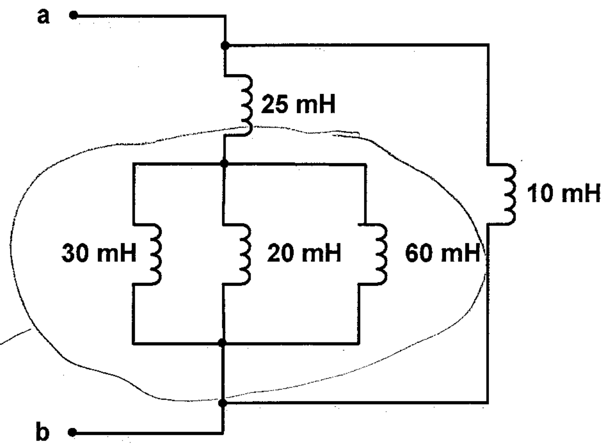
Key

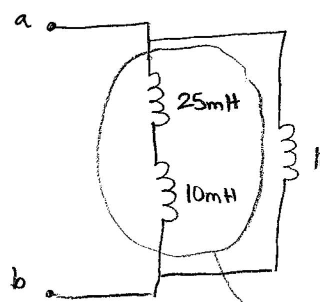
EE281A

Homework #10 -- 20 points Due: Friday, February 19, 2010

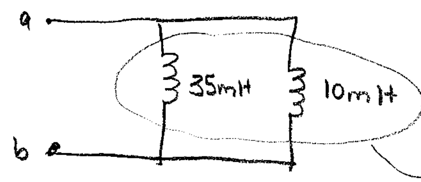
1. Simplify the following to a single equivalent component. (6 pts)



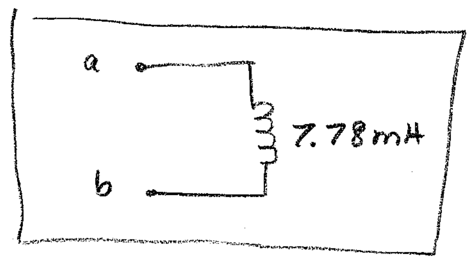
$$30 \| 20 \| 60 = \frac{1}{\frac{1}{30}+\frac{1}{20}+\frac{1}{60}} = 10\,\text{mH}$$



$25+10 = 35\,\text{mH}$



$$\frac{35(10)}{35+10} = 7.78\,\text{mH}$$



2. If the voltage across a 2-mH inductor is $v(t) = 2\cos(3t)$ V, find the current through the inductor as a function of t if the initial current at t=0 is 0.25 A. (4 pts)

$$i(t) = i(0) + \frac{1}{L}\int_0^t v\,d\lambda = 0.25 + \frac{1}{0.002}\int_0^t 2\cos(3\lambda)\,d\lambda$$

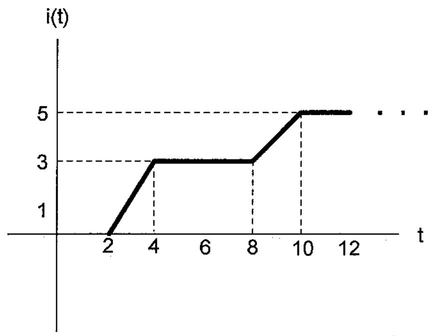
$$= 0.25 + 500\left(\frac{2}{3}\right)\sin(3\lambda)\Big|_0^t = \boxed{0.25 + \frac{1000}{3}\sin(3t) \quad \text{A}}$$

3. If the current through a 4-μH inductor is $i(t) = 2e^{-5t}$ A, find the voltage across the inductor as a function of t. (4 pts)

$$v = L\frac{di}{dt} = (4\times10^{-6})(-10e^{-5t})$$

$$= \boxed{-4\times10^{-5}e^{-5t} \quad \text{V}}$$

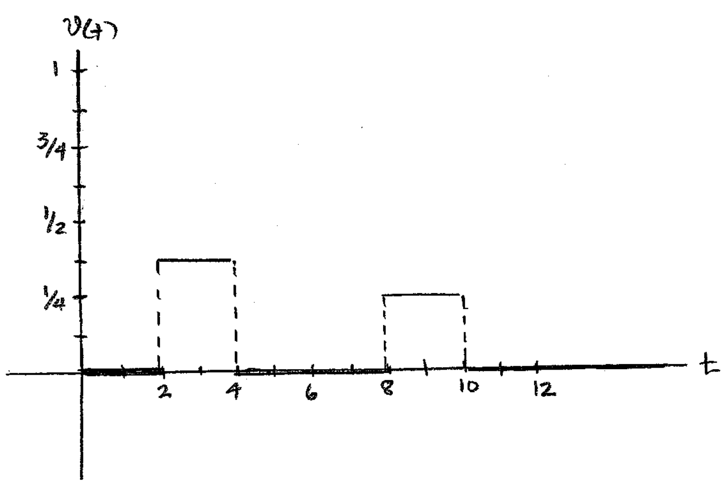
4. If the current through a 0.25-H inductor is given by the graph below, find the piecewise function that describes the voltage across the inductor and sketch the graph of the voltage. (6 pts)



$$v = L\frac{di}{dt}$$

$$\frac{di}{dt} = \begin{cases} 0 & , \quad 0 < t < 2 \\ 3/2 & , \quad 2 < t < 4 \\ 0 & , \quad 4 < t < 8 \\ 1 & , \quad 8 < t < 10 \\ 0 & \quad t > 10 \end{cases}$$

$$\boxed{v = \begin{cases} 0 & , \quad 0 < t < 2 \\ 3/8 & , \quad 2 < t < 4 \\ 0 & , \quad 4 < t < 8 \\ 1/4 & , \quad 8 < t < 10 \\ 0 & , \quad t > 10 \end{cases}}$$

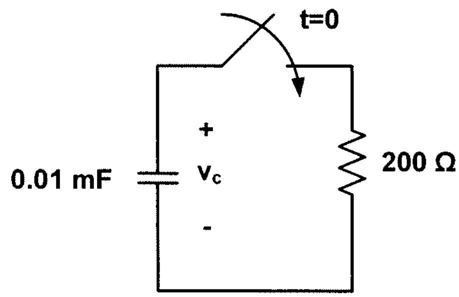


Key.

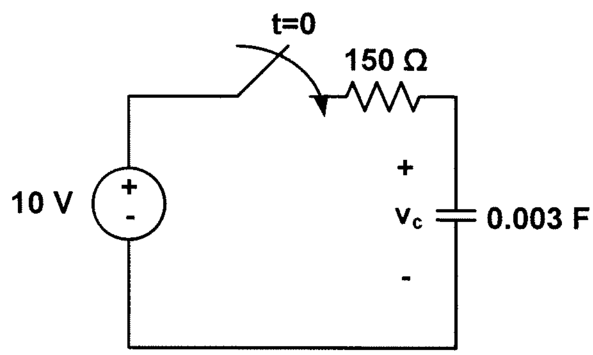
EE281B

Homework #11 -- 20 points Due: Friday, October 8, 2010

1. If the initial voltage across the capacitor is $v_c(0) = 45V$, find the time (in seconds) for the voltage v_c to be reduced to 10 V as the capacitor discharges. (7 pts)



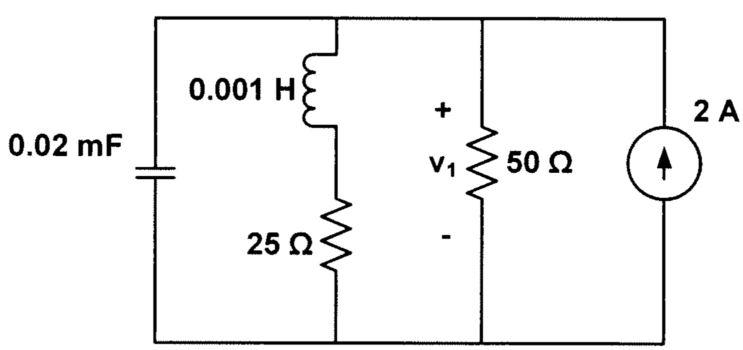
2. Consider the RC circuit in which v_c is the capacitor voltage.



   a. Write the differential equation in standard form used to find $v_c(t)$ for t>0. (3 pts)

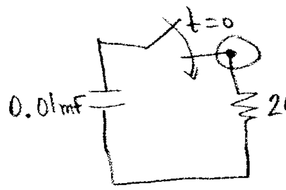
   b. If $v(0^+)=0$ V, find the solution of the differential equation (5 pts)

3. Solve the steady-state voltage v_1 in the following circuit. (5 pts)



EE281 Homework #11

1.



$RC = (0.01\times10^{-3})(200)$
$= 0.002$

$v_c(0^+) = 45V$

$$C\frac{dv_c}{dt} + \frac{v_c}{R} = 0$$

$$\frac{dv_c}{dt} + \frac{1}{RC}v_c = 0$$

$$\frac{dv_c}{dt} + 500\,v_c = 0$$

$$s + 500 = 0$$
$$s = -500$$

$$v_c(t) = K e^{-500t}$$

$$v_c(0) = 45 = Ke^0$$
$$K = 45$$

$$v_c(t) = 45e^{-500t}\ V$$

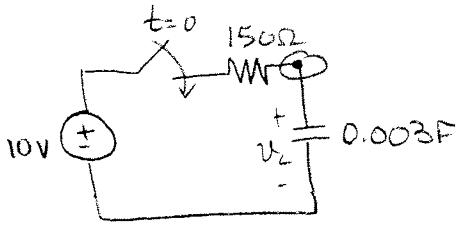
$$10 = 45e^{-500t}$$

$$\ln\left(\frac{10}{45}\right) = -500t$$

$$t = -\frac{1}{500}\ln\left(\frac{10}{45}\right)$$

$$\boxed{t = 3.01\ ms}$$

2.



$v(0^+) = 0\ V$

$$0.003\frac{dv_c}{dt} + \frac{v_c - 10}{150} = 0$$

$$\frac{dv_c}{dt} + \frac{1}{0.45}v_c = \frac{10}{0.45}$$

a) $$\boxed{\frac{dv_c}{dt} + 2.22\,v_c = 22.22}$$

b) $$\frac{dv_c}{dt} + 2.22v_c = 0$$

$$s + 2.22 = 0$$
$$s = -2.22$$

$$v_n = K_1e^{-2.22t}$$

$$v_p = K_2$$

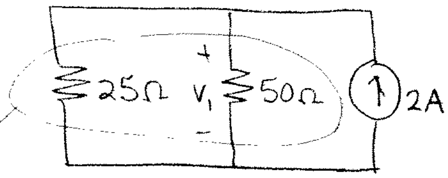
$$0 + 2.22K_2 = 22.22$$
$$K_2 = 10$$

$$v_c = K_1e^{-2.22t} + 10$$

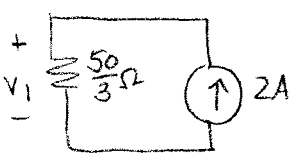
$$v_c(0^+) = 0 = K_1e^0 + 10$$
$$K_1 = -10$$

$$\boxed{v_c(t) = 10 - 10e^{-2.22t}\ V}$$

3.



$25 \| 50 = \frac{50}{3}$

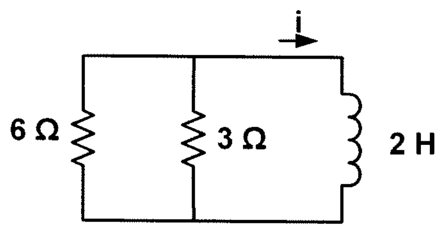


$$V_1 = \frac{50}{3}(2A) = \boxed{\frac{100}{3}\ V}$$

EE281 B

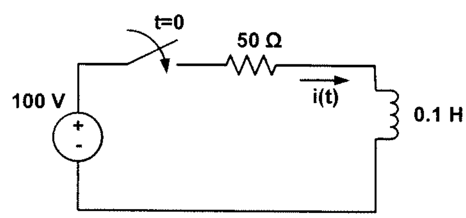
Homework #12 -- 20 points Due: Wednesday, October 13, 2010

1. Consider the source-free RL circuit in which i is the inductor current.



   a. Write the differential equation in standard form used to find i(t) for t>0. (4 pts)
   b. If $i(0^+)=2$ A, find the solution of the differential equation (i.e. the natural response of the circuit. (4 pts)
   c. What is the time constant, τ, for this problem? (2 pts)

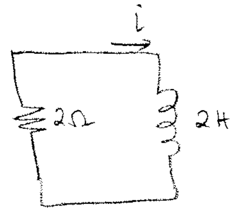
2. Consider the circuit shown below.



   a. What is the initial current in the loop, $i(0^+)$? (2 pts)

   b. Find the current, i, for t>0 as a function of t. (8 pts)

1.



$i(0^+) = 2A$

a) KVL: $L\frac{di}{dt} + Ri = 0$

$$\frac{di}{dt} + \frac{R}{L} i = 0$$

$$\boxed{\frac{di}{dt} + i = 0}$$

b) $s + 1 = 0$

$s = -1$

$i(t) = Ke^{-t}$

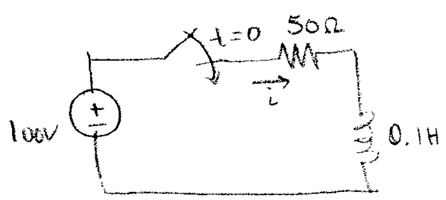
$i(0^+) = 2 = Ke^0$

$K = 2$

$$\boxed{i(t) = 2e^{-t} \text{ A}}$$

c) $\boxed{\tau = \frac{L}{R} = 1s}$

2.



a) $\boxed{i(0^+) = 0 \text{ A}}$

b) KVL: $-100 + 50i + 0.1\frac{di}{dt} = 0$

$$\frac{di}{dt} + 500i = 1000$$

$$\frac{di}{dt} + 500i = 0$$

$i_h = K_1 e^{-500t}$

$i_p = K_2$ plug into D.E.

$0 + 500K_2 = 1000$

$K_2 = 2$

$i(t) = K_1 e^{-500t} + 2$

$i(0^+) = 0 = K_1 e^0 + 2$

$K_1 = -2$

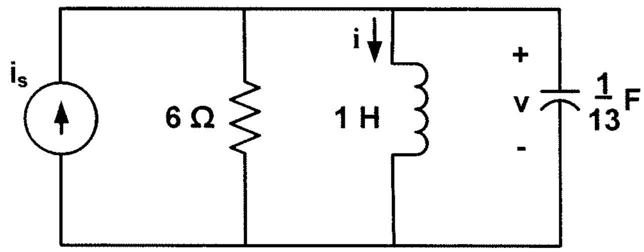
$$\boxed{i(t) = 2 - 2e^{-500t} \text{ A}}$$

Can use the form of the eqn given in the text rather than solving the D.E.

EE281 B

Homework #13 -- 20 points Due: Monday, October 18, 2010

1. In the following circuit $i_s = 250e^{-2t}$ V for t>0.



a. Write a KCL equation at the top node and then take the derivative to create a differential equation in terms of the voltage, v. (4 pts)

b. What are the initial current, $i(0^+)$, and initial voltage, $v(0^+)$, if the source has been off for a long time and is then turned on at t=0? (2 pts)

c. Find the form of the source-free response (do not solve for the constants). (4 pts)

d. Find the particular response, $v_p(t)$. (4 pts)

e. Find the complete solution, v(t). (Solve for all constants.) (4 pts)

f. Is the circuit overdamped, underdamped, or critically damped? (2 pts)

EE281 Hwk 13

1. a) $-i_s + \frac{v}{6} + \frac{1}{L}\int_{-\infty}^{t} v\,d\lambda + \frac{1}{13}\frac{dv}{dt} = 0$

$i_s = 250e^{-2t}$ A

$\frac{di_s}{dt} = -500e^{-2t}$ A

$$\boxed{\frac{d^2v}{dt^2} + \frac{13}{6}\frac{dv}{dt} + 13v = -500e^{-2t}}$$

b) $\boxed{\begin{array}{l} i(0^+) = 0 \\ v(0^+) = 0 \end{array}}$

c) $\frac{d^2v}{dt^2} + \frac{13}{6}\frac{dv}{dt} + 13v = 0$

$s^2 + \frac{13}{6}s + 13 = 0$

$s = \frac{-\frac{13}{6} \pm \sqrt{(\frac{13}{6})^2 - 4(13)}}{2} = -1.08 \pm j\,3.44$

$$\boxed{v_n(t) = K_1e^{-1.08t}\cos(3.44t) + K_2e^{-1.08t}\sin(3.44t)}$$

d) $v_p(t) = Ae^{-2t}$

$v_p' = -2Ae^{-2t}$

$v_p'' = 4Ae^{-2t}$

$4\cancel{Ae^{-2t}} + \frac{13}{6}(-2A\cancel{e^{-2t}}) + 13(\cancel{Ae^{-2t}}) = -500\cancel{e^{-2t}}$

$4A - \frac{13}{3}A + 13A = -500$

$A = -500/(4 - \frac{13}{3} + 13) = -39.47$

$$\boxed{v_p(t) = -39.47e^{-2t}}$$

$v(t) = K_1e^{-1.08t}\cos(3.44t) + K_2e^{-1.08t}\sin(3.44t) - 39.47e^{-2t}$

e) $v(0^+) = 0 = K_1 - 39.47$

$K_1 = 39.47$

$i(0) = 0 \rightarrow \frac{dv}{dt} = 0$

$v'(t) = K_1e^{-1.08t}(-3.44\sin(3.44t)) - 1.08K_1e^{-1.08t}\cos(3.44t) + K_2e^{-1.08t}(3.44\cos(3.44t))$
$- 1.08K_2e^{-1.08t}\sin(3.44t) + 78.94e^{-2t}$

f) $\boxed{\text{underdamped}}$

$v'(0) = 0 = -1.08K_1 + 3.44K_2 + 78.94$

$K_2 = (1.08(39.47) - 78.94)/3.44 = -10.56$

$$\boxed{v(t) = 39.47e^{-1.08t}\cos(3.44t) - 10.56e^{-1.08t}\sin(3.44t) - 39.47e^{-2t}\ \text{V}}$$

Key

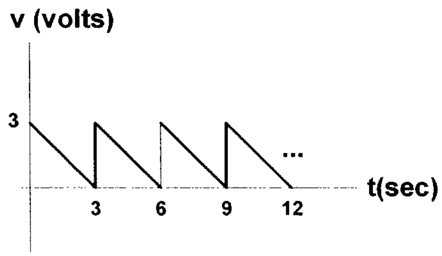
EE281 B

Homework #14 -- 20 points Due: Friday, October 22, 2010

1. Find the RMS value of each of the following signals. (4 pts each)

   a. $i(t) = 8\sin(500t - 15°)$ A $\boxed{I_{rms} = \frac{8}{\sqrt{2}} A}$

   b.



$$V_{rms} = \sqrt{\frac{1}{3}\int_0^3 (3-t)^2 dt} = \sqrt{\frac{1}{3}\left[-\frac{1}{3}(3-t)^3\Big|_0^3\right]}$$

$$= -\frac{1}{9}(0 - 3^3) = \frac{27}{9} = \boxed{3V}$$

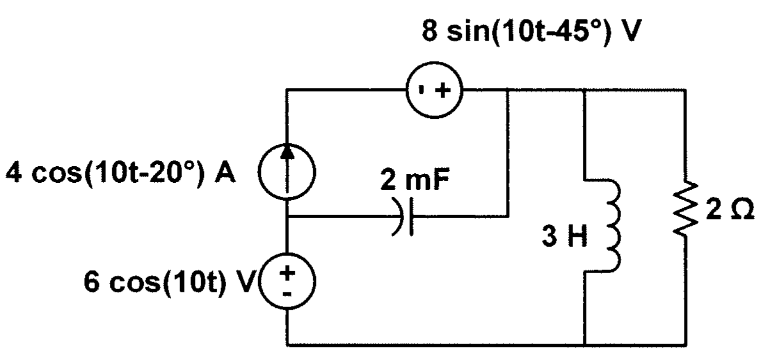
2. Write each of the following signals as phasors. (3 pts each)

   a. $v(t) = 12\cos(4t + 12°)$ V $\boxed{V = 12\angle 12° \ V}$

   b. $i(t) = 1.4\sin(20t + 30°)$ A $\boxed{I = 1.4\angle -60° A}$

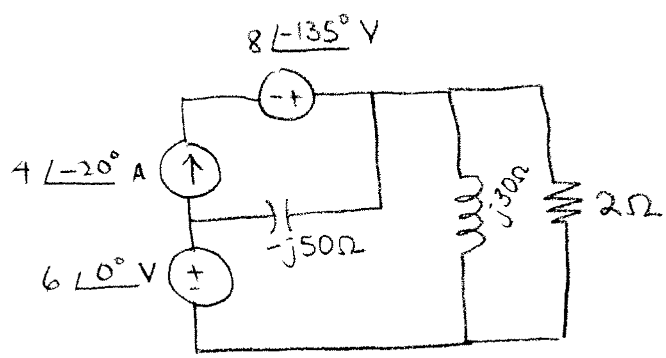
   $= 1.4\cos(20t - 60°)A$

3. Redraw the following circuit in the frequency domain (in terms of phasors and impedances). (6 pts)



$$\frac{1}{j\omega C} = -j\frac{1}{(2\times10^{-3})(10)} = -j50$$

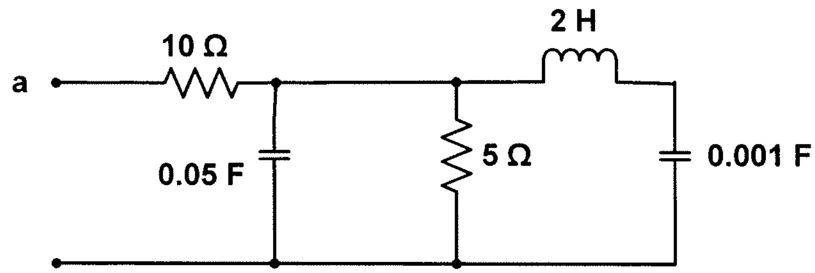
$$j\omega L = j(10)(3) = j30$$



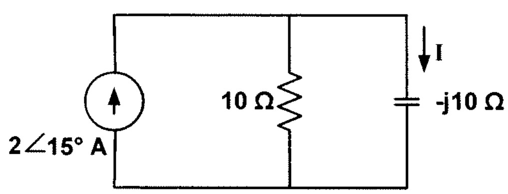
EE281 B

Homework #15 -- 20 points Due: Monday, October 25, 2010

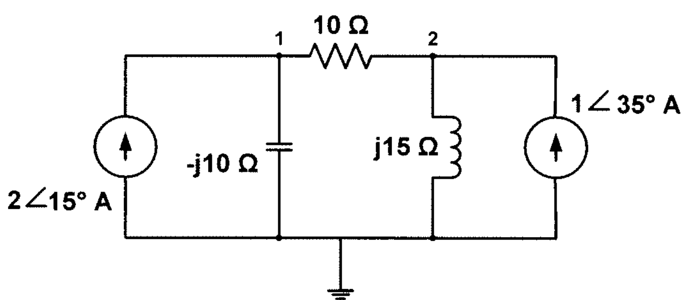
1. Write the following as a single impedance. Assume that ω=100 rad/sec. (6 pts)



2. Find the steady-state current i(t) in the following circuit if ω=100 rad/sec. (7 pts)



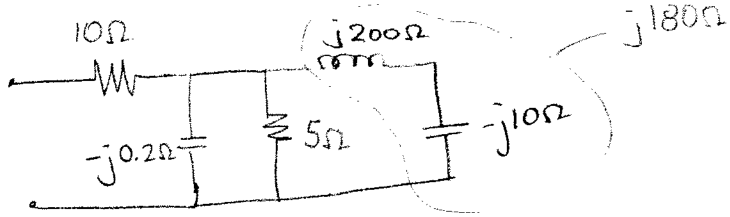
3. Set up the node voltage equations in matrix form for the following circuit. (7 pts)

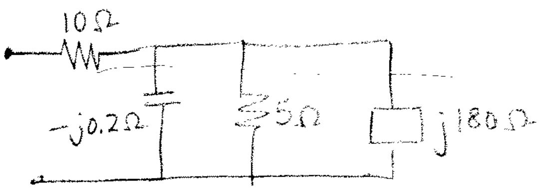


Homework #15

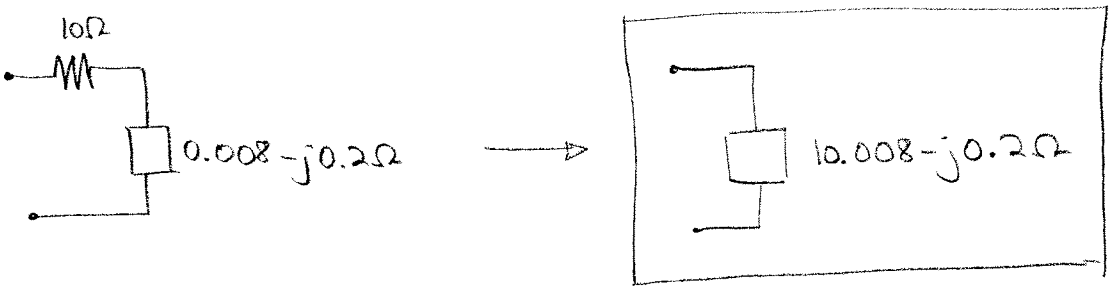
$\omega = 100\ \text{rad/sec.}$

1.





$$\frac{1}{\frac{1}{-j0.2} + \frac{1}{5} + \frac{1}{j180}} = 0.2\angle -87.71^\circ\ \Omega$$
$$= 0.008 - j0.2\ \Omega$$



$= 10.0\angle -1.14^\circ\ \Omega$

2. current divider:

$$I = \frac{10}{10 - j10}\left(2\angle 15^\circ\right) = 1.414\angle 60^\circ\ \text{A}$$

$$\boxed{i(t) = 1.414\cos(100t + 60^\circ)\ \text{A}}$$

3. node 1

$$-2\angle 15^\circ + \frac{V_1}{-j10} + \frac{V_1 - V_2}{10} = 0$$

① $\left(\frac{1}{10} + j\frac{1}{10}\right)V_1 - \frac{1}{10}V_2 = 2\angle 15^\circ$

node 2

$$\frac{V_2 - V_1}{10} + \frac{V_2}{j15} - 1\angle 35^\circ = 0$$

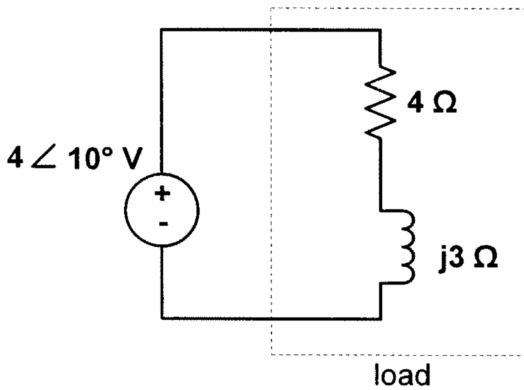
② $-\frac{1}{10}V_1 + \left(\frac{1}{10} - j\frac{1}{15}\right)V_2 = 1\angle 35^\circ$

$$\boxed{\begin{bmatrix} \frac{1}{10} + j\frac{1}{10} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{1}{10} - j\frac{1}{15} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2\angle 15^\circ \\ 1\angle 35^\circ \end{bmatrix}}$$

EE 281B

Homework #16 -- 20 points Due: Wednesday, October 27, 2010

1. Given the following circuit, find:



   a. The average power absorbed by the load. (4 pts)

   b. The reactive power absorbed by the load. (4 pts)

   c. The apparent power absorbed by the load. (3 pts)

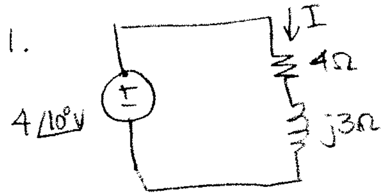
   d. The power factor of the load. (3 pts)

   e. Sketch the power triangle for the load. (3 pts)

2. What is the purpose of the large capacitor banks often found in factories which operate large induction motors? (3 pts)

Homework #16

1.



$Z = 4 + j3\,\Omega = 5\angle 36.87^\circ\ \Omega$

$I = \frac{V}{Z} = \frac{4\angle 10^\circ}{4+j3} = 0.8\angle -26.87^\circ\ A$

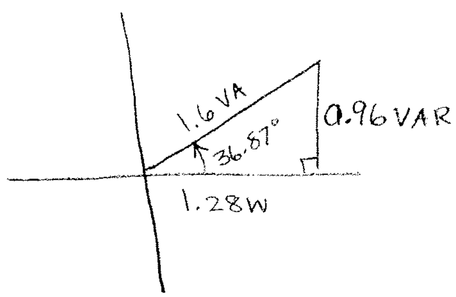
a) $P = \frac{1}{2} V_m I_m \cos\theta = \frac{1}{2}(4)(0.8)\cos(36.87^\circ) = \boxed{1.28\,W}$

b) $Q = \frac{1}{2} V_m I_m \sin\theta = \frac{1}{2}(4)(0.8)\sin(36.87^\circ) = \boxed{0.96\ VAR}$

c) $AP = \frac{P}{PF} = \frac{1.28}{\cos(36.87^\circ)} = \boxed{1.6\ VA}$

d) $PF = \cos\theta = \boxed{0.8 \text{ lagging}}$

e)



(Other formulas for P, Q + AP are also correct)

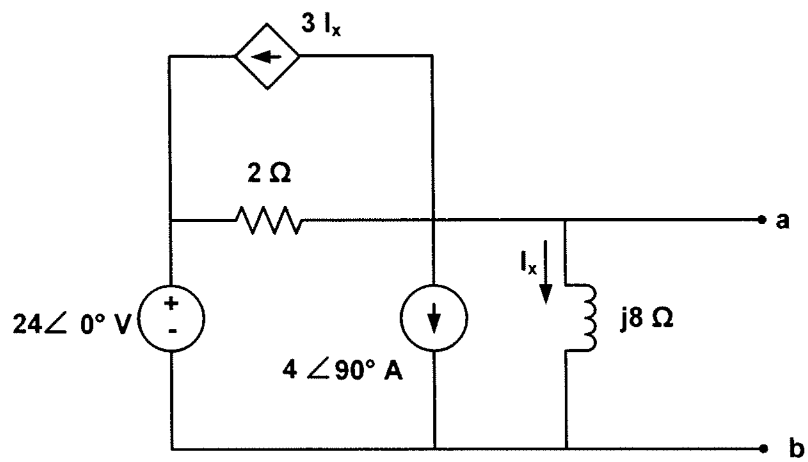
2. Power factor correction

This makes the load look more resistive by counteracting the positive reactive power of the motors with a negative reactive power from the capacitors.

EE 281B

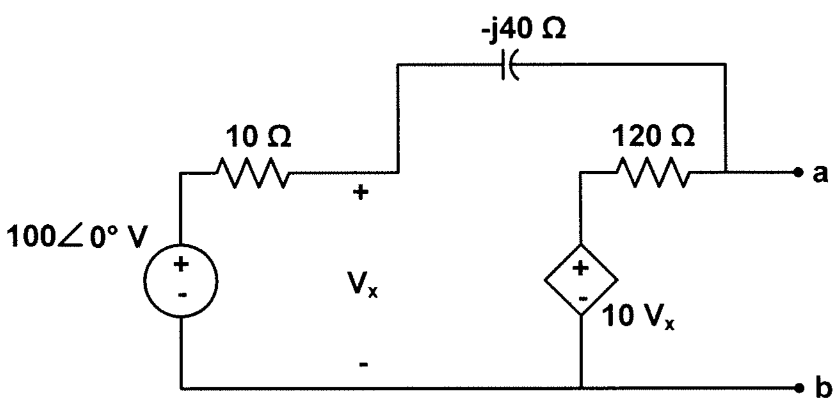
Homework #17 -- 20 points Due: Monday, November 8, 2010

1. Find the Thèvenin and Norton equivalents of the following circuit. (8 pts)



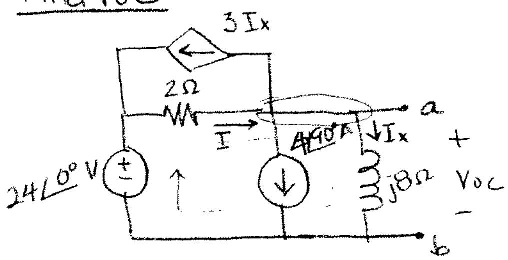
2. What load should be attached to the above circuit it maximize the power delivered to the load? (4 pts)

3. Find the Thèvenin equivalent of the following circuit. (8 pts)



EE281 Hwk #17

1. Find Voc



$V_{oc} = j8I_x$

KVL: $-24\angle 0° + 2I + j8I_x = 0$

① $2I + j8I_x = 24$

KVL: $-I + 4\angle 90° + I_x + 3I_x = 0$

② $-I + 4I_x = -4\angle 90° = -j4$

$$\begin{array}{r} 2I + j8I_x = 24 \\ 2(-I + 4I_x = -j4) \\ \hline (8+j8)I_x = 24 - j8 \end{array}$$

$I_x = \dfrac{24-j8}{8+j8} = 1 - j2\ A$

$V_{oc} = j8I_x = 16 + j8\ V$

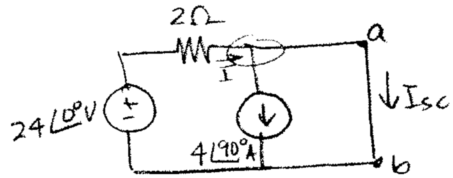
$= 17.9\angle 26.6°\ V$

$Z_{th} = \dfrac{V_{oc}}{I_{sc}} = \dfrac{16+j8}{12-j4} = 1 + j1\Omega$

$= 1.41\angle 45°\ \Omega$

Find Isc

$I_x = 0$



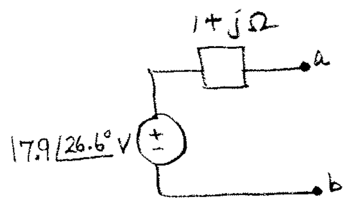
KVL: $-24\angle 0° + 2I = 0$

$I = 12\ A$

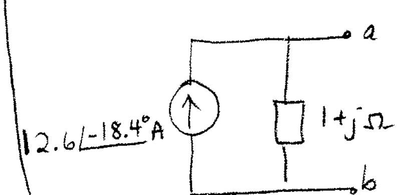
KCL: $-I + 4\angle 90° + I_{sc} = 0$

$I_{sc} = I - 4\angle 90° = 12 - j4\ A$

$= 12.6\angle -18.4°\ A$

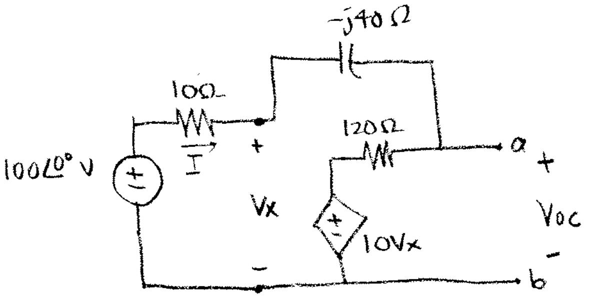


Thévenin

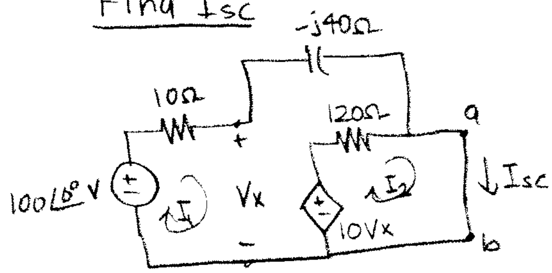


2. $\boxed{Z_{Load} = Z_{th}^* = 1 - j\ \Omega}$

3. Find V_{oc}



Find I_{sc}



$I_{sc} = I_2$

KVL:

$$-100\angle 0° + (130 - j40)I + 10V_x = 0$$

KVL: $-100\angle 0° + 10I + V_x = 0$

$$(130 - j40)I + 10V_x = 100\angle 0°$$

$$-10(\quad 10I + V_x = 100\angle 0°)$$

$$(30 - j40)I = -900$$

$$I = \frac{-900}{30 - j40} = 18\angle -126.9° \text{ A}$$

$$V_x = 100 - 10I = 258\angle 34.7° \text{ V}$$

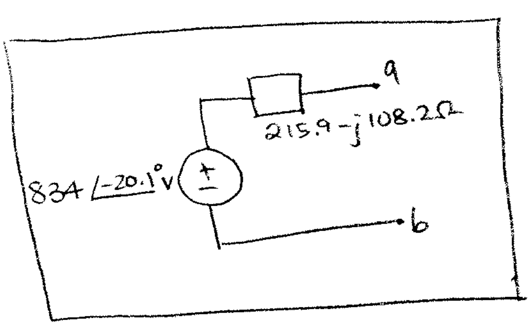
KVL: $120I + 10V_x - V_{oc} = 0$

$$V_{oc} = 120I + 10V_x = 834\angle -20.1° \text{ V}$$

$$= 783.2 - j286.6 \text{ V}$$

$$Z_{th} = \frac{V_{oc}}{I_{sc}} = 241.5\angle -26.6° \ \Omega$$

$$= 215.9 - j108.2 \ \Omega$$



m1: $-100\angle 0° + (10 - j40)I_1 + 120(I_1 - I_2) + 10V_x = 0$

m2: $-10V_x + 120(I_2 - I_1) = 0$

KVL: $-100\angle 0° + 10I_1 + V_x = 0$

$$V_x = 100 - 10I_1$$

(m2) $120(I_2 - I_1) - 1000 + 100I_1 = 0$

$$-20I_1 + 120I_2 = 1000$$

$$-I_1 + 6I_2 = 50$$

(m1) $(10 - j40)I_1 + 120(I_1 - I_2) + 1000 - 100I_1 = 100$

$$(30 - j40)I_1 - 120I_2 = -900$$

$$(3 - j4)I_1 - 12I_2 = -90$$

$$2(\quad -I_1 + 6I_2 = 50)$$

$$(3 - j4)I_1 - 12I_2 = -90$$

$$(1 - j4)I_1 = 10$$

$$I_1 = \frac{10}{1 - j4} = 2.4\angle 76.0°$$

$$I_2 = \frac{50 + I_1}{6} = 3.45\angle 6.5°$$

$$I_{sc} = 3.45\angle 6.5° \text{ A}$$

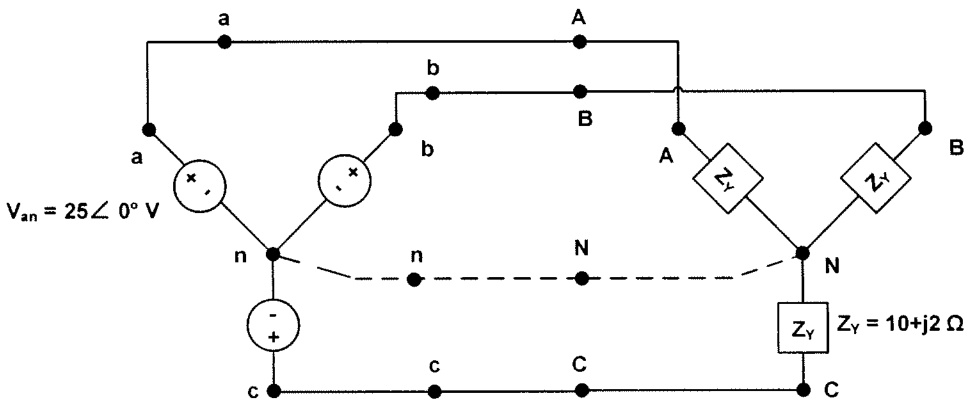
$$= 3.43 + j0.39 \text{ A}$$

EE 281B

Homework #18 -- 20 points Due: Friday, November 12, 2010

1. Each phase of a Y-connected load consists of a 50-Ω resistance in parallel with a 200-mH inductance. Find the impedance of an equivalent Δ-connected load. Assume 60-Hz operation. (4 pts)

2. For the following balanced three-phase system, assume a positive phase sequence and find:



a. The line-to-neutral voltages, V_{BN} and V_{CN}, at the load. (3 pts)

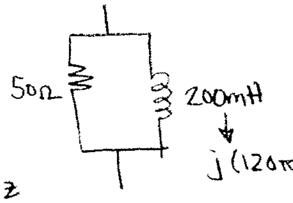
b. The line-to-line voltages, V_{ab}, V_{bc}, and V_{ca}. (5pts)

c. The line currents, I_{aA}, I_{bB}, and I_{cC}. (5 pts)

d. If, instead, the system had a negative phase sequence, find V_{BN} and V_{CN}. **(3pts)**

EE281 Homework #18

1. Z_Y:



$j(120\pi)(0.2) = j75.4\,\Omega$

$f = 60\text{ Hz}$

$\omega = 2\pi f = 120\pi \text{ rad/sec.}$

$$Z_Y = 50 \,\|\, j75.4 = \frac{50(j75.4)}{50 + j75.4}$$
$$= 41.67\angle 33.5^\circ\ \Omega$$
$$= 34.7 + j23.0\,\Omega$$

$$\boxed{\begin{aligned} Z_\Delta &= 3Z_Y \\ &= 125.01\angle 33.5^\circ\ \Omega \\ &= 104.2 + j69.1\,\Omega \end{aligned}}$$

2. a)
$$\boxed{\begin{aligned} V_{BN} &= 25\angle -120^\circ\ \text{V} \\ V_{CN} &= 25\angle -240^\circ\ \text{V} \end{aligned}}$$

b) $V_{ab} = V_{an}(\sqrt{3}\angle 30^\circ)$
$= (25\angle 0^\circ)(\sqrt{3}\angle 30^\circ)$

$$\boxed{\begin{aligned} V_{ab} &= 25\sqrt{3}\angle 30^\circ\ \text{V} \\ V_{bc} &= 25\sqrt{3}\angle -90^\circ\ \text{V} \\ V_{ca} &= 25\sqrt{3}\angle -210^\circ\ \text{V} \end{aligned}}$$

c) $I_{aA} = \dfrac{V_{AN}}{Z_Y} = \dfrac{25\angle 0^\circ\ \text{V}}{41.67\angle 33.5^\circ\ \Omega}$

$$\boxed{\begin{aligned} I_{aA} &= 0.6\angle -33.5^\circ\ \text{A} \\ I_{bB} &= 0.6\angle -153.5^\circ\ \text{A} \\ I_{cC} &= 0.6\angle -273.5^\circ\ \text{A} \end{aligned}}$$

d)
$$\boxed{\begin{aligned} V_{BN} &= 25\angle 120^\circ\ \text{V} \\ V_{CN} &= 25\angle 240^\circ\ \text{V} \end{aligned}}$$

equivalent angles are acceptable.

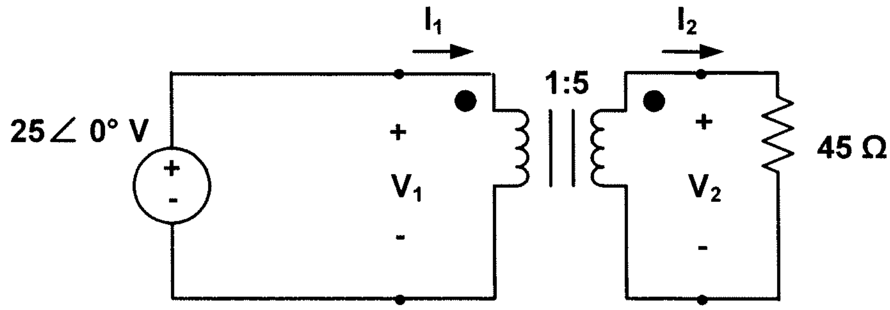
EE 281B

Homework #19 -- 20 points Due: Wednesday, November 17, 2010

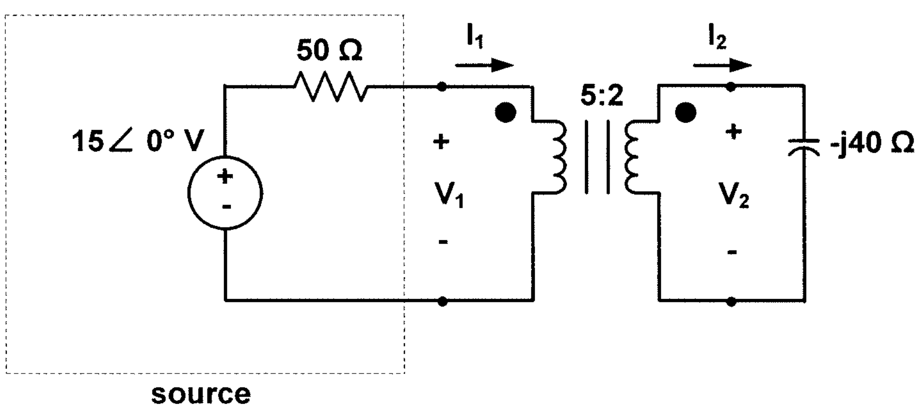
1. Find the turns ratio in an ideal transformer necessary to step up a 120-V_{rms} voltage to 5000 V_{rms}. (3 pts)

2. Find the turns ratio in an ideal transformer necessary to step down a 20-A_{rms} current to 2.5 A_{rms}. (3 pts)

3. Find the phasors $\mathbf{V_1}$, $\mathbf{V_2}$, $\mathbf{I_1}$, $\mathbf{I_2}$, and the average power delivered to the 45-Ω load. (8 pts)



4. Use impedance transformations (reflected impedances) to find the impedance seen by the source in the following circuit. (6 pts)



EE281 Homework #19

1. $V_2 = \frac{N_2}{N_1} V_1$

$5000 = \frac{N_2}{N_1}(120)$

$\frac{N_2}{N_1} = \frac{5000}{120} = \frac{250}{6} = \frac{125}{3}$

$\boxed{N_1 : N_2 = 3 : 125}$

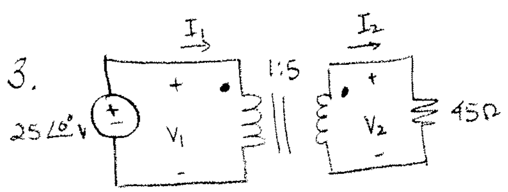
2. $I_2 = \frac{N_1}{N_2} I_1$

$2.5 = \frac{N_1}{N_2}(20)$

$\frac{N_1}{N_2} = \frac{2.5}{20} = \frac{25}{200} = \frac{1}{8}$

$\boxed{N_1 : N_2 = 1 : 8}$

3.



$\boxed{V_1 = 25\angle 0° \text{ V.}}$

$V_2 = \left(\frac{5}{1}\right)(25\angle 0°)$

$\boxed{V_2 = 125\angle 0° \text{ V}}$

$I_2 = \frac{V_2}{45\Omega} = \frac{125\angle 0°}{45} =$

$\boxed{I_2 = 2.78\angle 0° \text{ A}}$

$I_1 = \frac{N_2}{N_1} I_2 = \left(\frac{5}{1}\right) 2.78\angle 0°$

$\boxed{I_1 = 13.89\angle 0° \text{ A}}$

4. $Z_L' = \left(\frac{N_1}{N_2}\right)^2 Z_L$

$= \left(\frac{5}{2}\right)^2(-j40)$

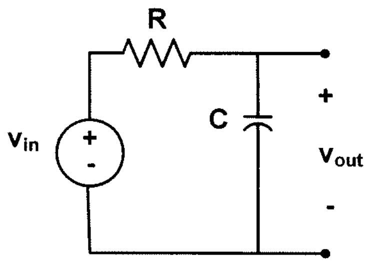
$\boxed{Z_L' = -j250\,\Omega}$

$= 250\angle -90°\,\Omega$

EE 281B

Homework #20 -- 20 points   Due: Monday, November 29, 2010

1. For the following lowpass filter:

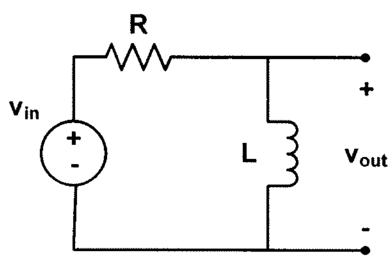


   a. Find the transfer function $H(j\omega)$, in terms of ω, R, and C. (3 pts)

   b. If R=100 Ω, find the value of C needed for a ½-power frequency of 5000 rad/sec. (3 pts)

   c. If $v_{in} = 20\cos(500t) + 20\cos(8000t)$ V, find $v_{out}(t)$ for the values of R and C from the previous part. (5 pts)

2. For the following filter, determine:



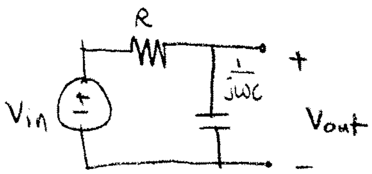
   a. The transfer function $H(j\omega)$, in terms of ω, R, and L. (4 pts)

   b. The ½-power frequency (in rad/sec) if R=1000 Ω and L=0.1 mH. (3 pts)

   c. Is this a lowpass or highpass filter? (2 pts)

Homework #20

1.



a) $H(j\omega) = \dfrac{1/j\omega C}{R + 1/j\omega C} = \boxed{\dfrac{1}{1 + j\omega RC}}$

b) ½ power freq $= \dfrac{1}{RC}$

$5000 = \dfrac{1}{100\,C}$

$\boxed{C = 2\mu F}$

c) $V_{in} = 20\cos(500t) + 20\cos(8000t)$

$\omega = 500$

$V_{out} = H(j500)\, 20\angle 0^\circ$

$= \dfrac{1}{1 + j(500)(100)(2\times 10^{-6})}\, 20\angle 0^\circ = 19.9\angle -5.71^\circ$

$V_{out} = 19.9\cos(500t - 5.71^\circ)$

$\omega = 8000$

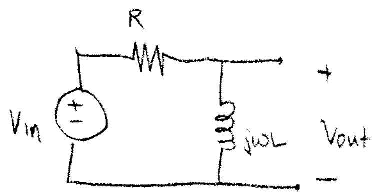
$V_{out} = H(j8000)\, 20\angle 0^\circ$

$= \dfrac{1}{1 + j(8000)(100)(2\times 10^{-6})}\, 20\angle 0^\circ = 10.60\angle -58.0^\circ$

$V_{out} = 10.60\cos(8000t - 58.0^\circ)$

$\boxed{V_{out}(t) = 19.9\cos(500t - 5.71^\circ) + 10.60\cos(8000t - 58.0^\circ)\ V}$

2.



a) $\boxed{H(j\omega) = \dfrac{j\omega L}{R + j\omega L}}$

b) $\omega = \dfrac{R}{L} = \dfrac{1000}{0.1 \times 10^{-3}} = \boxed{1 \times 10^{7} \text{ rad/sec}}$

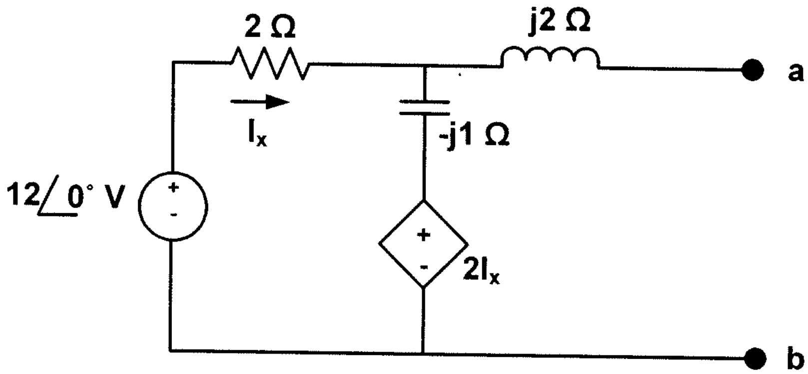
c) $|H(j\omega)| = \dfrac{\omega L}{\sqrt{R^2 + (\omega L)^2}}$

$|H(j0)| = 0$

$|H(j\infty)| = 1$

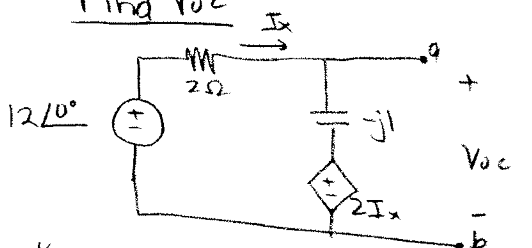
$\boxed{\text{highpass}}$

7. Find the Thévenin equivalent of the following circuit.



(20 pts)

Find Voc



KVL:

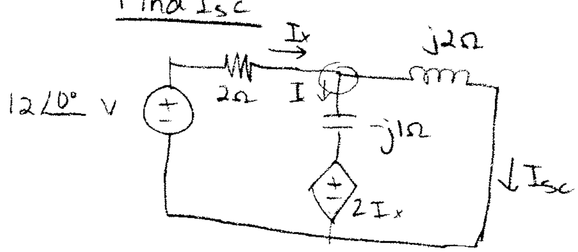
$$-12 + 2I_x - jI_x + 2I_x = 0$$

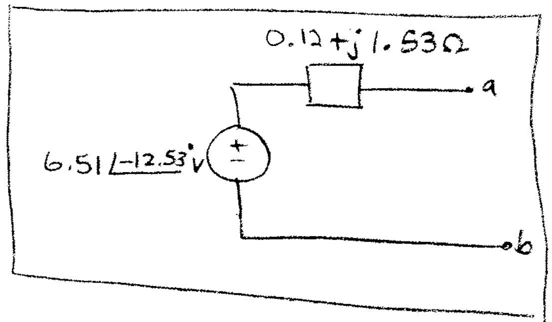
$$(4-j)I_x = 12$$

$$I_x = 12/(4-j) = 2.91\angle 14.04^\circ$$

$$V_{oc} = -jI_x + 2I_x = (2-j)I_x = 6.51\angle -12.53^\circ \text{ V}$$

Find Isc





KVL outer loop: $-12 + 2I_x + j2I_{sc} = 0$

① $2I_x + j2I_{sc} = 12 \Rightarrow I_x = \dfrac{12 - j2I_{sc}}{2} = 6 - jI_{sc}$

KCL: $I = I_x - I_{sc}$

KVL left loop: $-12 + 2I_x - jI + 2I_x = 0$

$$-12 + 4I_x - j(I_x - I_{sc}) = 0$$

② $(4-j)I_x + jI_{sc} = 12$

$$Z_{th} = \frac{V_{oc}}{I_{sc}} = \frac{6.51\angle -12.53^\circ}{4.24\angle -98.13^\circ} = 1.53\angle 85.6^\circ \Omega = 0.12 + j1.53\Omega$$

$$(4-j)(6-jI_{sc}) + jI_{sc} = 12$$

$$(-j(4-j)+j)I_{sc} = 12 - 6(4-j)$$

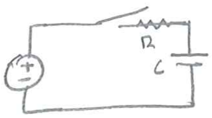
$$I_{sc} = \frac{12 - 6(4-j)}{-j(4-j)+j} = -0.6 - j4.2 = 4.24\angle -98.13^\circ \text{A}$$

Exam Practice

Final Exam Review (DM)

— Things to know — Final Exam (not on crib sheet

- P. Factor = cos (Θ) where Θ is angle of impedance, Z

- 

$V(t) = k_1 + k_2 e^{\frac{1}{RC}t}$

Leading = Θ is negative
Lagging =

- $I_{rms} = \frac{V_{rms}}{|Z|}$ so if up/side $V_s = 20\angle 60$ & $Z = 50\angle 45$, $I_{rms} = \frac{20}{50}$

- units of Q, apparent power, are <u>VA</u>

- $Z_L = j\omega L$
- $Z_C = \frac{1}{j\omega C}$

- phasor voltage written in terms of cosine $6(\sin(t+5)) = 6\cos(t-85)$
$= 6\angle -85$

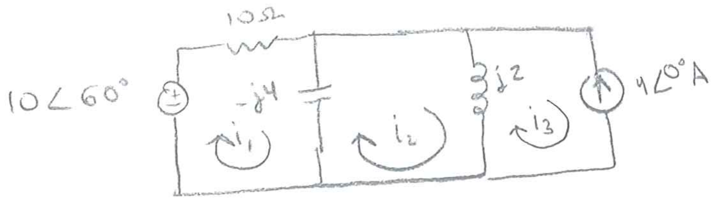
natural solution = homogeneous soln so $Ai + B\frac{di}{dt} = \emptyset$

particular solution = only right side of eqn so $f(\sin,\cos) = -A\cos(4t) + b\sin(4t)$
- same form as source takes

- underdamped - real & imaginary
- overdamped - real only
- critically - imaginary only

Exam 3 cont.

Review



Set up mesh eq's for $I_{1,2,3}$

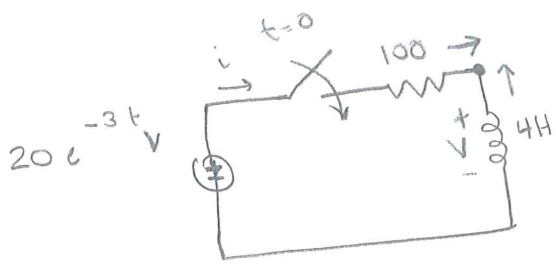
$10\angle 60 - 10 I_1 + j4(I_1 - I_2) = 0$

$j4(i_2 - i_1) - j2(i_2 - i_3) = 0$

$-j2(i_3 - i_2) = 0$

$i_3 = -4\angle 0° \text{ A}$

$$\begin{bmatrix} -10+j4 & -j4 & 0 \\ -j4 & j2 & j2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 10\angle 60 \\ 0 \\ -4\angle 0° \end{bmatrix}$$



KVL

$20e^{-3t} - 100i - 4\frac{di}{dt} = 0$

$\frac{di}{dt} + 25i = 5e^{-3t}$

$\uparrow$
$i_p(t)$

$i =$

$sV(s) + 25V(s) = 0$

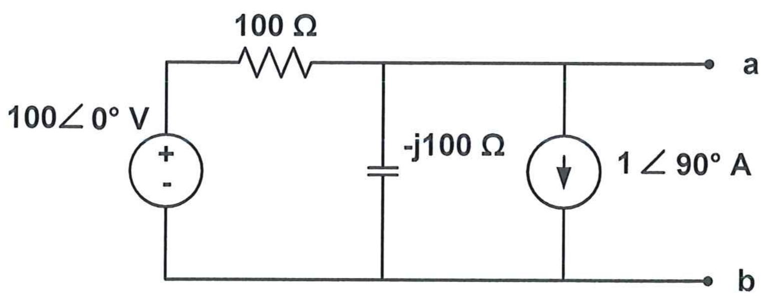
$V(s)(s+25) = 0 \Rightarrow i_1(t) = k_1 e^{-25t}$

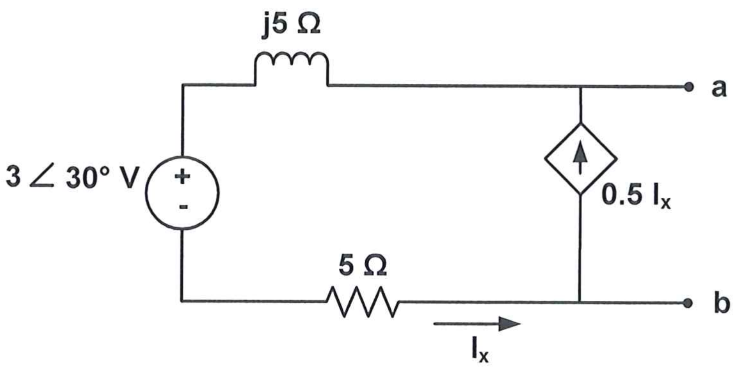
$i_p(0) = e^{-3t}$

Sample Problems – Exam 4

EE281

1. Find the Thevenin equivalents for each of the following circuits.





2. What load impedance should be placed between the terminals in each of the previous circuits to that maximum power is delivered to the load?

P_{max}, $Z = Z_{th}^*$

3. A Δ-connected load has an impedance Z_Δ=5-j2 Ω. What is the equivalent Y-connected load?

4. A balanced positive-phase sequence, Y-connected, 60-Hz, three-phase source has a line-to-line voltage of 440 V_{rms}. It is connected to a balanced three-phase, Y-connected load with an impedance Z_Y=5-j2 Ω. Assume $\mathbf{V_{an}}$ has a zero phase. Find the line-to-neutral voltage phasors, line-to-line voltage phasors, line current phasors, and the average and reactive power delivered to the load.

V_L

$V_{an} = \_\_\_\_ \angle 0°$

V_{ab}

V_{bc}

V_{ca}

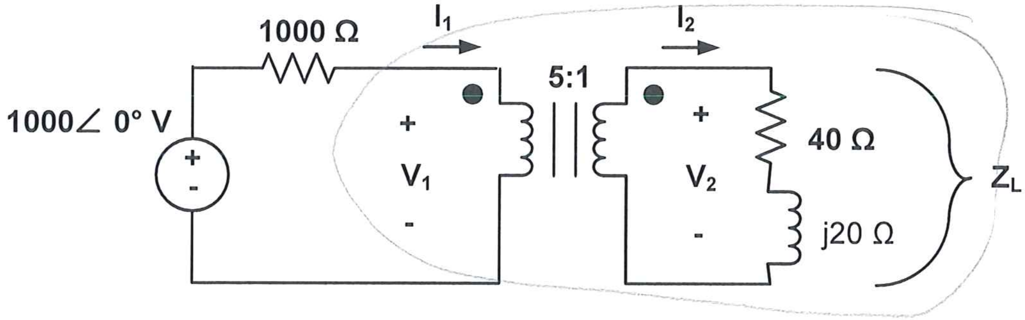
V_{aA}

V_{bB}

V_{cC}

5. Find the turns ratio necessary to step down a 440-V_{rms} AC source to 20 V_{rms}. Assume the transformer is ideal.

6. For the circuit below, find:

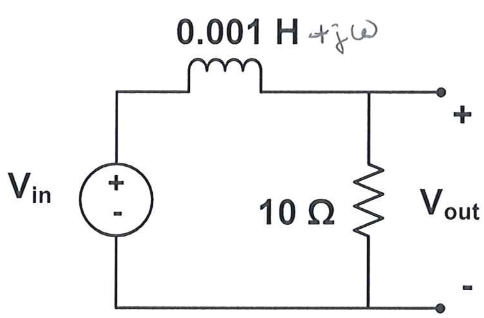


a. The total impedance seen by the 1000-V source.

b. The average power delivered to the load Z_L.

c. The phasors $\mathbf{V_1}$, $\mathbf{V_2}$, $\mathbf{I_1}$, and $\mathbf{I_2}$.

7. For the following filter, find:



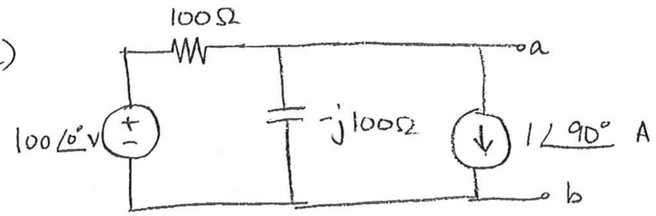
a. The transfer function $H(j\omega) = \frac{V_{out}}{V_{in}}$.

b. The half-power frequency in rad/sec.

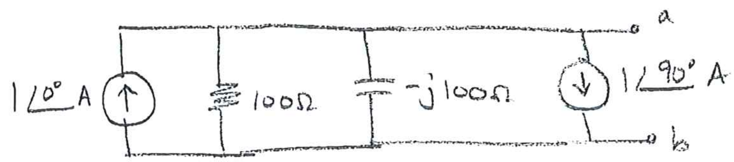
c. $v_{out}(t)$ if $v_{in}(t) = 50\cos(100t) + 50\cos(5000t - 45°)$ V.

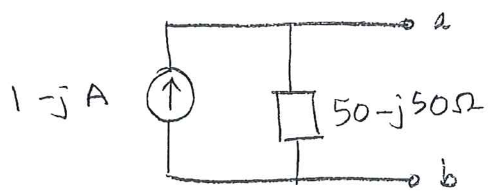
EE281 Sample Problems - Exam 4

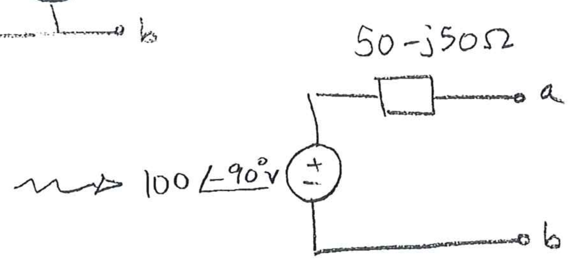
(1a)



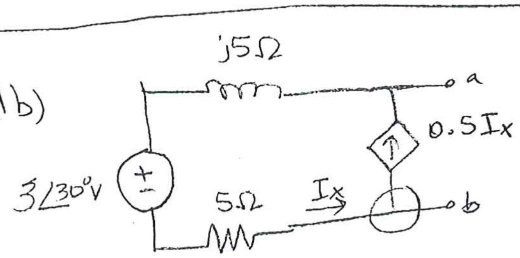
Using source transformations







(1b)



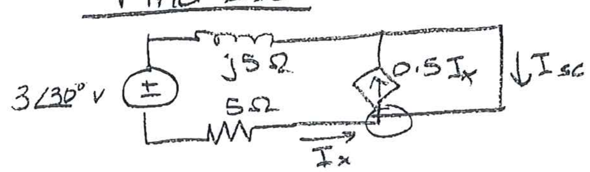
Find Voc

KCL: $I_x = 0.5 I_x \rightsquigarrow I_x = 0$

Thus no voltage drop across 5-Ω resistor or inductor.

$V_{oc} = 3\angle 30°$ V.

Find Isc



KCL: $I_x + I_{sc} - 0.5 I_x = 0$

$0.5 I_x + I_{sc} = 0$

KVL outer loop: $+3\angle 30° + 5I_x + j5I_x = 0$

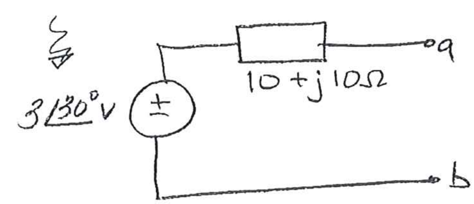
$(5+j5) I_x = -3\angle 30°$

$I_x = 0.424\angle 165°$

$I_{sc} = -0.5 I_x = 0.212\angle -15°$ A

$Z_{th} = V_{oc}/I_{sc} = \dfrac{3\angle 30°}{0.212\angle -15°} = 14.14\angle 45°\,\Omega$

$= 10 + j10\,\Omega$



(2a) $Z_L = Z_S^* = 50 + j50\,\Omega$

(2b) $Z_L = Z_S^* = 10 - j10\,\Omega$

(3) $Z_Y = \frac{Z_\Delta}{3} = \frac{5-j2}{3} = \frac{5}{3} - j\frac{2}{3}\,\Omega$

(4) $V_{ab} = V_{an}(\sqrt{3}\,\angle 30°)$

$$V_{an} = \frac{V_{ab}}{\sqrt{3}\,\angle 30°} = \frac{440\angle 30°}{\sqrt{3}\angle 30°} = 254.03\angle 0°\ V$$

$V_{an} = V_{AN} = 254.03\angle 0°\ V$

$V_{bn} = V_{BN} = 254.03\angle -120°\ V$

$V_{cn} = V_{CN} = 254.03\angle -240°\ V$

$V_{ab} = V_{AB} = 440\angle 30°\ V$

$V_{bc} = V_{BC} = 440\angle -90°\ V$

$V_{ca} = V_{CA} = 440\angle 150°\ V$

$$I_{aA} = \frac{V_{AN}}{Z_Y} = \frac{254.03\angle 0°}{5-j2} = 47.17\angle 21.8°\ A$$

$I_{bB} = 47.17\angle -98.2°\ A$

$I_{cC} = 47.17\angle 141.8°\ A$

$$P = 3V_{Y_{rms}} I_{L_{rms}} \cos\theta = 3(254.03)(47.17)\cos(-21.8°) = 33377\ W$$

$$Q = 3V_{Y_{rms}} I_{L_{rms}} \sin\theta = 3(254.03)(47.17)\sin(-21.8°) = -13350\ VAR$$

(5) $V_1 = \frac{N_1}{N_2} V_2$

$440 = \frac{N_1}{N_2}(20)$

$\frac{N_1}{N_2} = 22$

$N1 : N2 = 22 : 1$

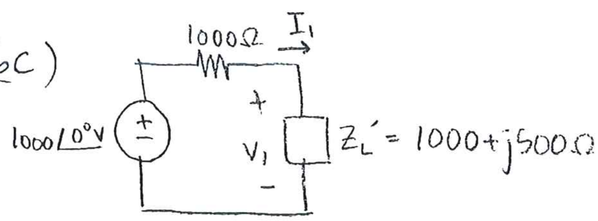
(6a) $Z_L = 40 + j20$

$$Z_L' = \left(\frac{N_1}{N_2}\right)^2 Z_L = \left(\frac{5}{1}\right)^2 (40+j20) = 1000 + j500\ \Omega$$

$$Z_L'' = 1000 + Z_L' = 2000 + j500\ \Omega$$

(6b) $P = I_{2rms}^2 R = \left(\frac{2.43}{\sqrt{2}}\right)^2 (40) = 118.1\ W$

(6c)



$$V_1 = \frac{1000+j500}{2000+j500}(1000\angle 0^\circ) = 542.3\angle 12.5^\circ\ V$$

$$I_1 = \frac{V_1}{Z_L'} = \frac{542.3\angle 12.5^\circ}{1000+j500} = 0.49\angle -14.0^\circ\ A$$

$$I_2 = \frac{N_1}{N_2} I_1 = \frac{5}{1}(0.49\angle -14.0^\circ) = 2.43\angle -14.0^\circ A$$

$$V_2 = \frac{N_2}{N_1} V_1 = \frac{1}{5}(542.3\angle 12.5^\circ) = 108.46\angle 12.5^\circ\ V$$

(7a) $$V_{out} = \frac{10}{10 + j\omega(0.001)} V_{in}$$

$$H(j\omega) = \frac{10}{10 + j0.001\omega}$$

(7b) $$|H(j\omega)| = \frac{10}{\sqrt{100 + (0.001\omega)^2}} = \frac{1}{\sqrt{2}}$$

$$200 = 100 + (0.001)^2\omega^2$$

$$\omega = 10{,}000\ \text{rad/sec.}$$

(7c) $\underline{\omega = 100}$

$V_{in_1} = 50\angle 0^\circ$

$$V_{out_1} = 50\angle 0^\circ \left(\frac{10}{10 + j0.001(100)}\right) = 49.998\angle -0.57^\circ$$

$\underline{\omega = 5000}$

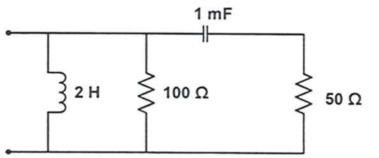
$V_{in_2} = 50\angle -45^\circ$

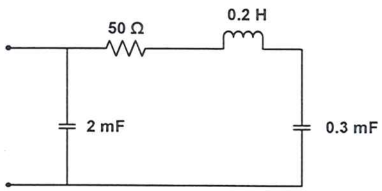
$$V_{out_2} = 50\angle -45^\circ \left(\frac{10}{10 + j0.001(5000)}\right) = 44.721\angle -71.57^\circ$$

$$v_{out}(t) = 49.998\cos(100t - 0.57^\circ) + 44.721\cos(5000t - 71.57^\circ)\ V$$

Exam 3 Extra Probs & Soln's

6. Write each of the following as a single impedance. Assume that ω=100 rad/sec.

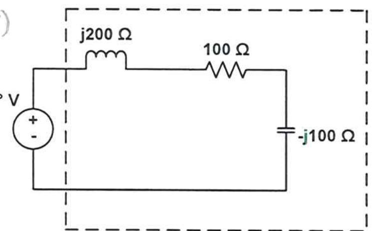




7. In the following circuit:

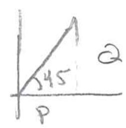
$10 \cos(\omega - 90°)$

$V_R =$



load

$(100 + 100j)\,\Omega$



a. Calculate the average power, reactive power, and apparent power absorbed by the load.

b. Draw the power triangle.

c. What is the power factor?

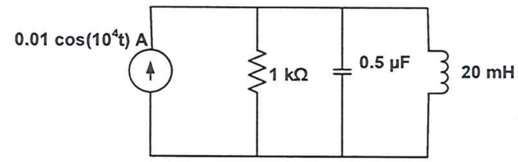
$P = I_{rms}^2 R = \frac{V_{rms}^2}{R} \quad \frac{\left(\frac{10}{\sqrt{2}}\right)^2}{} = \frac{100}{2} = \frac{50}{100}$

$PF = \frac{P}{S} \quad \cos\theta$

EE281 – Exam 3 – Sample Problems

$I = I \frac{\frac{1}{R_x}}{\frac{1}{R_{total}}}$

1. Redraw the following circuit in the frequency domain.

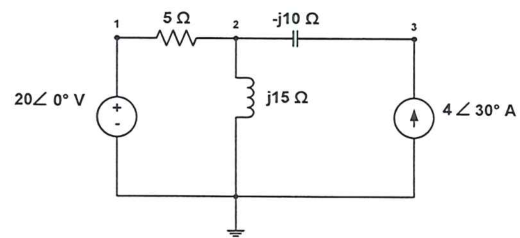


IR

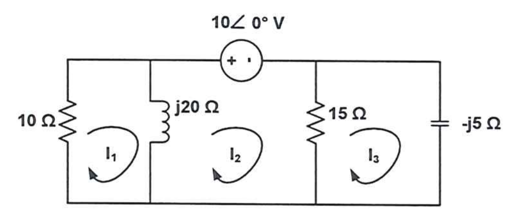
2. Find the steady-state current through each of the components (resistor, inductor, and capacitor) in the circuit in problem 1.

3. Find the steady-state voltage across each of the components (resistor, inductor, and capacitor) in the circuit in problem 1.

4. Set up the node voltage equations in matrix form to find the phasor node voltages V_1, V_2, and V_3. Do not solve.



5. Set up the mesh current equations in matrix form to find the phasor mesh currents I_1, I_2, and I_3. Do not solve.

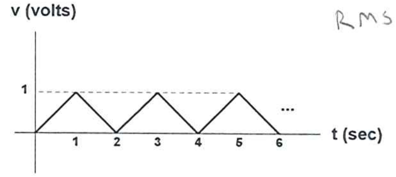


12. Find the RMS value of each of the following voltages.

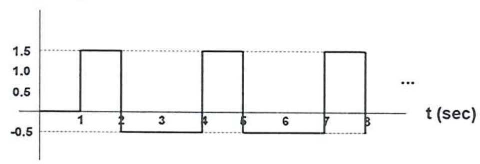
a. v(t)= 10 cos(2πt - 40°) V

b.

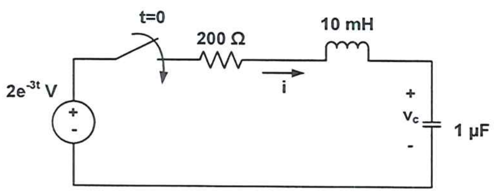
$RMS = \sqrt{\frac{1}{t}\int_{0}^{t} f^2 \, d\lambda}$



c.



13. Assuming that $v_c(0)=0$, find following for the circuit below:



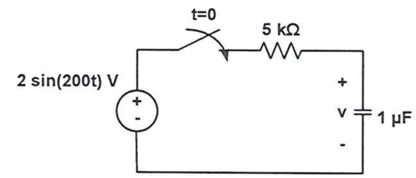
a. The form of the source-free (natural) solution of i(t).

b. The steady-state (particular) solution of i(t).

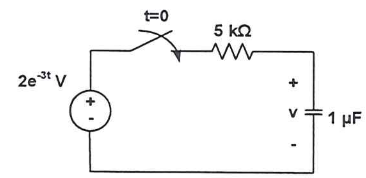
c. The form of the complete solution. You do not have to solve for the constants from the source-free portion.

d. Is this circuit overdamped, underdamped, or critically damped?

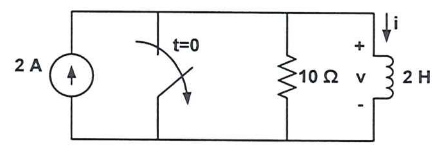
8. Find the steady-state voltage, v(t), in the following circuit.



9. Find the steady-state voltage (the particular response of the differential equation), v(t), in the following circuit.



10. Assuming that the switch has been closed for a long time prior to t=0, find an expression for i(t) and v(t) in the following circuit.

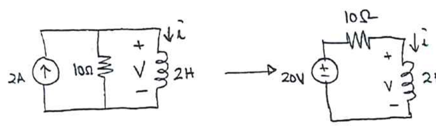


11. Write the following as phasors:

a. 20 cos(2πt - 30°)

b. 10 sin(20t + 8°)

Volts across component, real or img

10. 

$$-20 + 10i + 2\frac{di}{dt} = 0$$

$$\frac{di}{dt} + 5i = 10$$

$\frac{di}{dt} + 5i = 0$

$s + 5 = 0$

$s = -5$

$i_n(t) = Ke^{-5t}$

$i_p(t) = K_2$

$0 + 5K_2 = 10$

$K_2 = 2$

$i_p(t) = 2$

$i(t) = K_1 e^{-5t} + 2$

$i(0) = 0 = K_1 + 2 \rightarrow K_1 = -2$

$$\boxed{i(t) = 2 - 2e^{-5t}\ A}$$

$v(t) = L\frac{di}{dt} = 2(10e^{-5t})$

$$\boxed{v(t) = 20e^{-5t}\ V}$$

11. a) $20\angle -30^\circ$

b) $10\angle -82^\circ$

12. a) $\frac{10}{\sqrt{2}}$ V.

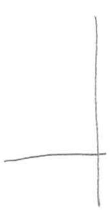
b) $V_{rms} = \sqrt{\frac{1}{2}\left[\int_0^1 t^2 dt + \int_1^2 (2-t)^2 dt\right]} = \sqrt{\frac{1}{2}\left[\frac{t^3}{3}\Big|_0^1 + \frac{(2-t)^3}{-3}\Big|_1^2\right]}$

$= \sqrt{\frac{1}{2}\left[\left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)\right]} = \frac{1}{\sqrt{3}}$ V

c) $V_{rms} = \sqrt{\frac{1}{3}\left[\int_1^2 (1.5)^2 dt + \int_2^4 (-0.5)^2 dt\right]}$

$= \sqrt{\frac{1}{3}\left[2.25t\Big|_1^2 + 0.25t\Big|_2^4\right]} = \sqrt{\frac{1}{3}[2.25 + 0.25(2)]}$

$= 0.957$ V

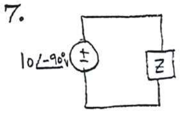


$P = I_{RMS}^2 R$

$Q = I_{RMS}^2 X$

$20\cos(15t + 30^\circ)$

$V_{RMS} = 20\angle 30^\circ$ V

7. 

$10\angle -90^\circ$; Z: $100 + j100\ \Omega = 141.4\angle 45^\circ\ \Omega$

$$\boxed{V_{rms} = \frac{10}{\sqrt{2}}\ V}$$ ✓

$V_R = \frac{100}{100+j100} 10\angle -90^\circ = 7.07\angle -135^\circ$ V

$V_L = \frac{j100}{100+j100} 10\angle -90^\circ = 7.07\angle -45^\circ$ V

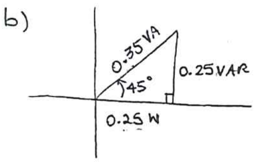
a) $P = \frac{V_{rms}^2}{R} = \frac{(7.07/\sqrt{2})^2}{100} = 0.25$ W

$Q = \frac{V_{rms}^2}{X} = \frac{(7.07/\sqrt{2})^2}{100} = 0.25$ VAR

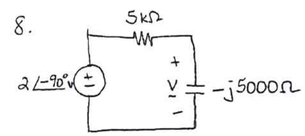
app power $= \sqrt{P^2 + Q^2} = 0.35$ VA

$P = I_{RMS}^2 R$

$Q = I_{RMS}^2 X$

b) 

c) PF: $\cos 45^\circ = \frac{1}{\sqrt{2}} = 0.707$

8. 

$V = \frac{-j5000}{5000 - j5000} 2\angle -90^\circ = 1.414\angle -135^\circ$ V

$$\boxed{v(t) = 1.414\cos(200t - 135^\circ)\ V}$$

9. $(1\times 10^{-6})\frac{dv}{dt} + \frac{V - 2e^{-3t}}{5000} = 0$

$$\boxed{v_p(t) = \frac{400}{197}e^{-3t}\ V}$$

$\frac{dv}{dt} + 200v = 400e^{-3t}$

$V_p(t) = Ae^{-3t}$

$-3Ae^{-3t} + 200Ae^{-3t} = 400e^{-3t}$

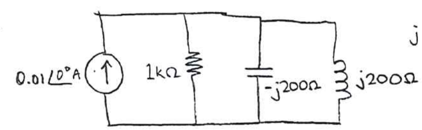
$197A = 400$

$A = 2.03 = 400/197$

EE281 - Exam 3

Sample Problems - solutions

1.



$j(10^4)(20\times10^{-3}) = j200$

$\dfrac{1}{j(10^4)(0.5\times10^{-6})} = -j200$

2. $I_R = \dfrac{\frac{1}{1000}}{\frac{1}{1000}+\frac{1}{-j200}+\frac{1}{j200}}(0.01\angle 0^\circ) = 0.01\angle 0^\circ\ A$

$\boxed{i_R(t) = 0.01\cos(10^4 t)\ A}$

$I_C = \dfrac{1/{-j200}}{\frac{1}{1000}+\frac{1}{-j200}+\frac{1}{j200}}(0.01\angle 0^\circ) = 0.05\angle 90^\circ\ A$

$\boxed{i_C(t) = 0.05\cos(10^4 t + 90^\circ)\ A}$

$I_L = \dfrac{1/j200}{1/1000 + 1/{-j200} + 1/j200}(0.01\angle 0^\circ) = 0.05\angle -90^\circ\ A$

$\boxed{i_L(t) = 0.05\cos(10^4 t - 90^\circ)\ A}$

3. $V_R = I_R Z_R = (0.01\angle 0^\circ)(1000) = 10\angle 0^\circ\ V$

$\boxed{v_R(t) = v_C(t) = v_L(t) = 10\cos(10^4 t)\ V}$

4. node 1

$V_1 = 20\angle 0^\circ$

node 2

$\dfrac{V_2 - V_1}{5} + \dfrac{V_2}{j15} + \dfrac{V_2 - V_3}{-j10} = 0$

$-\frac{1}{5}V_1 + (\frac{1}{5} + \frac{1}{30}j)V_2 - \frac{1}{10}jV_3 = 0$

node 3

$\dfrac{V_3 - V_2}{-j10} - 4\angle 30^\circ = 0$

$$\boxed{\begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{5} & \frac{1}{5}+j\frac{1}{30} & -j\frac{1}{10} \\ 0 & -j\frac{1}{10} & j\frac{1}{10} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 20\angle 0^\circ \\ 0 \\ 4\angle 30^\circ \end{bmatrix}}$$

5. mesh 1

$10I_1 + j20(I_1 - I_2) = 0$

$(10 + j20)I_1 - j20I_2 = 0$

mesh 2

$j20(I_2 - I_1) + 10\angle 0^\circ + 15(I_2 - I_3) = 0$

$-j20I_1 + (15 + j20)I_2 - 15I_3 = -10\angle 0^\circ$

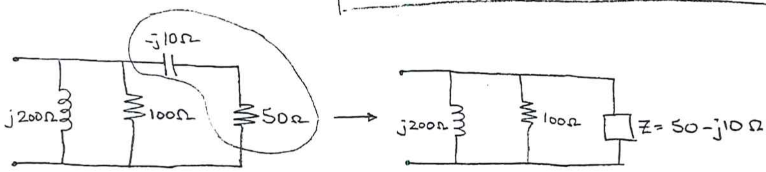
mesh 3

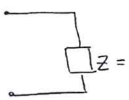
$-j5I_3 + 15(I_3 - I_2) = 0$

$-15I_2 + (15 - j5)I_3 = 0$

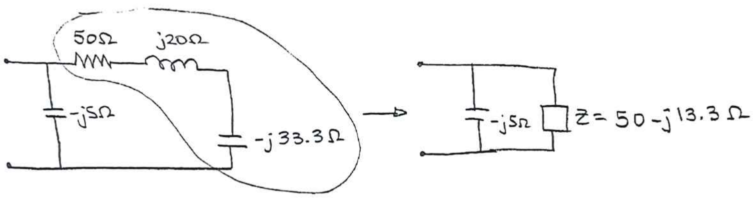
$$\boxed{\begin{bmatrix} 10+j20 & -j20 & 0 \\ -j20 & 15+j20 & -15 \\ 0 & -15 & 15-j5 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -10\angle 0^\circ \\ 0 \end{bmatrix}}$$

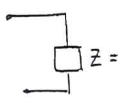
6.





$Z = \dfrac{1}{\frac{1}{j200} + \frac{1}{100} + \frac{1}{50-j10}} = \boxed{34.18\angle 2.26^\circ\ \Omega = 34.16 + j1.35\ \Omega}$





$Z = \dfrac{(-j5)(50-j13.3)}{-j5 + 50 - j13.3} = \boxed{0.44 - j4.84\,\Omega = 4.86\angle -84.8^\circ\ \Omega}$

13. a) $-2e^{-3t} + 200i + (10\times10^{-3})\frac{di}{dt} + \frac{1}{1\times10^{-6}}\int_{-\infty}^{t} i \, d\lambda = 0$

$$\frac{d^2i}{dt^2} + 20{,}000\frac{di}{dt} + (1\times10^{8})i = -600e^{-3t}$$

$$\frac{d^2i}{dt^2} + (2\times10^{4})\frac{di}{dt} + (1\times10^{8})i = 0$$

$$s^2 + (2\times10^4)s + (1\times10^8) = 0$$

$$s = \frac{-(2\times10^4) \pm \sqrt{(2\times10^4)^2 - 4(1\times10^8)}}{2} = -10{,}000, -10{,}000$$

$$\boxed{i_n(t) = K_1 e^{-10{,}000t} + K_2 t e^{-10{,}000t}}$$

b) $i_p(t) = Ae^{-3t}$

$$9Ae^{-3t} + 20000(-3Ae^{-3t}) + (1\times10^8)Ae^{-3t} = -600e^{-3t}$$

$$9A - 60{,}000A + (1\times10^8)A = -600$$

$$(99940009)A = -600 \rightarrow A = 6.004\times10^{-6}$$

$$\boxed{i_p(t) = (6.004\times10^{-6})e^{-3t}\ A}$$

c) $\boxed{i(t) = K_1 e^{-10{,}000t} + K_2 t e^{-10{,}000t} + (6.004\times10^{-6})e^{-3t}\ A}$

d) $\boxed{\text{critically damped}}$

Exam 3

P.F. = ?

1) a) $P.F. = \cos(7.12) \ , = .992$, lagging

b) RMS current

$$I_{rms} = \frac{V_{rms}}{|Z|} = \frac{20\angle 60^\circ}{|201.6\angle 7.13|} \quad \boxed{.0992} \ \cancel{32.9} \ A_{rms}$$

c) $$P_{real} = \frac{1}{2} V_m I_m \cos\theta$$

$$= \frac{1}{2}(20)(.0992)\cos(7.13) = \boxed{1.96\ W}$$

d) $$P_{apparent} = Q = \frac{1}{2}(20)(.992)\sin(7.13) = \boxed{0.123\ VA}$$

2) equivalent impedance $2\angle{-90}$ OK ✓

3) $v(t) = 6\sin(14t + 5^\circ)$ V

find RMS of $v(t)$ $= 6 \times \frac{1}{\sqrt{2}}$ OK ✓

phasor $6\angle 5^\circ$

find V_{rms}

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt}$$

$$= \sqrt{\frac{1}{1}\int_0^1 2^2\,dt + \frac{1}{1}\int_1^2 (4-2t)^2\,dt}$$

$$\sqrt{4 + \int_1^2 16 - 4t + 4t^2\,dt}$$

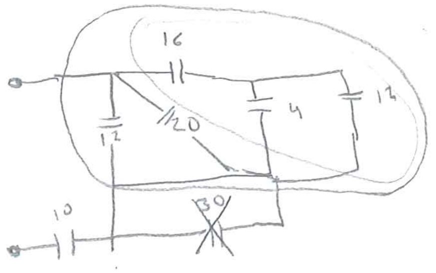
$$\sqrt{4 + 4\int_1^2 4 - t + t^2\,dt} = \sqrt{4 + 4\left[4t - \frac{t^2}{2} + \frac{t^3}{3}\right]_1^2}$$

$$= \sqrt{4 + 4\left[8 - 4 - \frac{4}{2} + \frac{1}{2} + \frac{8}{3} - \frac{1}{3}\right]} = 4.83$$

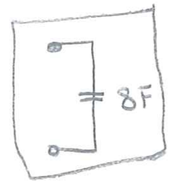
— Exam 2 — Review

1) $\left(\frac{1}{8}+\frac{1}{40}\right)^{-1}$ $\boxed{6.67\ \Omega}$ ✓ OK

2) Simplify:



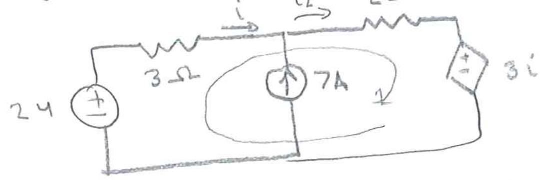
$\left(\frac{1}{16}+\frac{1}{16}\right)^{-1} = 8$



$12 + 20 + 8 = 40$

$\left(\frac{1}{40}+\frac{1}{10}\right)^{-1} = \boxed{8\,F}$

3) find i w/superpos.



first omit 7A: $24 - 3i - 2i - 3i = 0$

$24 = 8i \Rightarrow i = 3\,A$

2^{nd} omit 24V $\left.\begin{matrix} 3i + 2i_2 + 3i = 0 \\ i_2 = i + 7 \end{matrix}\right\}$ $6i + 2i + 14 = 0$

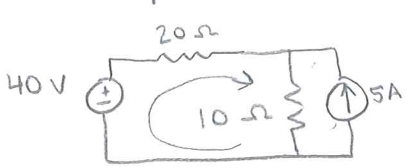
$8i = -14$

$i = -1.75$

$\boxed{i = 1.25\,A}$

---

4) solve DC steady state current i



$40 - 20i - 10(i + 5A) = 0$

$-10 - 30i = 0$ $\cancel{\boxed{i = 3\,A}}$ OK ✓

$\boxed{i = -\frac{1}{3}}$

5) 5mF capacitor $v(t>0)$ $i(t) = 8\sin(2t)\ \cancel{X}\ A$ $V(0) = 2V$

$V_c = v(t_0) + \frac{1}{c}\int_{t_0}^{t} i\,dx = 2 + \frac{1}{.005}\int_0^t 8\sin(2t)\,dt = 2 + 200\left[-\frac{8}{2}\cos(2t)\right]_0^t$

$2 + 200(+4)(1) - 800\cos(2t)$

$2 + 800 - 800\cos(2t)\ V$

$\boxed{v(t) = 802 - 800\cos(2t)\ V}$ ✓ OK

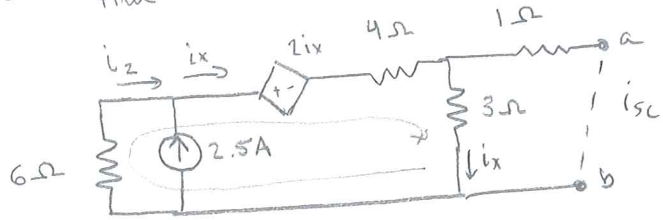
b) find $i(t)$ if $v = 8e^{-4t}\,V$

$i_c = C\frac{dv}{dt} = .005\left(-32e^{-4t}\right)\ A$

$\boxed{i(t) = -.16e^{-4t}\ A}$ ✓ OK

— Exam 2 —

6) find Thévenin



— find V.O.C. $\quad i_2 = i_x - 2.5A$

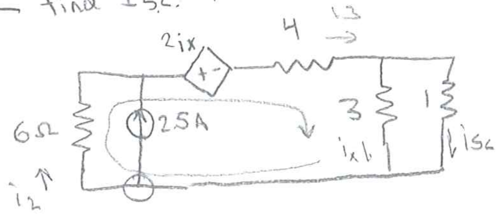
KVL: $\quad -2i_x - 4i_x - 3i_x - 6i_2 = 0$

$-9i_x - 6i_x - 15A = 0$

$-15i_x = 15A \Rightarrow i_x = -1A$

$V_{oc} = 3(-1) = \boxed{-3V}$

— find I_{sc}.



$-6i_2 - 2i_x - 4i_3 - 3i_x = 0$

$i_2 = i_x - 2.5 + i_{sc} = -2.5 + 4i_x$

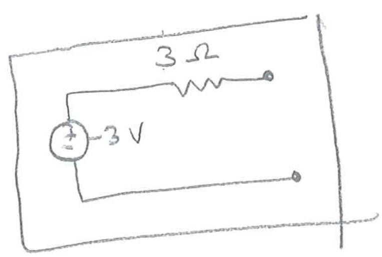
$i_3 = i_x + i_{sc} = 4i_x$

$i_{sc} = 3i_x$

$-6(-2.5 + 4i_x) - 5i_x - 4(4i_x) = 0$

$15 - 45i_x = 0 \Rightarrow i_x = \frac{1}{3}A$

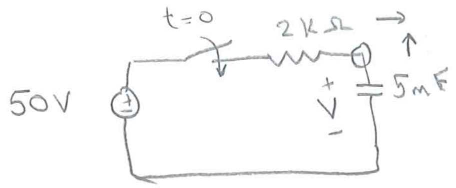
$\boxed{i_{sc} = 1A}$



$i = \frac{V}{R}$

$1 = \frac{-3}{R} \qquad R = -3$

7) $V(0^+) = 4V$ find $v(t)$ for $t \geq 0$ $V_{max} = ?$



$i = \frac{V}{R}$

KCL $\quad C\frac{dV}{dt} + \frac{V - V_S}{R} = 0$

$V_C = \frac{1}{C}\int_{t_0}^{t} i\,dx$

$\frac{dV}{dt} + \frac{V}{CR} = \frac{V_S}{CR}$

$\frac{dV}{dt} + \frac{1}{10}V = 5$

$V_C = k_1 e^{-0.1t}$

$V = k_1 e^{-0.1t} + k_2$

$V_p = k_2 \qquad \frac{1}{10}k_2 = 5$

$V_\infty = k_2 = 50V$

initial condition $\quad V(0) = 4 = 50 + k_1 e^{-0.1\cancel{t}^{\,0}}$

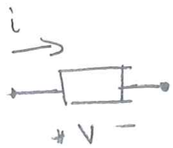
$-46 = k_1(1)$

$k_1 = -46$

$\boxed{v(t) = 50 - 46e^{-0.1t}}$

Exam 1 Review

1)



- power
- energy $0 < t < 3s$

$i = 2e^{-3t}\,A \qquad V = 6e^{-3t}\,V$

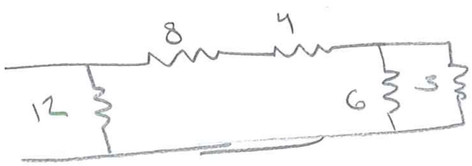
$P = Vi = 12e^{-3t}e^{-3t} = \boxed{12e^{-6t}\ W}$

$E = \int_0^t P\,dt = \int_0^3 12e^{-6t}\,dt = 12\left[\frac{1}{-6}e^{-6t}\right]_0^3$

OK

$= -2\left[e^{-18} - 1\right] = \boxed{2\,J}$

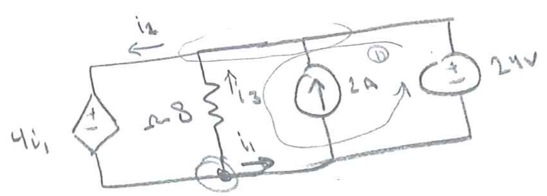
2)



$\left\{\left(\frac{1}{6}+\frac{1}{3}\right)^{-1} + 8 + 4\right\} = 2$

$14 \parallel 12 = \boxed{6.46\,\Omega} = R_{eq}$

3)



find i_2, i_3

KCL

$i_1 - i_2 + i_3 = 0$

$i_3 = i_2 - i_1$

$-3A = i_2 - i_1$

$4i_1 = 24V \qquad \boxed{i_1 = 6A \qquad i_2 = 3A}$

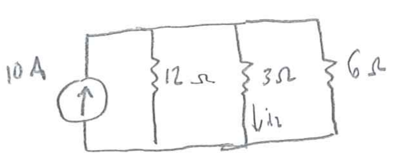
KVL ①

$24V + 8i_3 = 0$

$i_3 = -\frac{24}{8} = -3$

OK ✓

4) find i_2



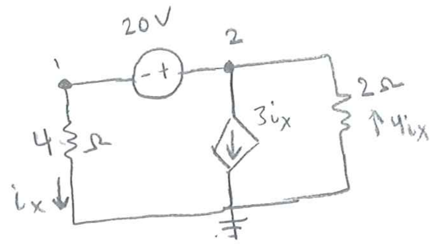
$\dfrac{10A\ \frac{1}{3}}{\frac{1}{12}+\frac{1}{3}+\frac{1}{6}} = \boxed{5.71\,A}$

OK ✓

5) Cu, wire, 10m, $d = 2.6mm$ resistivity $= 1.7 \times 10^{-8}\ \Omega \cdot m$ find resistance

$$R = \frac{1.7 \times 10^{-8}\ L}{A} = \boxed{.0320\ \Omega}$$

6) Solve node voltages V_1, V_2



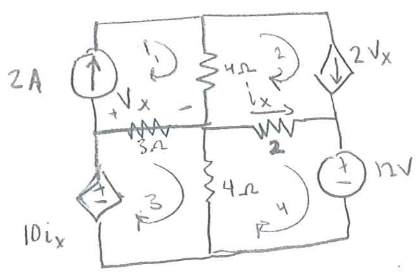
$V_2 = V_1 + 20$

$-20 - 4i_x - 8i_x = 0$

$-20 = 12 i_x \Rightarrow i_x = -1.67$ OK ✓

$V_1 = 4i_x = \boxed{-6.67}$ $\boxed{V_2 = 13.3}$

7) Set up eqns for mesh.



$i_1 = 2A$ $i_x = i_4 - i_2$

$i_2 = 2V_x$

$+12 + 4(i_4 - i_3) + 2(i_4 - i_2) = 0$

$+4(i_3 - i_4) + 3(i_3 - i_1) - 10(i_4 - i_2) = 0$

$+4(i_1 - i_2) + (i_1 - i_3)(3) = 0$

$+2(i_2 - i_4) + 4(i_2 - i_1) = 0$

$$\begin{bmatrix} 0 & -2 & -4 & 6 \\ -3 & 10 & 7 & -14 \\ 7 & -4 & -3 & 0 \\ -4 & 6 & 0 & -2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} -12 \\ 0 \\ 0 \\ \\ \end{bmatrix}$$ ✓

$$\begin{bmatrix} 0 & -1 & -2 & 3 \\ -3 & 0 & 7 & 4 \\ 7 & -4 & -3 & 0 \\ -2 & 3 & 0 & -1 \end{bmatrix} \Rightarrow \begin{matrix} 12 \\ 0 \\ 0 \\ 0 \end{matrix}$$

Practice for Final

$\boxed{B}$

Name Key

EE 281 A

Exam 2

Spring 2011

Show all your work. Circle or box your answer.

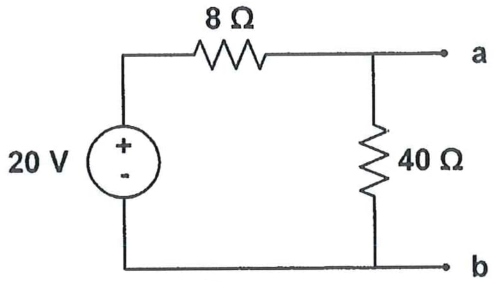
$$v_k = \frac{R_k}{R_1+R_2+R_3+\cdots+R_n} v_s \qquad i_k = \frac{\frac{1}{R_k}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\cdots+\frac{1}{R_n}} i_s$$

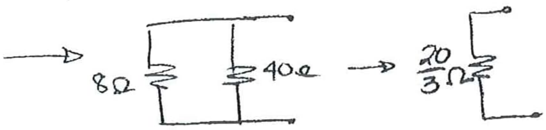
$$R = \frac{\rho L}{A}$$

$$i_C = C\frac{dv}{dt} \qquad v_C = \frac{1}{C}\int_{-\infty}^{t} i\, d\lambda = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i\, d\lambda$$

$$v_L = L\frac{di}{dt} \qquad i_L = \frac{1}{L}\int_{-\infty}^{t} v\, d\lambda = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v\, d\lambda \qquad P_{max} = \frac{v_{th}^2}{4R_{th}}$$

1. What load resistance should be placed between terminals a and b so that maximum power is delivered to the load?

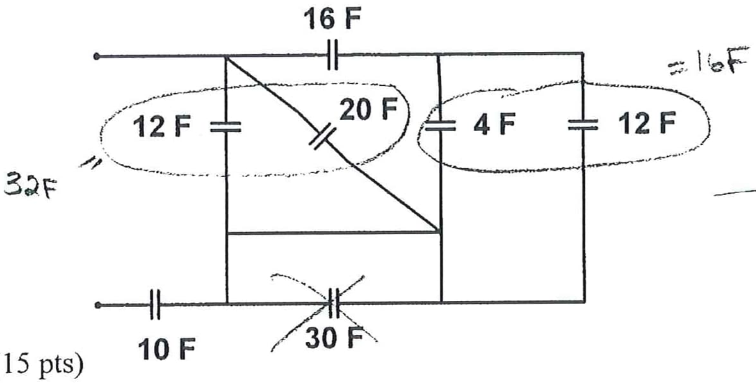




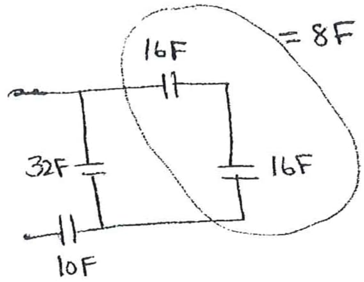
(10 pts)

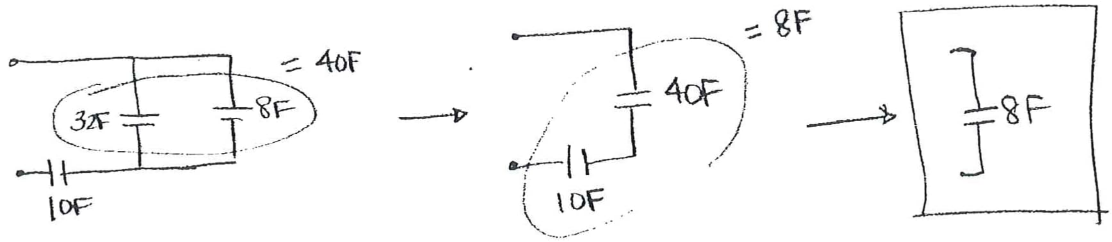
$\boxed{R_{th} = R_{load} = \frac{20}{3}\Omega}$

2. Simplify the following to a single equivalent component.

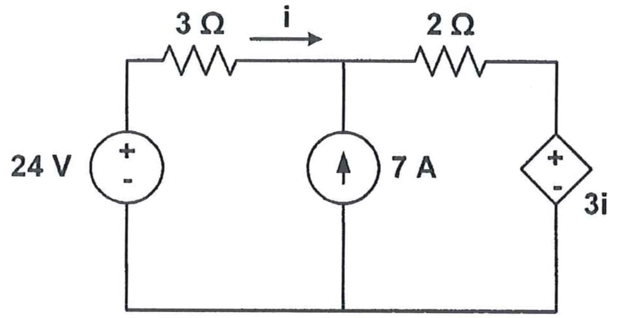


(15 pts)



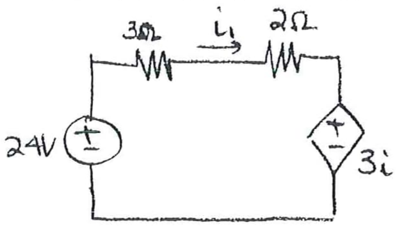


3. Use superposition to find i in the following circuit.



(15 pts)

① Keep 24-V source

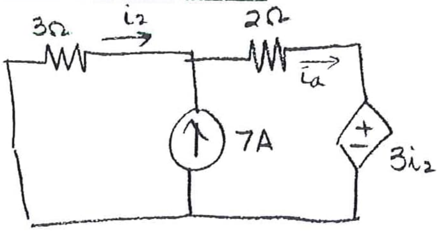


KVL: $-24 + 3i_1 + 2i_1 + 3i_1 = 0$

$8i_1 = 24$

$i_1 = 3A$

② Keep 7-A source



KCL:

$-i_2 + i_a - 7 = 0$

$i_a = 7 + i_2$

KVL: $3i_2 + 2i_a + 3i_2 = 0$

$6i_2 + 2(7 + i_2) = 0$

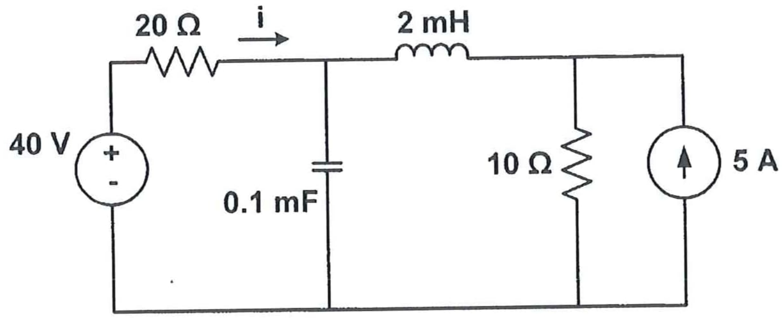
$8i_2 = -14$

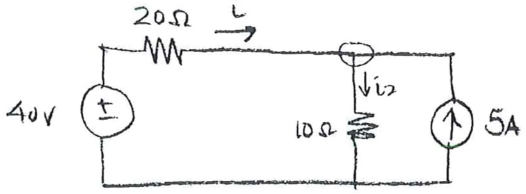
$i_2 = -\frac{14}{8} = -\frac{7}{4}$ A.

$i = i_1 + i_2 = 3 - \frac{7}{4} = \boxed{1.25\text{ A}}$

4. Solve for the DC steady-state current, i, in the following circuit.

(15 pts)





KCL: $i_2 - i - 5 = 0$

$i_2 = 5 + i$

KVL: $-40 + 20i + 10i_2 = 0$

$-40 + 20i + 10(5 + i) = 0$

$30i = -10$ $\boxed{i = -1/3\,A}$

5. For a 5-mF capacitor, calculate each of the following:

(10 pts)

a. The voltage for t>0 across the capacitor if the current through the capacitor is i(t) = 8 sin(2t) V and the initial voltage across the capacitor is v(0)=2 V.

$$V(t) = V(0) + \frac{1}{C}\int_0^t i\,d\lambda$$

$$= 2 + \frac{1}{0.005}\int_0^t 8\sin(2\lambda)\,d\lambda$$

$$= 2 + 200\left(-4\cos(2\lambda)\right)\Big|_0^t$$

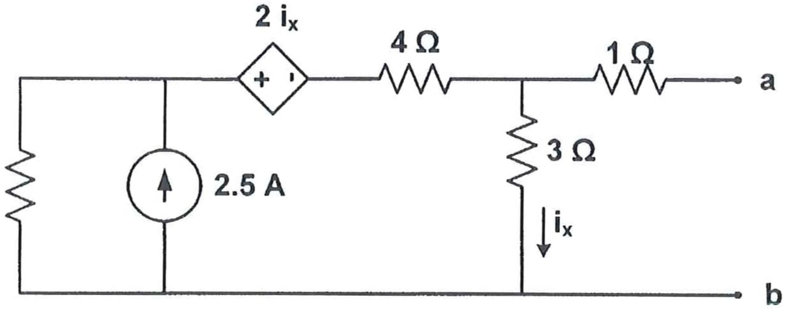
$$= 2 + 800(1 - \cos(2t)) = \boxed{802 - 800\cos(2t)\ V}$$

b. The current as a function of time through the capacitor if the voltage across the capacitor is $v(t) = 8e^{-4t}$ V.

$$i = C\frac{dv}{dt} = 0.005\left(-32e^{-4t}\right)$$

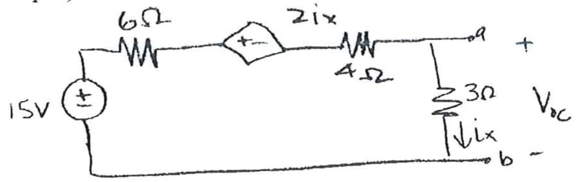
$$= \boxed{-0.16e^{-4t}\ A}$$

6. Find the Thévenin equivalent of the following circuit. Sketch the equivalent circuit.



(20 pts)

Find Voc



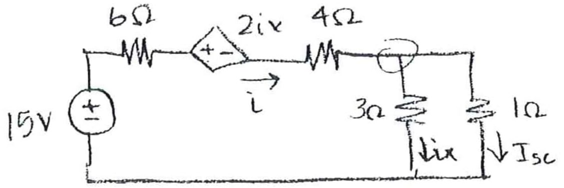
KVL: $-15 + 6i_x + 2i_x + 4i_x + 3i_x = 0$

$15i_x = 15$

$i_x = 1$

$\boxed{V_{oc} = 3i_x = 3\text{ V}}$ 7pts

Find Isc



KVL: $-15 + 6i + 2i_x + 4i + 3i_x = 0 \longrightarrow 10i + 5i_x = 15$

$i = \dfrac{15 - 5i_x}{10} = \dfrac{3 - i_x}{2}$

KCL: $-i + i_x + I_{sc} = 0$

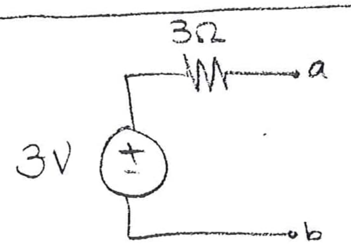
KVL: $3i_x = 1 \cdot I_{sc}$

$\dfrac{i_x - 3}{2} + i_x + I_{sc} = 0$

$\dfrac{(I_{sc}/3) - 3}{2} + \dfrac{I_{sc}}{3} + I_{sc} = 0$

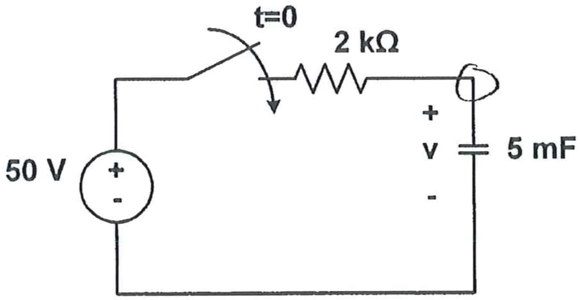
$\boxed{I_{sc} = \dfrac{3/2}{1/6 + 1/3 + 1} = 1\text{ A}}$ 7pts

$\boxed{R_{th} = \dfrac{V_{oc}}{I_{sc}} = \dfrac{3\text{V}}{1\text{A}} = 3\Omega}$ 3pts



3pts

7. If the initial voltage across the capacitor is $v(0^+)=4$ V, find the voltage, v(t), across the capacitor for t≥0. What is the maximum voltage across the capacitor for t>>0?



(15 pts)

KCL: $C\frac{dv}{dt} + \frac{V - V_s}{R} = 0$

$\frac{dv}{dt} + \frac{1}{RC}V = \frac{1}{RC}V_s$

$\frac{dv}{dt} + \frac{1}{10}V = 5$

$V_c = K_1 e^{-\frac{1}{10}t}$

$V_p = K_2 \rightsquigarrow \frac{1}{10}K_2 = 5$

$K_2 = 50$

$V(t) = K_1 e^{-\frac{1}{10}t} + 50$

$V(0) = 4 = K_1 + 50$

$K_1 = -46$

$\boxed{V(t) = 50 - 46e^{-\frac{1}{10}t}\ V}$

$\boxed{V_{max} = 50V}$

Practice for final

B

Name Key

EE 281 A

Exam 3

Spring 2011

Show all your work. Circle or box your answer.

$$v_k = \frac{R_k}{R_1+R_2+R_3+\cdots+R_n} v_s \qquad i_k = \frac{\frac{1}{R_k}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\cdots+\frac{1}{R_n}} i_s$$

$$R = \frac{\rho L}{A} \qquad P_{max} = \frac{v_{th}^2}{4R_{th}} \qquad e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

$$i_C = C\frac{dv}{dt} \qquad v_C = \frac{1}{C}\int_{-\infty}^{t} i\, d\lambda = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i\, d\lambda \qquad V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2 dt}$$

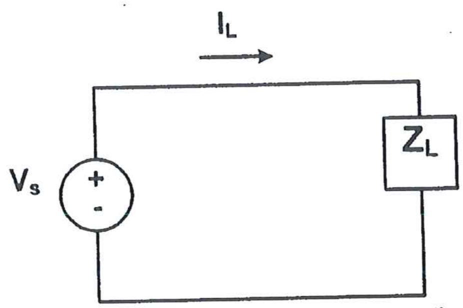
$$v_L = L\frac{di}{dt} \qquad i_L = \frac{1}{L}\int_{-\infty}^{t} v\, d\lambda = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v\, d\lambda$$

$$P = \frac{1}{2}V_m I_m \cos(\theta) = V_{rms} I_{rms} \cos(\theta) \qquad |S| = \frac{P}{PF} = \frac{1}{2}V_m I_m = V_{rms} I_{rms}$$

$$Q = \frac{1}{2}V_m I_m \sin(\theta) = V_{rms} I_{rms} \sin(\theta) \qquad x_p = \begin{cases} A \\ Ae^{bt} \\ A e^{at}\cos(bt) + Be^{at}\sin(bt) \end{cases}$$

$$x_n = \begin{cases} K_1 e^{s_1 t} + K_2 e^{s_2 t} \\ K_1 e^{st} + K_2 t e^{st} \\ K_1 e^{at}\cos(bt) + K_2 e^{at}\sin(bt) \end{cases}$$

1. In the following circuit, the source is a 60-Hz, 20-V_{rms} source and the load consists of a resistor and inductor in series with an impedance of $Z_L = 200+j25\ \Omega$.



$Z_L = 200 + j25 = 201.6 \angle 7.13° \Omega$

a. Find the power factor (include whether it is leading or lagging) of the load.

(5 pts)

$PF = \cos(7.13°) = \boxed{0.992 \text{ lagging}}$

b. Find the RMS current through the load Z_L. (Note: there will not be an angle associated with this current since you were not given one for the voltage source.)

(5 pts)

$$I_{rms} = \frac{V_{rms}}{|Z|} = \frac{20}{201.6} = \boxed{0.099\ A_{rms}}$$

c. Calculate the average (real) power absorbed by the load.

(5 pts)

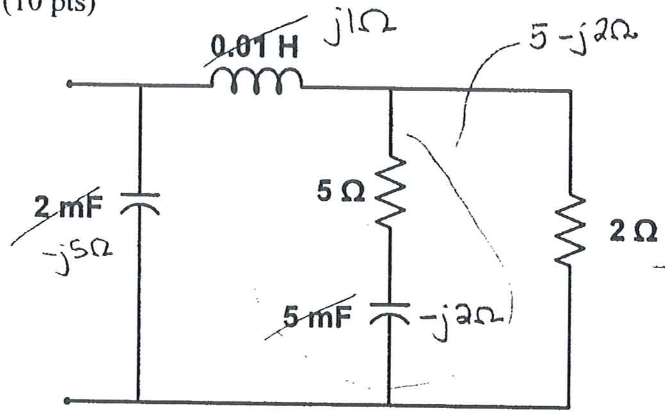
$$P = \frac{1}{2} V_m I_m \cdot PF = V_{rms} I_{rms} \cdot PF$$
$$= 20(0.099)(0.992) = \boxed{1.96\ W}$$

d. Find the apparent power absorbed by the load.

(5 pts)

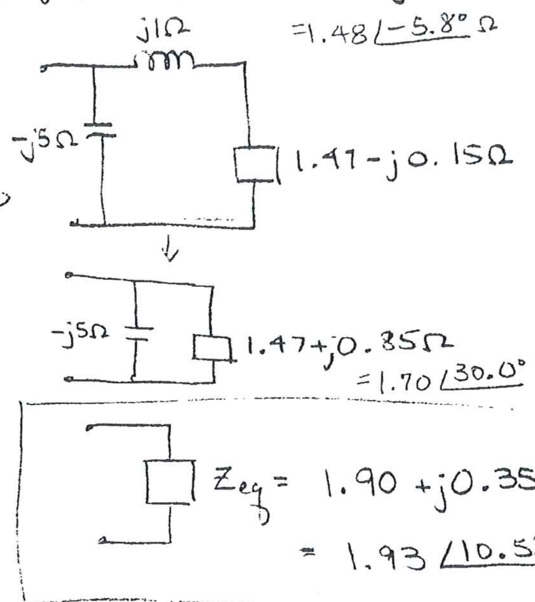
$$|S| = V_{rms} I_{rms} = 20(0.099) = \boxed{1.98\ VA}$$

2. Find the equivalent impedance, Z_{eq}, of the following. Assume that ω=100 rad/sec.
(10 pts)



$(5-j2)\parallel(2) = 1.47 - j0.15\Omega$

$= 1.48\angle -5.8^\circ\,\Omega$



$(1.47+j0.85)\parallel(-j5) =$

$$\boxed{Z_{eq} = 1.90 + j0.35\Omega = 1.93\angle 10.53^\circ\,\Omega}$$

3. Given $v(t) = 6\sin(14t + 5^\circ)$ V :

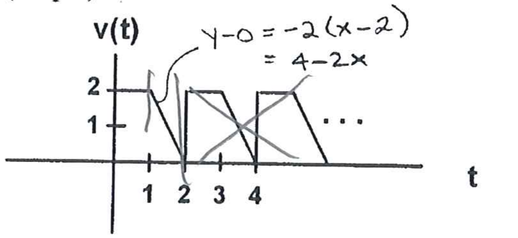
a. Find the RMS value of v(t).
(4 pts)

$$\boxed{V_{rms} = \frac{6}{\sqrt{2}} = 4.24\ V_{rms}}$$

b. Write v(t) as a phasor.
(4pts)

$$\boxed{\underset{\sim}{V} = 6\angle -85^\circ\ \text{V}}$$

4. Find the RMS value of the following period function:
(10 pts)



$0 \to 1$ $\quad V=2$

$1 \to 2$ $\quad V=4-2t$

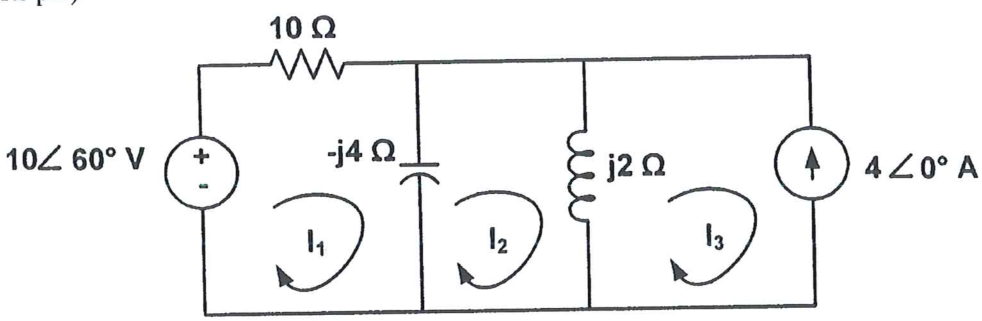
$$V_{rms} = \sqrt{\frac{1}{2}\left(\int_0^1 (2)^2 dt + \int_1^2 (4-2t)^2 dt\right)}$$

$$= \sqrt{\frac{1}{2}\left[\left(4t\Big|_0^1\right) + \frac{(4-2t)^3}{-6}\Big|_1^2\right]}$$

$$= \sqrt{\frac{1}{2}\left[4+\frac{8}{6}\right]} = \sqrt{\frac{1}{2}\left(\frac{16}{3}\right)} = \boxed{\sqrt{\frac{8}{3}} = 1.63\ V_{rms}}$$

5. Set up the equations in matrix form to find the phasor mesh currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the following circuit. Do not solve.

(12 pts)



m1: $-10\angle 60^\circ + 10I_1 + (-j4)(I_1 - I_2) = 0$

$(10-j4)I_1 + j4I_2 = 10\angle 60^\circ$

m2: $-j4(I_2 - I_1) + j2(I_2 - I_3) = 0$

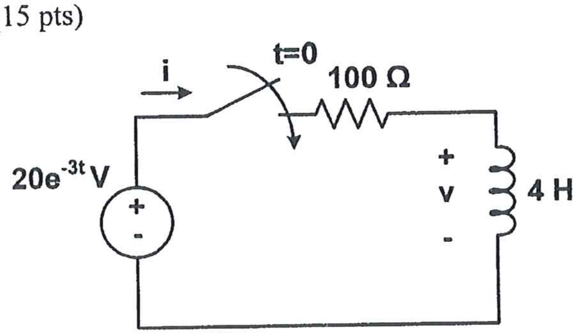
$j4I_1 + (-j2)I_2 - j2I_3 = 0$

m3: $I_3 = -4\angle 0^\circ$

$$\boxed{\begin{bmatrix} 10-j4 & j4 & 0 \\ j4 & -j2 & -j2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10\angle 60^\circ \\ 0 \\ -4\angle 0^\circ \end{bmatrix}}$$

6. Find the current, i(t), for t>0 in the following circuit. Solve for all the constants.

(15 pts)



KVL: $-20e^{-3t} + 100i + 4\frac{di}{dt} = 0$ (3pts)

$\frac{di}{dt} + 25i = 5e^{-3t}$

$s + 25 = 0 \rightarrow i_c(t) = K_1 e^{-25t}$ (4pts)

$i_p(t) = Ae^{-3t}$

$i_p' = -3Ae^{-3t}$

$-3A\cancel{e^{-3t}} + 25A\cancel{e^{-3t}} = 5\cancel{e^{-3t}}$

$22A = 5 \rightarrow A = 5/22$

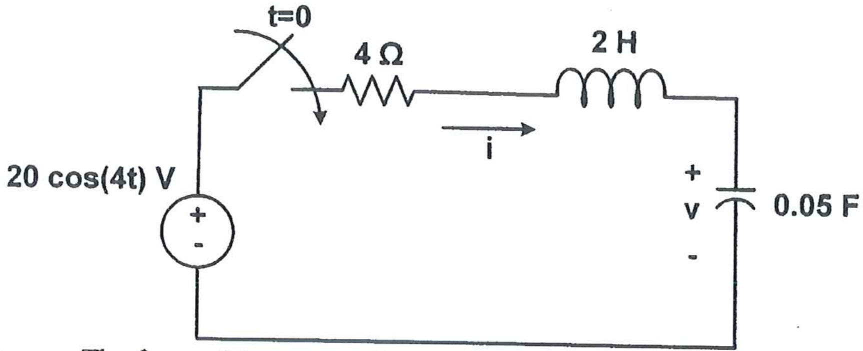
$i(t) = K_1 e^{-25t} + \frac{5}{22}e^{-3t}$

(3pts) $i(0) = 0 = K_1 + 5/22 \rightarrow K_1 = -5/22$

1pt. $\boxed{i(t) = \frac{5}{22}e^{-3t} - \frac{5}{22}e^{-25t}\ \text{A}}$

$= 0.23e^{-3t} - 0.23e^{-25t}$ A

7. Assuming that $v(0^+)=0$, use the circuit below to find:



a. (10 pts) The form of the source-free (natural) solution of i(t).

$$-20\cos(4t) + 4i + 2\frac{di}{dt} + \frac{1}{0.05}\int_{-\infty}^{t} i\, d\lambda = 0$$

$$\frac{d^2i}{dt^2} + 2\frac{di}{dt} + 10i = -40\sin(4t)$$

$$s^2 + 2s + 10 = 0$$

$$s = \frac{-2 \pm \sqrt{4 - 4(10)}}{2} = -1 \pm j3$$

$$\boxed{i_c(t) = K_1 e^{-t}\cos(3t) + K_2 e^{-t}\sin(3t)}$$

b. (8 pts) The steady-state (particular) solution of i(t).

$$i_p(t) = A\cos(4t) + B\sin(4t)$$

$$i_p' = -4A\sin(4t) + 4B\cos(4t)$$

$$i_p'' = -16A\cos(4t) - 16B\sin(4t)$$

$$-16A\cos(4t) - 16B\sin(4t) - 8A\sin(4t) + 8B\cos(4t) + 10A\cos(4t) + 10B\sin(4t) = -40\sin(4t)$$

$$\cos(4t)(-6A + 8B) + \sin(4t)(-6B - 8A) = -40\sin(4t)$$

$-6A + 8B = 0 \rightsquigarrow B = \frac{3}{4}A$

$-8A - 6B = -40$

$\left[-8 - 6\left(\frac{3}{4}\right)\right]B = -40$

$B = 3.2$

$A = 2.4$

$$\boxed{i_p(t) = 2.4\cos(4t) + 3.2\sin(4t)}$$

c. (4 pts) The other initial conditions: $i(0^+)$ and $\left.\frac{di}{dt}\right|_{t=0^+}$.

$\boxed{i(0^+) = 0\text{ A}}$

KVL: $-20 + 4\cancel{i(0)}^{\,0} + 2\left.\frac{di}{dt}\right|_{t=0} + \cancel{v(0)}^{\,0} = 0 \rightsquigarrow \boxed{\left.\frac{di}{dt}\right|_{t=0} = 10}$

d. (3 pts) Is this circuit overdamped, underdamped, or critically damped?

Practice for final

Name Key

$\boxed{B}$

EE 281 A

Exam 1

Spring 2011

Show all your work. Circle or box your answer.

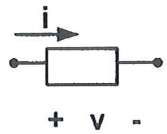
$$v_k = \frac{R_k}{R_1 + R_2 + R_3 + \cdots + R_n} v_s$$

$$i_k = \frac{\frac{1}{R_k}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}} i_s$$

$$R = \frac{\rho L}{A}$$

Sean
lindsey
chaffin

1. Calculate the power absorbed by the circuit element as a function of time. Calculate the energy delivered to the element on the interval $0 \leq t < 3$ s.



$i = 2e^{-3t}$ A $\qquad\qquad v = 6e^{-3t}$ V

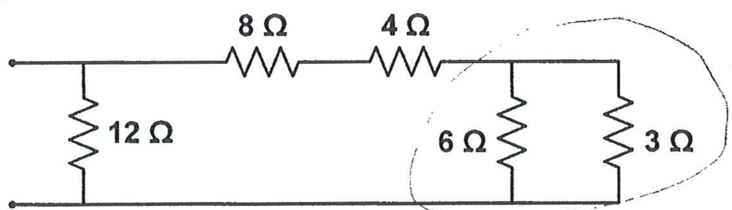
(10 pts)

$$p(t) = iv = (2e^{-3t})(6e^{-3t}) = \boxed{12e^{-6t}\ \text{W}}$$

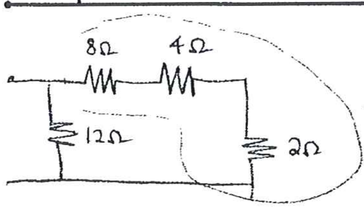
$$W(t) = \int_0^3 p(t)\,dt = \int_0^3 12e^{-6t}\,dt = \frac{12}{-6}e^{-6t}\Big|_0^3$$

$$= -2(e^{-18} - 1) \approx \boxed{2.0\ \text{J}}$$

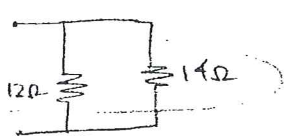
2. Simplify the following to a single equivalent resistance.
(15 pts)



$$= \frac{6(3)}{6+3} = 2\Omega$$



$$= 14\Omega$$



$$= \frac{12(14)}{12+14} = \frac{168}{26} = \boxed{6.46\Omega}$$

5. Calculate the resistance of a 10-m long piece of copper wire that has a diameter of 2.6 mm. The resistivity of copper is 1.7 x 10^{-8} Ω·m.

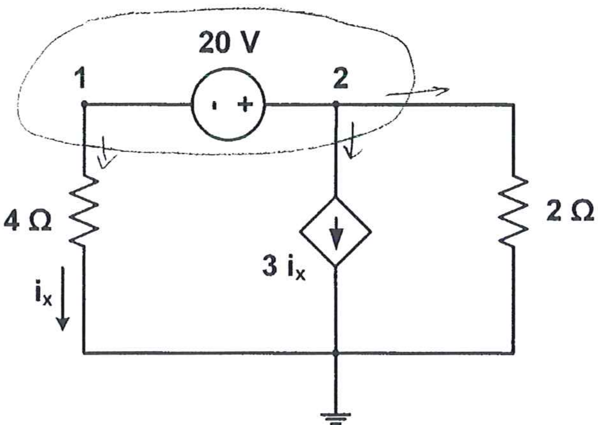
(10 pts)

$$R = \frac{\rho l}{A} = \frac{(1.7\times10^{-8}\ \Omega\text{-m})(10\text{m})}{\pi(1.3\times10^{-3})^2} = \boxed{0.032\ \Omega}$$

$$= 32\ m\Omega$$

6. Solve for the node voltages, v_1 and v_2, in the following circuit.

(15 pts)



Supernode (1, 2)

$$i_x = \frac{V_1}{4}$$

① $V_2 - V_1 = 20V$

KCL: $\frac{V_1}{4} + 3i_x + \frac{V_2}{2} = 0$

$$\frac{V_1}{4} + 3\left(\frac{V_1}{4}\right) + \frac{V_2}{2} = 0$$

② $V_1 + \frac{1}{2}V_2 = 0$

$$\boxed{\begin{aligned} V_1 &= -20/3\ V \\ V_2 &= \frac{40}{3}\ V \end{aligned}}$$

$= -6.67V$

$= 13.33V$

$$-V_1 + V_2 = 20$$
$$V_1 + \frac{1}{2}V_2 = 0$$

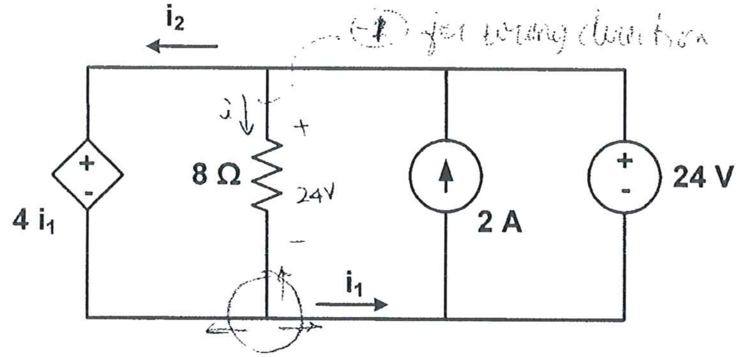
$$\frac{3}{2}V_2 = 20$$

$$V_2 = \frac{40}{3}$$

$$V_1 = -\frac{1}{2}V_2$$

$$V_1 = -\frac{20}{3}$$

3. Solve for i_1 and i_2 in the following circuit.
(15 pts)



KVL outer loop

$4i_1 = 24$

$\boxed{i_1 = 6A}$

$i = \frac{24}{8} = 3A$

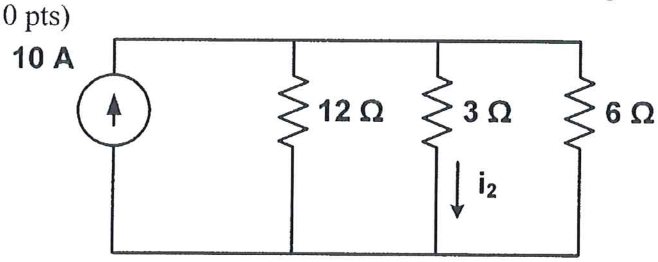
KCL bottom node

$-i_2 - 3 + i_1 = 0$

$i_2 = i_1 - 3 = 6 - 3 = 3$

$\boxed{i_2 = 3A}$

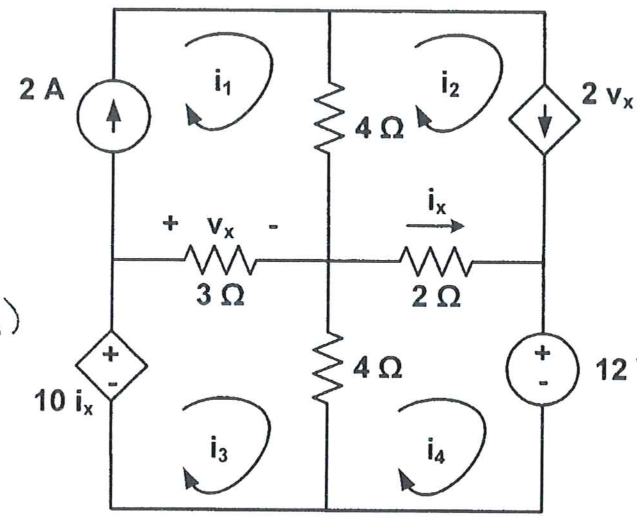
4. Use current divider to find i_2 in the following circuit.
(10 pts)



$$i_2 = \frac{1/3}{1/12 + 1/3 + 1/6}(10A) = \frac{10/3}{7/12} = \boxed{\frac{40}{7}A}$$

7. Set up the equations to find the mesh currents, i_1, i_2, i_3, and i_4, in the following circuit in matrix form. You must eliminate the control variables, v_x and i_x, from your equations.

(25 pts)



$V_x = 3(i_3 - i_1)$

$i_x = i_4 - i_2$

mesh1:

① $i_1 = 2A$

mesh 2:

$i_2 = 2V_x = 6(i_3 - i_1)$

② $6i_1 + i_2 - 6i_3 = 0$

mesh 3

$-10i_x + 3(i_3 - i_1) + 4(i_3 - i_4) = 0$

$-10(i_4 - i_2) + 7i_3 - 3i_1 - 4i_4 = 0$

③ $-3i_1 + 10i_2 + 7i_3 - 14i_4 = 0$

mesh 4

$4(i_4 - i_3) + 2(i_4 - i_2) + 12 = 0$

$-2i_2 - 4i_3 + 6i_4 = -12$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 6 & 1 & -6 & 0 \\ -3 & 10 & 7 & -14 \\ 0 & -2 & -4 & 6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ -12 \end{bmatrix}$$

Practice for final

Name Key

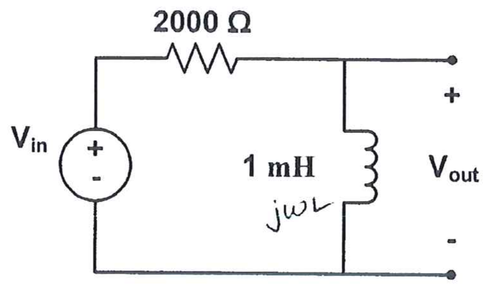
EE 281 B

Exam 4

Fall 2010

Show all your work. Circle or box your answer.

1. Find the transfer function $H(j\omega) = \frac{V_{out}}{V_{in}}$ for the following filter. Is this a highpass or lowpass filter?

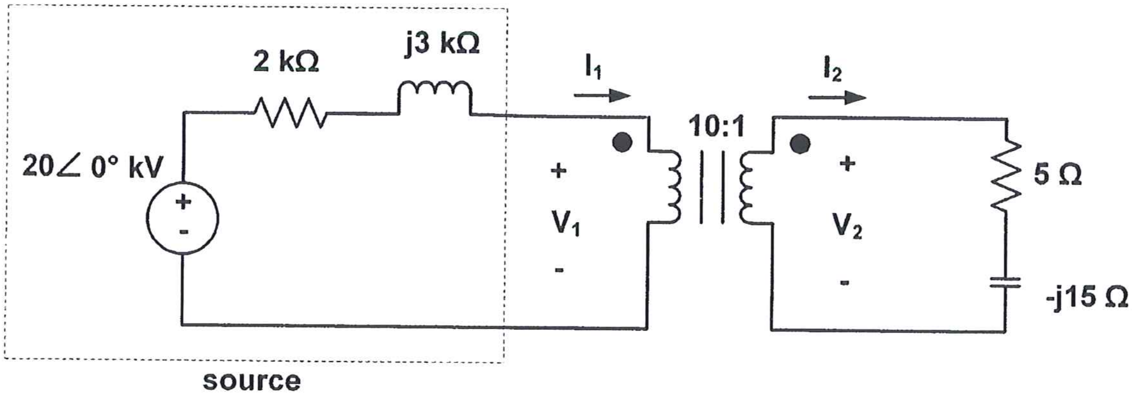


$$\boxed{H(j\omega) = \frac{j\omega L}{R + j\omega L} = \frac{j\omega(0.001)}{2000 + j\omega(0.001)}}$$

$\boxed{\text{highpass}}$

(15 pts)

2. Using impedance transformations (reflected impedances), find the equivalent impedance seen by the source in the following circuit.



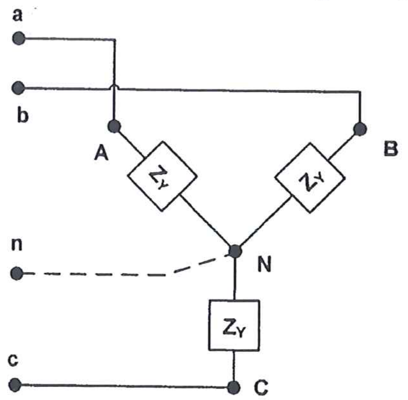
(10 pts) $Z_L' = \left(\frac{N_1}{N_2}\right)^2 Z_L = \left(\frac{10}{1}\right)^2(5 - j15) = \boxed{500 - j1500\ \Omega}$

3. Find the turns ratio of an ideal transformer necessary to step a 40-V_{rms} source up to 1000 V_{rms}.

(5 pts) $\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{1000}{40} = \frac{25}{1}$

$\boxed{N_1 : N_2 = 1:25}$

4. A balanced Y-connected load has an impedance of $Z_Y = 300 + j20\ \Omega$ for each phase. Assuming the phase sequence is positive and that $\mathbf{V_{AB}}$ is the reference, find the following when the line voltage is V_L=20 kV_{rms}.



$Z_Y = 300 + j20\ \Omega = 300.67\angle 3.81^\circ\ \Omega$

$V_{AB} = 20\angle 0^\circ\ kV_{rms}$.

$V_L = V_Y$

a. (3 pts) The phasor line-to-neutral voltage $\mathbf{V_{AN}}$.

$$V_{AN} = \frac{V_{AB}}{\sqrt{3}\angle 30^\circ} = \boxed{\frac{20}{\sqrt{3}}\angle -30^\circ\ V_{rms}}$$

b. (9 pts) The phasor currents $\mathbf{I_{AN}}$, $\mathbf{I_{BN}}$, and $\mathbf{I_{CN}}$.

$$I_{AN} = \frac{V_{AN}}{Z_Y} = \frac{20{,}000/\sqrt{3}\ \angle -30^\circ}{300 + j20} = \boxed{38.40\angle -33.8^\circ\ A}$$

$$\boxed{\begin{aligned} I_{BN} &= 38.40\angle -153.8^\circ\ A \\ I_{CN} &= 38.40\angle -273.8^\circ\ A \end{aligned}}$$

c. (5 pts) The phasor line current $\mathbf{I_{aA}}$.

$$\boxed{I_{aA} = I_{AN} = 38.40\angle -33.8^\circ\ A}$$

d. (5 pts) The average power, P, absorbed by the 3-phase load.

$$P = 3V_Y I_L \cos\theta = 3\left(\frac{20{,}000}{\sqrt{3}}\right)(38.4)\cos(3.81^\circ)$$

$$= \boxed{1327275\ W}$$

e. (3 pts) The impedance Z_Δ to create an equivalent Δ-connected load.

$$Z_\Delta = 3Z_Y = \boxed{900 + j60\ \Omega}$$

25

5. The transfer function for a particular first order filter is $H(j\omega) = \frac{V_{out}}{V_{in}} = \frac{j\omega(0.005)}{1+j\omega(0.005)}$. $=$

a. Find the half-power frequency of this filter.

(5 pts)

$\omega = \frac{1}{0.005} = \boxed{200 \text{ rad/sec}}$ $H(j\omega) = \frac{1}{\sqrt{2}}$

b. Find the output current, $v_{out}(t)$ if the input current is $v_{in}(t) = 20\cos(20t) + 20\cos(5000t - 45°)$ V.

(10 pts)

$\omega = 20$ $V_{in} = 20\angle 0°$

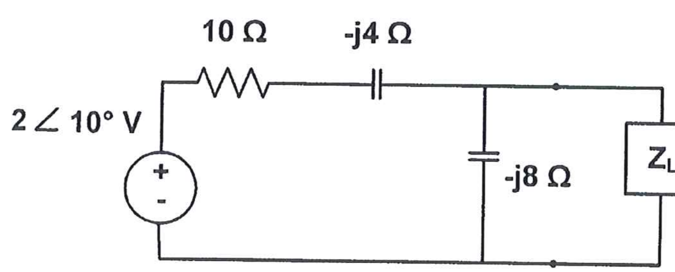
$V_{out} = \frac{j(20)(0.005)}{1+j(20)(0.005)} 20\angle 0° = 1.99\angle 84.3°$

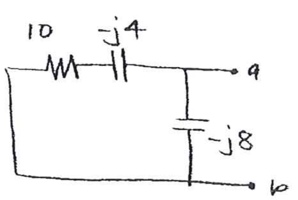
$\omega = 5000$ $V_{in} = 20\angle -45°$

$V_{out} = \frac{j(5000)(0.005)}{1+j(5000)(0.005)} 20\angle -45° = 19.98\angle -42.7°$

$\boxed{V_{out}(t) = 1.99\cos(20t + 84.3°) + 19.93\cos(5000t - 42.7°)\ \text{V}}$

6. For the following circuit, determine the value of Z_L that results in maximum average power transferred to Z_L.





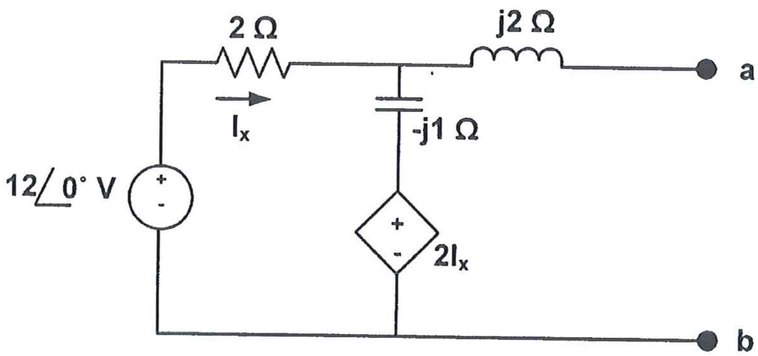
(10 pts)

$Z_{th} = (10 - j4) \| -j8 = \frac{(10-j4)(-j8)}{10-j4-j8} = 5.52\angle -61.6°\ \Omega$

$= 2.62 - j4.85\ \Omega$

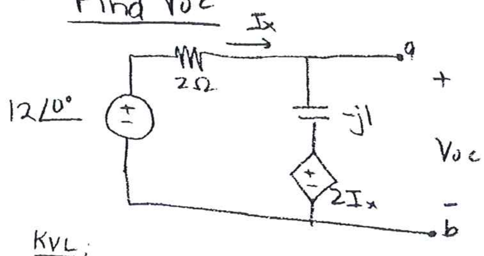
$Z_L = Z_{th}^* = \boxed{2.62 + j4.85\ \Omega}$

7. Find the Thévenin equivalent of the following circuit.



(20 pts)

Find V_{oc}



KVL:

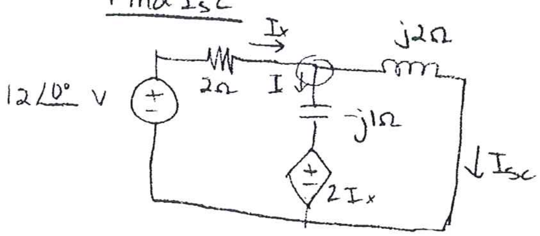
$$-12 + 2I_x - jI_x + 2I_x = 0$$

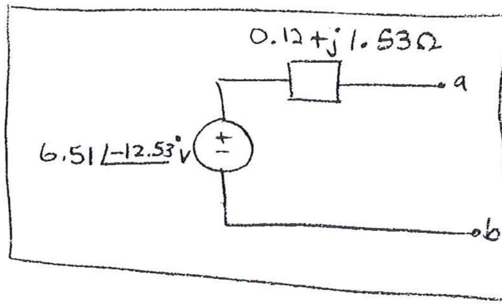
$$(4-j)I_x = 12$$

$$I_x = 12/(4-j) = 2.91\angle 14.04^\circ$$

$$V_{oc} = -jI_x + 2I_x = (2-j)I_x = 6.51\angle -12.53^\circ \text{ V}$$

Find I_{sc}





KVL outer loop: $-12 + 2I_x + j2I_{sc} = 0$

① $2I_x + j2I_{sc} = 12 \Rightarrow I_x = \dfrac{12 - j2I_{sc}}{2} = 6 - jI_{sc}$

KCL: $I = I_x - I_{sc}$

KVL left loop: $-12 + 2I_x - jI + 2I_x = 0$

$$-12 + 4I_x - j(I_x - I_{sc}) = 0$$

② $(4-j)I_x + jI_{sc} = 12$

$$(4-j)(6 - jI_{sc}) + jI_{sc} = 12$$

$$(-j(4-j) + j)I_{sc} = 12 - 6(4-j)$$

$$I_{sc} = \frac{12 - 6(4-j)}{-j(4-j) + j} = -0.6 - j4.2 = 4.24\angle -98.13^\circ \text{A}$$

$$Z_{th} = \frac{V_{oc}}{I_{sc}} = \frac{6.51\angle -12.53^\circ}{4.24\angle -98.13^\circ}$$

$$= 1.53\angle 85.6^\circ \Omega$$

$$= 0.12 + j1.53 \Omega$$

Spacer

Name Key

EE 281 B

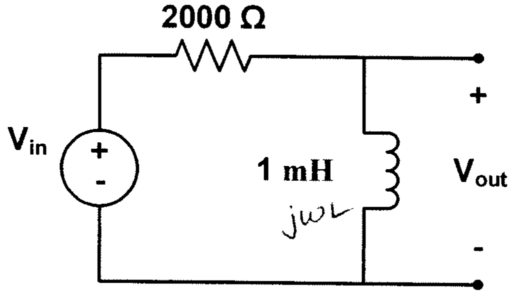
Exam 4

Fall 2010

Show all your work. Circle or box your answer.

1. Find the transfer function $H(j\omega) = \frac{V_{out}}{V_{in}}$ for the following filter. Is this a highpass or lowpass filter?

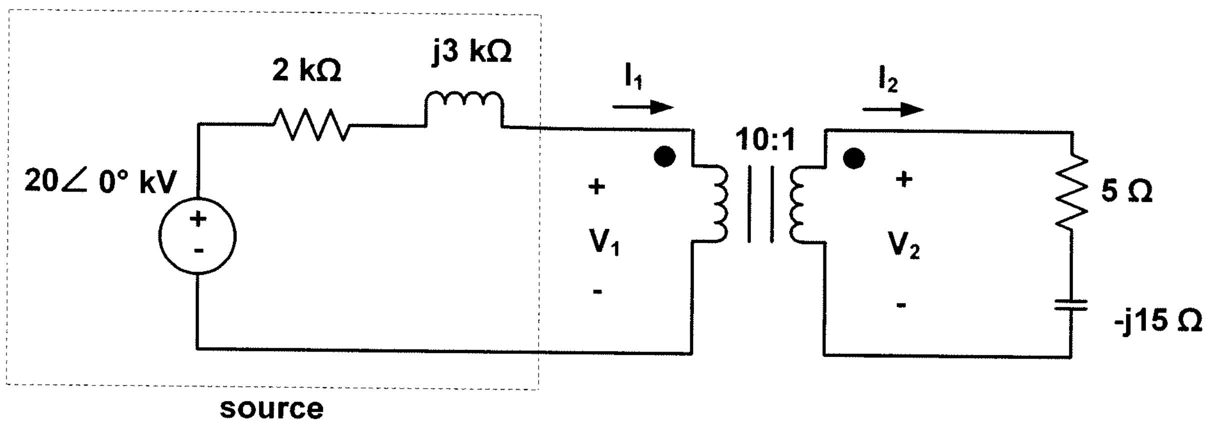
$$\boxed{H(j\omega) = \frac{j\omega L}{R + j\omega L} = \frac{j\omega(0.001)}{2000 + j\omega(0.001)}}$$



$\boxed{\text{highpass}}$

(15 pts)

2. Using impedance transformations (reflected impedances), find the equivalent impedance seen by the source in the following circuit.



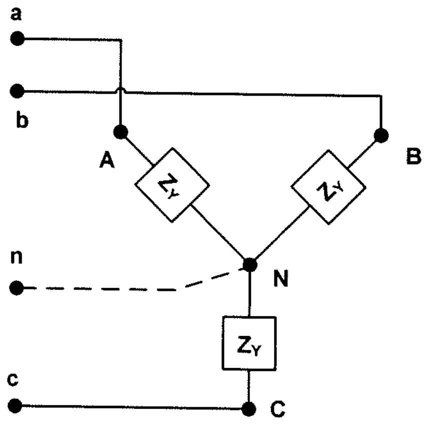
(10 pts) $Z_L' = \left(\frac{N_1}{N_2}\right)^2 Z_L = \left(\frac{10}{1}\right)^2 (5 - j15) = \boxed{500 - j1500\ \Omega}$

3. Find the turns ratio of an ideal transformer necessary to step a 40-V_{rms} source up to 1000 V_{rms}.

(5 pts) $\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{1000}{40} = \frac{25}{1}$

$\boxed{N_1 : N_2 = 1:25}$

4. A balanced Y-connected load has an impedance of $Z_Y = 300 + j20\ \Omega$ for each phase. Assuming the phase sequence is positive and that $\mathbf{V_{AB}}$ is the reference, find the following when the line voltage is $V_L = 20\ kV_{rms}$.



$Z_Y = 300 + j20\ \Omega = 300.67\angle 3.81^\circ\ \Omega$

$V_{AB} = 20\angle 0^\circ\ kV_{rms}$.

a. The phasor line-to-neutral voltage $\mathbf{V_{AN}}$.

(3 pts)

$$V_{AN} = \frac{V_{AB}}{\sqrt{3}\angle 30^\circ} = \boxed{\frac{20}{\sqrt{3}}\angle -30^\circ\ V_{rms}}$$

b. The phasor currents $\mathbf{I_{AN}}$, $\mathbf{I_{BN}}$, and $\mathbf{I_{CN}}$.

(9 pts)

$$I_{AN} = \frac{V_{AN}}{Z_Y} = \frac{20{,}000/\sqrt{3}\ \angle -30^\circ}{300 + j20} = \boxed{38.40\angle -33.8^\circ\ A}$$

$$\boxed{\begin{aligned} I_{BN} &= 38.40\angle -153.8^\circ\ A \\ I_{CN} &= 38.40\angle -273.8^\circ\ A \end{aligned}}$$

c. The phasor line current $\mathbf{I_{aA}}$.

(5 pts)

$$\boxed{I_{aA} = I_{AN} = 38.40\angle -33.8^\circ\ A}$$

d. The average power, P, absorbed by the 3-phase load.

(5 pts)

$$P = 3 V_Y I_L \cos\theta = 3\left(\frac{20{,}000}{\sqrt{3}}\right)(38.4)\cos(3.81^\circ)$$
$$= \boxed{1{,}327{,}275\ W}$$

e. The impedance Z_Δ to create an equivalent Δ-connected load.

(3 pts)

$$Z_\Delta = 3Z_Y = \boxed{900 + j60\ \Omega}$$

25

5. The transfer function for a particular first order filter is $H(j\omega) = \frac{V_{out}}{V_{in}} = \frac{j\omega(0.005)}{1+j\omega(0.005)}$.

a. Find the half-power frequency of this filter.

(5 pts)

$$\omega = \frac{1}{0.005} = \boxed{200 \text{ rad/sec}}$$

b. Find the output current, $v_{out}(t)$ if the input current is
$v_{in}(t) = 20\cos(20t) + 20\cos(5000t - 45°)$ V.

(10 pts)

$\omega = 20 \quad V_{in} = 20\angle 0°$

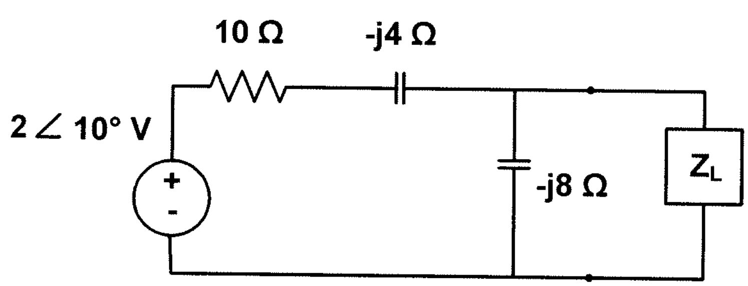
$$V_{out} = \frac{j(20)(0.005)}{1+j(20)(0.005)} 20\angle 0° = 1.99\angle 84.3°$$

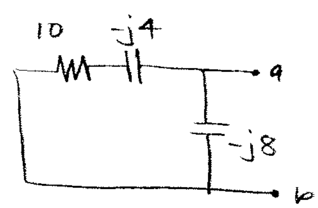
$\omega = 5000 \quad V_{in} = 20\angle -45°$

$$V_{out} = \frac{j(5000)(0.005)}{1+j(5000)(0.005)} 20\angle -45° = 19.98\angle -42.7°$$

$$\boxed{V_{out}(t) = 1.99\cos(20t + 84.3°) + 19.93\cos(5000t - 42.7°) \text{ V}}$$

6. For the following circuit, determine the value of Z_L that results in maximum average power transferred to Z_L.





(10 pts)

$$Z_{th} = (10 - j4) \parallel -j8 = \frac{(10-j4)(-j8)}{10-j4-j8} = 5.52\angle -61.6°\ \Omega$$

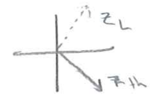
$$= 2.62 - j4.85\ \Omega$$

$$Z_L = Z_{th}^* = \boxed{2.62 + j4.85\ \Omega}$$

Exam 4

Formula Sheet - EE 281

maximize P, $Z_L = Z_{th}^*$



RMS of any sine wave $= \frac{a}{\sqrt{2}}$

$P_{avg} = I_{rms}^2 R$

$V_L = |V_{ab}|$
$= |V_{bc}|$
$= |V_{ca}|$

L H: $= Lj\omega$

C: $\frac{-j}{\omega C}$

$$v_k = \frac{R_k}{R_1+R_2+R_3+\cdots+R_n} v_s$$

$$i_k = \frac{\frac{1}{R_k}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\cdots+\frac{1}{R_n}} i_s$$

$$R = \frac{\rho L}{A}$$

$$P_{max} = \frac{v_{th}^2}{4R_{th}}$$

$$i_C = C\frac{dv}{dt} \qquad v_C = \frac{1}{C}\int_{-\infty}^{t} i\, d\lambda = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i\, d\lambda$$

$$v_L = L\frac{di}{dt} \qquad i_L = \frac{1}{L}\int_{-\infty}^{t} v\, d\lambda = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v\, d\lambda$$

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2 dt}$$

$$P = \frac{1}{2}V_m I_m \cos(\theta) = V_{rms} I_{rms}\cos(\theta) \qquad AP = \frac{P}{PF} = \frac{1}{2}V_m I_m = V_{rms} I_{rms}$$

$$Q = \frac{1}{2}V_m I_m \sin(\theta) = V_{rms} I_{rms}\sin(\theta)$$

$$x_n = \begin{cases} K_1 e^{s_1 t} + K_2 e^{s_2 t} \\ K_1 e^{st} + K_2 t e^{st} \\ K_1 e^{at}\cos(bt) + K_2 e^{at}\sin(bt) \end{cases} \qquad x_p = \begin{cases} A \\ Ae^{bt} \\ A\,e^{at}\cos(bt) + Be^{at}\sin(bt) \end{cases}$$

$$\omega = 2\pi f \qquad Z_\Delta = 3Z_Y$$

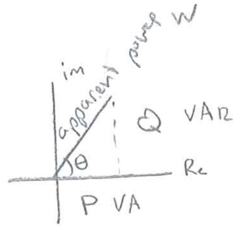
$$V_{ab} = V_{an} - V_{bn} = V_{an}\sqrt{3}\angle 30° \qquad I_{aA} = I_{AB} - I_{CA} = I_{AB}\sqrt{3}\angle -30°$$

$$P = 3V_{Yrms}I_{Lrms}\cos\theta = \frac{3V_Y I_L}{2}\cos\theta$$

reactive $\rightarrow Q = 3V_{Yrms}I_{Lrms}\sin\theta = \frac{3V_Y I_L}{2}\sin\theta$

$$V_2 = \frac{N_2}{N_1}V_1 \qquad I_2 = \frac{N_1}{N_2}I_1 \qquad Z_L' = \left(\frac{N_1}{N_2}\right)^2 Z_L$$

— Things to Know for Exam 3 —— EE 281 ——



Power factor = $\cos\theta$

ODE.
Volt source

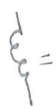
$$C\frac{dV}{dt} + \frac{V - V_s}{R} = 0$$

RMS of a cosine/sine = $\frac{\text{Amplitude}}{\sqrt{2}}$

$P = I_{rms}^2 R$ average power $= \frac{V_{Rms}^2}{R}$

$Q = I_{rms}^2 X$ reactive power $= \frac{V_{Rms}^2}{X}$

$AP = \cancel{\frac{P}{PF}} = \cancel{\frac{P}{\cos\theta}} = \sqrt{P^2 + Q^2}$

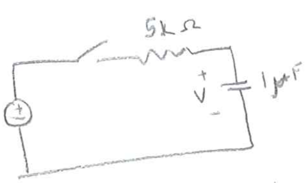
 $= \frac{1}{jC\omega}$  $= jL\omega$

$= \frac{1}{1\times10^{-6} \times 200} = -j5000\ \Omega$

$200\cos(200t + 90)\ V \Rightarrow 2\angle 90°\ V$

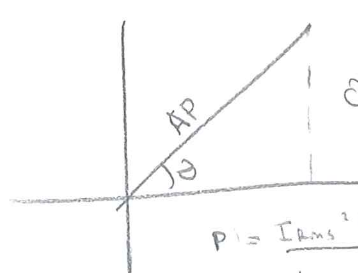
$$V_{ss} = \frac{-j5000}{5000 - j5000}(2\angle 90°) = 1.414 \angle -135\ V$$

$$= 1.414\cos(200t - 135)$$

$2e^{-3t}\ V$  =

$2\cos(3t + 0)$ $\quad 5000 + \frac{1}{1\times10^{-6}}\int_{-\infty}^{t} i\,dx = 2e^{-3t}$

imaginary current



$Q = I_{Rms}^2 X$
$= \frac{V_{Rms}^2}{R}$

$P = I_{Rms}^2 R$
$= V_{Rms}^2 \frac{1}{R}$

$PF = \cos\theta$