$$

Switch is closed $V_1 = 110 \angle 0^\circ \text{ Vrms}$

$$V_2 = \frac{N_2}{N_1} V_1 = \frac{1}{5} (110) = 22 \angle 0^\circ \text{ Vrms}$$

$$I_2 = \frac{V_2}{10 \Omega} = 2.2 \angle 0^\circ \text{ Arms}$$

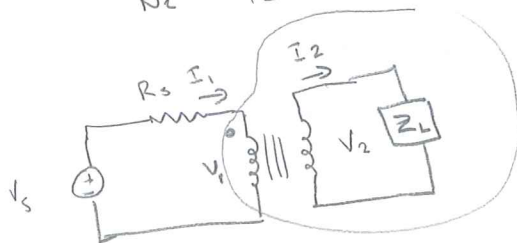
$$I_1 = \frac{N_2}{N_1} I_2 = \frac{1}{5} 2.2 \angle 0^\circ = .44 \angle 0^\circ \text{ Arms}$$

$$I_2 = \frac{N_1}{N_2} I_1$$

$$V_2 = \frac{N_2}{N_1} V_1$$

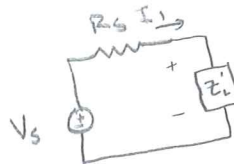
Find effective turns ratio required to step 10,000 Vrms to 120 Vrms

$$\frac{N_1}{N_2} = \frac{10,000}{120} = \frac{250}{3} \Rightarrow N_1 : N_2 = 250 : 3$$



Z_L'

$\Rightarrow$

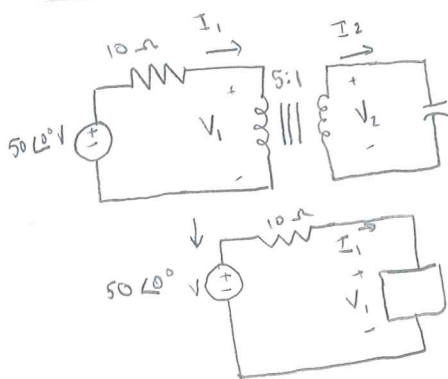


$$\begin{aligned} Z_L' &= \frac{V_1}{I_1} = \frac{N_2}{N_1} \frac{V_2}{I_2} \\ &= \frac{N_2}{N_1} \frac{N_2}{N_1} \frac{V_1}{I_1} \end{aligned}$$

$$Z_L' = \left(\frac{N_2}{N_1} \right)^2 \frac{V_1}{I_1}$$

$$Z_L' = \left(\frac{N_1}{N_2} \right)^2 Z_L$$

Notes 4-11-2011



Find V_1, V_2, I_1, I_2 , & the impedances seen by the 50-V source

$$Z_L' = \left(\frac{5}{1} \right)^2 (-j125) = -j125 \Omega$$

$$Z_T = 10 - j125 \Omega$$

$$I_1 = \frac{50 \angle 0^\circ}{10 - j125} = 0.4 \angle 85.4^\circ \text{ A}$$

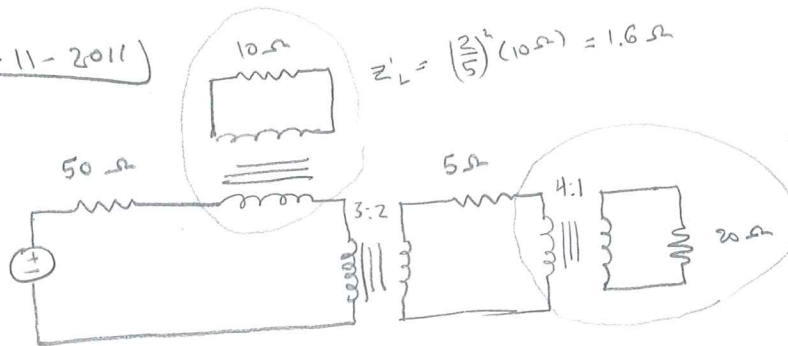
$$V_1 = I_1 (-j125) = 49.8 \angle -4.6^\circ \text{ V}$$

$$I_2 = \frac{N_1}{N_2} I_1 = 2 \angle 85.4^\circ \text{ V}$$

$$V_2 = \frac{N_2}{N_1} I_1 = 9.96 \angle -4.6^\circ \text{ V}$$

Notes 4-11-2011

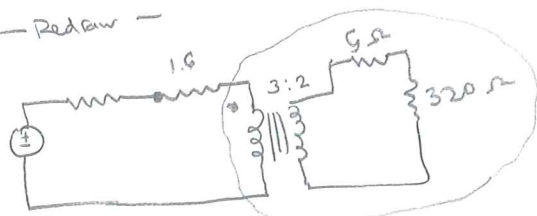
Find total impedance seen by the source



$$Z'_L = \left(\frac{2}{5}\right)^2 (10\Omega) = 1.6\Omega$$

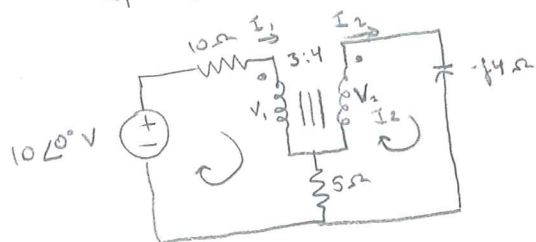
$$Z'_L = \left(\frac{4}{1}\right)^2 (20) = 320\Omega$$

Redraw



$$\left(\frac{3}{2}\right)^2 320 = Z'_L = 731.25$$

$$Z_T = 1.6 + 731.25 \approx 733\Omega$$



Find I_1 & I_2 using mesh analysis

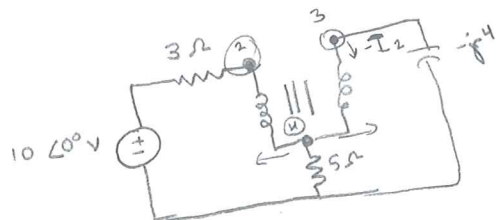
$$m1) -10\angle 0^\circ + 10I_1 + V_1 + 5(I_1 - I_2) = 0$$

$$m2) 5(I_2 - I_1) - V_2 - j4I_2 = 0$$

$$\text{Ideal transformer } V_1 = \frac{3}{4}V_2$$

$$I_2 = \frac{3}{4}I_1$$

Careful when using ideal transf. to ensure I 's are in direction



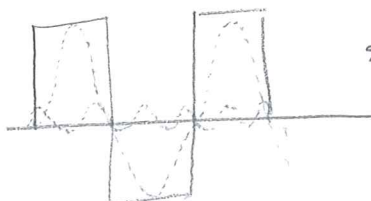
Node evaluation...

4-13-2011

6.1

We can build any waveform by adding sinusoids

• Square wave

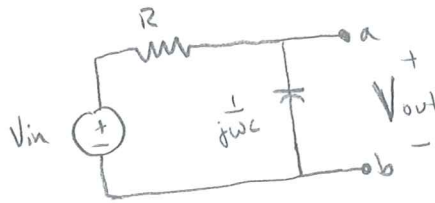
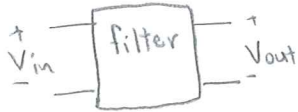


Σ dotted lines $\approx$ solid line

$$u_{sq} = \frac{4A}{\pi} \sin(\omega_0 t) + \frac{4A}{3\pi} \sin(3\omega_0 t) + \frac{4A}{5\pi} \sin(5\omega_0 t)$$

Notes 4-13-11

filter



(V. divider) $V_{out} = \frac{1/j\omega C}{R + 1/j\omega C} \Rightarrow H(j\omega) = \frac{V_{out}}{V_{in}} = \frac{1/j\omega C}{R + 1/j\omega C}$

$$H(j\omega) = \frac{1}{1 + j\omega RC}$$

$\omega = \text{rad/sec, angular frequency}$

$f = \text{frequency (Hz)}$

$$\omega = 2\pi f$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

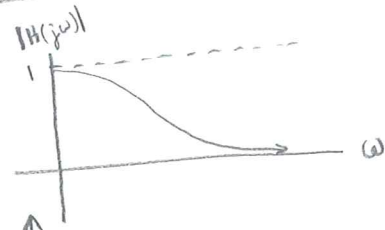
$\omega \rightarrow 0, |H(j\omega)| \rightarrow 1$

$\omega \rightarrow \infty, |H(j\omega)| \rightarrow 0$

$\omega \rightarrow 0, \angle \rightarrow 0$

$\omega \rightarrow \infty, \angle \rightarrow 90$

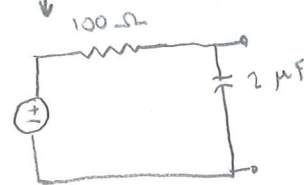
$$\angle H(j\omega) = -\tan^{-1}(\omega RC)$$



$\frac{1}{2}$ power frequency (cutoff frequency)

Set $|H(j\omega)| = \frac{1}{\sqrt{2}} \Rightarrow \text{solve for } \omega$

$$\omega_{\text{cutoff}} = \frac{1}{RC}$$



$$\omega_{\text{cutoff}} = \frac{1}{100(2 \times 10^{-6})} = 5000$$

① $V_{in}(t) = 10 \cos(2t) + 10 \cos(10,000t) \text{ V}$

Find $V_{out}(t)$

① $\omega = 2, V_{in,t} = 10 \cos(2t) \text{ V} = 10 \angle 0^\circ \text{ V}$

$$V_{out} = H(j\omega) V_{in} = \frac{1}{1 + j(2)(100)(2 \times 10^{-6})} 10 \angle 0^\circ$$

$$V_{out,1} = 9.99 \angle -0.023^\circ \text{ V} = 9.99 \cos(2t - 0.023^\circ) \text{ V}$$

② $\omega = 10,000, V_{in}(t) = 10 \cos(10,000t) \text{ V}$

$$V_{out,2} = \frac{1}{1 + j(10,000)(100)(2 \times 10^{-6})} 10 \angle 0^\circ$$

$$V_{out,2} = 4.47 \angle -63.43^\circ = 4.47 \cos(10,000t - 63.4^\circ) \text{ V}$$

$$V_{out} = V_{out,1} + V_{out,2}$$

other type of LowPass Filter



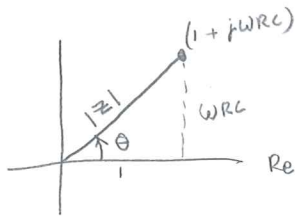
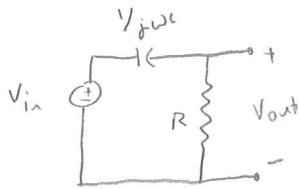
$$H(j\omega) = \frac{R}{R + j\omega L} = \frac{1}{1 + j\omega RL}$$

$$\omega_{\text{cutoff}} = \frac{R}{L}$$

$\omega \rightarrow 0, |H(j\omega)| \rightarrow 1$

$\omega \rightarrow \infty, |H(j\omega)| \rightarrow 0$

Exam 4 Wednesday



$$V_{out} = H(j\omega) V_{in}$$

$$V_{out} = \frac{R}{R + 1/j\omega C} V_{in}$$

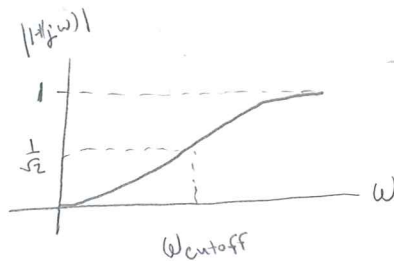
$$H(j\omega) = \frac{V_{out}}{V_{in}} = \frac{R}{R + 1/j\omega C} = \frac{j\omega RC}{1 + j\omega RC} = \frac{\omega RC \angle 90^\circ}{\sqrt{1 + (\omega RC)^2} \angle \tan^{-1}(\frac{\omega RC}{1})}$$

$$|H(j\omega)| = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}$$

$$\angle H(j\omega) = 90 - \tan^{-1}(\omega RC)$$

$$\omega \rightarrow 0 \quad |H(j\omega)| \rightarrow 0 \quad \angle H(j\omega) \rightarrow 90^\circ$$

$$\omega \rightarrow \infty \quad |H(j\omega)| \rightarrow 1 \quad \angle H(j\omega) \rightarrow 0^\circ$$



Cut-off frequency

$$|H(j\omega)| = \frac{1}{\sqrt{2}}$$

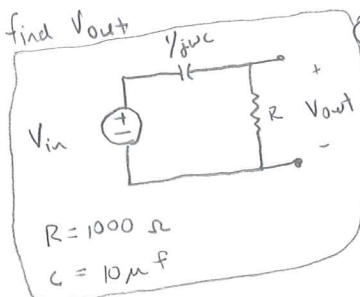
$$\omega_{cutoff} = \frac{1}{RC}$$

ex)

$$V_{in}(t) = 10 \cos(2t + 15^\circ) + 10 \cos(2000t) \text{ V}$$

$$\Rightarrow V_{in} = 10 \angle 15^\circ \text{ V}$$

find V_{out}



$$\textcircled{A} \omega = 2 \text{ rad/sec}$$

$$V_{out} = H(j\omega) V_{in}$$

$$V_{out} = \frac{j\omega RC}{1 + j\omega RC} = 0.2 \angle 103.9^\circ$$

$$V_{out}(t) = 0.2 \cos(2t + 103.9^\circ) \text{ V}$$

$$\textcircled{B} \omega = 2000 \text{ rad/sec} \rightarrow V_{in} = 10 \angle 0^\circ \text{ V}$$

$$V_{out} = H(j\omega) V_{in}$$

$$= \frac{j\omega RC}{1 + j\omega RC} = 9.99 \angle 2.86^\circ \text{ V}$$

$$V_{out} = 9.99 \cos(2000t + 2.86^\circ) \text{ V}$$

$$V_{out}(t) = 0.2 \cos(2t + 103.9^\circ) + 9.99 \cos(2000t + 2.86^\circ) \text{ V}$$

↓
Notes 4-22-11

Exam 4 Wed, Apr 20

- Thevenin & Norton Equivalents [source transformations
dependent sources
- High & low pass filters
- Ideal transformers → reflected impedance (impedance transformations)
- 3-phase
- Max power transfer

— distinguish between hi-pass & low pass

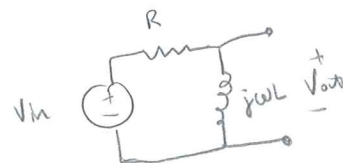
low pass: $\omega \rightarrow \infty$, $H(j\omega) = 0$

— Solve for t.f.

$$H(j\omega) = \frac{V_{out}}{V_{in}}$$

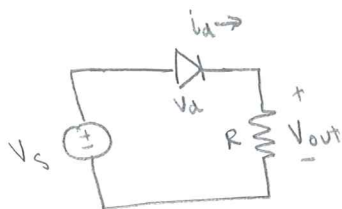
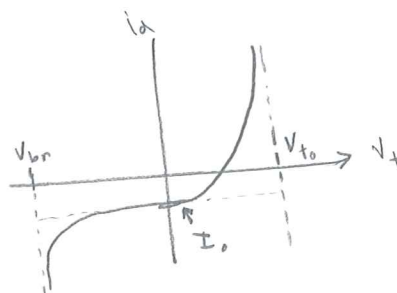
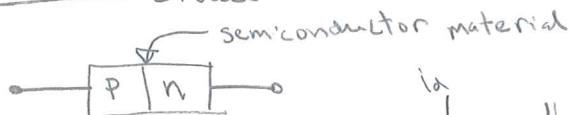
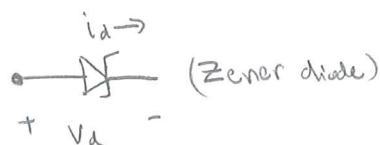
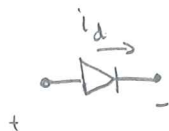
$$V_{out} = \frac{j\omega L}{R + j\omega L} V_{in}$$

↑ Voltage divider using impedances



Missed class 4-22-11

Diodes

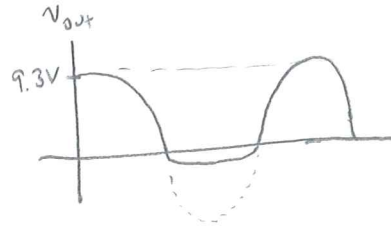
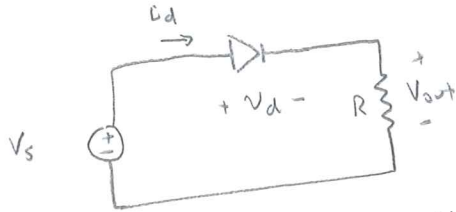


$V_s > 0 \rightarrow$ forward biased (right side of i_v curve)
 $V_s < 0 \rightarrow$ reverse biased (left side)

$V_s = 4V$ find R $i_d = 2mA$

(Saturation) current $I_s = -0.1mA$ $V_{to} = 0.7V$
 $V_{br} = -26V$

KVL $-V_s + V_d + Ri_d = 0$
 $-4 + 0.7 + R(0.002) = 0$
 $R = 1650 \Omega$



• half-wave rectifier cuts off neg. portion of cosine wave

$$V_s = 10 \cos(2t) \text{ V}$$

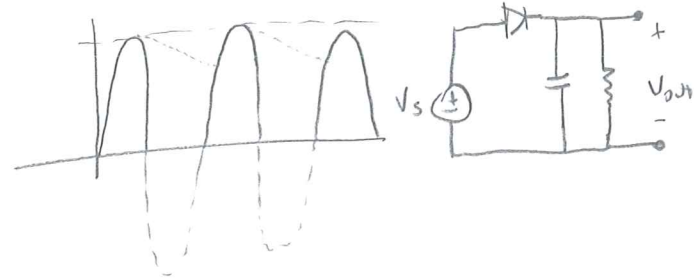
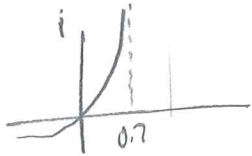
$$\downarrow$$

$$V_{s, \max} = 10 \text{ V}$$

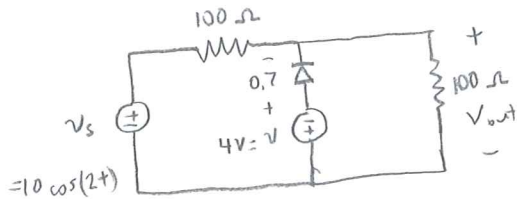
$$-V_s + v_d + \underbrace{i_d R}_{v_{out}} = 0$$

$$v_{out} = V_s - v_d$$

$$= V_s - 0.7$$



Clipper circuit

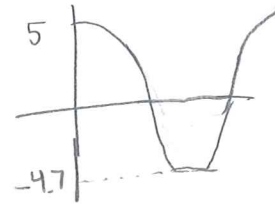


v_s (positive portion allowed through)

$$v_{out} = -v - 0.7$$

$$= -4.7 \text{ V}$$

clips off negative portion of curve $\rightarrow$



4-27-2011

Final Exam

Thévenin & Norton

- resistors
- impedances
- source transf.
- max power transfer

Mesh/Node:

- super nodes/meshes
- resistors/impedances

- Superposition
- 3-phase
- transfer functions/filters

Ideal transformers

Power • avg, reactive, apparent, P.F

RL/RC 1st order circuits } diff
RLC 2nd order } eq's

- Ohm's Law, KVL, KCL, current/voltage divider
- Diodes
- Parallel & Series R, C, L
- RMS, average value of a function

- Class is optional on Friday

Graded Exams

Name David Malawey

EE 281 A

Exam 4

Spring 2011

to your neighbor

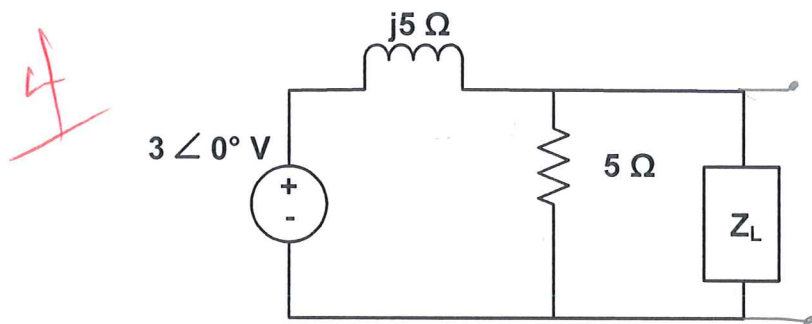
Show all your work. Circle or box your answer.

1

avg: 77.7%
high: 98%

92

1. For the following circuit, determine the value of Z_L that results in maximum average power transferred to Z_L .



$$Z_{th} = j5 \Omega \parallel 5 \Omega = \left(\frac{1}{5} + \frac{1}{j5} \right)^{-1} = 3.54 \angle 45^\circ$$

$$Z_L = Z_{th}^* = 3.54 \angle -45^\circ \checkmark$$

$$Z_L = 2 - 2j \quad (-1)$$

2. Find the effective turns ratio of an ideal transformer necessary to step a 440-V_{rms} source up to 2000 V_{rms}.

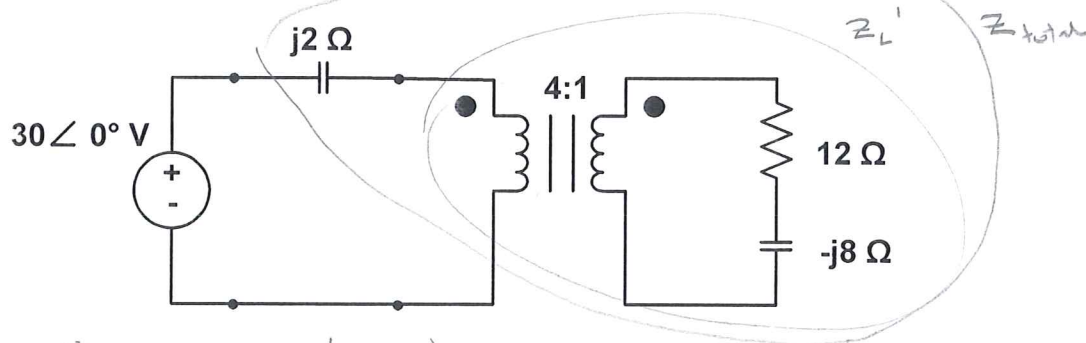
5
(5 pts)

$$\frac{V_2}{V_1} = \frac{2000}{440} = \frac{N_2}{N_1} = \frac{100}{22} = \frac{50}{11}$$

$$N_1 : N_2 = 11 : 50$$

3. Use impedance transformations (reflected impedances) to find the impedance seen by the 30-V source in the following circuit.

15
(15 pts)



$$Z_L' = \left(\frac{N_1}{N_2} \right)^2 Z_L$$

$$Z_L = (12 - j8) \Omega$$

$$Z_L' = 16 (12 - j8) \Omega$$

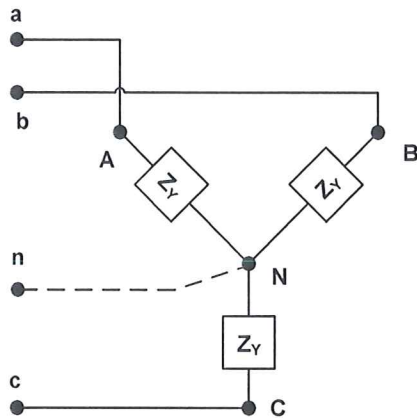
$$= (192 - 128j) \Omega$$

$$Z_{total} = 192 - 128j + j2 \Omega$$

$$Z_{total} = (192 - 126j) \Omega = 230 \angle -33.3^\circ$$

24

4. A balanced Y-connected load has an impedance of $Z_Y = 300 + j4 \Omega$ for each phase is connected to a 220-V_{rms} line. Assuming the phase sequence is positive, there is no line impedance, and that V_{ab} is the reference, find the following.



$$V_L = 220 V_{rms} = |V_{ab}|$$

$$V_{ab} = 220 \angle 0^\circ$$

- a. The phasor voltages V_{AN} and V_{BN} .
(4 pts)

$$V_{ab} = V_{an} \sqrt{3} \angle 30^\circ$$

$$V_{AN} = 127 \angle -30^\circ V_{rms}$$

$$V_{BN} = 127 \angle -150^\circ V_{rms}$$

- b. The phasor currents I_{aA} , I_{bB} , and I_{cC} .
(6 pts)

$$I_{aA} = \frac{V_{an}}{Z_Y} = \frac{127 \angle -30^\circ}{(300 + j4)} = 0.423 \angle -30.8^\circ A_{rms}$$

$$I_{bB} = \frac{V_{bn}}{Z_Y} = 0.423 \angle -150.8^\circ A_{rms}$$

$$I_{cC} = 0.423 \angle 89.2^\circ A_{rms}$$

- c. The phasor voltages V_{AB} and V_{CA} .
(4 pts)

$$V_{AB} = 220 \angle 0^\circ V_{rms}$$

$$V_{CA} = 220 \angle 120^\circ V_{rms}$$

- d. The average power, P , absorbed by the three-phase load. $Z = 300.027 \angle .764^\circ$
(6 pts)

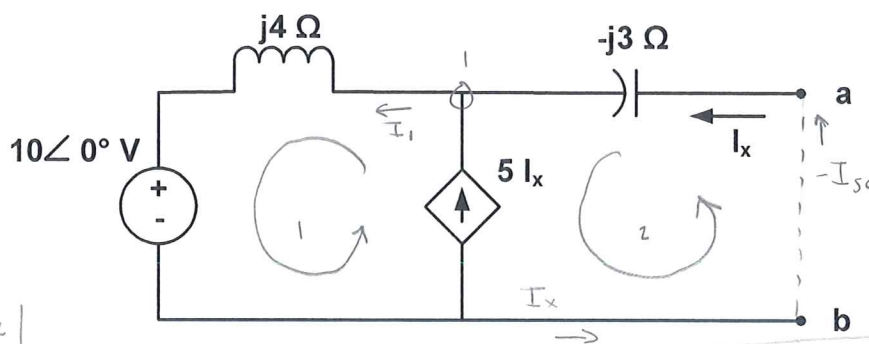
$$P_{avg} = 3 V_{Y,rms} I_{L,rms} \cos \theta = 3(127)(0.423) \cos (.764)$$

$$P_{avg} = 161 W$$

- e. The impedance Z_Δ to create an equivalent Δ -connected load.
(5 pts)

$$Z_\Delta = 3Z_Y = (900 + 12j) \Omega$$

5. Find the Thévenin equivalent of the following circuit. Sketch the equivalent circuit. (20 pts)



find I_{sc}

mesh 1

$$-j4I_1 - 10\angle 0^\circ = 0$$

$$5I_x = I_1 - I_2$$

$$5I_x = I_1 - I_x$$

$$I_1 = 6I_x$$

$$-j4(6I_x) - 10 = 0$$

$$I_x = \frac{10}{-24j} \quad I_{sc} = .417 \angle 90^\circ$$

KVL around outside +

$$-10 - 4I_1 + 3jI_x = 0$$

$$-10 - 24I_x + 3jI_x = 0$$

Node 1

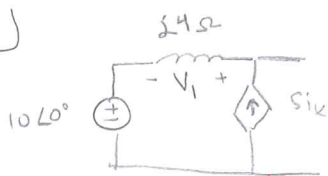
$$I_1 = 6I_x$$

$$(3j - 24)I_x = 10$$

$$I_x = .413 \angle -173^\circ$$

I did this part last, to double check. I'm more confident with this I_x but had no time to rework rest of problem

find V_{oc}

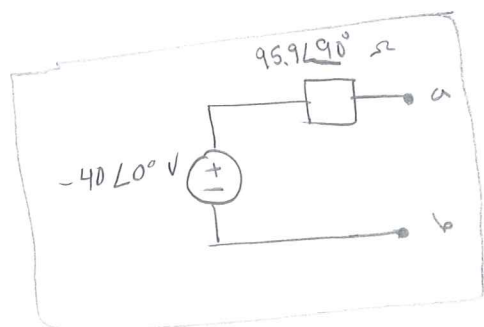


$$5i_x = \frac{-10\angle 0^\circ}{j4} \Rightarrow i_x = 2.5 \angle 90^\circ$$

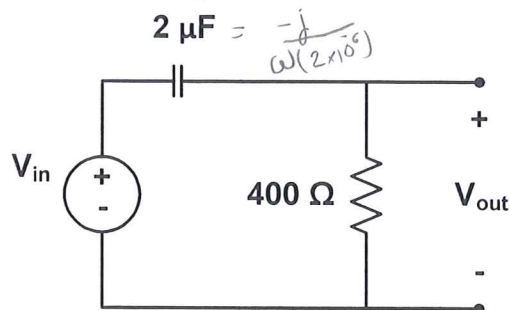
$$V_1 = (5)(2.5 \angle 90^\circ)(j4) = -50 \angle 0^\circ \text{ V}$$

$$V_{oc} = 10 \angle 0^\circ - 50 \angle 0^\circ = -40 \angle 0^\circ$$

$$I_{sc} = \frac{V_{oc}}{Z_{th}} \Rightarrow Z_{th} = 95.9 \angle 90^\circ$$



6. For the following filter find:



$$V_{out} = V_{in} \left(\frac{400}{400 - \frac{j}{\omega(2 \times 10^{-6})}} \right)$$

- a. The transfer function $H(j\omega) = \frac{V_{out}}{V_{in}}$
(8 pts)

$$H(j\omega) = \frac{400}{400 - \frac{j}{\omega 2 \times 10^{-6}}}$$

- b. The half-power frequency in radians/second.
(4 pts)

$$|H(j\omega)| = \frac{1}{\sqrt{2}} = \frac{400}{\sqrt{400^2 + \frac{1}{\omega^2(2 \times 10^{-6})^2}}} \Rightarrow \frac{400 + \frac{1}{\omega^2(2 \times 10^{-6})^2}}{2} = 400 \Rightarrow \frac{1}{\omega^2(2 \times 10^{-6})^2} = 400$$

$$\Rightarrow \omega^2 = \frac{1}{400(2 \times 10^{-6})^2} \Rightarrow \omega = \frac{1}{20(2 \times 10^{-6})}$$

$$\omega = 25,000 \text{ rad/sec}$$

- c. Is this a highpass or a lowpass filter?
(3 pts)

$$\omega \rightarrow \infty, |H(j\omega)| \rightarrow 1 \quad \text{high pass}$$

7. The transfer function for a particular first order filter is $H(j\omega) = \frac{V_{out}}{V_{in}} = \frac{R}{R + j\omega L}$.

- a. Find $|H(j\omega)|$, the magnitude of the transfer function, in terms of R, L, and ω .
(5 pts)

$$|H(j\omega)| = \frac{\sqrt{R^2}}{\sqrt{R^2 + \omega^2 L^2}} = \left(\frac{R^2}{R^2 + \omega^2 L^2} \right)^{1/2}$$

- b. Find the output current, $v_{out}(t)$ if $R=100 \Omega$, $L=50 \text{ mH}$, and the input voltage is $v_{in}(t) = 20 \cos(5t) + 20 \cos(4000t + 30^\circ) \text{ V}$.
(10 pts)

$$V_{out1} = 20 \angle 0^\circ \left(\frac{100}{100 + j(5)(.05)} \right) = 19.99 \angle -14.3^\circ$$

$$V_{out2} = 20 \angle 30^\circ \left(\frac{100}{100 + j(4000)(.05)} \right) = 8.94 \angle -33.4^\circ$$

$$V_{out} = 19.99 \cos(5t - 14.3^\circ) + 8.94 \cos(4000t - 33.4^\circ) \text{ V}$$

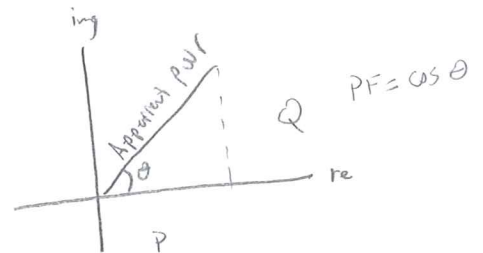
Name David Malawey

$$V_{rms} = \frac{Amp}{\sqrt{2}}$$

EE 281 A

Exam 3

Spring 2011



$$P = I_{rms}^2 R = \frac{V_{rms}^2}{R}$$

$$Q = I_{rms}^2 X = \frac{V_{rms}^2}{X}$$

Show all your work. Circle or box your answer.

$$V_k = \frac{R_k}{R_1 + R_2 + R_3 + \dots + R_n} V_s$$

$$i_k = \frac{\frac{1}{R_k}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}} i_s$$

$$R = \frac{\rho L}{A}$$

$$P_{max} = \frac{v_{th}^2}{4R_{th}}$$

$$e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)$$

$$i_C = C \frac{dv}{dt}$$

$$v_C = \frac{1}{C} \int_{-\infty}^t i \, d\lambda = v(t_0) + \frac{1}{C} \int_{t_0}^t i \, d\lambda$$

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T v^2 dt}$$

$$v_L = L \frac{di}{dt}$$

$$i_L = \frac{1}{L} \int_{-\infty}^t v \, d\lambda = i(t_0) + \frac{1}{L} \int_{t_0}^t v \, d\lambda$$

$$P = \frac{1}{2} V_m I_m \cos(\theta) = V_{rms} I_{rms} \cos(\theta)$$

$$|S| = \frac{P}{PF} = \frac{1}{2} V_m I_m = V_{rms} I_{rms}$$

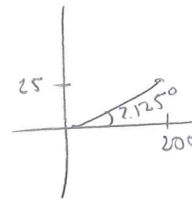
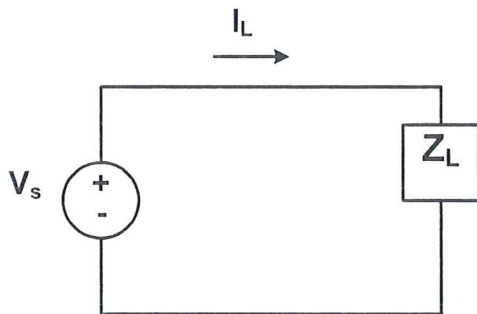
$$Q = \frac{1}{2} V_m I_m \sin(\theta) = V_{rms} I_{rms} \sin(\theta)$$

$$x_p = \begin{cases} A \\ Ae^{bt} \\ Ae^{at} \cos(bt) + Be^{at} \sin(bt) \end{cases}$$

$$x_n = \begin{cases} K_1 e^{s_1 t} + K_2 e^{s_2 t} \\ K_1 e^{st} + K_2 t e^{st} \\ K_1 e^{at} \cos(bt) + K_2 e^{at} \sin(bt) \end{cases}$$

65

1. In the following circuit, the source is a 60-Hz, 20-V_{rms} source and the load consists of a resistor and inductor in series with an impedance of $Z_L = 200 + j25 \Omega$.



- a. Find the power factor (include whether it is leading or lagging) of the load.
(5 pts)

$$PF = \cos \theta = .992 \text{ lagging} \checkmark$$

$$\theta = 7.125^\circ$$

- b. Find the RMS current through the load Z_L . (Note: there will not be an angle associated with this current since you were not given one for the voltage source.)
(5 pts)

$$I = \frac{V_{rms}}{Z} = \frac{20/\sqrt{2}}{(200 + j25 \Omega)} \quad -2$$

$$I_{RMS} = .070 \angle -7.125^\circ \text{ A}_{rms}$$

- c. Calculate the average (real) power absorbed by the load.
(5 pts)

$$P = \frac{V_{rms}^2}{R} = \frac{(20/\sqrt{2})^2}{200} = \frac{400}{200} = \frac{200}{200} = 1 \text{ W}$$

-2

- d. Find the apparent power absorbed by the load.
(5 pts)

$$AP = \sqrt{Q^2 + P^2}$$

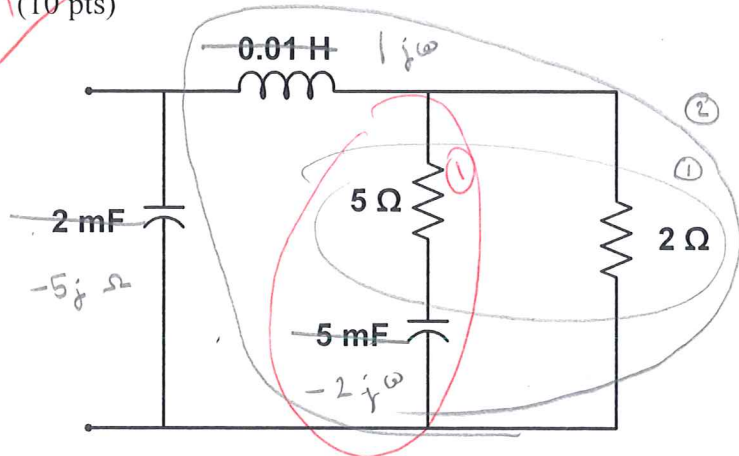
$$Q = \frac{(20/\sqrt{2})^3}{25j} = 8 \angle 90^\circ$$

$$AP = \sqrt{1 + 8^2} = 8.1$$

$$AP = \frac{(200/\sqrt{2})^2}{\sqrt{200^2 + 25^2}} = \frac{200}{201.6}$$

$$0.992 \text{ *units*}$$

2. Find the equivalent impedance, Z_{eq} , of the following. Assume that $\omega = 100$ rad/sec.
(10 pts)



Handwritten calculations for problem 2:

$$\frac{1}{\frac{1}{2} + -2j} = 1.414 \angle 45^\circ$$

$$j1 + 1.414 \angle 45^\circ = 2.24 \angle -26.6^\circ$$

$$\frac{1}{-5j + 2.24 \angle -26.6^\circ} =$$

Final result for problem 2:

$$Z_{eq} = 1.77 \angle -45.1^\circ$$

3. Given $v(t) = 6 \sin(14t + 5^\circ)$ V:

- a. Find the RMS value of $v(t)$.
(4 pts)

Handwritten calculation for problem 3a:

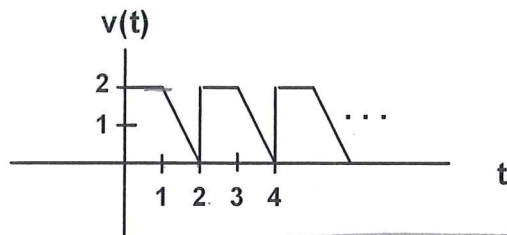
$$= \frac{6}{\sqrt{2}} = 4.24$$

- b. Write $v(t)$ as a phasor.
(4 pts)

Handwritten phasor for problem 3b:

$$6 \angle 5^\circ \text{ V}$$

4. Find the RMS value of the following period function:
(10 pts)



Handwritten calculation for problem 4:

$$V_{RMS} = \sqrt{\frac{1}{T} \int_0^T v^2 dt}$$

$$= \sqrt{\frac{1}{2} \left[\int_0^1 2^2 dt + \int_1^2 (2-2t)^2 dt \right]}$$

$$= \frac{1}{2} \left[4[t]_0^1 + \int_1^2 (4-4t+4t^2) dt \right]$$

Handwritten calculation for problem 4 (continued):

$$= \frac{1}{2} \left[4 + \left[4t - \frac{4t^2}{2} + \frac{4t^3}{3} \right]_1^2 \right] = \frac{1}{2} \left[4 + \left[8 - 8 + 10.67 - (4 - 2 + 1.25) \right] \right]$$

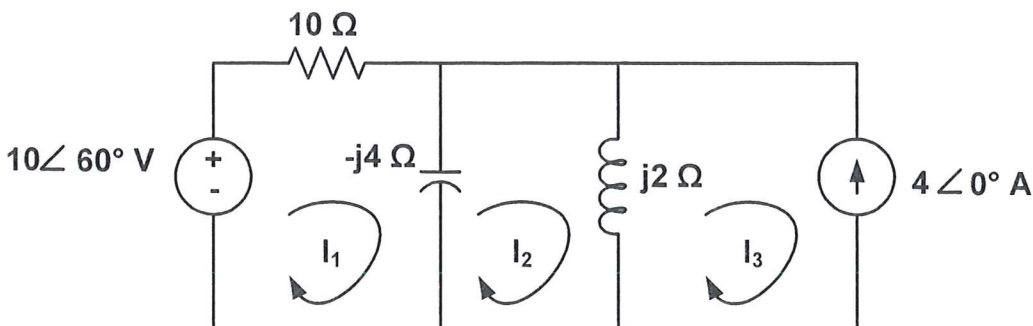
$$= \left[\frac{1}{2} (11.42) \right]^{1/2}$$

Handwritten result for problem 4:

$$= 2.39 \text{ V}_{RMS}$$

← this is too high.
Math error...

5. Set up the equations in matrix form to find the phasor mesh currents I_1 , I_2 , and I_3 in the following circuit. Do not solve. (12 pts)



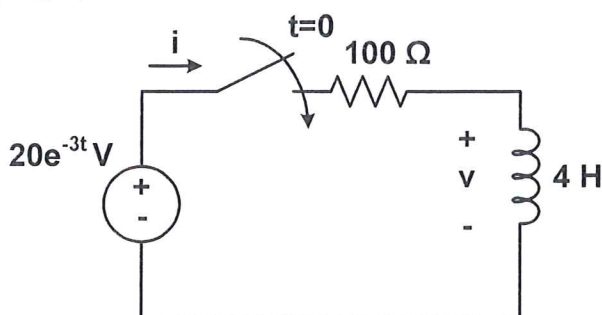
1) $-10\angle 60^\circ + 10i_1 - j4(i_1 - i_2) = 0$

2) $-j4(i_2 - i_1) + j2(i_2 - i_3) = 0$

3) $-4\angle 0^\circ A = i_3$

$$\begin{bmatrix} 10-j4 & +j4 & 0 \\ j4 & -j2 & -j2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -10\angle 60^\circ \\ 0 \\ -4\angle 0^\circ \end{bmatrix} A$$

6. Find the current, $i(t)$, for $t > 0$ in the following circuit. Solve for all the constants. $i = i(t_0) + \frac{1}{L} \int_{t_0}^t v d\lambda$ (15 pts)



$i_p) i_p = \frac{1}{4} \int_{-\infty}^0 20e^{-3t} V dt$

$= \frac{1}{4} \frac{20}{-3} e^{-3t}$

$i_p = -\frac{10}{6} e^{-3t}$

(-4)

$i_n) 10i + \frac{1}{4} \int_{-\infty}^0 i d\lambda = 0$

$10 \frac{di}{dt} + \frac{1}{4} i = 0$ (-3)

$10s + \frac{1}{4} = 0$

$1 + 40s = 0$

$s = -40$ X

$i_h = K_1 e^{-40t}$

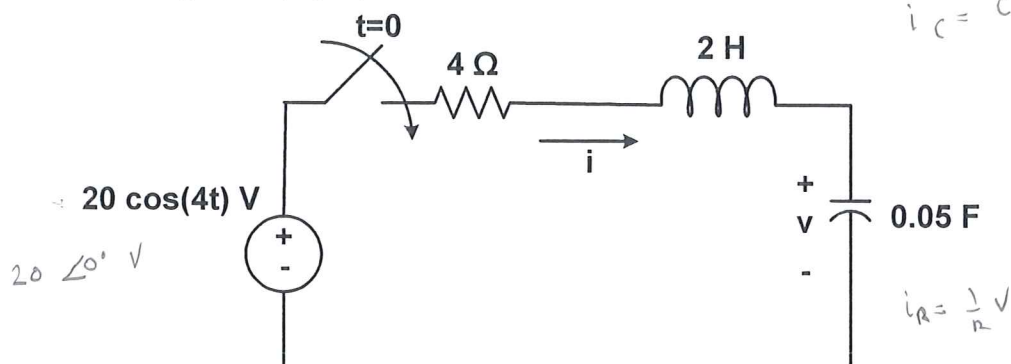
$i(0^+) = 0 = K_1 e^{-40 \cdot 0} - \frac{10}{6} e^{-3 \cdot 0}$

$\Rightarrow K_1 - \frac{10}{6} = 0 \Rightarrow K_1 = \frac{10}{6}$

$i(t) = \frac{5}{3} e^{-40t} - \frac{5}{3} e^{-3t}$

20

7. Assuming that $v(0^+) = 0$, use the circuit below to find:



- a. The form of the source-free (natural) solution of $i(t)$.
(10 pts)

$$0 = 4i + 2 \int i dt + 0.05 \frac{di}{dt}$$

$$0 = 4 \frac{di}{dt} + 2i + 0.05 \frac{d^2 i}{dt^2}$$

$$0.05 s^2 + 4s + 2 = 0$$

$$s^2 + 80s + 40 = 0$$

$$s = \frac{-80 \pm \sqrt{80^2 - 4(40)}}{2} = -40 \pm \frac{79}{2} = -40 \pm 39.5$$

$$s = -0.5, -79.5$$

$$i(t) = A e^{-0.5t} + B e^{-79.5t}$$

- b. The steady-state (particular) solution of $i(t)$.
(8 pts)

$$A e^{-0.5t} + B e^{-79.5t} = 20 e^{4t}$$

I don't know how to change the form of the source voltage
???

- c. The other initial conditions: $i(0^+)$ and $\frac{di}{dt}|_{t=0^+}$.
(4 pts)

$$i(0^+) = \frac{V_s}{R} = \frac{20 \cos 4t}{4} = 5 \cos 4t \text{ at } t=0 = 5 \cos(0) = 5 \text{ V}$$

$$\frac{di}{dt}(t=0) = 0 = \frac{d}{dt}(i(t)) \dots$$

- d. Is this circuit overdamped, underdamped, or critically damped?
(3 pts)

roots are real & distinct, overdamped

or by (a)

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EE 281 A

Exam 2

Spring 2011

Show all your work. Circle or box your answer.

$$V_k = \frac{R_k}{R_1 + R_2 + R_3 + \dots + R_n} V_s$$

$$i_k = \frac{\frac{1}{R_k}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}} i_s$$

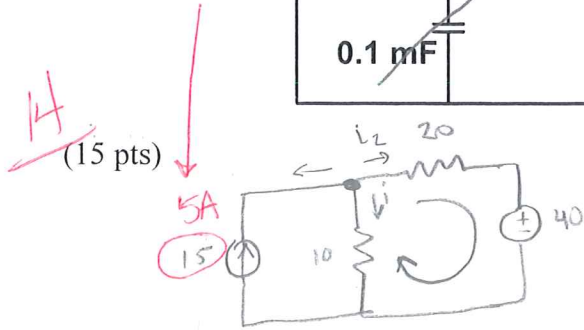
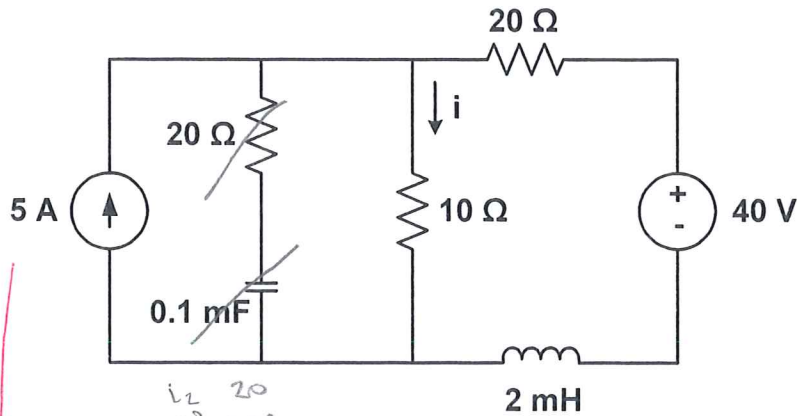
$$R = \frac{\rho L}{A}$$

$$i_C = C \frac{dv}{dt} \quad v_C = \frac{1}{C} \int_{-\infty}^t i \, d\lambda = v(t_0) + \frac{1}{C} \int_{t_0}^t i \, d\lambda$$

$$v_L = L \frac{di}{dt} \quad i_L = \frac{1}{L} \int_{-\infty}^t v \, d\lambda = i(t_0) + \frac{1}{L} \int_{t_0}^t v \, d\lambda$$

$$P_{\max} = \frac{v_{th}^2}{4R_{th}}$$

1. Solve for the DC steady-state current, i , in the following circuit.



KCL $15 = i + i_2$

KVL $-20i_2 - 40 + 10i = 0$

$i = 4 + 2i_2$

$i - 4 = 2i_2$

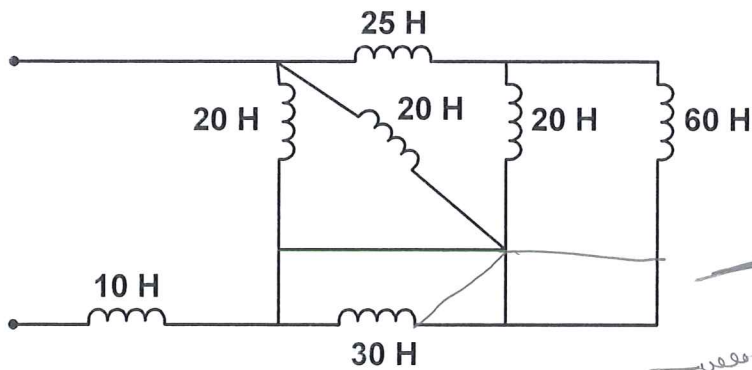
$\frac{i}{2} - 2 = i_2$

$15 = i + \frac{1}{2}i - 2$

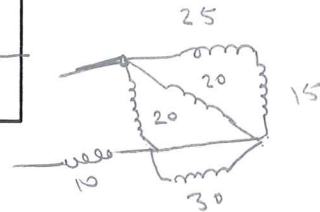
$17 = \frac{3}{2}i$

$i = \frac{34}{3} \text{ A}$

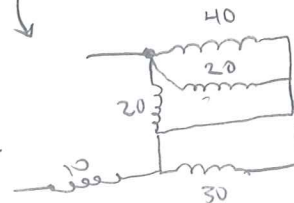
2. Simplify the following to a single equivalent component.



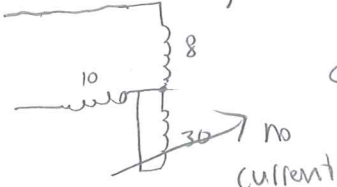
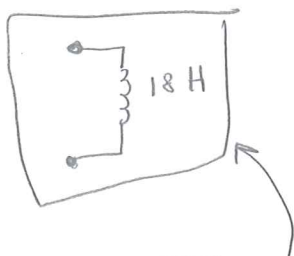
$\frac{1}{20} + \frac{1}{60} \rightarrow \frac{4}{60} \rightarrow \frac{2}{30} \rightarrow 15$



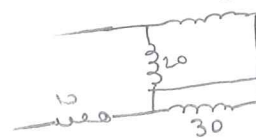
$\frac{1}{20} + \frac{1}{40} = \frac{3}{40} \rightarrow \frac{40}{3}$



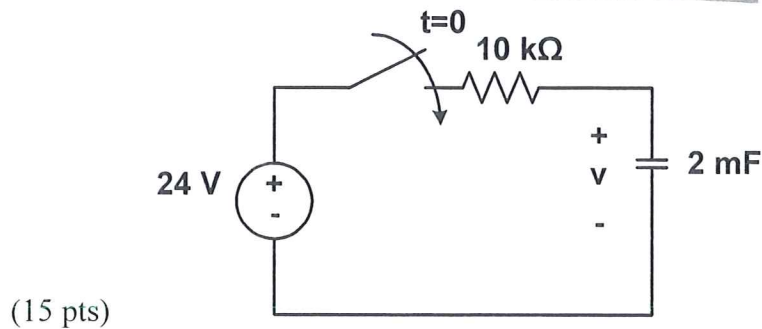
$\frac{3}{40} + \frac{1}{20} \rightarrow \frac{5}{40} \rightarrow \frac{40}{5} \rightarrow 8$



no current



3. If the initial voltage across the capacitor is $v(0^+) = 6$ V, find the voltage, $v(t)$, across the capacitor for $t \geq 0$. What is the maximum voltage across the capacitor for $t > 0$?



Max Voltage $V = 24$ V

$$\frac{1}{RC} = \frac{1}{10,000 (0.002)} = 0.05$$

$$\frac{v - v_s}{R} + C \frac{dv}{dt} = 0 \rightarrow \frac{v}{RC} + \frac{dv}{dt} = \frac{24}{RC}$$

$$\frac{v}{.05} + \frac{dv}{dt} = 480 \quad \text{Homogeneous solution: } \frac{v}{.05} = \frac{dv}{dt} \Rightarrow v(t) = k_1 + k_2 e^{-.05t}$$

Boundary Conditions:

$$v(0) = 6 \text{ V} = k_1 + k_2 (1)$$

$$v_{\infty} = 24 = k_1 + k_2 (0) \Rightarrow k_1 = 24 \text{ V}$$

$$\Rightarrow 24 + k_2 = 6 \Rightarrow k_2 = -18$$

$$v(t) = 24 - 18e^{-.05t} \text{ V}$$

4. For a 5-mH inductor, calculate each of the following:
(10 pts)

a. The current for $t > 0$ through the inductor if the voltage across the inductor is $v(t) = 4 \sin(5t)$ V and the initial current through the inductor is $i(0) = 0.5$ A.

$$i_L = i(t_0) + \frac{1}{L} \int_{t_0}^t v d\lambda$$

$$= 0.5 + \frac{1}{0.005} \int_0^t 4 \sin(5\lambda) d\lambda = 0.5 + 200 \left[-\frac{4}{5} \cos(5\lambda) \right]_0^t$$

$$= 0.5 - 160 \cos(5t) + 160 \cos(0)$$

$$i_L(t) = 155.8 - 160 \cos(5t) \text{ A}$$

-1

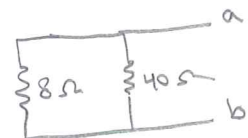
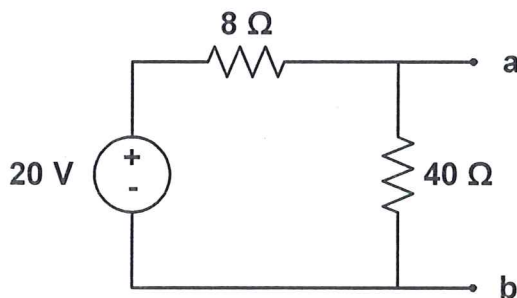
b. The voltage as a function of time across the inductor if the current through the inductor is $i(t) = 6e^{-5t}$ A.

$$V_L = L \frac{di}{dt} = 0.005 \frac{d}{dt} (6e^{-5t})$$

$$= 0.005 (-5) (6e^{-5t})$$

$$V(t) = -0.15 e^{-5t} \text{ V}$$

5. What load resistance should be placed between terminals a and b so that maximum power is delivered to the load?



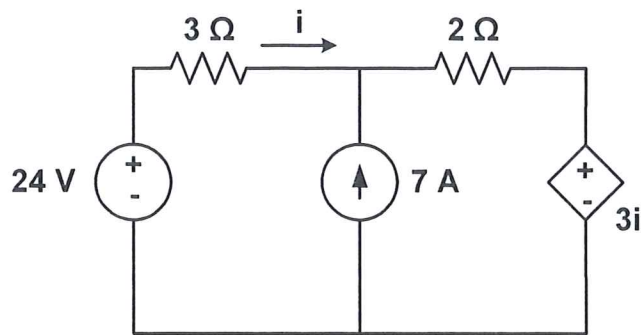
$$\frac{1}{6} + \frac{1}{40} = \frac{5+1}{40} \Rightarrow \frac{40}{6} = \frac{20}{3}$$

$$R_{\text{max power}} = R_{th} = \frac{20}{3} \Omega$$

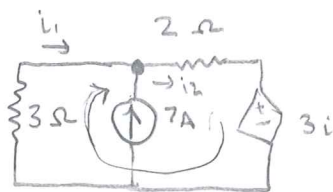
$$R = \frac{20}{3} \Omega$$

6. Use superposition to find i in the following circuit.

(15 pts)



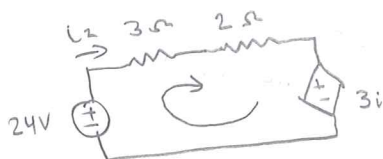
Keep 7A source



KCL $-7 - i_1 + i_2 = 0$
 $i_2 = 7 + i_1$

KVL $-3i_1 - 2i_2 - 3i_1 = 0$
 $-6i_1 = 2i_2$
 $-3i_1 = 2(7 + i_1)$
 $-3i_1 = 14 + 2i_1$
 $-i_1 = 14$
 $i_1 = -14$

Keep 24V src



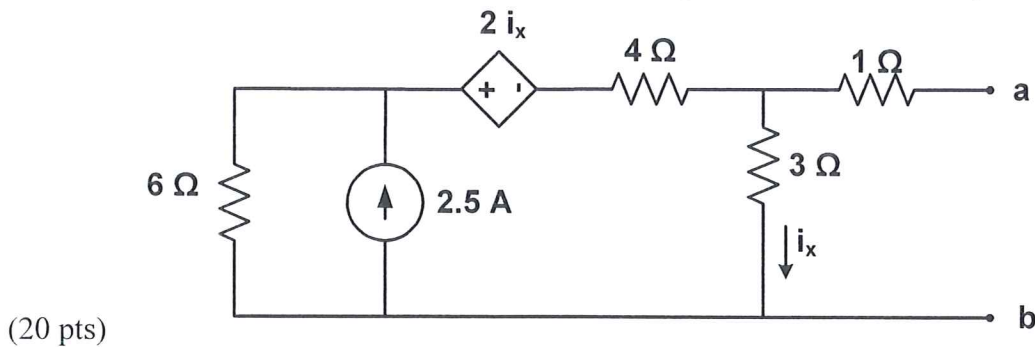
$24 - 3i_2 - 2i_2 - 3i_2 = 0$
 $24 = 8i_2$
 $i_2 = 3A$ ✓

$i = i_1 + i_2 = -14 + 3 = \boxed{-11A}$

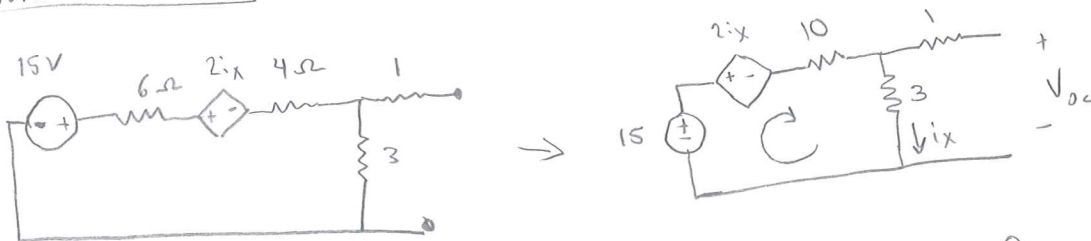
Note:

dependent source has been accounted for twice. I cannot remember the rules but this should be okay because in each case the dep. source was a function of the indep. source. (correct)!

7. Find the Thévenin equivalent of the following circuit. Sketch the equivalent circuit.



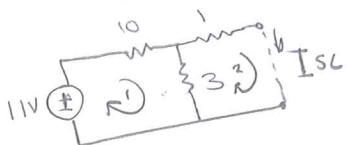
source transformation:



$$\begin{aligned} \text{KVL} \quad 15 - 2i_x + 10i_x - V_{oc} &= 0 \\ 15 = 12i_x + V_{oc} &\rightarrow 15 = 12i_x + 3i_x \\ V_{oc} = 3i_x &\rightarrow 15 = 15i_x \\ i_x &= 1 \end{aligned}$$

$$\therefore V_{oc} = 3V \checkmark$$

Now:



$$\left(10 + \frac{1}{\frac{1}{3} + 1}\right) \Omega = \left(10 + \frac{3}{4}\right) = 10.75$$

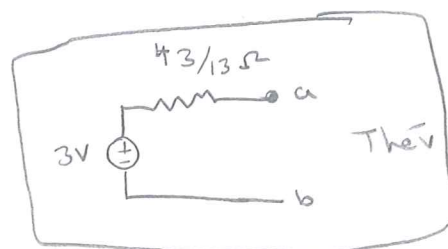
$$i = \frac{10.75}{11} = .977A$$

$$-i = i_{sc} + i_2$$

$$-.977 = \frac{V}{R} + \frac{V}{R} \quad \text{Set sources to zero}$$

$$\begin{aligned} &1 + \left(\frac{1}{10} + \frac{1}{3}\right)^{-1} \\ &= 1 + \left(\frac{13}{30}\right)^{-1} = \frac{43}{13} = R_{th} \end{aligned}$$

$$\begin{aligned} \text{mesh 1} \quad 11 - 10i_1 - 3(i_1 - i_{sc}) &= 0 \\ 11 - 13i_1 + 3i_{sc} &= 0 \\ \text{mesh 2} \quad -3(i_{sc} - i_1) + 1(i_{sc}) &= 0 \\ -2i_{sc} - 3i_1 &= 0 \end{aligned}$$



$$\begin{aligned} I_{sc} &= -10 \\ R_{th} &= \end{aligned}$$

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EE 281 A

Exam 1

Spring 2011

Power = Watts
Energy = $\int$ Power = Joules

Show all your work. Circle or box your answer.

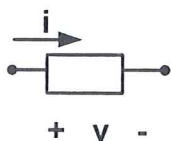
$$v_k = \frac{R_k}{R_1 + R_2 + R_3 + \cdots + R_n} v_s$$

$$i_k = \frac{\frac{1}{R_k}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}} i_s$$

$$R = \frac{\rho L}{A}$$

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1. Calculate the power absorbed by the circuit element as a function of time. Calculate the energy delivered to the element on the interval $0 \leq t < 3$ s.



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(10 pts)

$$i = 2e^{-3t} \text{ A}$$

$$v = 6e^{-3t} \text{ V}$$

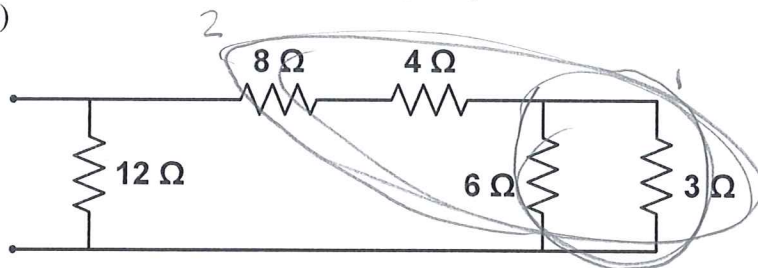
$$P = i v = 2e^{-3t} (6e^{-3t}) = 12e^{-6t} \text{ W}$$

$$W = \int_0^3 12e^{-6t} dt = -2e^{-6t} \Big|_0^3 = 2(1 - e^{-18}) \approx 1.99 \text{ J}$$

8

2. Simplify the following to a single equivalent resistance.

(15 pts)



$$\frac{1}{R_1} = \frac{1}{6} + \frac{1}{3} = \frac{1}{2} \Rightarrow R_1 = 2 \Omega$$

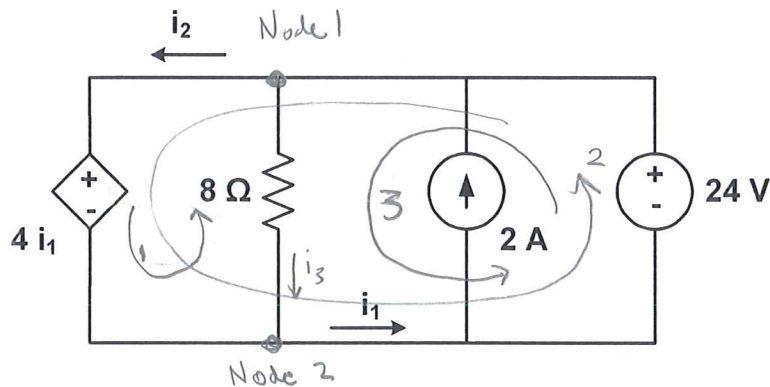
$$R_2 = 8 + 4 + 2 = 14 \Omega$$

$$\frac{1}{R_{eq}} = \frac{1}{12} + \frac{1}{14} = \frac{11}{84} \Rightarrow R_{eq} = \frac{84}{11} \approx 7.64 \Omega$$

$$R_{eq} = 7.64 \Omega$$

23

3. Solve for i_1 and i_2 in the following circuit.
(15 pts)



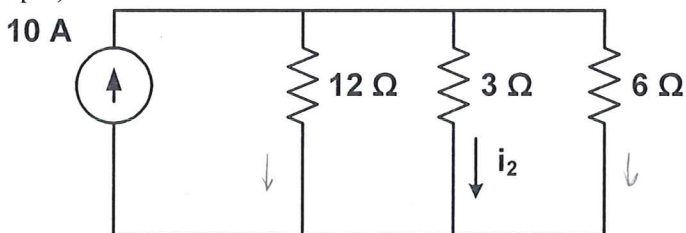
KVL 3 $24 - 8i_3 = 0$
 $i_3 = 3 \text{ A}$

KVL 2 $24 - 4i_1 = 0$
 $i_1 = \frac{24}{4} = 6 \text{ A}$ $i_1 = 6 \text{ A}$

Node 2 KCL $i_1 - i_3 - i_2 = 0$
 $6 - (3) = i_2$ $i_2 = 3 \text{ A}$

To double-check:
KVL 1
 $-4i_1 + 8i_3 = 0$
 $-4(6) + 8(3) = 0$
OK ✓

4. Use current divider to find i_2 in the following circuit.
(10 pts)



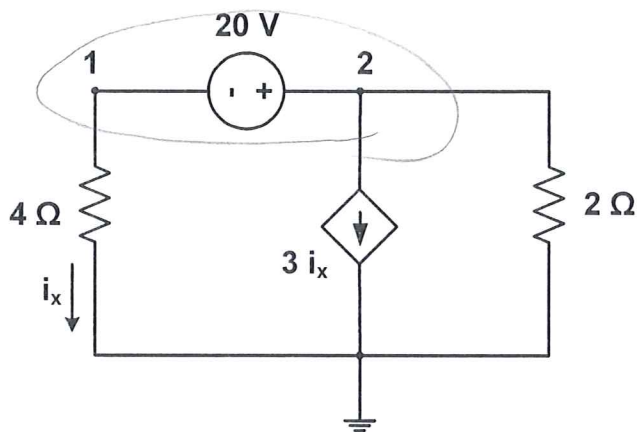
$R_k = 3 \Omega$

$i_2 = \frac{\frac{1}{3}}{\frac{1}{12} + \frac{1}{3} + \frac{1}{6}} (10 \text{ A}) = \boxed{i_2 = 5.71 \text{ A}}$

5. Calculate the resistance of a 10-m long piece of copper wire that has a diameter of 2.6 mm. The resistivity of copper is $1.7 \times 10^{-8} \Omega \cdot \text{m}$.
(10 pts)

$$R = \sigma \frac{L}{A} = 1.7 \times 10^{-8} \left(\frac{10}{\frac{\pi}{4} (.0026)^2} \right) = .0320 \Omega$$

6. Solve for the node voltages, v_1 and v_2 , in the following circuit.
(15 pts)



$$v_2 - v_1 = 20$$

Supermode (1 2)

$$i_x + 3i_x + \frac{v_2}{2} = 0$$

$$4i_x = -\frac{v_2}{2}$$

$$v_2 = -8i_x \quad (1)$$

$$\frac{v_1 - 0}{4} = i_x$$

$$v_1 = 4i_x \quad (2)$$

$$v = ir$$

$$i = \frac{v}{r}$$

$$v_2 = -8 \left(\frac{v_1}{4} \right)$$

$$v_2 = -4v_1$$

$$-4v_1 - v_1 = 20$$

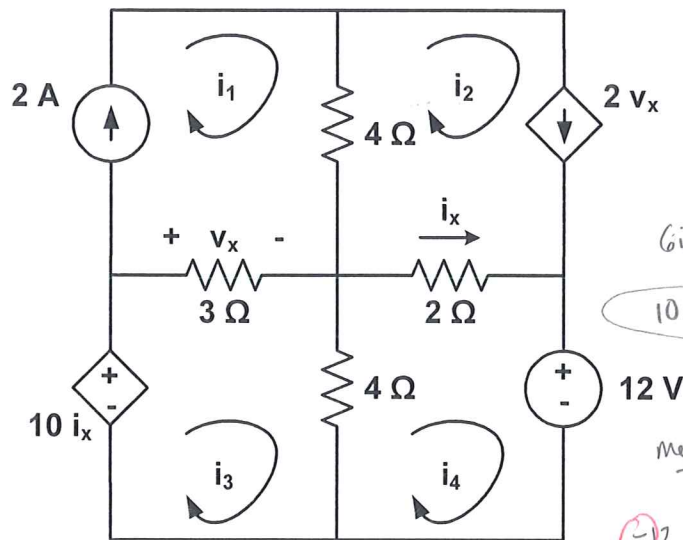
$$-5v_1 = 20$$

$$v_2 = 16 \text{ V}$$

$$v_1 = -4 \text{ V}$$

24

7. Set up the equations to find the mesh currents, i_1 , i_2 , i_3 , and i_4 , in the following circuit in matrix form. You must eliminate the control variables, v_x and i_x , from your equations. (25 pts)



Mesh 2

$$2v_x - 2(i_2 - i_4) - 4(i_2 - i_1) = 0$$

$$6i_1 - 6i_3 - 2i_2 - 2i_4 - 4i_2 + 4i_1 = 0$$

$$10i_1 - 6i_2 - 6i_3 - 2i_4 = 0$$

Mesh 4

$$-2 + 4(i_4 - i_3) + 2(i_4 - i_2) = 0$$

$$0i_1 - 2i_2 - 4i_3 + 6i_4 = 2$$

$$v_x = (i_1 - i_3)3$$

$$i_x = (i_4 - i_2)$$

Mesh 3

$$10i_x - 3(i_3 - i_1) - 4(i_3 - i_4) = 0$$

$$10i_4 - 10i_2 - 3i_3 + 3i_1 - 4i_3 + 4i_4 = 0$$

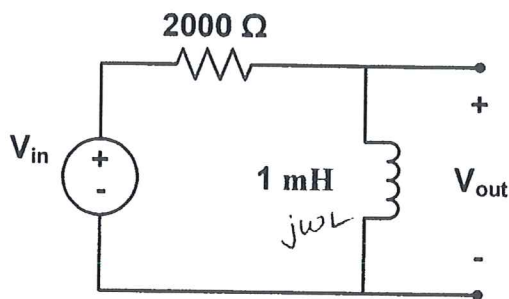
$$3i_1 - 10i_2 - 7i_3 + 14i_4 = 0$$

Mesh 1

$$i_1 = 2A$$

$$\begin{bmatrix} 5 & -3 & -3 & -1 \\ 0 & -1 & -2 & +3 \\ 3 & -10 & -7 & +14 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 0 \\ 2 \end{bmatrix}$$

1. Find the transfer function $H(j\omega) = \frac{V_{out}}{V_{in}}$ for the following filter. Is this a highpass or lowpass filter?

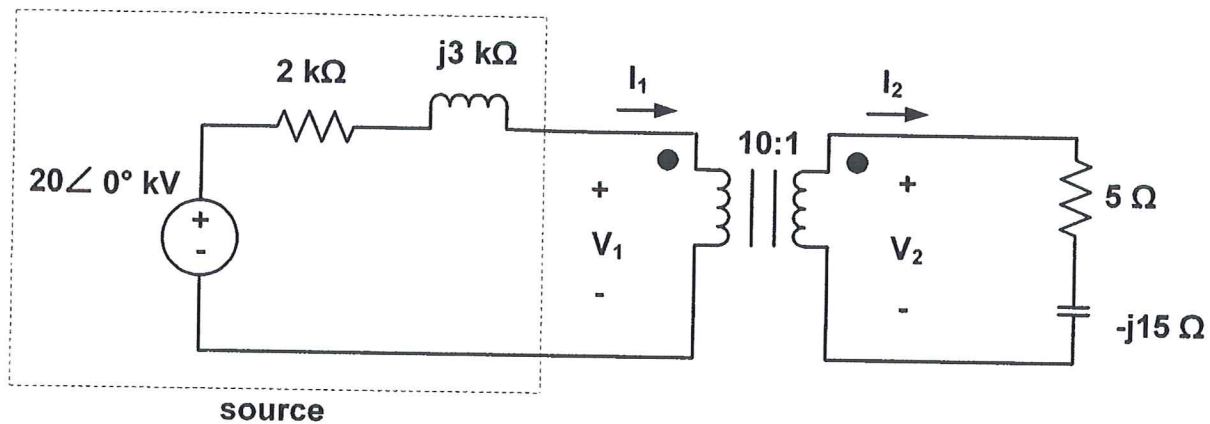


$$H(j\omega) = \frac{j\omega L}{R + j\omega L} = \frac{j\omega(0.001)}{2000 + j\omega(0.001)}$$

highpass

(15 pts)

2. Using impedance transformations (reflected impedances), find the equivalent impedance seen by the source in the following circuit.



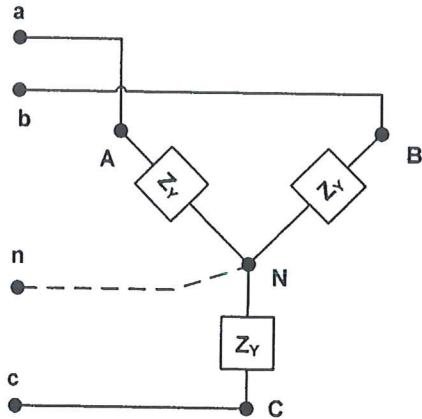
(10 pts) $Z_L' = \left(\frac{N_1}{N_2}\right)^2 Z_L = \left(\frac{10}{1}\right)^2 (5 - j15) = 500 - j1500 \Omega$

3. Find the turns ratio of an ideal transformer necessary to step a $40\text{-}V_{rms}$ source up to $1000\text{-}V_{rms}$.

(5 pts) $\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{1000}{40} = \frac{25}{1}$

$N_1 : N_2 = 1 : 25$

4. A balanced Y-connected load has an impedance of $Z_Y = 300 + j20 \Omega$ for each phase. Assuming the phase sequence is positive and that V_{AB} is the reference, find the following when the line voltage is $V_L = 20 \text{ kV}_{\text{rms}}$.



$$Z_Y = 300 + j20 \Omega = 300.67 \angle 3.81^\circ \Omega$$

$$V_{AB} = 20 \angle 0^\circ \text{ kV}_{\text{rms}}$$

- a. The phasor line-to-neutral voltage V_{AN} .
(3 pts)

$$V_{AN} = \frac{V_{AB}}{\sqrt{3} \angle 30^\circ} = \frac{20}{\sqrt{3}} \angle -30^\circ \text{ V}_{\text{rms}}$$

- b. The phasor currents I_{AN} , I_{BN} , and I_{CN} .
(9 pts)

$$I_{AN} = \frac{V_{AN}}{Z_Y} = \frac{20,000/\sqrt{3} \angle -30^\circ}{300 + j20} = 38.40 \angle -33.8^\circ \text{ A}$$

$$I_{BN} = 38.40 \angle -153.8^\circ \text{ A}$$

$$I_{CN} = 38.40 \angle -273.8^\circ \text{ A}$$

- c. The phasor line current I_{aA} .
(5 pts)

$$I_{aA} = I_{AN} = 38.40 \angle -33.8^\circ \text{ A}$$

- d. The average power, P , absorbed by the 3-phase load.
(5 pts)

$$P = 3 V_Y I_L \cos \theta = 3 \left(\frac{20,000}{\sqrt{3}} \right) (38.4) \cos(3.81^\circ) = 1,327,275 \text{ W}$$

- e. The impedance Z_Δ to create an equivalent Δ -connected load.
(3 pts)

$$Z_\Delta = 3 Z_Y = 900 + j60 \Omega$$

5. The transfer function for a particular first order filter is $H(j\omega) = \frac{V_{out}}{V_{in}} = \frac{j\omega(0.005)}{1+j\omega(0.005)}$.

- a. Find the half-power frequency of this filter.
(5 pts)

$$\omega = \frac{1}{0.005} = \boxed{200 \text{ rad/sec}}$$

- b. Find the output current, $v_{out}(t)$ if the input current is $v_{in}(t) = 20 \cos(20t) + 20 \cos(5000t - 45^\circ)$ V.
(10 pts)

$$\omega = 20 \quad V_{in} = 20 \angle 0^\circ$$

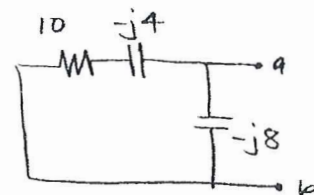
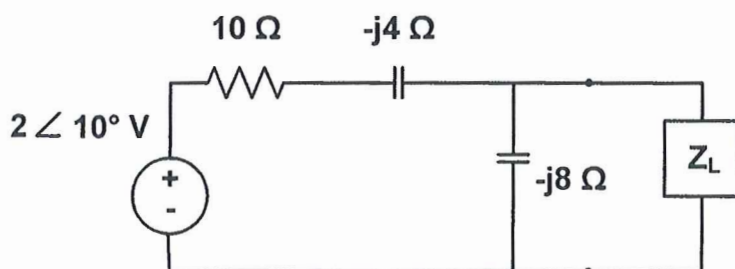
$$V_{out} = \frac{j(20)(0.005)}{1+j(20)(0.005)} 20 \angle 0^\circ = 1.99 \angle 84.3^\circ$$

$$\omega = 5000 \quad V_{in} = 20 \angle -45^\circ$$

$$V_{out} = \frac{j(5000)(0.005)}{1+j(5000)(0.005)} 20 \angle -45^\circ = 19.98 \angle -42.7^\circ$$

$$\boxed{v_{out}(t) = 1.99 \cos(20t + 84.3^\circ) + 19.93 \cos(5000t - 42.7^\circ) \text{ V}}$$

6. For the following circuit, determine the value of Z_L that results in maximum average power transferred to Z_L .



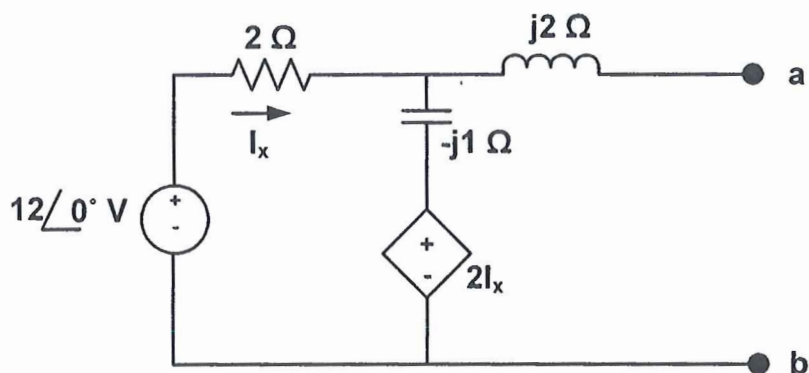
(10 pts)

$$Z_{Th} = (10 - j4) \parallel -j8 = \frac{(10 - j4)(-j8)}{10 - j4 - j8} = 5.52 \angle -61.6^\circ \Omega$$

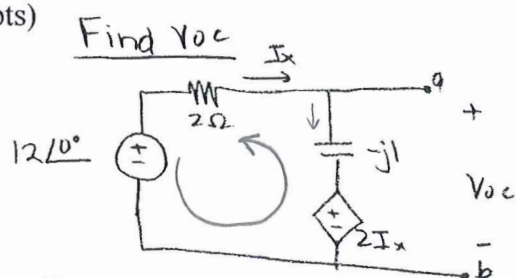
$$= 2.62 - j4.85 \Omega$$

$$Z_L = Z_{Th}^* = \boxed{2.62 + j4.85 \Omega}$$

7. Find the Thévenin equivalent of the following circuit.



(20 pts)



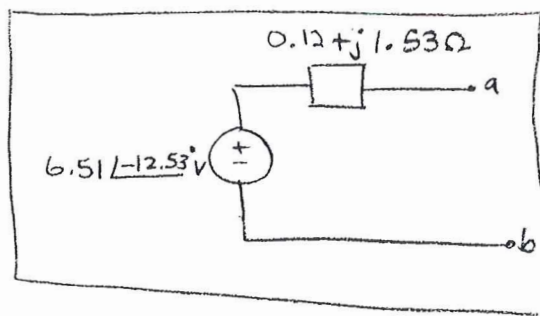
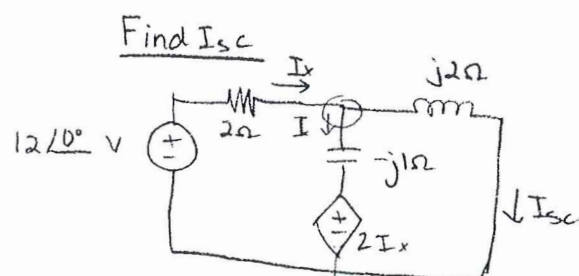
KVL:

$$-12 + 2I_x - jI_x + 2I_x = 0$$

$$(4-j)I_x = 12$$

$$I_x = 12 / (4-j) = 2.91 \angle 14.04^\circ$$

$$V_{oc} = -jI_x + 2I_x = (2-j)I_x = 6.51 \angle -12.53^\circ \text{ V}$$



KVL outer loop: $-12 + 2I_x + j2I_{sc} = 0$

① $2I_x + j2I_{sc} = 12 \Rightarrow I_x = \frac{12 - j2I_{sc}}{2} = 6 - jI_{sc}$

KCL: $I = I_x - I_{sc}$

KVL left loop: $-12 + 2I_x - jI + 2I_x = 0$

$$-12 + 4I_x - j(I_x - I_{sc}) = 0$$

② $(4-j)I_x + jI_{sc} = 12$

$$= 1.53 \angle 85.6^\circ \Omega \quad (4-j)(6 - jI_{sc}) + jI_{sc} = 12$$

$$= 0.12 + j1.53 \Omega \quad (-j(4-j) + j)I_{sc} = 12 - 6(4-j)$$

$$I_{sc} = \frac{12 - 6(4-j)}{-j(4-j) + j} = -0.6 - j4.2 = 4.24 \angle -98.13^\circ \text{ A}$$