2025.03 Outline of math for engineers. I produced this table with ChatGPT to sort out what topics are covered in Calculus I, II, and III. Also included is Differential Equations, a required course for Mechanical Engineers. -DM

Math for Engineers

Sure! Here's an updated table including **Differential Equations**, which is often taken after Calculus 3 or alongside it in engineering programs.

Course	Key Topics
Calculus 1 (Differential Calculus)	**Limits and Continuity**: Limits, L'Hôpital's Rule, Continuity & Discontinuities. **Derivatives**: Definition, Differentiation Rules (Power, Product, Quotient, Chain), Implicit Differentiation, Applications (Tangent Lines, Optimization, Related Rates). **Basic Integration**: Antiderivatives, Definite & Indefinite Integrals, Fundamental Theorem of Calculus.
Calculus 2 (Integral Calculus & Series)	**Advanced Integration Techniques**: Integration by Parts, Trigonometric Integrals & Substitutions, Partial Fractions, Improper Integrals. **Applications of Integration**: Area Between Curves, Volume (Disk, Washer, Shell Methods), Arc Length, Surface Area, Work, Center of Mass. **Sequences and Series**: Convergence & Divergence, Series Tests (Comparison, Ratio, Root, Integral, Alternating), Power Series, Taylor/Maclaurin Series.
Calculus 3 (Multivariable Calculus)	**Vectors and Geometry in Space**: 3D Coordinate Systems, Dot & Cross Products, Equations of Lines & Planes. **Partial Derivatives and Multiple Integrals**: Partial Derivatives, Gradient Vectors, Directional Derivatives, Tangent Planes, Double & Triple Integrals, Cylindrical & Spherical Coordinates. **Vector Calculus**: Vector Fields, Line Integrals, Green's Theorem, Stokes' Theorem, Divergence Theorem.
Differential Equations	**First-Order Differential Equations**: Separable, Exact, Homogeneous, Linear Equations, Applications (Mixing Problems, Population Growth, Cooling). **Higher-Order Differential Equations**: Second-Order Linear Equations, Method of Undetermined Coefficients, Variation of Parameters, Applications (Mechanical Vibrations, Electrical Circuits). **Laplace Transforms and Systems of Equations**: Laplace Transforms, Inverse Laplace, Convolution, Systems of Linear Differential Equations, Eigenvalues & Eigenvectors. **Fourier Series & Partial Differential Equations**: Fourier Series, Heat Equation, Wave Equation, Laplace's Equation.

J. Hake
Office: SM276
Phone: 513-4843

COURSE SYLLABUS

MTH:210-Single Variable Calculus Summer-2008

A. Course Prerequisite: MTH-160 and Mth170 with a grade of C or better, or Mth-185 with a grade of C or better, or satisfactory score on placement test.

B. Text: Calculus, Sixth Edition, James Stewart, Brooks/Cole Publishing Company, Pacific Grove, 2008.

C. Course Coverage: Chapters 1-6 (With certain exclusions)

D. Course objectives:

1.The student will learn the concept of a limit, the continuity of a function, and the role each plays in the study and application of mathematics.

2. The student will be able to differentiate algebraic and trigonometric functions, and use the derivatives in selected applications including related rates, the graph of a function, and optimization problems.

3. The student will learn and employ concepts regarding the anti-differentiation of a function and its relationships to the definite integral, and use integration techniques in selected applications including the area between two curves, volumes of solids of revolution, work and others.

E. Tests: There will be at least three 60 minute tests during the semester. Each test may cover material no later than two days previous to the test. There may be review time during class the meeting previous to the test. Each test is worth 100 points and must be taken in pencil only. There are no make-ups.

1 of 4 tests will be dropped

F. W Grades: Any student desiring a **W** grade must initiate this her/himself on or before Friday July 11 , 2008.

G. Homework: Homework is the key to success in this (or any other) course. I will collect homework during this summer session. HOWEVER: If you don't do your homework, you WILL NOT pass this class!

H. Quizzes: There will be a brief homework quiz often. Not only do quizzes provide an incentive to do homework, but they also supply you with extra points with which to determine your final grade.

I. Attendance: Attendance is mandatory and will be recorded each time class meets. When you miss class, you cheat yourself and you don't know what is going on. If someone has a borderline grade, full attendance will be used to tip the scale in his/her favor, while poor attendance will tip the scales in a different direction.

J. Semester Grades: Each student's semester grade will be based on an accumulated point basis and then converted to a percentage. Letter grades will be awarded in accordance with the following:

90-100%............A
80-89%..............B
70-79%..............C
60-69%..............D
Below 60%.........F

K. Final Exam: There will be a 200 point final exam during the time established by the Instruction Office. -ATTENDANCE IS MANDATORY—(July 24th)

L. Office Hours: I will be on campus Monday through Thursday from 12:30 until 5:30 pm. If I am not in my office, you might find me in SM 246. Also available by appointment.

M. Access Office: The ACCESS OFFICE (513-4551) has been designated by the college as the primary office to guide, counsel, and assist students with disabilities. If you receive services through the ACCESS OFFICE and require accommodations for this class, make an appointment with me **as soon as possible** to discuss your approved accommodation needs. Bring your Instructor Notification Memo provided by the ACCESS OFFICE to the appointment. I will hold any information you share with me in strictest confidence unless you give me permission to do otherwise. If you have not made contact with the ACCESS OFFICE and have reasonable accommodation needs (volunteer note taker, extended time for tests, seating arrangements, etc.), I will be happy to refer you. The ACCESS OFFICE will require appropriate documentation of disability. If you have a disability and have no need for accommodations, the use of the ACCESS OFFICE is VOLUNTARY.

N. Course Repeats: Any student who is attempting this course for a third time (or more) is required to make an appointment with the Mathematics Department chair and with me to develop a contract for success which will outline the student's plan to make this attempt a successful one.

O. I Grades: **I** grades are discretionary grades which can be awarded by the course instructor. These two grades are described in the current St. Louis Community College catalogue.

P. Our Electronic Age: **Does Not Exist** once we cross the threshold of the classroom. Please turn **OFF** all beepers, phones, or any other electronic devices over which we have no control. This means that calculators are the **only** electronic device allowed, and even then, turn the sound off if there is any.

Homework MTH210

1.1= 1-15odd,19, 21-27all, 27-57odd,61-67odd.

1.2= 1-19odd,20, 21, 25.

COLUMBIA COLLEGE OCT. 2009

COURSE	ISBN	TITLE	EDITION	NEW	USED	PUB	AUTHOR
ACCT 281	9780077303204	FUND. ACCOUNTING PRINCIPLES	19TH 09	$204.80	$157.70	MCG	WILD
	9780073366364	FUND. OF ACCT. WK. PPR.	19TH 09	$67.15	$51.70	MCG	WILD
ASTR 108	9781429205191	DISCOVERING THE UNIVERSE	8TH 08	$127.40	$99.10	MPS	COMINS
CJAD 101	9780135130308	CRIMINAL JUSTICE TODAY	10TH 09	$134.80	$103.80	PH	SCHMALLEGER
COLL 103	9780073376387	CONCEPTS OF WELLNESS & FITNESS	8TH 09	$80.50	$62.00	MCG	CORBIN
COLL 108	9780073535692	BEYOND FEELINGS	8TH 08	$55.50	$42.75	MCG	RUGGIERO
ECON 294	9780073287126	MICROECONOMY TODAY	11TH 08	$150.35	$115.80	MCG	SCHILLER
EDUC 251	9780976423317	FIRST DAYS OF SCHOOL	4TH 09	$32.95	$25.40	WONG	WONG
EDUC 300	9780131950849	UNDERSTANDING BY DESIGN	2ND 06	$42.35	$32.60	PH	WIGGINS
EDUC 351	9780205491001	HOW TO DEV. A PROF. PORTFOLIO	4TH 07	$42.50	$32.70	PH	CAMPBELL
	9780976423317	FIRST DAYS OF SCHOOL	4TH 09	$32.95	$25.40	WONG	WONG
EDU/PSY 392	9780073382616	ADOLESCENCE	12TH 08	$146.60	$112.90	MCG	SANTROCK
ENGL 107	9780393978827	NORTON SAMPLER	6TH 03	$58.25	$44.85	NORTON	COOLEY
	9780205309023	ELEMENTS OF STYLE	4TH 00	$9.95	$7.70	PH	STRUNK
	9781413033816	LEAST YOU SHOULD KNOW ABOUT ENG.	10TH 09	$73.65	$56.70	CENGAGE	WILSON
EDUC 362	9780879860967	A LINK TO THE PAST: ENGAGING ETC.	2002	$22.00	$15.85	NCSS	YELL
ENGL 111	9780877798095	MERRIAM-WEBSTER COLLEGIATE DICT.	11TH 05	$26.95	$20.80	MERRIAM	DICTIONARY
	9780205651719	LITTLE BROWN HANDBOOK	11TH 10	$74.70	$56.05	PH	FOWLER
	9780312472078	BRIEF BEDFORD READER	10TH 09	$49.25	$37.95	MPS	KENNEDY
ENGL 210	9780558279530	INTRODUCTION TO LITERATURE	1ST 09	$55.50	$42.75	PH	STRATMAN
	9780486440989	GREAT SHORT STORIES	2005	$3.50	$2.70	DOVER	NEGRI
HIST 122	9780073307022	UNFINISHED NATION: CONCISE ETC. V2	5TH 08	$67.85	$52.25	MCG	BRINKLEY
	9780312459680	READING THE AMERICAN PAST V2	4TH 09	$20.55	$15.85	MPS	JOHNSON
HIST 342	9780395868492	MAJOR PROBLEMS IN CIVIL WAR	2ND 98	$69.40	$51.35	CENGAGE	PERMAN
	9780072418156	AMERICAN ILLIAD	2ND 02	$55.85	$43.00	MCG	ROLAND
HUMS 310							
HUMS 345	9780495100638	PROMOTING COMMUNITY CHANGE	4TH 08	$119.90	$92.30	CENGAGE	HOMAN
MATH 104	9780618611324	ALGEBRA: INTRO. & INTERM. SSM	4TH 07	$55.50	$42.75	CENGAGE	AUFMANN
	9780618609536	ALGEBRA: INTRO. & INTERMEDIATE	4TH 07	$179.80	$138.45	CENGAGE	AUFMANN
MATH 106	9780618611324	ALGEBRA: INTRO. & INTERM. SSM	4TH 07	$55.50	$42.75	CENGAGE	AUFFMANN
	9780618609536	ALGEBRA: INTRO. & INTERMEDIATE	4TH 07	$179.80	$138.45	CENGAGE	AUFFMANN
MATH 215	9780547167022	CALCULUS	9TH 10	$219.90	$169.30	CENGAGE	LARSON

		9780547213095	CALCULUS S.G. VI	9TH 10	$53.10	$40.90	CENGAGE	LARSON
	MATH 235	9780547167022	CALCULUS	9TH 10	$219.90	$169.30	CENGAGE	LARSON
		9780547213101	CALCULUS VII	9TH 10	$53.10	$40.90	CENGAGE	LARSON
	MATH 250	9780321422095	ELEMENTARY STATISTICS	7TH 08	$138.40	$106.55	PH	WEISS
	MATH 370	9780495108245	FIRST COURSE IN DIFF. EQUATIONS	9TH 09	$191.45	$147.45	CENGAGE	ZILL
		9780495385660	FIRST COURSE IN DIFF. EQUATIONS S.G.	9TH 09	$59.25	$45.65	CENGAGE	ZILL
	MGMT 265	9780073524931	LAW FOR BUSINESS	10TH 09	$206.20	$158.75	MCG	BARNES
	MKTG 310	9780138157180	MARKETING: INTRODUCTION	9TH 09	$171.25	$131.85	PH	ARMSTRONG
	MGMT364	9780471410690	ON STAFFING	2003	$75.00	$57.75	WILEY	BURKHOLDER
	PHIL 330	9780872204645	NICOMACHEAN ETHICS	2ND 99	$13.95	$10.75	HACKETT	ARISTOTLE
		9780061311598	GROUNDWORK OF METAPHYSICS	1ST 64	$13.00	$10.05	HARPER	KANT
		. 9780872206052	UTILITARIANISM	2ND 01	$4.45	$3.45	HACKETT	MILLER
	OPTIONAL	9780073386713	ELEMENTS OF MORAL PHILOSOPHY	6TH 10	$47.30	$36.40	MCG	RACHELS
	PHED 100		NO TEXT					
	PSYC 325	9781433805615	PUBLICATION MANUAL OF APA	6TH 10	$28.50	$21.75	APA	AMER PSYCH
		9780534609764	RESEARCH METHODS IN PSCHOLOGY	8TH 06	$169.20	$130.30	CENGAGE	ELMES
	PSYC 330	9780077236359	HUMAN DEVELOPMENT	10TH 07	$158.95	$122.40	MCG	PAPALIA
	RELI 101	9780130923868	EXPLORING RELIGIOUS MEANING	6TH 03	$66.45	$51.20	PH	MONK
	SOCI 321	9780495391029	CRIMINOLOGY	10TH 09	$157.20	$121.05	CENGAGE	SIEGEL
		9780495504207	CRIMINOLOGY S.G.	10TH 09	$58.25	$44.85	CENGAGE	SIEGEL
			* TO BE SELECTED					

– find gradient, or ∇f –

$f_x + f_y + f_z$

13.7 – find normal vector to function f –

∇f

13.7 – find 'unit' normal vector to function –

13.6 – find directional derivative in V direction –

$\nabla f \cdot U, \quad U = \frac{V}{\|V\|}$

13.6 – find directional deriv. in Q direction –

$\nabla f \cdot U, \quad U = \langle Q - P \rangle / \| \langle Q - P \rangle \|$

13.6 – find $D_u f(x, y)$ –

- $f_x(x, y)\cos\theta + f_y(x, y)\sin\theta$ By definition
- or, $U \cdot \nabla f$

– find dw/dt –

=

13.7 – find tangent plane to a func. at point –

$V = \langle x - x_0, y - y_0, z - z_0 \rangle$ plug in (a, b, c) for x_0, y_0

find ∇f, find $V \cdot \nabla f$, set $= 0$, reduce

13.7 – find Symmetric eq. of normal line

- find tan plane – x, y, z coefficients $= \langle a, b, c \rangle$
- $\frac{x-a}{x_0} = \frac{y-b}{y_0} = \frac{z-c}{z_0}$

13.8 – relative extrema –

- $d = f_{xx} f_{yy} - (f_{xy})^2$ $\quad d > 0$ & $f_{xx} > 0 \Rightarrow$ relative min
- $d > 0$ & $f_{xx} < 0$, rel. max, $d < 0 \Rightarrow$ saddle point

13.8
- 1st set $f_{xx} = f_{xy} = 0$ & solve for x & y

quotient rule

$\frac{g'(x)h(x) - h'(x)g(x)}{(h(x))^2}$

product rule

$f'(g) + f(g')$

— implicit diff. — 13.5

$\frac{dy}{dx} = -\frac{F_x(x,y)}{F_y(x,y)}$

- Partial deriv. is the same

— Chain Rule — 13.5

$\frac{dw}{ds} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s}$

— calculate error — 13.4

$\partial F = \frac{\partial F}{\partial x} dx + \frac{\partial F}{\partial y} dy.$

dx & dy are component err.

— total differential — 13.4

same as above

— Notation — 13.3

$\frac{\partial}{\partial x} f(x,y) = f_x(x,y)$

Tests: There will be (at least) three one-hour (approx.) tests worth 100 points each. These will be announced in advance and will cover only material since the previous test. It is department policy that no make-ups are allowed. If a student misses ONE test, I will replace that grade with his/her raw score on the final exam (DON'T LET THIS HAPPEN!!). Any subsequent missed exams will be counted as zeros.

Final Exam: There will be a comprehensive final exam (worth 200 pts.) given during finals week at the time designated by the instruction office (TBA at a later date). A portion of this exam may be designed by the department as a standardized test and may be in multiple choice format. Attendance to the final exam is mandatory.

Grading: Your semester grade will be calculated from a total of your accumulated points (a percentage thereof) from:

A) Two (or more) take-home assignments collectively worth 200 points (+).
B) Three (or more) regular semester tests worth 100 points each, 300 total pts (+).
C) Regularly scheduled quizzes worth 25 points each (150 total pts).
D) The comprehensive final exam worth 200 points.
E) Periodic "special assignments" will generate "bonus" points that are added to your total, but not counted as part of the %. These are FREE points!!
(The assignments are NOT optional, but they DO generate bonus points!!).

The grading scale is:

90% and up	=	A
80% -- 89%	=	B
70% -- 79%	=	C
60% -- 69%	=	D
Below 60%	=	F

(Note: there are a total of (at least) 850 pts. all semester.)

"W" Grade: Any student desiring a "W" grade (withdrawal) must initiate this themselves on or before Fri July11th 2007 (see me 1st !!). There could be repercussions as to financial aid, etc. and the student should (also) see an academic advisor 1st.

"PR" Grade: The "PR" grade is described in the current SLCC catalog, and will be awarded only under very special circumstances. To be eligible the student must have taken all the exams, not missed more than two class meetings, have maintained at least a 50% average and must take the final exam. A "PR" grade must be requested in writing (a form would be provided). There may be further repercussions as to transferability, financial aid or other considerations and the student should see an academic advisor 1st.

"I" Grade: The incomplete grade is also described in the catalog and also is reserved for special circumstances, such as catastrophic health concerns, etc. that might force a student to miss a significant portion of the coursework, whereby the student might complete the work later as agreed by the instructor (documentation required).

STUDENTS ENROLLED IN

MTH 210
(Mr. Hake's Calculus)

Where can you go to get Help?

Summer Schedule

Supplemental Instruction (SI) is available, and it's **FREE!!**

Your SI Leader, Linda Schmitt, will help you and your classmates improve learning skills in Calculus in a comfortable, small-group format.

Sessions are scheduled for:

Mon.	**12:30 - 1:30**
Tues.	**8:30 - 10, 12:30 - 1:30**
Wed.	**12:30 - 1:30**
Thurs.	**8:30 - 10, 12:30 - 1:30**

Linda will be in class:

Mon.	**11:30 - 12:20**
Tues.	**11:30 - 12:20**
Wed.	**11:30 - 12:20**
Thurs.	**11:30 - 12:20**

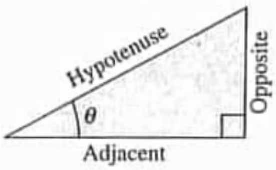
See how helpful SI can be!
Sessions are held in the

ACADEMIC SUPPORT CENTER
Room 130 in the Student Center, lower level
(Across the hall from the New Bookstore)

Reference

Definition of the Six Trigonometric Functions

Right triangle definitions, where $0 < \theta < \pi/2$.

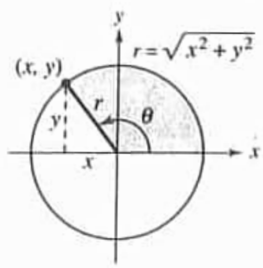


$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$ $\quad \csc\theta = \dfrac{\text{hyp}}{\text{opp}}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$ $\quad \sec\theta = \dfrac{\text{hyp}}{\text{adj}}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$ $\quad \cot\theta = \dfrac{\text{adj}}{\text{opp}}$

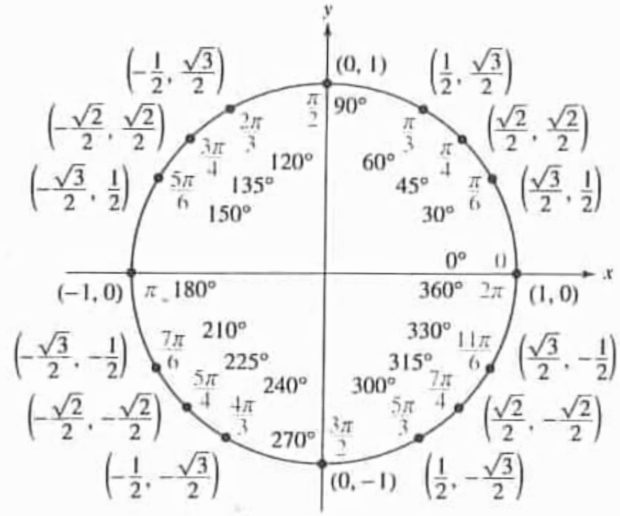
Circular function definitions, where θ is any angle.



$\sin\theta = \dfrac{y}{r}$ $\quad \csc\theta = \dfrac{r}{y}$

$\cos\theta = \dfrac{x}{r}$ $\quad \sec\theta = \dfrac{r}{x}$

$\tan\theta = \dfrac{y}{x}$ $\quad \cot\theta = \dfrac{x}{y}$



Reciprocal Identities

$\sin x = \dfrac{1}{\csc x}$ $\quad \sec x = \dfrac{1}{\cos x}$ $\quad \tan x = \dfrac{1}{\cot x}$

$\csc x = \dfrac{1}{\sin x}$ $\quad \cos x = \dfrac{1}{\sec x}$ $\quad \cot x = \dfrac{1}{\tan x}$

Tangent and Cotangent Identities

$\tan x = \dfrac{\sin x}{\cos x}$ $\quad \cot x = \dfrac{\cos x}{\sin x}$

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$ $\quad 1 + \cot^2 x = \csc^2 x$

Cofunction Identities

$\sin\left(\dfrac{\pi}{2} - x\right) = \cos x$ $\quad \cos\left(\dfrac{\pi}{2} - x\right) = \sin x$

$\csc\left(\dfrac{\pi}{2} - x\right) = \sec x$ $\quad \tan\left(\dfrac{\pi}{2} - x\right) = \cot x$

$\sec\left(\dfrac{\pi}{2} - x\right) = \csc x$ $\quad \cot\left(\dfrac{\pi}{2} - x\right) = \tan x$

Reduction Formulas

$\sin(-x) = -\sin x$ $\quad \cos(-x) = \cos x$

$\csc(-x) = -\csc x$ $\quad \tan(-x) = -\tan x$

$\sec(-x) = \sec x$ $\quad \cot(-x) = -\cot x$

Sum and Difference Formulas

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$

$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$

$\tan(u \pm v) = \dfrac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$

Double-Angle Formulas

$\sin 2u = 2 \sin u \cos u$

$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$

$\tan 2u = \dfrac{2\tan u}{1 - \tan^2 u}$

Power-Reducing Formulas

$\sin^2 u = \dfrac{1 - \cos 2u}{2}$

$\cos^2 u = \dfrac{1 + \cos 2u}{2}$

$\tan^2 u = \dfrac{1 - \cos 2u}{1 + \cos 2u}$

Sum-to-Product Formulas

$\sin u + \sin v = 2\sin\left(\dfrac{u+v}{2}\right)\cos\left(\dfrac{u-v}{2}\right)$

$\sin u - \sin v = 2\cos\left(\dfrac{u+v}{2}\right)\sin\left(\dfrac{u-v}{2}\right)$

$\cos u + \cos v = 2\cos\left(\dfrac{u+v}{2}\right)\cos\left(\dfrac{u-v}{2}\right)$

$\cos u - \cos v = -2\sin\left(\dfrac{u+v}{2}\right)\sin\left(\dfrac{u-v}{2}\right)$

Product-to-Sum Formulas

$\sin u \sin v = \dfrac{1}{2}[\cos(u-v) - \cos(u+v)]$

$\cos u \cos v = \dfrac{1}{2}[\cos(u-v) + \cos(u+v)]$

$\sin u \cos v = \dfrac{1}{2}[\sin(u+v) + \sin(u-v)]$

$\cos u \sin v = \dfrac{1}{2}[\sin(u+v) - \sin(u-v)]$

$\cancel{\int \dfrac{du}{\sqrt{a^2-u^2}} = \arcsin\dfrac{u}{a} + C}$ $\quad$ if $\log_a y = x$, then $y = a^x$

© Houghton Mifflin Company, Inc.

$\int \dfrac{du}{a^2+u^2} = \dfrac{1}{a}\arctan\dfrac{u}{a}$

Basic Differentiation Rules

1. $\frac{d}{dx}[cu] = cu'$
2. $\frac{d}{dx}[u \pm v] = u' \pm v'$
3. $\frac{d}{dx}[uv] = uv' + vu'$
4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$
5. $\frac{d}{dx}[c] = 0$
6. $\frac{d}{dx}[u^n] = nu^{n-1}u'$
7. $\frac{d}{dx}[x] = 1$
8. $\frac{d}{dx}[|u|] = \frac{u}{|u|}(u'), \quad u \neq 0$
9. $\frac{d}{dx}[\ln u] = \frac{u'}{u}$
10. $\frac{d}{dx}[e^u] = e^u u'$
11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$
12. $\frac{d}{dx}[a^u] = (\ln a)a^u u'$
13. $\frac{d}{dx}[\sin u] = (\cos u)u'$
14. $\frac{d}{dx}[\cos u] = -(\sin u)u'$
15. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$
16. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$
17. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$
18. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$
19. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$
20. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$
21. $\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$
22. $\frac{d}{dx}[\operatorname{arccot} u] = \frac{-u'}{1+u^2}$
23. $\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2-1}}$
24. $\frac{d}{dx}[\operatorname{arccsc} u] = \frac{-u'}{|u|\sqrt{u^2-1}}$
25. $\frac{d}{dx}[\sinh u] = (\cosh u)u'$
26. $\frac{d}{dx}[\cosh u] = (\sinh u)u'$
27. $\frac{d}{dx}[\tanh u] = (\operatorname{sech}^2 u)u'$
28. $\frac{d}{dx}[\coth u] = -(\operatorname{csch}^2 u)u'$
29. $\frac{d}{dx}[\operatorname{sech} u] = -(\operatorname{sech} u \tanh u)u'$
30. $\frac{d}{dx}[\operatorname{csch} u] = -(\operatorname{csch} u \coth u)u'$
31. $\frac{d}{dx}[\sinh^{-1} u] = \frac{u'}{\sqrt{u^2+1}}$
32. $\frac{d}{dx}[\cosh^{-1} u] = \frac{u'}{\sqrt{u^2-1}}$
33. $\frac{d}{dx}[\tanh^{-1} u] = \frac{u'}{1-u^2}$
34. $\frac{d}{dx}[\coth^{-1} u] = \frac{u'}{1-u^2}$
35. $\frac{d}{dx}[\operatorname{sech}^{-1} u] = \frac{-u'}{u\sqrt{1-u^2}}$
36. $\frac{d}{dx}[\operatorname{csch}^{-1} u] = \frac{-u'}{|u|\sqrt{1+u^2}}$

Basic Integration Formulas

1. $\int kf(u)\,du = k\int f(u)\,du$
2. $\int [f(u) \pm g(u)]\,du = \int f(u)\,du \pm \int g(u)\,du$
3. $\int du = u + C$
4. $\int a^u\,du = \left(\frac{1}{\ln a}\right)a^u + C$ 5.5
5. $\int e^u\,du = e^u + C$
6. $\int \sin u\,du = -\cos u + C$
7. $\int \cos u\,du = \sin u + C$
8. $\int \tan u\,du = -\ln|\cos u| + C$
9. $\int \cot u\,du = \ln|\sin u| + C$
10. $\int \sec u\,du = \ln|\sec u + \tan u| + C$
11. $\int \csc u\,du = -\ln|\csc u + \cot u| + C$
12. $\int \sec^2 u\,du = \tan u + C$
13. $\int \csc^2 u\,du = -\cot u + C$
14. $\int \sec u \tan u\,du = \sec u + C$
15. $\int \csc u \cot u\,du = -\csc u + C$
16. $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$ 5.7
17. $\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$ 5.7
18. $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$ 5.7

alternate forms of 8-11 on 340

19. $\int u^n\,du = \frac{u^{n+1}}{n+1} \quad n \neq -1$
20. $\int \frac{du}{u} = \ln|u| + C$

© Houghton Mifflin Company, Inc.

Definitions of the Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \operatorname{csch} x = \frac{1}{\sinh x}, \quad x \neq 0$$

$$\cosh x = \frac{e^x + e^{-x}}{2} \qquad \operatorname{sech} x = \frac{1}{\cosh x}$$

$$\tanh x = \frac{\sinh x}{\cosh x} \qquad \coth x = \frac{1}{\tanh x}, \quad x \neq 0$$

Hyperbolic Identities

$$\cosh^2 x - \sinh^2 x = 1 \qquad \sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$$

$$\tanh^2 x + \operatorname{sech}^2 x = 1 \qquad \sinh(x - y) = \sinh x \cosh y - \cosh x \sinh y$$

$$\coth^2 x - \operatorname{csch}^2 x = 1 \qquad \cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$$

$$\cosh(x - y) = \cosh x \cosh y - \sinh x \sinh y$$

$$\sinh^2 x = \frac{-1 + \cosh 2x}{2} \qquad \cosh^2 x = \frac{1 + \cosh 2x}{2}$$

$$\sinh 2x = 2 \sinh x \cosh x \qquad \cosh 2x = \cosh^2 x + \sinh^2 x$$

THEOREM 5.18 Derivatives and Integrals of Hyperbolic Functions

Let u be a differentiable function of x.

$$\frac{d}{dx}[\sinh u] = (\cosh u)u' \qquad \int \cosh u \, du = \sinh u + C$$

$$\frac{d}{dx}[\cosh u] = (\sinh u)u' \qquad \int \sinh u \, du = \cosh u + C$$

$$\frac{d}{dx}[\tanh u] = (\operatorname{sech}^2 u)u' \qquad \int \operatorname{sech}^2 u \, du = \tanh u + C$$

$$\frac{d}{dx}[\coth u] = -(\operatorname{csch}^2 u)u' \qquad \int \operatorname{csch}^2 u \, du = -\coth u + C$$

$$\frac{d}{dx}[\operatorname{sech} u] = -(\operatorname{sech} u \tanh u)u' \qquad \int \operatorname{sech} u \tanh u \, du = -\operatorname{sech} u + C$$

$$\frac{d}{dx}[\operatorname{csch} u] = -(\operatorname{csch} u \coth u)u' \qquad \int \operatorname{csch} u \coth u \, du = -\operatorname{csch} u + C$$

THEOREM 5.19 Inverse Hyperbolic Functions

Function	*Domain*
$\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right)$	$(-\infty, \infty)$
$\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right)$	$[1, \infty)$
$\tanh^{-1} x = \dfrac{1}{2} \ln \dfrac{1 + x}{1 - x}$	$(-1, 1)$
$\coth^{-1} x = \dfrac{1}{2} \ln \dfrac{x + 1}{x - 1}$	$(-\infty, -1) \cup (1, \infty)$
$\operatorname{sech}^{-1} x = \ln \dfrac{1 + \sqrt{1 - x^2}}{x}$	$(0, 1]$
$\operatorname{csch}^{-1} x = \ln\left(\dfrac{1}{x} + \dfrac{\sqrt{1 + x^2}}{\lvert x \rvert}\right)$	$(-\infty, 0) \cup (0, \infty)$

THEOREM 5.20 Differentiation and Integration Involving Inverse Hyperbolic Functions

Let u be a differentiable function of x.

$$\frac{d}{dx}[\sinh^{-1} u] = \frac{u'}{\sqrt{u^2 + 1}} \qquad \frac{d}{dx}[\cosh^{-1} u] = \frac{u'}{\sqrt{u^2 - 1}}$$

$$\frac{d}{dx}[\tanh^{-1} u] = \frac{u'}{1 - u^2} \qquad \frac{d}{dx}[\coth^{-1} u] = \frac{u'}{1 - u^2}$$

$$\frac{d}{dx}[\operatorname{sech}^{-1} u] = \frac{-u'}{u\sqrt{1 - u^2}} \qquad \frac{d}{dx}[\operatorname{csch}^{-1} u] = \frac{-u'}{\lvert u \rvert \sqrt{1 + u^2}}$$

$$\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln\left(u + \sqrt{u^2 \pm a^2}\right) + C$$

$$\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left\lvert \frac{a + u}{a - u} \right\rvert + C$$

SHOULD BE $\left|\frac{u+a}{u-a}\right|$

$$\int \frac{du}{u\sqrt{a^2 \pm u^2}} = -\frac{1}{a} \ln \frac{a + \sqrt{a^2 \pm u^2}}{\lvert u \rvert} + C$$

SEE #24 PA21

11.2 - 11.7

Parallel vectors $U = cV$ can be multiplied by a const. to make equal

orthogonal vectors $U \cdot V = 0$

Triangle vectors dot products >0 ⇒ acute angles

Vector direction cosines : $\cos\alpha = \frac{i}{\|U\|}$

orthogonal components : $W_2 = U - W_1$

$W_1 = \left(\frac{U \cdot V}{\|V\|^2}\right) V$

cross product $U \times V = \langle (U_2V_3 - U_3V_2), -(U_1V_3 - U_3V_1), (U_1V_2 - U_2V_1) \rangle$

dot product $U \cdot V = (U_1V_1 + U_2V_2 + U_3V_3)$ scalar

Volume by triple scalar product

adjacent vectors U, V, W

$V = |U \cdot (V \times W)| = \begin{vmatrix} U & U & U \\ V & V & V \\ W & W & W \end{vmatrix}$

Line

- parametric $x = x_1 + at$ $y = y_1 + bt$ $z = z_1 + ct$
- symmetric $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$

Plane

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ └ std ┘

$ax + by + cz + d = 0$ └ general ┘

- finding equation :

$n = U \times V$ where U & V are adjacent vectors on the plane

Angle between 2 planes

$$\cos\theta = \frac{|n_1 \cdot n_2|}{\|n_1\| \, \|n_2\|}$$

Line of intersection (2 planes)

① $\cos\theta = \frac{|n_1 \cdot n_2|}{\|n_1\| \, \|n_2\|}$

② line eqn : solve eqn's simultaneously
solve 1 variable for another
plug in

③ $n_1 \times n_2$ = vector parallel to intersection

Point of intersection :

- Line & Plane ① get parametric line eq.
② plug values into line for x, y, z, of plane
③ solve for t
④ plug t value into line eq.

Dist Point to Plane

P = point Q = point on plane

$$d = \frac{|\vec{PQ} \cdot n|}{\|n\|}$$

Dist Point to line

$$d = \frac{\|\vec{PQ} \times U\|}{\|U\|}$$

u = direction vector for line.

Ellipsoid/sphere

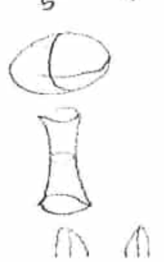
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Hyperboloid of 1 sheet

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

Hyperboloid of 2 sheets

$$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$



Elliptic cone

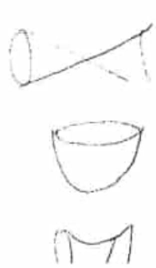
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$

Elliptic Paraboloid

$$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

Hyperbolic Paraboloid

$$z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$



Surface of revolution : to resolve: solve as func. of revolution variable & plug into equation

$y^2 + z^2 = [r(x)]^2$ (about x-axis)

cylindrical to rectangular $x = r\cos\theta$ $y = r\sin\theta$ $z = z$

rect. to cyl $r^2 = x^2 + y^2$ $\tan\theta = \frac{y}{x}$ $z = z$

spher. to rec. $x = \rho\sin\phi\cos\theta$ $y = \rho\sin\phi\sin\theta$ $z = \rho\cos\phi$

rect to spher $\rho^2 = x^2 + y^2 + z^2$ $\tan\theta = \frac{y}{x}$ $\phi = \arccos\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$

sph. to cyl. $r^2 = \rho^2\sin^2\phi$ $\theta = \theta$ $z = \rho\cos\phi$

cyl to spher. $\rho = \sqrt{r^2 + z^2}$ $\theta = \theta$ $\phi = \arccos\left(\frac{z}{\sqrt{r^2 + z^2}}\right)$

$(r \geq 0)$

$r = c$ cyl

$\rho = c$ spher.

$\theta = c$ plane

$\phi = c$ cone

$r^2 = z$

Circle

$r = c\sin\theta$ centered on y, radius c

7J: Cylinder

Partial Fractions

$$\frac{1}{x^2-5x+6} = \frac{A}{x-3} + \frac{B}{x-2} \Rightarrow 1 = A(x-2)+B(x-3)$$

$$\frac{2x^3-4x-8}{(x^2+x)(x^2+4)} \Rightarrow \frac{A}{x} + \frac{B}{(x-1)} + \frac{Cx+D}{x^2+4}$$

$$\frac{8x^3+13x}{(x+2)^2} \Rightarrow \frac{Ax+B}{(x+2)^2} + \frac{Cx+D}{(x+2)^2}$$

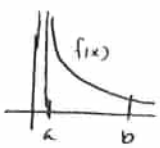
By Parts

$$\int u\,dv = vu - \int v\,du$$

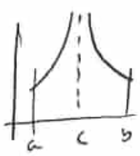
L'Hopital's

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

$$\int_a^b = \lim_{c \to b^-} \int_a^c f(x)\,dx$$



$$\int_a^b = \lim_{c \to a^+} \int_c^b f x$$



$$= \lim \int_a^c + \int_c^b$$

Trig substitution

special formulus

$$\int \sqrt{a^2-u^2}\,du = \frac{1}{2}\left(a^2 \arcsin\frac{u}{a} + u\sqrt{a^2-u^2}\right) + C$$

$$\int \sqrt{u^2-a^2}\,du = \frac{1}{2}\left(u\sqrt{u^2-a^2} - a^2 \ln\left|u+\sqrt{u^2-a^2}\right|\right) + C \quad u>a$$

$$\int \sqrt{u^2+a^2}\,du = \frac{1}{2}\left(u\sqrt{u^2+a^2} + a^2 \ln\left|u+\sqrt{a^2+u^2}\right|\right) + C$$

Wallis' Formulas

n is odd

$$\int_0^{\pi/2} \cos^n x\,dx = \left(\frac{2}{3}\right)\left(\frac{4}{5}\right)\left(\frac{6}{7}\right)\cdots\left(\frac{n-1}{n}\right)$$

n is even

$$\int_0^{\pi/2} \cos^n x\,dx = \left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{5}{6}\right)\cdots\left(\frac{n-1}{n}\right)\left(\frac{\pi}{2}\right)$$

< or $\sin^n$ >

Calc III notes

15.1 Vector fields

Concepts: Definition of vector field, inverse square field, conservative vector field, curl of a vector field, Divergence, relationship Divergence Vs. Curl

Uses: gradient, test for conservative (plane), test for conservative (space) potential function(f) for F.

Tasks: sketch vectors in a field, verify conservative field, find potential function, find curl of F at a point, find curl F for a field, find divergence, find curl(FxG)

15.2 Line integrals

Concepts: piecewise smooth parametrization, orientation of parametrization, definition of line integral, evaluation of a line integral as a definite integral, integrating a line integral over a path (this just mean the line integral has several sections (C1, C2, C3)) line integral of a vector field, line integral in differential form

Uses: magnitude of (r'(t)) relationship of ds to dt, tangent line to a path C (called T)

f(x, y, z) is the density of the wire.
C is the curve, given by a function r(t)

15.3 Conservative vector fields

Concepts: Fundamental theorem of line integrals, independence of path

15.4 Green's Theorem

Concepts: green's theorem

15.5 Parametric Surfaces

Concepts: definition of parametric surface, normal vector to a parametric surface, area of a parametric surface

Use: cross product, magnitude of cross product,

Curl $F(x,y,z) = \nabla \times F(x,y,z) = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)i - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)j + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)k$

$\nabla F = \frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k$

div. $F(x,y,z) = \nabla \cdot F = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

Potential function of F

$\int M\,dx, \int N\,dy, \int P\,dz$

collect val

conservative

$x = r\cos\theta \quad y = r\sin\theta$

15.6

use 14.5 surface area $S = \int_R \int \sqrt{1+(f_x)^2+(f_y)^2}\, dA$

if ⓐ $z = g(x,y)$ surface

ⓑ g, g_x, g_y continuous on R

ⓒ $f(x,y,z)$ function

then $\int_S \int f(x,y,z)\, dS$ = 3 forms   1 form is $z = g(x,y)$

1) $z = g(x,y)$ $\int_R \int f(x,y,\underbrace{g(x,y)}_{z})\sqrt{1+g_x(x,y)^2+[g_y(x,y)]^2}\, dA$ R in x-y plane

2) $y = g(x,z)$ .... R in xz plane

Ex 3 gone through

orientation of a surface $z = g(x,y)$ $G(x,y,z) = z - g(x,y)$

then 1. $N = \dfrac{\nabla G(x,y,z)}{\|\nabla G(x,y,z)\|} = \dfrac{-g_x i - g_y j + g_z k}{\sqrt{1+(g_x)^2+(g_y)^2}}$ = upward unit normal vector

2). $N = -\nabla G(x,y,z)$ ... downward

In parametric form $N = \dfrac{r_u \times r_v}{\|r_u \times r_v\|}$ upward

surface: 1. $z = g(x,y) \Rightarrow G(x,y,z) = z - g(x,y)$

Flux integral   volume of fluid $= \int_S \int F \cdot dS$

$z = g(x,y)$ $G(x,y,z) = z - g(x,y)$ in parametric form:

$N\,dS = \dfrac{\nabla G}{\|\nabla G\|}\, dS = \dfrac{\nabla G}{\sqrt{g_x^2+g_y^2+1}}$ $N\,dS = \nabla G\, dA$ 1. $N = r_u \times r_v$ ....

$\int_S \int F \cdot N\,dS = \int_R \int F\, [-g_x i - g_y j + k]\, dA$ (oriented upward)

parametric $\int_S \int F \cdot N\,dS = \int_D \int F \cdot \dfrac{r_u \times r_v}{\cancel{\|r_u \times r_v\|}} \cancel{\|r_u \times r_v\|}\, dA = \int_D \int F \cdot (r_u \times r_v)\, dA$

15.5)

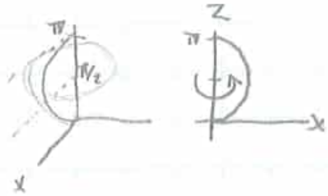
21) cylinder $x^2+y^2=16$ find vector-valued func

let $x=4\cos u$ $y=4\sin u$, $z=v$ (height) $r(u,v)= 4\cos u\, i + 4\sin u\, j + v k$

25) plane $z=4$, inside circ. $x^2+y^2=9$ $z=4$

$v\cos u\, i + v\sin u\, j$ $0\le v\le 3$ $0\le u\le 2\pi$

29) $x=\sin z$ $0\le z\le \pi$ about z axis



let $z=u$ $x=f(u)\cos v$ $y=f(u)\sin v$

$x=\sin u\cos v$ $y=\sin u\sin v$ $0\le u\le \pi$

$0\le v\le 2\pi$

33) $r(u,v)=2u\cos v\, i+3u\sin v\, j+u^2k$ xyz: $(0,6,4)$ Tan. plane eq.?

$r_u = 2\cos v\, i + 3\sin v\, j + 2u k$

$r_v = 2u(-\sin v)i + 3u(\cos v)j + (0)k$ $(u,v)=?$ $(2,\pi/2)$ think values to get $x=0, y=6, z=4$

$r_u(2,\pi/2) = 0i+3j+4k$

$r_v(\ ) = 2(2)(-\sin\pi/2)i + 3(2)\cos v$ $r_u\times r_v = \begin{vmatrix} i & j & k\\ 0 & 3 & 4\\ -4 & 0 & 0\end{vmatrix} = i(0)-j(16)+k(12)$

$= -4i + (0)j + 0k$ also $= \langle 0,4,-3\rangle$

Tan. plane: $0(x-0)+4(y-6)+(-3)(z-4) = 4y-3z = 24-12 = 12$

39) Area; Part of cone

$r(u,v) = au\cos v\, i + au\sin v\, j + uk$ where $0\le u\le b$ $0\le v\le 2\pi$

$r_u = a\cos v\, i + a\sin v\, j + k$

$r_v = au(-\sin v)i + au\cos v\, j + 0k$ $r_u\times r_v = i(-au\cos v) - j(au\sin v) + k(\overbrace{a^2u\cos^2 v + a^2u\sin^2 v}^{a^2u})$

$\|r_u\times r_v\| = \sqrt{(a^2u)^2 + a^4u^2} = \sqrt{a^2u^2(1+a^2)} = au(\sqrt{1+a^2})$

$A=\int_D\int \|r_u\times r_v\|\, dA = \int_0^b\int_0^{2\pi} au\sqrt{1+a^2} = 2\pi\sqrt{1+a^2}\int_0^b u\,du = 2\pi a\sqrt{1+a^2}\,\frac{b^2}{2}$

15.4 homework 1-27 odds

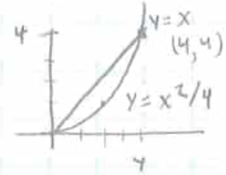
1) $\iint_R \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\, dA$ C: 

$M = xy$ $M = y^2$ $N = x^2$

$\frac{\partial N}{\partial x} = 2x$ $\frac{\partial M}{\partial y} = 2y$

$= \int_0^4\int_0^4 2x - 2y\, dy\,dx =$

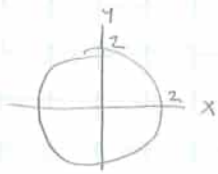
$= \int_0^4 2xy - \frac{\cancel{2}y^2}{\cancel{2}}\Big|_0^4 dx$ $\int_0^4 (8y - 16)\,dx = \frac{8x^2}{2} - 16x\Big|_0^4 = 4(16) - 16(4) = \boxed{0}$

3) $M = y^2$ $N = x^2$ $\frac{\partial N}{\partial x} = 2x$ $\frac{\partial M}{\partial y} = 2y$ C = 

$\int_0^4\int_{x^2/4}^{x} 2x - 2y\,dy\,dx = \int_0^4 \left[2xy - y^2\right]_{x^2/4}^{x}$

$= \int_0^4 2x^2 - x^2 - 2x\frac{(x)^2}{4} + \frac{x^4}{16} = \int_0^4 \left(\frac{x^4}{16} - \frac{x^3}{2} + x^2\right)dx = \frac{x^5}{5\cdot 16} - \frac{x^4}{8} + \frac{x^3}{3}\Big|_0^4$

$12.8 - 32 + \frac{64}{3} = \boxed{2.13}$

5) $M = xe^y$ $N = e^x$ 

$4\int_0^2\int_0^{2-x^2} dy\,dx$

$\boxed{15.3}$ homework 1-35 odds

1) $F(x,y) = x^2 i + xy j$ $M = x^2$ $N = xy$

a) $r_1(t) = ti + t^2 j$ $0 \le t \le 1$ $x = t$ $y = t^2$ $dx = dt$ $dy = 2t\,dt$

$\int_0^1 (t^2) + (t^3 2t)\,dt = \int_0^1 2t^4 + t^2\,dt = \frac{2t^5}{5} + \frac{t^3}{3}\Big|_0^1 = \frac{2}{5} + \frac{1}{3} = \frac{5+6}{15} = \boxed{\frac{11}{15}}$

b) $r_2(\theta) = \sin\theta i + \sin^2\theta j$ $0 \le \theta \le \frac{\pi}{2}$ $x = \sin\theta$ $y = \sin^2\theta$ $dx = \cos\theta$ $dy =$

$d \sin\theta \sin\theta$

$\cos\theta \sin\theta + \sin\theta\cos\theta$

$\int_0^{\frac{\pi}{2}} \left[\sin^2\theta \cos\theta + \sin^3\theta\,(2\cos\theta\sin\theta)\right] d\theta$

$= \int_0^{\frac{\pi}{2}} (\sin^2\theta\cos\theta + 2\sin^4\theta\cos\theta)\,d\theta$

$\frac{\sin^3\theta}{3} + \frac{2\sin^5\theta}{5}\Big|_0^{\pi/2}$ $\frac{1}{3} + \frac{2}{5} = \frac{5+6}{15} = \boxed{\frac{11}{15}}$

5) $F(x,y) = e^x \sin y\, i + e^x \cos y\, j$ $\frac{\partial N}{\partial x} = e^x \cos y$ $\frac{\partial M}{\partial y} = e^x \cos y$ conservative

$M = e^x \sin y$ $N = e^x \cos y$

9) $F(x,y,z) = y^2 z i + 2xyz j + xy^2 k$ $\frac{\partial P}{\partial y} = 2yx$ $\frac{\partial N}{\partial z}$ $2yx$ $\frac{\partial M}{\partial y} = 2yz = \frac{\partial N}{\partial x}$ $\frac{\partial P}{\partial x} = y^2 = \frac{\partial M}{\partial z}$

$M = y^2 z$ $N = 2xyz$ $P = xy^2$ — conservative —

11) $F(x,y) = 2xy i + x^2 j$ $\frac{\partial N}{\partial x} = 2x = \frac{\partial M}{\partial y}$ cons. ✓

a) $ti + t^2 j$, $0 \le t \le 1$ ⇒ change path $ti + tj$ $0 \le t \le 1$ $\int_0^1 (2t^2 + t^2)\,dt = t^3 \Big|_0^1 = \boxed{1}$

b) $ti + t^3 j$ ⇒ $\int_c =$ (still goes (0,0) to (0,1)) $\int = \boxed{1}$

13) NOT conservative

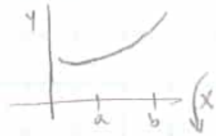
$\frac{\partial N}{\partial x} = -1$ $\frac{\partial M}{\partial y}$ 1

15.5 Parametric Surfaces

Plane curve $r(t) = x(t)i + y(t)j$

If $z = f(x,y)$

THEN $r(x,y) = xi + yj + f(x,y)k$

surface of revolution a)  Let $x = u$ $y = f(x)$

b)  $x = f(z)$

c)  $y = u$ $f(u)\cos v$ $z = f(u)\sin v$

$r(u,v)$ $x(u,v)i + y(u,v)j + z(u,v)k$

$\frac{\partial r(u,v)}{\partial u} = r_u = \frac{\partial x}{\partial u} i + \frac{\partial y}{\partial u} j + \frac{\partial z}{\partial u} k$ $\qquad \frac{\partial r}{\partial v} = r_v = \frac{dx}{dv} i \ldots j \ldots k$

Normal vector $= r_u(u_0,v_0) \times r_v(u_0,v_0) = \begin{vmatrix} i & j & k \\ \frac{\partial x}{\partial u} & \frac{dy}{du} & \frac{dz}{du} \\ \frac{dx}{dv} & \frac{dy}{dv} & \frac{dz}{dv} \end{vmatrix}$

$r_u \times r_v =$ upward 

$r_v \times r_u =$ downward

* not a unit vector, $\frac{N}{\|N\|} =$ unit vect.

Tangent plane: $N\langle a,b,c\rangle$, point (x,y,z)

- dot product $a(x_1 - x_1) + b(y - y_1) + c(z - z_1) = 0$

— Area of a parametric surface —

$S = \int_S \int dS = \int_D \int \|r_u \times r_v\| \, dA$ in the uv plane

$= \int_R \int \sqrt{1 + (f_x)^2 + (f_y)^2} \, dA$

14.5 $z = f(x,y)$ $r(x,y) = xi + yj + zk$

$r_x = i + f_x(x,y)k$

$r_y = j + f_y(x,y)k$

$r_x \times r_y \begin{vmatrix} i & j & k \\ 1 & 0 & f_x \\ 0 & 1 & f_y \end{vmatrix} = -f_x i - f_y j + k$

$\|r_x \times r_y\| = \sqrt{1 + (f_x(x,y))^2 + (f_y(x,y))^2}$

2) $u\cos v i + u\sin v j + u k$

$x^2 + y^2 = u^2\cos^2 v + u^2\sin^2 v = u^2$ $z = u$

$z^2 = u^2$ $x^2 + y^2 = z^2$

3) $x = \rho\sin\phi\cos\theta$

$y = \rho\sin\phi\sin\theta$ $x^2 + y^2 + z^2 = 4$

$z = \rho\cos\phi$

7) $r(u,v) = 2\cos u i + v j + 2\sin u k$ $\quad x = 2\cos u$ $z = 2\sin u$ $x^2 + z^2 = 2^2(1)$ $\underline{x^2 + z^2 = 4}$

v is on y, right circular cylinder

9) $s(u,v) = u\cos v i + u\sin v j - u^2 k$ flipped down

10) $s(u,v) = u\cos v i + u^2 j + u\sin v k$ pointed along y-axis

* hyperbolic properties *

15) graph $r = 2\sinh u\cos v i + \sinh u \sin v j + \cosh u k$

$x^2 + 4y^2 = \ldots 4\sinh^2 u(\cos^2 v + \sin^2 v) = 4\sinh u$ $\quad x^2 + y^2 + 4z^2 = 4\sinh^2 u + 4\cosh^2 u =$

$-(x^2 + 4y^2) + 4z^2 = -4\sinh^2 u + 4\cosh^2 u = 4$

 ← me

$\frac{z^2}{1} - \frac{x^2}{4} - \frac{y^2}{1} = 1$

15.4 NOTES

— Green's Theorem —

4) M & N must have continuous partial derivatives in an open region containing R

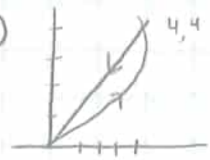
1) simply connected region 3) oriented counterclockwise

2) Piecewise smooth boundary

Then: $\int_C M\,dx + N\,dy = \int_R\int \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$ if $\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = 1$, $\iint$ = area

choose: $M = -\frac{y}{2}$ $N = \frac{x}{2}$ $A = \frac{1}{2}\int_C (x\,dy - y\,dx)$

3)



$y = \frac{x^2}{4}$ a) let $x = t$ $r(t) = \begin{cases} t\,i + \frac{1}{4}t^2; & 0 \le t \le 4 \\ (8-t)i + (8-t); & 4 \le t \le 8 \end{cases}$ $\int_C y^2\,dx + x^2\,dy$ ① $x = t$ $dx = dt$

$y = x$ $y = \frac{1}{4}t^2$ $dy = \frac{1}{4}(2)t\,dt = \frac{1}{2}t\,dt$ $\int_0^4 (\frac{1}{4}t^2)^2\,dt + \cancel{x^2}\, t^2(\frac{1}{2}t\,dt)$

$= \int_0^4 (\frac{1}{16}t^4 + \frac{1}{2}t^3)dt = \frac{1}{16}t^5/5 + \frac{1}{2}t^4/4 \Big]_0^4 = 44.8$

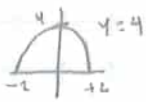
② $x = (8-t)$ $dx = -dt$ $y = (8-t)$ $dy = -dt$ $\int_4^8 (8-t)^2(-dt) + (8-t)^2$ ...... $= \frac{2(8-t)^3}{3}\Big|_4^8 - 42.667$

① + ② = $44.8 - 42.667 = 32/15$ $\int_C = 2.133$ $\int_C = \int_R\int \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$

$\int_C = \int_R\int (2x - 2y)\,dy\,dx = \int_0^4 \int_{\frac{1}{4}x^2}^{x} (2x - 2y)\,dy\,dx = \int_0^4 \left[2xy - y^2\right]_{\frac{1}{4}x^2}^{x} dx = \int_0^4 2x^2 - x^2 - \left[\frac{2x^3}{4} - \frac{x^4}{16}\right] dx$

$= \int_0^4 \left(x^2 - \frac{x^3}{2} + \frac{x^4}{16}\right) dx = \left[\frac{x^3}{3} - \frac{1}{2}\frac{x^4}{4} + \frac{1}{16}\frac{x^5}{5}\right]_0^4 = \boxed{2.133 \text{ (same)}}$

11) $\int_C 2xy\,dx + (x+y)\,dy$



$y = 4 - x^2$ $\frac{\partial M}{\partial y} = 2x$ $\frac{\partial N}{\partial x} = 1$ $= \int_R\int \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA = \int_{-2}^{2}\int_0^{4-x^2} (1 - 2x)\,dy\,dx$

$\int_{-2}^{+2} [y - 2xy]_0^{4-x^2} dx = \int_{-2}^{+2} \left[(4 - x^2) - 2x(4 - x^2)\right] dx = \int_{-2}^{+2} 4 - x^2 - 8x + 2x^3)dx = 4x - \frac{8x^2}{2} - \frac{x^3}{3} + \frac{2x^4}{4}\Big]_{-2}^{2}$

$= \frac{32}{3}$

15) $\int_C 2\arctan\frac{y}{x}\,dx + \ln(x^2 + y^2)\,dy$ C: $x = 4 + 2\cos\theta$ $y = 4 + \sin\theta$ $\frac{\partial M}{\partial y} = (2)\frac{\frac{\partial}{\partial y}(\frac{y}{x})}{1 + (\frac{y}{x})^2} = \frac{2(\frac{1}{x})}{1 + \frac{y^2}{x^2}} = \boxed{\frac{2\frac{1}{x}}{\frac{x^2+y^2}{x^2}}}$

$= \frac{2x}{x^2+y^2}$ $\frac{\partial N}{\partial x} = \frac{1}{x^2+y^2}(2x) = \boxed{\frac{2x}{x^2+y^2}}$ $\boxed{\int_R\int \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\,dA = 0}$

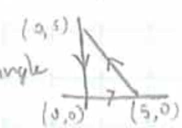
19) $\int_C xy\,dx + (x+y)\,dy$ $x^2 + y^2 = 1$ $x^2 + y^2 = 9$

$\frac{\partial M}{\partial x} = x$ $\frac{\partial N}{\partial y} = 1$ $\int_R\int \left(\frac{\partial M}{\partial x} - \frac{\partial N}{\partial y}\right) dA$

$= \int_R\int (1 - x)\,dA \Rightarrow$ polar $\rightarrow \int_0^{2\pi}\int_1^3 (1 - r\cos\theta)\,r\,dr\,d\theta$ $\iint (r - r^2\cos\theta)\,dr\,d\theta = \int \frac{r^2}{2} - \frac{r^3}{3}\cos\theta\Big|_1^3 d\theta$

$= \int\left(\frac{9}{2} - \frac{27}{3}\cos\theta - \frac{1}{2} - \frac{1}{3}\cos\theta\right) d\theta = \int_0^{2\pi} (4 - \frac{26}{3}\cos\theta)\,d\theta$ $\left[4\theta - \frac{26}{3}\sin\theta\right]_0^{2\pi}$ $(8\pi - 0) - (0 - 0) = \boxed{8\pi}$

23) $F(x,y) = (x^{1/2} - 3y)\,i + (6x + 5\sqrt{y})$; triangle



$\frac{\partial M}{\partial y} = -3$ $\frac{\partial N}{\partial x} = 6$ $\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$

$= \int_R\int 6 - (-3)\,dA = \iint 9\,dA$ $A = \frac{1}{2}bh = \frac{1}{2}5 \cdot 5 = 9\left(\frac{25}{2}\right) = \boxed{\frac{225}{2}}$

15.3 NOTES

a) C- Piecewise smooth curve $r(t) = x(t)i + y(t)j + z(t)k$ $a \le t \le b$

b) $F(x,y,z) = Mi + Nj + Pk$ F is conservative M, N, P are continuous

— thm 15.5 — then c) $\int_C F\cdot dr \int_C \nabla f\cdot dr = f(x(b), y(b)\ z(b)) - f(x(a), y(a)\ z(a))$

11) $F(x,y) = 2xyi + x^2j$ a) $r_1 t = ti + t^2 j$ $0 \le t \le 1$ b) $r_2(t) = ti + t^3 j$ $0 \le t \le 1$

$\frac{\partial M}{\partial y} = 2x$ $\frac{\partial N}{\partial x} = 2x$ $r_1' = i + 2tj$ $F\cdot t = 2(t)(t^2)i + t^2 j = 2t^3 i + t^2 j$ $F\cdot dr = 2t^3 + 2t^3 = 4t^3$

$\int_0^1 4t^3 dt = \frac{4t^4}{4}\Big|_0^1 = \boxed{1}$ $r_2'(t) = i + 3t^2 j$ $F(t) = 2t(t^3)i + t^2 j$ $2t^4 i + t^2 j$ $F\cdot dr = 2t^4 + 3t^4 = 5t^4$

$\int_0^1 5t^4 dt = 5t^5/5\Big|_0^1 = \boxed{1}$ $t=0$ $(0,0)$ $t=1, (1,1)$ $f = x^2y + k$ $[x^2y]_{(0,0)}^{(1,1)} = 1 - 0 = 1$

13) $F(x,y) = yi - xj = Mi + Nj$ $\frac{\partial m}{\partial y} = +1$ $\frac{\partial n}{\partial x} = -1$ NOT CONSERVATIVE a) $r_1(t) = ti + tj$ $0 \le t \le 1$

$r_1'(t) = i + j$ $F(t) = y - x = ti - tj$ $F\cdot dr = t - t = \boxed{0}$ b) $r_2(t) = ti + t^2 j$ $r'(t) = i + 2tj$

$F(t) = t^2 i - tj$ $F\cdot dr = t^2 - 2t^2 = -t^2$ $\int_0^1 -t^2 dt = -t^3/3\Big|_0^1 = -\frac{1}{3}$ c) $r_3(t) = ti + t^3 j$ $0 \le t \le 1$

$r'(t) = i + 3t^2 j$ $F(t) = t^3 i - tj$ $F\cdot dr = t^3 - 3t^3 = -2t^3$ $\int_0^1 -2t^3 dt = -\frac{2t^4}{4}\Big|_0^1 = -1/2$

15) $\int_C y^2dx + 2xydy$ $F(x,y) = y^2 i + 2xyj$ $\frac{\partial m}{\partial y} = 2y$ $\frac{\partial N}{\partial x} = 2y$ CONSERVATIVE

RECOVER POTENTIAL FUNCTION $\int y^2 dx = y^2x + g(y)$ $\int 2xydy = y^2x + h(x)$ $f(x,y) = y^2x + k$

$y^2x\Big|_{0,0}^{4,4} = 4^2(4) - 0 = \boxed{64}$

19) $f(x\ y\ z) = yzi + xzj + xyk$ curl $F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz & xy \end{vmatrix} = i(x-x) - j(y-y) + k(z-z)$

$\int yzdx = xyz + g(y,z)$ $\int xz\, dy = xyz + h(x,z)$ $\int xy\, dz = xyz + J(h,y)$

$\boxed{f(x,y,z) = xyz + k}$ a) $r_1(t) = ti + 2j + tk$ $0 \le t \le 4$ $t=0\ (0,2,0)$ $t=4\ (4,2,4)$ $[xyz]_{(0,2,0)}^{(4,2,4)} = \boxed{32 - 0}$

b) $r_2 t = t^2 i + tj + t^2 k$ $0 \le t \le 2$ $t=0 \Rightarrow (0,0,0)$ $t=2$ $(4,2,4)$ $[xyz]_{(0,0,0)}^{(4,2,4)} = \boxed{32 - 0}$

29) $\int_C e^x\sin y dx + e^x\cos y dy$ $\frac{\partial m}{\partial y} = e^x\cos y$ $\frac{\partial m}{\partial x}$ $e^x\cos y$ CONSERVATIVE $f_x = e^x\sin y$ $f_y = e^x\cos y$

$\int e^x\sin y dx = e^x\sin y + g(y)$ $\int e^x\cos y dy = e^x\sin y + h(x)$ $f(x,y) = e^x\sin y + k$

Cycloid $x = \theta - \sin\theta$ $y = 1 - \cos\theta$ $(0,0)$ $(2\pi, 0)$ $e^x\sin y\Big|_{0,0}^{2\pi,0} = e^{2\pi}\sin 0 - e^0\sin 0 = \boxed{0}$

35) Work $F(x,y) = 9x^2y^2 i + (6x^3y - 1)j$ $\frac{\partial m}{\partial y} = 18x^2y$ $\frac{\partial N}{\partial x}$ $18x^2y$ conservative

$\int 9x^2y^2 dx = \frac{9}{3}x^3y^2 = 3x^3y^2 + g(y)$ $\int(6x^3y - 1)dy = \frac{6x^3y^2}{2} - y + h(x)$ $f(x,y) = 3x^3y^2 - y + k$

$P(0,0)$ $Q = (5,9)$ $[3x^3y^2 - y]_{(0,0)}^{(5,9)} = [(3)(5)^3(9)^2 - 9 - 0(0)] = \boxed{30,366}$

15.2 homework 43-59 odds

43) $F(x,y) = x^2 i + xy j$ $M = x^2$ $N = xy$ $x = 2t$ $y = (t-1)$ $dx = 2dt$ $dy = 1dt$

a) $r_1(t) = 2t i + (t-1) j$ $1 \le t \le 3$ $\int F \cdot dr = \int (2t)^2 (2) + (2t)(t-1)$ dt

$= \int_1^3 8t^2 + 2t^2 - 2t$ $\int_1^3 10t^2 - 2t$ $= \frac{10t^3}{3} - \frac{2t^2}{2} \Big|_1^3$ $\frac{270}{3} - \frac{10}{3} - \frac{18}{2} + 1 = \frac{236}{3}$

b $r_2(t) = \ldots$ both paths join (2,0) & (6,2), opposite directions

45) $\int_c F \cdot dr = 0$ 45) $F(x,y) = yi - xj$ C: $r(t) = ti - 2tj$ $M = y$ $N = -x$ $x = t$ $y = -2t$ $dx = 1dt$ $dy = -2dt$

$\int_0^1 [-2t(1) + -t(-2)] dt = \int_0^1 -2t + 2t \; dt = \underline{\int 0 dt}$

49) C: $x = 2t$ $y = 10t$ $0 \le t \le 1$ $\int_c (x + 3y^2) dy$ $dy = 10dt$ $\int_0^1 [2t + 3(10t)^2] 10dt$

$= 10\int_0^1 (2t + 300t^2) dt = 10 \left[t^2 + \frac{300t^3}{3} \right]_0^1 = 10[101] = \boxed{1010}$

51) $\int_c xy dx + y dy = \int_0^1 20t^2 (2dt) + 10t(10dt) = \int_0^1 (40t^2 + 100t) dt = \frac{40t^3}{3} + \frac{100t^2}{2} \Big|_0^1$

$= \frac{40}{3} + 50 = \boxed{\frac{190}{3}}$

53) $\int_c (2x-y) dx + (x+3y) dy$ C: axis (0,0) to (0,5) $x = t$ $y = 0$ $0 \le t \le 5$ $dx = 1dt$ $dy = 0$

$\int_0^5 (2t - 0) 1dt + (\quad) 0 = \int_0^5 2t dt = \frac{2t^2}{2} = \boxed{25}$

57) C: $y = 1 - x^2$ (0,1) to (1,0) $y = 1 - t^2$ $x = t$ $0 \le t \le 1$ $dy = -2t\,dt$ $dx = 1dt$

$\int_0^1 2t - (1-t^2) dt + (t + 3 - 3t^2) - 2t dt = \int_0^1 (2t - 1 + t^2 - 2t^2 - 6t + 6t^3) dt$

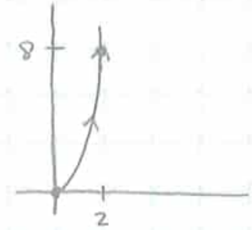
$\int_0^1 (6t^3 - t^2 - 4t - 1) dt = \frac{6t^4}{4} - \frac{t^3}{3} - \frac{4t^2}{2} - t \Big|_0^1 = \frac{6}{4} - \frac{1}{3} - 2 - 1 = \frac{3}{2} - \frac{1}{3} - 3 = \frac{9 - 2 - 18}{6} = \boxed{-\frac{11}{6}}$

59) $x = t$, $y = 2t^2$ (0,0) to (2,8) $0 \le t \le 2$ $dx = 1dt$ $dy = 4t dt$

$\int_0^2 (2t - 2t^2) dt + (t + 6t^2) 4t dt = \ldots \boxed{\frac{316}{3}}$

15.2) homework 35-59 odds

35) $F(x,y) = -xi - 2yj$ C: $y = x^3$ (0,0) to (2,8)



$y = t^3$ $x = t$ $0 \le t \le 2$ $M = -x$ $N = -2y$

$dy = 3t^2$ $dx = 1$

$\int_c F \cdot dr = \int_c M\,dx + N\,dy$

$= \int_0^2 -x\,dx + -2y\,dy$

$= \int_0^2 [-t(1) + (-2t^3)(3t^2)]\,dt$

$= \int_0^2 -6t^5 - t\ dt$

$= -\frac{6}{6}t^6 - \frac{t^2}{2} \Big|_0^2$

$= -64 - \frac{4}{2} = \boxed{-66}$

37 parametrization must be rewritten so that t is continuous

$r_1 =$ $0 \le t \le 1$

$r_2 =$ $1 \le t \le 2$

$r_3 =$ $2 \le t \le 3$

37) $F(x,y) = 2xi + yj$ clwise

$M = 2x$ $N = y$

C_1 $r = ti$

$x = t$ $y = 0$ $dx = 1\,dt$

$\int_{C_1} F \cdot dr = \int_0^1 2t(1)\,dt =$

$t^2 \big|_0^1 = \boxed{1}$

$C_2 = r = 1i + tj$ $0 \le t \le 1$ $x' = 0$ $y' = 1$

$\int_{C_2} F\,dr = \int_0^1 2(0) + t(1)\,dt$ $+ \frac{t^2}{2}\Big|_0^1 = \boxed{\frac{1}{2}}$

$C_3 = r = (1-t)i\ \ (1-t)j$ $0 \le t \le 1$ $x' = -1$ $y' = -1$

$\int_{C_3} F \cdot dr = \int_0^1 2(1-t)(-1\,dt) + (1-t)(-1\,dt)$

$= \int_0^1 -4(1-t)\,dt = -4\left[t - \frac{t^2}{2}\right]_0^1$

$= -4\left[1 - \frac{1}{2}\right] = \boxed{-2}$

$\int_0^1 -2(1-t) + (1-t)\ dt$

$= -2\int_0^1 (2-2t)\,dt$

$= -2\int_0^1 2t - \cancel{2}\frac{t^2}{\cancel{2}}$

$= -2\int_0^1 2 - 1$

39) $F(x,y,z) = xi + yj - 5zk$

$C: r(t) = 2\cos t\,i + 2\sin t\,j + tk$ $0 \le t \le 2\pi$

$M = x$ $N = y$ $P = -5z$

$x = 2\cos t$ $y = 2\sin t$ $z = t$

$dx = -2\sin t\,dt$ $dy = 2\cos t\,dt$ $dz = 1\,dt$

$\int_0^{2\pi} [2\cos t(-2\sin t) + 2\sin t(2\cos t) + -5t]\,dt$

$\int_0^{2\pi} [-\cos t\sin t(4) + \cos t\sin t(4) - 5t]\,dt$

$= \left[-5\frac{t^2}{2}\right]_0^{2\pi} = -\frac{5}{2}(4\pi^2) = \boxed{-10\pi^2}$

41) $F(x,y,z) = -150zk$ $= -150$ $dz = 0$

$C: r = 3\cos t\,i + 3\sin t\,j + \frac{5}{\pi}tk$ $0 \le t \le 2\pi$

$M = 0$ $N = 0$ $P = -150z$

$x = 3\cos t\,i$ $y = 3\sin t\,j$ $z = \frac{5}{\pi}t$

dx dy $dz = \frac{5}{\pi}\,dt$

$\int_c M\,dx + N\,dy + P\,dz = \int_0^{2\pi} -150\left(\frac{5}{\pi}\right)dt$

$= -150\left(\frac{5}{\pi}\right)(2\pi) = \boxed{1500\ \text{ft} \cdot \text{lb}}$

15.2 homework 19-31 odds 35-59 odds

19) $\int_c (2x+y^2-z)\,ds$ $\quad r = \begin{cases} ti + 0j + 0k & 0 \le t \le 1 & r' = \sqrt{1} \\ i \quad 0j + tk & 0 \le t \le 1 & = \sqrt{1} \\ i + \quad tj + k & 0 \le t \le 1 & = \sqrt{1} \end{cases}$

$\int_{c_1} = \int_0^1 2t(1)\,dt = \frac{2t^2}{2} = \boxed{1}$

$\int_{c2} = \int_0^1 2i - tk\ dt = 2t - \frac{t^2}{2}\Big|_0^1 = 2 - \frac{1}{2} = \boxed{3/2}$ $\qquad \frac{6+9+3}{6} = \boxed{\frac{23}{6}}$

$\int_{c3} = \int_0^1 2\cancel{t} + t^2 - 1\ dt = \int_0^1 t^2 + 1\,(dt) = \frac{t^3}{3} + t\Big|_0^1 = \frac{1}{3} + 1 = \boxed{\frac{4}{3}}$

23) $r(t) = \cos t\,i + \sin t\,j$ $\quad P(x,y) = x + y \quad 0 \le t \le \pi$

$r'(t) = -\sin t\,i + \cos t\,j$ $\quad \int_0^\pi \cos t + \sin t\ (1)\,dt = \sin t - \cos t\Big|_0^\pi = [0 - -1] - [0 - 1]$

$\|r'(t)\| = \sqrt{1}$

$= 1 + 1 = \boxed{2}$

25) $r(t) = t^2 i + 2t j + tk$ $\quad P(x,y,z) = kz\ (k > 0) \quad 1 \le t \le 3$

$r'(t) = 2t\,i + 2j + k$

$\|r'\| = \sqrt{4t^2 + 4 + 1} = \sqrt{4t^2+5}$ $\quad mass = \int_c P(x,y,z)\,ds = \int_1^3 kz\,ds = \int_1^3 kt\sqrt{4t^2+5}\ dt$

$u = \sqrt{4t^2+5}\quad du = 8t$ $\quad = \frac{1}{8}k\int_1^3 8t\sqrt{4t^2+5}\ dt = \frac{k(4t^2+5)^{3/2}}{8}\left(\frac{2}{3}\right)\Big|_1^3$

$= \frac{k(4t^2+5)^{3/2}}{12}\Big|_1^3 = \frac{k}{12}\left[(36+5)^{3/2} - (9)^{3/2}\right]$

27) $\int_C F \cdot dr$ $\quad F(x,y) = xy\,i + y\,j$ $\quad C: r(t) = 4t\,i + t\,j \quad 0 \le t \le 1 \quad r'(t) = 4i + j$

$x = 4t \quad y = t$

$\int 4t^2 i + t j$

$\int_c F \cdot dr = \int_0^1 (16t^2 + t)\,dt = \frac{16t^3}{3} + \frac{t^2}{2}\Big|_0^1 = \frac{16}{3} + \frac{1}{2} =$

$F(t) \cdot r'(t)$

29) $F(x,y) = 3x\,i + 4y\,j$ $\quad C: r(t) = 2\cos t\,i + 2\sin t\,j \quad 0 \le t \le \frac{\pi}{2}$

$r'(t) = -2\sin t\,i + 2\cos t\,j$

$[3(2\cos t)i + 4(2\sin t)](-2\sin t\,i + 2\cos t\,j)$

$\int_0^{\pi/2} -12\cos t\sin t + 16\sin t\cos t) = \int_0^{\pi/2} 4\sin t\cos t\ dt \quad 2\int_0^{\pi/2} \sin 2t\ dt = \frac{2}{2}\left[-\cos 2t\right]_0^{\pi/2}$

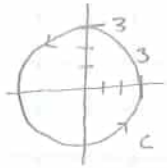
$= [-\cos\pi + \cos 0] = [- -1 + 1] = \boxed{2}$

31) $F(x,y,z) = (x^2y)i + (x-z)j + (xyz)k$ $\quad C: r(t) = ti + t^2 j + 2k \quad 0 \le t \le 1 \quad 1i + 2tj + 0k$

$\int_0^1 t^4 i + (t-2)j + (2t^3)k \cdot [1, 2t, 0]$

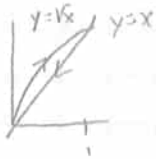
$\int_0^1 (t^4 + 2t^2 - 4t)\,dt = \frac{t^5}{5} + \frac{2t^3}{3} - \frac{4t^2}{2}\Big|_0^1 = \frac{1}{5} + \frac{2}{3} - \frac{4}{2} = \frac{3}{15} + \frac{10 - 30}{} = \boxed{\frac{-17}{15}}$

15.2 homework   1-31 odds, 35-59 odds

1) 

$x^2+y^2=9$ parametric form $r(t) = x(t)i + y(t)j$

$y = 3\cos t \quad x = 3\sin t \qquad r(t) = 3\cos t(i) + 3\sin t(j) \quad 0 \le t \le 2\pi$

5) 

$r(t) = \begin{cases} ti + \sqrt{t}j & 0 \le t \le 1 \\ (2-t)i + (2-t)j, & 1 \le t \le 2 \end{cases}$

$t=1 \quad x=1 \ y=1$
$t=2 \quad x=0, y=0$

9) $\int_C (x^2+y^2+z^2)\,ds \quad C: r(t) = \sin t\, i + \cos t\, j + 8t\, k \qquad 0 \le t \le \pi/2 \qquad r'(t) = \cos t\, i - \sin t\, j + 8k$

evaluate $\int_0^{\pi/2} \sin^2 t\, i + \cos^2 t\, j + 64t^2 k$

$\|r'(t)\| = \sqrt{(\cos^2 t) + (\sin^2 t) + 64} = \sqrt{1+64}$

$\int_0^{\pi/2} (1 + 64t^2)(\sqrt{65})\,dt = \sqrt{65}t + \frac{64\sqrt{65}t^3}{3} \Big|_0^{\pi/2} = \frac{\pi}{2}\sqrt{65} + \frac{64\sqrt{65}\pi^3}{3 \cdot 8}$

$\boxed{= \sqrt{65}\left(\frac{\pi}{2} + \frac{8\pi^3}{3}\right)}$

11) $\int_C x^2+y^2\,ds \qquad$ x-axis from $x=0$ to $x=3 \qquad r(t) = ti \quad 0 \le t \le 3 \quad r't = i + 0k$

$\|r'\| = \sqrt{1^2+0^2} \quad r'(t) = 1$

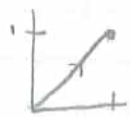
$= \int_0^3 (t^2+0)\sqrt{1+0}\,dt = \frac{t^3}{3}\Big|_0^3 = \frac{27}{3} = 9$

13) C:  $x^2+y^2=1 \qquad r(t) = \cos t\, i + \sin t\, j, \qquad r't = \sin t\, i - \cos t\, j$

$0 \le t \le \pi/2 \qquad \|r'(t)\| = 1$

$\int_0^{\pi/2} \cos^2 t\, i + \sin^2 t\, j\,(1)\,dt =$

$= \int_0^{\pi/2} (1)(1)\,dt = \boxed{\pi/2}$

15) $\underbrace{\int_C x + 4\sqrt{y}\,ds}_{\text{the field}} \quad \underbrace{C: \text{line from } (0,0) \text{ to } (1,1)}_{\text{path}}$ 

$x = t$
$y = t$

$r = ti + tj \quad 0 \le t \le 1$

$r' = i + j \quad \|r'\| = \sqrt{2}$

$\int_0^1 (t + 4\sqrt{t})(\sqrt{2})\,dt = \sqrt{2}\left[\frac{1^2}{2} + \frac{4(1)^{3/2}}{}\left(\frac{2}{3}\right)\right] = \frac{\sqrt{2}}{2} + \frac{8\sqrt{2}}{3} = \frac{19\sqrt{2}}{6}$

17) $\int_C x + 4\sqrt{y}\,ds$ 

$r = \begin{cases} ti & 0 \le t \le 1 \\ (2-t)i + (t-1)j & 1 \le t \le 2 \\ (3-t)j & 2 \le t \le 3 \end{cases} \qquad \|r'(t)\| = \begin{cases} \sqrt{1^2} & 0 \le t \le 1 \\ \sqrt{2} \\ \sqrt{-1^2} \end{cases}$

$\int_{C_1} = \int_0^1 t\,dt = \boxed{1/2}$

$\int_{C_2} = \int_1^2 \left[(2-t) + 4\sqrt{t-1}\right]\sqrt{2}\,dt = \left[2t - \frac{t^2}{2} + 4(t-1)^{3/2}\left(\frac{2}{3}\right)\right]_1^2 \sqrt{2} \qquad \frac{14\sqrt{2}}{3} - \frac{3\sqrt{2}}{2} \qquad \boxed{\frac{19\sqrt{2}}{6}}$

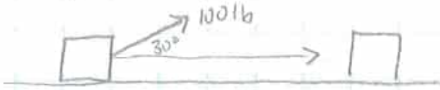
$\int_{C_3} = \int_2^3 0 + 4(3-t)^{1/2}\,dt = -4\left[(3-t)^{3/2}\frac{2}{3}\right]_2^3 = -\frac{8}{3}\left[0^{3/2} - 1^{3/2}\right] = \boxed{+\frac{8}{3}}$

$\int_C x + 4\sqrt{y}\,ds = \frac{19(1+\sqrt{2})}{6}$

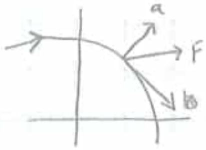
15.2 Notes

line integrals are single integrals so there must be only 1 var

Line integrals over vector fields $\quad W = F \times D$



$W = 100\,lb \cos 30° (10\,ft) = 866\ Ft \cdot lb$



$r(t) \quad r'(t) \quad \frac{r'(t)}{\|r'(t)\|} = T \qquad F \cdot T$ is tangential force

$W = \int_c F(x,y,z) \cdot T(x,y,z)\, dS$

$F \cdot T\, dS = F \cdot \underbrace{\frac{r'(t)}{\|r'(t)\|}}_{\text{unit tangent vector}} \|r'(t)\|\, dt = F \cdot r'(t)\, dt = F \cdot dr$

abs. value $\quad$ magnitude

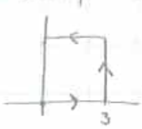
$|r| \quad \|r\|$

Line integrals in different Form $\quad F(x,y) = Mi + Nj \quad$ OR $\quad F(x,y,z) = Mi + Nj + Pk$

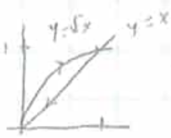
$\int_c F \cdot dr = \int F \cdot \frac{dr}{dt}\, dt = \int_a^b (Mi + Nj + Pk) \cdot (x'(t) + y'(t) + z'(t))\, dt$

$= \int_a^b \left(M \frac{dx}{dt} + N \frac{dy}{dt} + P \frac{dz}{dt} \right) dt = \int_a^b M\,dx + N\,dy + P\,dz$

1) $x^2 + y^2 = 9 \quad y = 3\cos t \quad x = 3\sin t \quad r(t) = 3\cos t\, i + 3\sin t\, j \qquad 0 \le t \le 2\pi$

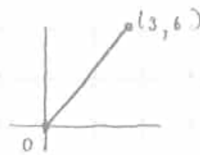
3) 

$r(t) = \begin{cases} ti + 0j & 0 \le t \le 3 \\ 3i + (t-3)j & 3 \le t \le 6 \\ (9-t)i + 3j & 6 \le t \le 9 \\ 0i + (12-t)j & 9 \le t \le 12 \end{cases}$

5) 

$r(t) = \begin{cases} ti + \sqrt{t}\, j & 0 \le t \le 1 \\ (2-t)i + (2-t)j & 1 \le t \le 2 \end{cases}$

7) $\int_c (x-y)\, dS \qquad C: r(t) = 4ti + 3tj \qquad 0 \le t \le 2$



$r'(t) = 4i + 3j$

$\|r'(t)\| = \sqrt{x'^2 + y'^2} = \sqrt{25} = 5$

$\int_c (x-y)\, ds = \int_0^2 (4t - 3t) 5\, dt = \int_0^2 5t\, dt = 10$

9) $\int_c (x^2 + y^2 + z^2)\, ds \qquad C: \overset{i}{\sin t} + \overset{j}{\cos t} + \overset{k}{8t} \qquad 0 \le t \le \pi/2 \qquad r'(t) = \cos t\, i - \sin t\, j + 8k$

$\sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2} = \sqrt{\cos^2 t + \sin^2 t + 64} = \sqrt{65}$

$= \int_0^{\pi/2} (\underbrace{\sin^2 t + \cos^2 t}_{(1)} + 64t^2) \sqrt{65}\, dt = \int_0^{\pi/2} (1 + 64t^2)\sqrt{65}\, dt = \sqrt{65}\left[\frac{\pi}{2} + \frac{64}{3}\left(\frac{\pi^3}{8}\right) \right]$

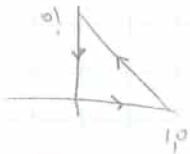
13) C: 

$r(t) = \cos t\, i + \sin t\, j \quad 0 \le t \le \pi/2 \quad r' = -\sin t\, i + \cos t\, j \quad \|r'(t)\| = \ldots 1$

$\int_0^{\pi/2} (\cos^2 t + \sin^2 t)(1)\, dt = \boxed{\frac{\pi}{2}}$

15.2 | Notes

17) $\int (x+4\sqrt{y})\,ds$



$r(t): \begin{cases} t\,i & 0 \le t \le 1 \\ (2-t)i + (t-1)j & 1-2 \\ (3-t)j & 2-3 \end{cases}$

① $\int_c (x+4\sqrt{y})\,ds$ $\sqrt{\quad} = 1$ $\int_0^1 (t+0)(1)\,dt = \frac{1^2}{2}$

② $\int_{c2} x+4\sqrt{y}\,dS$ $\sqrt{(x'(t))^2+(y'(t))^2} = \sqrt{-1^2+1^2} = \sqrt{2}$

$\int_1^2 (2-t)+4\sqrt{t-1}\,\sqrt{2}\,dT = \sqrt{2}\left[2t - \frac{t^2}{2} + 4(t-1)^{3/2}\frac{2}{3}\right]_1^2 = \ldots\ \sqrt{2}\left(\frac{19}{6}\right)$

③ $\int_{c3} (x+4\sqrt{y})\,dS$ $\sqrt{\quad} = 1$ $-\int_2^3 (0+4\sqrt{3-t})(-)\,dt = \left[-4(3-t)^{3/2}\frac{2}{3}\right]_2^3 = 8/3$

$\int_c = \Sigma\left(\frac{1}{2} + \sqrt{2}\,\frac{19}{6} + 8/3\right)$

25) $r(t) = t^2 i + 2t j + t k$ $\quad \rho(xyz) = kz \quad 1 \le t \le 3$

$r'(t) = 2t i + 2j + k \qquad \|r'(t)\| = \sqrt{4t^2+4+1} = \sqrt{4t^2+5}$

$m = \int_1^3 \rho\,dS = \int_1^3 (kz)\sqrt{4t^2+5}\,dt = \int \frac{k}{8}\sqrt{4t^2+5}\,(8)t\,dt = \frac{k}{8}(4t^2+5)^{3/2}\frac{2}{3}\Big]_1^3$

$= k/12\,[41\sqrt{41} - 27]$

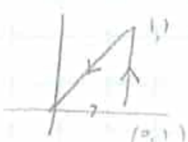
31) Evaluate $\int_c F\cdot dr$

$F(xyz) = x^2y\,i + (x-z)j + (xyz)k \qquad C: r(t) = t i + t^2 j + 2k \quad 0 \le t \le 1$

$r'(t) = i + 2tj + 0k \qquad F(x(t), y(t), z(t)): t^2 t^2 i\ (t-2)j + (t)(t^2)(2)k = t^4 i + (t-2)j + 2t^3 k$

$F\cdot dr = t^4 + 2t^2 - 4t + 0 \qquad \int_0^1 (t^4 + 2t^2 - 4t)\,dt = \frac{t^5}{5} + \frac{2t^3}{3} - \frac{4t^2}{2}\Big|_0^1 = -17/15$

37) find W $\quad f(x,y) =$



$r(t) = \begin{cases} t\,i & 0 \le t \le 1 \\ i + (t-1)j & 1 \le t \le 2 \\ (3-t)i + (3-t)j & 2 \le t \le 3 \end{cases}$

work: $\int_c F\cdot dr$

① C_1 ... $r'(t) = i$

$F(x(t), y(t)) = 2t\,i + 0$

$F\cdot r'(t) = 2t$

$W \int_0^1 2t\,dt = t^2\Big|_0^1 = \boxed{+1}$

② $r' = 2$ $\quad F(x(t), y(t)) = 2(1)i + (t-1)j \qquad F\cdot r'(t) = t-1 \qquad W = \int_1^2 (t-1)\,dt = \frac{t^2}{2} - t\Big|_1^2 = \boxed{+\frac{1}{2}}$

③ C_3 $\quad r(t) = (3-t)i + (3-t)j \qquad r'(t) = -i - j \qquad F\ x(t), y(t) = 2(3-t)i + (3-t)j$

$F\cdot r'(t) = -2(3-t) + (3-t)j = -9+3t \qquad$ Work: $\int_2^3 (-9+3t)\,dt = \ldots\ \boxed{-3/2}$

$\text{Work}_{(tot)} = 1 + \frac{1}{2} - 3/2 = 0$

51) $\int_c xy\,dx + y\,dy \quad x = 2t\ dx = 2dt \quad y = 10t\ dy = 10dt \quad 0 \le t \le 1 \quad x = \frac{1}{5}y\ dx = \frac{1}{5}dy \quad y = 5x\ dy = 5dx$

$0 \le x \le 2 \quad \int_0^2 (x)(5x)\,dx + 5x(5dx) = \int_0^2 (5x^2 + 25x)\,dx = \frac{5x^3}{3} + \frac{25x^2}{2}\Big|_0^2 = 190/3$

$\int_0^1 (2t)(10t)(2dt) + (10t)(10dt) = \int (40t^2 + 100t)\,dt = \frac{40t^3}{3} + \frac{100t^2}{2}\Big|_0^1 = 190/3$

15.1 39-75 odds

39) $F(x,y) = \overset{M}{\frac{x}{x^2+y^2}} i + \overset{N}{\frac{y}{x^2+y^2}} j$

$\frac{\partial N}{\partial x} = \frac{0(x^2+y^2) - y(2x)}{(x^2+y^2)^2}$

Quotient Rule

$f(x) = \frac{g(x)}{h(x)}$

$f'(x) = \frac{g' x hx - g x h' x}{h x^2}$

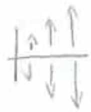
$\frac{\partial N}{\partial x} = \frac{-2xy}{(x^2+y^2)^2}$ $\frac{\partial M}{\partial y} = \frac{-2xy}{(x^2+y^2)^2}$ CONSERVATIVE

$\int M dx = \frac{1}{2}\int \frac{2x}{x^2+y^2} dx = \frac{1}{2} \ln(x^2+y^2) +$

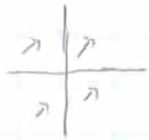
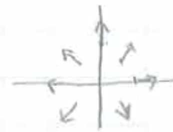
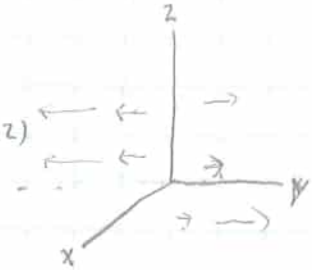
$\int N dy = \frac{1}{2} \ln(x^2+y^2) + g(x)$

$\boxed{f(x,y) = \frac{1}{2} \ln(x^2+y^2) + K}$

15.1 | 1-15 odds 21-75 odds homework

1) $F(x,y) = xj$ $\boxed{c}$  3) $F(x,y) = xi + 3y j$ $\boxed{b}$

5) $F = \langle x, \sin y \rangle$ $\boxed{a}$

7) $f(x,y) = i + j$  9) $xi + yj$  $f(x,y,z)$ 11) $3y j$ 

21) $f(x,y) = 5x^2 + 3xy + 10y^2$ $\quad \nabla f = f_x i + f_y j = (10x+3y)i + (3x+20y)j$

$f_x = 10x + 3y \quad f_y = 3x + 20y$

23) $f(x,y,z) = z - ye^{x^2}$ $\quad \underline{f_x = -2xye^{x^2}}$ $\quad \underline{f_y = -e^{x^2}}$ $\quad \underline{f_z = 1}$ $\rightarrow (\ )i\ (\ )j\ (\ )k$

25) $g(x,y,z) = xy \ln(x+y)$ $\quad g_x = y\ln(x+y) + xy\left(\frac{1}{x+y}\right)$ $\quad g_y = x\ln(x+y) + \frac{xy}{x+y}$ $\quad g_z = 0$

$G(x,y,z) = \left[y\ln(x+y) + \frac{xy}{x+y}\right]i + \left[x\ln(x+y) + \frac{xy}{x+y}\right]j$

27) $F(x,y) = 12xyi + 6(x^2+y)j$ $\quad \frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}$ $\quad 6x^2 + 6y \rightarrow 12x = 12x$ $\boxed{\text{conservative}}$

29) $F(x,y) = \sin y i + x\cos y j$ $\quad \frac{\partial N}{\partial x} = \cos y$ $\quad \frac{\partial M}{\partial y} = \cos y$ $\boxed{\text{CONSERVATIVE}}$

31) $F(x,y) = 5y^2(3yi - xj) = \underbrace{15y^3 i}_{M} - \underbrace{5xy^2 j}_{N}$ $\quad \frac{\partial N}{\partial x} = -5y^2$ $\quad \frac{\partial M}{\partial y} = 45y^2$ $\boxed{\text{Not conserv.}}$

33) $F(x,y) = \frac{2}{y^2}e^{2x/y}(yi - xj)$ $\quad \left(\frac{2}{y}e^{\frac{2x}{y}}\right)i\ \left(\frac{-2x}{y^2}e^{2x/y}\right)j$ $\quad \frac{\partial N}{\partial x} = \frac{-2}{y^2}\left(e^{2x/y}\right) + \frac{2}{y}\left(e^{2x/y}\right)\left(\frac{-2x}{y^2}\right)$

$\frac{\partial M}{\partial y} = \frac{-2y}{y^3}\ \frac{-4y}{y^3}\left(e^{2x/y}\right)$ $\quad = -2\left(\frac{e^{2x/y}}{y^2}\right) + \frac{-4x}{y}\left(\frac{e^{2x/y}}{y^2}\right)$

$\frac{-2}{y^2}e^{2x/y} \ \Big|\ \frac{-4x}{y^3}e^{2x/y}$ $\quad -\left(\frac{2y+4x}{y}\right)\left(\frac{e^{2x/y}}{y^2}\right)$

$\frac{2}{y}\left(\frac{-2x}{y^2}\right)e^{2x/y}$ $\quad = \frac{-2(y+2x)}{y^3}e^{2x/y}$

$\boxed{\text{conservative}}$

35) $M = 2xy \quad N = x^2$

$\frac{\partial x}{\partial N} = 2x \quad \frac{\partial y}{\partial x} = 2x$ $\boxed{\text{yes,}}$ $\boxed{f = x^2y + k}$

$\nabla f(x,y) = 2xyi + x^2 j$ $\quad f_x(x,y) = 2xy \quad f_y(x,y) = x^2$ $\quad \boxed{\begin{array}{l}\int f_x(x,y)\,dx = x^2y + g(y) \\ \int f_y(x,y)\,dy = x^2y + h(x)\end{array}}$

37) $F(x,y) = xe^{x^2y}(2yi + xj)$ $\quad M = 2yxe^{x^2y} \quad N = x^2e^{x^2y}$

$\frac{\partial N}{\partial x} = 2xe^{x^2y} + 2xye^{x^2y}(x^2) = 2x + 2x^3y(e^{x^2y})$

$\frac{\partial M}{\partial y} = 2xe^{x^2y} + 2yx(x^2)(e^{x^2y})$ $\boxed{\text{conservative}}$ $\quad \int M\,dx = \int 2yxe^{x^2y}\,dx = \boxed{e^{x^2y} + c}$

$\int e^u\,du = e^u + c \quad u = x^2y \quad du = 2xy$

$\boxed{f(x,y) = e^{x^2y} + k}$ $\quad \int N\,dy \quad \int x^2e^{x^2y}\,dy = \boxed{e^{x^2y} + c}$

15.1 Vector fields:

- vector field over Q

$F(x,y,z)$ (Space) $= Mi + Nj + Pk$ M, N, P are functions

- gradient $\nabla f(x,y,z)$

$= f_x(x,y,z)i + f_y(x,y,z)j + f_z(x,y,z)k$

- inverse square field $F(x,y,z) = \frac{k}{\|r\|^2}u$ and $r(t) = x(t)i + y(t)j + z(t)k$

$u = r/\|r\|$, a unit vector in r direction

- F is CONSERVATIVE if there exists a differentiable function f sutch that $F = \nabla f$
  - $F(x,y) = Mi + Nj$ is conservative if $\frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}$ [Plane]
  - $F(x,y,z)$ is conserv. only if $\frac{\partial P}{\partial y} = \frac{\partial N}{\partial z}$, $\frac{\partial P}{\partial x} = \frac{\partial M}{\partial z}$, & $\frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}$

15.1 Vector Fields Notes

1. $F(x,y) = Mi + Nj$ (Plane) $\quad$ M, N, P are functions of (x,y,z)
2. $F(x,y,z) = Mi + Nj + Pk$ (space)

Gradient $f(x,y,z) = \nabla f(x,y,z) = f_x i + f_y j + f_z k$

1) $f(x,y) = xj$ $\quad$ c $\quad$ 2) pointed in x dir, magnitude of y component

3) $F(x,y) = xi + 3yj$ $\quad \|F\| = \sqrt{x^2+(3y)^2}$ $\quad \sqrt{x^2+9y^2} = c$

$x^2 + 9y^2 = c^2$ $\quad c=3,$ $\quad \frac{x^2}{9} + \frac{y^2}{1} = 1$ $\quad \boxed{b}$

5) $f(x,y) = \langle x, \sin y \rangle$ $\quad (x,y) = (2,0)$ $\quad f(x,y) = \langle 2,0 \rangle$ $\quad (2, -\frac{\pi}{2}) \Rightarrow \langle 2,-1 \rangle$ $\quad \boxed{a}$

$(2, \pi/2) \Rightarrow \langle 2,1 \rangle$ $\quad (2,\pi) \Rightarrow \langle 2,0 \rangle$

25) $g(x,y,z) = xy\ln(x+y)$ $\quad f_x = (xy)\left(\frac{1}{x+y}\right) + \ln(x+y)(y)$ $\quad f_z = 0$

$f_y = ($ " " " (x)

$G(x,y,z) = \nabla g(x,y,z) = f_x i + f_y j$

33) $F(x,y) = \frac{2}{y^2} e^{\frac{2x}{y}} (yi - xj) = e^{\frac{2x}{y}}\left(\frac{2}{y}\right)i - e^{\frac{2x}{y}}\left(\frac{2x}{y^2}\right)j$

$\frac{\partial M}{\partial y} = e^{2x/y}\left(\frac{2}{-y^2}\right) + \frac{2}{y}(e^{2x/y})\left(\frac{2x}{-y^2}\right) = e^{2x/y}\left(-\frac{2}{y^2} - \frac{4x}{y^3}\right) = e^{2x/y}\left(\frac{-2y-4x}{y^3}\right)$

$\frac{\partial N}{\partial x} = e^{2x/y}\left(\frac{-2}{y^2}\right) + \left(\frac{-2x}{y^2}\right)(e^{2x/y})\left(\frac{2}{y}\right) = e^{2x/y}\left(-\frac{2}{y^2} - \frac{4x}{y^3}\right) = e^{2x/y}\left(\frac{-2y-4x}{y^3}\right)$

conservative. $\quad$ 1. $f(x,y) = \int e^{2x/y}\left(\frac{2}{y}\right)dx = e^{2x/y} + g(y)$ $\quad f(x,y) = e^{2x/y} + K$

2. $f(x,y) = \int e^{2x/y}\left(\frac{-2x}{y^2}\right)dy = e^{2x/y} + h(x)$

35) $F(x,y) = 2xyi + x^2 j$ $\quad \frac{\partial M}{\partial y} = 2x$ $\quad \frac{\partial N}{\partial x} = 2x$ $\quad$ conservative

1) $f(x,y) = \int 2xy\,dx = x^2y + g(y)$ $\quad$ 2 versions $\quad f(x,y) = x^2y + K$

2) $f(x,y) = \int x^2 dy = x^2y + h(x)$

39) $F(x,y)$ $\frac{xi+yj}{x^2+y^2} = \frac{x}{x^2+y^2}i + \frac{y}{x^2+y^2}j$ $\quad \frac{\partial}{\partial y}\left(\frac{x}{x^2+y^2}\right) = \frac{\partial}{\partial y}(x)(x^2+y^2)^{-1} = (x)(-1)(x^2+y^2)^{-2}(2y)$

$\frac{-2xy}{(x^2+y^2)^2}$ $\quad \frac{\partial}{\partial x}\left(\frac{y}{x^2+y^2}\right) = \frac{\partial}{\partial y}(y)(x^2+y^2)^{-1} = y(-1)(x^2+y^2)^{-2}(2x) = \frac{-2xy}{(x^2+y^2)^2}$

1. $f(x,y) = \frac{1}{2}\int \frac{2x}{x^2+y^2}dx = \frac{1}{2}\ln(x^2+y^2) + g(y)$ $\quad$ 2. $f(x,y) = \frac{1}{2}\int \frac{2y}{x^2+y^2}dy = \frac{1}{2}\ln(x^2+y^2) + h(x)$

$f(x,y) = \frac{1}{2}\ln(x^2+y^2) + K$

15.1 Notes 7242 physics 24 pin

43) $F(x,y,z) = xyz\,i + y\,j + z\,k$ Point$(1,2,1)$ curl $F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xyz & y & z \end{vmatrix}$

$= i(0-0) - j(0-xy) + k(0-xz) = xyj - xzk = \boxed{2j-k}$

Vector NOT CONSERVATIVE if curl is non-zero

55) $F(\ ) = \frac{1}{y}i - \frac{x}{y^2}j + (2z-1)k$

curl $= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{1}{y} & \frac{-x}{y^2} & (\ \ ) \end{vmatrix} = i(0-0) - j(0-0) + k\left(-\frac{1}{y^2} - \left(-\frac{1}{y^2}\right)\right) = 0$ CONSERVATIVE

① $f(x,y,z) = \int \frac{1}{y}\,dx = \frac{x}{y} + g(y,z) + K_1$

② $f(x,y,z) = \int \frac{-x}{y^2}\,dy = \frac{x}{y} + h(x,z) + K_2$

③ $f(x,y,z) = \int (2z-1)\,dz = z^2 - z + P(x,y) + K_3$

61) $f(x,y,z)$ Pt$(1,2,1)$ $= xyz\,i + y\,j + z\,k$ Div $F(\ ) = \frac{\partial}{\partial x}(xyz) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(z)$

$\nabla \cdot F = yz + 1 + 1 = yz + 2$

71) curl $F(x,y,z)$ then curl(curl $F(x,y,z)$)

curl $= i(0-0) - j(0-xy) + k(0-xz) = xyj - xzk$

75) $F(x,y,z) = xyz\,i + y\,j + z\,k$ DIV(curl F) $= \nabla \cdot (\text{curl } F) = \frac{\partial M}{x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

$= 0 + x + (-x) = 0$ Definition of Div(curl)

15.2 Notes — Line Integrals —

Piecewise Smooth curve parametric form $r(t) = x(t)i + y(t)j + z(t)k$

① $\frac{dx}{dt}, \frac{dy}{dt}, \ \& \ \frac{dz}{dt}$ are continuous on $[a,b]$ ② are NOT simultaneously zero

$\int_C (1)\,dS$ = Arc Length

14.7/ 1-27 odds

1) $\int_0^4\int_0^{\pi/2}\int_0^2 r\cos\theta \, dr\,d\theta\,dz = \iint_0 \frac{4}{2}\cos\theta \, d\theta\,dz = 2\int_0^4 \sin\theta \Big|_0^{\pi/2} dz = 2\int_0^4 (1-0)\,dz$
$= 2(4) = \boxed{8}$

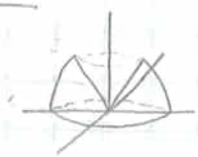
5) $\int_0^{2\pi}\int_0^{\pi/4}\int_0^{\cos\phi} \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta = \iint_0 \frac{1}{3}\cos^3\phi \sin\phi \, d\phi\, d\theta \quad -\frac{1}{3}\int u^3\,du$

$= (-)\frac{1}{3}\iint \cos^3\phi(-\sin\phi)\,d\phi\,d\theta = -\frac{1}{3}\int \frac{\cos^4\phi}{4}\Big|_0^{\pi/4} d\theta = -\frac{1}{12}\int_0^{2\pi}(.25-1)\,d\theta$
$= \frac{-1}{12} \quad -.75(2\pi) = \frac{3(2)\pi}{4(12)} = \pi\left(\frac{1}{4}\left(\frac{1}{2}\right)\right) = \boxed{\frac{\pi}{8}}$

11) $\int_0^{2\pi}\int_{\pi/6}^{\pi/2}\int_0^4 \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta = \iint \frac{4^3}{3}\sin\phi\, d\phi\, d\theta = \frac{64}{3}\int_0^{2\pi} -[\cos\frac{\pi}{2} - \cos\frac{\pi}{6}]\,d\theta$

$\Rightarrow \frac{64}{3}\int_0^{2\pi}\frac{\sqrt{3}}{2}\,(d\theta) = \frac{64}{3}\left(\frac{\sqrt{3}}{\cancel{2}}\right)(\cancel{2}\pi) = \frac{64\sqrt{3}\pi}{3}$

$\frac{\pi}{6} \le \phi \le \pi/2 \qquad 0 \le \theta \le 2\pi$

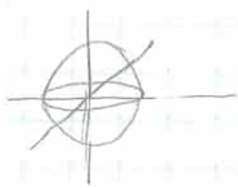


13) $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_0^{\sqrt{16-x^2-y^2}} x\,dz\,dy\,dx$ | cyl and spher. coords.

$z^2 = 16 - x^2 - y^2$

$16 = x^2 + y^2 + z^2$

$-4+x^2 \le y^2 \le 4+x^2$



$\int_0^4\int_0^{2\pi}\int_0^{\sqrt{4-x^2-y^2}} dz\,d\theta\,dr$

cylindrical

spherical

$\int\int\int$

$\int_0^{2\pi}\int_0^2\int_{r^2}^4 r^2\cos\theta\, dz\,dr\,d\theta$

14.7 Notes

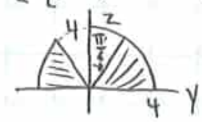
5) $\int_0^{2\pi}\int_0^{\pi/4}\int_0^{\cos\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta = \iint \frac{\rho^3}{3}\Big|_0^{\cos\phi} \sin\phi \, d\phi \, d\theta = \iint -\frac{1}{3}\cos^3\phi \, (-\sin\phi) \, d\phi \, d\theta$

$= -\frac{1}{3}\int \frac{\cos^4\phi}{4}\Big|_0^{\pi/4} d\theta = \left(\frac{1}{4}\right)\left(-\frac{1}{3}\right)\int\left[\frac{1}{4}-1\right] d\theta = \frac{-1}{12}\left(-\frac{3}{4}\right)\theta\Big|_0^{2\pi} = \frac{3(2\pi)}{12(4)} = \frac{\pi}{8}$

7) $\int_0^4\int_0^2\int_0^{\pi/2} re^r \, d\theta \, dr \, dz = \frac{\pi}{2}\iint re^r \, dr \, d\theta$ TABLES #82 $= \frac{\pi}{2}\int (r-1)e^r \Big|_0^2 dz$

$= \frac{\pi}{2}\int\left[(2-1)e^2 - (0-1)e^0\right] dz = \frac{\pi}{2}\left(2e^2 - e^2 + 1\right) dz = \frac{\pi}{2}\left[(2-1)e^2 - e^2 + z\right]_0^4 = \pi(e^4+3)$

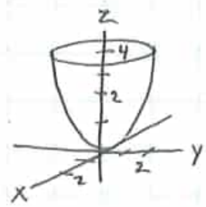
11) $\int_0^{2\pi}\int_{\pi/6}^{\pi/2}\int_0^2 \underbrace{\rho^2 \sin\phi \, d\rho \, d\phi \, d\theta}_{dV}$ ρ_0^4 tells us its inside a sphere

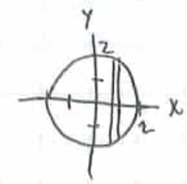


$= \iint \frac{\rho^3}{3}\Big|_0^4 \sin\phi \, d\phi \, d\theta = \frac{64}{3}\iint \sin\phi \, d\phi \, d\theta = \frac{64}{3}\int\left[-\cos\phi\right]_{\pi/6}^{\pi/2} d\theta = \frac{64}{3}\int\left[\cos\frac{\pi}{2} + \cos\frac{\pi}{6}\right] d\theta$

$= \frac{64}{3}\left(\frac{\sqrt{3}}{2}\right)(2\pi)$ $\boxed{\frac{64\sqrt{3}\pi}{3}}$

13) $\int_{-2}^{+2}\int_{-\sqrt{4-x^2}}^{\sqrt{}}\int_{x^2y^2}^{4} x \, dz \, dy \, dx$ $x^2+y^2 \le z \le 4$



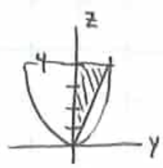


$r^2 \le z \le 4$ $0 \le r \le 2$ $0 \le \theta \le 2\pi$

$\int_0^{2\pi}\int_0^2\int_{r^2}^4 (r\cos\theta)(dz)(r \, dr \, d\theta) = \iint (4-r^2)(r^2\cos\theta \, dr \, d\theta) = \iint (4r^2 - r^4) \, dr \cos\theta \, d\theta = \int\left[\frac{4r^3}{3} - \frac{r^5}{5}\right]_0^2$

$= \frac{64}{15}\int_0^{2\pi}\cos\theta \, d\theta = \frac{64}{15}\left[\sin\theta\right]_0^{2\pi} = 0$

Spherical coordinates



① $0 \le \phi \le \arctan\frac{1}{2}$ $\rho = ? = 4\sec\phi$

$\int_0^{2\pi}\int_0^{\arctan\frac{1}{2}}\int_0^{4\sec\phi} (\rho\sin\phi\cos\phi)(\rho^2\sin\phi \, d\rho \, d\phi \, d\theta)$

② $\arctan\frac{1}{2} \le \phi \le \pi/2$ $0 \le \theta \le 2\pi$ $z = x^2+y^2$ $z = r^2$ $z = \rho\cos\phi$ $r^2 = \rho^2\sin^2\phi$

$\rho\cos\phi = \rho^2\sin^2\phi$ $\rho = \frac{\cos\phi}{\sin^2\phi} = \cot\phi\csc\phi$ $\int_0^{2\pi}\int_{\arctan(\frac{1}{2})}^{\pi/2}\int_0^{\cot\phi\csc\phi} (\quad)(\quad)$

$0 \le \rho \le \cot\phi\csc\phi$

21. mass – cyl. coords $0 \le z \le 9 - x - 2y$ $x^2+y^2 \le 4$ $\rho(\ ) = k\sqrt{x^2+y^2} \Rightarrow$ cyl, $k\sqrt{r^2} = kr$

$m = \int_0^{2\pi}\int_0^2\int_0^{9-r\cos\theta-2\sin\theta} (kr) \, dz \, r \, dr \, d\theta$

$= \iint (kr^2)(9 - r\cos\theta - 2r\sin\theta) \, dr \, d\theta = \iint k(9r^2 - r^3\cos\theta - 2r^3\sin\theta) \, dr \, d\theta$

$= k\int \frac{9r^3}{3} - \frac{r^4}{4}\cos\theta - \frac{2}{4}r^4\sin\theta\Big|_0^2 d\theta = k\int_0^{2\pi} \frac{9(8)}{3} - \frac{16}{4}\cos\theta - \frac{2(16)}{4}\sin\theta \, d\theta$

$= k\int 24 - 4\cos\theta - 8\sin\theta \, d\theta = k\int 24\theta - 4\sin\theta \Big]_0^{2\pi} = k\left[(48\pi) - 0 + (8)(\cos 2\pi)\right]$

.... $= 48k\pi$

14.6) Homework 1-51 odds

1) $\int_0^3\int_0^2\int_0^1 (x+y+z)\,dx\,dy\,dz = \int_0^3\int_0^2 \left(\frac{x^2}{2}+yx+zx\right)\Big|_0^1 dy\,dz$

$= \int_0^3 \frac{1}{2}y + \frac{y^2}{2} + zy\Big|_0^2 dz = \int_0^3 1+2+2z\,dz = 3z + \frac{2z^2}{2}\Big|_0^3 = 9+9 = \boxed{18}$

– 3, 5, skip –

7) $\int_0^4\int_0^{\pi/2}\int_0^{1-x} x\cos y\,dz\,dy\,dx = \iint x\cos y(1-x) \Rightarrow \iint_0 x\cos y - x^2\cos y\,dy\,dx = \int x\sin y - x^2\sin y\Big|_0^{\pi/2} dx$

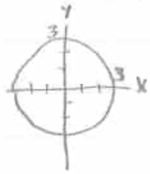
$= \int_0^4 x(1) - x^2(1)\,dx = \frac{x^2}{2} - \frac{x^3}{3}\Big|_0^4 = \frac{16}{2} - \frac{64}{3} = \boxed{\frac{-40}{3}}$

9, skip

~~11)~~ $\int_0^2\int_0^{\sqrt{4-x^2}}\int_1^4 \frac{x^2\sin y}{z}\,dz\,dy\,dx = \int_0^2\int_0^{\sqrt{4-x^2}} x^2\sin y\ln\left(\frac{4}{1}\right) dy\,dy = \int_0^2 -\cos y\,x^2\ln(4)\Big|_0^{\sqrt{4-x^2}}$ still GOOD ✓

$= \int_0^2 -\cos\sqrt{4-x^2} + \cos(0)\left(\ln(4)x^2\right)dx = \int_0^2 \ln 4x^2\left(1-\cos\sqrt{4-x^2}\right)dx =$ Now use calculator? ≈ 2.442 in answers

15) $z = 9-x^2-y^2$, $z=0$ set up triple integral



$x^2+y^2=9 \quad y^2 = 9-x^2 \quad y = \sqrt{9-y^2}$

$\int_0^{9-x^2-y^2}\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} dy\,dx\,dz$

$\boxed{\int_{-3}^3\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_0^{9-x^2-y^2} dz\,dy\,dx}$

17) $\int_{-2}^2\int_0^{4-y^2}\int_0^x dz\,dx\,dy = \int_{-2}^2\int_0^{4-y^2} x\,dx\,dy \quad \int_{-2}^2 \frac{(4-y^2)^2}{2}dy = \int_{-2}^2 (y^4-8y^2+16)dy$

$= 2\left[\frac{y^5}{5} - \frac{8y^3}{3} + 16y\right]_0^2 = \frac{32}{5} - \frac{8(8)}{3} + 32 = \boxed{\frac{256}{15}}$ easy!

23) $\int_0^4\int_0^{(4-x)/2}\int_0^{(12-3x-6y)/4} dz\,dy\,dx$

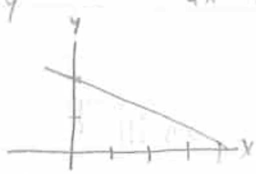
$\Rightarrow dy\,dx\,dz \quad y = \frac{12-3x-4z}{6}$

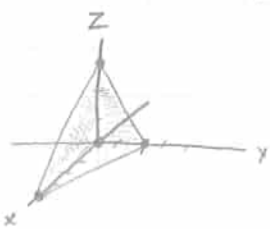
$4z = 12-3x-6y \quad z = 3 - \frac{3}{4}x - \frac{3}{2}y$

$12 = 3x+6y+4z$

$0 \le y \le \frac{4-x}{2}$

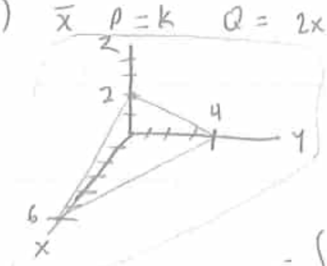
$0 \le y \le 2 - \frac{1}{2}x$





$\boxed{\int_0^3\int_0^{4-2y}\int_0^{(12-3x-4z)/6} dy\,dx\,dz}$

33) $\bar{x}$ $\rho = k$ $Q = 2x+3y+6z = 12$ $x=0$ $y=0$, $z=0$ $\int_0^6\int_0^{4-\frac{2}{3}x}\int_0^{(12-2x-3y)/6} dz\,dy\,dx$



$z = -\frac{1}{3}x - \frac{1}{2}y + 2$

$y = 4 - \frac{2}{3}x$

$\int_0^6\int_0^{4-\frac{2}{3}x} 2 - \frac{x}{3} - \frac{y}{3}\,dy\,dx \quad \int_0^6 2y - \frac{xy}{3} - \frac{y^2}{6}\Big|_0^{4-2/3x} dx$

$= \int_0^6 8 - \frac{4}{3}x - \frac{x(4-2/3x)}{3} - \frac{(4-\frac{2}{3}x)^2}{6}dx$

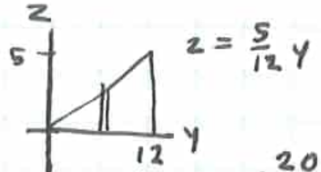
$= \int_0^6 8 - \frac{4}{3}x - \frac{4}{3}x + \frac{\frac{2}{3}x^2}{3} - \frac{\left(16+\frac{4}{9}x^2 - \frac{4}{3}x\right)}{6}dx$

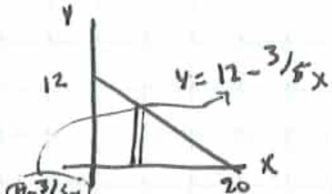
$\int_0^6 8 - \frac{8}{3}x + \frac{2}{9}x^2 - \frac{16}{6} - \frac{4x^2}{54} + \frac{4}{18}x\,dx$

$\int_0^2 \frac{16}{3} - \frac{22}{9}x + \frac{4}{27}x^2\,dx = \frac{16}{3}(2) - \frac{22}{9}\frac{(4)}{2} + \frac{4}{27}\left(\frac{8}{3}\right)$

14.6] NOTES

47) Plane





$\rho = k$

$m = \iiint \rho \, dV = \iiint k \, dz\,dy\,dx = k \int_0^{20} \int_0^{12-3/5x} \int_0^{5/12 y} = 200k$

$M_{yz} = k \int_0^{20} \int_0^{12-\frac{3}{5}x} \int_0^{\frac{5}{12}y} x \, dz\,dy\,dx = 1000k$

$M_{xz} = k \int \int \int (y) \, dz\,dy\,dx = 1200k$

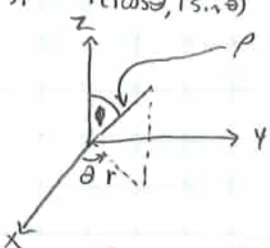
$M_{xy} = k \iiint (z) \, dz\,dy\,dx = 250k \quad \bar{x} = \frac{M_{yz}}{m} = 5.0 \quad \bar{y} = \frac{M_{xz}}{m} = 6.0 \quad \bar{z} = \frac{M_{xy}}{m} = 1.25$

14.7] Cylindrical coordinates —Review Section 11.7—

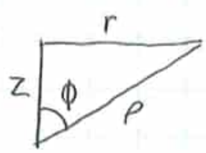
$x = r\cos\theta \quad y = r\sin\theta \quad r^2 = x^2+y^2 \quad \tan\theta = \frac{y}{x} \quad z = z$

$$\iiint_Q f(x,y,z)\,dV = \int_{\theta_1}^{\theta_2} \int_{g_1(\theta)}^{g_2(\theta)} \int_{h_1(r\cos\theta,\, r\sin\theta)}^{h_2(r\cos\theta,\, r\sin\theta)} f(r\cos\theta, r\sin\theta, z)(r\,dz\,dr\,d\theta)$$

Spherical coordinates



$0 \le \phi \le \pi$



$\frac{r}{\rho} = \sin\phi$

$\frac{z}{\rho} = \cos\phi$

$z = \rho\cos\phi$

$r = \rho\sin\phi$

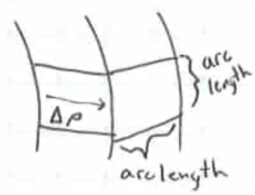
$x = r\cos\theta = \rho\sin\phi\cos\theta \qquad r^2 = x^2+y^2$

$y = r\sin\theta = \rho\sin\phi\sin\theta \qquad \rho^2 = x^2+y^2+z^2$

$\tan\theta = \frac{y}{x} \quad \phi = ? \quad \cos\phi = \frac{z}{\rho} \quad \phi = \arccos\left(\frac{z}{\rho}\right) = \arccos\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$

$\iiint_Q f(x,y,z) = \int_{\theta_1}^{\theta_2}\int_{\phi_1}^{\phi_2}\int_{\rho_1}^{\rho_2} f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\,(dV)$

$* \ \underbrace{\rho^2 \sin\phi \, d\rho\, d\phi\, d\theta}_{dV}$



$\Delta s = \rho\Delta\theta \qquad \Delta V = (\Delta\rho)(\rho\Delta\phi)(\rho\Delta\theta\sin\phi) = \rho^2\sin\phi\,\Delta\rho\,\Delta\phi\,\Delta\theta$

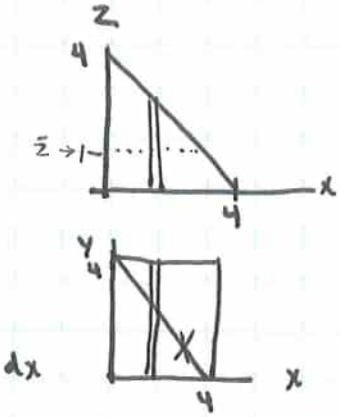
3) $\int_0^{\pi/2}\int_0^{2\cos^2\theta}\int_0^{4r^2} r\sin\theta\,dz\,dr\,d\theta \quad \iint z\Big|_0^{4-r^2} r\sin\theta\,dr\,d\theta = \iint (4-r^2)\,r\sin\theta\,dr\,d\theta$

$= \iint (4r - r^3)\,dr\,\sin\theta\,d\theta = \int \left[\frac{4r^2}{2} - \frac{r^4}{4}\right]_0^{2\cos^2\theta} \sin\theta\,d\theta = \int \left[2(4\cos^4\theta) - \frac{1}{4}(16\cos^8\theta)\right]\sin\theta\,d\theta$

$= -\int (8\cos^4\theta - 4\cos^8\theta)(-\sin\theta\,d\theta) = \left[-8\frac{\cos^5\theta}{5} - 4\frac{\cos^9\theta}{9}\right]_0^{\pi/2} = -\left[\frac{8}{5}(0) - \frac{4}{9}(0)\right]$

$+ \left[\frac{8}{5}(1) - \frac{4}{9}(1)\right] = \frac{52}{45}$

14.6 NOTES



35) Find $\bar{z}$ $\rho = kx$ $z = 4-x$ $z=0$ $y=0$ $y=4$ $x=0$

#51 image $m = \iiint \rho \, dz\,dy\,dx$

$= \int_0^4 \int_0^4 \int_0^{4-x} kx \, dz\,dy\,dx = \int_0^4 \int_0^4 kx(4-x) \, dy\,dx = \int_0^4 4kx(4-x)\,dx$

$= \int_0^4 4k(4x - x^2)\,dx = 4k\left[\frac{4}{2}x^2 - \frac{x^3}{3}\right]_0^4 = \boxed{128k/3}$ m

$M_{xy} = \iiint z\rho \, dV = \int_0^4\int_0^4\int_0^{4-x} zkx \, dz\,dy\,dx = \iint kx \frac{z^2}{2}\Big|_0^{4-x} dy\,dx = \iint \frac{kx}{2}(4-x)^2 \, dy\,dx$

$= \int \frac{kx}{2}(4-x)^2 y\Big|_0^4 dx = \int_0^4 \frac{4kx}{2}(16 - 8x + x^2)\,dx = 2k\left[\frac{16x^2}{2} - \frac{8x^3}{3} + \frac{x^4}{4}\right]_0^4 = \frac{128k}{3}$

51) a) $\rho = k$ $I_x = \iiint_Q (y^2+z^2)\,\rho(x,y,z)\,dV = \int_0^4\int_0^4\int_0^{4-x}(y^2+z^2)\,k\,dz\,dy\,dx$

$= k\iint \left(y^2 z + \frac{z^3}{3}\right)\Big|_0^{4-x} dy\,dx = k\iint \left[y^2(4-x) + \frac{1}{3}(4-x)^3\right]dy\,dx = k\int \left[\frac{y^3}{3}(4-x) + \frac{1}{3}y(4-x)^3\right]_0^4 dx$

$= k\int \left[\frac{64}{3}(4-x) + \frac{4}{3}(4-x)^3\right]dx = k\left[-\frac{64}{3}\frac{(4-x)^2}{2} - \frac{4}{3}\frac{(4-x)^4}{4}\right]_0^4$ $I_x = I_z = 256k$

$I_y = \int_0^4\int_0^4\int_0^{4-x} k(x^2+z^2)\,dz\,dy\,dx = \ldots \frac{512k}{3}$

b) $\rho = ky$ $I_x = \int_0^4\int_0^4\int_0^{4-x}(y^2+z^2)(ky)\,dz\,dy\,dx$ $I_x = I_z = \frac{2048k}{3}$

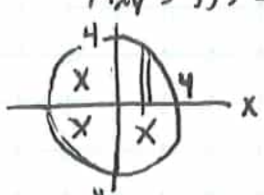
$I_y = \int_0^4\int_0^4\int_0^{4-y}(x^2+z^2)(ky)\,dz\,dy\,dx = \frac{1024k}{3}$

45) $z = \sqrt{4^2 - x^2 - y^2}$ $z^2 = 4^2 - x^2 - y^2$ $x^2+y^2+z^2 = 16$ sphere $r=4$ $z=0$

$\bar{x} = \bar{y} = 0$ volume sphere $= \frac{4}{3}\pi r^3 \Rightarrow \frac{1}{2}V = \frac{1}{2}\cdot\frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3$ $\rho = k$ $m = \frac{2}{3}k\pi r^3$

$r = 4$ $m = \frac{2}{3}k\pi 4^3 = \frac{128}{3}\pi k$ find $\bar{z}$, M_{xy}

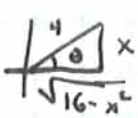
$M_{xy} = \iiint z\rho\,dV = \iiint zk\,dz\,dy\,dx = \int_0^4 \int_0^{\sqrt{4^2-x^2}} \int_0^{\sqrt{}} zk\,dz\,dy\,dx$



$= 4k\iint \frac{z^2}{2}\Big|_0^{\sqrt{}} dy\,dx = \frac{4k}{2}\iint (4^2 - x^2 - y^2)\,dy\,dx = 2k\int\left[4^2 y - x^2 y - \frac{y^3}{3}\right]_0^{\sqrt{4^2-x^2}} dx$

$= 2k$ (plugged last step) $= 2k\int \frac{2}{3}(4^2-x^2)(\sqrt{4^2-x^2})\,dx = \frac{4k}{3}\int (4^2-x^2)^{3/2}\,dx$

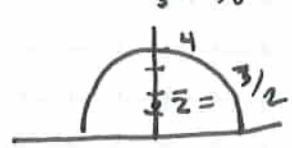
$\sqrt{a^2-u^2} = \sqrt{4^2-x^2}$ $x = 4\sin\theta$



$dx = 4\cos\theta\,d\theta$ $\sqrt{} = 4\cos\theta$

$= \frac{4k}{3}\int (4\cos\theta)^3(4\cos\theta\,d\theta)$ $0 \le x \le 4$ $0 \le \theta \le \pi/2$ $= \frac{4}{3}k\int_0^{\pi/2} 4^4\cos^4\theta\,d\theta =$ WALLIS $\frac{4k}{3}\,4^4\left(\frac{3\pi}{6}\right) = 64\pi k$

$\bar{z} = \frac{64\pi k(3)}{128\pi k} = \frac{3}{2}$



14.6] Triple Integrals — Notes —

3 variables: $f(x,y,z)$ elemental volume $\Delta V_i = \Delta x_i \Delta y_j \Delta z_k$

Volume of $Q = \iiint_Q dV = \iiint_Q dx\,dy\,dz$ 6 ways to set up...

Definition of triple integral: $\iiint_Q$ (p1024) ... $\int_a^b \int_{h_1(x)}^{h_2(x)} \int_{g_2(x,y)}^{g_1(x,y)} f(x,y,z)\,dz\,dy\,dx$

3-D 1. determine inner limits 1st

middle 2nd

outer 3rd

1. Vol $\iiint_Q dV$ 2 Mass $= \iiint_Q \rho(x,y,z)\,dV$ 3. 1st moment about a plane

ⓐ $M_{yz} = \iiint_Q (x)\,\rho(x,y,z)\,dV$

" xz " (y) "

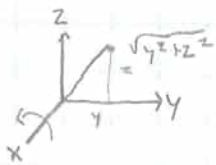
" xy " (z) "

4. ctr of mass

$\bar{x} = \dfrac{M_{yz}}{m}$ $\bar{y} = \dfrac{M_{xz}}{m}$, etc

5. second moments about a line

$I_x = \iiint_Q y^2+z^2\,\rho(x,y,z)\,dV$



EX) $I_{xy} = \iiint_Q z^2 \rho\,dV$ $I_x = I_{xy} + I_{xz}$

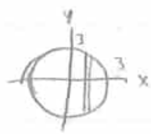
5) $\int_1^4\int_0^1\int_0^x 2ze^{-x^2}\,dy\,dx\,dz = \int_1^4\int_0^1 2ze^{-x^2}\,y\Big|_0^x\,dx\,dz = \left(-\tfrac{1}{2}\right)\iint 2ze^{-x^2}\,{-2}x\,dx\,dz$

$-\frac{1}{2}\int 2ze^{-x^2}\Big|_0^1 dz = -\frac{1}{2}\int_1^4 2z(e^{-1}-e^0)\,dz = -\frac{1}{2}\left(\frac{1}{e}-1\right)\int_1^4 2z\,dz = -\frac{1}{2}\left(\frac{1}{e}-1\right)z^2\Big|_1^4$

$= -\frac{1}{2}\left(\frac{1}{e}-1\right)(16-1) = \frac{15}{2}\left(1-\frac{1}{e}\right)$

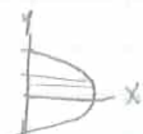
15) Paraboloid

$z = 9-x^2-y^2$ $z=0$



$V = \int_{-3}^{+3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_0^{9-x^2-y^2} dz\,dy\,dx$

17) Volume $z=x$, $z=0$, $x=4-y^2$



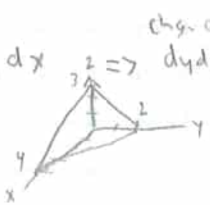
$V = \iiint dz\,dx\,dy$

$= \int_{-2}^{2}\int_0^{4-y^2}\int_0^x dz\,dx\,dy$

$\iint x\,dx\,dy = \int \frac{x^2}{2}\Big|_0^{4-y^2} dy = \frac{1}{2}\int_{-2}^{+2}(4-y^2)^2\,dy = \frac{1}{2}\int(16-8y^2+y^4)\,dy$

$= \frac{1}{2}\left[16y - \frac{8y^3}{3} - \frac{y^5}{5}\right]_{-2}^{2} = 256/15$

23) $\int_0^4\int_0^{(4-x)/2}\int_0^{(12-3x-6y)/4} dz\,dy\,dx$ chg. order => $dy\,dx\,dz$



$y = -\frac{1}{2}x+2 = \frac{(4-x)}{2}$

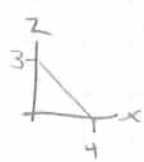
$z = (12-3x-6y)/4$

$3x+6y+4z = 12$

$z=3$
$y=2$
$x=4$

$6y = 12-3x-4z$

$y = \dfrac{12-3x-4z}{6}$



$z = -\frac{3}{4}x+3$

$4z = -3x+12$

$x = (-4z+12)/3$

→

14.5) Notes

17) $2\pi a \left[a - \sqrt{a^2-b^2} \right]$

$2\pi(5)\left[5 - \sqrt{25-4}\right]$ = ... 10π (x2 for whole sphere)

21) polar coord's

$x^2+y^2=4$

$S = \int_0^{2\pi}\int_0^2 \sqrt{1+4r^2}\; r\,dr\,d\theta = \int_0^{2\pi}\frac{1}{8}\int_0^2 (1+4r^2)^{1/2}\, 8r\,dr\,d\theta$

$= \int_0^{2\pi} \frac{1}{8}\; 1+4r^2{}^{3/2}\, \frac{2}{3}\Big|_0^2 \,d\theta = \frac{1}{12}(2\pi)(1+4r^2)^{3/2}\Big|_0^2 = \frac{\pi}{6}\left[17\sqrt{17}-1\right]$

23) $f(x,y) = 4-x^2-y^2$ $\quad 0 \le x \le 1 \quad 0 \le y \le 1$ $\qquad S = \int_0^1\int_0^1 \sqrt{1+4x^2+4y^2}\; dy\,dx$

Calculator (both sets of limits are constant)

∫(∫((1+4x^2+4y^2)^(1/2), y, 0, 1), x, 0, 1) = 1.8616

33) $f(x,y) = e^{xy}$ $\quad R: 0 \le x \le 4 \quad 0 \le y \le 10$ $\qquad f_x = e^{xy}(y) \quad (f_x)^2 = y^2e^{2xy}$

$f_y = e^{xy}(x) \quad f_y^{\,2} = x^2e^{2xy}$

$\int \sqrt{1+y^2e^{2xy}+x^2e^{2xy}}$

$S = \int_0^4\int_0^{10} \sqrt{1+e^{2xy}(x^2+y^2)}\; dy\,dx$

14.6) Triple

14.5| Notes

3) $f(x,y) = 8+2x+2y$ $\quad f_x = 2 \quad f_y = 2$ $\quad \sqrt{1+f_x^2+f_y^2}$

$R = x^2+y^2 \le 4$ $\quad \sqrt{2^2+2^2+1} = \sqrt{9} = 3$

$S = \int_{-2}^{+2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} 8\,dy\,dx \implies S = \int_0^{2\pi}\int_0^2 (3)r\,dr\,d\theta = \int_0^{2\pi} \frac{3r^2}{2}\,d\theta$

$= 6(2\pi) = 12\pi$

5) $f(x,y) = 9-x^2 = z$

$f_x = -2x \quad f_y = 0 \quad \sqrt{1+f_x^2+f_y^2} = \sqrt{1+4x^2}$

$S = \int_0^3\int_0^3 \sqrt{1+4x^2}\,dy\,dx = \int_0^3 y\sqrt{1+4x^2} = \int_0^3 3\sqrt{1+4x^2}\,dx$

$= (3)\left(\frac{1}{2}\right)\int_0^3 \sqrt{(1)^2+(2x)^2}\,(2\,dx) = \frac{3}{2}\left[\frac{1}{2}u\sqrt{a^2+u^2}\ a^2\ln(u+\sqrt{u^2+a^2}\right]$

$2x\sqrt{1+4x^2} + \ln|2x+\sqrt{1+4x^2}\Big|_0^3$

$\frac{3}{4}\left[6\sqrt{37}\ \ln(6+\sqrt{37})\right] \approx 29.24$

9) $f(x,y) = \ln(\sec x) \quad 0 \le x \le \frac{\pi}{4} \quad 0 \le y \le \tan x$

$f_x = \frac{1}{\sec x}(\sec x \tan x) \quad f_x = \tan x \quad f_y = 0$

$\sqrt{1+\tan^2 x} = \sqrt{\sec^2 x} = \sec x \quad S = \int_0^{\pi/4}\int_0^{\tan x} \sec x\,dy\,dx = \int_0^{\pi/4} \sec x \tan x\,dx = \sec x\Big|_0^{\pi/4}$

$= \sqrt{2} - 1$

13) $f(x,y)\ \sqrt{a^2-x^2-y^2} \quad R: x^2+y^2 \le b^2 \quad b \le a$

$f_x = \frac{1}{2}(a^2-x^2-y^2)^{-1/2}(-2x) \quad S = \int_{-b}^{b}\int_{-\sqrt{b-x^2}}^{\sqrt{b^2-x^2}} \frac{a}{\sqrt{a^2-x^2-y^2}}\,dy\,dx$

$f_y = \frac{1}{\cancel{2}}(a^2-x^2-y^2)^{-1/2}(-\cancel{2}y)$

$S = \int_0^{2\pi}\int_0^b \frac{a}{\sqrt{a^2-r^2}}\,r\,dr\,d\theta = \int_0^{2\pi}\left(-\frac{1}{2}\right)\int_0^b a(a^2-r^2)^{-1/2}(-2r\,dr)\,d\theta$

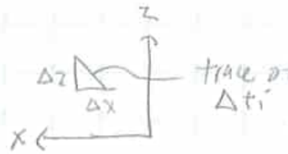
$= \int_0^{2\pi}\left(\frac{-a}{2}\right)\frac{(a^2-r^2)^{1/2}}{1/2}\,d\theta = \int_0^{2\pi}(-a)\left[a^2-b^2\right]^{1/2} - (a^2-0)^{1/2}\,]\,d\theta$

$= 2\pi(-a)\,((a^2-b^2)^{1/2} - a) = 2\pi a\left(a - \sqrt{a^2-b^2}\right]$

14.5 Surface Area Notes

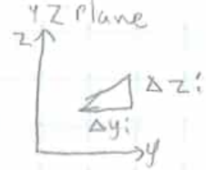
$z = f(x,y)$ formula for $A_{\text{tangent plane}}$

a) slope $= \frac{\Delta z}{\Delta x}$ $\Delta z = \text{slope}\, \Delta x_i$ $\Delta z = f_x(x_i, y_i)\, \Delta x_i$



Vector form $u = \Delta x_i\, i + f_x(x_i, y_i)\, \Delta x_i\, k$

b) YZ plane



$v = \Delta y_i\, j + f_y(x_i, y_i)\, \Delta y_i\, k$

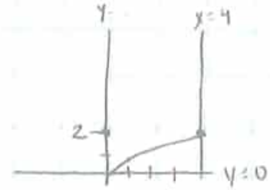
Th 11-8 $\|u \times v\|$ area of Δt_i (tangent plane)

surface area $= \sum_{i=1}^{n} \Delta S_i \approx \sum_{i=1}^{n} \sqrt{(f_x)^2 + (f_y)^2 + 1}\, \Delta A_i$

$S.A. = \int_R \int dS =$

$= \int_R \int \sqrt{(f_x)^2 + (f_y)^2 + 1}\, dA$

14.4 homework 7-39 odds



13) $y=\sqrt{x}$ $y=0$, $x=4$ $\rho=kxy$

$m=\int_0^4\int_0^{\sqrt{x}} kxy\,dy\,dx$

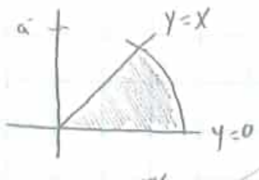
$m=\int_0^4 \frac{kxy^2}{2}\Big|_0^{\sqrt{x}}dx = \int_0^2 \int \frac{kx^2}{2}dx = \frac{kx^3}{6}\Big|_0^4 = \frac{k64}{6} = \boxed{\frac{32k}{3}}$

$M_x=\int_0^4\int_0^{\sqrt{x}} kxy^2\,dy\,dx = \int_0^4 \frac{kxy^3}{3}\Big|_0^{\sqrt{x}}dx = \int_0^4 \frac{kx^{2.5}}{3}dx = \frac{k\,x^{3.5}}{3(3.5)}\Big|_0^4 = \frac{128k}{10.5}\ \boxed{\frac{256k}{21}}$

$M_y=\int_0^4\int_0^{\sqrt{x}} kx^2y\,dy\,dx = \int_0^4 \frac{kx^2x}{2}dx = \frac{kx^4}{8}\Big|_0^4 = \frac{256k}{8} = 32k$

$\bar{x}=\frac{32k}{32k/2} = 2$ $\quad\bar{y}=\frac{256k}{21}\frac{3}{32k} = 8\frac{3}{21} = \boxed{\frac{8}{7}}$

21) $y=\sqrt{a^2-x^2}$ $y=0$ $y=x$ $\rho=k\sqrt{x^2+y^2}$ $\quad r\cos\theta, r\sin\theta$ $\quad\int_\alpha^\beta\int_{g_1(\theta)}^{g_2(\theta)} f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$



$m=\int_0^{\pi/4}\int_0^{\sqrt{a^2-x^2}} k\sqrt{r^2\cos^2\theta+r^2\sin^2\theta}\;r\,dr\,d\theta = \int_0^{\pi/4}\int_0^{r} k\sqrt{r^2(\cos^2\theta+\sin^2\theta)}\;r\,dr\,d\theta$

$=\iint k\,r(r)\,dr\,d\theta = \int_0^{\frac{\pi}{4}} \frac{kr^3}{3}\Big|_0^{\sqrt{a^2-x^2}}d\theta = \int_0^{\frac{\pi}{4}}\frac{(a^2-x^2)^{3/2}}{3}d\theta$

$=3\int_0^{\pi/4}(a^2-r^2\cos^2\theta)\,d\theta = 3a^2\theta$

$m=\int_0^{\pi/4}\int_0^a k\,r\,dr\,d\theta = \int_0^{\pi/4}\frac{a^2}{2}k\,d\theta = \frac{\pi a^2 k}{8}$

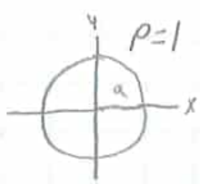
$M_x=\int_0^{\pi/4}\int_0^a kr\sin\theta\, r\,dr\,d\theta = \int_0^{\pi/4}\frac{ka^3}{3}\sin\theta\,d\theta = -\frac{ka^3}{3}\cos\theta\Big|_0^{\pi/4} = -\frac{ka^3}{2}\left(\frac{\sqrt{2}}{2}-1\right)$

$\bar{y}=\dfrac{-\frac{ka^3}{3}\left(\frac{\sqrt{2}-2}{2}\right)}{\frac{\pi a^2 k}{8}} = \frac{ka^3}{6}(2-\sqrt{2})\frac{8}{\pi a^2 k} = \boxed{\frac{4}{3\pi}a(2-\sqrt{2})}$

$\bar{x}$ skipped

— Moments of inertia: $I_x=\iint_R (y^2)\rho(x,y)\,dA$ —

• sum of I_x & I_y $=I_0$



29) $\rho=1$ $\quad y^2=r^2\sin^2\theta$

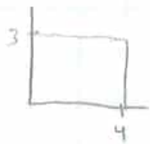
$I_x=\int_0^{2\pi}\int_0^a r^2\sin^2\theta\, r\,dr\,d\theta = \iint r^3\sin^2\theta\,dr\,d\theta = \int_0^{2\pi}\frac{a^4}{4}\sin^2\theta\,d\theta$

$=\frac{a^4}{4}(4)\left(\frac{1}{2}\right)\left[\theta-\sin\theta\cos\theta\right]_0^{\pi/2} = \frac{a^4}{2}\left[\frac{\pi}{2}-1(0)\right]-\left[0-0\right]\ \boxed{I_x=\frac{\pi a^4}{4}}$

$I_y=\frac{a^4}{2}\left[\theta+\sin\theta\cos\theta\right]_0^{\pi/2} = \frac{a^4}{2}\left(\pi/2+0\right)-(0) = \frac{\pi a^4}{4}$ $\quad\boxed{I_0=\frac{\pi a^4}{2}}$

14.4) homework 1-39 odds

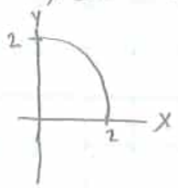
1) find mass $\rho(x,y) = xy$ $\quad 0 \le x \le 4 \quad 0 \le y \le 5$



$m = \iint_R \rho(x,y)\,dA = \int_0^4\int_0^3 xy\,dy\,dx = \int_0^4 \frac{xy^2}{2}\Big|_0^3 dx$

$= \int_0^4 \frac{x(9)}{2}\,dx = \frac{x^2}{2}\left(\frac{9}{2}\right)\Big|_0^4 = \frac{16}{2}\,\frac{9}{2} = 4(9) = \boxed{36}$

3) $x \ge 0 \quad 0 \le y \le \sqrt{4-x^2}$



$y = \sqrt{4-x^2}$

$y^2 = 4 - x^2$

$y^2 + x^2 = 4$

$\int_0^{\pi/2}\int_0^2 r\cos\theta\, r\sin\theta\, r\,dr\,d\theta = \int_0^{\pi/2}\int_0^2 r^3\cos\theta\sin\theta\,dr\,d\theta$

$= \int_0^{\pi/2} \frac{r^4}{4}\cos\theta\sin\theta\Big|_0^2 d\theta = \int_0^{\pi/2}\frac{16}{4}\cos\theta\sin\theta\,d\theta$

$= 4\int_0^{\pi/2}$

$dv = \sin\theta\,d\theta \quad v = -\cos\theta$

$u = \cos\theta \quad du = \sin\theta$

$\cos\theta\sin\theta = \qquad = \frac{1}{2}\sin 2u$

$\int u\,dv = uv - \int v\,du \qquad \int \cos\theta\sin\theta\,d\theta = -\cos^2\theta - \int -\cos\theta\sin\theta$

U-sub-stitution

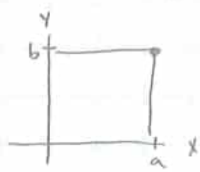
$\frac{1}{2}\int \sin 2\theta\,d\theta \qquad u = 2\theta \quad f(u) = \sin u \quad F(u) = -\cos u \qquad \int f(u)\,du = F(u) + C$

$\uparrow \quad \uparrow \qquad du = 2\,d\theta$

$2 \quad 2$

$2\left(\frac{1}{2}\right)\int \sin 2\theta\, 2\,d\theta = -\cos(2\theta)\Big]_0^{\pi/2} = \boxed{2}$

5) R = rectangle



a) $\rho = k \quad m = \int_0^a\int_0^b k\,dy\,dx = \int_0^a ky\Big|_0^b dx$

$= bk\Big|_0^a = abk$

$M_x = \iint_R y\rho(x,y)\,dA$

moment: $M_x = \int_0^a\int_0^b yk\,dy\,dx = \int_0^a \frac{y^2}{2}k\Big|_0^b dx \qquad M_y = \frac{a^2bk}{2}$

$= \frac{b^2k}{2}a = \frac{ab^2k}{2} \qquad M_x = \frac{kab^2}{2}$

$(\bar{x},\bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right) = \left(\frac{\frac{ab^2k}{2}}{abk}, \frac{\frac{a^2bk}{2}}{abk}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$

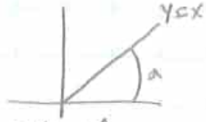
b) $\rho = ky \quad m = \int_0^a\int_0^b ky\,dy\,dx = \int_0^a \frac{kb^2}{2}dx = \frac{kab^2}{2} \qquad M_x = \int_0^a \frac{kb^3}{3}dx = \frac{kab^3}{3} \quad M_y = \int_0^a\int_0^b xyk\,dy\,dx$

$= \int_0^a \frac{y^2}{2}kx\Big|_0^b dx = \frac{b^2}{2}k\frac{x^2}{2}\Big|_0^a = \frac{b^2a^2k}{4} \qquad (\bar{x},\bar{y}) = \frac{\frac{kab^3}{3}}{\frac{kab^2}{2}}, \frac{\frac{ka^2b^2}{4}}{\frac{kab^2}{2}} = \left(\frac{2}{3}a, \frac{1}{2}b\right)$

c) $\bar{x},\bar{y} = \left(\frac{1}{2}a, \frac{2}{3}b\right)$

14.4)

21) $y=\sqrt{a^2-x^2}$ $\quad 0\le y\le x \quad \rho=k$



$m=\rho A = k\,\frac{\pi a^2}{8}$

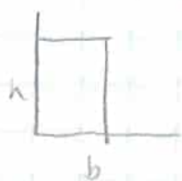
$M_x = \int_R\int (y)\rho(x,y)\,dA = \int_0^{\pi/4}\int_0^a (r\sin\theta)k(r\,dr\,d\theta) = \iint kr^2\sin\theta\,dr\,d\theta$

$\int \frac{r^3}{3}\Big|_0^a \sin\theta\,d\theta \qquad ka^3/_3\,(-\cos\theta)\Big|_0^{\pi/4} = \frac{ka^3}{3}\left(\frac{2-\sqrt{2}}{2}\right) = \frac{ka^3(2-\sqrt{2})}{6}$?

$M_y = \int_R\int x(r\cos\theta)(r\,dr\,d\theta) \qquad \int_0^{\pi/4} ka^3/3\,\cos\theta\,d\theta \qquad k\frac{a^3}{3}\sin\theta\Big|_0^{\pi/4} \quad \frac{ka^3}{3}\,\frac{1}{\sqrt{2}} = \frac{ka^3}{3}\,\frac{\sqrt{2}}{2}$

$= \frac{ka^3\sqrt{2}}{6} \qquad \bar{x} = M_y/m = \frac{\sqrt{2}ka^3}{6}\,\frac{8}{ka^2\pi} \quad \frac{4\sqrt{2}a}{3\pi} \qquad \bar{y} = M_x/m = ka^3\frac{(2-\sqrt{2})}{6}\,\frac{8}{k\pi a^2} = \frac{4a(2-\sqrt{2})}{3\pi}$

27) rectangle $\rho=1$ $\quad m=\rho A = bh$



$I_x = \int_R\int (y^2)\rho(x,y)\,dA = \int_0^b\int_0^h y^2(1)\,dy\,dx \quad \int_0^b \frac{y^3}{3}\Big|_0^h dx$

$=\int_0^b \frac{h^3}{3}\,dx = \frac{bh^3}{3}$

$I_y = \int_R\int x^2\rho(x,y)\,dA = \int_0^b\int_0^h x^2(1)\,dy\,dx = \frac{hb^3}{3}$

$\bar{\bar{x}} = \sqrt{\frac{I_y}{m}} = \sqrt{\frac{hb^3}{3}\,\frac{1}{bh}} = \boxed{\sqrt{\frac{b^2}{3}}} = \frac{b}{\sqrt{3}} \qquad \bar{\bar{y}} = \sqrt{\frac{I_x}{m}} = \sqrt{\frac{bh^3}{3}\,\frac{1}{bh}} = \boxed{\frac{h}{\sqrt{3}}}$

29) Circle $\rho=1$ radius $=r$ $\quad m=\pi r^2 = \pi a^2$

$I_x = \int_R\int y^2\rho(x,y)\,dA = \int_0^{2\pi}\int_0^r (r\sin\theta)^2(1)\,r\,dr\,d\theta = \int_0^{2\pi}\int_0^a \sin^2\theta\, r^3\,dr\,d\theta$

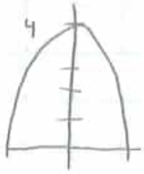
$\int_0^{2\pi} \frac{r^4}{4}\Big|_0^a \sin^2\theta\,d\theta = \frac{a^4}{4}\int_0^{2\pi}\sin^2\theta\,d\theta = \frac{a^4}{4}\left[\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right]_0^{2\pi}$

$= \frac{a^4}{4}\left[\frac{1}{2}(2\pi) - \frac{1}{4}\sin(4\pi)\right) - \left(\frac{1}{2}(0) - \frac{1}{4}\sin(0)\right)\right] = \frac{a^4\pi}{4}$

$\bar{\bar{x}} = \bar{\bar{y}} = \sqrt{\frac{I_x}{m}} = \sqrt{\frac{a^4\pi}{4}\,\frac{1}{a^2\pi}} = \sqrt{\frac{a^2}{4}} = \boxed{\frac{a}{2}}$

31) $I_x = I_y \qquad I_0 = \frac{1}{8}\pi a^4 = \frac{a^4\pi}{16} \qquad I_0 = I_x + I_y$

35)



$I_x = \int_R\int (y)^2\rho(x,y)\,dy\,dx \qquad \rho = kx \qquad I_x = \int_0^2\int_0^{4-x^2}(kx)y^2\,dy\,dx$

$= \int_0^2 kx\frac{y^3}{3}\Big]_0^{4-x^2}dx = \int_0^2 \frac{kx}{3}(4-x^2)^3\,dx = \int_0^2 \frac{k}{3}\left[64x - 48x^3 + 12x^5 - x^7\right]dx$

$= \frac{k}{3}\left[64x^2/_2 - 48\frac{x^4}{4} + 12x^5 - \frac{x^8}{8}\right]_0^2 = \frac{32k}{3}$

$I_y = \int_0^2\int_0^{4-x^2}(x^2)kx\,dy\,dx = kx\int_0^2 x^2 y\Big|_0^2 dx = kx\int_0^2 2x^2\,dx = kx\left[\frac{2}{3}x^3\right]_0^2$

$\int k(4x^3 - x^5)\,dx = k\left[\frac{4x^4}{4} - \frac{x^6}{6}\right]_0^2 = \frac{16k}{3}$

$m = \int_0^2\int_0^{4-x^2}(kx)\,dy\,dx = 4k \qquad \bar{\bar{x}} = \sqrt{\frac{I_y}{m}} = \sqrt{\frac{16k}{3(4k)}} = \sqrt{\frac{4}{3}} \quad \frac{2}{\sqrt{3}}$

$\bar{\bar{y}} = \sqrt{\frac{I_x}{m}} = \sqrt{\frac{32k}{3(4k)}} = \frac{4}{\sqrt{3}} \qquad I_0 = ? = I_x + I_y = \frac{32k}{3} + \frac{16k}{3}$

14.4 Notes

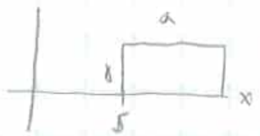
3) $\bar{x} = \frac{n_y}{m} = \frac{ka^2b}{2}\ \frac{1}{kab} = \frac{a}{2}$

$\bar{y} = \frac{n_x}{m} = \frac{kab}{3}\ \frac{2}{kab^2} = \frac{2}{3}b$

5) b) $\rho = ky$ $(\bar{x},\bar{y}) = (\frac{a}{2}, \frac{2}{3}b)$

c) $\rho = kx$ $(\bar{x},\bar{y}) = (\frac{2a}{3}, \frac{b}{2})$

9) $f(x,y) = \ln|\sec x|$



a) $\rho = K$ $(\bar{x},\bar{y}) = (\frac{a}{2}+5, \frac{b}{2})$

b) $\rho = ky$ $(\bar{x},\bar{y}) = (\frac{a}{2}+5, \frac{2b}{3})$

c) $\rho = kx$ $(\bar{x},\bar{y}) = (?, \frac{b}{2})$ $m = \int_R\int (kx)\,dy\,dx = \int_5^{5+a}\int_0^b (kx)\,dy\,dx$

$kb\int_5^{a+5} x\,dx = kb\ \frac{x^2}{2}\Big|_5^{a+5} = kb[(a+5)^2 - (5)^2] = \ldots\ \frac{kba}{2}(a+10)$

$M_y = \int_5^{a+5}\int_0^b (kx^2)\,dy\,dx = kb\int_5^{a+5} x^2\,dx = kb\ \frac{x^3}{3}\Big]_5^{a+5} = \frac{kba}{3}\ \cancel{a^3}^2 + 15a^{\cancel{2}} + 75\cancel{a} +$

$\bar{x} = \frac{M_y}{m} = \frac{\frac{kba}{3}(a^2+15a+75)}{\frac{kba}{2}(a+10)} = \frac{2}{3}\ \frac{a^2+15a+75}{a+10}$

13) $y = \sqrt{x}$ $y=0$ $x=4$ $\rho = kxy$

$M = \int_R\int \rho(x,y)\,dA = \int_0^4\int_0^{\sqrt{x}} kxy\,dy\,dx$

$= \int (kx)\frac{y^2}{2}\Big|_0^{\sqrt{x}}\,dx = \int k\frac{x^2}{2}\,dx = \frac{k}{2}\int x^2\,dx = \frac{k}{2}\Big[\frac{x^3}{3}\Big]_0^4 = \frac{64k}{6} = \frac{32k}{3}$

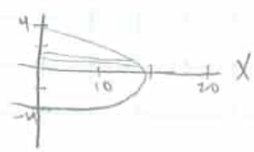
$M_x = \int_0^4\int_0^{\sqrt{x}} (kxy)\,y\,dy\,dx = \int_0^4 (kx)\frac{y^3}{3}\Big|_0^{\sqrt{x}} dx = \frac{1}{3}\int_0^4 k\,x^{5/2}\,dx\ \ \frac{k}{3}[x^{7/2}]_0^4 = \ldots\ \frac{256k}{21}$

$M_y = \int_0^4\int_0^{\sqrt{x}} (kxy)\,x\,dy\,dx = \int kx^2\frac{y^2}{2}\Big|_0^{x} dx = \int kx^2\frac{x}{2}dx = \int k\frac{1}{2}x^3 dx$

$= \ldots\ 32k$ $\quad \bar{x} = \frac{M_y}{m}\ \frac{32k}{32k/3} = 3$ $\quad \bar{y} = \frac{M_x}{m}\ \frac{256k}{21}\ \frac{3}{32k} = 8/7$

17) $x = 16 - y^2$ $x = 0$ $\rho = kx$

$\bar{y} = 0$



$m = \int_R\int \rho(x,y)\,dx\,dy = \int_{-4}^4\int_0^{16-y^2} (kx)\,dx\,dy$

$= \int_{-4}^{+4} k\frac{x^2}{2}\Big|_0^{16-y^2} dy = \int \quad = \ldots\ \frac{1}{2}k\Big[256y - \frac{32y^3}{3} + \frac{y^5}{5}\Big]_{-4}^{+4}$

$M_y = \int_{-4}^4\int_0^{16-y^2} x(kx)\,dx = \int_{-4}^{+4} k\frac{x^3}{3}\Big|_0^{16-y^2} dy = \int_{-4}^4 (k)(\frac{1}{3})[4096 - 768y^2 + 48y^4 - y^6]\,dy$

$k(\frac{1}{3})[4096y - \frac{768}{3}y^3 + \frac{48y^5}{5} - \frac{y^7}{7}]_{-4}^4$ $\quad \bar{X} = \frac{M_y}{m} = \frac{499\ k}{\frac{8192k}{15}} = 9.143$

21)

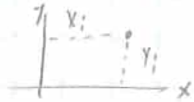
14.4) Notes

CENTER OF MASS & MOMENTS OF INERTIA

Laminar sections

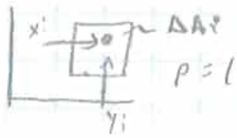
- constant density $\rho = k$ $A = \int_R \int dA$ ① $m = \int_R \int \rho(x,y)\, dA$
- Variable density $\rho(x,y)$

moments & center of mass



Point Mass, m_i Moment about x-axis $= m_i y_i$

" "   y axis $m_i x_i$



$\rho = (x_i, y_i)$ Moment about x-axis $= (\text{mass})(y_i) = \rho(x_i y_i)\Delta A_i y_i$

Moment about y-axis $= \rho(x_i y_i)\,\Delta A_i x_i$

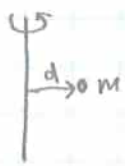
Extend to the entire area ② $M_x = \int_R \int y\, \rho(x,y)\, dA$ $M_y = \int_R \int x\, \rho(x,y)\, dA$

③ C. of mass $(\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right)$

Moments OF INERTIA

mass $F = ma$ $K.E. = \frac{1}{2}mv^2$

Rotational Motion – moment (I), $\tau = I\alpha$ $K. = \frac{1}{2}I\omega^2$



$I = md^2$ M_x & M_y are 1st moments $M_x = \int_R \int (\underline{y})\rho(x,y)\, dA$

Moments of Inertia are 2nd Moments

1. $I_x = \int_R \int (y)^2 \rho(x,y)\, dA$
2. $I_y = \int_R \int (x)^2 \rho(x,y)\, dA$
3. Polar Moment of Inertia $= \longrightarrow I_0 = \int_R \int \overbrace{(x^2+y^2)}^{r^2} \rho(x,y)\, dA = I_x + I_y$

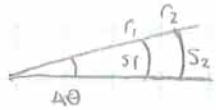
C. of mass : Radius of Gyration $(\bar{\bar{r}})$ $\bar{\bar{r}} = \sqrt{\frac{I}{m}}$

1) rotation about X-axis $\bar{\bar{y}} = \sqrt{\frac{I_x}{m}}$ 2) Rotation about y axis $\bar{\bar{x}} = \sqrt{\frac{I_y}{m}}$

3) Rotation about z axis (not in book) $\bar{\bar{r}} = \sqrt{\frac{I_0}{m}} = \sqrt{\frac{I_x + I_y}{m}}$

14.3 Polar Co-ordinates — Notes —

standard conversions $x = r\cos\theta$ $y = r\sin\theta$ $r^2 = x^2+y^2$ $\tan\theta = y/x$

A of polar sector  $\theta_2 - \theta_1 = \Delta\theta$ $\boxed{\text{ARC LENGTH } S = r\theta}$

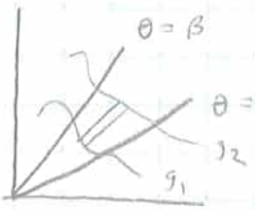
$s_1 = r_1\Delta\theta$ $S_2 = r_2\Delta\theta$ $\frac{s_1+S_2}{2} = \frac{r_1+r_2}{2}\Delta\theta$ = radius to center of the element

$*\Delta\theta = r\Delta\theta$

$\Delta A = \left(\frac{s_1+s_2}{2}\right)\Delta r = r\Delta\theta\,\Delta r$ $\boxed{dA = r\,dr\,d\theta}$ $dA = dy\,dx$ $\boxed{\text{OR}}$ $dA = dx\,dy$

$dA = r\,dr\,d\theta$ OR $dA = r\,d\theta\,dr$ $dy\,dx \neq dr\,d\theta$

$$\int_r\int f(x,y)\,dA = \int_\alpha^\beta \int_{g_1(\theta)}^{g_2(\theta)} f(\cos\theta, \sin\theta)\overbrace{(r\,dr\,d\theta)}^{dA} = \int_{r'}^{r_2} \int_{h(r)}^{h_2(r)} f(r\cos\theta, r\sin\theta)(r\,d\theta\,dr)$$

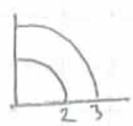


⇦ since α & θ are constant, make them the <u>outer</u> limits $(dr\,d\theta)$

r - simple region

⇦ order of Integration: $d\theta\,dr$ mixed up

θ - simple region

11.) $\int_0^{\pi/2}\int_2^3 \sqrt{9-r^2}\,r\,dr\,d\theta$  $= \int -\frac{1}{2}\int (9-r^2)^{\frac{1}{2}}\underbrace{(-2r\,dr)}\,d\theta$

$= \int -\frac{1}{2}(9-r^2)^{3/2}\left(\frac{2}{3}\right)\Big|_2^3\,d\theta$ $= \int -\frac{1}{3}\cancel{(9-9)^{3/2}} + \frac{1}{3}(9-4)^{3/2}\,]\,d\theta = \int_0^{\pi/2}\left(\frac{1}{3}\right)(5)\sqrt{5}\,d\theta$

$= \frac{5\sqrt{5}}{3}\theta\Big|_0^{\pi/2} = \boxed{\frac{5\sqrt{5}\pi}{6}}$

13) $\int_0^{\pi/2}\int_0^{1+\sin\theta} \theta r\,d\theta = \int_0^{\pi/2} \frac{r^2}{2}\Big|_0^{1+\sin\theta}\theta\,d\theta = \int\left(\frac{1}{2}\right)(1+\sin\theta)^2\,\theta\,d\theta$

$= \int \frac{1}{2}(1+2\sin\theta+\sin^2\theta)\,\theta\,d\theta = \frac{1}{2}\int\left[\theta + 2\theta(\sin\theta) + \theta\left[\frac{1-\cos 2\theta}{2}\right]\right]d\theta$

$= \frac{1}{2}\frac{\theta^2}{2}\Big|_0^{\pi/2} + \frac{1}{2}\int 2\theta\sin\theta\,d\theta + \frac{1}{2}\frac{1}{2}\frac{\theta^2}{2}\Big|_0^{\pi/2} - \frac{1}{4}\int\theta\cos 2\theta\,d\theta$

$= \frac{\pi^2}{16} + \frac{1}{2}\int + \frac{\pi^2}{32} \Rightarrow \frac{3\pi^2}{32}$ $\frac{1}{2}\int_0^{\pi/2} 2\theta\sin\theta\,d\theta = \int\theta\sin\theta\,d\theta = [\sin\theta - \theta\cos\theta]_0^{\pi/2} = 1$

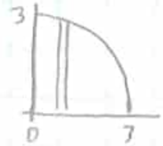
TABLES

#52) $\int u\sin u\,du = \sin u - u\cos u + C$ $-\frac{1}{4}\int\theta\cos 2\theta\,d\theta = \frac{1}{4}\frac{1}{2}\frac{1}{2}\int 2\theta\cos 2\theta\,(2d\theta)$

#53, $= -\frac{1}{16}[\cos 2\theta + (2\theta)\sin 2\theta]_0^{\pi/2} = \frac{1}{8}$ $\iint = \frac{3\pi^2}{32} + 1 + \frac{1}{8}$

14.3) Notes

17) $\int_0^3 \int_0^{\sqrt{9-x^2}} \underbrace{(x^2+y^2)}_{r^2}^{3/2} \underbrace{dy\,dx}_{r\,dr\,d\theta}$



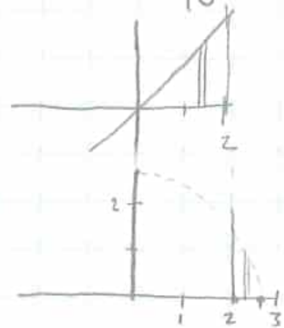
change limits, $\int_0^{\pi/2}\int_0^3 (r^2)^{3/2}\, r\,dr\,d\theta$

$= \iint r^4\,dr\,d\theta = \int \frac{r^5}{5}\Big|_0^3 d\theta = \int_0^{\pi/2} \frac{243}{5}\, d\theta = \frac{243\pi}{10}$

21) $\int_0^2\int_0^x \sqrt{x^2+y^2}\,dy\,dx + \int_2^{2\sqrt2}\int_0^{\sqrt{8-x^2}} \sqrt{x^2+y^2}\,dy\,dx$

$= \int_0^{\pi/4}\int_0^{2\sqrt2} \sqrt{r^2}\, r\,dr\,d\theta = \int_0^{\pi/4} \frac{r^3}{3}\Big|_0^{2\sqrt2} d\theta$

$= \int_0^{\pi/4} \frac{16\sqrt2}{3}\, d\theta = \frac{16\sqrt2\,\pi}{12} = \frac{4\sqrt2\,\pi}{3}$



$0 \le y \le x$

$0 \le x \le 2$

together forms 1/8 of a circle

$0 \le r \le 2\sqrt2,\quad 0 \le \theta \le \frac{\pi}{4}$

25) $f(x,y) = \arctan y/x$

R: $x^2+y^2 \ge 1$

$x^2+y^2 \ge 4$

$0 \le y \le x$

$\int_0^{\frac{1}{\sqrt2}}\int_{\sqrt{1-y^2}}^{\sqrt{4-y^2}} \arctan(y/x)\,dx\,dy + \int_{\frac{1}{\sqrt2}}^{\sqrt2}\int_y^{\sqrt{4-y^2}} \arctan(y/x)\,dx\,dy$

$1 \le r \le 2,\ 0 \le \theta \le \pi/4 \qquad \tan\theta = y/x$

$\int_0^{\pi/4}\int_1^2 r\,dr\,d\theta \qquad \boxed{\arctan(\tan\theta) = \arctan(y/x)}$

$= \int \frac{r^2}{2}\Big|_1^2 \theta\,d\theta = \int (2-\tfrac12)\theta\,d\theta = \tfrac32\, \frac{\theta^2}{2}\Big|_0^{\pi/4} = \frac{3}{2(2)}\left(\frac{\pi}{4}\right)^2 = \boxed{3\pi^2/64}$

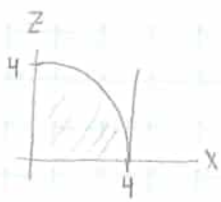
31) INSIDE THE HEMISPHERE & INSIDE THE CYLINDER

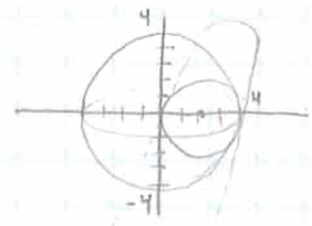
$z = \sqrt{16-x^2-y^2}$

$x^2+y^2-4x = 0$

$(x^2-4x+4)+y^2 = 4$

$(x-2)^2+y^2 = 4 \quad c(2,0)\ r=2$





$V = \iint (z)\, r\,dr\,d\theta \qquad z(\text{sphere}) = \sqrt{16-x^2-y^2} = \sqrt{16-r^2}$

$V = \int_0^{\pi}\int_0^{4\cos\theta} \sqrt{16-r^2}\; r\,dr\,d\theta$

$-\frac12\int (16-r^2)^{\frac12}(-2r\,dr) = -\frac12 (16-r^2)^{3/2}\left(\frac23\right) = -\frac13 (16-r^2)^{3/2}\Big|_0^{4\cos\theta}$

$= -\frac13\left[(16-(4\cos\theta)^2)^{3/2} - (16-0)^{3/2}\right] = -\frac13\left[(16-16\cos^2\theta)^{3/2} - 64\right] = \ldots -\frac13(64)\left[(\sin^2\theta)^{3/2} - 1\right]$

$= -\frac13(64)\left[\sin^3\theta - 1\right] \qquad 2\int_0^{\pi/2}\left(-\frac13\right)(64)(\sin^3\theta - 1)\,d\theta = \frac{-128}{3}\int_0^{\pi/2}(\sin^3\theta-1)\,d\theta$

$= \frac{128}{\pi}\int_0^{\pi/2}(1-\sin^3\theta)\,d\theta$ WALLIS' FMLA $= \frac{128}{3}\left(\frac{\pi}{2} - \frac23\right)$

14.1) ~~#6~~ 1-63 odds

1) $\int_0^x (2x-y)\,dy = 2xy - \frac{1}{2}y^2\Big]_0^x = 2x(x) - \frac{1}{2}(x)^2 = \boxed{\frac{3}{2}x^2}$

3) $\int_1^{2y} \frac{y}{x}\,dx \quad y>0 \quad = y\ln x\Big|_1^{2y} = y\ln(2y) - y\ln 1$

$= y\ln 2y \;(y>0)$

5) $\int_0^{\sqrt{4-x^2}} x^2 y\,dy = \frac{1}{2}x^2y^2\Big|_0^{\sqrt{4-x^2}} = \frac{1}{2}x^2(4-x^2) = \boxed{2x^2 - \frac{1}{2}x^4}$

7) $\int_{e^y}^{y} \frac{y\ln x}{x}\,dx \quad y>0 \quad y\int \frac{\ln x}{} \frac{dx}{x} = y\frac{(\ln x)^2}{2}\Big|_{e^y}^{y} = \frac{y}{2}(\ln y)^2 - \frac{y}{2}(\ln e^y)^2$

$= \frac{y}{2}\ln y^2 - \frac{y}{2}y^2 = \boxed{\frac{y}{2}(\ln y^2 - y^2)} \quad y>0$

review this problem 9) $\int_0^{x^3} ye^{-y/x}\,dy =$ int. by parts $u=y \quad du = dy \quad dv = e^{-y/x}dy$

$\int u\,dv = uv - \int v\,du = y(-xe^{-y/x}) - \int -xe^{-y/x}dy = -xye^{-y/x} - \left(\frac{1}{x}(xe^{-y/x})\right)$

$= -xye^{-y/x} - e^{-y/x}$

$\int u\,dv \quad = \quad uv \quad - \int v\,du$

$u = y$

$du = dy$

$dv = e^{-y/x}dy$

$v = -xe^{-y/x}$

$\int_0^{x^3} ye^{-y/x}\,dy = \left[-xye^{-y/x}\right]_0^{x^3} + x\int_0^{x^3} e^{-y/x}\,dy$

$= -x(x^3)e^{-x^3/x} + \left[x(x)e^{-y/x}\right]_0^{x^3}$

$= -x^4e^{-x^2} + x^2e^{-x^2}$

$\boxed{= e^{-x^2}(x^2 - x^4)}$ X wrong

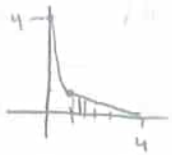
11) $\int_0^1\int_0^2 (x+y)\,dy\,dx = \int_0^1\left[\frac{1}{2}x^2 + xy\right]_0^2 dx = \int_0^1 (2+2x)\,dx = 2x + x^2\Big|_0^1 = \boxed{3}$

skip 13) $\int_0^\pi\int_0^{\sin x} (1+\cos x)\,dy\,dx = \int_0^\pi y + y\cos x\Big|_0^{\sin x} dx = \int_0^\pi \sin x + \sin x\cos x\,dx$

$= \int_0^\pi \sin x + \frac{1}{2}\sin(2x)\,dx = \left[-\cos x + \frac{1}{2}\sin^2 x\right]_0^\pi$

14.1) Notes

35) $\sqrt{x}+\sqrt{y}=2$ $\quad x=0\ y=0 \quad \sqrt{y}=2-\sqrt{x}$

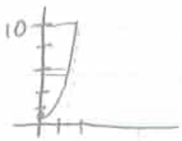


$y=(2-\sqrt{x})^2$ or $x=(2-\sqrt{y})^2$

$A=\int_0^4\int_0^{(2-\sqrt{x})^2} dy\,dx = \int_0^4 y\Big|_0^{(2-\sqrt{x})^2} dx = \int_0^4 (4-4\sqrt{x}+x)\,dx$

$=\left(4x-\frac{4x^{3/2}}{3/2}+\frac{x^2}{2}\right)\Big|_0^4 = 4x - 8/3\,x\sqrt{x}+\frac{1}{2}x^2)\Big|_0^4 = \boxed{8/3}$

45) $\int_1^{10}\int_0^{\ln y} f(x,y)\,dx\,dy$ $\quad 0\le x\le \ln y \quad 1\le y\le 10 \quad x=\ln y \quad y=e^x \quad x=\ln 10 \quad x\cong 2.3$

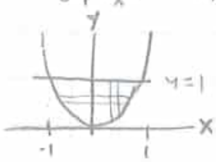


Change to vertical element $dy\,dx$ $\quad e^x\le y\le 10 \quad 0\le x\le \ln 10$

$\int_0^{\ln 10}\int_{e^x}^{10} f(x,y)\,dy\,dx$

47) $\int_{-1}^1\int_{x^2}^1 f(x,y)\,\underbrace{dy\,dx}^{\text{vertical element}}$ $\quad x^2\le y\le 1 \quad -1\le x\le +1 \quad x^2=y \quad x=\pm\sqrt{y}$

$\int_0^1\int_{-\sqrt{y}}^{\sqrt{y}} f(x,y)\,dx\,dy$



51) $\int_0^1\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} dx\,dy$ $\quad -\sqrt{1-y^2}\le x\sqrt{1-y^2} \quad x=\sqrt{1-y^2} \quad x^2=1-y^2 \quad x^2+y^2=1$

$\int_0^1\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} dx\,dy = \int_0^1 x\Big|_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} dy = \int_0^1(\sqrt{1-y^2}+\sqrt{1-y^2})\,dy = \int_0^1 2\sqrt{1-y^2}\,dy$

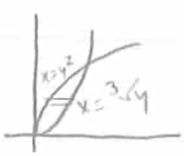
$\int\sqrt{a^2-u^2}\,du = \frac{1}{2}(u\sqrt{a^2-u^2}+a^2\arcsin\frac{u}{a})+\cancel{C}$

$\int_0^1 2\sqrt{1-y^2}\,dy = 2(\frac{1}{2})(y\sqrt{1-y^2}+(1)^2\arcsin|y/1|)\Big|_0^1$

$=[1\sqrt{1-1}+\arcsin(1)]-[0+\arcsin(0)] = \boxed{\pi/2}$ $\quad 0\le y\le\sqrt{1-x^2} \quad -1\le x\le +1$

$\int_{-1}^{+1}\int_0^{\sqrt{1-x^2}} dy\,dx = \frac{\pi}{2}$ $\quad \int_0^1\int_{y^2}^{\sqrt[3]{y}} dx\,dy \quad y^2\le x\le\sqrt[3]{y} \quad 0\le y\le 1$ ① $x=y^2$ ② $x=\sqrt[3]{y}$

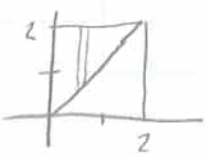
$y=\pm\sqrt{x}$ $\quad y=x^3$



$\int_0^1\int_{y^2}^{\sqrt[3]{y}} dx\,dy = \int_0^1 \Big|_x^{\sqrt[3]{y}} dy = \int_0^1(\sqrt[3]{y}-y^2)\,dy = \frac{y^{4/3}}{4/3}-\frac{y^3}{3}\Big|_0^1 = \boxed{\frac{5}{12}}$

vertical element: $\int_0^1\int_{x^3}^{\sqrt{x}} dy\,dx = \boxed{5/12}$ SAME

61) $\int_0^2\int_x^2 x\sqrt{1-y^3}\,dy\,dx = ?$ $\quad$ Horizontal element $\quad 0\le x\le y \quad 0\le y\le 2$



$x\le y\le 2$

$0\le x\le 2$

$\int_0^2\int_0^y (x)\sqrt{1+y^3}\,dx\,dy = \int_0^2\sqrt{1+y^3}\,\frac{x^2}{2}\Big|_0^y dy = \frac{1}{2}\int_0^2(1+y^3)^{\frac{1}{2}}(y^2dy)$

$1/3 \quad 3$

$=\frac{1}{6}(1+y^3)^{3/2}\,\frac{2}{3}\Big|_0^2 = \frac{26}{9}$

63) $\int_0^1\int_y^1 \sin(x^2)\,dx\,dy$

$y\le x\le 1$

$0\le y\le 1$

$=f$

Vertical element $\Rightarrow 0\le y\le x \quad 0\le x\le 1$

$\int_0^1\int_0^x \sin(x)^2\,dy\,dx = \int_0^1 \sin(x)^2\,y\Big|_0^x dx$

$=\frac{1}{2}\int_0^1 \sin(x^2)(2x\,dx) = -\frac{1}{2}\cos(x)^2\Big|_0^1$

$=-\frac{1}{2}\cos(1)+\frac{1}{2}\underbrace{\cos 0}_{1} = -0.27+0.5 = 0.23$

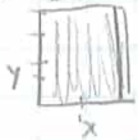
14.2 | 33-41 odds   49-55 odds

33) $z = xy \quad z = 0 \quad y = x \quad x = 1$

$$\int_0^1 \int_0^x xy \, dy\,dx = \int_0^1 \left[\tfrac{1}{2} x y^2 \right]_0^x dx = \int_0^1 \tfrac{1}{2} x^3 \, dx = \tfrac{1}{8} x^4 \Big]_0^1$$

$\boxed{= \tfrac{1}{8}}$ ✓ Great Job!

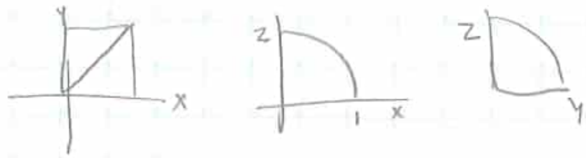
35) $z = 0 \quad z = x^2 \quad x = 0 \quad x = 2 \quad y = 0 \quad y = 4$



$$\int_0^2 \int_0^4 x^2 \, dy\,dx = \int_0^2 x^2 y \Big]_0^4 dx = \int_0^2 4x^2 \, dx$$

$$= \int_0^2 \tfrac{4}{3} x^3 \Big]_0^2 = \tfrac{4}{3}(2)^3 = \tfrac{4}{3}(8) = \boxed{\tfrac{32}{3}} \checkmark$$

37) $x^2 + z^2 = 1 \quad y^2 + z^2 = 1$, 1st octant



$z = \sqrt{1-x^2} \qquad x^2 + 1 = y^2 + 1 \quad x^2 = y^2 \quad x = y$

$$2\int_0^1\int_0^y \sqrt{1-x^2}\, dy\,dx = 2\int_0^1 y\sqrt{1-x^2} \Big|_0^x dx = 2\int_0^1 x\sqrt{1-x^2}\, dx = (-1)\int_0^1 -2x\sqrt{1-x^2}\, dx$$

$$= (-)\tfrac{2}{3} \, \frac{\sqrt{1-x^2}^{(3/2)}}{} \Big]_0^1 = (-)\tfrac{2}{3}\sqrt{0} - \left[(-)\tfrac{2}{3} \, 1^{3/2}\right] = \boxed{+\tfrac{2}{3}} \checkmark$$

39) $z = x + y \quad x^2 + y^2 = 4$, 1st octant

$x^2 = 4 - y^2$

$x = \sqrt{4-y^2}$

$$\int_0^2 \int_0^{\sqrt{4-y^2}} x + y \, dx\,dy = \int_0^2 \tfrac{1}{2}x^2 + yx \Big]_0^{\checkmark} dy$$

$$= \int_0^2 \tfrac{1}{2}(4-y^2) + y\sqrt{4-y^2}\, dy = \int_0^4 2 - \tfrac{y^2}{2} + y\sqrt{4-y^2}\, dy$$

$$= 2y + (-\tfrac{1}{2})(\tfrac{1}{3})y^3 \Big]_0^2 + \tfrac{-1}{2}\int_0^2 -2y\sqrt{4-y^2}\, dy$$

$$+ \tfrac{-1}{2}\sqrt{4-y^2}^{3/2} \Big]_0^2$$

$$= 2(2) - \tfrac{1}{6}(8) \cancel{- \tfrac{1}{2}\sqrt{0}} (-) - \tfrac{1}{2}\sqrt{4}^{3/2}$$

$$= 4 - \tfrac{4}{3} + \tfrac{1}{2}(2)^{3/2}$$

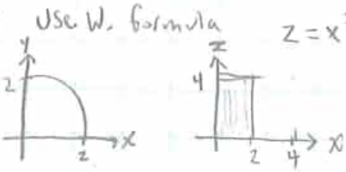
$$= \tfrac{8}{3} + \tfrac{1}{2}(2\sqrt{2}) = \tfrac{8}{3} + \tfrac{2\sqrt{2}}{1} \quad X$$

Wallis' Formula

n is odd: $(n \geq 3)$ $\quad \int_0^{\pi/2} \cos^n x\, dx = \left(\tfrac{2}{3}\right)\left(\tfrac{4}{5}\right)\left(\tfrac{6}{7}\right)\cdots\left(\tfrac{n-1}{n}\right)$

n is even: $(n \geq 2)$ $\quad \int_0^{\frac{\pi}{2}} \cos^n x\, dx = \left(\tfrac{1}{2}\right)\left(\tfrac{3}{4}\right)\left(\tfrac{5}{6}\right)\cdots\left(\tfrac{n-1}{n}\right)\left(\tfrac{\pi}{2}\right)$

41) Use W. formula $\quad z = x^2 + y^2 \quad x^2 + y^2 = 4 \quad z = 0$



$$4\int_0^2\int_0^{\sqrt{4-x^2}} x^2 + y^2 \, dy\,dx = 4\int_0^2 \left[x^2 y + \tfrac{1}{3}y^3 \right]_0^{\sqrt{4-x^2}} dx$$

$$= 4\int_0^2 x^2\sqrt{4-x^2} + \tfrac{1}{3}\sqrt{4-x^2}^3 \, dx \qquad x = 2\sin\theta$$

$$= 4\int_0^2 4\sin^2\theta\sqrt{4-4\sin^2\theta} + \tfrac{1}{3}(4-4\sin^2\theta)^{3/2} \, d\theta$$

$$4\int_0^2 4\sin^2\theta\sqrt{4\cos^2\theta} + \tfrac{1}{3}(2\cos\theta)^3$$

$$4\sin^2\theta \, 2\cos\theta + \tfrac{1}{3}(2\cos\theta)^3$$

$$\sin^2\theta(8\cos\theta) + \tfrac{8}{3}\cos^3\theta$$

$$\rightarrow 4\int_0^{\pi/2} (1-\cos^2\theta)(8\cos\theta) + \tfrac{8}{3}\cos^3\theta)\, d\theta$$

$$\int_0^{\pi/2} (8\cos\theta - 8\cos^3\theta + \tfrac{8}{3}\cos^3\theta)\, d\theta$$

$$32\int_0^{\pi/2} (\cos\theta - \cos^3\theta + \tfrac{1}{3}\cos^3\theta)$$

14.2 | 1-29 odds 33-41 odds 49-55 odds

1) $\int_0^4\int_0^2 (x+y)\,dy\,dx = \int_0^4 xy+\frac{1}{2}y^2\Big|_0^2\,dx = \int_0^4 2x+2\,dx = x^2+2x\Big|_0^4$
$= 16+8 = \boxed{24}$

3) $\frac{1}{2}\int_0^4\int_0^2 x^2y\,dy\,dx = \frac{1}{2}\int_0^4 x^2\left(\frac{1}{2}\right)y^2\Big|_0^2\,dx = \frac{1}{2}\int_0^4 2x^2\,dx = \frac{1}{2}\left[\frac{2}{3}x^3\right]_0^4 \quad 64\left(\frac{2}{3}\right)$
skip. copied wrong prob.

5) skip

7) $\int_0^2\int_0^1 (1+2x+2y)\,dy\,dx \quad \int_0^2 y+2xy+y^2\Big|_0^1\,dx = \int_0^2 2+2x\,dx = 2x+x^2\Big|_0^2 = \boxed{8}$
$z=1+2x+2y$ 

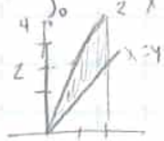
9) $\int_0^6\int_{y/2}^3 (x+y)\,dx\,dy = \int_0^6 \frac{1}{2}x^2+yx\Big|_{y/2}^3\,dy = \int_0^6 \left[\frac{9}{2}+3y\right]-\left(\frac{1}{2}\left(\frac{y^2}{4}\right)+\frac{y^2}{2}\right]dy$
$= \int_0^6 \frac{9}{2}+3y-\frac{y^2}{8}-\frac{4y^2}{8}\,dy = \int_0^6\left(\frac{9}{2}+3y-\frac{5y^2}{8}\right)dy = \left[\frac{9}{2}y+\frac{3}{2}y^2-\frac{5}{8}\left(\frac{1}{3}\right)y^3\right]_0^6$
$27+\frac{3}{2}(36)-45 = \boxed{36}$

11) $\int_{-a}^a\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} (x+y)\,dy\,dx \quad \int_{-a}^a xy+\frac{1}{2}y^2\Big]_{*}^{*} \Rightarrow \int_{-a}^a x\sqrt{a^2-x^2} - x-\sqrt{} + \frac{1}{2}(a^2-x^2) - \frac{1}{2}(\quad)\,dx$

How to do this step? $= \int_{-a}^a 2x\sqrt{a^2-x^2}\,dx = \left[-\frac{2}{3}(a^2-x^2)^{3/2}\right]_{-a}^a = 0$

13) $\int_R\int xy\,dA$ rectangle 

$= \int_0^3\int_0^5 xy\,dy\,dx = \int_0^3 \frac{1}{2}xy^2\Big|_0^5\,dx$
$\int_0^3 \frac{25}{2}x\,dx = \frac{1}{2}\left(\frac{25}{2}\right)x^2\Big|_0^3 = \frac{25}{4}(9) = \boxed{\frac{225}{4}}$

15) 

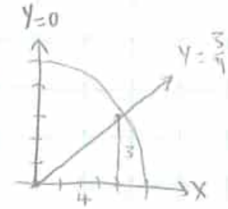
$\cancel{\int_R\int \frac{y}{x^2+y^2}\,dA = \int_0^2\int_x^{2x}\frac{y}{x^2+y^2}\,dy\,dx} = \int_0^2\frac{1}{2}\int_x^{2x}\frac{2y}{x^2+y^2}\,dy\,dx$
$\int_0^2 \frac{1}{2}\left[\ln(x^2+y^2)\right]_x^{2x}\,dx = \frac{1}{2}\int_0^2 \ln(x^2+4x^2)-\ln(x^2+x^2)\,dx = \frac{1}{2}\int_0^2\left(\ln 5x^2-\ln 2x^2\right)dx$
$= \frac{1}{2}\ln\frac{5}{2}\int_0^2 dx = (2)\frac{1}{2}\left(\ln\frac{5}{2}\right) = \boxed{\ln\frac{5}{2}}$

? 17) $\int_R\int -2y\ln x\,dA \quad y=4-x^2 \quad y=4-x$ 

$\int_0^1\int_{4-x}^{4-x^2} -2y\ln x\,dy\,dx$
$= \int_0^1 -y^2\ln x\Big|_{4-x}^{4-x^2}dx = \int_0^1 (4-x^2)^2-(4-x)^2(-\ln x)\,dx = \int_0^1 \cancel{16}+x^4-8x^2\cancel{-16}-x^2+8x$
$-\int_0^1(\ln x)(x^4-9x^2+8x)\,dx$ #89 tables

14.2 HW 19–29 odds 33–41 odds 49–55 odds

7



19) $\iint_R x\,dA$ circ in Q1. $y=\sqrt{25-x^2}$ $3x-4y=0$ $y=0$

$y^2=25-x^2$ $y=\frac{3}{4}x$

$y^2+x^2=25$ $r^2=25$

$\int_0^{\arctan\frac{3}{4}}\int_0^5 (25-r^2)\,r\,dr\,d\theta = \int\int_0^5 25r-r^3\,dr\,d\theta = \int_0^{.64} \frac{25}{2}r^2-\frac{1}{4}r^4\Big]_0^5 d\theta = \int_0^{.64}(625/4)\,d\theta$

$\frac{625}{4}(\pi/2-.64)$ book does it 145.3

21) $z=y/2$ $0\le x\le 4$ $0\le y\le 2$ $\int_0^4\int_0^2 \frac{1}{2}y\,dy\,dx = \int_0^4\left[\frac{1}{2}\frac{1}{2}y^2\right]_0^2 dx$

$=\int_0^4 \frac{1}{4}(4)\,dx = \left[1x\right]_0^4 = \boxed{4}$

23) $z=4-x-y$ $y=x$ $y=2$

$\int_0^2\int_0^2 4-x-y\,dx\,dy = \int_0^2\left[4x-\frac{1}{2}x^2-yx\right]_0^2 dy$

$=\int_0^2 8-\frac{1}{2}(8)-y(2) = \int_0^2 4-2y = 4y-y^2\Big]_0^2 = 8-4=\boxed{4}$

25) $2x+3y+4z=12$ $0\le x\le 6$ $\int_0^6\int_0^4 3-\frac{1}{2}x-\frac{3}{4}y\,dy\,dx = \int_0^6 3y-\frac{1}{2}xy-\frac{3}{4}\left(\frac{1}{2}\right)y^2\Big]_0^4 dx$

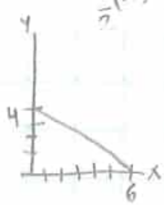
$4z=12-2x-3y$ $0\le y\le 4$ $=\int_0^6 12-2x-\frac{3}{2}(16)\,dx = 12x-2\left(\frac{1}{2}\right)x^2-24x\Big|_0^6$

$z=3-\frac{1}{2}x-\frac{3}{4}y$

$\frac{1}{2}x+\frac{3}{4}y=3$ ① set $z=0$

$\frac{3}{4}y=3-\frac{1}{2}x$ ② solve for y

$y=4-\frac{1}{2}\left(\frac{4}{3}\right)x$ $y=4-\frac{2}{3}x$

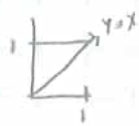


view graph from x-y plane ($z=0$)

27) $z=1-xy$

$y=x$

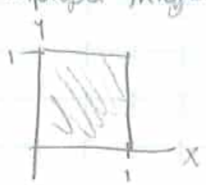
$y=1$



$\int_0^1\int_0^y 1-xy\,dx\,dy$ $\int_0^1 x-\frac{y}{2}x^2\Big|_0^y dy = \int_0^1 y-\frac{y^3}{2}\,dy$

$=\frac{y^2}{2}-\frac{1}{4}\frac{y^4}{2}\Big|_0^1 = \frac{1}{2}-\frac{1}{8}=\boxed{\frac{3}{8}}$ ✓!

29) Improper Integral $z=\frac{1}{(x+1)^2(y+1)^2}$ $0\le x\le\infty$ $0\le y\le\infty$



$\int_0^\infty\int_0^\infty \frac{1}{(x+1)^2(y+1)^2}\,dy\,dx = \int_0^\infty\left[-\frac{1}{(x+1)^2(y+1)}\right]_0^\infty dx =$

$=-\left[\frac{1}{(x+1)^2(\infty)}\right]-(-)\left[\frac{1}{(x+1)^2(0+1)}\right] = \int_0^\infty \frac{1}{(x+1)^2}\,dx = = -\frac{1}{(x+1)}\Big]_0^\infty$

$=-\frac{1}{\infty}-(-)\frac{1}{1}=\boxed{1}$

14.2 Notes

29) $\frac{1}{(x+1)^2(y+1)^2}$ $\quad 0 \le x \le \infty \quad 0 \le y \le \infty \qquad \int_0^\infty \int_0^\infty \frac{1}{(x+1)^2(y+1)^2}\,dy\,dx$

$= \int_0^\infty \frac{1}{(x+1)^2}(y+1)^{-2}\,dy\,dx \quad = \iint_0^\infty \frac{1}{(x+1)^2}(y+1)^{-2}\,dy\,dx$

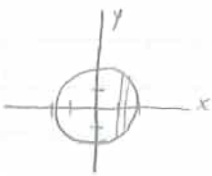
$\int_0^\infty \text{ // } \left[\frac{-1}{(y+1)}\right]_0^\infty dx$

$= \text{ // } \left[\frac{1}{\infty+1} + \frac{1}{0+1}\right] dx$

$= \int_0^\infty \frac{1}{(x+1)^2}(1)\,dx = \frac{-1}{x+1}\Big|_0^\infty = \frac{-1}{\infty+1} + \frac{1}{0+1} = 1.0$

* circle - things work well with polar coordinates *

31) $z = 4 - x^2 - y^2 \quad z = 0 \quad x^2+y^2 = 4 \quad \int_{-2}^{+2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (4-x^2-y^2)\,dy\,dx$

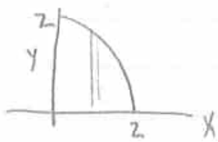


change to polar:

$\int_0^{2\pi}\int_0^2 (4-r^2)\,r\,dr\,d\theta = \int\int (4r - r^3)\,dr$

$= \int \left(\frac{4r^2}{2} - \frac{r^4}{2}\right)\Big|_0^2 d\theta = \_\, 4\theta\,\Big|_0^{2\pi} = 8\pi$

39) $z = x+y \quad x^2+y^2 = 4 \quad V = \int\int z\,dy\,dx = \iint (x+y)\,dy\,dx$

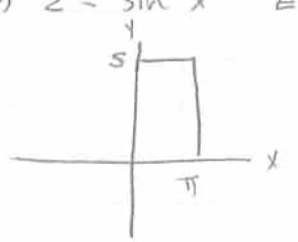


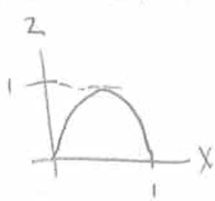
$= \int_0^2 \int_0^{\sqrt{4-x^2}} (x+y)\,dy\,dx = \int_0^2 \left(xy + \frac{y^2}{2}\right)\Big|_0^{\sqrt{4-x^2}} dx$

$= \int_0^2 \left[x\sqrt{4-x^2} + \frac{4-x^2}{2}\right] dx = \int_0^2 \left(2 - \frac{x^2}{2} + \sqrt{4-x^2}\;x\right) dx$

$= \int 2\,dx - \frac{1}{2}\int x^2\,dx - \frac{1}{2}\int (4-x^2)^{\frac{1}{2}}(-2x\,dx) = 2x - \frac{1}{2}\frac{x^3}{3} - \frac{1}{2}\frac{(4-x^2)^{3/2}}{3/2}\Big|_0^2 \quad \frac{16}{3}$

42) $z = \sin^2 x \quad z = 0 \quad 0 \le x \le \pi \quad 0 \le y \le 5$





$V = \iint z\,dx\,dy = \int_0^5 \int_0^\pi \sin^2 x\,dx\,dy$

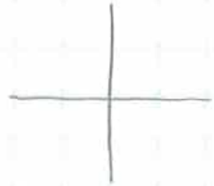
integration tables

do left side then double it

$0, \to \pi/2$ for limits

14-1 39-63 odds

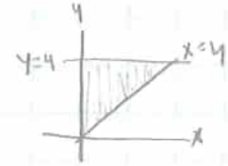
~~39)~~ $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



$\int_0^a \int_0^{b/a\sqrt{a^2-x^2}} dy\,dx$

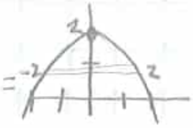
skip

41) sketch region & switch order $\int_0^4 \int_0^y f(x,y)\,dx\,dy$



$\int_0^4 \int_0^x f(x,y)\,dy\,dx$

43) $\int_{-2}^{2} \int_0^{\sqrt{4-x^2}} f(x,y)\,dy\,dx =$



$\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} f(x,y)\,dx\,dy$

$y = \sqrt{4-x^2}$

$0 \le y \le \sqrt{4-x^2} \quad -2 < x < 2$

$y = \sqrt{4-x^2} \quad y^2 = 4-x^2 \quad x^2 = \sqrt{4-y^2}$

45) $\int_0^{10} \int_0^{\ln y} f(x,y)\,dx\,dy \qquad 0 \le x \le \ln y \quad 0 \le y \le 10$

$\int \int_{e^x}^{10} f(x,y)\,dy\,dx \qquad x = \ln y \quad e^x = e^{\ln y} = y$

47) $\int_{-1}^{1} \int_{x^2}^{1} f(x,y)\,dy\,dx \qquad x^2 \le y \le 1 \quad -1 \le x \le 1$

$y = x^2 \quad x = \sqrt{y}$

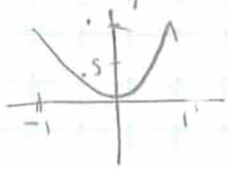
$\int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} dx\,dy$

49) $\int_0^1 \int_0^2 dy\,dx$

$= \int_0^2 \int_0^1 dx\,dy \quad \boxed{2}$

51) $\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} dx\,dy \quad = -\sqrt{1-y^2} \le x \le \sqrt{1-y^2}$

$\int_{-1}^{1} \int_0^{\sqrt{1-x^2}} dy\,dx$



$x^2 = 1-y^2 \quad y = \sqrt{1-x^2}$

$= \int_{-1}^{1} y \Big|_0^{\sqrt{1-x^2}} dx \quad = \int_{-1}^{1} \sqrt{1-x^2}\,dx$

$= \frac{1}{2}\left(x\sqrt{1-x^2} + \arcsin x\right)\Big|_{-1}^{1}$

$= \frac{1}{2}\left(\sqrt{0} + \arcsin 1\right) - \left(\sqrt{0} + \arcsin(-1)\right)$

$\frac{1}{2}\left(\frac{\pi}{2} - (-)\frac{\pi}{2}\right) = \frac{1}{2}\pi = \boxed{\frac{\pi}{2}}$

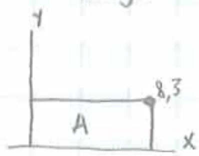
14.11 homework 25-63 odds

25) $\int_1^\infty \int_0^{1/x} y\,dy\,dx = \int_1^\infty \frac12 y^2 \Big|_0^{1/x} dx = \int_1^\infty \frac12 x^{-2}\,dx = \frac12(-1)x^{-1}\Big|_1^\infty$

$= -\frac12\left(\frac1x\right)\Big|_1^\infty = 0-\left(-\frac12\right) = \boxed{\frac12}$

27) $\int_1^\infty\int_1^\infty \frac{1}{xy}\,dx\,dy \quad \iint x^{-1}y^{-1}\,dy = \int_1^\infty \frac1y \ln x\Big|_1^\infty dy = \int_1^\infty \left[\frac1y\infty - \frac1y(0)\right]dy$

diverges

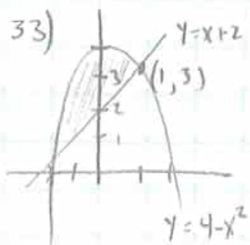
29) 

$\int_0^3\int_0^8 dx\,dy = \int_0^3 x\Big|_0^8 dy = 8y\Big|_0^3 = 8(3) = \boxed{24}$

31) 

$\int_0^2\int_0^{4-x^2} dy\,dx = \int_0^2 y\Big|_0^{4-x^2}dx = \int_0^2 4-x^2\,dx = 4x-\frac13x^3\Big|_0^2$

$4(2)-\frac13(2)^3 - \left[4(0)-\frac13(0)^3\right] = 8-\frac13(8) = \frac23(8) = \boxed{\frac{16}{3}}$

33) 

$\int_{-2}^1\int_{x+2}^{4-x^2} dy\,dx = \int_{-2}^1 (4-x^2)-(x+2)\,dx = \int_{-2}^1 -x^2-x+2\,dx$

$= -\frac13x^3 - \frac12x^2+2x\Big|_{-2}^1 = -\frac13(1)^3-\frac12(1)^2+2(1) - \left[-\frac13(-2)^3 - \frac12(-2)^2+2(-2)\right]$

$= -\frac13-\frac12+2 - \left(\frac13(8) - \frac12(4) - 4\right) = -\frac56+2-\frac83+2+4 = 8-\frac{21}{6}$

$= 8-\frac72 = \boxed{\frac92}$

35) $\sqrt x+\sqrt y = 2 \quad x=0, y=0$

$\sqrt x - 2 = \sqrt y$

$y = (\sqrt x-2)^2$

$\int_0^4\int_0^{(\sqrt x-2)^2} dy\,dx = \int_0^4 (\sqrt x-2)^2\,dx$

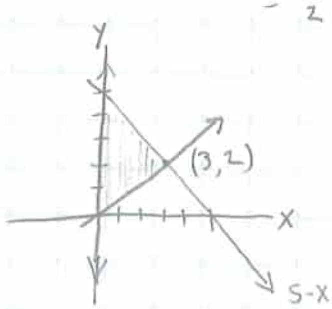
$= \int_0^4 x-4\sqrt x+4\,dx = \frac12x^2 - 4\left(\frac23\right)x^{3/2}+4x\Big]_0^4$

$= \frac12(16) - \frac83(8)+16 = 24-\frac{64}{3} = \boxed{\frac83}$

37) $2x-3y=0 \quad y=\frac23x$

$x+y=5 \quad y=5-x$

$y=0 \quad y=0$



$\int_0^3\int_{2/3x}^{5-x} dy\,dx = \int_0^3 5-x-\left(\frac23x\right)\,dx$

$=\int_0^3 5-\frac53x = 5x-\frac53\left(\frac12\right)x^2\Big]_0^3$

$15-\frac56(9) = 15-\frac{5\cdot\cancel{9}3}{2\cancel{6}} = 15-\frac{15}{2} = \frac{15}{2}$

finish this

14.1 15-63 odds

15) $\int_0^1\int_0^x \sqrt{1-x^2}\,dy\,dx = \int_0^1 \sqrt{1-x^2}(y)\Big|_0^x dx = \int_0^1 x\sqrt{1-x^2}\;\; dx$

$= -\frac{1}{2}\left(\frac{2}{3}\right)(1-x^2)^{3/2}\Big|_0^1$

$\int u\,dv = uv - \int v\,du$

$u = \sqrt{1-x^2} \quad du = \frac{2}{3}(1-x^2)^{3/2}$

$dv = x\,dx \quad v = \frac{1}{2}x^2$

$= \left(\frac{1}{2}x^2\sqrt{1-x^2}\right) + \int \frac{1}{\cancel{2}}x^2\left(\frac{\cancel{2}}{3}\right)(1-x^2)^{3/2}$

$+ \int \frac{1}{3}x^2(1-x^2)^{3/2}$

13) $\int_0^\pi\int_0^{\sin x}(1+\cos x)\,dy\,dx$

$y + (\cos x)y\Big]_0^{\sin x}$

$= \int_0^\pi \sin x + \cos x \sin x\,dx$

17) $\int_1^2\int_0^4 (x^2-2y^2+1)\,dx\,dy$

$\frac{1}{3}x^3 - 2y^2x + x\Big]_0^4 = \frac{1}{3}(64) - 2y^2(4) + 4 = \int_1^2\left(\frac{76}{3} - 8y^2\right)dy$

$= \frac{76}{3}y - \frac{8}{3}y^3\Big]_1^2 \quad \frac{76}{3}(2-1) - \frac{8}{3}(8-1) = \frac{76}{3} - \frac{56}{3} = \boxed{\frac{20}{3}}$

19) $\int_0^1\int_0^{\sqrt{1-y^2}} (x+y)\,dx\,dy = \int_0^1 \frac{1}{2}x^2 + yx\Big]_0^{\sqrt{1-y^2}} dy = \int_0^1 \frac{1}{2}(1-y^2) + y\left(\sqrt{1-y^2}\right)\,dy$

$\int_0^1 \frac{1}{2} - \frac{1}{2}y^2 + y\sqrt{1-y^2}\,dy \quad = \frac{1}{2}y - \frac{1}{6}y^3 + -\frac{1}{3}(1-y^2)^{3/2}\Big]_0^1$

$u = 1-y^2 \quad du = -2y\,dy \quad = \frac{1}{2} - \frac{1}{6} \cancel{-\frac{1}{3}(0)} - \left(-\frac{1}{3}\right)(1)$

$= \frac{1}{2} - \frac{1}{6} + \frac{1}{3} = \frac{3}{6} - \frac{1}{6} + \frac{2}{6} = 4/6 = \boxed{2/3}$

$\int \sqrt{u}\,du = \int -2y\sqrt{1-y^2}\,dy$

$-\frac{1}{2}\int u^{1/2}du = \frac{-1}{\cancel{2}}\left(\frac{\cancel{2}}{3}u^{3/2}\right) =$

$= -\frac{1}{3}u^{3/2} = -\frac{1}{3}(1-y^2)^{3/2}$

21) $\int_0^2\int_0^{\sqrt{4-y^2}} \frac{2}{\sqrt{4-y^2}}\,dx\,dy = \int_0^2 \frac{2x}{\sqrt{4-y^2}}\Big|_0^{\sqrt{4-y^2}} dy = \int_0^2 \frac{2\cancel{\sqrt{4-y^2}}}{\cancel{\sqrt{4-y^2}}}\,dy = 2y\Big|_0^2 = \boxed{4}$

close

23) $\int_0^{\pi/2}\int_0^{\sin\theta} \theta r\,dr\,d\theta = \int_0^{\pi/2} \frac{1}{2}\theta r^2\Big|_0^{\sin\theta} d\theta = \int_0^{\pi/2} \frac{1}{2}\theta\sin^2\theta\,d\theta$

$\int_0^{\pi/2} \frac{1}{2}\theta\left(\frac{1-\cos 2\theta}{2}\right)d\theta = \frac{1}{4}\int_0^{\pi/2} \theta - \theta\cos 2\theta\,d\theta \quad \frac{1}{4}\left(\left[\frac{1}{2}\theta^2\right]_0^{\pi/2} - [-1]\right) = \frac{1}{4}\left(\frac{1}{2}\frac{\pi^2}{4} + 1\right)$

$\frac{1}{2}\int_0^{\pi/2} 2\theta\cos 2\theta\,d\theta = \frac{1}{2}\left[\cos 2\theta + 2\theta\sin 2\theta\right]_0^{\pi/2}$

$\frac{1}{2}(-1 + \pi(0) - 1) = \frac{1}{2}(-2) = -1$

$\frac{1}{2}(\cos\pi + \pi\sin\pi - \cos 0 + 0)$

$= \frac{1}{4}\left(\frac{\pi^2}{8} + 1\right)$

$= \frac{\pi^2}{32} + 1/4$

$\int_0^1 x\sqrt{1-x^2}\,dx$

$U = 1-x^2 \quad du = 2x$

$V = \frac{1}{2}x^2 \quad dV = x\,dx$

$\boxed{\int u\,dv = uv - \int v\,du}$

$1-x^2\left(\frac{1}{2}x^2\right)$

14.1 Homework

#15

$\sqrt{(1-x^2)}$

$(1-x^2)^{1/2}$

$\frac{2}{3}(1-x^2)^{3/2}$

$\int_0^1 x\sqrt{1-x^2}\,dx$

$u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}\int_0^1 \sqrt{u}\,du$

$u = (1-x^2)$

$-\frac{1}{2} \quad du = \frac{-2x\,dx}{-2}$

$\int_0^1 -\frac{1}{2}\sqrt{u}\,du$

$-\frac{1}{2}\int_0^1 \sqrt{u}\,du$

$-\frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_0^1$

$-\frac{1}{2}\left[\frac{2}{3}(1-x^2)^{3/2}\right]_0^1$

$\cancel{-\frac{1}{3}(1-1)^{3/2}} - -\frac{1}{3}(1-0)^{3/2}$

$\boxed{+\frac{1}{3}}(1)$

14.1 Notes

7) $\int_{e^y}^{y} \frac{y \ln x}{x}\, dx = y \int \ln x \left(\frac{dx}{x}\right) = y \frac{(\ln x)^2}{2} = \frac{1}{2} y (\ln x)^2 \Big|_{e^y}^{y}$

$= \frac{1}{2} y (\ln y)^2 - \frac{1}{2} y \underbrace{(\ln e^y)^2} = \frac{1}{2} y (\ln y)^2 - \frac{1}{2} y^3$

9) $\int_0^{x^3} y e^{-y/x}\, dy$ INTEGRATION BY PARTS $u = y$ $du = dy$ $dv = e^{-y/x} dy$ $v =$

$\frac{\partial(-y/x)}{\partial y} = \frac{-1}{x}$ $\quad v = \int dv = \int e^{-y/x} dy = (-x) \int e^{-y/x} \left(-\frac{1}{x}\right) dy = (-x) e^{-y/x}$

$= \int u\, dv = uv - \int v\, du = y(-x e^{-y/x}) - \int (-x) e^{-y/x} dy + x(-x) \int e^{-y/x} \left(-\frac{1}{x}\right) dy$

$-x^2 e^{-y/x} = -xy e^{-y/x} - xy e^{-y/x} = e^{-y/x}(-xy - x^2) = -e^{-y/x}(xy + x^2)$

$\int_0^{x^3} = \left[-e^{-y/x}(xy + x^2)\right]_0^{x^3} = \left[-e^{x^3/x}(x(x^3)) + x^2\right] - \left[-e^0 (0 + x^2)\right]$

$-e^{-x^2}(x^4 + x^2) + x^2$

17. $\int_1^2 \int_0^4 (x^2 - 2y^2 + 1)\, dx\, dy \qquad \int_1^2 \left[\frac{1}{3} x^3 - 2y^2 x + x\right]_0^4 dy$

$= \int_1^2 \left[\frac{64}{3} - 8y^2 + 4\right] dy = \int_1^2 \left(\frac{76}{3} - 8y^2\right) dy = \frac{76}{3} y - \frac{8y^3}{3}\Big]_1^2 = \frac{20}{3}$

23. $\int_0^{\pi/2} \int_0^{\sin\theta} \theta r\, dr\, d\theta = \int_0^{\pi/2} \frac{\theta r^2}{2}\Big|_0^{\sin\theta} d\theta \quad \int_0^{\pi/2} \frac{\theta}{2} \sin^2\theta\, d\theta = \int \frac{\theta}{2}\left(\frac{1 - \cos 2\theta}{2}\right) d\theta =$

$\int \frac{1}{4} \theta\, d\theta - \frac{1}{4} \int \theta \cos 2\theta\, d\theta = \frac{1}{2} \frac{\theta^2}{2} \Big|_0^{\pi/2} = \frac{\pi^2}{32} \qquad \iint = \frac{\pi^2}{32} + \frac{1}{8}$

$-\frac{1}{4} \int \theta \cos 2\theta\, d\theta$ #53 $\int u \cos u\, du = \cos u + u \sin u + C$

$-\frac{1}{4} \int \theta \cos 2\theta\, d\theta$

$-\frac{1}{4} \frac{1}{2} \frac{1}{2} \int (2\theta) \cos 2\theta\, (2\, d\theta)$

$= \frac{-1}{16} \left[\cos 2\theta + 2\theta \sin 2\theta\right] \Big|_0^{\pi/2} = \frac{1}{8}$

25) $\int_1^{\infty} \int_0^{1/x} y\, dy\, dx = \int_1^{\infty} \frac{1}{2} y^2 \Big|_0^{1/x} dx = \int_1^{\infty} \frac{1}{2} \left(\frac{1}{x}\right)^2 dx = \int_1^{\infty} \frac{1}{2} \frac{1}{x^2} dx = \frac{1}{2} \frac{x^{-1}}{-1} = \frac{-1}{2x} \Big|_1^{\infty}$

$= -\frac{1}{2(\infty)} + \frac{1}{2(1)} = -0 + \frac{1}{2} = \frac{1}{2}$

27) $\int_1^{\infty} \int_1^{\infty} \frac{1}{xy}\, dx\, dy = \int_1^{\infty} \frac{1}{y} \underbrace{\int_1^{\infty} \frac{dx}{x}} dy = \int_1^{\infty} \frac{1}{y} \ln x \Big|_1^{\infty} dy = \int_1^{\infty} \frac{1}{y} [\underbrace{\ln(\infty)}_{\infty} - \ln(1)]\, dy$

Ch 14.1 — Iterated Integrals —

$F(x,y) = x^2+y^2+10 \quad f_x = 2x \quad f_y = 2y$

$\int f_x\,dx = \int 2x\,dx =$ ① x^2 + constant of integration $= x^2 + g(y) \leftarrow V1$

② $\int f_y\,dy = \int 2y\,dy = y^2 + h(x) \leftarrow V2$

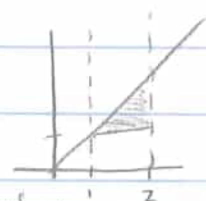
$F(x,y) = x^2+y^2+K$

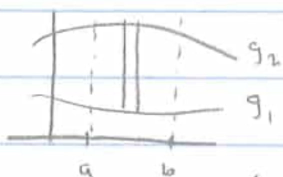
$\int_{h_1(y)}^{h_2(y)} f_x(x,y)\,dx = f(h_2(y),y) - f(h_1(y),y)$ ⇦ gray box p 982

$\int_{g_1(x)}^{g_2(x)} f_y(x,y)\,dy =$

$\iint = 3?$ $\quad \iint \underbrace{(2x^2y^{-2}+2y)}_{z}\,\underbrace{dy\,dx}_{da} = \text{volume}$

$\int_1^2\int_1^x (z)\,dy\,dx \quad y=x \quad 1 \le y \le x \quad 1 \le x \le 2$





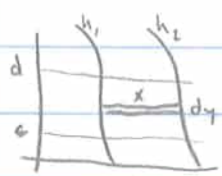
$dA = y\,dx$ Vertical element order of integration is $dy\,dx$

* work problems from inside out

vertically simple region

$A = \int_a^b [g_2(x) - g_1(x)]\,dx \qquad A = \int_a^b \int_{g_1(x)}^{g_2(x)} dy\,dx$

$= y\Big|_{g_1}^{g_2} dx = \int_a^b [g_2(x) - g_1(x)]$



horizontally simple region

$dA = x\,dy \qquad A = \int_c^d [h_2(y) - h_1(y)]\,dy$

horiz. element order of int must be dx, dy $\quad A = \int_c^d \int_{h_1(y)}^{h_2(y)} dx\,dy$

314-344-5600

$1\frac{1}{4}''$

homework

13.8 | 1-33 odds 43-61 odds

1) $z = g(x,y) = (x-1)^2 + (y-3)^2$ Z minimum = 0 at (1, 3, 0)

3) $f(x,y) = \sqrt{x^2+y^2+1}$ z min = 1 at (0, 0, 1)

5) $f(x,y) = x^2+y^2+2x-6y+6$

$= (x^2+2x+1)(+y^2-6y+9) = -6+1+9$

$(x+1)^2+(y-3)^2 = 4$ z min. = 4 at (-1, 3, 4)

7) $f(x,y) = 2x^2+2xy+y^2+2x-3$

$f_x = 4x+2y+2$ $f_y = 2x+2y$

$2x = -2y$ $x = -y$

$-4y+2y+2=0$, $2y=2$, $\boxed{y=1, x=-1}$ $z=-4$ $(-1,1,-4)$

$f_{xx} = 4$ $f_{yy} = 2$ $f_{xy} = 2$ $d = f_{xx}f_{yy} - (f_{xy})^2$

$d = 4(2) - (2^2) = 4, > 0$ $f_{xx} > 0$, $\boxed{\text{relative minimum}}$

15) $g(x,y) = 4-|x|-|y|$ max @ z = 4

18) $f(x,y) = y^3-3yx^2-3y^2-3x^2+1$

$f_x = -6yx-6x$ $f_{xx} = -6y-6$ $f_{xy} = -6x$

$f_y = 3y^2-3x^2-6y$ $f_{yy} = 6y-6$

$d = f_{xx}f_{yy} - (f_{xy})^2 =$

$(-6x)(y+1) = 0$

$x=0$ or $y=-1$ ⇒ plug those in

13.8 | Notes

25) $f(x,y) = x^3 - 3xy + y^3$

1) $f_x = 3x^2 - 3y = 0$

2 $f_y = -3x + 3y^2 = 0$

① $3x^2 - 3y = 0$ ② $-3x + 3y^2 = 0$

$3x^2 = 3y$ $\quad -x + y^2 = 0$

$x^2 = y$ $\quad -x + (x^2)^2 = 0$

$x^4 - x = 0$

$x(x^3 - 1) = 0$

$x(x-1)(x^2+x+1) = 0$

$x = 0$ $\quad x = \frac{-1 \pm \sqrt{3}i}{2}$ Real Roots Only

$x = 1$

$x = 0$ $\quad x = 1$

$y = 0$ $\quad y = 1$

$(0,0,1)$ $\quad (1,1,-1)$

a) $f_{xx} = 6x$ $\quad f_{yy} = 6y$ $\quad f_{xy} = -3$ $\quad (0,0)$ $f_{xx} = 0$ $\quad f_{yy} = 0$ $\quad f_{xy} = -3$

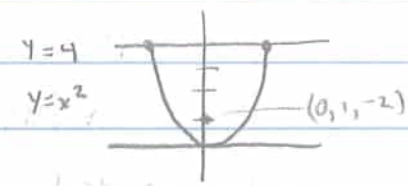
$d = f_{xy} \cdot f_{yy} - (f_{xy})^2 = -9 < 0$ SADDLE POINT

b) $(1,1,-1)$ $f_{xy} = 6$ $\quad f_{yy} = 6$ $\quad f_{xy} = -3$, $d = 27, > 0$

47) $f(x,y) = (x-1)^2 (y+4)^2$ minimum $z = 0$, where

$(x\,y\,z) = (1, a, 0) \Leftarrow$ min. lines $\Rightarrow (x\,y\,z) = (b, -4, 0)$

55) $f(x,y) = 3x^2 + 2y^2 - 4y$



R = region in xy plane bounded by $y = x^2$ & $y = 4$

$f_x = 6x = 0 \Rightarrow x = 0$ $\quad (x,y,z) = (0,1,-2)$

$f_y = 4y - 4 = 0 \Rightarrow y = 1$

$f_{xx} = 6$ $\quad f_{yy} = 4$ $\quad f_{yx} = 0$ $\quad d = f_{xx} f_{xy} - (f_{xy})^2 = 24 > 0$ Rel, Min

a) on line $y = 4$ $\quad -2 \le x \le 2$

$f(x,y) = 3x^2 + 2y^2 - 4y$ $\qquad f(x,4)$ max $x = \pm 2$ $\quad z = 28$

$f(x,4) = 3x^2 + 2(4)^2 - 4(4)$ $\qquad$ Min $x = 0$ $\quad z = 16$

$= 3x^2 + 16$

b) on parabola $y = x^2$ $\quad -2 \le x \le 2$ $\quad f(x,y) = 3x^2 + 2y^2 - 4y$

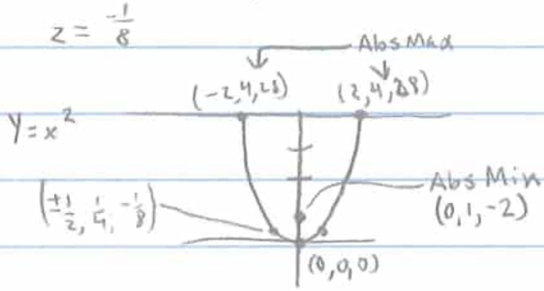
$f(x, x^2) = \ldots -x^2 + 2x^4$ $\quad x = \pm 2$ $\quad f(x,x^2) = 28$

$df(x, x^2) = -2x + 8x^3 = 0$

$x = \pm \frac{1}{2}$ $\qquad 2x(4x^2 - 1)$

$(2x+1)(2x-1)$

$z = \frac{-1}{8}$



Back →

13.8 Notes

TH 13.13 <u>Second Partials Test</u> $z = f(x,y)$

IF ① $f_x(a,b) = 0$ AND ② $f_y(a,b) = 0$ ③ f has continuous second partials

on an open region containing (a,b) f_x & f_y MUST EXIST or test not good

$d = f_{xx}(a,b) f_{yy}(a,b) - [f_{xy}(a,b)]^2$

1. $d > 0$ & f_{xx} (or f_{xy}) > 0, RELATIVE MIN @ (A,b)
2. $d > 0$ & $f_{(x,y)}$ " < 0, Relative Max @ (a,b)
3. $d < 0$, $\Rightarrow$ saddle pt. at (a,b)
4. $d = 0$ test fails

1) $g(x,y) = (x-1)^2 + (y-3)^2$ $z = (x-1)^2 + (y-3)^2$, z min $= 0$

$(x,y,z) = (1,3,0)$

5) $f(x,y) = x^2 + y^2 + 2x - 6y + 6$ complete the sq.

$(x^2 + 2x + 1) + (y^2 - 6y + 9)$ $= -6 + 1 + 9$

$(x+1)^2 + (y-3)^2 = 4$, $z = (x+1)^2 + (y-3)^2 - 4$

z minimum $= -4$ @ $(-1, 3, -4)$

7) $f(x,y) = 2x^2 + 2xy + y^2 + 2x - 3$ 1) $f_x = 4x + 2y + 2$ 2) $f_y = 2x + 2y$ SOLVE SIMULT. both $= 0$

② $2x = -2y$, $x = -y$ ① $4(-y) + 2y + 2 = 0$ $-2y = -2$, $y = 1$, $\Rightarrow x = -1$

$(-1, 1, -4)$ $f_{xx} = 4$ $f_{yy} = 2$ $f_{xy} = 2$ $d = f_{xx}f_{yy} - (f_{xy})^2 = 4(2) - (2)^2 = 4, > 0$

$d > 0$, $f_{xx} > 0$, <u>Rel Min Point</u>

15) $g(x,y) = 4 - |x| - |y|$ $g(0,0) = 4$ $(x,y,z) = (0,0,4)$ Rel Max

18) $f(x,y) = y^3 - 3yx^2 - 3y^2 - 3x^2 + 1$ $f_x = -6yx - 6x$ $f_y = 3y^2 - 3x^2 - 6y$ solve for both are $= 0$

1. $(-6x)(y-1) = 0 \Rightarrow x = 0, y = -1$

2. $3y^2 - 3x^2 - 6y = 0$ $(3y)(y-2) = 0 \Rightarrow y = 2, y = 0$

ⓑ $y = -1$ $3y^2 - 3x^2 - 6y = 0$ $3(1)^2 - 3(x^2 - 6(-1) = 0$ $-3x^2 - 3 - 6 = -9$

$x^2 = 3$ $x = \pm\sqrt{3}$ <u>3 cases</u> a) $x = 0, y = 2$
b) $x = \pm\sqrt{3}, y = -1$
c) $x = y = 0$

ⓐ $f_{xx} = -6(y-1)$ $f_{xy} = -6x = f_{yx}$
$f_{yy} = 6y - 6$
$d = f_{xx}f_{yy} - (f_{xy})^2 = \dots -108$
$d < 0$
$(0, 2, -3)$

ⓑ ... $d = -108$ saddle pts. $(\pm\sqrt{3}, -1, -3)$

ⓒ ~~$(0,0)$ $d = 36$, $f_{xx} = -6$, $(0,0,1)$ Rel Max~~

13.8 Notes

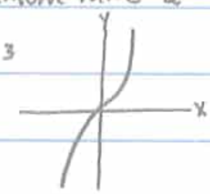
Extrema of Functions of 2 variables

TH 13.15 Extreme Value thm.

if: $f(x,y)$ is ① continuous ② defined of a closed bounded region in xy plane

then: there is at least 1 point in the region (R) where f takes on a minimum value & " " a maximum value

$y = x^3$

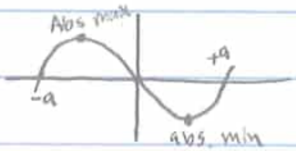


① open interval $-\infty < x < \infty$ ② closed interval $0 \le x \le 1$

Absolute extrema VS relative extrema

$y = x^2$

$(x,y) = (0,0)$ is a minimum



closed interval, absolute values $-a \le x \le a$

open interval, relative values

Critical points: f defined on open region R

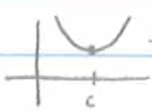
1) $f_x(x_0,y_0) = 0$ & $f_y(x_0,y_0) = 0$

2) $f_x(x_0,y_0)$ or $f_y(x_0,y_0)$ DNE or undefined

Critical #'s

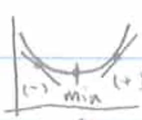
1) $f'(c) = 0$

2) $f'(c)$ is undefined



$f''c > 0$ concave upward

$f''c < 0$ concave down



inflection pt

2nd Derivative Test

$y = f(x)$ set 1st derivative equal to 0

1) $f'(c) = 0 \Rightarrow$ max, min, or inflection pt.

2) $f''(c)$ exists on an interval containing c

a) if $f''(c) > 0$ Then $f(c)$ is relative min

B) if $f''(c) < 0$, relative max

c) if $f''(c) = 0$, Test fails. Go back to 1st deriv. test

13.7 homework 17–33 odds, 41–55 odds, skip 47

17) $z=\sqrt{x^2+y^2}$ $P(3,4,5)$ find tang. plane to graph at point

$z^2=x^2+y^2$ $\nabla f = 2x+2y-2z$

$x^2+y^2-z^2=0$ $\nabla f_{(3,4,5)} = 6i+8j-10k$

$V=\langle x-x_0 i,\ y-y_0 j,\ z-z_0 k\rangle = \langle x-3i,\ y-4j,\ z-5k\rangle$

$V\cdot\nabla f = 6x-18+8y-32-10z+50 = 6x+8y-10z=0$

$\boxed{3x+4y-5z=0}$

19) $g(x,y)=x^2-y^2$ $P(5,4,9)$ $z=x^2-y^2$ $x^2-y^2-z=0$

$\nabla g = 2xi-2yj-k$ $\nabla g_{(P)} = 10i-8j-k$ $V=\langle x-5i,\ y-4j,\ z-9\rangle$

$V\cdot\nabla g = 10x-50-8y+32-z+9,$ $10x-8y-z-9=0$ $\boxed{10x-8y-z=9}$

21) $z=e^x(\sin y+1)$ $(0,\frac{\pi}{2},2)$ $V=\langle x-0i,\ y-\frac{\pi}{2}j,\ z-2k\rangle$

$e^x(\sin y+1)-z=0$

$\nabla f = e^x(\sin y+1)i\ \ e^x\cos y\, j-1k$ $\nabla f_{(P)} = 1(\sin\frac{\pi}{2}+1)i+1\cos\frac{\pi}{2}j-k$

$\nabla f_{(P)} = 2i+0-k$ $\nabla f\cdot V = 2x-z+2=0$ $\boxed{2x-z=-2}$

23) skip, 25) skip 27) skip

29) find eq. of tan. plane & find symm. equations of normal line

$x^2+y^2+z=9$ $(1,2,4)$

$\nabla f = 2xi+2yj+k$ $\nabla f_{(1,2,4)} = 2i+4j+k$ $V=\langle x-1,\ y-2,\ z-4\rangle$

$\nabla f\cdot V = 2x-2+4y-8+z-4 = \boxed{2x+4y+z=14}$ ← tan plane

sym. form of normal line: $\nabla f = 2i+4j+k$ $\langle 2,4,1\rangle$

$$\frac{x-1}{2}=\frac{y-2}{4}=\frac{z-4}{1}$$

?

41) $\underbrace{x^2+y^2=5}_{}$ $f(x)$ $z=x$ $g(x)$ $(2,1,2)$

$\nabla f\times\nabla g$

$\nabla f = 2xi+2yj$ $g(x)=x-z$

$$\begin{vmatrix} i & j & k \\ 4 & 2 & 0 \\ 1 & 0 & -1 \end{vmatrix}$$

$\nabla g = 1i-1k$

$\nabla f(2,1,2) = (4i+2j)$

$\nabla g(2,1,2) = i-k$

13.7 | homework 1-33 odds, 41-55 odds, skip 47

1) describe the level surface $F(x,y,z)=0$

$F_{(\ )} = 3x - 6y + 3z - 15$, a plane in space

3) $F_{(xyz)} = 4x^2 + 9y^2 - 4z^2 = 0$ elliptic cone lying on z-axis

5) surface; $x+y+z=4$ $\overset{P.}{(2,0,2)}$ find normal unit vector

$\nabla f = i + j + k$ $\quad \frac{\nabla f}{\|\nabla f\|} = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}(i, j, k)}$

7) $z = \sqrt{x^2+y^2}$ $\quad z^2 = x^2+y^2$ $\quad x^2+y^2-z^2=0$ $\quad P(3,4,5)$

$\nabla f = 2xi + 2yj + -2zk$ $\quad \nabla f_{(P)} = 6i + 8j - 10k$

$\frac{\nabla f}{\|\nabla f\|} = \frac{6}{\sqrt{200}} i + \frac{8}{\sqrt{200}} j + \frac{-10}{\sqrt{200}} k = \frac{6}{10\sqrt{2}} i + \frac{8}{10\sqrt{2}} j \ \frac{-10}{10\sqrt{2}} k$

$= \frac{3}{5\sqrt{2}} i + \frac{4}{5\sqrt{2}} j - \frac{5}{5\sqrt{2}} k$

$= \frac{\sqrt{2}}{5 \cdot 2}(3i + 4j - 5k)$

9) $x^2y^4 - z = 0$ $\quad (1,2,16)$

$\nabla f = y^4 2x\, i + 4x^2y^3 j - 1k$

$\nabla f(1,2,16) = (2)16\, i + 4(2)^3 - 1k = 32i + 32j - k = \frac{\sqrt{32^2+32^2+1}}{32^2+32^2+1}(32i, 32j, -k)$

$\boxed{= \frac{\sqrt{2049}}{2049}(32i, + 32j, -k)}$

11) $\ln\left(\frac{x}{y-z}\right) = 0$ $\quad (1,4,3)$

$\nabla f = \frac{\frac{(1)(y-z) - 0(x)}{(y-z)^2}}{\left(1\right)} i \quad \frac{\frac{0-(1x)}{(y-z)^2}}{\left(1\right)} j \quad \frac{\frac{(-)-x}{(y-z)^2}}{\left(1\right)}$ $\quad \nabla f_{(1,4,3)} = \frac{1}{1} i \ \frac{-1}{1} j \ \frac{+1}{1} k$

$= i, -j, +k$

$\frac{1}{\sqrt{3}}$ $\boxed{\frac{\sqrt{3}}{3} i - j + k}$

13) skip

15) $z = 25 - x^2 - y^2$ $\quad P(3,1,15)$ $\quad f_{(xyz)} = 25 - x^2 - y^2 - z$

$\nabla z = -2xi - 2yj - 1k$ $\quad \nabla z_{(P)} = -6i - 2j - k$ $\quad \frac{\nabla z}{\|\ \|} =$

$V = \langle x - x_0 i + y - y_0 j + z - z_0 k \rangle$

$V = \langle x - 3i + y - 1j + z - 15k \rangle$ $\quad V \cdot \nabla z = -6x + 18 - 2y + 2 - z + 15$

$= \boxed{-6x - 2y - z + 35}$

13.7 Tangent Planes & Normal Lines

$z = x(f, y)$

level surface $\underline{f = (x, y, z) = 0}$ $\quad F(x,y,z) = f(x,y) - z$ $\quad$ consider ~~level~~ surface to be a level surface of F given by ~~=~~ $F(x y z) = 0$

TH 11.12 p794 $\quad m = \langle a, b, c \rangle$ Point $\langle x, y, z \rangle$

Fig 11.48 General point $\quad \vec{PQ}$ (from specific to general point

$m \cdot \vec{PQ} = 0 \quad a(x - x_1) + b(y - y_1) + c(z - z_1)$

- ∇f is normal to the tangent plane
- ∇f is normal to every vector in the tangent plane

1. write the equation of a vector in the tangent plane

   $V = (x - x_0)i + (y - y_0)j + (z - z_0)k$

2. the dot product of ∇f & v must be zero. they are orthogonal

   $\nabla f \cdot v = 0$

3. equation of the tangent Plane $F_x(x_0, y_0, z_0)(x - x_0) + F_y(x_0, y_0, z_0)(y - y_0) + F_z(x_0, y_0, z_0)(z - z_0)$

TH 13.13

Eq. of a normal line to a surface

1) calculate the gradient $\nabla f(x, y, z)$

TH 11.11 P798 line parallel to $V \langle a, b, c \rangle$ & passing thru $P(x, y, z)$

Parametric form: $x = x_1 + at$ $\quad$ Symetric form: a, b, c, must be Non-zero

$y = y_1 + bt \qquad \dfrac{x - x_1}{a} = \dfrac{y - y_1}{b} = \dfrac{z - z_1}{c}$

$z = z_1 + ct$

cross product $u \times v = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$ $\quad$ Angle of inclination of a plane is defined as the $\underset{\text{acute}}{\hat{<}}$ b/w the given plane & xy plane

TH 11.5 $\angle$ between 2 vectors $\quad \cos\theta = \dfrac{u \cdot v}{\|u\| \|v\|}$

$\cos\theta = \dfrac{m \cdot k}{\|m\| \underbrace{\|k\|}_{1}} = \dfrac{|\nabla f \cdot k|}{\|\nabla f\|} \quad \theta \to 0 - 180^\circ$

7) $z = \sqrt{x^2 + y^2}$ $\quad P(3,4,5)$ $\quad f(x,y,z) = \sqrt{x^2+y^2} - z = (x^2+y^2)^{1/2} - z$

$\nabla f(x,y,z) = f_x i + f_y j + f_z k$

$\nabla f = \frac{1}{2}(x^2+y^2)^{-1/2}(2x)i + \frac{1}{2}(x^2+y^2)^{-1/2}(2y)j - k$

$\nabla f(3,4,5) = 3/5\, i + 4/5\, j - k \quad n = \dfrac{\nabla f}{\|\nabla f\|} = \dfrac{3/5\, i + 4/5\, j - k}{\sqrt{\frac{3}{5}^2 + \frac{4}{5}^2 + (-1)^2}} = \dfrac{(\quad)}{\sqrt{2}} = \dfrac{\sqrt{2}}{10}(3i + 4j - 5k)$

13.7) Notes

11) $\ln\left(\frac{x}{y-z}\right) = 0$ Pt. $(1,4,3)$ $f(x,y,z) = \ln x - \ln(y-z)$

$\nabla f(\ ) = \frac{1}{x} i - \frac{1}{y-z} j - \frac{1}{y-z}(-1) k$

$\nabla f(1,4,3) = i - j + k$ $n = \frac{\Delta f}{\|\Delta f\|} = \frac{i-j+k}{\sqrt{3}}$

15) $z = 25 - x^2 - y^2$ $P(3,1,15)$ $f(x,y,z) = 25 - x^2 - y^2 - z$

$\nabla f(\ ) = -2xi - 2yj - k$

$\nabla(3,1,15) = -6i - 2j - 15k$ $V = (x-x_0)i + (y-y_0)j + (z-z_0)k$

$= (x-3)i + (y-1)j + (z-15)k$

$\nabla f \cdot u = -6(x-3) + (-2)(y-1) + (-1)(z-15) = 0$

$-6x - 2y - z + 35 = 0$

29) $x^2 + y^2 + z = 9$ $P(1,2,4)$

$f(x,y,z) = x^2 + y^2 + z - 9 = 0$ $\nabla f(\ ) = 2xi + 2yj + k$

$\nabla f(1,2,4) = 2i + 4j + k$ $V = (x-1)i + (y-2)j + (z-4)k$

$\nabla f \cdot V = 2(x-1) + 4(y-2) + 1(z-4) = 0$ ← eq of tangent line

$\boxed{2x + 4y + z = 14}$

$\nabla f\ = 2i + 4j + k = \langle 2,4,1 \rangle$ Symmetric form of normal line

$\frac{x-1}{2} = \frac{y-2}{4} = \frac{z-4}{1}$

43) ① $x^2 + z^2 = 25$ ② $y^2 + z^2 = 25$ $P(3,3,4)$

$f(x,y,z) = x^2 + z^2 - 25$ $g(x,y,z) = y^2 + z^2 - 25 = 0$

$\nabla f(\ ) = 2xi + 2zk$ $\nabla g(\ ) = 2yj + 2zk$

$\nabla f(3,3,4) = 6i + 8k$ $\nabla g(3,3,4) = 6j + 8k$

$\nabla f \times \nabla g = \begin{vmatrix} i & j & k \\ 6 & 0 & 8 \\ 0 & 6 & 8 \end{vmatrix} = 48i - 48j + 36k$ Tang. line: $\frac{x-3}{4} = \frac{y-3}{4} = \frac{z-4}{-3}$

∇ direction numbers $-12(4,4,-3)$

49) inclination of a plane $\cos\theta = \frac{|\nabla f \cdot k|}{\|\nabla f\|} = \frac{|-1|}{\sqrt{12^2 + 8^2 + 1^2}}$

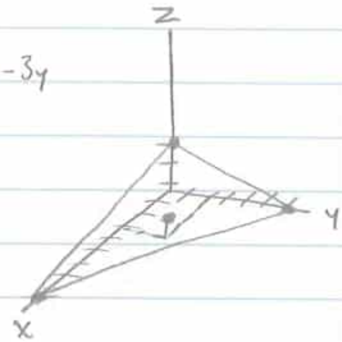
surface: $3x^2 + 2y^2 - z = 15$

$f(x,y,z) = 3x^2 + 2y^2 - z - 15 = 0$ $\cos\theta = \frac{1}{209} = 0.0697$

$\nabla f(\ ) = 6xi + 4yj - 1k$ $\theta = \arccos(0.0697)$ degrees $= 86.03°$

$\nabla f(2,2,5) = 12i + 8j - k$

13.6 | Homework 39–49 odds, 55–63 odds



39) Sketch $F(x,y) = 3 - \frac{x}{3} - \frac{y}{2}$, $6z = 18 - 2x - 3y$

$18 = 6z + 2x + 3y$

$(3, 9, 6)$

41) $D_u f(3,2)$ $u = \cos\theta i + \sin\theta j$

a) $\theta = \frac{4\pi}{3}$ $\quad U = -\frac{1}{2} i - \frac{\sqrt{3}}{2} j$

$\nabla f = -\frac{1}{3} i + (-)\frac{1}{2} j$ $\quad U \cdot \nabla f = \frac{1}{6} i + \frac{\sqrt{3}}{4} j$

43) $f(x,y) = 3 - \frac{x}{3} - \frac{y}{2}$ $\quad 6z = 18 - 2x - 3y$

$\nabla f = -\frac{1}{3} i - \frac{1}{2} j$ $\quad D_u f(3,2)$ $\quad U = \frac{V}{\|V\|}$

a) V is $(1,2)$ to $(-2,6) = \langle -3, 4 \rangle$ $\quad U = \frac{-3}{\sqrt{25}} i + \frac{4}{5} j = -\frac{3}{5} i + \frac{4}{5} j$

$\nabla f \cdot u = \frac{1}{5} - \frac{4}{10} = \boxed{-\frac{1}{5}}$ ✓

b) V is $\langle 1, 3 \rangle$ $\quad U = \frac{1}{\sqrt{10}} i + \frac{3}{\sqrt{10}} j$ $\quad \nabla f \cdot U = \frac{-1}{3\sqrt{10}} - \frac{3}{2\sqrt{10}} = \frac{-2-9}{6\sqrt{10}} = \boxed{\frac{-11\sqrt{10}}{60}}$

45) max value of directional d. @ $(3,2)$

$\nabla f = -\frac{1}{3} i - \frac{1}{2} j$ $\quad U = \sqrt{\frac{1}{9} + \frac{1}{4}} = \sqrt{\frac{4+9}{36}} = \sqrt{\frac{13}{36}} = \boxed{\frac{\sqrt{13}}{6}}$

47) skip

49) skip

55) Find Normal vector to the level curve

$f(x,y) = x^2 + y^2$ $\quad P(3,4)$ $\quad c = 25$

$x^2 + y^2 = c = 25$ $\qquad x^2 + y^2 = 25$

$\nabla f = 2x i + 2y j$ $\quad \nabla f(3,4) = \boxed{6i + 8j}$

57) $f(x,y) = \frac{x}{x^2+y^2} = \frac{1}{2}$ $\quad P(1,1)$ $\quad c = \frac{1}{2}$

$\nabla f = \frac{1(x^2+y^2) - 2x(x)}{(x^2+y^2)^2} i$, $\nabla f(1,1) = \frac{1+1(-1)2}{4} = (\cancel{1i}\ \ 0i \ \ \frac{(-1)}{2} j) = \boxed{-\frac{1}{2} j}$

$+ \frac{(0)(x^2+y^2) - 2y(x)}{(\ \ )^2} j = (1,1) = \frac{2}{4} = (-\frac{1}{2} j)$

59) $4x^2 - y = 6$ $\quad (2,10)$ use ∇ to find Normal vector

$f: 4x^2 - y - 6 = 0$ $\quad \nabla f = 8x i - j$ $\quad \nabla f(2,10) = 16i - j$

$U = \frac{16}{\sqrt{257}} i - \frac{1}{\sqrt{257}} j = \boxed{\frac{\sqrt{257}}{257}(16i - j)}$

61) $9x^2 + 4y^2 = 40$ $\quad (2,-1)$

$\nabla f = 18x i + 8y j$

$\nabla f(2,-1) = 32i - 8j$ $\quad \frac{\nabla f}{\|\nabla f\|} = \frac{32}{\sqrt{1088}} i - \frac{8}{\sqrt{1088}} j$

13.6 homework

1-47 odds, 55-63 odds

1) $f(x,y) = 3x - 4xy + 5y$ $\quad P(1,2),\ v = \frac{1}{2}(i + \sqrt{3}\,j)$ directional derivative of f @ P, in V direction

$\nabla f(x,y) = f_x(x,y)i + f_y(x,y)j$

$(3-4y)i + (-4x+5)j = (-5)i + (1)j \cdot u = \frac{-5}{2} + \frac{\sqrt{3}}{2} = \boxed{\frac{\sqrt{3}-5}{2}}$

$u = \frac{v}{\|v\|} = \dfrac{\frac{1}{2}i + \frac{\sqrt{3}}{2}j}{\sqrt{\frac{1}{4}+\frac{3}{4}}} = \dfrac{\frac{1}{2}i + \frac{\sqrt{3}}{2}j}{1}$

correct!

3) $f(x,y) = xy$ $\quad P(2,3)$ $\quad v = i + j$

$\nabla f = yi + xj = 3i + 2j \cdot U = \frac{3\sqrt{2}}{2} + \frac{2\sqrt{2}}{2} = \boxed{\frac{5\sqrt{2}}{2}}$

$U = \frac{v}{\|v\|} = \frac{i+j}{\sqrt{2}} \quad \frac{\sqrt{2}}{2}i + \frac{\sqrt{2}}{2}j$

5) $g(x,y) = \sqrt{x^2+y^2}$ $\quad P(3,4)$ $\quad v = 3i - 4j$

$\nabla f = f_x i + f_y j = \frac{1}{2}(x^2+y^2)^{-1/2}(2x) + \frac{1}{2}(x^2+y^2)^{-1/2}(2y) = \frac{x}{\sqrt{x^2+y^2}}i + \frac{y}{\sqrt{x^2+y^2}}j$

$\nabla f(3,4) = \frac{3}{\sqrt{9+16}}i + \frac{4}{\sqrt{25}}j = \frac{3}{5}i + \frac{4}{5}j \cdot U = \frac{-3}{25} + \frac{-4}{25} = \boxed{\frac{-7}{25}}$

$U = \frac{v}{\|v\|} = \frac{3i-4j}{\sqrt{25}} = \frac{-1}{5}$

7) $h(x,y) = e^x \sin y$ $\quad P = (1, \frac{\pi}{2})$ $\quad v = -i$ $\quad U = -1$

$\nabla h = e^x \sin y\, i + e^x \cos y\, j$ $\quad \nabla h_{(1,\frac{\pi}{2})} = e\sin(\frac{\pi}{2})i + e\cos(\frac{\pi}{2})j$

$= ei, \cdot U = \boxed{-e}$

9) $f(x,y,z) = xy + yz + xz$ $\quad P(1,1,1)$ $\quad v = 2i + j - k$

$\nabla f = (y+z)i + (x+z)j + (y+x)k \quad \nabla f(1,1,1) = 2i + 2j + 2k$ $\quad \boxed{\frac{2\sqrt{6}}{3}}$

$U = \frac{v}{\|v\|} = \frac{2i + 1j - 1k}{\sqrt{2^2+(1)^2+(-1)^2}} = \frac{2i+j+k}{\sqrt{6}} \quad \frac{2}{\sqrt{6}} + \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{6}} \quad \frac{4}{\sqrt{6}} = \frac{4\sqrt{6}}{6} =$

11) $h(x,y,z) = x \arctan yz$ $\quad P(4,1,1)$ $\quad v = \langle 1,2,-1 \rangle$

$\nabla h = \arctan yz\, i + x\frac{z}{1+(yz)^2}j + \frac{xy}{1+(yz)^2}k$

$\nabla h(4,1,1)\ \arctan(1)i + \frac{4}{1+1}j + \frac{4}{1+1}k = \left[\frac{\pi}{2}i + 2j + 2k\right]$

$U = \frac{v}{\|v\|} = \left[\frac{i}{\sqrt{6}} + \frac{2j}{\sqrt{6}} + \frac{-k}{\sqrt{6}}\right] \quad \frac{\pi}{4\sqrt{6}} + \frac{4}{\sqrt{6}} - \frac{2}{\sqrt{6}} = \frac{\pi}{4\sqrt{6}} + \frac{2}{\sqrt{6}} = \frac{\pi}{4\sqrt{6}} + \frac{8}{4\sqrt{6}}$

$= \boxed{\frac{\pi + 8(\sqrt{6})}{24}}$

13.6 Homework

13) $u = \cos\theta\, i + \sin\theta\, j = \frac{1}{\sqrt{2}} i + \frac{1}{\sqrt{2}} j$

$f(x,y) = x^2 + y^2 \quad \theta = \frac{\pi}{4} \qquad u \cdot \nabla f = \frac{2}{\sqrt{2}} x + \frac{2}{\sqrt{2}} y = 2\left(\frac{\sqrt{2}}{2} x + \frac{\sqrt{2}}{2} y\right)$

$\nabla f = f_x(x,y) i + f_y(x,y) j \qquad = \boxed{\sqrt{2}(x+y)}$

$\nabla f = 2x i + 2y j$

15) $f(x,y) = \sin(2x - y) \quad \theta = -\frac{\pi}{3} \qquad u = \frac{1}{2} i, -\frac{\sqrt{3}}{2} j$

$\nabla f = 2(\cos(2x-y)) i + (-)\cos(2x-y) j$

$u \cdot \nabla f = \cos(2x-y) + \frac{\sqrt{3}}{2}\cos(2x-y) = \boxed{\frac{2+\sqrt{3}}{2}\cos(2x-y)}$

17) find directional d of func. P in Q direction

$f(x,y) = x^2 + 4y^2 \quad P(3,1) \quad Q(1,-1) \quad v = \langle -2,-2 \rangle \quad u = \frac{-2-2}{\sqrt{8}} = -\frac{1}{\sqrt{2}} i - \frac{1}{\sqrt{2}} j$

$\nabla f = 2x i + 8y j$

$\nabla f_{(P)} = 6i + 8j \qquad \nabla f \cdot u = -\frac{14}{\sqrt{2}} = \frac{-14(\sqrt{2})}{2} = \boxed{-7\sqrt{2}}$

19) $h(x,y,z) = \ln(x+y+z) \quad P(1,0,0) \quad Q(4,3,1)$

$\nabla h \ \frac{1}{(x+y+z)} i + \frac{1}{(x+y+z)} j + \frac{1}{(x+y+z)} k \qquad \cancel{u = \frac{4}{\sqrt{26}} i, \frac{3}{\sqrt{26}} j, \frac{1}{\sqrt{26}} k}$

$\nabla h_{(1,0,0)} = i + j + k \qquad u \cdot \nabla h = \frac{7}{\sqrt{19}} = \boxed{\frac{7\sqrt{19}}{19}}$

$v = (3,3,1) \quad u = \frac{3}{\sqrt{19}} i + \frac{3}{\sqrt{19}} j + \frac{1}{\sqrt{19}} k$

21) $f(x,y) = 3x - 5y^2 + 10 \quad (2,1)$ find gradient.

$\nabla f = 3i + (-10)y j \qquad \nabla f_{(3,1)} = 3i - 10j \qquad \cancel{\|\nabla f_{(3,1)}\| = \sqrt{9+100}}$

$\cancel{= \sqrt{109}}$ Not necessary

23) $z = \cos(x^2+y^2) \quad (3,-4)$

$\nabla z = 2x(-\sin x^2+y^2) i + 2y(-\sin x^2+y^2) j$

$\nabla z_{(3,-4)} = 6(-\sin(25)) i + (-8)(-\sin(25)) j$

$= -6\sin 25\, i + 8\sin 25\, j$

13.6 / Homework

25) $w = 3x^2y - 5yz + z^2$ $(1,1,-2)$ find $\Delta w(1,1,2)$

$\Delta w = 6xy\hat{i} + 3x^2 - 5z\,j + [(-5y) + 2z]k$

$\Delta w_{1,1,2} = 6i + 13j - 9k$

27) Find dir. deriv. at P in Q direction

$g(x,y) = x^2+y^2+1$ $P(1,2)$ $Q(3,6)$ $V = \langle 2,4 \rangle$ $U = \frac{2}{\sqrt{20}} i, \frac{4}{\sqrt{20}} j$

$\nabla g = 2x\,i + 2y\,j = 2i + 4j \cdot \frac{1}{\sqrt{5}} i + \frac{2}{\sqrt{5}} j = \frac{2\sqrt{5}}{5} + \frac{8\sqrt{5}}{5} = \boxed{2\sqrt{5}}$

29) $f(x,y) = e^{-x}\cos y$ $P(0,0)$ $Q(2,1)$ $V = \langle 2,1 \rangle$ $U = \frac{2}{\sqrt{5}} i + \frac{1}{\sqrt{5}} j$

$\nabla f = -e^{-x}\cos y\, i + -e^{-x}\sin y = -1\cos(0)\, i - 1\sin(0)\, j$

$-1i \cdot U = -\frac{2}{\sqrt{5}} = \boxed{-\frac{2\sqrt{5}}{5}}$

31) $h(x,y) = x\tan y$ $P(2, \frac{\pi}{4})$

$\nabla h = \tan y\, i + x\sec^2 y\, j$ $\nabla h(p) = 1i + 2(2)j = \boxed{i + 4j}$

$\|\nabla h\| = \sqrt{1^2+4^2} = \boxed{\sqrt{17}}$

33) $g(x,y) = \ln(x^2+y^2)^{1/3} = \frac{1}{3}\ln(x^2+y^2)$ $P(1,2)$

$\nabla g = \frac{1}{3}\frac{2x}{(x^2+y^2)} i + \frac{1}{3}\left(\frac{2y}{x^2+y^2}\right) = \frac{2}{3(5)} i + \frac{4}{3(5)} j = \boxed{\frac{2}{15} i + \frac{4}{15} j}$

$\|\nabla g\| = \sqrt{\frac{4}{225} + \frac{16}{225}} = \sqrt{\frac{20}{225}} = \sqrt{\frac{4}{45}} = \frac{2}{3\sqrt{5}} \boxed{\frac{2\sqrt{5}}{15}}$

35) $f(x\,y\,z) = \sqrt{x^2+y^2+z^2}$ $P(1,4,2)$

$\nabla f = \frac{1}{2}\frac{1}{(x^2+y^2+z^2)^{1/2}}(2x)\, i + \frac{y}{\sqrt{}} j + \frac{z}{\sqrt{}} k = \boxed{\frac{1}{\sqrt{21}} i \frac{4}{\sqrt{21}} j \frac{2}{\sqrt{21}} k}$

$\|\nabla f\| = \sqrt{\frac{1}{21} + \frac{16}{21} + \frac{4}{21}} = \sqrt{\frac{21}{21}} = 1$

did not need to plug #s here

37) $f(x\,y\,z)$ xe^{yz} $P(2,0,-4)$

$\nabla f = e^{yz}\, i + (yz)xe^{yz}\, j + (y)xe^{yz}\, k < \|\nabla f\| = \sqrt{e^{4zz\,0} + xe^{2yz\,0} + xe^{2yz\,0}}$

$= \sqrt{1 + 2(1) + 2(1)}$
8 8

$= \sqrt{65}$

$\sqrt{65}$

13.6 | Directional Derivatives (Notes)

Quiz 13.1-5
Wednesday
8-2

$z = f(x,y)$ f_x = slope in x-dir f_y = slope in y-dir

-specify direction-

$u = \cos\theta\, i + \sin\theta\, j$ (UNIT VECTOR)

Point 13.43-44. line through pt. parallel to u.

Slope of Secant line $= \dfrac{f(x_0 + t\cos\theta,\ y_0 + t\sin\theta) - f(x_0, y_0)}{t} = \dfrac{\Delta z}{\text{length of PQ in xy plane}}$

$x = x_0 + t\cos\theta$ $y = y_0 + t\sin\theta$

Directional Derivative $= D_u$. $D_u f(x,y) = \lim_{t \to 0} \dfrac{f(x_0 + t\cos\theta,\ y_0 + t\sin\theta) - f(x,y)}{t}$

TH 13.9 -- $D_u f(x,y) = f_x(x,y)\cos\theta + f_y(x,y)\sin\theta$

GRADIENT OF A FUNCTION OF 2 VARIABLES

GRADIENT OF $f(x,y)$ = grad $f(x,y) = \nabla f(x,y)$ Del f of x & y

$\nabla f(x,y) = f_x(x,y)\,i + f_y(x,y)\,j$

$D_u f(x,y) = \nabla f(x,y) \cdot u$ (scalar)

The Direction of Maximum Increase is Given By $\nabla f(x,y)$

The max. value of $D_u f(x,y)$ is given by the magnitude of the gradient

$D_u f(x,y)\ \text{MAX} = \|\nabla f(x,y)\|$

13.6 Notes

3) $f(x,y) = xy \quad P(2,3) \quad v = i + j \qquad u = \frac{v}{\|v\|} = \frac{i+j}{\sqrt{2}} = \frac{\sqrt{2}}{2}i + \frac{\sqrt{2}}{2}j$

$\nabla f(x,y) = yi + xj \qquad \nabla f(2,3) = 3i + 2j$

$D_u f(2,3) = \nabla f(2,3) \cdot u = \left[\frac{\sqrt{2}}{2}i + \frac{\sqrt{2}}{2}j\right] \cdot \left[3i + 2j\right]$

$= \frac{3\sqrt{2}}{2} + \frac{2\sqrt{2}}{2} = \frac{5\sqrt{2}}{2}$ (scalar)

11) $h(x,y,z) = x\arctan yz \quad P(4,1,1) \qquad v = \langle 1, 2, -1 \rangle$

$u = \frac{v}{\sqrt{6}} \qquad \nabla h = \arctan yz\, i + \frac{(x)(z)}{1+(yz)^2} j + \frac{(x)(y)}{1+(yz)^2} k$

☆ $\nabla h(4,1,1) = \arctan(1)i + \frac{4(1)}{1+1} j + \frac{4(1)}{1+1} k$

$= \frac{\pi}{4} i + 2j + 2k \qquad D_u h(4,1,1) = \nabla h(4,1,1) \cdot u$

$= \left[\frac{\pi}{4} i + 2j + 2k\right] \cdot \left[\frac{1}{\sqrt{6}} i + \frac{2}{\sqrt{6}} j - \frac{1}{\sqrt{6}} k\right]$

$= \frac{\pi}{4\sqrt{6}} + \frac{4}{\sqrt{6}} + \frac{-2}{\sqrt{6}} = \frac{(\pi+8)\sqrt{6}}{24}$

13.6 | Notes

17) $f(x,y) = x^2 + 4y^2 \Rightarrow \nabla f(x,y) = f_x i + f_y j = 2xi + 8yj$

$P = (3,1) \quad Q = (1,-1) \quad \nabla f(3,1) = 6i + 8j$

$\vec{PQ} = (1-3)i + (-1-1)j$

$-2i - 2j = V$

$u = \frac{V}{\|V\|} = \frac{-2i-2j}{\sqrt{8}} \quad \frac{-2i-2j}{2\sqrt{2}} = \frac{-i}{\sqrt{2}} \frac{-j}{\sqrt{2}}$

$D_u f(3,1) = \nabla f(3,1) \cdot u = (6i + 8j) \cdot \left[\frac{-i}{\sqrt{2}} \ \frac{-j}{\sqrt{2}}\right] = \frac{-14\sqrt{2}}{2} = 7\sqrt{2}$

25) $W = 3x^2y - 5yz + z^2 \quad (1,1,-2) \quad \nabla w = W_x i + W_y j + W_z k =$

$\nabla W = (6xy)i \ (3x^2 - 5z)j + (-5y + 2z)k$

$\nabla w(1,1,-2) = 6(1)(1)i + 3(1) - 5(-2)j + (-5(1) + 2(-2))k$

$6i \quad +13j \quad -9k$

31) $h(x,y) \quad P(2, \pi/4) = x\tan y$

$\nabla h(x,y) = h_x i + h_y j$

$= \tan y\, i + x\sec^2 y\, j$

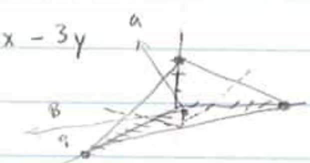
$\nabla h(2, \pi/4) = \tan\frac{\pi}{4} i + 2(\sec^2\frac{\pi}{4})j$

$= (1)i + (4)j$

Direction of max increase is $\nabla h = i + 4j \quad D_u \max = \|\nabla h(x,y)\| = \sqrt{1^2 + 4^2} = \sqrt{17}$

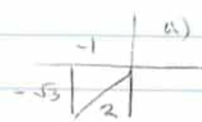
39) $f(x,y) = 3 - \frac{x}{3} - \frac{y}{2} \quad z = 3 - \frac{x}{3} - \frac{y}{2} \quad 6z = 18 - 2x - 3y$

$6z + 2x + 3y = 18 \quad z = 3,\ x = 9 \quad y = 6$



41) find $D_u f(3,2) \quad u = \cos\theta i + \sin\theta j$

a) $\theta = \frac{4\pi}{3} \quad u = -\frac{1}{2}i - \frac{\sqrt{3}}{2}j$



$\nabla f(x,y) = -\frac{1}{3}i - \frac{1}{2}j = \nabla f(3,2)$

$D_u f\left(\frac{4\pi}{3}\right) f(3,2) = \left[-\frac{1}{3}i - \frac{1}{2}j\right] \cdot \left[-\frac{1}{2}i - \frac{3}{\sqrt{2}}j\right] = \frac{2 + 3\sqrt{3}}{12} \cong 0.5997$

b) $\theta = \frac{-\pi}{6} \quad u = \frac{\sqrt{3}}{2}i - \frac{1}{2}j$

$D_u\left(\frac{\pi}{6}\right) f(3,2) = \left(-\frac{1}{3}i - \frac{1}{2}j\right) \cdot \left(\frac{\sqrt{3}}{2}i - \frac{1}{2}j\right) = \frac{3 - 2\sqrt{3}}{12} \cong -0.04$

13.6 Notes

45) $D_u(\max) = \|\nabla f\| = \|-\frac{1}{3}i - \frac{1}{2}j\| = \sqrt{(-\frac{1}{3})^2 + (\frac{1}{2})^2} = \frac{\sqrt{13}}{6} \cong 0.6009$

57) $f(x,y) = \frac{x}{x^2+y^2}$ level curves: $\frac{x}{x^2+y^2} = c = \frac{1}{2}$ $\quad x^2+y^2 = 2x \quad x^2-2x+y^2=0$

complete square

$p = (1,1)$

$x^2 - 2x + 1 + y^2 = 1$

$(x-1)^2 + y^2 = 1$ circle $c(1,0)$ $r=1$

Gradient $\nabla f(x,y) = (f_x i + f_y j)$ $\quad f_x = \frac{(x^2+y^2)(1) - (x)(2x)}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$

$=$

$f_x(1,1) = \frac{(1^2-1^2)}{(1+1)^2} = 0 \quad f_y = \frac{(x^2+y^2)(0) - (x)(2y)}{(x^2+y^2)^2} = \frac{-2xy}{(x^2+y^2)^2} = f_y(1,1) = \frac{-2}{4} = -\frac{1}{2}$

$= -\frac{1}{2}j$

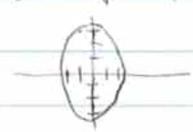
61) $9x^2 + 4y^2 = 40$ Point $(2,-1)$ $\quad f(x,y) = 9x^2 + 4y^2 - 40$

$\nabla f(x,y) = 18x i + 8y j$

$\nabla f(2,-1) = 36j + (-8)j$ Unit vector $\frac{\nabla f}{\|\nabla f\|}$ $\quad \frac{(2,1)}{\|\nabla f(2,-1)\|} = \frac{36i-8j}{\sqrt{36^2+(-8)^2}} = \frac{\sqrt{85}(9i-2j)}{85}$ $= 0.976i$ $- 0.217j$

Ellipse Point $(2,-1)$ $\quad x=0 \quad y = \pm 3.16$

$9(2)^2 + 4(-1)^2 = 36 + 4 = 40 \quad y=0 \quad x = \pm 2.11$



13.5 | 1-53 odds

Chain Rule

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$y = f(u)$ is a dif-able func. of u

$u = g(x)$ is a dif-able func of x

then $y = f(g(x))$

1) $w = x^2 + y^2 \quad x = e^t \quad y = e^{-t}$

$w = e^{2t} + e^{-2t}$

$\frac{dw}{dt} = 2(e^{2t} - e^{-2t})$

3) $w = x \sec y \quad x = e^t \quad y = \pi - t$

$w = e^t \sec(\pi - t) \quad \frac{dw}{dt} = e^t \sec(\pi - t) + (-1)(\sec(\pi - t))(\tan(\pi - t))e^t$ ← simplify

5) $w = xy \quad x = 2\sin t \quad y = \cos t$

$\frac{dw}{dt} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial t} = y \cdot (2)(\cos t) + x(-\sin t) = 2\cos^2 t - 2\sin^2 t$ ~~No~~ yes

$w = 2\sin t \cos t \quad \frac{dw}{dt} = 2\cos t(\cos t) + 2(-\sin t)(\sin t) = 2(\cos^2 t - \sin^2 t)$

$= \boxed{2(\cos 2t)}$

7) $w = x^2 + y^2 + z^2 \quad x = e^t \cos t,\ y = e^t \sin t,\ z = e^t$

Why doesn't this way work?

$\frac{dw}{dt} = \frac{dw}{dx} \cdot \frac{dx}{dt} + \frac{dw}{dy} \cdot \frac{dy}{dt}$

$2x \cdot (e^t \cos t + (-\sin t) e^t) + 2y \cdot (e^t \sin t (\cos t) e^t) + 2z(e^t)$

$= 2 \cdot [x e^t (\cos t - \sin t) + y e^t (\sin t + \cos t) + z e^t]$

$= 2e^t [x(\cos t - \sin t) + y(\sin t + \cos t) + z]$ — NO —

$w = (e^t \cos t)^2 + (e^t \sin t)^2 + (e^t)^2$

$= e^{2t}(\cos^2 t + \sin^2 t + 1) = \boxed{e^{2t}(2)} \quad \frac{dw}{dt} = \boxed{4e^{2t}}$

9) skip

11) $x_1 = 10\cos 2t \quad y_1 = 6\sin 2t \quad x_2 = 7\cos t \quad y_2 = 4\sin t \quad t = \frac{\pi}{2}$

~~21~~ 13.5 / Notes

study this prob.

21) $W = \arctan \frac{y}{x}$ $x = r\cos\theta$ $y = r\sin\theta$ $\frac{d}{dx}[\arctan(u)] = \frac{u'}{1+u^2}$ $u = \frac{y}{x}$

$\frac{\partial u}{\partial x} = \frac{-y}{x^2}$ $\frac{du}{dy} = \frac{1}{x}$ a) $\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r}$

$$\frac{\partial w}{\partial r} = \frac{\frac{-y}{x^2}}{1+\left(\frac{y}{x}\right)^2}(\cos\theta) + \frac{\frac{1}{x}}{1+\left(\frac{y}{x}\right)^2}\sin\theta = \frac{\frac{-y\cos\theta}{x^2}}{\frac{x^2+y^2}{x^2}} + \frac{\frac{\sin\theta}{x}}{\frac{x^2+y^2}{x^2}}$$

$$= \frac{-y\cos\theta}{x^2y^2} + \frac{\sin\theta}{x^2y^2}$$

SUBSTITUTE

$$\frac{\partial w}{\partial r} = \frac{-r\sin\theta(\cos\theta)}{r^2} + \frac{r\cos\theta(\sin\theta)}{r^2} = 0$$

— SUB. FIRST: —

$$W = \arctan\frac{r\cos\theta}{r\sin\theta} = \arctan(\tan\theta) = \theta \qquad \frac{\partial w}{\partial r} = 0$$

$$\frac{\partial w}{\partial \theta} = 1$$

29) IMPLICIT DIF. $\ln\sqrt{x^2+y^2} + xy - 4 = 0$

$$F_x = \frac{1}{\sqrt{x^2+y^2}}\left[\tfrac{1}{2}(x^2+y^2)^{-1/2}(2x)\right] + y = \frac{x}{x^2+y^2} + y = \frac{x+x^2y+y^3}{x^2+y^2}$$

$$F_y = \frac{y}{x^2+y^2} + x = \frac{y+y^2x+x^3}{x^2+y^2}$$

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{x+x^2y+y^3}{\cancel{x^2+y^2}}\cdot\frac{\cancel{x^2+y^2}}{y+y^2x+x^3} = \frac{-(x+x^2y+y^3)}{y+y^2x+x^3}$$

old school implicit diff.

$$\tfrac{1}{2}\ln(x^2+y^2) + xy - 4 = 0$$

$$\frac{1}{2}\frac{1}{x^2+y^2}(2x+2yy') + [xy' + y(1)] - 0 = 0 \qquad \frac{2yy'}{2(x^2+y^2)} + xy' = \frac{-2x}{2(x^2+y^2)} - y = \frac{-x-yx^2-y^3}{x^2+y^2}$$

$$y'\left[\frac{y+x^3+xy^2}{x^2+y^2}\right] = \frac{-(x+x^2y+y^3)}{(y+y^2x+x^3)}$$

33) $\tan(x+y) + \tan(y+z) = 1$

$$f(x,y,z) = \tan(x+y) + \tan(y+z) + (-1) = 0 \qquad F_x = \sec^2(x+y) \quad F_y = \sec^2(x+y) + \sec^2(y+z)$$

$$F_z = \sec^2(y+z) \qquad \frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \qquad \frac{\partial z}{\partial y} = \frac{-F_y}{F_z} \qquad \frac{\partial z}{\partial x} = \frac{-\sec^2(x+y)}{\sec^2(y+z)}$$

$$\frac{\partial z}{\partial y} = \frac{-\sec^2(x+y) - \sec^2(y+z)}{\sec^2(y+z)}$$

13.5 | Notes

39. $xyz + xzw - yzw + w^2 - 5 = 0$

$F_x = yz + zw \quad F_y = xz - zw \quad F_z = xy + xw - yw \quad F_w = xz - yz + 2w$

$\frac{\partial w}{\partial x} = -\frac{F_x}{F_w} = \frac{-z(y+w)}{xz-yz+2w} \quad \frac{\partial w}{\partial y} = -\frac{F_y}{F_w} = \frac{-z(x-w)}{(\quad)} \quad -\frac{F_z}{F_w} = \frac{-(xy+xw-yw)}{(\quad)}$

IMPLICIT DIFF $\frac{\partial w}{\partial x}$ (1 of 3 partial d.'s)

$(yz)\frac{\partial x}{\partial x} + \left[(xz)\frac{\partial w}{\partial x} + (zw)\frac{\partial x}{\partial x}\right] - yz\frac{\partial w}{\partial x} + 2w\frac{\partial w}{\partial x} = 0$

$yz + xz\frac{\partial w}{\partial x} + zw - yz\frac{\partial w}{\partial x} + 2w\frac{\partial w}{\partial x} = 0 \quad (xz - yz + 2w)\frac{\partial w}{\partial x} = -yz - zw$

$\frac{\partial w}{\partial x} = \frac{-yz - zw}{(\quad)} = \frac{-z(y+w)}{xz - yz + 2w}$

51) volume & surface area, right circ. cylinder

radius increasing 6in/min height decreasing 4in/min

$R = 12in. \quad H = 36 in \quad$ 1. volume $= V = \pi r^2 h \quad \frac{dV}{dt} = \frac{\partial V}{\partial r}\frac{dr}{dt} + \frac{\partial V}{\partial h}\frac{dh}{dt}$

$= 2\pi rh\frac{dr}{dt} + \pi r^2\frac{dh}{dt} \quad \frac{dV}{dt} = 2\pi(12)(36)(6) + \pi(12)^2(-4) = 4608\pi \text{ in}^3/\text{min}$

2. surf. area $= 2\pi rh + 2\pi r^2 = 2\pi(rh + r^2) \quad \frac{dA}{dt} = \frac{\partial A}{\partial r}\frac{dr}{dt} + \frac{\partial A}{\partial h}\frac{dh}{dt}$

$= 2\pi(h+2r)\frac{dr}{dt} + 2\pi(r)\frac{dh}{dt} = 624\pi \text{ in}^2/\text{min}$

13.5 notes (2)

Ex 7. $F(x,y,z) = 3x^2z - x^2y^2 + 2z^3 + 3yz - 5$

F_x, F_y, F_z

1. $\frac{\partial z}{\partial x} = \frac{-F_x}{F_z}$ 2. $\frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$

3). $W = x\sec y$

$x = e^t \quad y = \pi - t$

$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt}$

$= \sec y\, e^t + x(\sec y \tan y)(-1)$

$\frac{\partial w}{\partial t} = e^t \sec(\pi - t) + (-)e^t \sec(\pi-t)\tan(\pi-t)$

$= e^t\sec(\pi-t)\left(1-\tan(\pi-t)\right)$

Substitution 1st

$W = e^t\sec(\pi-t)$

$\frac{dw}{dt} = e^t\sec(\pi-t)\tan(\pi-t) +$

$+ \sec(\pi-t)e^t$

$= e^t\sec(\pi-t)\,[1-\tan(\pi-t)]$

7) $W = x^2+y^2+z^2 \quad x = e^t\cos t \quad y = e^t\sin t \quad z = e^t$

a) $\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$

$= 2x[e^t(-\sin t) + \cos t(e^t)]$

$+ 2y[e^t(\cos t) + \sin t(e^t)]$

$2z(e^t)$

substitute for terms of $t(x,y,z)$

$= 2e^{2t}[\cos^2 t - \cos t\sin t + \sin t\cos t + \sin^2 t + 1]$

$= 4e^{2t}$

b) sub. first: $W = x^2+y^2+z^2 \quad w = [e^{2t}\cos^2 t + e^{2t}\sin^2 t + e^{2t}]$

$= e^{2t}[\cos^2 t + \sin^2 t + 1]$

$W = 2e^{2t} \quad \frac{dw}{dt} = 4e^{2t}$

15) $W = x^2+y^2 \quad x = s+t \quad y = s-t$ Point. $s=2, t=-1$

$\frac{dw}{ds} = \frac{\partial w}{\partial x}\frac{dx}{ds} + \frac{\partial w}{\partial y}\frac{dy}{ds} = 2x(1) + (2y)(1) = 2x+2y = 2[(s+t)+(s-t)] = 4s$

$\frac{dw}{ds} = 4(2) = 8$ $\quad \frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} = 2x(1) + 2y(-1) = 2(x-y)$

$= 2(s+t-(s-t)) = 2t2 = 4(-1) = -4$

13.5 | Chain rule Notes

Case I : $W = f(x,y)$

x & y are funcs of a single independent variable, t

$x = g(t) \quad y = h(t)$

Want $\frac{dw}{dt}$ 1. Use thm. 13.6

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt}$$

2. Substitute First.

Substitute $x = g(t)$ AND $y = h(t)$ into W

$W = m(t) \Rightarrow \frac{dw}{dt}$

Case II : $W = f(x,y)$

$x = g(s,t) \quad y = h(s,t)$

1. substitute $x = g(s,t)$

$y = h(s,t)$ into $w \Rightarrow W = m(s,t)$

$\Rightarrow \frac{\partial w}{\partial s}, \frac{\partial w}{\partial t}$

2. Chain rule Th 13.7

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s}$$

$$\frac{\partial w}{\partial t} = \frac{\partial x}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t}$$

— Implicit differentiation — P142 implicit diff.

expl. form : $y = f(x)$ $W = f(x,y)$

$y^3x + y^2x^2 + x^4y^4 = 16$

$F(x,y) = 0$ Right side $\overset{\text{Must}}{=} 0$

$$F_x(x,y)\frac{dx}{dx} + F_y(x,y)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{F_x(x,y)}{F_y(x,y)} \quad F_y(x,y) \neq 0$$

13.4/ 1-39 odds homework

1) $z = 3x^2y^3$ Total differential $dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy$

$\frac{\partial z}{\partial x} = 6xy^3 \quad \frac{\partial z}{\partial y} = 9x^2y^2 \quad \boxed{dz = 6xy^3 dx + 9x^2y^2 dy}$

3) $z = \frac{-1}{x^2+y^2} \quad d_x = \frac{0(x^2+y^2) - (2x)(-1)}{(x^2+y^2)^2} \quad d_y = \frac{0(x^2+y^2) - (2y)(-1)}{(x^2+y^2)^2}$

$dz = \boxed{\frac{2x}{(x^2+y^2)^2} dx + \frac{2y}{(x^2+y^2)^2} dy}$ ← can simplify more

5) $z = x\cos y - y\cos x \quad \frac{\partial z}{\partial x} = \cos y - (1)\overset{(-)}{y}\sin x \quad \frac{\partial z}{\partial y} = x(-\sin y) - \cos x$

$dz = (\cos y + y\sin x)\, dx - (x\sin y + \cos x)\, dy$

7) $z = e^x \sin y \quad \frac{\partial z}{\partial x} = e^x \sin y \quad \frac{\partial z}{\partial y} = e^x \cos y \quad dz = (e^x \sin y)\, dx + (e^x \cos y)\, dy$

9) $w = 2z^3 y \sin x \quad \frac{\partial w}{\partial x} = 2z^3 y \cos x \quad \frac{\partial w}{\partial y} = 2z^3 \sin x \quad \frac{\partial w}{\partial z}\ 6z^2 y \sin x$

$dw = (2z^3 y\cos x)^{dx} + (2z^3 \sin x)^{dy} + (6z^2 \sin x)^{dz}$

11) $f(x,y) = 9 - x^2 - y^2 \quad f(1,2) = 9 - 1 - 4 = 4 \quad f(1.05, 2.1) = 3.4875$

$df = -2x\,dx - 2y\,dy \quad -2(.05) - 2(2)(.1) = -0.1 - 0.4 = \boxed{-0.5 = dz}$

$\Delta z = 3.4875 - 4 = \boxed{-0.5125}$

13) $f(x,y) = x\sin y \quad df = \sin y\, dx + x\cos y\, dy = \sin(2)(.05) + \cos(2)(0.1) = \cancel{1.01684}$ 0.0038502

$1\sin 2 = \cancel{0.0349}$ 0.909297 $\quad \Delta z = \cancel{0.0035}$ 0.002927

$1.05\sin 2.1 = \cancel{0.03848}$ 0.906369

15) $f(x,y) = 3x - 4y \quad df = 3dx - 4dy = 3(.05) - 4(0.1) = -0.25$

$3(1) - 4(2) = -5 \quad 3(1.05) - 4(2.1) = -5.25 \quad \Delta z = -0.25 = dz$

17) find $z = f(x,y)$ & use total diff to approximate qty.

$\sqrt{5.05^2 + 3.1^2} - \sqrt{5^2 + 3^2} \quad 0.09463$

19) skip

13.4) homework 21-39 odd

21) total differential ~~of~~ a func. of 2 variables is ; if $z = f(x,y)$

& Δx & Δy are increments of x & y, x & y are independent vars, total differential is

$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy$

23) linear approximation of $z = f(x,y)$ at point $P(x_0, y_0)$

is the approx of Δz by dz, ~~dz~~ dz represents change in height of a plane

tangent to ~~the~~ the surface at point P.

25) $A = lh$ errors Δl & Δh find dA

$dA = \frac{\partial A}{\partial h} dh + \frac{\partial A}{\partial l} dl = h\,dl + l\,dh =$ top right reps difference

27) skip 29) skip

31) $V = \pi r^2 h$ $r \pm 4\%$ $h \pm 2\%$

$dV = \frac{dV}{dr} dr + \frac{dV}{dh} dh = 2\pi r h\, dr + \pi r^2 dh$ -

$2\pi r h\ 4\% + \pi r^2\ 2\%$

$= 2(.04) + (0.02) = 0.10$

$\pm 10\%$ error

13.3 / homework 31-47 odd, 51-67, 73-85

? 31) $f = \sqrt{x+y}$ $f_x = \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x+y} - (\sqrt{x+y})}{\Delta x}$ $\lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x+y}}{\Delta x} = \frac{x+\Delta x+y}{\Delta x\sqrt{x+\Delta x+y}}$

33) $f(x,y) = \arctan\frac{y}{x}$ $(2,-2)$ $f_x = \frac{-\frac{y}{x^2}}{1+\frac{y^2}{x^2}} = \frac{-y}{x^2}\cdot\frac{(x^2+x^2)}{y^2} = \frac{-2x^2}{yx^2}$ $\frac{u'}{1+u^2}$

$\boxed{f_y} = \frac{\frac{1}{x}}{1+\frac{y^2}{x^2}} = \frac{\frac{1}{x}}{\frac{x^2+y^2}{x^2}}$ $\frac{2}{4+4} = \boxed{\frac{1}{4}}$ $= \frac{-y}{\cancel{x^2}} \cdot \frac{\cancel{(x^2)}}{(y^2+x^2)}$ $\frac{-y}{y^2+x^2}$ $\frac{2}{4+4} = \frac{1}{4}$

skip → 35) $f(x,y) = \frac{xy}{x-y}$ $\overset{x\ y}{(2,-2)}$ f_x $\frac{u'}{1+u^2} = \frac{\frac{y(x-y)-xy(1)}{(x-y)^2}}{1+\frac{(xy)^2}{(x-y)^2}} = \frac{\frac{y(x-y)-xy}{\cancel{(x-y)^2}}}{\frac{(x-y)^2+(xy)^2}{\cancel{(x-y)^2}}}$ $\boxed{\frac{g'(x)h(x)-g(x)h'(x)}{(h(x))^2}}$

$= \frac{y(x-y)-(xy)}{(x-y)^2+(xy)^2} = \frac{-2(4)-(-4)}{(4)^2+(-4)^2} = \frac{-8+4}{32} = -\frac{4}{32} = -\frac{1}{8}$

37) $g(x,y) = 4-x^2-y^2$ pt$\overset{x\ y\ z}{(1,1,2)}$

$g_x(x,y) = -2x$ $\boxed{-2}$ $g_y(1,1) = -2y = \boxed{-2}$

39) $z = e^{-x}\cos y$ $(0,0,1)$ $z_x = (-1)e^{-x} = -1(1) = \boxed{-1}$

$z_y = 1(-\sin y) = \boxed{-1}$

41) 43) skip

45) $f(x,y) = x^2+4xy+y^2-4x+16y+3$ $f_x = 2x+4y-4$

$f_y = 4x+2y+16$

$2x+4y-4 = 4x+2y+16$

$2y = 2x+20$

$y = x+10$ $2x+4(x+10)-4 = 0$

$2x+4x+40-4 = 0$

$6x = -36$

$\boxed{x=-6 \quad y=4}$

13.3) homework 47, 51-67 odd, 73-85 odd

~~$f_{(x,y)} \int_x^y (t^2-1)\,dt \qquad f_x(x,y) = (x^2-y)-(y^2-1) = x^2-y^2$~~

47) skip

49) b is slope in x direction, f_x a) is slope in y direction, f_y

51) $w = \sqrt{x^2+y^2+z^2}$ $\quad \frac{1}{2}\sqrt{x^2+y^2+z^2}^{-1}(2z) = \frac{\cancel{2}z}{\cancel{2}\sqrt{x^2+y^2+z^2}}$ & same for x, y, respectively

53) $f_{(x,y,z)} = \ln\sqrt{x^2+y^2+z^2}$ $\quad f_z\ \frac{z}{\sqrt{x^2+y^2+z^2}} \cdot \frac{1}{\sqrt{x^2+y^2+z^2}} = \frac{z}{x^2+y^2+z^2}$ ‖

55) $H(x,y,z) = \sin(x+2y+3z) \quad H_x = \cos(x+2y+3z) \quad H_y = 2\sin(x+2y+3z)$, etc.

57) $f_{(x,y,z)} = \sqrt{3x^2+y^2-2z^2}$ $(1,-2,1)$ $\quad H_x = \left(\frac{1}{2}\right)\frac{6x}{\sqrt{3x^2+y^2-2z^2}} = \frac{\cancel{6}\,3}{\sqrt{3+4-2}} = \frac{3}{\sqrt{5}} = \boxed{\frac{3\sqrt{5}}{5}}$

59) skip

61) $z = x^2-2xy+3y^2$ $\quad$ find 4 second partial derivatives $\quad z_y = -2x+6y$

$z_x = 2x-2y \qquad \frac{\partial^2 z}{\partial x^2} = 2 \qquad \frac{\partial^2 z}{\partial y^2} = 6 \qquad z_{yx} = -2$

$z_{xy} = -2$

? 63) $z = \sqrt{x^2+y^2} \quad \frac{\partial z}{\partial x} = \frac{1}{2}\frac{1}{\sqrt{x^2+y^2}}(2x) = \frac{x}{\sqrt{x^2+y^2}} \quad \frac{\partial^2 z}{\partial y\partial x} = -\frac{1}{2}\frac{1}{x^2+y^2}(2y) = \frac{-y}{x^2+y^2}$

65) $z = e^x \tan y \quad \frac{\partial z}{\partial x} = (1)e^x \quad \cancel{\frac{\partial^2 z}{\partial x^2}}$ product rule $f'g + fg' = (1)e^x\tan y + 0(e^x)$

$\boxed{\frac{\partial^2 z}{\partial x^2} = \boxed{e^x \tan y}} \qquad \boxed{\frac{\partial^2 z}{\partial y\partial x}} = (0)\tan y + \boxed{e^x \sec^2 y}$

$\frac{g'xhx - gxh'x}{(hx)^2}$

$\frac{\partial^2 z}{\partial y^2} = (0)\sec^2 y + e^x[\sec^2 y + \tan^2 y](2) = \boxed{2e^x(\sec^2 y + \tan^2 y)}$

67) $z = \arctan\frac{y}{x} \quad \frac{\partial z}{\partial x} = \frac{(-1)\frac{y}{x^2}}{1+\frac{y^2}{x^2}} = \frac{\frac{-y}{x^2}}{\frac{x^2+y^2}{x^2}} = \frac{-y\cancel{x^2}}{\cancel{x^2}(x^2+y^2)} = \frac{-y}{x^2+y^2} \qquad \frac{u'}{1+u^2}$

$\frac{\partial^2 z}{\partial x^2} : (2) \quad \frac{0(2x) - (-y)(2x)}{(x^2+y^2)^2} = \boxed{\frac{+2xy}{(x^2+y^2)^2}} \qquad \frac{\partial z}{\partial y} = \frac{\frac{1}{x}}{1+\frac{y^2}{x^2}} = \frac{x^2}{\cancel{x}(x^2+y^2)}$

$\frac{\partial^2 z}{\partial y^2} = \frac{0(x^2+y^2) - x(2y)}{(x^2+y^2)^2} = \boxed{\frac{-2xy}{(x^2+y^2)^2}} \qquad \boxed{\frac{\partial^2 z}{\partial_{xy}} = \text{skip}}$

13.3| 73-85 odd

73. $f(x,y,z) = xyz$ $\quad f_x = yz \quad f_{xy} = z \quad f_{xyy} = 0$

$f_y = xz \quad f_{yx} = z \quad f_{yxy} = 0$ etc.

13.3 homework 5-47 odd 51-67 odds 73-85 odds

5) $f(x,y) = 2x - 3y + 5$ $\quad f_x \frac{\partial f}{\partial x} = 2, \quad f_y \frac{\partial f}{\partial y} = -3$

7) $z = x\sqrt{y}$ $\quad Z_x = \sqrt{y}$ $\quad Z_y = \frac{1}{2} x y^{-1/2}$

$\frac{\partial z}{\partial x}$ $\quad \frac{\partial z}{\partial y}$

Product Rule

$$\boxed{(f \cdot g)' = f' \cdot g + f \cdot g'}$$

9) $z = x^2 - 5xy + 3y^2$ $\quad \frac{\partial z}{\partial x} = 2x - 5y$ $\quad \frac{\partial z}{\partial y} = -5x + 6y$ $\quad f' \ln(u) = \frac{u'}{u}$

11) $z = x^2 e^{2y}$ $\quad \frac{\partial z}{\partial x} = 2xe^{2y}$ $\quad \frac{\partial z}{\partial y} = x^2 e^{2y}(2)$

13) $z = \ln(x^2+y^2)$ $\quad \frac{\partial z}{\partial x} = \frac{2x}{(x^2+y^2)}$ $\quad \frac{\partial z}{\partial y} = \frac{2y}{(x^2+y^2)}$

Quotient Rule

? - 15) $z = \ln \frac{x+y}{x-y}$ $\quad \frac{\partial z}{\partial x} = \frac{1}{\frac{x+y}{x-y}} \cdot \frac{(1)(x-y) - (x+y)(1)}{(x-y)^2}$

$$= \frac{-(x+y)}{x-y} = \frac{1}{\quad}$$

$$f'(x) = \frac{g'(x)h(x) - g(x)h'(x)}{(h(x))^2}$$

where $f(x) = \frac{g(x)}{h(x)}$

17) $z = \frac{x^2}{2y} + \frac{4y^2}{x}$ $\quad \frac{\partial z}{\partial x} = \frac{x}{y} + \frac{2y^2}{x^2}(-1)$ $\quad \frac{x^3}{yx^2} - \frac{2y^3}{x^2y} = \frac{x^3 - 2y^3}{x^2y}$

$$\frac{\partial z}{\partial y} = \frac{-x^2}{2y^2} + \frac{8y}{x} = \frac{-x^3 + 8y(2y^2)}{2y^2x} = \frac{16y^3 - x^3}{2y^2x}$$

19) $h(x,y) = e^{-(x^2+y^2)}$ $\quad h_x(x,y) = -2xe^{-(x^2+y^2)}$ $\quad h_y(x,y) = -2ye^{-(x^2+y^2)}$

21) $f(x,y) = \sqrt{x^2+y^2}$ $\quad (x^2+y^2)^{1/2}$ $\quad f_x(x,y) = \left(\frac{1}{\cancel{2}}\right)\frac{1}{\sqrt{x^2+y^2}}(\cancel{2}x) = \frac{x}{\sqrt{x^2+y^2}}$

$$f_y(x,y) = \frac{y}{\sqrt{x^2+y^2}}$$

23) $z = \tan(2x-y)$ $\quad Z_x = \sec^2(2x-y)(2)$ $\quad Z_y = -\sec^2(2x-y)$

25) $z = e^y \sin(xy)$ $\quad Z_x = \cos(xy)$ $\quad Z_y = (1)e^y \sin(xy) + e^y \cos(xy)(x)$

$$e^y(\sin xy + x\cos xy)$$

27) $f = \int_x^y (t^2-1)\,dt$ $\quad \cancel{f_x(x,y) = 1 - x^2}$ $\quad f_x(x,y) = \cancel{(y^2-1) - (x^2-1)}\ (\quad) - (x^2-1)$

$$= (1-x^2)$$

$$f_y(x,y) = (y^2-1)$$

29) $f = 2x + 3y$ use limit def. of partial deriv. to find f_x & f_y

$$f_x(x,y) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x} = \lim_{\Delta x \to 0} \frac{2(x+\Delta x) + 3y - (2x+3y)}{\Delta x} = \boxed{\lim_{\Delta x \to 0} \frac{2\Delta x}{\Delta x} = 2}$$

good

$$f_y(x,y) \lim_{\Delta y \to 0} \frac{(2x + 3(y+\Delta y)) - (2x+3y)}{\Delta y} = \boxed{3}$$

13.3 | Notes

75) $f(x,y,z)\ e^{-x}\sin yz$

$f_x = -e\sin yz$

$f_{xy} = -e^{-x}\cos yz\,(z) = -ze^{-x}\cos yz$

$f_{xyy} = -ze^{-x}\,{-\sin yz}\,(z) = +\boxed{z^2e^{-x}\sin yz}$

$f_y = e^{-x}\cos yz\,(z) = ze^{-x}\cos yz$

$f_{yx} = -ze^{-x}\cos yz$

$f_{yxy} = -ze^{-x}(-\sin yz)\ z = \boxed{z^2e^{-x}\sin yz}$

79) Laplace equation

$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0 \qquad z = e^x\sin y$

$\frac{\partial z}{\partial x} = e^x\sin y \qquad \frac{\partial z}{\partial y} = e^x\cos y$

$\frac{\partial^2 z}{\partial x^2} = e^x\sin y \qquad \frac{\partial^2 z}{\partial y^2} = e^x(-\sin y)$

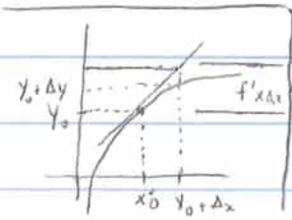
$+ = 0$

! 13.4

13.4 | Notes

Differentials

$y = f(x) \quad \frac{dy}{dx} = f'x \quad dy = f'(x)\,dx$ Differential form



$\Delta y = f(x+\Delta x) - f(x)$

$\Delta y \approx f'x\,\Delta x = dy$

$z = f(x,y)$

Increment of x is Δx

" " of y is Δx

Increment of z $\quad \Delta z = (x+\Delta x, y+\Delta y) - f(x,y)$

TOTAL DIFFERENTIAL

$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy \qquad dx \approx \Delta x$

$dy \approx \Delta y$

$w = f(x,y,z)$

$dw = \frac{\partial w}{\partial x}dx + \frac{\partial w}{\partial y}dy \quad \frac{\partial w}{\partial z}dz$

error terms.

$\Delta z = f_x(x_0,y_0)\Delta x + f_y(x_0,y_0)\Delta y + \epsilon_1\Delta x\ \epsilon_2\Delta y$

ϵ_1 & $\epsilon_2 \to 0$ as Δx & $\Delta y \to 0,0$

13.3 | notes Partial Derivatives

Ex 2 $f(x,y)\ xe^{x^2y}$

$f_y = xe^{x^2y}(x^2)$ — Deriv. of exponent w/respect to 'y'

product rule, chain rule

$f_x = (x)e^{x^2y}(2xy) + e^{x^2y}(1)$

$z = f(x,y)$ two partial derivatives, gives slope in x & y directions

area:

$A = ab\sin\theta$ parallelogram

— Higher order partial derivatives —

① $\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2} = f_{xx}$ ② $\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2} = f_{yy}$

③ $\frac{\partial f}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y\,\partial x}$ "∂" notation has R to L order

$= f_{xy}$ subscript not. has L to R order

④ $\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x\,\partial y} = f_{yx}$

Ex 7 $f_{xy} = f_{yx} = 6y + 20xy$ — for most continuous functions, mixed derivatives are equal

Ex 8 $f(x,y,z) = ye^x + x\ln z$

$\frac{\partial f}{\partial x}\ ye^x + \ln z \quad \frac{\partial f}{\partial z} = \frac{x}{z} \quad \frac{\partial}{\partial z}\left(\frac{\partial f}{\partial x}\right) = \frac{1}{z} \quad \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial z}\right) = \frac{1}{z}$ SAME

$\frac{\partial}{\partial z}\left(\frac{\partial}{\partial z}\left(\frac{\partial f}{\partial x}\right)\right) = f_{zz} = \frac{-1}{z^2} \quad \frac{\partial}{\partial z}\left(\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial z}\right)\right) = f_{zxz} = \frac{-1}{z^2}$

13.3 | HW

11) $z = x^2e^{2y} \quad \frac{\partial z}{\partial x} = 2xe^{2y} \quad \frac{\partial z}{\partial y} = x^2e^{2y}(2)$

15) $z = \ln\frac{x+y}{x-y} = \ln(x+y) - \ln(x-y)$

$\frac{\partial z}{\partial x} = \frac{1}{x+y}(1) - \frac{1}{x-y}(1) = \frac{(x-y)-(x+y)}{(x+y)(x-y)} = \frac{-2y}{x^2y^2}$

$\frac{\partial z}{\partial y} = \frac{1}{x+y}(1) - \frac{1}{x-y}(-1) = \frac{2x}{x^2y^2}$

~~19)~~

13.3 examples

19) $h(x,y) = e^{-(x^2+y^2)}$ $\quad \frac{\partial h}{\partial x} = e^{-(x^2+y^2)}(-2x)$ $\quad \frac{\partial h}{\partial y} = e^{-(x^2+y^2)}(-2y)$

25) $z = e^y \sin xy$ $\quad \frac{\partial z}{\partial x} = e^y(\cos xy)(y)$ $\quad \frac{\partial z}{\partial y}$ prod. rule $= e^y \cos xy (x) + (\sin xy)e^y$

study this prob.

33) $f(x,y) = \arctan x/y$

form: $\boxed{\frac{\partial}{\partial x}(\arctan x) = \frac{u'}{1+u^2} \quad u = y/x \quad \frac{\partial u}{\partial x} = \frac{-y}{x^2} \quad \frac{\partial u}{\partial y} = \frac{1}{x}}$

$(2, -2, -\frac{\pi}{4})$ $\quad \frac{\partial f}{\partial x} = \frac{\frac{-y}{x^2}}{1+\left(\frac{y}{x}\right)^2} = \frac{\frac{-y}{x^2}}{\frac{x^2+y^2}{x^2}} = \frac{-y}{x^2+y^2}$

$\frac{\partial f}{\partial x}(2,-2) = \frac{-(-2)}{(2)^2+(-2)^2} = \frac{2}{4+4} = \frac{1}{4}$

$\frac{\partial f}{\partial y} = \frac{\frac{1}{x}}{1+\left(\frac{y}{x}\right)^2} = \frac{x}{x^2+y^2}$ $\quad \frac{\partial f}{\partial y}(2,-2) \; \frac{2}{4+4} = \frac{1}{4}$

61) $z = \arctan y/x$ $\boxed{\frac{\partial z}{\partial x} = \frac{-y}{x^2+y^2}}$ $\quad \frac{\partial^2 z}{\partial x^2} = \frac{(x^2+y^2)(0) - (-y)(2x)}{(x^2+y^2)^2} = \frac{2xy}{(x^2+y^2)^2}$

$\frac{\partial z}{\partial y} = \frac{x}{x^2+y^2}$ $\quad \frac{\partial^2 z}{\partial y^2} = \frac{(x^2+y^2)(0) - x(2y)}{(x^2+y^2)^2} \longrightarrow = \frac{-2xy}{}$

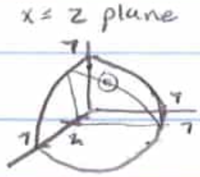
quotient rule

$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{-y}{x^2+y^2}\right) = \frac{x^2+y^2(-1) - (-y)(2y)}{x^2+y^2} \quad \frac{y^2-x^2}{(x^2+y^2)^2}$

$\frac{\partial^2 z}{\partial x \partial y} = \frac{y^2-x^2}{(x^2+y^2)^2}$

41) $z = \sqrt{49-x^2-y^2}$ sphere $(2,3,6)$

$x = z$ plane $\quad z = \sqrt{49-x^2-y^2} = \sqrt{49-(2)^2-y^2} = \sqrt{45-y^2}$



$\frac{\partial z}{\partial y} = \frac{\partial(45-y^2)^{\frac{1}{2}}}{\partial y} = \frac{1}{2}(45-y^2)^{-\frac{1}{2}}(-2y) = \frac{-y}{\sqrt{45-y^2}}$

$\frac{\partial z}{\partial y}(3) = \frac{-3}{\sqrt{45-9}} = \frac{-3}{6} = -\frac{1}{2}$

53) $F(x,y,z) = \ln\sqrt{x^2+y^2+z^2}$ $\quad \frac{\partial F}{\partial x} = \frac{1}{\sqrt{x^2+y^2+z^2}} \; \frac{1}{2}(x^2+y^2+z^2)^{-1/2}(2x)$ } long way to write problem

$= \frac{x}{\sqrt{}\sqrt{}} = \frac{x}{x^2+y^2+z^2}$

$F(\ldots) = \frac{1}{2}\ln(x^2+y^2+z^2)$, $\quad \frac{\partial F}{\partial x} = \frac{1}{2}\left(\frac{1}{x^2+y^2+z^2}\right)(2x)$ $\quad \frac{\partial F}{\partial y} = \frac{y}{x^2+y^2+z^2}$

$\frac{\partial F}{\partial z} = \frac{z}{x^2+y^2+z^2}$

↑ log. properties

quotient rule

4) $w = \frac{x+y}{z-2y}$ $\quad \frac{\partial w}{\partial x} = \frac{1}{z-2y}$ $\quad \frac{\partial w}{\partial y}$ $\frac{(z-2y)(1)-(x+y)(-2)}{z-2y^2} = \frac{z+2x}{(\ )^2}$

$\frac{\partial w}{\partial z}$ $\frac{(z-2y)(0)-(x+y)(1)}{(\quad)^2} = \frac{-x-y}{(\ )^2}$ $\quad dw = \frac{\partial w}{\partial x}dx + \frac{\partial w}{\partial y}dy + \frac{\partial w}{\partial z}dz$

$= \frac{1}{z-2y}dx + \frac{z+2x}{(z-2y)^2}dy + \frac{-x-y}{(z-2y)^2}dz$

5) $z = x\cos y - y\cos x$ $\quad dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy = [\cos y - y(-\sin x)]dx + [x(-\sin y) - \cos x]dy = \cos y + y\sin x(dx) + \ldots$

9) $W = 2z^3 y\sin x$

$dw = \frac{\partial w}{\partial x}dx + \frac{\partial w}{\partial y}dy + \frac{\partial w}{\partial z}dz$

$dw = 2z^3 y\cos x\,dx + 2z^3\sin x\,dy + 6z^2 y\sin x\,dz$

11) $f(x,y) = 9 - x^2 - y^2$

a) $f(1,2) = 9 - 1 - 4 = 4.0$

$f(1.05, 2.1) = 9 - 1.05^2 - (2.1)^2 = 3.4875$

$\Delta z = 3.4875 - 4 = -0.5125$ Actual change

b) $z = 9 - x^2 - y^2$

$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$

$dz = -2x\,dx - 2y\,dy$

$dz = -2(1)(.05) - 2(2)(.1)$

$= .10 - 0.40 = -.50$ Approximation

19) $\frac{1-(3.05)^2}{5.95} - \frac{1-3^2}{6^2} = -0.012$

Let $z = \frac{1-x^2}{y^2}$ $\quad x=3 \quad dx = 0.05$, $y=6 \quad dy = 0.05$

$z = \frac{1}{y^2} - \frac{x^2}{y^2} = y^{-2} - x^2y^{-2}$

$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$

$dz = -2xy^{-2}dx + [-2y^{-3} - x^2(-2)y^{-3}]dy$

$dz = \frac{-2x}{y^2}dx + \quad dy$

$dz = \frac{-2(3)}{(6)^2}(.003) + \frac{(-2)(1-(3)^2)}{(6)^3}(-0.05) = -0.012$

31) VOLUME – Right Circular Cylinder

$V = \pi r^2 h$ $\quad$ 4% error in r, 2% error in h ⇦ relative errors, need relative terms

$dV = \frac{\partial V}{\partial r}dr + \frac{\partial V}{\partial h}dh$

$dV = (2\pi rh)dr + (\pi r^2)dh$

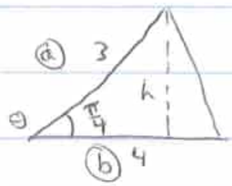
$\frac{dV}{V} = \frac{(2\pi rh)dr + (\pi r^2)dh}{\pi r^2 h} = 2\frac{dr}{r} + \frac{dh}{h} = 2(.04) + (.02)$

$= 0.10$

Rel. error $= \pm 10\%$

13.4) Notes

32) area triangle



$A = \frac{1}{2}bh \quad h = a\sin\theta$

$A = \frac{1}{2}ba\sin\theta$ (3 variables)

$$dA = \frac{\partial A}{\partial a}da + \frac{\partial A}{\partial b}db + \frac{\partial A}{\partial \theta}d\theta$$

$$dA = \frac{1}{2}b\sin\theta\, da + \frac{1}{2}a\sin\theta\, db + \frac{1}{2}ab\cos\theta\, d\theta$$

* angle Δ's must be in rad. for these types of problems.

$da = db = \frac{1}{16}$ inch

$d\theta = 0.02$ radians

$$dA = \frac{1}{2}(4)\sin\frac{\pi}{4}\left(\frac{1}{16}\right) + \frac{1}{2}(3)\sin\frac{\pi}{4}\left(\frac{1}{16}\right) + \frac{1}{2}(3)(4)\cos\frac{\pi}{4}(.02) = .23959$$

$$dA \cong \pm .24 \text{ in}^2$$

13.2 | Homework 33-57 odds

33. $f(x,y) = \begin{cases} \dfrac{x^2+2xy^2+y^2}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$ discuss continuity

~~continuous~~

Not continuous

$g(x,y) = \begin{cases} \dfrac{x^2+2xy^2+y^2}{x^2+y^2} & xy \neq 00 \\ 1 & xy = 1 \end{cases}$ continuous

$\lim\limits_{x,y \to 00} \dfrac{x^2+2xy^2+y^2}{x^2+y^2}$ $\lim\limits_{\substack{x=0 \\ y\to 0}} \dfrac{y^2}{y^2} = 1$ $\lim\limits_{\substack{y=0 \\ x\to 0}} \dfrac{x^2}{x^2} = 1$ $\lim = 1$

35) 37) 39) Skip

41) $\lim\limits_{xy \to 00} \dfrac{\sin(x^2+y^2)}{x^2+y^2}$ $x = r\cos\theta$, $y = r\sin\theta$ $\lim\limits_{r\to 0} \dfrac{\sin(r^2)}{(r^2)} = \dfrac{0}{0}$

$\boxed{\text{l hopital's rule: } \lim\limits_{x\to c} \dfrac{f(x)}{g(x)} = \lim\limits_{x\to c} \dfrac{f'(x)}{g'(x)}}$ $\boxed{r = \sqrt{x^2+y^2}}$ $x = r\cos\theta$, $y = r\sin\theta$

$\dfrac{(2r)\cos r^2}{(2r)}$ $\lim\limits_{r\to 0} \cos(r^2) = \boxed{1}$

43) $\lim\limits_{xy \to 00} \dfrac{x^3+y^3}{x^2+y^2}$ $\lim\limits_{r\to 0} \dfrac{\cancel{r^2}(x+y)}{\cancel{r^2}}$ $\lim\limits_{r\to 0} \dfrac{x+y}{0} = 0$

$\dfrac{r^3\cos^3\theta + r^3\sin^3\theta}{r^2}$ $\lim\limits_{r\to 0} \dfrac{r^3(\cos^3\theta+\sin^3\theta)}{\cancel{r^2}} = 0$ $\cos^2\theta + \sin^2\theta = 1$

45) $\lim\limits_{(x,y)\to 00} \dfrac{x^2y^2}{x^2+y^2}$ $\dfrac{r^2(\cos^2\theta+\sin^2\theta)}{\cancel{r^2}} = 1$

47) $\lim\limits_{xy \to 00} (x^2+y^2)(\ln)(x^2+y^2) = \lim\limits_{x,y\to 00} r^2 \ln(r^2)$ = indeterminate

$\lim\limits_{r\to 0} \dfrac{\ln(r^2)}{\frac{1}{r^2}} = \lim\limits_{r\to 0} \dfrac{\frac{2r}{r^2}}{\cdot 2\left(\frac{1}{r}\right)}$ $\dfrac{\frac{2r}{r^2} \cdot \frac{r}{2}}{\frac{2}{r}} = 1$

$\frac{1}{2}r^{-1/2}$ $-\frac{1}{4}r^{-3/2}$

$\lim\limits_{r\to 0} \dfrac{\frac{1}{r^2}(2r)}{-2/r^3}$ $\dfrac{1}{r} \cdot \dfrac{r^3}{-2} = -r^2$ $\lim\limits_{r\to 0} r^2 = 0$

49) $f(x,y,z) = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$ continuity:

$\lim\limits_{\substack{z=0 \\ y=0 \\ x\to 0}}$ $x, y, z \neq 0$

51) $f(x,y,z)$ $\dfrac{\sin z}{e^x+e^y}$ continuous

53) $\dfrac{\sin xy}{xy}$ $xy \neq 0$ lim continuous

55) $f(g) = (3x-2y)^2$ continuous

57) $f(g) = \dfrac{1}{4-(x^2+y^2)}$ $x^2+y^2 \neq 4$ elsewhere continuous

13.2| homework 1-57 odds

1) $\lim_{(x,y)\to 2,3} x = 2$ 3) skip } both use proofs?

5) $\lim [5-3] = 2$ 7) $\lim [5 \cdot 3] = 15$

9) $\lim_{(x,y)\to(2,1)} (x+3y^2) = 5$ continuous

11) $\lim_{x,y\to 2,4} \frac{x+y}{x-y} = \frac{6}{-2} = -3$ continuous except $x \neq y$,

13) $\lim_{x,y\to(0,1)} \frac{\arcsin(x/y)}{1+xy} = 0$ $xy \neq -1$ $y \neq 0$ $\left|\frac{x}{y}\right| \leq 1$

15) $\lim_{x,y\to(-1,2)} e^{xy} = e^{-2}$ continuous

17) $\lim_{x,y,z\to(1,2,5)} \sqrt{x+y+z}$ $\sqrt{8}$ $2\sqrt{2}$ $(x+y+z) \geq 0$

19) $\lim_{(x,y)\to(0,0)} \frac{x+y}{x^2+y}$ $x \to 0$ $\frac{x+0}{x^2+0}$ $\frac{x+0}{x^2+0}$ $\lim \frac{1}{x}$ DNE as a finite #

21) $\lim_{x,y\to 1,1} \frac{xy-1}{1+xy} = 0$ 23) $\lim_{x,y,z\to 0,0,0} \frac{xy+yz+xz}{x^2+y^2+z^2}$ $x=0$ $\frac{yz}{y^2+z^2} =$

$(0,0,z)\to(0,0,0)$ $\frac{0}{z^2} = 0$ $x=y=z$ $(x,y,z)\to(0,0,0)$ $\frac{(x^2)^3}{(x^2)^3} = 1$ limit DNE

25) $\lim_{x,y\to 0,0} e^{xy} = 1$, continuous

? 27) $\lim_{x,y\to 0,0} \ln(x^2+y^2)$

29) $f(x,y) = \frac{xy}{x^2+y^2}$ does not exist. 31) skip

approaches different values from different paths

x	y	z
1	0	0
.5	0	0
.1	0	0
.01	0	0
.001	0	0
1	1	.5
.5	.5	.5
.1	.1	.5
.01	.01	.5
.001	.001	0.5

Homework

13.1| 1-37 odds, 45-55 odds, 69-73 odds,

1. $x^2z+yz-xy=10$ z is a func. of x & y

3. $\frac{x^2}{4}+\frac{y^2}{9}+z^2=1$ z is NOT a func. of x & y

5) $f(x,y)=x/y$ a) $3/2$ b) $-1/4$, c) 6 d) $5/y$ e) $x/2$ f) $5/t$

7) $f(x,y)=xe^y$ a) 5 b) $3e^2=22.17$ c) 0.73 d) $5e^y$ e) xe^2 te^t

9) skip 11) skip

13) $g(x,y)=\int_x^y (2t-3)\,dt$

a) (0,4) $\int_0^4 (2t-3)\,dt$

15) a) $z=x^2-2y$

$\frac{f(x+\Delta x,y)-f(x,y)}{\Delta x}$ $\frac{2x^2+x\Delta x+\Delta x^2-2y-(x^2-2y)}{\Delta x}$ $\rightarrow$ $\boxed{2x+\Delta x \quad \Delta x\neq 0}$

17) $f(x,y)=\sqrt{4-x^2-y^2}$ describe domain & range

R: $0\le z\le 2$

D: $(x^2+y^2)\le 4$

19) $z=\arcsin(x+y)$

Range $\left(-\frac{\pi}{2}\le z\le\frac{\pi}{2}\right)$

Dom. $-1\le(x+y)\le 1$

21) $z=\ln(4-x-y)$

Domain $(x+y)<4$

Range $(-\infty,\infty)$

23) $z=\frac{x+y}{xy}$

D: $x\neq 0$ $y\neq 0$

R: $(-\infty,\infty)$ $\mathbb{R}$

25) $z=e^{x/y}$ Dom: $y\neq 0$ Range: $\mathbb{R}$

27) $g(x,y)=\frac{1}{xy}$ $x\neq 0$ $y\neq 0$ $\mathbb{R}\neq 0$

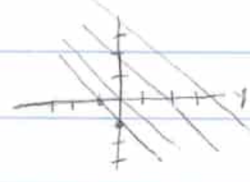
31) $z=5$

33)

35 – 41) skip

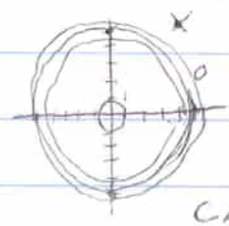
45) c 47) b

49)



$c=-1,0,2,4$

51) $z=\sqrt{25-x^2-y^2}$ $c=0,1,2,3,4$



$4=25-x^2-y^2$

$x^2+y^2=21$

$9=$

CAN DO THIS

13.2 examples. p902

11) $\lim_{(x,y)\to(2,4)} \frac{x+y}{x-y}$ $\quad \frac{2+4}{2-4} \quad \frac{6}{-2} \quad \boxed{-3}$ direct sub.

13) $\lim_{(x,y)\to(0,1)} \frac{\arcsin\left(\frac{x}{y}\right)}{1+xy} = \frac{\arcsin\left(\frac{0}{1}\right)}{1+0(1)} = \arcsin(0) = 0$

15) $\lim_{\to(1,2)} e^{xy} \quad e^{(-1)(2)} \quad e^{-2}$

19) $\lim_{(x,y)\to(0,0)} \frac{x+y}{x^2+y}$ direct sub. won't work.

$\lim_{(x\to 0)(\to(0,0)} \frac{x+0}{x^2+0} = \lim \frac{1}{x} = \frac{1}{0}$ DNE. as a finite #

23) $\lim_{(x\,y\,z)\to(0,0,0)} \frac{xy+yz+xz}{x^2+y^2+z^2} = 0/0$

pick 2 pathways

1. $x=0 \quad y=0$

$\lim_{(0,0,z)\to(0,0,0)} = \frac{0+0+0}{0+0+z^2} \quad \frac{0}{z^2} = 0$

2. $x=y=z$

$\lim_{(x,x,x)} \frac{x^2+x^2+x^2}{x^2+x^2+x^2} = 1$

D.N.E.
2 pathways
2 limiting values

39) $\lim_{x\,y\to 0,0} f(x,y) \quad \frac{10xy}{2x^2+3y^2}$

① $x=0$

$\lim_{(0,y)\to(0,0)} \frac{0}{0+3y^2} = 0$ | $\lim_{(x,x)\to(0,0)} \frac{10x^2}{2x^2+3x^2} = 2$ Limit D.N.E.

41) $\lim_{(x,y)\to 00} \frac{\sin(x^2+y^2)}{x^2+y^2} = \frac{0}{0}$ $\boxed{x=r\cos\theta \quad y=r\sin\theta \quad x^2+y^2=r^2 \quad \tan\theta = y/x}$

rewrite: $\lim_{r\to 0} \frac{\sin r^2}{r^2} = \frac{0}{0}$

L'Hopital's rule

$\frac{(\cos r^2)(2r)}{2r} \quad \lim_{r\to 0} \cos r^2 \quad \cos(0) = 1$

43. $\lim_{x,y\to 00} \frac{x^3+y^3}{x^2+y^2} = \frac{0}{0} \quad \lim_{r\to 0} \frac{r\cos\theta^3 + r\sin\theta^3}{r^2} = \lim_{r\to 0} \frac{r^3(\cos^3\theta+\sin^3\theta)}{r^2} = \lim_{r\to 0} r(\cos^3\theta+\sin^3\theta) = 0$

47. $\lim_{(x,y)\to(0,0)} x^2+y^2 \ln(x^2+y^2) = 0(-\infty)$ INDETERMINATE

$\lim_{r\to 0} r^2 \ln(r^2) = 0(-\infty)$, L'Hopitals, $\lim_{r\to 0} \frac{\ln(r^2)}{\frac{1}{r^2}} = \frac{-\infty}{\infty}$, $\frac{\frac{1}{r^2}(2r)}{\frac{-2}{r^3}} = -r^2$ (back)
incorrect form

13.1

Level surfaces — examples

$f(x,y,z) = c$

13. $g(x,y) = \int_x^y (2\tau - 3)\, d\tau$

b) $g(1,4) = \int_1^4 (2\tau-3)\, d\tau = [\tau^2 - 3\tau]_1^4 = 4-(-2) = 6$

15. $f(x,y) = x^2 - 2y$

$$\frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$$

$$= \frac{[(x+\Delta x)^2 - 2y] - [x^2 - 2y]}{\Delta x}$$

$$= \frac{x^2 + 2(\Delta x)x + (\Delta x)^2 - 2y - x^2 + 2y}{\Delta x}$$

$$\frac{2(\Delta x)x + (\Delta x)^2}{\Delta x} = 2x + \Delta x$$

$\lim_{\Delta x \to 0} (2x + \Delta x) = \underline{\underline{2x}}$ partial derivative w/ respect to Δx

$$\frac{\partial f(x,y)}{\partial x} = 2x$$

17. $f(x,y) = \sqrt{4 - x^2 - y^2}$

$4 - (x^2 + y^2)$

Domain: $x^2 + y^2 \le 4$

Range: $0 \le z \le 2$

19. $f(x,y) = \arcsin(x+y)$

$\boxed{y = \arcsin(x) \quad \text{Domain } [-1,+1] \quad \text{Range } \left[-\frac{\pi}{2}, +\frac{\pi}{2}\right]}$

Domain: $\{(x,y) \; -1 \le x+y \le +1$

Range $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ $\;-\frac{\pi}{2} \le z \le \frac{\pi}{2}$

21. $f(x,y) = \ln(4-x-y)$

$y = \ln x$

domain: $x > 0$

$4 - x - y > 0$

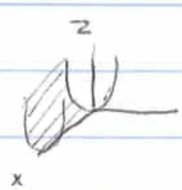
D: $x + y < 4$ R: $(-\infty, \infty)$

Range: $(-\infty, \infty)$ for $\ln x$

25. $f(x,y) = e^{x/y}$

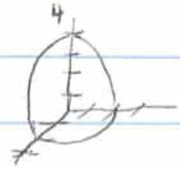
Domain $y \neq 0$

Range ~~$(0,\infty)$~~, $z > 0$



mixed up w/ 33

35. $4 - x^2 - y^2$



Set $z = 0$

$y^2 + x^2 = 4$

Domain = x-y plane

53) $f(x,y) = y^2$

$z = y^2$

45) $f(x,y) = e^{1-x^2-y^2}$

$z = e^{1-x^2-y^2}$

for level curves, let z = const.

$\ln c = \cancel{\ln e}^{\,1-x^2-y^2}$

$x^2 + y^2 = 1 - \ln c$ CIRCLE

46 $f(x,y) = e^{1-x^2+y^2}$

$z = e^{1-x^2+y^2}$

$\ln c = 1 - x^2 + y^2$

$x^2 - y^2 = 1 - \ln c$

hyperbola

13.1 8-17-09 Notes

$y = f(x)$

— 3D surface —

$z = f(x,y) = x^3 + 2y^2 + xy \Rightarrow$ x & y are independent var's
z is dependent var.

$w = f(x,y,z) = x^2 + 4y - z^2$ any # of independent var's

— Domain & Range —

Ex 1 $f(x,y) = \dfrac{\sqrt{x^2+y^2-9}}{x}$

- $x \neq 0$
- Real #'s $x^2+y^2 \geq 9$ domain outside/on boundry of circle rad. 3

$f(x,y,z) \dfrac{x}{\sqrt{9-x^2-y^2-z^2}}$ domain: inside a sphere radius 3

- $x^2+y^2+z^2 < 9$

—Polynomial Functions —

$cx^m y^n$ c - real #'s
m & n - positive integers only

Rational fractions: ratio of 2 polynomials

p812-813
surfaces in space
equations

$f(x,y) = z = \sqrt{16-4x^2-y^2}$ ← only top half of ellipsoid

$z^2 = 16-4x^2-y^2$

$z^2+4x^2+y^2 = 16$

$\dfrac{z^2}{16} + \dfrac{x^2}{4} + \dfrac{y^2}{16} = 1$ ← describes ellipsoid

domain: set $z = 0$, $\dfrac{x^2}{4} + \dfrac{y^2}{10} = 1$

13.1| Notes

49. $z = x+y$ PLANE

$z = c$

$x+y = c$ STRAIGHT LINES

$c = -1, 0, 2, 4$

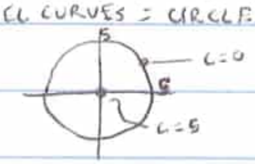
$x+y = -1, 0, 2, 4$

$y = -x-1, -x, -x+2 \ldots$

51. $z = \sqrt{25-x^2-y^2}$

TOP HALF of SPHERE

LEVEL CURVES = CIRCLES



53. $f(x,y) = xy$

$z = xy$, $c = xy$

$y = \frac{c}{x}$ ← eq. for level curves

$y = \frac{1}{x}$

69. $f(x,y,z) = x - 2y + 3z \quad c=6$

$c = 6 = x - 2y + 3z$

73 $f(x,y,z) = 4x^2 + 4y^2 - z^2 \quad c=0$

$c=0 = 4x^2, 4y^2 - z^2$

CIRCULAR CONE

13.

13.2| Notes

limits & continuity

δ - neighborhood

Point of interest (x_0, y_0)

Use dist. formula

$\{ (x,y) : \sqrt{(x-x_0)^2 - (y-y_0)^2} < \delta$
$\leq \delta$

DEFINITION OF THE LIMIT OF A FUNCTION OF 2 Variables

$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$ IF for each $\epsilon > 0$ There exists a $\delta > 0$ Such that

$|f(x,y) - L| < \epsilon$ whenever

$0 < \sqrt{(x-x_0)^2 - (y-y_0)^2}$

P. 59 Properties of Limits

approach method. - shows when limits DO NOT exist
- cannot show when limits exist

continuity if func. is continuous at a point,

the value of the p

Cal III Book

29. $\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_0^4 xyz\,dz\,dy\,dx$, $\int_{-3}^{3}\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}}\int_0^4 xyz\,dz\,dx\,dy$

$\int_{-3}^{3}\int_0^4\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} xyz\,dy\,dz\,dx$, $\int_0^4\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} xyz\,dy\,dx\,dz$

$\int_0^4\int_{-3}^{3}\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} xyz\,dx\,dy\,dz$, $\int_{-3}^{3}\int_0^4\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} xyz\,dx\,dz\,dy$

31. $\int_0^1\int_0^{\sqrt{1-x}}\int_0^{1-y} dz\,dy\,dx$, $\int_0^1\int_0^{1-z}\int_0^{1-y^2} dx\,dy\,dz$

$\int_0^1\int_0^{1-y}\int_0^{1-y^2} dx\,dz\,dy$, $\int_0^1\int_0^{1-y^2}\int_0^{1-y} dz\,dx\,dy$

$\int_0^1\int_0^{2z-z^2}\int_0^{1-z} 1\,dy\,dx\,dz + \int_0^1\int_{2z-z^2}^1\int_0^{\sqrt{1-x}} 1\,dy\,dx\,dz$,

$\int_0^1\int_{\sqrt{1-x}}^1\int_0^{1-z} 1\,dy\,dz\,dx + \int_0^1\int_0^{1-\sqrt{1-x}}\int_0^{\sqrt{1-x}} 1\,dy\,dz\,dx$

33. $m = 8k$
$\bar{x} = \frac{3}{2}$

35. $m = 128k/3$
$\bar{z} = 1$

37. $m = k\int_0^b\int_0^b\int_0^b xy\,dz\,dy\,dx$

$M_{yz} = k\int_0^b\int_0^b\int_0^b x^2y\,dz\,dy\,dx$

$M_{xz} = k\int_0^b\int_0^b\int_0^b xy^2\,dz\,dy\,dx$

$M_{xy} = k\int_0^b\int_0^b\int_0^b xyz\,dz\,dy\,dx$

39. $\bar{x}$ will be greater than 2, and $\bar{y}$ and $\bar{z}$ will be unchanged.

41. $\bar{x}$ and $\bar{z}$ will be unchanged, and $\bar{y}$ will be greater than 0.

43. $(0, 0, 3h/4)$ **45.** $\left(0, 0, \frac{3}{2}\right)$ **47.** $\left(5, 6, \frac{5}{4}\right)$

49. (a) $I_x = 2ka^5/3$, $I_y = 2ka^5/3$, $I_z = 2ka^5/3$ (b) $I_x = ka^8/8$, $I_y = ka^8/8$, $I_z = ka^8/8$

51. (a) $I_x = 256k$, $I_y = 512k/3$, $I_z = 256k$ (b) $I_x = 2048k/3$, $I_y = 1024k/3$, $I_z = 2048k/3$

53. Proof **55.** $\int_{-1}^1\int_{-1}^1\int_0^{1-x} (x^2+y^2)\sqrt{x^2+y^2+z^2}\,dz\,dy\,dx$

57. (a) $m = \int_{-2}^2\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_0^{4-x^2-y^2} kz\,dz\,dy\,dx$

(b) $\bar{x} = \bar{y} = 0$, by symmetry.

$\bar{z} = \frac{1}{m}\int_{-2}^2\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_0^{4-x^2-y^2} kz^2\,dz\,dy\,dx$

(c) $I_z = \int_{-2}^2\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_0^{4-x^2-y^2} kz(x^2+y^2)\,dz\,dy\,dx$

59. See "Definition of Triple Integral" on page 1024 and Theorem 14.4, "Evaluation by Iterated Integrals" on page 1025.

61. (a) Solid B

(b) Solid B has the greater moment of inertia because it is more dense.

(c) Solid A will reach the bottom first. Since Solid B has a greater moment of inertia, it has a greater resistance to rotational motion.

63. $\frac{13}{3}$ **65.** $\frac{3}{2}$

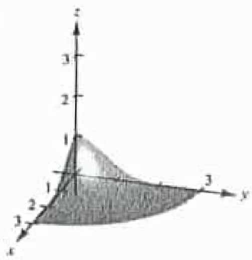
67. Q: $3z^2 + y^2 + 2x^2 \le 1$; $4\sqrt{6}\pi/45 \approx 0.684$

69. $a = 2, \frac{16}{3}$ **71.** Putnam problem B1, 1965

Section 14.7 (page 1040)

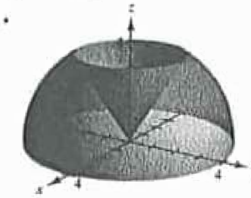
1. 8 **3.** $\frac{52}{45}$ **5.** $\pi/8$ **7.** $\pi(e^4 + 3)$

9.



$(1 - e^{-9})\pi/4$

11.



$64\sqrt{3}\pi/3$

13. Cylindrical: $\int_0^{2\pi}\int_0^2\int_{r^2}^4 r^2\cos\theta\,dz\,dr\,d\theta = 0$

Spherical: $\int_0^{2\pi}\int_0^{\arctan(1/2)}\int_0^{4\sec\phi} \rho^3\sin^2\phi\cos\theta\,d\rho\,d\phi\,d\theta$
$+ \int_0^{2\pi}\int_{\arctan(1/2)}^{\pi/2}\int_0^{\cot\phi\csc\phi} \rho^3\sin^2\phi\cos\phi\,d\rho\,d\phi\,d\theta = 0$

15. Cylindrical: $\int_0^{2\pi}\int_0^a\int_a^{a+\sqrt{a^2-r^2}} r^2\cos\theta\,dz\,dr\,d\theta = 0$

Spherical: $\int_0^{\pi/4}\int_0^{2\pi}\int_{a\sec\phi}^{2a\cos\phi} \rho^3\sin^2\phi\cos\theta\,d\rho\,d\theta\,d\phi = 0$

17. $(2a^3/9)(3\pi - 4)$ **19.** $(2a^3/9)(3\pi - 4)$

21. $48k\pi$ **23.** $\pi r_0^2h/3$ **25.** $(0, 0, h/5)$

27. $I_z = 4k\int_0^{\pi/2}\int_0^{r_0}\int_0^{h(r_0-r)/r_0} r^3\,dz\,dr\,d\theta$
$= 3mr_0^2/10$

29. Proof **31.** $16\pi^2$ **33.** $k\pi a^4$

35. $(0, 0, 3r/8)$ **37.** $k\pi/192$

39. Rectangular to cylindrical: $r^2 = x^2 + y^2$
$\tan\theta = y/x$
$z = z$

Cylindrical to rectangular: $x = r\cos\theta$
$y = r\sin\theta$
$z = z$

41. $\int_{\theta_1}^{\theta_2}\int_{g_1(\theta)}^{g_2(\theta)}\int_{h_1(r\cos\theta,\, r\sin\theta)}^{h_2(r\cos\theta,\, r\sin\theta)} f(r\cos\theta, r\sin\theta, z)r\,dz\,dr\,d\theta$

43. (a) r constant: right circular cylinder about z-axis
θ constant: plane parallel to z-axis
z constant: plane parallel to xy-plane

(b) ρ constant: sphere
θ constant: plane parallel to z-axis
ϕ constant: cone

45. $\frac{1}{2}\pi^2a^4$

Section 14.8 (page 1047)

1. $-\frac{1}{2}$ **3.** $1 + 2v$ **5.** 1 **7.** $-e^{2u}$

1. $\cdot = \dfrac{\Delta\theta r_2^2}{2} - \dfrac{\Delta\theta r_1^2}{2} = \Delta\theta\left(\dfrac{r_1 + r_2}{2}\right)(r_2 - r_1) = r\Delta r\Delta\theta$

Section 14.4 (page 1015)

1. $m = 36$ **3.** $m = 2$

5. (a) $m = kab$, $(a/2, b/2)$ (b) $m = kab^2/2$, $(a/2, 2b/3)$
(c) $m = ka^2b/2$, $(2a/3, b/2)$

7. (a) $m = kbh/2$, $(b/2, h/3)$ (b) $m = kh^2b/6$, $(b/2, h/2)$
(c) $m = khb^2/4$, $(7b/12, h/3)$

9. (a) $(a/2 + 5, b/2)$ (b) $(a/2 + 5, 2b/3)$
(c) $\left(\dfrac{2(a^2 + 15a + 75)}{3(a + 10)}, \dfrac{b}{2}\right)$

11. (a) $m = k\pi a^2/2$, $(0, 4a/3\pi)$
(b) $m = \dfrac{ka^4}{24}(16 - 3\pi)$, $\left(0, \dfrac{a}{5}\left[\dfrac{15\pi - 32}{16 - 3\pi}\right]\right)$

13. $m = 32k/3$, $\left(3, \frac{8}{7}\right)$ **15.** $m = k\pi/2$, $(0, (\pi + 2)/(4\pi))$

17. $m = 8192k/15$, $\left(\frac{64}{7}, 0\right)$ **19.** $m = kL/4$, $(L/2, 16/(9\pi))$

21. $m = \dfrac{k\pi a^2}{8}$, $\left(\dfrac{4\sqrt{2}a}{3\pi}, \dfrac{4a(2 - \sqrt{2})}{3\pi}\right)$

23. $m = \dfrac{k}{4}(1 - e^{-4})$, $\left(\dfrac{e^4 - 5}{2(e^4 - 1)}, \dfrac{4}{9}\left[\dfrac{e^6 - 1}{e^6 - e^2}\right]\right)$

25. $m = k\pi/3$, $(81\sqrt{3}/(40\pi), 0)$

27. $\bar{\bar{x}} = \sqrt{3}b/3$, $\bar{\bar{y}} = \sqrt{3}h/3$ **29.** $\bar{\bar{x}} = a/2$, $\bar{\bar{y}} = a/2$ **31.** $\bar{\bar{x}} = a/2$, $\bar{\bar{y}} = a/2$

33. $I_x = kab^4/4$
$I = kb^2a^3/6$
$(3kab^4 + 2ka^3b^2)/12$
$\bar{\bar{x}} = \sqrt{3}a/3$
$\bar{\bar{y}} = \sqrt{2}b/2$

35. $I_x = 32k/3$
$I_y = 16k/3$
$I_0 = 16k$
$\bar{\bar{x}} = 2\sqrt{3}/3$
$\bar{\bar{y}} = 2\sqrt{6}/3$

37. $I_x = 16k$
$I_y = 512k/5$
$I_0 = 592k/5$
$\bar{\bar{x}} = 4\sqrt{15}/5$
$\bar{\bar{y}} = \sqrt{6}/2$

39. $I_x = 3k/56$
$I_y = k/18$
$I_0 = 55k/504$
$\bar{\bar{x}} = \sqrt{30}/9$
$\bar{\bar{y}} = \sqrt{70}/14$

41. $2k\displaystyle\int_{-b}^{b}\int_0^{\sqrt{b^2-x^2}} (x - a)^2\,dy\,dx = \dfrac{k\pi b^2}{4}(b^2 + 4a^2)$

43. $\displaystyle\int_0^4\int_0^{\sqrt{x}} kx(x - 6)^2\,dy\,dx = \dfrac{42{,}752k}{315}$

45. $\displaystyle\int_0^a\int_0^{\sqrt{a^2-x^2}} k(a - y)(y - a)^2\,dy\,dx = ka^5\left(\dfrac{7\pi}{16} - \dfrac{17}{15}\right)$

47. $\bar{y}$ will increase. **49.** $\bar{x}$ and $\bar{y}$ will both increase.

51. See definition on page 1011. **53.** Answers will vary.

55. $L/3$ **57.** $L/2$

Section 14.5 (page 1022)

1. 6 **3.** 12π **5.** $\frac{3}{4}\left[6\sqrt{37} + \ln\left(\sqrt{37} + 6\right)\right]$

7. $\frac{4}{27}\left(31\sqrt{31} - 8\right)$ **9.** $\sqrt{2} - 1$ **11.** $\sqrt{2}\pi$

13. $2\pi a\left(a - \sqrt{a^2 - b^2}\right)$ **15.** $48\sqrt{14}$ **17.** 20π

19. $\displaystyle\int_0^1\int_0^x \sqrt{5 + 4x^2}\,dy\,dx = \dfrac{27 - 5\sqrt{5}}{12} \approx 1.3183$

21. $\displaystyle\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \sqrt{1 + 4x^2 + 4y^2}\,dy\,dx$
$= \dfrac{\pi}{6}\left(17\sqrt{17} - 1\right) \approx 36.1769$

23. $\displaystyle\int_0^1\int_0^1 \sqrt{1 + 4x^2 + 4y^2}\,dy\,dx \approx 1.8616$ **25.** e

27. 2.0035 **29.** $\displaystyle\int_{-1}^{1}\int_{-1}^{1} \sqrt{1 + 9(x^2 - y)^2 + 9(y^2 - x)^2}\,dy\,dx$

31. $\displaystyle\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \sqrt{1 + e^{-2x}}\,dy\,dx$

33. $\displaystyle\int_0^4\int_0^{10} \sqrt{1 + e^{2xy}(x^2 + y^2)}\,dy\,dx$

35. If f and its first partial derivatives are continuous on the closed region R in the xy-plane, then the area of the surface S given by $z = f(x, y)$ over R is
$$\int_R\int \sqrt{1 + [f_x(x, y)]^2 + [f_y(x, y)]^2}\,dA.$$

37. 16 **39.** (a) 30,415.74 ft^3 (b) 2081.53 ft^2

41. (a) $812\pi\sqrt{609}$ cm^3 (b) $100\pi\sqrt{609}$ cm^2

Section 14.6 (page 1032)

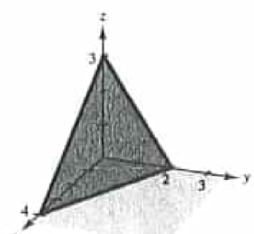
1. 18 **3.** $\frac{1}{10}$ **5.** $\frac{15}{2}(1 - 1/e)$ **7.** $-\frac{40}{3}$ **9.** $\frac{128}{15}$

11. 2.44167 **13.** $V = \displaystyle\int_0^4\int_0^{4-x}\int_0^{4-x-y} dz\,dy\,dx$

15. $V = \displaystyle\int_{-3}^{3}\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}}\int_0^{9-x^2-y^2} dz\,dx\,dy$

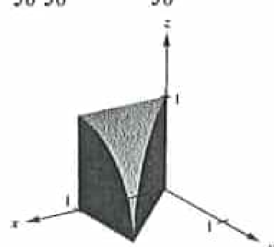
17. $\frac{256}{15}$ **19.** $4\pi a^3/3$ **21.** $\frac{256}{15}$

23.



$\displaystyle\int_0^3\int_0^{(12-4z)/3}\int_0^{(12-4z-3x)/6} dy\,dx\,dz$

25.

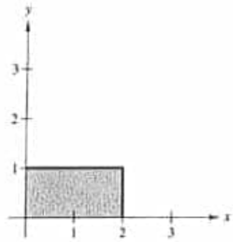


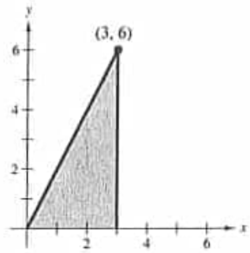
$\displaystyle\int_0^1\int_0^x\int_0^{\sqrt{1-y^2}} dz\,dy\,dx$

27. $\displaystyle\int_0^1\int_0^x\int_0^3 xyz\,dz\,dy\,dx$, $\displaystyle\int_0^1\int_y^1\int_0^3 xyz\,dz\,dx\,dy$,
$\displaystyle\int_0^1\int_0^3\int_0^x xyz\,dy\,dz\,dx$, $\displaystyle\int_0^3\int_0^1\int_0^x xyz\,dy\,dx\,dz$,
$\displaystyle\int_0^3\int_0^1\int_y^1 xyz\,dx\,dy\,dz$, $\displaystyle\int_0^1\int_0^3\int_y^1 xyz\,dx\,dz\,dy$

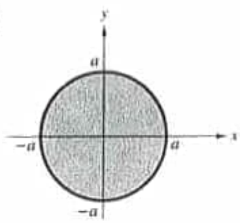
Section 14.2 (page 997)

1. 24 (approximation is exact)
3. Approximation: 52; Exact: $\frac{160}{3}$ **5.** 400; 272
7. 8 **9.** 36





11. 0



13. $\displaystyle\int_0^3\int_0^5 xy\,dy\,dx = \frac{225}{4}$

$\displaystyle\int_0^5\int_0^3 xy\,dx\,dy = \frac{225}{4}$

15. $\displaystyle\int_0^2\int_x^{2x} \frac{y}{x^2+y^2}\,dy\,dx = \ln\frac{5}{2}$

$\displaystyle\int_0^2\int_{y/2}^{y} \frac{y}{x^2+y^2}\,dx\,dy + \int_2^4\int_{y/2}^{2} \frac{y}{x^2+y^2}\,dx\,dy = \ln\frac{5}{2}$

17. $\displaystyle\int_0^1\int_{4-x}^{4-x^2} -2y\ln x\,dy\,dx = \frac{26}{25}$

$\displaystyle\int_3^4\int_{4-y}^{\sqrt{4-y}} -2y\ln x\,dx\,dy = \frac{26}{25}$

19. $\displaystyle\int_0^3\int_{4y/3}^{\sqrt{25-y^2}} x\,dx\,dy = 25$

$\displaystyle\int_0^4\int_0^{3x/4} x\,dy\,dx + \int_4^5\int_0^{\sqrt{25-x^2}} x\,dy\,dx = 25$

21. 4 **23.** 4 **25.** 12 **27.** $\frac{3}{8}$ **29.** 1 **31.** 8π

33. $\displaystyle\int_0^1\int_0^x xy\,dy\,dx = \frac{1}{8}$ **35.** $\displaystyle\int_0^2\int_0^4 x^2\,dy\,dx = \frac{32}{3}$

37. $\displaystyle 2\int_0^1\int_0^x \sqrt{1-x^2}\,dy\,dx = \frac{2}{3}$

39. $\displaystyle\int_0^2\int_0^{\sqrt{4-x^2}} (x+y)\,dy\,dx = \frac{16}{3}$

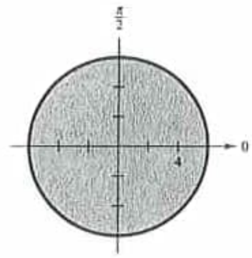
41. 8π **43.** $81\pi/2$ **45.** 1.2315 **47.** Proof
49. $1 - e^{-1/4} \approx 0.221$ **51.** $\frac{1}{3}\left[2\sqrt{2} - 1\right]$ **53.** 2 **55.** $\frac{8}{3}$
57. See "Definition of Double Integral" on page 992. The double integral of a function $f(x, y) \ge 0$ over the region of integration yields the volume of that region.
59. kB **61.** 25,645.24 **63.** Proof; $\frac{1}{5}$ **65.** Proof; $\frac{7}{27}$
67. 2500 m^3 **69.** (a) 1.784 (b) 1.788
71. (a) 11.057 (b) 11.041 **73.** d

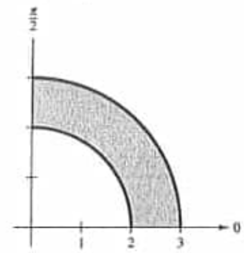
75. False: $\displaystyle V = 8\int_0^1\int_0^{\sqrt{1-y^2}} \sqrt{1-x^2-y^2}\,dx\,dy.$

77. $\frac{1}{2}(1-e)$ **79.** R: $x^2 + y^2 \le 9$ **81.** ≈ 0.82736
83. Putnam problem A2, 1989

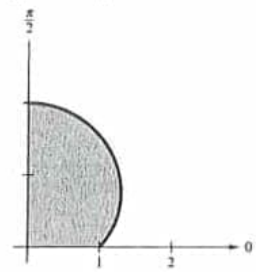
Section 14.3 (page 1006)

1. Rectangular **3.** Polar
5. The region R is a half-circle of radius 8. It can be described in polar coordinates as
$R = \{(r, \theta): 0 \le r \le 8, 0 \le \theta \le \pi\}$.
7. The region R is a cardioid with $a = b = 3$. It can be described in polar coordinates as
$R = \{(r, \theta): 0 \le r \le 3 + 3\sin\theta, 0 \le \theta \le 2\pi\}$.
9. 0 **11.** $5\sqrt{5}\pi/6$





13. $\frac{9}{8} + 3\pi^2/32$ **15.** $a^3/3$ **17.** $243\pi/10$ **19.** $\frac{2}{3}$



21. $\displaystyle\int_0^{\pi/4}\int_0^{2\sqrt{2}} r^2\,dr\,d\theta = \frac{4\sqrt{2}\pi}{3}$

23. $\displaystyle\int_0^{\pi/2}\int_0^{2} r^2(\cos\theta + \sin\theta)\,dr\,d\theta = \frac{16}{3}$

25. $\displaystyle\int_0^{\pi/4}\int_1^{2} r\theta\,dr\,d\theta = \frac{3\pi^2}{64}$ **27.** $\frac{1}{8}$ **29.** $\frac{250\pi}{3}$

31. $\frac{64}{9}(3\pi - 4)$ **33.** $2\sqrt{4 - 2\sqrt[3]{2}}$ **35.** 1.2858
37. 9π **39.** $3\pi/2$ **41.** π
43. Let R be a region bounded by the graphs of $r = g_1(\theta)$ and $r = g_2(\theta)$ and the lines $\theta = a$ and $\theta = b$. When using polar coordinates to evaluate a double integral over R, R can be partitioned into small polar sectors.
45. r-simple regions have fixed bounds for θ and variable bounds for r.
θ-simple regions have variable bounds for θ and fixed bounds for r.
47. Insert a factor of r; Sector of a circle **49.** 56.051 **51.** c
53. False: Let $f(r, \theta) = r - 1$ and let R be a sector where $0 \le r \le 6$ and $0 \le \theta \le \pi$.
55. (a) 2π (b) $\sqrt{2\pi}$ **57.** 486,788

59. (a) $\displaystyle\int_2^4\int_{y/\sqrt{3}}^{y} f\,dx\,dy$

(b) $\displaystyle\int_{2/\sqrt{3}}^{2}\int_2^{\sqrt{3}x} f\,dy\,dx + \int_2^{4/\sqrt{3}}\int_x^{\sqrt{3}x} f\,dy\,dx + \int_{4/\sqrt{3}}^{4}\int_x^4 f\,dy\,dx$

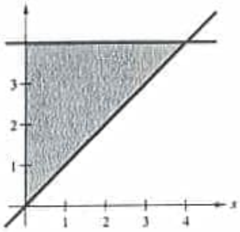
(c) $\displaystyle\int_{\pi/4}^{\pi/3}\int_{2\csc\theta}^{4\csc\theta} f\,r\,dr\,d\theta$

Chapter 14

Section 14.1 (page 988)

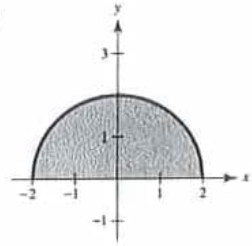
1. $3x^2/2$ **3.** $y \ln(2y)$ **5.** $(4x^2 - x^4)/2$
7. $(y/2)[(\ln y)^2 - y^2]$ **9.** $x^2(1 - e^{-x^2} - x^2e^{-x^2})$ **11.** 3
13. 2 **15.** $\frac{1}{3}$ **17.** $\frac{20}{3}$ **19.** $\frac{2}{3}$ **21.** 4 **23.** $\pi^2/32 + \frac{1}{8}$
25. $\frac{1}{2}$ **27.** Diverges **29.** 24 **31.** $\frac{16}{3}$ **33.** $\frac{9}{2}$ **35.** $\frac{8}{3}$
37. 5 **39.** πab

41.



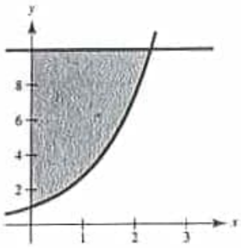
$$\int_0^4 \int_x^4 f(x, y)\, dy\, dx$$

43.



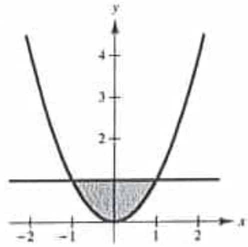
$$\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} f(x, y)\, dx\, dy$$

45.



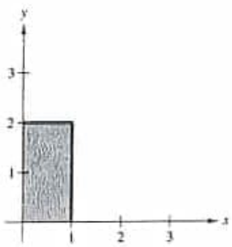
$$\int_0^{\ln 10} \int_{e^x}^{10} f(x, y)\, dy\, dx$$

47.



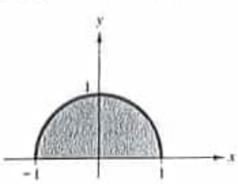
$$\int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} f(x, y)\, dx\, dy$$

49.



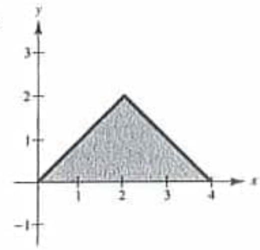
$$\int_0^1 \int_0^2 dy\, dx = \int_0^2 \int_0^1 dx\, dy = 2$$

51.



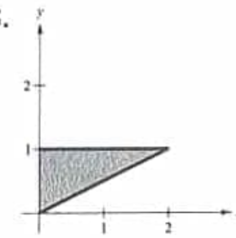
$$\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} dx\, dy = \int_{-1}^1 \int_0^{\sqrt{1-x^2}} dy\, dx = \frac{\pi}{2}$$

53.



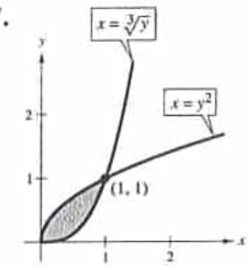
$$\int_0^2 \int_0^x dy\, dx + \int_2^4 \int_0^{4-x} dy\, dx = \int_0^2 \int_y^{4-y} dx\, dy = 4$$

55.



$$\int_0^2 \int_{x/2}^1 dy\, dx = \int_0^1 \int_0^{2y} dx\, dy = 1$$

57.

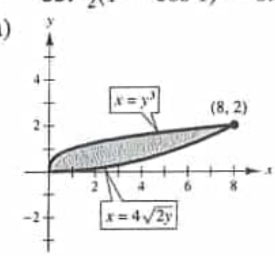


$$\int_0^1 \int_{y^2}^{\sqrt[3]{y}} dx\, dy = \int_0^1 \int_{x^3}^{\sqrt{x}} dy\, dx = \frac{5}{12}$$

59. The first integral arises using vertical representative rectangles. The second two integrals arise using horizontal representative rectangles.
Value of the integrals: $15{,}625\pi/24$

61. $\frac{26}{9}$ **63.** $\frac{1}{2}(1 - \cos 1) \approx 0.230$ **65.** $\frac{1664}{105}$ **67.** $(\ln 5)^2$

69. (a)



(b) $\displaystyle\int_0^8 \int_{x^2/32}^{\sqrt[3]{x}} (x^2y - xy^2)\, dy\, dx$ (c) 67,520/693

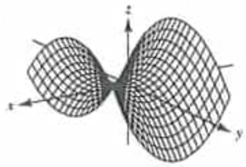
71. 20.5648 **73.** $15\pi/2$

75. An iterated integral is integration of a function of several variables. Integrate with respect to one variable while holding the other variables constant.

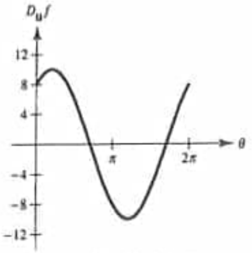
77. If all four limits of integration are constant, the region of integration is rectangular.

79. True

53. (a)



(b) $D_{\mathbf{u}}f(4, -3) = 8\cos\theta + 6\sin\theta$



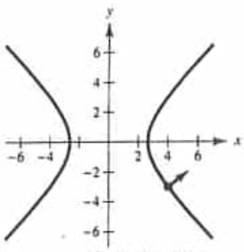
Generated by Mathematica

(c) $\theta \approx 2.21$, $\theta \approx 5.36$
Directions in which there is no change in f

(d) $\theta \approx 0.64$, $\theta \approx 3.79$
Directions of greatest rate of change in f

(e) 10; Magnitude of the greatest rate of change

(f)

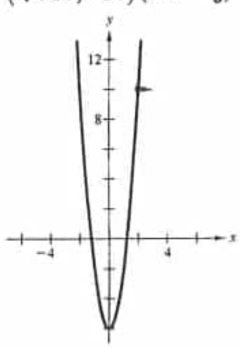


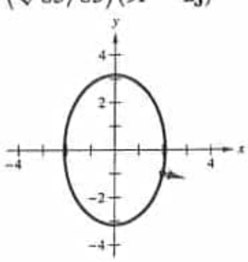
Generated by Mathematica

Orthogonal to the level curve

55. $6\mathbf{i} + 8\mathbf{j}$ **57.** $-\frac{1}{2}\mathbf{j}$

59. $(\sqrt{257}/257)(16\mathbf{i} - \mathbf{j})$ **61.** $(\sqrt{85}/85)(9\mathbf{i} - 2\mathbf{j})$





63. $\frac{1}{625}(7\mathbf{i} - 24\mathbf{j})$

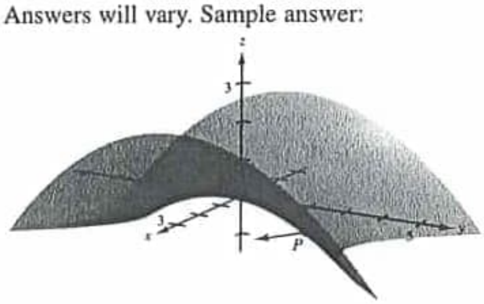
65. The directional derivative of $z = f(x, y)$ in the direction of $\mathbf{u} = \cos t\mathbf{i} + \sin t\mathbf{j}$ is

$$D_{\mathbf{u}}f(x, y) = \lim_{t\to 0} \frac{f(x + t\cos\theta, y + t\sin\theta) - f(x, y)}{t}$$

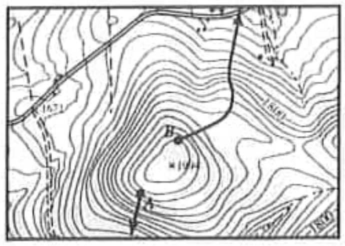
if the limit exists.

67. Let $f(x, y)$ be a function of two variables and let $\mathbf{u} = \cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j}$ be a unit vector.
(a) If $\theta = 0°$, then $D_{\mathbf{u}}f = \partial f/\partial x$.
(b) If $\theta = 90°$, then $D_{\mathbf{u}}f = \partial f/\partial y$.

69. Answers will vary. Sample answer:

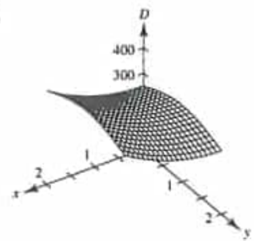


71.



73. $y^2 = 10x$

75. (a)



(b) Graph $-D$.
(c) 315 m
(d) 60.0
(e) 55.5
(f) $60.0\mathbf{i} + 55.5\mathbf{j}$

77. True **79.** True **81.** $f(x, y, z) = e^x\cos y + \frac{1}{2}z^2 + C$

Section 13.7 (page 949)

1. The level surface can be written as $3x - 5y + 3z = 15$, which is an equation of a plane in space.

3. The level surface can be written as $4x^2 + 9y^2 - 4z^2 = 0$, which is an elliptic cone that lies on the z-axis.

5. $(\sqrt{3}/3)(\mathbf{i} + \mathbf{j} + \mathbf{k})$ **7.** $(\sqrt{2}/10)(3\mathbf{i} + 4\mathbf{j} - 5\mathbf{k})$

9. $(\sqrt{2049}/2049)(32\mathbf{i} + 32\mathbf{j} - \mathbf{k})$ **11.** $(\sqrt{3}/3)(\mathbf{i} - \mathbf{j} + \mathbf{k})$

13. $(\sqrt{113}/113)(-\mathbf{i} - 6\sqrt{3}\mathbf{j} + 2\mathbf{k})$ **15.** $6x + 2y + z = 35$

17. $3x + 4y - 5z = 0$ **19.** $10x - 8y - z = 9$

21. $2x - z = -2$ **23.** $3x + 4y - 25z = 25(1 - \ln 5)$

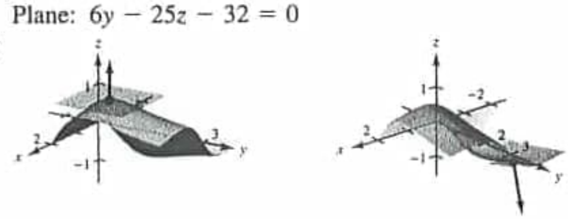
25. $x - 4y + 2z = 18$ **27.** $x + y + z = 1$

29. $2x + 4y + z = 14$
$\dfrac{x-1}{2} = \dfrac{y-2}{4} = \dfrac{z-4}{1}$

31. $3x + 2y + z = -6$
$\dfrac{x+2}{3} = \dfrac{y+3}{2} = \dfrac{z-6}{1}$

33. $x - y + 2z = \pi/2$
$(x - 1)/1 = (y - 1)/-1 = (z - \pi/4)/2$

35. (a) Line: $x = 1, y = 1, z = 1 - t$
Plane: $z = 1$
(b) Line: $x = -1, y = 2 + \frac{6}{25}t, z = -\frac{4}{5} - t$
Plane: $6y - 25z - 32 = 0$
(c)



(d) At $(1, 1, 1)$, the tangent plane is parallel to the xy-plane, implying that the surface is level. At $(-1, 2, -\frac{4}{5})$, the function does not change in the x-direction.

37. See "Definition of Tangent Plane and Normal Line" on page 944.

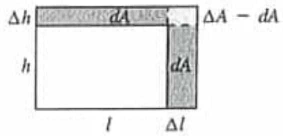
39. For a sphere, the common object is the center of the sphere. For a cylinder, the common object is its axis.

41. (a) $\dfrac{x-2}{1} = \dfrac{y-1}{-2} = \dfrac{z-2}{1}$ (b) $\dfrac{\sqrt{10}}{5}$, not orthogonal

43. (a) $\dfrac{x-3}{4} = \dfrac{y-3}{4} = \dfrac{z-4}{-3}$ (b) $\dfrac{16}{25}$, not orthogonal

45. (a) $\dfrac{y-1}{1} = \dfrac{z-1}{-1}$, $x = 2$ (b) 0, orthogonal

25. $dA = h\,dl + l\,dh$



27.

Δr	Δh	dV	ΔV	$\Delta V - dV$
0.1	0.1	4.7124	4.8391	0.1267
0.1	−0.1	2.8274	2.8264	−0.0010
0.001	0.002	0.0565	0.0565	0.0001
−0.0001	0.0002	−0.0019	−0.0019	0.0000

29. (a) $dz = -0.04\,dx + 0.64\,dy$
(b) $dz = \pm 0.17$; $dz/z = 2.1\%$

31. 10% **33.** $dC = \pm 0.24418$; $dC/C = 19\%$

35. (a) $V = 18 \sin\theta$ ft^3; $\theta = \pi/2$
(b) 1.047 ft^3

37. 1% **39.** $L \approx 8.096 \times 10^{-4} \pm 6.6 \times 10^{-6}$ microhenrys

41. Answers will vary.
Example:
$\varepsilon_1 = \Delta x$
$\varepsilon_2 = 0$

43. Answers will vary.
Example:
$\varepsilon_1 = y\,\Delta x$
$\varepsilon_2 = 2x\,\Delta x + (\Delta x)^2$

45. Proof

47. Answers will vary. For example, we can use the equation $F = ma$.
Then $dF = (\partial F/\partial m)\,dm + (\partial F/\partial a)\,da = a\,dm + m\,da$.
We can estimate the possible propagated errors when given the error in measurement.

Section 13.5 (page 929)

1. $2(e^{2t} - e^{-2t})$ **3.** $e^t \sec(\pi - t)[1 - \tan(\pi - t)]$

5. $2\cos 2t$ **7.** $4e^{2t}$ **9.** $3(2t^2 - 1)$

11. $-11\sqrt{29}/29 \approx -2.04$

13. $\dfrac{8\sin t\cos t(4\sin^2 t\cos^2 t - 3)}{(4\sin^2 t\cos^2 t + 1)^2}$; 0

15. $\partial w/\partial s = 4s, 8$
$\partial w/\partial t = 4t, -4$

17. $\partial w/\partial s = 2s\cos 2t, 0$
$\partial w/\partial t = -2s^2\sin 2t, -18$

19. $\partial w/\partial r = 0$
$\partial w/\partial \theta = 8\theta$

21. $\partial w/\partial r = 0$
$\partial w/\partial \theta = 1$

23. $\dfrac{\partial w}{\partial s} = t^2(3s^2 - t^2)$
$\dfrac{\partial w}{\partial t} = 2st(s^2 - 2t^2)$

25. $\dfrac{\partial w}{\partial s} = \dfrac{te^{(s-t)/(s+t)}(s^2 + 4st + t^2)}{(s+t)^2}$
$\dfrac{\partial w}{\partial t} = \dfrac{se^{(s-t)/(s+t)}(s^2 + t^2)}{(s+t)^2}$

27. $\dfrac{3y - 2x + 2}{2y - 3x + 1}$ **29.** $-\dfrac{x + y(x^2 + y^2)}{y + x(x^2 + y^2)}$

31. $\dfrac{\partial z}{\partial x} = \dfrac{-x}{z}$
$\dfrac{\partial z}{\partial y} = \dfrac{-y}{z}$

33. $\dfrac{\partial z}{\partial x} = \dfrac{-\sec^2(x + y)}{\sec^2(y + z)}$
$\dfrac{\partial z}{\partial y} = -1 - \dfrac{\sec^2(x + y)}{\sec^2(y + z)}$

35. $\partial z/\partial x = -x/(y + z)$
$\partial z/\partial y = -z/(y + z)$

37. $\partial z/\partial x = -(ze^{xz} + y)/xe^{xz}$
$\partial z/\partial y = -e^{-xz}$

39. $\dfrac{\partial w}{\partial x} = \dfrac{-yz - zw}{xz - yz + 2w}$
$\dfrac{\partial w}{\partial y} = \dfrac{-xz + zw}{xz - yz + 2w}$
$\dfrac{\partial w}{\partial z} = \dfrac{yw - xy - xw}{xz - yz + 2w}$

41. $\dfrac{\partial w}{\partial x} = \dfrac{y\sin xy}{z}$
$\dfrac{\partial w}{\partial y} = \dfrac{x\sin xy - z\cos yz}{z}$
$\dfrac{\partial w}{\partial z} = -\dfrac{y\cos yz + w}{z}$

43. 1; $xf_x(x, y) + yf_y(x, y) = \dfrac{xy}{\sqrt{x^2 + y^2}} = 1f(x, y)$

45. 0; $xf_x(x, y) + yf_y(x, y) = \dfrac{xe^{x/y}}{y} - \dfrac{xe^{x/y}}{y} = 0$

47. $dw/dt = \partial w/\partial x \cdot dx/dt + \partial w/\partial y \cdot dy/dt$

49. The explicit form of a function of two variables is of the form $z = f(x, y)$, as in $z = x^2 + y^2$. The implicit form of a function of two variables is of the form $F(x, y, z) = 0$, as in $z - x^2 - y^2 = 0$.

51. 4608π in.^{3}/min; 624π in.^{2}/min

53. $(\sqrt{2}/10)(15 + \pi)$m^2/hr **55.** $28m$ cm^2/sec

57. (a) Proof
(b) $d\theta/dx = (2\cos^2\theta - 2x\cos\theta\sin\theta)/(x^2 + 8)$
(c) $x = 2\sqrt{2}$

59. Proof **61.** (a) Proof (b) Proof **63.** Proof

Section 13.6 (page 940)

1. $(\sqrt{3} - 5)/2$ **3.** $5\sqrt{2}/2$ **5.** $-\frac{7}{25}$ **7.** $-e$ **9.** $2\sqrt{6}/3$

11. $\boxed{(8 + \pi)\sqrt{6}/24}$ **13.** $\sqrt{2}(x + y)$

15. $[(2 + \sqrt{3})/2]\cos(2x - y)$

17. $-7\sqrt{2}$ **19.** $7\sqrt{19}/19$ **21.** $3\mathbf{i} - 10\mathbf{j}$

23. $-6\sin 25\mathbf{i} + 8\sin 25\mathbf{j} \approx 0.7941\mathbf{i} - 1.0588\mathbf{j}$

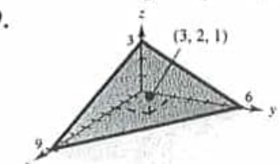
25. $6\mathbf{i} + 13\mathbf{j} - 9\mathbf{k}$ **27.** $2\sqrt{5}$ **29.** $-2\sqrt{5}/5$

31. $\tan y\mathbf{i} + x\sec^2 y\mathbf{j}$, $\sqrt{17}$

33. $\dfrac{2}{3(x^2 + y^2)}(x\mathbf{i} + y\mathbf{j})$, $\dfrac{2\sqrt{5}}{15}$ **35.** $\dfrac{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{\sqrt{x^2 + y^2 + z^2}}$, 1

37. $e^{yz}\mathbf{i} + xze^{yz}\mathbf{j} + xye^{yz}\mathbf{k}$; $\sqrt{65}$

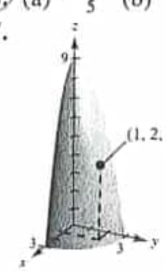
39.



41. (a) $(2 + 3\sqrt{3})/12$
(b) $(3 - 2\sqrt{3})/12$

43. (a) $-\frac{1}{5}$ (b) $-11\sqrt{10}/60$ **45.** $\sqrt{13}/6$

47.



49. $-2\mathbf{i} - 4\mathbf{j}$, $2\sqrt{5}$

51. (a) Answers will vary. Example: $-4\mathbf{i} + \mathbf{j}$
(b) $-\frac{2}{5}\mathbf{i} + \frac{1}{10}\mathbf{j}$ (c) $\frac{2}{5}\mathbf{i} - \frac{1}{10}\mathbf{j}$
The direction opposite that of the gradient

55. $H_x(x, y, z) = \cos(x + 2y + 3z)$
$H_y(x, y, z) = 2\cos(x + 2y + 3z)$
$H_z(x, y, z) = 3\cos(x + 2y + 3z)$

57. $f_x = 3\sqrt{5}/5; f_y = -2\sqrt{5}/5; f_z = -2\sqrt{5}/5$

59. $f_x = 0; f_y = 0; f_z = 1$

61. $\dfrac{\partial^2 z}{\partial x^2} = 2$, $\dfrac{\partial^2 z}{\partial y^2} = 6$, $\dfrac{\partial^2 z}{\partial y \partial x} = \dfrac{\partial^2 z}{\partial x \partial y} = -2$

63. $\dfrac{\partial^2 z}{\partial x^2} = \dfrac{y^2}{(x^2 + y^2)^{3/2}}$, $\dfrac{\partial^2 z}{\partial y^2} = \dfrac{x^2}{(x^2 + y^2)^{3/2}}$, $\dfrac{\partial^2 z}{\partial y \partial x} = \dfrac{\partial^2 z}{\partial x \partial y} = \dfrac{-xy}{(x^2 + y^2)^{3/2}}$

65. $\dfrac{\partial^2 z}{\partial x^2} = e^x \tan y$, $\dfrac{\partial^2 z}{\partial y^2} = 2e^x \sec^2 y \tan y$, $\dfrac{\partial^2 z}{\partial y \partial x} = \dfrac{\partial^2 z}{\partial x \partial y} = e^x \sec^2 y$

67. $\dfrac{\partial^2 z}{\partial x^2} = \dfrac{2xy}{(x^2 + y^2)^2}$, $\dfrac{\partial^2 z}{\partial y^2} = \dfrac{-2xy}{(x^2 + y^2)^2}$, $\dfrac{\partial^2 z}{\partial y \partial x} = \dfrac{\partial^2 z}{\partial x \partial y} = \dfrac{y^2 - x^2}{(x^2 + y^2)^2}$

69. $\partial z/\partial x = \sec y$
$\partial z/\partial y = x \sec y \tan y$
$\partial^2 z/\partial x^2 = 0$
$\partial^2 z/\partial y^2 = x \sec y(\sec^2 y + \tan^2 y)$
$\partial^2 z/\partial y \partial x = \partial^2 z/\partial x \partial y = \sec y \tan y$
No values of x and y exist such that $f_x(x, y) = f_y(x, y) = 0$.

71. $\partial z/\partial x = (y^2 - x^2)/[x(x^2 + y^2)]$
$\partial z/\partial y = -2y/(x^2 + y^2)$
$\partial^2 z/\partial x^2 = (x^4 - 4x^2y^2 - y^4)/[x^2(x^2 + y^2)^2]$
$\partial^2 z/\partial y^2 = 2(y^2 - x^2)/(x^2 + y^2)^2$
$\partial^2 z/\partial y \partial x = \partial^2 z/\partial x \partial y = 4xy/(x^2 + y^2)^2$
No values of x and y exist such that $f_x(x, y) = f_y(x, y) = 0$.

73. $f_{xyy}(x, y, z) = f_{yxy}(x, y, z) = f_{yyx}(x, y, z) = 0$

75. $f_{xyy}(x, y, z) = f_{yxy}(x, y, z) = f_{yyx}(x, y, z) = z^2 e^{-x} \sin yz$

77. $\partial^2 z/\partial x^2 + \partial^2 z/\partial y^2 = 0 + 0 = 0$

79. $\partial^2 z/\partial x^2 + \partial^2 z/\partial y^2 = e^x \sin y - e^x \sin y = 0$

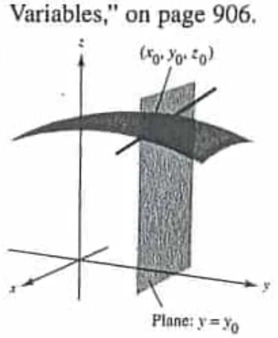
81. $\partial^2 z/\partial t^2 = -c^2 \sin(x - ct) = c^2(\partial^2 z/\partial x^2)$

83. $\partial^2 z/\partial t^2 = -c^2/(x + ct)^2 = c^2(\partial^2 z/\partial x^2)$

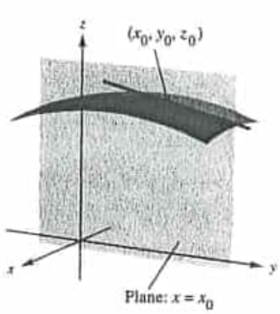
85. $\partial z/\partial t = -e^{-t} \cos x/c = c^2(\partial^2 z/\partial x^2)$

87. See "Definition of Partial Derivatives of a Function of Two Variables," on page 906.

89.

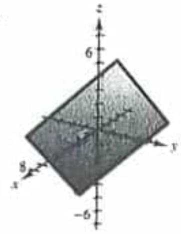


$\partial f/\partial x$ represents the slope of the curve formed by the intersection of the surface $z = f(x, y)$ and the plane $y = y_0$ at any point on the curve.



$\partial f/\partial y$ represents the slope of the curve formed by the intersection of the surface $z = f(x, y)$ and the plane $x = x_0$ at any point on the curve.

91.



93. (a) $\partial C/\partial x = 183$, $\partial C/\partial y = 237$
(b) The fireplace-insert stove results in the cost increasing at a higher rate because the coefficient of y is greater in magnitude than the coefficient of x.

95. An increase in either the charge for food and housing or the tuition will cause a decrease in the number of applicants.

97. $\partial T/\partial x = -2.4°$ per m, $\partial T/\partial y = -9°$ per m

99. $T = PV/(nR) \Rightarrow \partial T/\partial P = V/(nR)$
$P = nRT/V \Rightarrow \partial P/\partial V = -nRT/V^2$
$V = nRT/P \Rightarrow \partial V/\partial T = nR/P$
$\partial T/\partial P \cdot \partial P/\partial V \cdot \partial V/\partial T = -nRT/(VP) = -nRT/(nRT) = -1$

101. (a) $\partial z/\partial x = -0.04$; $\partial z/\partial y = 0.64$
(b) For every decrease of 0.04 gallon of whole milk there is an increase of one gallon of skim milk. For every increase of 0.64 gallon of whole milk there is an decrease of one gallon of reduced-fat milk.

103. False; Let $z = x + y + 1$. **105.** True

107. (a) $f_x(x, y) = \dfrac{y(x^4 + 4x^2y^2 - y^4)}{(x^2 + y^2)^2}$
$f_y(x, y) = \dfrac{x(x^4 - 4x^2y^2 - y^4)}{(x^2 + y^2)^2}$
(b) $f_x(0, 0) = 0$, $f_y(0, 0) = 0$
(c) $f_{xy}(0, 0) = -1$, $f_{yx}(0, 0) = 1$
(d) f_{xy} or f_{yx} or both are not continuous at $(0, 0)$.

109. (a) Proof (b) $f_y = (x, y)$ does not exist when $y = -x$.

Section 13.4 (page 921)

1. $dz = 6xy^3\,dx + 9x^2y^2\,dy$

3. $dz = 2(x\,dx + y\,dy)/(x^2 + y^2)^2$

5. $dz = (\cos y + y \sin x)\,dx - (x \sin y + \cos x)dy$

7. $dz = (e^x \sin y)\,dx + (e^x \cos y)\,dy$

9. $dw = 2z^3 y \cos x\,dx + 2z^3 \sin x\,dy + 6z^2 y \sin x\,dz$

11. (a) $f(1, 2) = 4, f(1.05, 2.1) = 3.4875$, $\Delta z = -0.5125$
(b) $dz = -0.5$

13. (a) $f(1,2) \approx 0.90930, f(1.05, 2.1) \approx 0.90637, \Delta z \approx -0.00293$
(b) $dz \approx 0.00385$

15. (a) $f(1, 2) = -5, f(1.05, 2.1) = -5.25$, $\Delta z = -0.25$
(b) $dz = -0.25$

17. 0.094 **19.** -0.012

21. If $z = f(x, y)$ and Δx and Δy are increments of x and y, and x and y are independent variables, then the total differential of the dependent variable z is
$dz = (\partial z/\partial x)\,dx + (\partial z/\partial y)\,dy = f_x(x, y)\,\Delta x + f_y(x, y)\,\Delta y$.

23. The approximation of Δz by dz is called a linear approximation, where dz represents the change in height of a plane that is tangent to the surface at the point $P(x_0, y_0)$.

29.

(x, y)	$(1, 0)$	$(0.5, 0)$	$(0.1, 0)$	$(0.01, 0)$	$(0.001, 0)$
$f(x, y)$	0	0	0	0	0

$y = 0$: 0

(x, y)	$(1, 1)$	$(0.5, 0.5)$	$(0.1, 0.1)$
$f(x, y)$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

(x, y)	$(0.01, 0.01)$	$(0.001, 0.001)$
$f(x, y)$	$\frac{1}{2}$	$\frac{1}{2}$

$y = x$: $\frac{1}{2}$
Limit does not exist.
Continuous except at $(0, 0)$

31.

(x, y)	$(1, 1)$	$(0.25, 0.5)$	$(0.01, 0.1)$
$f(x, y)$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$

(x, y)	$(0.0001, 0.01)$	$(0.000001, 0.001)$
$f(x, y)$	$-\frac{1}{2}$	$-\frac{1}{2}$

$x = y^2$: $-\frac{1}{2}$

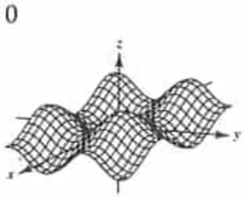
(x, y)	$(-1, 1)$	$(-0.25, 0.5)$	$(-0.01, 0.1)$
$f(x, y)$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

(x, y)	$(-0.0001, 0.01)$	$(-0.000001, 0.001)$
$f(x, y)$	$\frac{1}{2}$	$\frac{1}{2}$

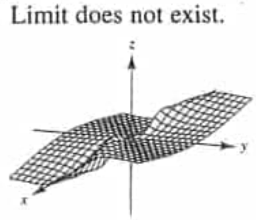
$x = -y^2$: $\frac{1}{2}$
Limit does not exist.
Continuous except at $(0, 0)$

33. f is continuous except at $(0, 0)$.
g is continuous.
f has a removable discontinuity at $(0, 0)$.

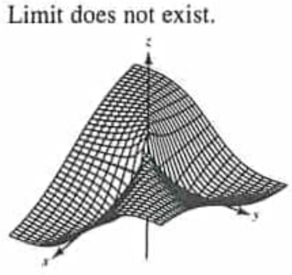
35. 0



37. Limit does not exist.



39. Limit does not exist.



41. 1 **43.** 0 **45.** 0 **47.** 0

49. Continuous except at $(0, 0, 0)$

51. Continuous **53.** Continuous **55.** Continuous

57. Continuous for $y \neq 3x/2$ **59.** (a) $2x$ (b) -4

61. (a) $2 + y$ (b) $x - 3$ **63.** True

65. False: let $f(x, y) = \begin{cases} \ln(x^2 + y^2), & x \neq 0, y \neq 0 \\ 0, & x = 0, y = 0 \end{cases}$.

67. See "Definition of the Limit of a Function of Two Variables," on page 897; show that the value of $\lim_{(x, y) \to (x_0, y_0)} f(x, y)$ is not the same for two different paths to (x_0, y_0).

69. No: the existence of $f(2, 3)$ has no bearing on the existence of the limit as $(x, y) \to (2, 3)$.

71. (a) $(1 + a^2)/a, \quad a \neq 0$ (b) Limit does not exist.
(c) No, the limit does not exist. Different paths result in different limits.

73. 0 **75.** $\pi/2$ **77.** Proof

Section 13.3 (page 912)

1. $f_x = (4, 1) < 0$

3. $f_y = (4, 1) > 0$

5. $f_x(x, y) = 2$
$f_y(x, y) = -3$

7. $\partial z/\partial x = \sqrt{y}$
$\partial z/\partial y = x/(2\sqrt{y})$

9. $\partial z/\partial x = 2x - 5y$
$\partial z/\partial y = -5x + 6y$

11. $\partial z/\partial x = 2xe^{2y}$
$\partial z/\partial y = 2x^2e^{2y}$

13. $\partial z/\partial x = 2x/(x^2 + y^2)$
$\partial z/\partial y = 2y/(x^2 + y^2)$

15. $\partial z/\partial x = -2y/(x^2 - y^2)$
$\partial z/\partial y = 2x/(x^2 - y^2)$

17. $\partial z/\partial x = (x^3 - 4y^3)/(x^2y)$
$\partial z/\partial y = (-x^3 + 16y^3)/(2xy^2)$

19. $h_x(x, y) = -2xe^{-(x^2+y^2)}$
$h_y(x, y) = -2ye^{-(x^2+y^2)}$

21. $f_x(x, y) = x/\sqrt{x^2 + y^2}$
$f_y(x, y) = y/\sqrt{x^2 + y^2}$

23. $\partial z/\partial x = 2\sec^2(2x - y)$
$\partial z/\partial y = -\sec^2(2x - y)$

25. $\partial z/\partial x = ye^y \cos xy$
$\partial z/\partial y = e^y(x \cos xy + \sin xy)$

27. $f_x(x, y) = 1 - x^2$
$f_y(x, y) = y^2 - 1$

29. $f_x(x, y) = 2$
$f_y(x, y) = 3$

31. $f_x(x, y) = 1/(2\sqrt{x + y})$
$f_y(x, y) = 1/(2\sqrt{x + y})$

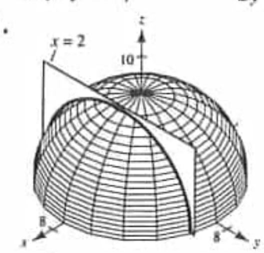
33. $\partial z/\partial x = \frac{1}{4}$
$\partial z/\partial y = \frac{1}{4}$

35. $\partial z/\partial x = -\frac{1}{4}$
$\partial z/\partial y = \frac{1}{4}$

37. $g_x(1, 1) = -2$
$g_y(1, 1) = -2$

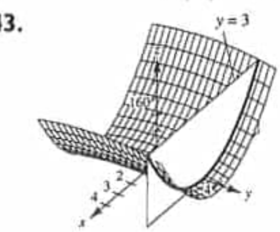
39. $\partial z/\partial x = -1$
$\partial z/\partial y = 0$

41.



$-\frac{1}{2}$

43.



18

45. $x = -6, y = 4$ **47.** $x = 1, y = 1$

49. (a) f_y (b) f_x
f_x represents the slope in the x-direction, and f_y represents the slope in the y-direction.

51. $\dfrac{\partial w}{\partial x} = \dfrac{x}{\sqrt{x^2 + y^2 + z^2}}$
$\dfrac{\partial w}{\partial y} = \dfrac{y}{\sqrt{x^2 + y^2 + z^2}}$
$\dfrac{\partial w}{\partial z} = \dfrac{z}{\sqrt{x^2 + y^2 + z^2}}$

53. $F_x(x, y, z) = \dfrac{x}{x^2 + y^2 + z^2}$
$F_y(x, y, z) = \dfrac{y}{x^2 + y^2 + z^2}$
$F_z(x, y, z) = \dfrac{z}{x^2 + y^2 + z^2}$

2009 Fall
Math 245
Calculus for Engineering
Columbia College in Rolla, Missouri

Syllabus

Quiz 1 = 24/30 80%
Test 1 = 80/100 after +5 curve 80%
Quiz 2 = 38/50 76%
Test 2 = 68/100 after +6 curve 68%
Homework = /25

Course Syllabus

« Return to Previous Page

Total /405

Print this Syllabus

Rolla Campus
2303 N. Bishop Ave./Hwy. 63
Rolla, MO 65401
(573) 341-3350

A = 322 pts
B = 282 pts

I have 210, need 72 (B) or 112 (A)

20 + 52 25 + 87

Course Syllabus
09 / 11 - Early Fall Session
August - October 2009

Course Prefix and Number:	MATH 245 A
Course Title:	Calculus and Analytic Geometry, IIB
Semester Credit Hours:	3
Class Day and Time:	Mon Wed 7:45 pm-10:15 pm
Instructor:	**Fred Browning** *Instructor*
	Home Phone: (573) 265-1444 **CougarMail:** frbrowning@cougars.ccis.edu

Catalog Description

The fourth course in a four part Calculus sequence. Topics include: parametric equations and polar coordinates, vectors and the geometry of space, vector functions, partial derivatives and their applications. Prerequisite: MATH 235 with a score of C or higher.

Prerequisites/Corequisites

MATH 235 with a score of C or higher.

Assessment

Material from this course may be tested on the Major Field Test (MFT) administered during the Culminating Experience course for the degree.

Text

Calculus with Analytic Geometry Eighth edition
Author: Larson, Hostetler, and Edwards (Houghton Mifflin Company)
ISBN: 0-618-50298-X

Course Objectives

- To use calculus to formulate and solve problems and communicate solutions to others.
- To use technology as an integral part of the process of formulation, solution, and communication.
- To understand calculus from numerical, graphical, symbolic, and analytical perspectives.
- To understand and appreciate the connections between mathematics and other disciplines.

Additional Instructor Objectives

The Catalog Description and the Measurable Learning Outcomes are incorrect for the block of material covered by this course. The first two outcomes are covered in Math 235. Outcomes related to multiple integration and vector analysis have not been included.

The student will be able to:

1) Represent curves, surfaces, and their properties in space with vectors and vector-valued functions;
2) Evaluate partial derivatives, directional derivatives, and gradients;
3) Calculate multiple integrals in rectangular, cylindrical, and spherical coordinates;
4) Evaluate line and surface integrals; and,
5) Apply Green's Theorem, Divergence Theorem, and Stoke's Theorem to vector analysis problems.

Measurable Learning Outcomes

· Apply basic calculus concepts to parametric and polar curves to determine arc length, surface area of revolution, and other geometric characteristics.
· Use vectors in two and three dimensions to describe lines and planes in space.
· Use vector functions to describe curves in space.
· Use vector-valued functions to describe the motion of objects through space.
· Apply basic ideas of differential calculus to functions of several variables.

Instructional Methods

In class activities will consist primarily of lectures. The concepts will be presented and a large number of problems will be solved. I teach by example. I use the problem solutions to illustrate concepts and to develop procedures and methods needed for effective problem solving.

Graded Activities

Chapter Tests - 4 Total 400 Points
Description - Chapter tests will be given upon completion of each of the text chapters. The tests will consist of problems. The tests will be either open book or students will be allowed to use a formula sheet. Make-up tests will be given if the instructor or site director feels that the absence was justified.

Quizzes - 2 to 4 Total 50 to 100 Points
Description - Quizzes will be given to encourage students to come to class and to stay current on problem assignments. Unannounced quizzes may be given at any time - especially when attendance is low. Quizzes will be open book. There will be no make-up assignments for quizzes.

Homework 25 to 50 Points
Description - A few homework assignments will be collected and graded. Homework will not be collected unannounced. Problems to be turned in will be given a due date. Homework must be submitted when it is due.

Grading Scale

90 - 100	**A**
80 - 89	**B**
70 - 79	**C**
60 - 69	**D**
0 - 59	**F**

Additional Information / Instructions

Grade Determination:
At the end of the course, the sum of points earned from tests, quizzes, and homework will be divided by the total number of possible points in order to calculate numerical averages for each student. In order to determine final letter grades for the course, the numerical averages will be rounded up if the score is within one-half of one percent of the next higher grade. For example, 89.0 to 89.4% is a "B" and 89.5 to 89.9% is an "A".

Calculator Policy:
A graphing calculator is required for any of the calculus courses. Graphing calculators that will perform symbolic integration and differentiation can not be used during tests and quizzes. During tests and quizzes, TI-84 or lower number calculators are acceptable.

Prerequisites:
The prerequisite for the course is Math 235 with a grade of C or better. This is a firm prerequisite that must be met in order to gain entry into this course. No substitutions will be allowed.

Transfer Policy:
Math 215 and Math 226 are transferable to UMR in place of Math 008 or Math 014.
Math 226 and Math 235 are transferable to UMR in place of Math 021 or Math 015.
Math 235 and Math 245 are transferable to UMR in place of Math 022.

Weekly Activities and Assignments

Week 1
Vector-Valued Functions
Activities: Introduction to vector-valued functions.
Differentiation and integration of vector-valued functions.
Velocity and acceleration problems.
Reading: ~~Chapter 12, Sections 1, 2, and 3.~~ skip
Assignments: Problem Assignments:
Section 12.1 1-37 odds, 45-79 odds.
Section 12.2 1-25 odds, 29-43 odds, 49-67 odds.
Section 12.3 1-27 odds.

Week 2
Vector-Valued Functions
Activities: Continue with vector-valued functions.
Determination of tangent and normal vectors.
Calculation of arc length and curvature.
Reading: ~~Chapter 12, Sections 4 and 5.~~ skip
Assignments: Problem Assignments:
Section 12.4 5-17 odds, 21-57 odds.
Section 12.5 1-15 odds (skip 7), 21-45 odds.
Examinations: Chapter 12, Sections 1 through 5.

Week 3
Functions of Several Variables
Activities: Introduction to functions of several variables - continuity, limits, partial derivatives, and differentials.
Reading: Chapter 13, Sections 1 through 5.
Assignments: Problem Assignments:
Section 13.1 1-37 odds, 45-55 odds, 69-73 odds.
Section 13.2 1-57 odds.
Section 13.3 5-47 odds, 51-67 odds, 73-85 odds.
Section 13.4 1-39 odds.
Section 13.5 1-53 odds.

Week 4
Functions of Several Variables
Activities: Continue with functions of several variables - gradients, tangent planes, normal lines, and extrema of functions of two independent variables.
Reading: Chapter 13, Sections 6 through 8.
Assignments: Problem Assignments:
Section 13.6 1-49 odds, 55-63 odds.
Section 13.7 1-33 odds, 41-55 odds (skip 47).
Section 13.8 1-33 odds, 43-61 odds.

Week 5
Multiple Integration
Activities: Introduction to multiple integration - area and volume problems, use of polar coordinates, center of mass and moment of inertia problems.
Reading: Chapter 14, Sections 1 through 4.
Assignments: Problem Assignments:
Section 14.1 1-63 odds.
Section 14.2 1-29 odds, 33-41 odds, 49-55 odds.
Section 14.3 9-41 odds.
Section 14.4 1-39 odds.
Examinations: Chapter 13, Sections 1 through 8.

Week 6
Multiple Integration
Activities: Continue with multiple integration - surface area problems, introduction to triple integrals, and triple integrals in cylindrical and spherical coordinates.
Reading: Chapter 14, Sections 5 through 7.
Assignments: Problem Assignments:
Section 14.5 1-33 odds.
Section 14.6 1-51 odds.
Section 14.7 1-27 odds.

Week 7
Vector Analysis
Activities: Introduction to vector analysis - vector fields, line integrals, and Green's Theorem. Search
Reading: Chapter 15, Sections 1 through 4.
Assignments: Problem Assignments:
Section 15.1 1-15 odds, 21-75 odds.
Section 15.2 1-31 odds, 35-59 odds.
Section 15.3 1-35 odds.
Section 15.4 1-27 odds.
Examinations: Chapter 14, Sections 1 through 7.

Week 8
Vector Analysis
Activities: Continue with vector analysis - parametric surfaces, surface integrals, Divergence Theorem, and Stoke's Theorem.
Reading: Chapter 15, Sections 5 through 8.
Assignments: Problem Assignments:
Section 15.5 1-41 odds.
Section 15.6 1-29 odds.
Section 15.7 1-17 odds.
Section 15.8 1-19 odds.
Examinations: Chapter 15, Sections 1-8.

Library Resources

Columbia College Resources - Online databases are available at http://www.ccis.edu/offices/library/index.asp. You may access them from off-campus using your eServices login and password when prompted.

Course Policies and Procedures

Attendance

Columbia College Policy - Columbia College students are expected to attend all classes and laboratory periods for which they are enrolled. Students are directly responsible to instructors for class attendance and work missed during an absence for any cause. If absences jeopardize progress in a course, an instructor may withdraw a student from the course with a grade of "F" or "W" at the discretion of the instructor.

Campus Policy - Columbia College students are expected to attend all classes and laboratory periods for which they are enrolled. Students are directly responsible to instructors for class attendance and for work missed during an absence for any cause. Make-up work will not be given if the absence is deemed unexcused by the instructor.

If absences jeopardize progress in a course, an instructor may withdraw a student from that course. When a student cannot attend class, he/she must notify the instructor or campus office. A student who misses one-fourth of class instructional time (two class meetings during an eight-week session, or ten hours) may be deemed to have incurred excessive absences. Unless absences are a direct result of mitigating (i.e. unavoidable and excusable) circumstances, the student may be withdrawn from the course and awarded a grade of "F" or "W" (at the instructor's discretion)

Academic Integrity

Columbia College Policy - Columbia College students must fulfill their academic obligations through honest, independent effort. Dishonesty is considered a serious offense subject to strong disciplinary actions. Activities which constitute academic dishonesty include plagiarism, unauthorized joint effort on exams or assignments, falsification of forms or records, providing false or misleading information, or aiding another in an act of academic dishonesty. Possible penalties for these activities are discussed in detail in the AHE Degree Completion Catalog.

Campus Policy - An additional aspect of academic integrity is the realization by students that they are enrolling in college-level courses that require college-level work for instructors with college-level expectations. In order for a Columbia College degree to retain legitimacy and value, it must represent an indication that the graduate completed a rigorous, thought-provoking, meaningful curriculum and developed effective communication and critical-thinking skills.

Class Conduct and Personal Conduct

Columbia College Policy - Students must conduct themselves so others will not be distracted from the pursuit of learning. Students may be disciplined for any conduct which constitutes a hazard to the health, safety, or well-being of members of the college community or which is deemed detrimental to the college's interests. Discourteous or unseemly conduct may result in a student being asked to leave the classroom. Examples of misconduct and possible disciplinary actions are described in the AHE Degree Completion Catalog.

Cal II Notes

11.7 Cylindrical coordinates

Polar, (r, θ)

Cylindrical (r, θ, z)

conversions: rectangular (x, y, z)

cylindrical (r, θ, z)

1) Cylindrical to rectangular

$x = r\cos\theta \quad y = r\sin\theta \quad z = z$

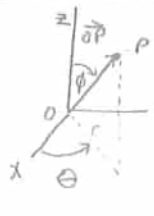
2) Rectangular to cylindrical

$r^2 = x^2 + y^2 \quad \tan\theta = y/x \quad z = z$

vertical plane, $\theta = c$ horizontal planes $z = c$

— spherical coordinates —

1 distance & 2 angles



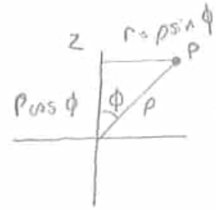
radius $\pm$

$\theta = \pm$

ρ is always $\oplus$

ϕ always $\oplus$

$0° \le \phi \le 180°$



Conversions:

sph. to rectangular ①

$x = r\cos\theta = \rho\sin\phi\cos\theta$

$y = r\sin\theta = \rho\sin\phi\sin\theta$

$z = \rho\cos\phi$

② to spherical from rectangular

$\rho^2 = x^2 + y^2 + z^2$

$\tan\theta = y/x$

$z = \rho\cos\phi$

$\cos\phi = \frac{z}{\rho} = \frac{z}{\sqrt{x^2+y^2+z^2}}$

$\phi = \arccos\left(\frac{z}{\rho}\right) = \arccos\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$

③ spherical to cylindrical

$r^2 = \rho^2\sin^2\phi$

$\theta = \theta$

$z = \rho\cos\theta$

④ Cylindrical to spherical

$\rho = \sqrt{r^2+z^2}$

$\theta = \theta$

$\phi = \arccos\left(\frac{z}{\rho}\right) = \arccos\left(\frac{z}{\sqrt{r^2+z^2}}\right)$

11) $(2, -2, -4)$ to cylindrical (x y z)

$r^2 = x^2 + y^2$

$r^2 = 8$

$r = \pm 2\sqrt{2}$

θ, $\tan\theta = \frac{y}{x} = \frac{-2}{2} = -1$

$\theta = \frac{3\pi}{4}, -\frac{\pi}{4}$ Quadrant 4

$(r, \theta, z) = (2\sqrt{2}, -\frac{\pi}{4}, -4)$

$= (-2\sqrt{2}, \frac{3\pi}{4}, -4)$

17) $y = x^2$

$r\sin\theta = (r\cos\theta)^2$

$\sin\theta = r\cos^2\theta$

$r = \frac{\sin\theta}{\cos^2\theta} = \frac{\sin\theta}{\cos\theta}\frac{1}{\cos\theta} = \tan\theta\sec\theta$

21) $r = 2$

$x^2 + y^2 = r^2$

$x^2 + y^2 = 4$ (generating curve)

23) $\theta = \frac{\pi}{6}$ plane

$\tan\theta = y/x$

$\frac{1}{\sqrt{3}} = y/x$

$x = \sqrt{3}y$

25) $r = 2\sin\theta$ Circle in 2d

$r^2 = 2r\sin\theta$

$x^2 + y^2 = 2r\sin\theta$

$x^2 + y^2 - 2y = 0$

$x^2 + y^2 - 2y + 1 = 1$

$x^2 (y-1)^2 = 1 \quad C = (0, 1)$

11.7) continued

27) $r^2+z^2=4$

$x^2+y^2+z^2=4$

sphere, $r=2$

31) (ρ,θ,ϕ) spherical

(x,y,z) rectangular

$(-2, 2\sqrt{3}, 4)$

$\rho^2 = x^2+y^2+z^2$

$\rho^2 = 4+12+16 = 32$

$\rho = \sqrt{32}$

$\tan\theta = y/x$

$\tan\theta = \frac{2\sqrt{3}}{-2} = -\sqrt{3}$

$\theta = \frac{2\pi}{3}$

$\phi = \arccos\left(\frac{z}{\rho}\right)$

$\phi = \arccos\left(\frac{4}{4\sqrt{2}}\right)$

$\phi = \arccos\frac{1}{\sqrt{2}}$

$\phi = 45^\circ$ $\left(\sqrt{32}, \frac{2\pi}{3}, \frac{\pi}{4}\right)$

35) (ρ,θ,ϕ) $\left(4, \frac{\pi}{6}, \frac{\pi}{4}\right)$

(x,y,z)? $\Rightarrow$ $(\sqrt{6}, \sqrt{2}, 2\sqrt{2})$

$x = \rho\sin\phi\cos\theta = 4\sin\frac{\pi}{4}\cos\frac{\pi}{6} = \sqrt{6}$

$y = \rho\sin\phi\sin\theta = 4\sin\frac{\pi}{4}\sin\frac{\pi}{6} = 4\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) = \sqrt{2}$

$z = \rho\cos\phi = 4\cos\frac{\pi}{4} = 4\frac{1}{\sqrt{2}} = 2\sqrt{2}$

45) $x^2+y^2=9$

$x = \rho\sin\phi\cos\theta$

$y = \rho\sin\phi\sin\theta$

$(\rho\sin\phi\cos\theta)^2 + (\rho\sin\phi\sin\theta)^2 = 9$

$\rho^2\sin^2\phi\,(\cos^2\theta+\sin^2\theta) = 9$

$\rho^2\sin^2(1) = 9$

$\rho\sin\phi = 3$, $\boxed{\rho = 3\csc\phi}$

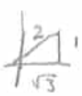
47) $x^2+y^2 = 2z^2$ $\rho^2\sin^2\phi\,\underbrace{(\sin^2\theta+\cos^2\theta)}_{1} = 2(\rho\cos\phi)^2$

$\rho^2\sin^2\phi = 2\rho^2\cos^2\phi$

$\frac{\sin^2\phi}{\cos^2\phi} = 2$ $\tan^2\phi = 2$ $\tan\phi = \pm\sqrt{2}$

51) $\phi = \frac{\pi}{6}$

$\phi = \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$



$\cos\phi = \left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$

$\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{z^2}{x^2+y^2+z^2}$

$\frac{3}{4} = \frac{z^2}{x^2+y^2+z^2}$

$3x^2+3y^2+3z^2 = 4z^2$

$3x^2+3y^2-z^2=0$

53) $\rho = 4\cos\phi$

$\sqrt{x^2+y^2+z^2} = 4\frac{z}{\sqrt{x^2+y^2+z^2}}$

$\sqrt{\ }\sqrt{\ } = 4z$

$x^2+y^2+z^2-4z=0$

$x^2+y^2+(z^2-4z+4) = 4$

$x^2+y^2+(z-2)^2 = 4$

61) Cyl to sphe.

$(r,\theta,z) = \left(4, \frac{-\pi}{6}, 6\right)$

$\rho = \sqrt{r^2+z^2} = \sqrt{24+36} = 2\sqrt{13}$

$\theta = \frac{-\pi}{6}$ $\phi = \arccos\left(\frac{z}{\rho}\right) = \arccos\frac{6}{2\sqrt{13}}$

$\theta = 33.69^\circ$ (ρ,θ,ϕ) $\left(2\sqrt{13}, \frac{-\pi}{6}, 3..1^\circ\right)$

101) $x^2+y^2=4y$

a) cyl. coord.

$r^2 = 4r\sin\theta$

$(\rho\sin\phi\cos\theta)^2 + (\rho\sin\phi\sin\theta)^2 = 4(1)$

$\rho^2(1)+\rho^2(1) = 4\rho^2\sin\theta\sin^2\phi$

$\rho\sin\phi = 4\sin\theta$

$\rho = \frac{4\sin\theta}{\sin\phi} = \boxed{4\sin\theta\csc\phi}$

11.5

TEMA

13) pt $(2,3,4)$ ‖ to plane x-z & yz

vertical

direction vector $v = k$ $\langle 0,0,1\rangle$

$\boxed{x=2 \quad y=3, \quad z=4+t}$

15) pt $(2,3,4)$ perpendicular to $3x+2y-z=6$

$\boxed{x=2+3t \quad y=3+2t \quad z=4-t}$

17) $(5,-3,4)$ ‖ to $v=\langle 2,-1,3\rangle$

$\boxed{x=5+2t \quad y=-3-t \quad z=4+3t}$

19) $(2,1,2)$ parallel to line $x=-t \quad y=1+t, \quad z=-2+t$

$\boxed{x=2-t \quad y=1+t \quad z=2+t}$

21) $x=3-t \quad y=-1+2t \quad z=-2$

pt: $(3,-1,-2)$ $V=\langle -1,2,0\rangle$

23) $\frac{x-7}{4} = \frac{y+6}{2} = z+2$

direction vector $\langle 4,2,1\rangle$ pt: $(7,-6,-2)$

25) are any parallel?/identical?

L_2 parallel to L_3 $\quad$ L_1 identical to L_2

27)

$x=4t+2 \quad y=3, \quad z=-t+1 \quad \langle 4,0,-1\rangle$

$x=2s+2 \quad y=2s+3 \quad z=s+1 \quad \langle 2,2,1\rangle$

$2s+3=3$

$2s=0$

$s=0$ $x=2, y=3, z=1$ pt $(2,3,1)$

$t=0$ $2, 3, 1$

$\boxed{\text{yes, @ } (2,3,1)}$ $\cos\theta = \frac{|u\cdot v|}{\|u\|\|v\|} = \frac{|7|}{\sqrt{17}\sqrt{9}} = \frac{7}{3\sqrt{17}}$ $\frac{7\sqrt{17}}{51}$

28) $x = -3t+1 \quad y = 4t+1, \quad z = 2t+4$

$x = 3s+1 \quad y = 2s+4 \quad z = -s+1$

$-3t+1 = 3s+1 \qquad 4t+1 = 2s+4 \qquad 2t+4 = -s+1$

29) $\frac{x}{3} = \frac{y-2}{-1} = z+1 \;\Big|\; \frac{x-1}{4} = y+2 = \frac{z+3}{-3}$

$x = 3s \quad y = 2-s \quad z = -1+t \;|\; x = 1+4t, \; y = -2+s, \; z = -3-3t$

i) $3s = 1+4t \qquad 3s = 1+4(4-s) \qquad 3s = 17-4s$

$7s = 17$

ii) $2-s = -2+t \qquad t = 4-s \qquad \boxed{s = \frac{17}{7}}$

iii) $-1+s \quad -3+3t \qquad t = \frac{28-17}{7} \qquad \boxed{t = \frac{11}{7}}$

Do Not Intersect

$s = -2+3t$

$\frac{17}{7} = -\frac{14}{7} + \frac{33}{7}$

$\frac{17}{7} \neq \frac{19}{7}$

33) a) coordinates of 3 pts P, Q, R,

$4x - 3y - 6z = 6$

P $(3, 1, \frac{1}{2})$ $\quad \overline{PQ} = \langle 3, 3, \frac{1}{2} \rangle$

Q $(6, 4, 1)$ $\quad \overline{PR} = \langle -3, 1, -\frac{5}{2} \rangle$

R $(0, 2, -2)$ $\quad \overline{PQ} \times \overline{PR} = \langle -\frac{15}{2} - \frac{1}{2}, \; \overset{(-)}{\frac{-15}{2} - \frac{3}{2}}, \; 3+9 \rangle$

$\langle \cancel{\frac{-16}{2}}\, 8, \; +9 \; +12 \rangle$

cross product is salar product of eqn $\langle 4, -3, -6 \rangle$ & (-2)

35) pt $(2,1,2)$ perpendicular to $n = i$ $\quad \langle 1, 0, 0 \rangle$

$x = 2 \qquad \langle 0, 1, -1 \rangle$

Notes | 11.5 |

line

83) Plane! $2x - 2y + z = 12$ $\quad x - \frac{1}{2} = \frac{y + \frac{3}{2}}{-1} = \frac{z+1}{2}$

$x = t + \frac{1}{2} \quad y = -\frac{3}{2} - t \quad z = -1 + 2t$

$\underbrace{2(t + \tfrac{1}{2}) - 2(-t - \tfrac{3}{2}) + (2t - 1) = 12}_{t = 3/2}$

$x = 2 \quad y = -3 \quad z = 2$ Point of intersection $(2, -3, 2)$

87) Dist from pt to plane

$P\ (0, 0, 0) \quad 2x + 3y + z = 12$ $\quad d = \frac{|\vec{PQ} \cdot n|}{\|n\|}$ $\quad PQ \cdot n = \begin{matrix} \langle -6\,0\,0 \rangle \\ \langle 2\,3\,1 \rangle \end{matrix} = -12$

n = normal vector $\langle 2, 3, 1 \rangle$ pt. in plane $(6, 0, 0)$ Q (can be any) $\quad d = \frac{12}{\sqrt{14}} = 3.2$

$\vec{PQ} = (-6, 0, 0) \quad \|n\| = \sqrt{14}$

95) Dist Between a pt. & a line

Pt. $(1, 5, -2)$ Q (given)

line ($x = 4t - 2$, $y = 3$, $z = -t + 1$) - get a pt on line $t = 0$, $(-2, 3, 1)$ $\quad \vec{PQ}\ \langle 3, 2, -3 \rangle$ $\quad U$ = direction vector for line

$\vec{PQ} \times U = \langle -2, -9, -8 \rangle \quad \|\vec{PQ} \times U\| = \sqrt{4 + 81 + 64} = \sqrt{149}$ $\quad U = \langle 4, 0, -1 \rangle$

$\|U\| = \sqrt{17}$

$D = \frac{\|\vec{PQ} \times U\|}{\|U\|} = \frac{\sqrt{149}}{\sqrt{17}} = \boxed{2.96}$

11.6 - Notes -

- surfaces in space -

1) sphere $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2$

2) planes $ax+by+cz+d=0$

3) cylindrical surfaces

4) Quadric surfaces general eq: $Ax^2+By^2+Cz^2+Dxy+Exz+Fyz+Gx+Hy+Iz+J=0$

5) surface of revolution $x^2+y^2=rz^2$ about x axis $y^2+z^2=[r x]^2$ about y: $x^2+z^2=(r(y))^2$

P818 1) C

2) $15x^2-4y^2+15z^2=-4$

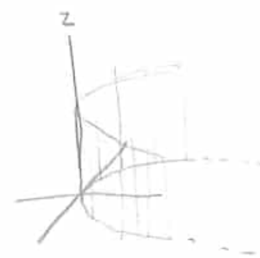
$-\frac{15}{4}x^2+y^2+-\frac{15}{4}z^2=1$ e hyperboloid of 2 sheets

3) / by 4, 1 sheet hyperboloid

4) b

5) $y=x^2+\frac{1}{4}z^2$ elliptic paraboloid

6) $z=-x^2+\frac{1}{4}y^2$



9) $y^2+z^2=9$

generating curve, circle in yz plane, $r=3$

11) $x^2-y=0$

$y=x^2$ parabola in xy plane

13) $4x^2+y^2=4$ ellipse

$\frac{x^2}{1}+\frac{y^2}{4}=1$

21) $16x^2-y^2+16z^2=4$ hyperboloid of 1 sheet, about y axis

$4x^2+4z^2-\frac{y^2}{4}=1$

$\frac{x^2}{1/4}+\frac{z^2}{1/4}-\frac{y^2}{4}=1$

25) $x^2-y^2+z=0$

$z=y^2-x^2$

hyp. paraboloid

27) $z^2=x^2+\frac{y^2}{4}$

$x^2+\frac{y^2}{4}-z^2=0$

29) $16x^2+9y^2+16z^2-32x-36y+36=0$

ellipsoid, not at origin

$16x^2-32x$ $9y^2-36y$ $16z^2=-36$

$16(x^2-2x+1)+9(y^2-4y+4)+16z^2=-36+36+16$

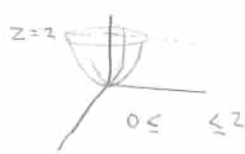
$16(x-1)^2+9(y-2)^2+(4z)^2=16$

$\frac{(x-1)^2}{1}+\frac{(y-2)^2}{16/9}+\frac{z^2}{1}=1$ c $(1,2,0)$

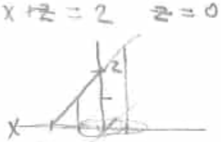
41) ① $z=2\sqrt{x^2+y^2}$ ② $z=2$

$z^2=4x^2+4y^2$

$4x^2+4y^2-z^2=0$



43) $x^2+y^2=1$ $x+z=2$ $z=0$



45) $z^2=4y$ $z=\pm 2\sqrt{y}$

axis of revolution: $\boxed{y}$

$x^2+z^2=[r(y)]^2$

$=(\pm 2\sqrt{y})^2=4y$

$x^2+z^2=4y$

47) $z=2y$ axis of rev. $\boxed{z}$

$x^2+y^2=(r(z))^2=\left(\frac{z}{2}\right)^2=\frac{1}{4}z^2$

$y=\frac{z}{2}$, $x^2+y^2=\frac{1}{4}z^2$

$x^2+y^2-\frac{z^2}{4}=0$

51) $x^2+y^2-2z=0$

find eq of generating curve

$x^2+y^2=2z$ $2z=(r(z))^2$

$r(z)=\sqrt{2z}$ ① $x=\sqrt{2z}$ ② $y=\sqrt{2z}$

11.6| Practice

1) $\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{9} = 1$

ellipsoid (c)

2) $15x^2 - 4y^2 + 15z^2 = -4$

$-\frac{15x^2}{4} + \frac{y^2}{1} - \frac{15z^2}{4} = 1$

hyperboloid 1 sheet (f)

3) $x^2 - \frac{y^2}{4} + z^2 = 1$ (e)

4) $4x^2 + 9z^2 - y^2 = 0$

elliptic cone y axis (b)

5) $4x^2 - 4y + z^2 = 0$

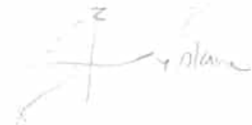
$y = x^2 + \frac{z^2}{4}$

elliptic paraboloid (d)

6) $x^2 + \frac{y^2}{4} = z$ (A)

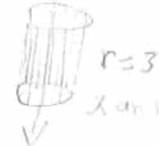
elliptic thing saddle (z)

7) $z = 3$



9) $y^2 + z^2 = 9$

cylinoid



11) $x^2 - y = 0$

$y = x^2$

parabola

11

10.2 $\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right]$ $\tan^2\theta + 1 = \sec^2\theta$ $\sin^2\theta + \cos^2\theta = 1$ $\sin^2\theta = \frac{1-\cos 2\theta}{2}$ $\cos^2\theta = \frac{1+\cos 2\theta}{2}$

curve C given by $x = f(t)$ & $y = g(t)$

— Arc length —

$s = \int_a^b \sqrt{(f'(t))^2 + [g'(t)]^2}\, dt$

$s = \int_{x_0}^{x_1} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dt$

— surface Area —

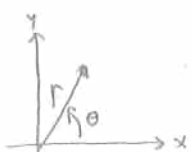
$S = 2\pi \int_a^b g(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ (about x-axis)

10.3 - make x, y, t chart. find duplicates of (x, y)

- take $\frac{dy}{dx}$, plug in t values to get slopes

$y - k = (\text{slope})(x - h)$ = tangent lines

10.4



Pt: (R, θ) or $(-r, \theta + \pi n)$

$x = r\cos\theta$

$y = r\sin\theta$

product Rule $(fg)' = f' \cdot g + g' f$

$\frac{f'g - g'f}{fg}$

Graphs $r = 2$, same as $x^2 + y^2 = 4$

$\theta = \pi/3$, line

— horiz. tan lines —

set $\frac{dy}{d\theta} = 0$, not $\frac{dx}{d\theta}$

— vert —

opposite above

10.5 Area POLAR

$A = \frac{1}{2}\int_\alpha^\beta [f(\theta)]^2 d\theta = \frac{1}{2}\int_\alpha^\beta r^2 d\theta$

ARC LENGTH POLAR

$s = \int_\alpha^\beta \sqrt{(f(\theta))^2 + [f'(\theta)]^2}\, d\theta = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

intersect: set $R_1 = R_2$

10.6 ellipse:

$a^2 = b^2 + c^2 \Rightarrow b^2 = (a^2)(1 - e^2)$ Eccentricity $e = \frac{c}{a}$

hyperbola

$c^2 = a^2 + b^2 \Rightarrow b^2 = a^2(e^2 - 1)$

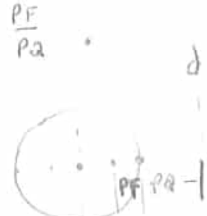
$\boxed{e}$ $0 < e < 1$ ellipse

$e = 1$ parabola

$e > 1$ hyperbola

$r = \frac{ed}{1 + e\cos\theta}$

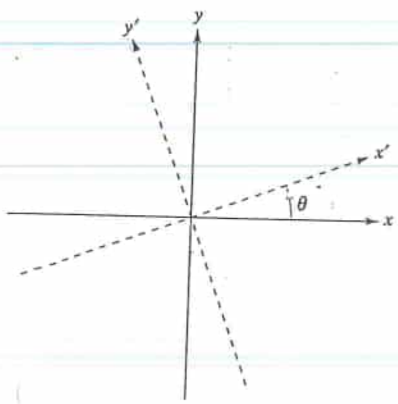
$\frac{PF}{Pd}$



APPENDIX E

Rotation and the General Second-Degree Equation

Rotation of Axes • Invariants Under Rotation



After rotation of the x- and y-axes counterclockwise through an angle θ, the rotated axes are denoted as the x'-axis and y'-axis.
Figure A.41

Rotation of Axes

In Section 9.1, you learned that equations of conics with axes parallel to one of the coordinate axes can be written in the general form

$$Ax^2 + Cy^2 + Dx + Ey + F = 0. \qquad \text{Horizontal or vertical axes}$$

Here you will study the equations of conics whose axes are rotated so that they are *not* parallel to the x-axis or the y-axis. The general equation for such conics contains an *xy-term.*

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \qquad \text{Equation in } xy\text{-plane}$$

To eliminate this xy-term, you can use a procedure called **rotation of axes.** You want to rotate the x- and y-axes until they are parallel to the axes of the conic. (The rotated axes are denoted as the x'-axis and the y'-axis, as shown in Figure A.41.) After the rotation has been accomplished, the equation of the conic in the new $x'y'$-plane will have the form

$$A'(x')^2 + C'(y')^2 + D'x' + E'y' + F' = 0. \qquad \text{Equation in } x'y'\text{-plane}$$

Because this equation has no $x'y'$-term, you can obtain a standard form by completing the square.

The following theorem identifies how much to rotate the axes to eliminate an xy-term and also the equations for determining the new coefficients A', C', D', E', and F'.

THEOREM A.1 Rotation of Axes

The general equation of the conic

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0,$$

where $B \neq 0$, can be rewritten as

$$A'(x')^2 + C'(y')^2 + D'x' + E'y' + F' = 0$$

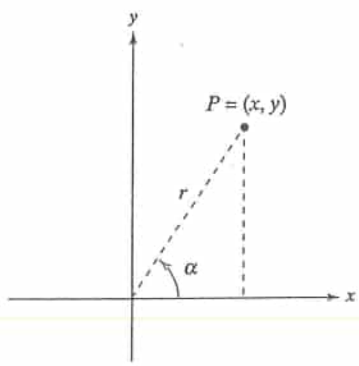
by rotating the coordinate axes through an angle θ, where

$$\cot 2\theta = \frac{A - C}{B}.$$

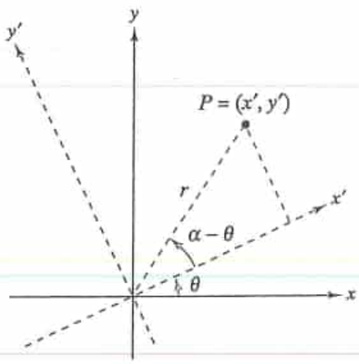
The coefficients of the new equation are obtained by making the substitutions

$$x = x' \cos\theta - y' \sin\theta$$

$$y = x' \sin\theta + y' \cos\theta.$$



Original: $x = r\cos\alpha$
$y = r\sin\alpha$



Rotated: $x' = r\cos(\alpha - \theta)$
$y' = r\sin(\alpha - \theta)$

Figure A.42

Proof To discover how the coordinates in the xy-system are related to the coordinates in the $x'y'$-system, choose a point $P = (x, y)$ in the original system and attempt to find its coordinates (x', y') in the rotated system. In either system, the distance r between the point P and the origin is the same, and thus the equations for x, y, x', and y' are those given in Figure A.42. Using the formulas for the sine and cosine of the difference of two angles, you obtain

$$\begin{aligned} x' &= r\cos(\alpha - \theta) = r(\cos\alpha\cos\theta + \sin\alpha\sin\theta) \\ &= r\cos\alpha\cos\theta + r\sin\alpha\sin\theta = x\cos\theta + y\sin\theta \\ y' &= r\sin(\alpha - \theta) = r(\sin\alpha\cos\theta - \cos\alpha\sin\theta) \\ &= r\sin\alpha\cos\theta - r\cos\alpha\sin\theta = y\cos\theta - x\sin\theta. \end{aligned}$$

Solving this system for x and y yields

$$x = x'\cos\theta - y'\sin\theta \quad \text{and} \quad y = x'\sin\theta + y'\cos\theta.$$

Finally, by substituting these values for x and y into the original equation and collecting terms, you obtain the following.

$$\begin{aligned} A' &= A\cos^2\theta + B\cos\theta\sin\theta + C\sin^2\theta \\ C' &= A\sin^2\theta - B\cos\theta\sin\theta + C\cos^2\theta \\ D' &= D\cos\theta + E\sin\theta \\ E' &= -D\sin\theta + E\cos\theta \\ F' &= F \end{aligned}$$

Now, in order to eliminate the $x'y'$-term, you must select θ such that $B' = 0$, as follows.

$$\begin{aligned} B' &= 2(C - A)\sin\theta\cos\theta + B(\cos^2\theta - \sin^2\theta) \\ &= (C - A)\sin 2\theta + B\cos 2\theta \\ &= B(\sin 2\theta)\left(\frac{C - A}{B} + \cot 2\theta\right) = 0, \quad \sin 2\theta \neq 0 \end{aligned}$$

If $B = 0$, no rotation is necessary, because the xy-term is not present in the original equation. If $B \neq 0$, the only way to make $B' = 0$ is to let

$$\boxed{\cot 2\theta = \frac{A - C}{B}, \qquad B \neq 0.}$$

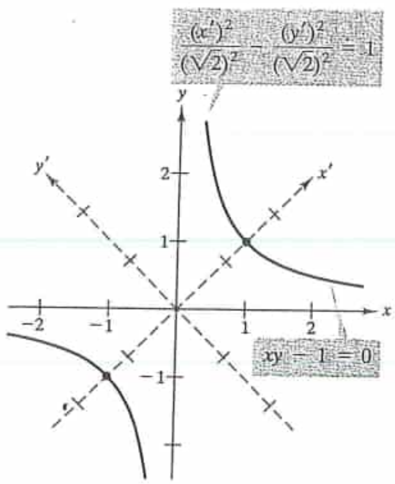
Thus, you have established the desired results.

EXAMPLE 1 Rotation of a Hyperbola

Write the equation $xy - 1 = 0$ in standard form.

Solution Because $A = 0$, $B = 1$, and $C = 0$, you have (for $0 < \theta < \pi/2$)

$$\cot 2\theta = \frac{A - C}{B} = 0 \quad \Longrightarrow \quad 2\theta = \frac{\pi}{2} \quad \Longrightarrow \quad \theta = \frac{\pi}{4}.$$



Vertices:
$(\sqrt{2}, 0)$, $(-\sqrt{2}, 0)$ in $x'y'$-system
1), $(-1, -1)$ in xy-system
ure A.43

The equation in the $x'y'$-system is obtained by making the following substitutions.

$$x = x'\cos\frac{\pi}{4} - y'\sin\frac{\pi}{4} = x'\left(\frac{\sqrt{2}}{2}\right) - y'\left(\frac{\sqrt{2}}{2}\right) = \frac{x' - y'}{\sqrt{2}}$$

$$y = x'\sin\frac{\pi}{4} + y'\cos\frac{\pi}{4} = x'\left(\frac{\sqrt{2}}{2}\right) + y'\left(\frac{\sqrt{2}}{2}\right) = \frac{x' + y'}{\sqrt{2}}$$

Substituting these expressions into the equation $xy - 1 = 0$ produces

$$\left(\frac{x' - y'}{\sqrt{2}}\right)\left(\frac{x' + y'}{\sqrt{2}}\right) - 1 = 0$$

$$\frac{(x')^2 - (y')^2}{2} - 1 = 0$$

$$\frac{(x')^2}{(\sqrt{2})^2} - \frac{(y')^2}{(\sqrt{2})^2} = 1. \qquad \text{Standard form}$$

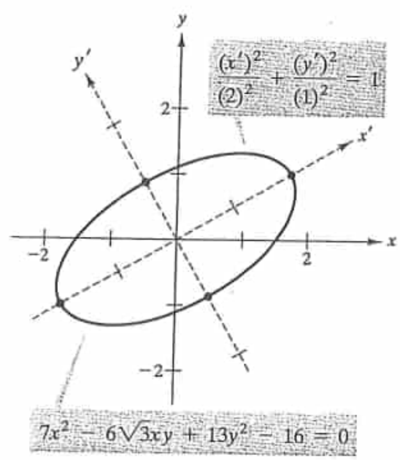
This is the equation of a hyperbola centered at the origin with vertices at $(\pm\sqrt{2}, 0)$ in the $x'y'$-system, as shown in Figure A.43.

EXAMPLE 2 Rotation of an Ellipse

Sketch the graph of $7x^2 - 6\sqrt{3}xy + 13y^2 - 16 = 0$.

Solution Because $A = 7$, $B = -6\sqrt{3}$, and $C = 13$, you have (for $0 < \theta < \pi/2$)

$$\cot 2\theta = \frac{A - C}{B} = \frac{7 - 13}{-6\sqrt{3}} = \frac{1}{\sqrt{3}} \quad \Longrightarrow \quad \theta = \frac{\pi}{6}.$$



Vertices:
$(\pm 2, 0)$, $(0, \pm 1)$ in $x'y'$-system
$(\pm\sqrt{3}, \pm 1)$, $\left(\pm\frac{1}{2}, \pm\frac{\sqrt{3}}{2}\right)$ in xy-system
re A.44

Therefore, the equation in the $x'y'$-system is derived by making the following substitutions.

$$x = x'\cos\frac{\pi}{6} - y'\sin\frac{\pi}{6} = x'\left(\frac{\sqrt{3}}{2}\right) - y'\left(\frac{1}{2}\right) = \frac{\sqrt{3}x' - y'}{2}$$

$$y = x'\sin\frac{\pi}{6} + y'\cos\frac{\pi}{6} = x'\left(\frac{1}{2}\right) + y'\left(\frac{\sqrt{3}}{2}\right) = \frac{x' + \sqrt{3}y'}{2}$$

Substituting these expressions into the original equation eventually simplifies (after considerable algebra) to

$$4(x')^2 + 16(y')^2 = 16$$

$$\frac{(x')^2}{(2)^2} + \frac{(y')^2}{(1)^2} = 1. \qquad \text{Standard form}$$

This is the equation of an ellipse centered at the origin with vertices at $(\pm 2, 0)$ and $(0, \pm 1)$ in the $x'y'$-system, as shown in Figure A.44.

In writing Examples 1 and 2, we chose the equations such that θ would be one of the common angles 30°, 45°, and so forth. Of course, many second-degree equations do not yield such common solutions to the equation

$$\cot 2\theta = \frac{A - C}{B}.$$

Example 3 illustrates such a case.

EXAMPLE 3 Rotation of a Parabola

Sketch the graph of $x^2 - 4xy + 4y^2 + 5\sqrt{5}y + 1 = 0$.

Solution Because $A = 1$, $B = -4$, and $C = 4$, you have

$$\cot 2\theta = \frac{A - C}{B} = \frac{1 - 4}{-4} = \frac{3}{4}.$$

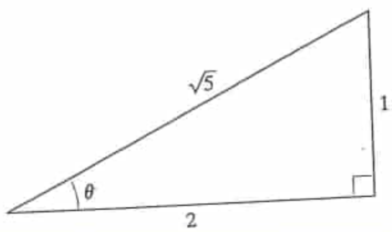


Figure A.45

The trigonometric identity $\cot 2\theta = (\cot^2\theta - 1)/(2\cot\theta)$ produces

INVERT, $\tan 2\theta = \frac{4}{3} = 1.33$

$$\cot 2\theta = \frac{3}{4} = \cancel{\frac{\cot^2\theta - 1}{2\cot\theta}}$$

DO NOT USE

$2\theta = \tan^{-1}\left(\frac{4}{3}\right) = .9273$ radians

from which you can obtain the equation

$$6\cot\theta = 4\cot^2\theta - 4 \implies 4\cot^2\theta - 6\cot\theta - 4 = 0$$
$$(2\cot\theta - 4)(2\cot\theta + 1) = 0.$$

Considering $0 < \theta < \pi/2$, it follows that $2\cot\theta = 4$. Thus,

$$\cot\theta = 2 \implies \theta \approx 26.6°.$$

From the triangle in Figure A.45, you can obtain $\sin\theta = 1/\sqrt{5}$ and $\cos\theta = 2/\sqrt{5}$. Consequently, you can write the following.

$$x = x'\cos\theta - y'\sin\theta = x'\left(\frac{2}{\sqrt{5}}\right) - y'\left(\frac{1}{\sqrt{5}}\right) = \frac{2x' - y'}{\sqrt{5}}$$

$$y = x'\sin\theta + y'\cos\theta = x'\left(\frac{1}{\sqrt{5}}\right) + y'\left(\frac{2}{\sqrt{5}}\right) = \frac{x' + 2y'}{\sqrt{5}}$$

Substituting these expressions into the original equation produces

$$\left(\frac{2x' - y'}{\sqrt{5}}\right)^2 - 4\left(\frac{2x' - y'}{\sqrt{5}}\right)\left(\frac{x' + 2y'}{\sqrt{5}}\right) + 4\left(\frac{x' + 2y'}{\sqrt{5}}\right)^2 +$$
$$5\sqrt{5}\left(\frac{x' + 2y'}{\sqrt{5}}\right) + 1 = 0$$

which simplifies to

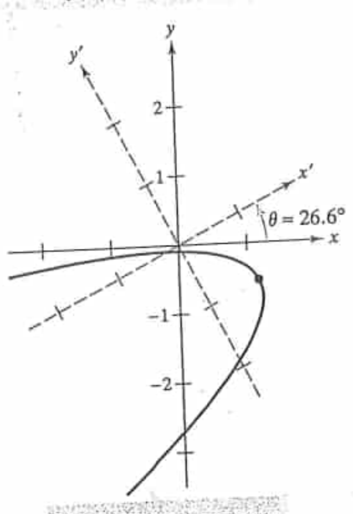
$$5(y')^2 + 5x' + 10y' + 1 = 0.$$

By completing the square, you can obtain the standard form

$$5(y' + 1)^2 = -5x' + 4$$

$$(y' + 1)^2 = 4\left(-\frac{1}{4}\right)\left(x' - \frac{4}{5}\right). \quad \text{Standard form}$$

$x^2 - 4xy + 4y^2 + 5\sqrt{5}y + 1 = 0$



$(y' + 1)^2 = 4\left(-\frac{1}{4}\right)\left(x' - \frac{4}{5}\right)$

Vertex: $\left(\frac{4}{5}, -1\right)$ in $x'y'$-system

$\left(\frac{13}{5\sqrt{5}}, -\frac{6}{5\sqrt{5}}\right)$ in xy-system

Figure A.46

The graph of the equation is a parabola with its vertex at $\left(\frac{4}{5}, -1\right)$ and its axis parallel to the x'-axis in the $x'y'$-system, as shown in Figure A.46.

COMPLIMENTARY COPY NOT FOR SALE COMPLIMENTARY COPY NOT FOR SALE COMPLIMENTARY COPY NOT FOR SALE

Invariants Under Rotation

In Theorem A.1, note that the constant term $F' = F$ is the same in both equations. Because of this, F is said to be **invariant under rotation.** Theorem A.2 lists some other rotation invariants. The proof of this theorem is left as an exercise (see Exercise 34).

THEOREM A.2 Rotation Invariants

The rotation of coordinate axes through an angle θ that transforms the equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ into the form

$$A'(x')^2 + C'(y')^2 + D'x' + E'y' + F' = 0$$

has the following rotation invariants.

1. $F = F'$
2. $A + C = A' + C'$
3. $B^2 - 4AC = (B')^2 - 4A'C'$

You can use this theorem to classify the graph of a second-degree equation *with* an xy-term in much the same way you do for a second-degree equation *without* an xy-term. Note that because $B' = 0$, the invariant $B^2 - 4AC$ reduces to

$$B^2 - 4AC = -4A'C' \qquad \text{Discriminant}$$

which is called the **discriminant** of the equation

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0.$$

Because the sign of $A'C'$ determines the type of graph for the equation

$$A'(x')^2 + C'(y')^2 + D'x' + E'y' + F' = 0$$

the sign of $B^2 - 4AC$ must determine the type of graph for the original equation. This result is stated in Theorem A.3.

THEOREM A.3 Classification of Conics by the Discriminant

The graph of the equation

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

is, except in degenerate cases, determined by its discriminant as follows.

1. *Ellipse or circle* $\quad B^2 - 4AC < 0$
2. *Parabola* $\quad B^2 - 4AC = 0$
3. *Hyperbola* $\quad B^2 - 4AC > 0$

EXAMPLE 4 Using the Discriminant

Classify the graph of each of the following equations.

a. $4xy - 9 = 0$

b. $2x^2 - 3xy + 2y^2 - 2x = 0$

c. $x^2 - 6xy + 9y^2 - 2y + 1 = 0$

d. $3x^2 + 8xy + 4y^2 - 7 = 0$

Solution

a. The graph is a hyperbola because

$$B^2 - 4AC = 16 - 0 > 0.$$

b. The graph is a circle or an ellipse because

$$B^2 - 4AC = 9 - 16 < 0.$$

c. The graph is a parabola because

$$B^2 - 4AC = 36 - 36 = 0.$$

d. The graph is a hyperbola because

$$B^2 - 4AC = 64 - 48 > 0.$$

EXERCISES FOR APPENDIX E

In Exercises 1–12, rotate the axes to eliminate the xy-term. Give the resulting equation and sketch its graph showing both sets of axes.

1. $xy + 1 = 0$
2. $xy - 4 = 0$
3. $x^2 - 10xy + y^2 + 1 = 0$
4. $xy + x - 2y + 3 = 0$
5. $xy - 2y - 4x = 0$
6. $13x^2 + 6\sqrt{3}xy + 7y^2 - 16 = 0$
7. $5x^2 - 2xy + 5y^2 - 12 = 0$
8. $2x^2 - 3xy - 2y^2 + 10 = 0$
9. $3x^2 - 2\sqrt{3}xy + y^2 + 2x + 2\sqrt{3}y = 0$
10. $16x^2 - 24xy + 9y^2 - 60x - 80y + 100 = 0$
11. $9x^2 + 24xy + 16y^2 + 90x - 130y = 0$
12. $9x^2 + 24xy + 16y^2 + 80x - 60y = 0$

In Exercises 13–18, use a graphing utility to graph the conic. Determine the angle θ through which the axes are rotated. Explain how you used the utility to obtain the graph.

13. $x^2 + xy + y^2 = 10$
14. $x^2 - 4xy + 2y^2 = 6$
15. $17x^2 + 32xy - 7y^2 = 75$
16. $40x^2 + 36xy + 25y^2 = 52$
17. $32x^2 + 50xy + 7y^2 = 52$
18. $4x^2 - 12xy + 9y^2 + (4\sqrt{13} + 12)x - (6\sqrt{13} + 8)y = 91$

In Exercises 19–26, use the discriminant to determine whether the graph of the equation is a parabola, an ellipse, or a hyperbola.

19. $16x^2 - 24xy + 9y^2 - 30x - 40y = 0$
20. $x^2 - 4xy - 2y^2 - 6 = 0$
21. $13x^2 - 8xy + 7y^2 - 45 = 0$
22. $2x^2 + 4xy + 5y^2 + 3x - 4y - 20 = 0$
23. $x^2 - 6xy - 5y^2 + 4x - 22 = 0$
24. $36x^2 - 60xy + 25y^2 + 9y = 0$
25. $x^2 + 4xy + 4y^2 - 5x - y - 3 = 0$
26. $x^2 + xy + 4y^2 + x + y - 4 = 0$

In Exercises 27–32, sketch the graph (if possible) of the degenerate conic.

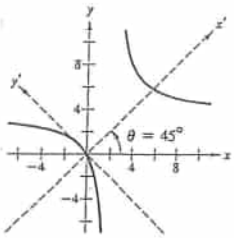
27. $y^2 - 4x^2 = 0$
28. $x^2 + y^2 - 2x + 6y + 10 = 0$
29. $x^2 + 2xy + y^2 - 1 = 0$
30. $x^2 - 10xy + y^2 = 0$
31. $(x - 2y + 1)(x + 2y - 3) = 0$
32. $(2x + y - 3)^2 = 0$

33. Show that the equation $x^2 + y^2 = r^2$ is invariant under rotation of axes.
34. Prove Theorem A.2.

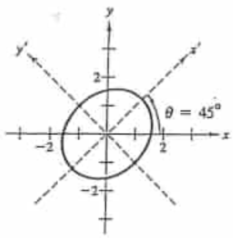
The symbol indicates an exercise in which you are instructed to use graphing technology or a symbolic computer algebra system. The solutions of other exercises may also be facilitated by use of appropriate technology.

A blue number indicates that a detailed solution can be found in the Study and Solutions Guide.

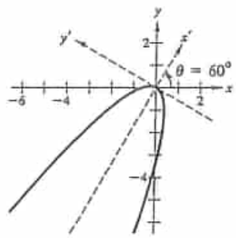
5. $\dfrac{(x' - 3\sqrt{2})^2}{16} - \dfrac{(y' - \sqrt{2})^2}{16} = 1$



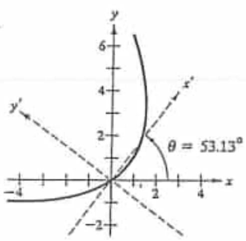
7. $\dfrac{(x')^2}{3} + \dfrac{(y')^2}{2} = 1$



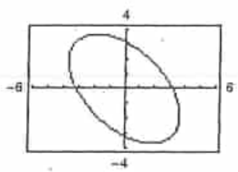
9. $x' = -(y')^2$



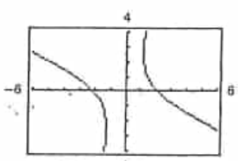
11. $y' = \dfrac{(x')^2}{6} - \dfrac{x'}{3}$



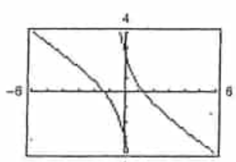
13. $\theta = 45°$



15. $\theta \approx 26.57°$

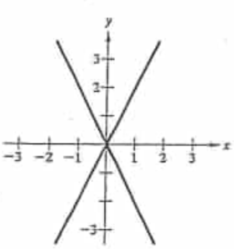


17. $\theta \approx 31.72°$

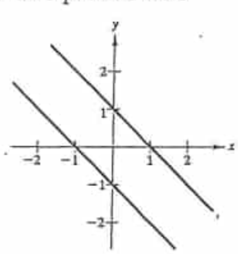


19. Parabola 21. Ellipse 23. Hyperbola 25. Parabola

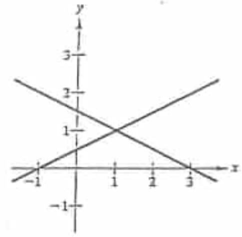
27. Two lines



29. Two parallel lines



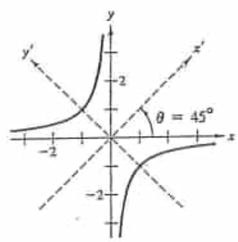
31. Two lines



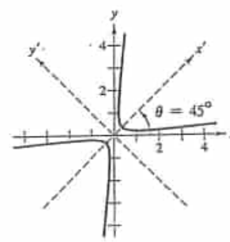
33. Proof

APPENDIX E *(page A56)*

1. $\dfrac{(y')^2}{2} - \dfrac{(x')^2}{2} = 1$



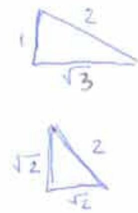
3. $\dfrac{(x')^2}{1/4} - \dfrac{(y')^2}{1/6} = 1$



9.7 | n'th Taylor Polynomial for f (at c)

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \dots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

$\frac{x}{r}$ sin csc
cos sec
tan cot



n'th Maclaurin polynomial for f (at 0)

$$P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots + \frac{f^{n}(0)}{n!}x^n$$

9.8 | Power Series

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots a_n x^n$$

at center c:

$$\sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_1(x-c) + a_2(x-c)^2 + \dots a_n(x-c)^n$$

- convergence
  1) only at c
  2) converges for $|x-c| < R$ $\quad \lim_{n\to\infty}\left|\frac{U_{n+1}}{U_n}\right|$
  3) converges absolutely
- Test $\lim_{n\to\infty}\frac{a_{n+1}}{a_n} > 1$, diverge

9.9 | Representation of f by Power Series

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r} \quad |r| < 1$$

- never converges at endpoints

— Geometric Power Series —

9.10 |
9 | Taylor & Maclaurin Series

every convergent power series form: if f is $f(x) = \sum a_n (x-c)^n$, $a_n = \frac{f^{(n)}(c)}{n!}$

and $f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2$ ... as above

Taylor series ~~at c~~ for f(x) at c, MacLaurin series for f at c = 0

10.1) Eccentricity, $e = \frac{c}{a}$ close to 1 = elongated

center, focus, vertex

Ellipse

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$ Major axis horizontal

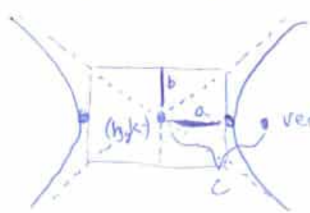
for major axis vertical

foci = $(h, k \pm c)$ with $c^2 = a^2 - b^2$

quadratic $\left(\frac{b}{2}\right)^2$ to add

Hyperbola

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$ transverse horizontal



Parabola

$(x-h)^2 = 4p(y-k)$
(vertical axis)

directrix $y = k - p$

horiz = focus $(h, k+p)$

vert = focus $(h+p, k)$

$x = x'\cos\theta - y'\sin\theta$
$y = x'\sin\theta + y'\cos\theta$

Rotating: $\cot 2\theta = \frac{A-C}{B}$ for $\theta = \frac{\pi}{4}$ $x = \frac{x'-y'}{\sqrt{2}}$ $y = \frac{x'+y'}{\sqrt{2}}$

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

$A'(x')^2 + C'(y')^2 + D'x' + E'y' + F' = 0$

Elipse, $B^2 - 4AC < 0$ ellipse
$= 0$ parabola
> 0 hyperbola

Degenerate sys. FAILS

$\cot 2\theta = \frac{-1}{\sqrt{3}}$, $\theta = 60°$

$\sqrt{3}$, $\theta = 30$

Columbia College
Rolla Campus
2303 N. Bishop Ave./Hwy. 63
P.O. Box 1701
Rolla, MO 65402-1701
(573) 341-3350

Course Syllabus
08 / 12 - Late Fall Session
October - December 2008

Course Prefix and Number:	MATH 235 A
Course Title:	Calculus and Analytic Geometry, IIA
Semester Credit Hours:	3
Class Day and Time:	Mon Wed 5:00 PM-7:30 PM
Instructor:	**Fred Browning** *Instructor*
	Home Phone: (573) 265-1444 **CougarMail:** frbrowning@cougars.ccis.edu

Catalog Description

The third course in a four part Calculus sequence. Topics include: techniques of integration, improper integrals, sequences and series. Prerequisite: MATH 226 with a score of C or higher.

Prerequisites/Corequisites

MATH 226 with a score of C or higher.

Assessment

Material from this course may be tested on the Major Field Test (MFT) administered during the Culminating Experience course for the degree.

Text

Calculus with Analytic Geometry Eighth edition
Author: Larson, Hostetler, and Edwards (Houghton Mifflin Company)
ISBN: 0-618-50298-X

Course Objectives

· To use calculus to formulate and solve problems and communicate solutions to others.
· To use technology as an integral part of the process of formulation, solution, and communication.
· To understand calculus from numerical, graphical, symbolic, and analytical perspectives.
· To understand and appreciate the connections between mathematics and other disciplines.

Additional Instructor Objectives

The Catalog Description and the Measurable Learning Outcomes are Incorrect for the block of material covered by this course.
The student will be able to:
1) Find limits of sequences and series and calculate power series representations of functions;
2) Represent curves, including conic sections, with parametric equations;
3) Calculate arc length and area using rectangular and polar coordinates;
4) Represent curves and surfaces in space using rectangular, cylindrical, and spherical coordinates;
5) Perform basic operations with vectors; and,
6) Represent curves, surfaces, and their properties in space with vectors and vector-valued functions.

Measurable Learning Outcomes

· Use the Substitution Rule and the Integration by Parts formula to evaluate trigonometric and rational functions.
· Explain the special method required to integrate trigonometric and rational functions.
· Apply numerical methods of integration such as Simpson's Rule and the Trapezoidal Rule to approximate definite integrals.
· Classify improper integrals and distinguish between convergent and divergent improper integrals.
· Explore geometric applications of integration, such as the length, the area of a surface, as well as their applications to physics, engineering, economics, and biology.
· Apply various tests for convergence to distinguish between absolutely and conditionally convergent and divergent numeric series.
· Find the radius and interval of convergence of power series.
· Find Taylor and Maclaurin series for certain classes of functions.
· Explore applications of Taylor series and polynomials to approximate functions and definite integrals, to evaluate limits, and solve initial value problems.

Instructional Methods

In-class activities will consist primarily of lectures. The concepts will be presented and a large number of problems will be solved. I teach by example. I use the problem solutions to illustrate concepts and to develop procedures and methods needed for effective problem solving.

Graded Activities

Chapter Tests - 5 Total 500 Points
Description - Chapter tests will be given upon completion of each of the text chapters. The tests will consist of problems. The tests will be either open book or students will be allowed to use a formula sheet. Make-up tests will be given if the instructor or site director feels that the absence was justified.

Quizzes - 2 to 4 Total 50 to 100 Points
Description - Quizzes will be given to encourage students to come to class and to stay current on problem assignments. Unannounced quizzes may be given at any time - especially when attendance is low. Quizzes will be open book. There will be no make-up assignments for quizzes.

Homework 25 to 50 Points
Description - A few homework assignments will be collected and graded. Homework will not be collected unannounced. Problems to be turned in will be given a due date. Homework must be submitted when it is due.

Grading Scale

90 - 100	**A**
80 - 89	**B**
70 - 79	**C**
60 - 69	**D**
0 - 59	**F**

Additional Information / Instructions

Grade Determination:
At the end of the course, the sum of points earned from tests, quizzes, and homework will be divided by the total number of possible points in order to calculate numerical averages for each student. In order to determine final letter grades for the course, the numerical averages will be rounded up if the score is within one-half of one percent of the next higher grade. For example, 89.0 to 89.4% is a "B" and 89.5 to 89.9% is an "A".

Calculator Policy:
A graphing calculator is required for any of the calculus courses. Graphing calculators that will perform symbolic integration and differentiation can not be used during tests and quizzes. During

tests and quizzes, TI-84 or lower number calculators are acceptable.

Prerequisites:
The prerequisite for the course is Math 226. Prerequisites may be met by completing two semesters of calculus with grades of C or better. (Math 008 and Math 021 or Math 014 and Math 015 at UMR.)

Transfer Policy:
Math 215 and Math 226 are transferable to UMR in place of Math 008 or Math 014.
Math 226 and Math 235 are transferable to UMR in place of Math 021 or Math 015.
Math 235 and Math 245 are transferable to UMR in place of Math 022.

Weekly Activities and Assignments

Week 1
Infinite Series

Activities: Survey of the methods used to determine convergence and divergence of infinite series.
Reading: Chapter 9, Sections 1 through 6.
Assignments: Problem Assignments:
Section 9.1 1-19 odds, 25-93 odds.
Section 9.2 1-27 odds, 33-71 odds.
Section 9.3 1-17 odds, 21-41 odds, 61-71 odds, 79-89 odds.
Section 9.4 3-43 odds.
Section 9.5 11-61 odds.
Section 9.6 13-67 odds.

Week 2
Infinite Series

Activities: Continue with the study of infinite series.
Power series, Taylor and Maclaurin Series, and the representation of functions by infinite series.
Reading: Chapter 9, Sections 7 through 10.
Assignments: Problem Assinments:
Section 9.7 1-7 odds, 13-29 odds, 35, 41-51 odds.
Section 9.8 1-47 odds, 49-56 all.
Section 9.9 1-27 odds, 31-34 all, 35-43 odds, 53-57 odds.
Section 9.10 1-59 odds, skip 11, 13, and 35.
Examinations: Chapter 9, Sections 1 through 6. B

Week 3
Conic Sections and Parametric Equations

Activities: Conic sections including rotation of axes.
Representation of equations in parametric form.
Reading: Chapter 10, Sections 1,2, and 3.
Appendix E
Assignments: Problem Assignments:
Section 10.1 1-8 all, 9-63 odds, 67-75 odds.
Appendix E 1-31 odds.
Section 10.2 1-53 odds.
Section 10.3 1-55 odds, 63-71 odds.
Examinations: Chapter 9, Sections 7 through 10, Chapter 10, Section 1, and Appendix E. C

Week 4
Polar Coordinates

Activities: Introduction to polar coordinates and polar graphs.
Area and arc length in polar coordinates.
Polar equations of conics.
Reading: Chapter 10, Sections 4, 5, and 6.
Assignments: Problem Assignments:
Section 10.4 1-91 odds
Section 10.5 1-39 odds, 45-59 odds.

Section 10.6 1, 7-43 odds.

Week 5
Vectors and the Geometry of Space
Activities: Introduction to vectors in a plane and in space.
Basic vector operations including dot product and cross product.
Reading: Chapter 11, Sections 1 through 4.
Assignments: Problem Assignments:
Section 11.1 1-75 odds.
Section 11.2 1-87 odds.
Section 11.3 1-49 odds.
Section 11.4 1-15 odds, 21, 23, 27-47 odds.
Examinations: Chapter 10, Sections 2 through 6. C ?

Week 6
Vectors and the Geometry of Space
Activities: Lines, planes, and surfaces in space.
Introduction to cylindrical and spherical coordinates.
Reading: Chapter 11, Sections 5, 6, and 7.
Assignments: Problem Assignments:
Section 11.5 1-99 odds.
Section 11.6 1-29 odds, 41-51 odds.
Section 11.7 1-71 odds, 87-103 odds.

Ch 11.1 – 11.6?

Week 7
Vector-Valued Functions
Activities: Introduction to vector-valued functions.
Differentiation and integration of vector-valued functions.
Velocity and acceleration problems.
Reading: Chapter 12, Sections 1, 2, and 3.
Assignments: Problem Assignments:
Section 12.1 1-37 odds, 45-79 odds.
Section 12.2 1-25 odds, 29-43 odds, 49-67 odds.
Section 12.3 1-27 odds.
Examinations: Chapter 11

Week 8
Vector-Valued Functions
Activities: Continue with vector-valued functions.
Determination of tangent and normal vectors.
Calculation of arc length and curvature.
Reading: Chapter 12, Sections 4 and 5.
Assignments: Problem Assignments:
Section 12.4 5-17 odds, 21-57 odds.
Section 12.5 1-15 odds, skip 7, 21-45 odds.
Examinations: Chapter 12

Library Resources

Columbia College Resources - Online databases are available at http://www.ccis.edu/offices/library/resources.asp. You may access them from off-campus using your eServices login and password when prompted.

Campus Resources - Library Resources: Primary Library Resources available to the Columbia College Rolla Campus includes access as guest patron to the Wilson Library at Missouri University of Science & Technology, access to the Main Post Library at Fort Leonard Wood, and access to Stafford Library on Home Campus. Electronic resources available include Congressionsal Quarterly, EBSCOhost, Lexis-Nexis, Gale Resource Center, Infotrac, JSTOR, ProQuest, Newsbank, and others, all available though our library services on the college's web site. The online databases are available at: http://www.ccis.edu/offices/library/resources.asp, and instructions for their use are available online and as handouts in the Computer Lab. Use your eServices ID and password for off-campus access: if you have not yet activated you eServices account, you will need to do so as soon as possible.

Columbia College
Rolla Campus
2303 N. Bishop Ave./Hwy. 63
P.O. Box 1701
Rolla, MO 65402-1701
(573) 341-3350

Course Syllabus
08 / 11 - Early Fall Session
August - October 2008

Course Prefix and Number:	MATH 226 A
Course Title:	Calculus and Analytic Geometry, 1B
Semester Credit Hours:	3
Class Day and Time:	Mon Wed 5:00 pm-7:30 pm
Instructor:	**Fred Browning** *Instructor* **Home Phone:** (573) 265-1444 **CougarMail:** frbrowning@cougars.ccis.edu

Catalog Description

The second course in a four part calculus sequence. Topics include: the integral and its application, transcendental functions, and integration by parts. Prerequisite: MATH 215 with a score of C or higher. G.E.

Prerequisites/Corequisites

MATH 215 with a score of C or higher.

Assessment

Material from this course may be tested on the Major Field Test (MFT) administered during the Culminating Experience course for the degree.

Text

Calculus with Analytic Geometry Eighth edition
Author: Larson, Hostetler, and Edwards (Houghton Mifflin Company)
ISBN: 0-618-50298-X

Course Objectives

· To use calculus to formulate and solve problems and communicate solutions to others.
· To use technology as an integral part of the process of formulation, solution and communication.
· To understand calculus topics from numerical, graphical, symbolic, and analytical perspectives.
· To understand and appreciate the connections between mathematics and other disciplines.

Additional Instructor Objectives

The Measurable Learning Outcomes do not correspond exactly to the block of material covered by this course. The first three outcomes are covered in Math 215. The remaining outcomes are covered in this course. This course also includes introductory topics in infinite series.
The student will be able to:
1) Calculate derivatives and integrals of logarithmic and exponential functions;
2) Apply integration techniques such as trigonometric substitutions, integration by parts, and partial fractions;
3) Apply L'Hopital's Rule to determine limits of indeterminate forms;
4) Evaluate improper integrals; and,
5) Apply various tests to determine convergence or divergence of sequences and series.

Measurable Learning Outcomes

· Compute definite integrals as the limit of Riemann sums and approximate integrals using finite Riemann sums.
· Evaluate definite integrals using geometric interpretation together with other basic properties of integrals.
· Evaluate definite and indefinite integrals using the Fundamental Theorem of Calculus and the method of substitution.
· Compute areas and volumes using definite integrals.
· Identify the natural exponential and logarithmic functions as inverses of each other and find their derivatives and integrals.
· Solve exponential growth and decay problems arising from biology, physics, chemistry, and other sciences.
· Compute derivatives and integrals of functions containing inverse trigonometric functions.
· Analyze various indeterminate forms and apply L'Hospital's rule to evaluate limits of such forms.
· Use the Substitution Rule and the Integration by Parts formula to evaluate indefinite and definite integrals.

Instructional Methods

In-class activities will consist primarily of lectures. The concepts will be presented and a large number of problems will be solved. I teach by example. I use the problem solutions to illustrate concepts and to develop procedures and methods for effective problem solving.

Graded Activities

Chapter Tests - 4 Total 400 Points
Description - Chapter tests will be given upon completion of each of the text chapters. The tests will consist of problems. The tests will be either open book or students will be allowed to use a formula sheet. Make-up tests will be given if the instructor or site director feels that the absence was justified.

Quizzes - 2 to 4 Total 50 to 100 Points
Description - Quizzes will be given to encourage students to come to class and to stay current on problem assignments. Unannounced quizzes may be given at any time - especially when attendance is low. Quizzes will be open book. There will be no make-up assignments for quizzes.

Homework 25 to 50 Points
Description - A few homework assignments will be collected and graded. Homework will not be collected unannounced. Problems to be turned in will be given a due date. Homework must be submitted when it is due.

Grading Scale

90 - 100	**A**
80 - 89	**B**
70 - 79	**C**
60 - 69	**D**
0 - 59	**F**

Additional Information / Instructions

Grade Determination:
At the end of the course, the sum of points earned from tests, quizzes, and homework will be divided by the total number of possible points in order to calculate numerical averages for each student. In order to determine final letter grades for the course, the numerical averages will be rounded up if the score is within one-half of one percent of the next higher grade. For example, 89.0 to 89.4% is a "B" and 89.5 to 89.9% is an "A".

Calculator Policy:
A graphing calculator is required for any of the calculus courses. Graphing calculators that will

perform symbolic integration and differentiation can not be used during tests and quizzes. During tests and quizzes, TI-84 of lower number calculators are acceptable.

Prerequisites:
The prerequisite for the course is Math 215. The prerequisite may be met by completing one semester of calculus with a grade of C or better. (Math 008 or Math 014 at UMR.)

Transfer Policy:
Math 215 and Math 226 are transferable to UMR in place of Math 008 or Math 014.
Math 226 and Math 235 are transferable to UMR in place of Math 021 or Math 015.
Math 235 and Math 245 are transferable to UMR in place of Math 022.

Weekly Activities and Assignments

Week 1 MW 5-7:30 pm
Logarithmic, Exponential, and Other Transcendental Functions
Activities: Differentiation and integration of logarithmic and exponential functions.
Reading: Chapter 5, Sections 1 through 5.
Assignments: Problem Assignments:
Section 5.1 7-87 odds (skip 35), 93,95,97.
Section 5.2 1-73 odds.
Section 5.3 1-51 odds (skip 45), 59-65 odds, 71-89 odds.
Section 5.4 1-61 odds, 85-115 odds.
Section 5.5 1-73 odds.

Week 2
Logarithmic, Exponential, and Other Transcendental Functions
Activities: Differentiation and integration of inverse trigonometric functions and hyperbolic functions.
Reading: Chapter 5, Sections 6 through 8.
Assignments: Problem Assignments:
Section 5.6 5-27 odds, 31, 33, 41-65 odds.
Section 5.7 1-57 odds, 63, 65, 67.
Section 5.8 1-89 odds (skip 31, 33, 37, 83).
Examinations: Chapter 5, Sections 1 through 8.

Week 3
Applications of Intergation
Activities: Area and volume problems.
Reading: Chapter 7, Sections 1 through 3.
Assignments: Problem Assignments:
Section 7.1 1-51 odds.
Section 7.2 1-37 odds.
Section 7.3 1-37 odds.

Week 4
Applications of Integration
Activities: Determination of arc length, areas of surfaces of revolution, and centroids.
Reading: Chapter 7, Sections 4 and 6.
Assignments: Problem Assignments:
Section 7.4 1-27 odds, 39-45 odds.
Section 7.6 1-27 odds, 43, 45, 49, 51, 57.
Examinations: Chapter 7, Sections 1 through 4 and 6.

Week 5
Integration Techniques
Activities: Basic integration rules, integration by parts, trigonometric integrals, and trigonometric substitutions.
Reading: Chapter 8, Sections 1 through 4.
Assignments: Problem Assignments:
Section 8.1 1-73 odds.
Section 8.2 1-63 odds.
Section 8.3 5-71 odds.

Section 8.4 5-53 odds.

Week 6
Integration Techniques
Activities: Partial fractions, integration tables, indeterminate forms, and improper integrals.
Reading: Chapter 8, Sections 5 through 8.
Assignments: Problem Assignments:
Section 8.5 1-31 odds, 41, 43, 45.
Section 8.6 1-49 odds.
Section 8.7 5-53 odds.
Section 8.8 1-47 odds.

Week 7
Infinite Series
Activities: Survey of the methods used to determine convergence and divergence of infinite series.
Reading: Chapter 9, Sections 1 through 3.
Assignments: Problem Assignments:
Section 9.1 1-19 odds, 25-93 odds.
Section 9.2 1-27 odds, 33-71 odds.
Section 9.3 1-17 odds, 21-41 odds, 61-71 odds, 79-89 odds.
Examinations: Chapter 8, Sections 1 through 8.

Week 8
Infinite Series
Activities: Survey of the methods used to determine convergence and divergence of infinite series.
Reading: Chapter 9, Sections 4 through 6.
Assignments: Problem Assignments:
Section 9.4 3-43 odds.
Section 9.5 11-61 odds.
Section 9.6 13-67 odds.
Examinations: Chapter 9, Sections 1 through 6.

Library Resources

Columbia College Resources - Online databases are available at http://www.ccis.edu/offices/library/resources.asp. You may access them from off-campus using your eServices login and password when prompted.

Campus Resources - Library Resources: Primary Library Resources available to the Columbia College Rolla Campus includes access as guest patron to the Wilson Library at University of Missouri - Rolla, access to the Main Post Library at Fort Leonard Wood, and access to Stafford Library on Home Campus. Electronic resources available include Congressionsal Quarterly, EBSCOhost, Lexis-Nexis, Gale Resource Center, Infotrac, JSTOR, ProQuest, Newsbank, and others, all available though our library services on the college's web site. The online databases are available at: http://www.ccis.edu/offices/library/resources.asp, and instructions for their use are available online and as handouts in the Computer Lab. Use your eServices ID and password for off-campus access: if you have not yet activated you eServices account, you will need to do so as soon as possible.

Course Policies and Procedures

Attendance

Columbia College Policy - Columbia College students are expected to attend all classes and laboratory periods for which they are enrolled. Students are directly responsible to instructors for class attendance and work missed during an absence for any cause. If absences jeopardize progress in a course, an instructor may withdraw a student from the course with a grade of "F" or "W" at the discretion of the instructor.

Campus Policy - Attendance Policy: Columbia College students are expected to attend all classes and laboratory periods for which they are enrolled. Students are directly responsible to instructors for class attendance and for work missed during an absence for any cause. If absences jeopardize progress in a course an instructor may withdraw a student from that course. The

BOOK

From Theorem 11.13, you can determine that the distance between the p $Q(x_0, y_0, z_0)$ and the plane given by $ax + by + cz + d = 0$ is

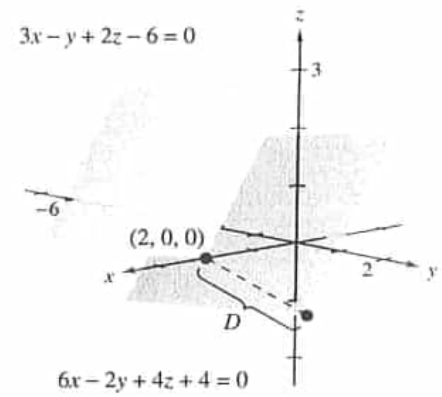
$$D = \frac{|a(x_0 - x_1) + b(y_0 - y_1) + c(z_0 - z_1)|}{\sqrt{a^2 + b^2 + c^2}}$$

or

$$D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$ Distance between a point and a plane

where $P(x_1, y_1, z_1)$ is a point in the plane and $d = -(ax_1 + by_1 + cz_1)$.

EXAMPLE 6 Finding the Distance Between Two Parallel Planes



The distance between the parallel planes is approximately 2.14.
Figure 11.53

Find the distance between the two parallel planes given by

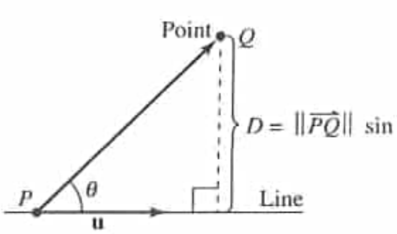
$$3x - y + 2z - 6 = 0 \quad \text{and} \quad 6x - 2y + 4z + 4 = 0.$$

Solution The two planes are shown in Figure 11.53. To find the distance betw the planes, choose a point in the first plane, say $(x_0, y_0, z_0) = (2, 0, 0)$. Then, from second plane, you can determine that $a = 6$, $b = -2$, $c = 4$, and $d = 4$, and conc that the distance is

$$\begin{aligned} D &= \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}} \\ &= \frac{|6(2) + (-2)(0) + (4)(0) + 4|}{\sqrt{6^2 + (-2)^2 + 4^2}} \\ &= \frac{16}{\sqrt{56}} = \frac{8}{\sqrt{14}} \approx 2.14. \end{aligned}$$ Distance between a point and a plane

The formula for the distance between a point and a line in space resembles for the distance between a point and a plane—except that you replace the dot pro with the length of the cross product and the normal vector **n** with a direction ve for the line.

> **THEOREM 11.14 Distance Between a Point and a Line in Space**
>
> The distance between a point Q and a line in space is given by
>
> $$D = \frac{\|\overrightarrow{PQ} \times \mathbf{u}\|}{\|\mathbf{u}\|}$$
>
> where **u** is a direction vector for the line and P is a point on the line.



The distance between a point and a line
Figure 11.54

Proof In Figure 11.54, let D be the distance between the point Q and the gi line. Then $D = \|\overrightarrow{PQ}\| \sin\theta$, where θ is the angle between **u** and $\overrightarrow{PQ}$. By Theo 11.8, you have

$$\|\mathbf{u}\| \|\overrightarrow{PQ}\| \sin\theta = \|\mathbf{u} \times \overrightarrow{PQ}\| = \|\overrightarrow{PQ} \times \mathbf{u}\|.$$

Consequently,

$$D = \|\overrightarrow{PQ}\| \sin\theta = \frac{\|\overrightarrow{PQ} \times \mathbf{u}\|}{\|\mathbf{u}\|}.$$

In Exercises 13–20, find a set of parametric equations of the line.

13. The line passes through the point $(2, 3, 4)$ and is parallel to the xz-plane and the yz-plane.

14. The line passes through the point $(-4, 5, 2)$ and is parallel to the xy-plane and the yz-plane.

15. The line passes through the point $(2, 3, 4)$ and is perpendicular to the plane given by $3x + 2y - z = 6$.

16. The line passes through the point $(-4, 5, 2)$ and is perpendicular to the plane given by $-x + 2y + z = 5$.

17. The line passes through the point $(5, -3, -4)$ and is parallel to $\mathbf{v} = \langle 2, -1, 3\rangle$.

18. The line passes through the point $(-1, 4, -3)$ and is parallel to $\mathbf{v} = 5\mathbf{i} - \mathbf{j}$.

19. The line passes through the point $(2, 1, 2)$ and is parallel to the line $x = -t,\ y = 1 + t,\ z = -2 + t$.

20. The line passes through the point $(-6, 0, 8)$ and is parallel to the line $x = 5 - 2t,\ y = -4 + 2t,\ z = 0$.

In Exercises 21–24, find the coordinates of a point P on the line and a vector v parallel to the line.

21. $x = 3 - t,\quad y = -1 + 2t,\quad z = -2$

22. $x = 4t,\quad y = 5 - t,\quad z = 4 + 3t$

23. $\dfrac{x-7}{4} = \dfrac{y+6}{2} = z + 2$

24. $\dfrac{x+3}{5} = \dfrac{y}{8} = \dfrac{z-3}{6}$

In Exercises 25 and 26, determine if any of the lines are parallel or identical.

9 $13 - 8(-\frac{1}{2})$

25. L_1: $x = 6 - 3t,\quad y = -2 + 2t,\quad z = 5 + 4t$ $\langle -3, 2, 4\rangle \times (-2)$
L_2: $x = 6t,\quad y = 2 - 4t,\quad z = 13 - 8t$ $\langle 6, -4, -8\rangle$
L_3: $x = 10 - 6t,\quad y = 3 + 4t,\quad z = 7 + 8t$
L_4: $x = -4 + 6t,\quad y = 3 + 4t,\quad z = 5 - 6t$

26. L_1: $\dfrac{x-8}{4} = \dfrac{y+5}{-2} = \dfrac{z+9}{3}$
L_2: $\dfrac{x+7}{2} = \dfrac{y-4}{1} = \dfrac{z+6}{5}$
L_3: $\dfrac{x+4}{-8} = \dfrac{y-1}{4} = \dfrac{z+18}{-6}$
L_4: $\dfrac{x-2}{-2} = \dfrac{y+3}{1} = \dfrac{z-4}{1.5}$

In Exercises 27–30, determine whether the lines intersect, and if so, find the point of intersection and the cosine of the angle of intersection.

27. $x = 4t + 2,\quad y = 3,\quad z = -t + 1$
$x = 2s + 2,\quad y = 2s + 3,\quad z = s + 1$

28. $x = -3t + 1, y = 4t + 1, z = 2t + 4$
$x = 3s + 1, y = 2s + 4, z = -s + 1$

29. $\dfrac{x}{3} = \dfrac{y-2}{-1} = z + 1,\quad \dfrac{x-1}{4} = y + 2 = \dfrac{z+3}{-3}$

30. $\dfrac{x-2}{-3} = \dfrac{y-2}{6} = z - 3,\quad \dfrac{x-3}{2} = y + 5 = \dfrac{z+2}{4}$

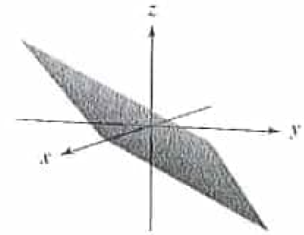
In Exercises 31 and 32, use a computer algebra system to gr[aph] the pair of intersecting lines and find the point of intersectio[n].

31. $x = 2t + 3, y = 5t - 2, z = -t + 1$
$x = -2s + 7, y = s + 8, z = 2s - 1$

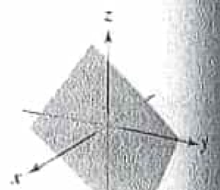
32. $x = 2t - 1, y = -4t + 10, z = t$
$x = -5s - 12, y = 3s + 11, z = -2s - 4$

Cross Product **In Exercises 33 and 34, (a) find the coordinat[es] of three points P, Q, and R in the plane, and determine th[e] vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. (b) Find $\overrightarrow{PQ} \times \overrightarrow{PR}$. What is the relations[hip] between the components of the cross product and the coeffi[-] cients of the equation of the plane? Why is this true?**

33. $4x - 3y - 6z = 6$



34. $2x + 3y + 4z = 4$



In Exercises 35–40, find an equation of the plane pass[ing] through the point perpendicular to the given vector or line.

Point	*Perpendicular to*
35. $(2, 1, 2)$	$\mathbf{n} = \mathbf{i}$
36. $(1, 0, -3)$	$\mathbf{n} = \mathbf{k}$
37. $(3, 2, 2)$	$\mathbf{n} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$
38. $(0, 0, 0)$	$\mathbf{n} = -3\mathbf{i} + 2\mathbf{k}$
39. $(0, 0, 6)$	$x = 1 - t, y = 2 + t, z = 4 - 2t$
40. $(3, 2, 2)$	$\dfrac{x-1}{4} = y + 2 = \dfrac{z+3}{-3}$

In Exercises 41–52, find an equation of the plane.

41. The plane passes through $(0, 0, 0)$, $(1, 2, 3)$, and $(-2, 3, 3)$.

42. The plane passes through $(2, 3, -2)$, $(3, 4, 2)$, and $(1, -1, 0)$.

43. The plane passes through $(1, 2, 3)$, $(3, 2, 1)$, and $(-1, -2, 2)$.

44. The plane passes through the point $(1, 2, 3)$ and is parallel to the yz-plane.

45. The plane passes through the point $(1, 2, 3)$ and is parallel to the xy-plane.

46. The plane contains the y-axis and makes an angle of $\pi/6$ with the positive x-axis.

Th' 'ane contains the lines given by

$$\frac{x - \,}{-2} = y - 4 = z \quad \text{and} \quad \frac{x - 2}{-3} = \frac{y - 1}{4} = \frac{z - 2}{-1}.$$

The plane passes through the point (2, 2, 1) and contains the line given by

$$\frac{x}{2} = \frac{y - 4}{-1} = z.$$

The plane passes through the points (2, 2, 1) and (−1, 1, −1) and is perpendicular to the plane $2x - 3y + z = 3$.

The plane passes through the points (3, 2, 1) and (3, 1, −5) and is perpendicular to the plane $6x + 7y + 2z = 10$.

The plane passes through the points (1, −2, −1) and (2, 5, 6) and is parallel to the x-axis.

The plane passes through the points (4, 2, 1) and (−3, 5, 7) and is parallel to the z-axis.

Exercises 53 and 54, sketch a graph of the line and find the ...nts (if any) where the line intersects the xy-, xz-, and yz-planes.

$x = 1 - 2t, \quad y = -2 + 3t, \quad z = -4 + t$

$\frac{x - 2}{3} = y + 1 = \frac{z - 3}{2}$

Exercises 55 and 56, find an equation of the plane that ...tai... all the points that are equidistant from the given points.

(2, 2, 0), (0, 2, 2)

(−3, 1, 2), (6, −2, 4)

Exercises 57–62, determine whether the planes are parallel, ...hogonal, or neither. If they are neither parallel nor ...hogonal, find the angle of intersection.

$5x - 3y + z = 4$
$x + 4y + 7z = 1$

58. $3x + y - 4z = 3$
$-9x - 3y + 12z = 4$

$x - 3y + 6z = 4$
$5x + y - z = 4$

60. $3x + 2y - z = 7$
$x - 4y + 2z = 0$

$x - 5y - z = 1$
$5x - 25y - 5z = -3$

62. $2x - z = 1$
$4x + y + 8z = 10$

...xercises 63–70, label any intercepts and sketch a graph of ...plane.

$4x + 2y + 6z = 12$

$3x + 6y + 2z = 6$

$2x - y + 3z = 4$

$2x - y + z = 4$

$y + z = 5$

$x + 2y = 4$

$x = 5$

In Exercises 71–74, use a computer algebra system to graph the plane.

71. $2x + y - z = 6$

72. $x - 3z = 3$

73. $-5x + 4y - 6z = -8$

74. $2.1x - 4.7y - z = -3$

In Exercises 75 and 76, determine if any of the planes are parallel or identical.

75. P_1: $3x - 2y + 5z = 10$
P_2: $-6x + 4y - 10z = 5$
P_3: $-3x + 2y + 5z = 8$
P_4: $75x - 50y + 125z = 250$

76. P_1: $-60x + 90y + 30z = 27$
P_2: $6x - 9y - 3z = 2$
P_3: $-20x + 30y + 10z = 9$
P_4: $12x - 18y + 6z = 5$

In Exercises 77–80, describe the family of planes represented by the equation, where c is any real number.

77. $x + y + z = c$

78. $x + y = c$

79. $cy + z = 0$

80. $x + cz = 0$

In Exercises 81 and 82, find a set of parametric equations for the line of intersection of the planes.

81. $3x + 2y - z = 7$
$x - 4y + 2z = 0$

82. $6x - 3y + z = 5$
$-x + y + 5z = 5$

In Exercises 83–86, find the point(s) of intersection (if any) of the plane and the line. Also determine whether the line lies in the plane.

83. $2x - 2y + z = 12, \quad x - \frac{1}{2} = \frac{y + (3/2)}{-1} = \frac{z + 1}{2}$

84. $2x + 3y = -5, \quad \frac{x - 1}{4} = \frac{y}{2} = \frac{z - 3}{6}$

85. $2x + 3y = 10, \quad \frac{x - 1}{3} = \frac{y + 1}{-2} = z - 3$

86. $5x + 3y = 17, \quad \frac{x - 4}{2} = \frac{y + 1}{-3} = \frac{z + 2}{5}$

In Exercises 87–90, find the distance between the point and the plane.

87. (0, 0, 0)
$2x + 3y + z = 12$

88. (0, 0, 0)
$8x - 4y + z = 8$

89. (2, 8, 4)
$2x + y + z = 5$

90. (3, 2, 1)
$x - y + 2z = 4$

p646

Summary of Tests for Series

Test	Series	Condition(s) of Convergence	Condition(s) of Divergence	Comment
nth-Term	$\sum_{n=1}^{\infty} a_n$		$\lim_{n\to\infty} a_n \neq 0$	This test cannot be used to show convergence.
Geometric Series	$\sum_{n=0}^{\infty} ar^n$	$\lvert r\rvert < 1$	$\lvert r\rvert \geq 1$	Sum: $S = \dfrac{a}{1-r}$
Telescoping Series	$\sum_{n=1}^{\infty} (b_n - b_{n+1})$	$\lim_{n\to\infty} b_n = L$		Sum: $S = b_1 - L$
p-Series	$\sum_{n=1}^{\infty} \dfrac{1}{n^p}$	$p > 1$	$p \leq 1$	
Alternating Series	$\sum_{n=1}^{\infty} (-1)^{n-1} a_n$	$0 < a_{n+1} \leq a_n$ and $\lim_{n\to\infty} a_n = 0$		Remainder: $\lvert R_N\rvert \leq a_{N+1}$
Integral (f is continuous, positive, and decreasing)	$\sum_{n=1}^{\infty} a_n$, $a_n = f(n) \geq 0$	$\int_1^{\infty} f(x)\,dx$ converges	$\int_1^{\infty} f(x)\,dx$ diverges	Remainder: $0 < R_N < \int_N^{\infty} f(x)\,dx$
Root	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} < 1$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} > 1$	Test is inconclusive if $\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = 1$.
Ratio	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \left\lvert \dfrac{a_{n+1}}{a_n} \right\rvert < 1$	$\lim_{n\to\infty} \left\lvert \dfrac{a_{n+1}}{a_n} \right\rvert > 1$	Test is inconclusive if $\lim_{n\to\infty} \left\lvert \dfrac{a_{n+1}}{a_n} \right\rvert = 1$.
Direct Comparison ($a_n, b_n > 0$)	$\sum_{n=1}^{\infty} a_n$	$0 < a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ converges	$0 < b_n \leq a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges	
Limit Comparison ($a_n, b_n > 0$)	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \dfrac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ converges	$\lim_{n\to\infty} \dfrac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ diverges	

Σa_n is absolutely convergent if $\Sigma |a_n|$ converges

$x^n \to x^n \ln x$

CALCI

Things We All Should Know From Calc1

put a unit on everything

1. Domains and ranges of functions.
2. Domain of f+g, f-g, f*g, f/g $x \in$ f & g, f & g, f & g, f & g such that $g(x) \neq 0$
3. Domain of f(g(x))= $\{x \text{ in } D_g | g(x) \text{ is in } D_f\}$
4. Limits
5. Limit Laws
6. Continuity $\lim_{x \to a} f(x) = f(a)$ AND $\lim_{x \to a} f(x)$ exists AND $f(a)$ exists.

CONTINUOUS ON DOMAIN
polynomials, root funcs, trig funcs, rational funcs

7. The derivative $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $\lim_{a \to b} \frac{f(a) - f(b)}{a-b}$ IF it exists
8. Differentiation formulas.
9. Derivatives of trigonometric functions.
10. The product rule. $\frac{d}{dx} f(x) \cdot g(x) = f(x) \frac{d}{dx} g(x) - g(x) \frac{d}{dx} f(x)$
11. The quotient rule. $\frac{b\,d\,top - top\,d\,bottom}{bottom^2}$
12. THE CHAIN RULE $[f(g(x))]' = f'(g(x)) \cdot g'(x)$ $\frac{dy}{dx}$

A FUNC IS DIFFERENTIABLE IF Lim from (-) EQUALS LIM FROM (+)

line eq. = $y - (y_1) = m(x - x_1)$

$\frac{d}{dx}$
$\sin x = \cos x$ $\sec x = \tan x \sec x$
$\cos x = -\sin x$ $\csc x = -\cot x \csc x$
$\tan x = \sec^2 x$ $\cot x = -\csc^2 x$

13. Implicit differentiation.
14. Differentiation in terms of time.
15. Limits at infinity. multiply by $\frac{1}{x}$ $\frac{1}{\sqrt{x^2}}$
16. Optimization problems.
17. Antiderivatives.

Rolle's: $c \in (a,b)$ s.t. $f'(c) = 0$ if $f(a) = f(b)$ & continuous & diff $[a,b]$ (a,b)

18. The integral $\lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x = \int_a^b f(x) dx$
19. $\int_{t_1}^{t_2} v(t) dt$ versus $\int_{t_1}^{t_2} |v(t)| dt$ how are these different.

net ΔA = Rieman Sum
top - bottom vs total area

20. The fundamental theorem pts 1 and 2

$$\frac{d}{dx} \int_a^x f(t) dt = f(x) \quad \text{and} \quad \int_a^b f(x) dx = F(b) - F(a)$$

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$

21. The indefinite integral. (+C!)
22. Integration with substitution. – plug u into integral limits
23. Area between two curves. (High minus low)
24. Volumes by discs and washers. discs = $\pi r^2 \Delta t$
25. 1 by Cylinders. cylinders $2\pi r h \Delta r$
26. You're a piece of work.
27. One just like #19 in 6.5

avg density = $\frac{\int_a^b f(x) dx}{(b-a)}$

assum./Want set $\varepsilon > 0$
$0 < |x - a| < \delta$
$|f(x) - L| \leq \varepsilon$

$W = F \cdot d$ $\boxed{W_{a \to b} = \int_a^b f(x) dx}$
$J = Nm$
$1N =$

Midpoint:
$\int_a^b f(x) dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(\bar{x}_i) \Delta x$
$\bar{x}_i = \frac{x_{i-1} + x_i}{2}$

$\frac{d}{dx}(g(x)) = \frac{d}{dx} \int_a^x f(t) dt = f(x)$

Homework MTH210

1.1= 1-15odd,19, 21-27all, 27-57odd,61-67odd.
1.2= 1-19odd,20, 21, 25.
1.3=1-25odd,29-47odd,51-57odd,59,63.
1.4=1,19,21,23.
2.1=1-9odd
2.2=1-17odd,21,25,27,37,40.
2.3=1(a,d,f),3-29odd,33,37,39,43,45,47,60.
2.4=1,2,3,11,19,21,23.
2.5=1-9odd,10,11,13,14,15-37odd,40,41,47,49,53,64(ab)
3.1=1-45odd,46.
3.2=1-27odd,31,33,35,39,41,48,49.
3.3=1-21odd,22,23-41odd,49,51,53,55,57,58,61,73,75,84.
3.4=1-15odd,19,21-29odd,33,35,37,39,41,45,50.
3.5=1-43odd,53,57,61,62,70,75,77,79.
3.6=1-31odd,39,42,45,53.
3.7=1,5-15odd,18,19,21,29.
3.8=1-19odd,20,23,25,27,31,39.
4.1=1,2,3,5,9,11,13,15-21odd,25-41odd,45,47,51,53,55,64,65,66.
4.2=1-15odd,19,23,31.
4.3=1-21odd,27,29-45odd,49,51,53,59,64.
4.4=1-29odd,35,43,45,49,51,60.
4.5=1-29eeo,33,35,39,41,43,47,49.
4.7=1-25odd,27,29,30,31-37odd. (more later)

4.9=1-43odd,47,51-57odd,61,63,64.

5.1=1,3,5,11,13,17,18,19,22.

5.2=1-9odd,17-29odd,33,37,41,,47,49,53,55.

5.3=1-35odd, 45, 47,49,51,53(a,b),57.

7/14/08 5.4=1-41odd,45,46,47-59odd,63.

5.5=1-29odd,35-49odd,61.

6.1= 1-27 odd

David Malaway

Test#1 MTH-210

1.1-3.1

Show Your Work

6-12-08

1. State the domain of the following in interval notation and state or show why.

a. $f(x)=1+\frac{1}{2}\left(x+\sqrt[10]{18}\right)^{47}$ $\left(x+\sqrt[10]{18}\right)^{47}$ x

D_f: $\mathbb{R}$ because its a ~~sum and a product of~~ polynomials

rational func, product of sum of polynomials

5

b. $f(x)=\frac{x}{\sqrt{3-x}}$ 7-23

D_f: ~~$\mathbb{R}$ because~~ $(-\infty, 3)$ because it is a quotient & radical of polynomials and $D: x > 3$ is undefined

5

2. Let $f(x)=\sqrt{4-7x}$. Determine each of the following algebraically. [not with a graph]

a. $f(x+2) = \sqrt{4-(7(x+2))}$

$= \sqrt{4-7x-14} = \sqrt{-7x-10}$

5

b. $f(x^2) = \sqrt{4-7x^2}$

5

c. $[f(x)]^2 = \left(\sqrt{4-7x}\right)^2 = 4-7x$

5

27.5
28
13.5
11.5
3.5

$\frac{84}{95} = 88\%$ B

3. Classify the following function. (what kind of function is it?)

$f(x)=\frac{x^2+e}{x-\pi}$ quotient of polynomials = rational

2.5

27.5

4. Determine algebraically whether the function $f(x)=\sqrt{1-x^2}$ is even, odd, or neither. $f(-x)=\sqrt{1-x^2}=f(x)$, $\therefore$ it is even

$-f(x)=(-)\sqrt{1-x^2}$

5

even $f(x)=f(-x)$

5. A rectangle has perimeter, 24m. Express the area A of the rectangle as a function of l, the length of one of its sides. 4

24 m $A(l)=\boxed{l(12-l)}=\boxed{-l^2+12l}$ $-16+12(4)=$

5 $24-2l=2l_2$ $\left(\frac{24-2l}{2}\right)l_1$ oops $\boxed{A(l)=-l^2+12l}$

$2l_2\div l=l_2$, $l_2\times l_1=A$

6. Find $f\circ g$ if $f(x)=\sqrt{3-x^2}$ and $g(x)=x+1$.

$f\circ g=\sqrt{3-(x+1)^2}=\sqrt{3-(x^2+2x+2)}=\boxed{\sqrt{-x^2-2x+1}}$

3.5 1

$\cancel{\sqrt{-(x^2+2x-1)}}$

7. Express $G(x)=\sqrt{\tan x+1}$ in the form $f\circ g$.

$g(x)=\tan x$

$f(x)=\sqrt{x+1}$

5

8. The point $P(12,4)$ lies on the curve $f(x)=\sqrt{x+4}$.

a. If Q is the point $(x,\sqrt{x+4})$, find the slope of the secant line through PQ (correct to 6 decimal places) for each value of x,

i. x=11.99 $y=3.998749805$

$\frac{y_2-y_1}{x_2-x_1}=\frac{-0.001250195}{-0.01}$

5

$M_{PQ}=0.1250195$

ii. x=12.01 $y=4.001249805$

$\frac{y_2-y_1}{x_2-x_1}=\frac{0.00124980}{0.01}$

$M_{PQ}=0.124980$

b. Use the results from 8a. to predict the value of the slope of the line tangent to $f(x)$ at x=12.

5 0.125 $\frac{1}{8}$ ~~7~~ 24

28

9. Use limit laws to find the limit. (Justify each step)

$\lim_{x\to 2}[7x(4x-7)]$ $= \lim_{x\to 2} 7x \left(\lim_{x\to 2}(4x-7)\right) = \lim_{x\to 2} 7x \cdot \left(\lim_{x\to 2} 4x - \lim_{x\to 2} -7\right)$

product of limits — distributive property

$= 14(8-(-7))$ $= 14 \cdot 15 = \boxed{210}$

limit of polynomials (continuous)

$x \to a = f(a)$

(1)

3

10. Find the value of the limit.

$\lim_{x\to 2}\frac{\frac{1}{x}-\frac{1}{5}}{x-5}$ $= \lim_{x\to 2}\frac{\frac{5-x}{5x}}{x-5} = \lim_{x\to 2}\frac{5-x}{5x(x-5)} = \lim_{x\to 2}\frac{(5-x)}{-5x(5-x)} = \lim_{x\to 2}\frac{1}{-5x}$

$= \frac{1}{-5(2)} = -\frac{1}{10}$

5

11. Find the value of the limit.

$\lim_{x\to\frac{\pi}{2}}\frac{\cos x}{\tan x}$ $= \lim_{x\to\frac{\pi}{2}}\frac{\cos x}{\sin x/\cos x} = \lim_{x\to\frac{\pi}{2}} \sin x = \frac{1}{1} = 1$

$= \frac{\sin}{\cos}$

$\frac{\frac{\sqrt{2}}{2}}{\frac{1}{\sqrt{2}}} = \frac{\sqrt{2}}{2}\sqrt{2} = \frac{2}{2} = 1$

#7-24

2

12. Find the value of the limit.

$\lim_{h\to 0}\frac{(3+h)^2-9}{h}$ $= \lim_{h\to 0}\frac{\cancel{9}+6h+h^2-\cancel{9}}{h} = \lim_{h\to 0}\frac{h^2-6h}{h} = \lim_{h\to 0} h-6$

$= -6$

+

3.5

13.5

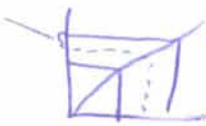
13. Use the strict definition of a limit to show that $\lim_{x\to 2} 4x = 8$. (be mindful to do all of the steps)

Let $\varepsilon > 0$ Assume $0 < |x-2| < \delta$ $|f(x) - L| < \delta$ $|f(x) - L| < \varepsilon$

$|4x - 8| < \delta$? → so $4|x-2| < 4\delta$ $4\delta = \varepsilon$

$4x - 8 = 4(x-2)$ $\delta = \frac{1}{4}\varepsilon$

ε



2.5 $|4x-8| < \varepsilon$

14. **Use** the definition of continuity to **explain** why this function is

a. continuous at x=3

if $f(a) = \lim_{x\to a} f(x)$

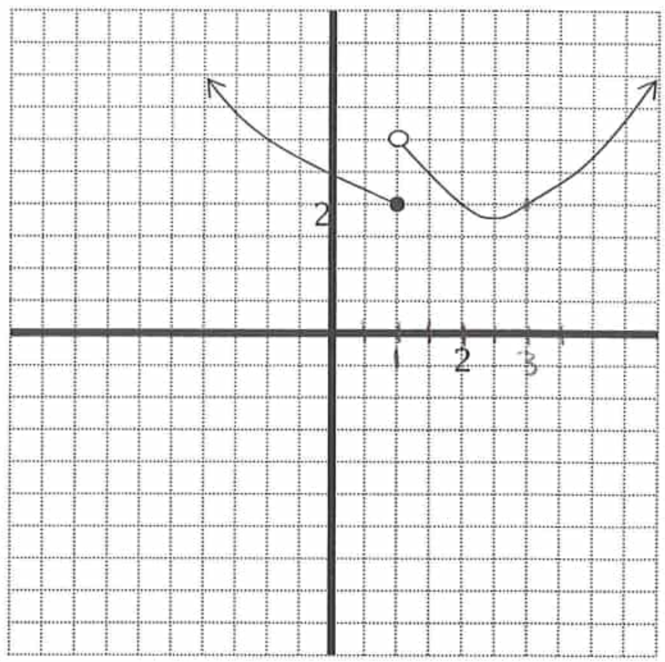
then it is continuous.

2.5 $f(a) = ?$ $\lim_{x\to a} f(x) = ?$

b. discontinuous at x=1

if $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x) = ?$

then it is discontinuous.



15. Given $f(x) = \begin{cases} x^3 + 15 & x \le -3 \\ x^2 - c & x > -3 \end{cases}$, determine the value of c so that f is continuous everywhere.

must be:

$x^2 - c = 8$ @ $x = 3$, so $9 - c = \cancel{24}$ -12 (42) $-c = \cancel{15}$ 33 -21 $c = \cancel{-15}$ -33 21

2.5

$(3^3 + 15)$

$= (27 + 15) = -12$

$= \cancel{24}$ (42)

11.5

16. Find the slope of the line tangent to the function $f(x) = 3x^2 - 1$ at the point $(2,11)$, and also write the equation of that line in $y = mx + b$ form.

$y - f(a) = f'(a)(x-a)$

$m = f'(a)$

check
f the pt. $f(2) = 3(4) - 1 = 11$
is in the func.

$0 = 3x^2 - 1 \qquad -\frac{1}{3} = x^2 \qquad x = \sqrt{-\frac{1}{3}}$

a a+h

$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

$m = \lim_{h \to 0} \frac{3(a+h)^2 - 1 - (3a^2 - 1)}{h}$

$= \frac{3(a^2 + 2ah + h^2) - 1 - 3a^2 + 1}{h}$

$= 3a^2 + 6ha + 3h^2 - 1 + 1 - 3a^2$

$\lim$

$\frac{f(x+h) - f(x)}{h}$

$\lim_{h \to 0} \frac{6ha + 3h^2}{h}$ or $\lim_{h \to 0} \frac{6(h2) + 3h^2}{h}$

$= \lim_{h \to 0} \frac{12h + 3h^2}{h} = \lim_{h \to 0} 3h + 12 = -12$?

$m = -12$

$y -$

$y = -12(x) + b$

&

$y = 3(2)^2 - 1 = 12(x-2)$

$y - 3(4) - 1 = 12(x-2)$

$y - (11) = 12x - 24$

$y = 12x - 24 + 11$

$y = 12x - 13$

$m = +12$?

3.5

KEY

<u>Test#1 MTH-210</u>

1.1-3.1

Show Your Work

1. State the domain of the following in interval notation <u>and</u> state or show why.

a. $f(x)=1+\frac{1}{2}\left(x+\sqrt[10]{18}\right)^{47}$

D_f : $\mathbb{R}$ (polynomial)

b. $f(x)=\frac{x}{\sqrt{3-x}}$ $\quad 3-x>0 \quad x<3$

D_f : $(-\infty, 3)$

2. Let $f(x)=\sqrt{4-7x}$. Determine each of the following algebraically. [not with a graph]

a. $f(x+2) = \sqrt{4-7(x+2)}$

$= \sqrt{4-7x-14} = \boxed{\sqrt{-10-7x}}$

b. $f(x^2) = \boxed{\sqrt{4-7x^2}}$

c. $[f(x)]^2 = \left(\sqrt{4-7x}\right)^2 = \boxed{4-7x}$

3. Classify the following function. (what kind of function is it?)

$f(x)=\frac{x^2+e}{x-\pi}$

Rational

4. Determine algebraically whether the function $f(x)=\sqrt{1-x^2}$ is even, odd, or neither.

$f(-x)=\sqrt{1-(-x)^2}=\sqrt{1-x^2}=f(x)$

$f(-x)=f(x)$ Even

5. A rectangle has perimeter, 24m. Express the area A of the rectangle as a function of l, the length of one of its sides.

L, W

$2L+2W=P$

$2L+2W=24$

$\Rightarrow W=\frac{24-2L}{2}=12-L$

$A=L\cdot W \Rightarrow$

$A(L)=L(12-L)$

$\boxed{A(L)=12L-L^2}$

6. Find $f\circ g$ if $f(x)=\sqrt{3-x^2}$ and $g(x)=x+1$.

$f(g(x))=f(x+1)=\sqrt{3-(x+1)^2}$

$=\sqrt{3-x^2-2x-1}=\sqrt{-x^2-2x+2}$

7. Express $G(x)=\sqrt{\tan x+1}$ in the form $f\circ g$.

$f(x)=\sqrt{x}$

$g(x)=\tan x+1$

8. The point $P(12,4)$ lies on the curve $f(x)=\sqrt{x+4}$.

a. If Q is the point $(x,\sqrt{x+4})$, find the slope of the secant line through PQ (correct to 6 decimal places) for each value of x.

i. $x=11.99$

ii. $x=12.01$

$m_{PQ}=\frac{\sqrt{11.99+4}-4}{11.99-12}=\frac{-0.00125019\text{5}}{-0.01}=.125019$

$m_{PQ}=\frac{\sqrt{12.01+4}-4}{12.01-12}=\frac{.001249980\text{47}}{.01}=.124990749$

b. Use the results from 8a. to predict the value of the slope of the line tangent to $f(x)$ at $x=12$.

0.125 or $\frac{1}{8}$

16. Find the slope of the line tangent to the function $f(x)=3x^2-1$ at the point $(2,11)$, and also write the equation of that line in $y=mx+b$ form.

$$m=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$$

$$=\lim_{h\to 0}\frac{3(2+h)^2-1-[11]}{h}$$

$$=\lim_{h\to 0}\frac{3[4+4h+h^2]-1-11}{h}$$

$$=\lim_{h\to 0}\frac{12+12h+3h^2-12}{h}=\lim_{h\to 0}\frac{12h+3h^2}{h}$$

$$=\lim_{h\to 0}12+3h=\boxed{12=m}$$

$$y-11=12(x-2)\to y=12x-24+11$$

$$\boxed{y=12x-13}$$

Test 2 prac.

$x^2 + xy + y^2 = 3 \quad (1,1)$

$y' = 2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0 \qquad y - 1 = (-1)(x-1)$

$x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - y \qquad y = -x + 1 + 1 = \boxed{-x+2}$

$\frac{dy}{dx} = \frac{-2x-y}{(2y+x)} = \frac{-3}{3} = -1 = m$

$y'\sin = \cos \qquad y'\cot = -\csc^2$

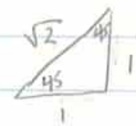
$y'\cos = -\sin \qquad y'\csc = -\csc\cot$

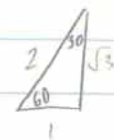
$y'\tan = \sec^2$

$y'\sec = -\sec\tan$

$\sin 45 = \sin\frac{\pi}{4} = \sqrt{2}/2$

$\sin 30 = \sin\frac{\pi}{6} = 0.5$





#1 Use Eq. #2 to find slope of tan line to $y = \frac{x-1}{x-2}$ @ $(3,2)$ $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\lim_{h\to 0}\frac{\frac{3+h-1}{3+h-2} - \frac{x-1}{x-2}}{h} = \frac{\frac{h-2}{h-1} - \frac{2(h-1)}{(h-1)}}{h} = \frac{h-2-2h+2}{h^2+h} = \frac{-h}{h^2+h} = \frac{-1}{h+1} = -1$

$\text{Eq 1}: \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} \qquad \text{Eq 2}: \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

#2 $\lim_{h\to 0}\frac{\cos(\pi+h)+1}{h} = \frac{\cos(x+h)-\cos(x)}{h} \qquad f = \cos x \qquad a = \pi$

#4 Diff. a) $y(t) = 6t^{-9} \quad y'(t) = -54t^{-10}$ b) $y = \frac{r^2}{1+\sqrt{r}} = \left(\frac{r^2}{1+r^{\frac{1}{2}}}\right)$

didn't have to simp. this much. $\to$ $y' = \frac{(1+r^{\frac{1}{2}})(2r) - (\frac{1}{2}r^{-\frac{1}{2}})(r^2)}{(1+\sqrt{r})^2} = \frac{2r + 2r^{\frac{3}{2}} - \frac{1}{2}r^{\frac{3}{2}}}{(1+\sqrt{r})^2} = \frac{2r + \frac{3}{2}r^{3/2}}{(1+\sqrt{r})^2}$

3 find deriv. using definition.

$f(t) = 5t - 9t^2 \quad f'(t) = \lim_{h\to 0}\frac{5(t+h) - 9(t+h)^2 - (5t - 9t^2)}{h} = \lim_{h\to 0}\frac{5t + 5h - 9t^2 - 18th - 9h^2 - 5t + 9t^2}{h}$

$= \frac{5h - 18th - 9h^2}{h} = 5 - 18t - 9h = \boxed{5 - 18t}$

state domain of $f: \mathbb{R}$ & $f': \mathbb{R}$

5 $t^3\cos t$

$3t^2(\cos t) + (-\sin t(t^3)) = 3t^2\cos t - t^3\sin t$

David Malawey

Test#2 MTH-210

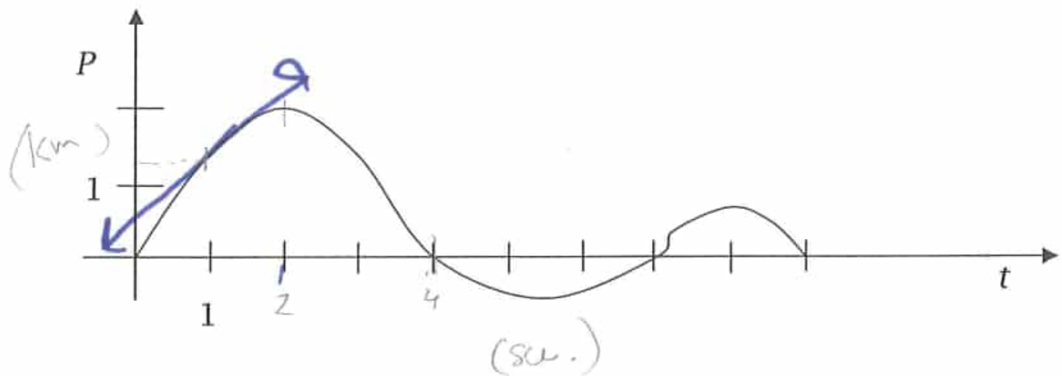
3.1-3.8

Show Your Work

14
41
20.5
18
4

$\frac{97.5}{110} = 89\%$ B

1. The following is a position function $P(t)$ (in km) of an object moving along a line



a. Use the graph to ***estimate*** the velocity at t=1 sec

3

$P'(1) \approx 1 \frac{km}{s}$ $V = \frac{d}{dt} P(t) = \text{slope} = 1$

b. State ***a*** time interval over which the object's velocity is negative.

3

$2 < t < 4$

$(2, 4)$

2. **Use the *definition* of the derivative (!)** to find $f'(x)$ if $f(x) = 4x^2 - 5x + 2$.

$f'(x) = \lim_{h \to 0} \frac{4(x+h)^2 - 5(x+h) + 2 - [4x^2 - 5x + 2]}{h} = \lim_{h \to 0} \frac{4x^2 + 8xh + 4h^2 - 5x - 5h + 2 - 4x^2 + 5x - 2}{h}$

$= \lim_{h \to 0} \frac{8xh + 4h^2 - 5h}{h} = \lim_{h \to 0} 8x + 4h - 5 = \boxed{8x - 5}$

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

5

7-24 $= \lim \frac{4(x+h)^2 - 5(x+h) + 2 - (4x^2 - 5x + 2)}{h}$ simplify

3. Given $f(x) = \sqrt{1-3x}$, $\frac{1}{2}(1-3x)^{-\frac{1}{2}}(-3)$ 7-24

3

a. Find and state the domain of f without using a calculator.

$1 - 3x \geq 0$

$-3x \geq -1$ $D(f): [\frac{1}{3}, +\infty)$

$x \leq \frac{-1}{-3}$

$x \leq \frac{1}{3}$ $(-\infty, \frac{1}{3}]$

$1 - 3x \geq 0$

$1 \geq 3x$

$\frac{1}{3} \geq x$

14

3b. Find the derivative of f. $f(x) = \sqrt{1-3x} = (1-3x)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(1-3x)^{-\frac{1}{2}}(-3) = -\frac{3}{2}(1-3x)^{-\frac{1}{2}}$

3.5

$\frac{3 \cdot 1}{2\sqrt{1-3x}}$ $\frac{3}{2\sqrt{1-3x}}$

3c. Find and state the domain of f' without using a calculator.

$x \geq \frac{1}{3}$ & $x > \frac{1}{3}$ ← b/c $2(0) = 0$

2.5 $1-3x > 0$
$1 > 3x$
$\frac{1}{3} > x$ $x < \frac{1}{3}$

$\therefore D(f') = (\frac{1}{3}, +\infty)$ $(-\infty, \frac{1}{3})$

4. Over what interval is $f(x) = \sqrt{1-3x}$ differentiable?

5 but wrong! $(\frac{1}{3}, +\infty)$ $-\infty, \frac{1}{3}$

5. Over what interval is $f(x) = \sqrt{1-3x}$ continuous?

5 but wrong! $[\frac{1}{3}, +\infty)$

$f'(x) =$
$2\sqrt{1-3x} \neq 0$
$\sqrt{1-3x} \geq 0$
$3x \leq 1$
$x \leq \frac{1}{3}$
$\boxed{(-\infty, \frac{1}{3})}$

7-24

6. If $f(x) = (2x+1)^2(x+2)$, find $f'(3)$.

$f'(x) = 2(2x+1)(2)[x+2] + [1](2x+1)^2$
$= 4(2x+1)(x+2) + (2x+1)(2x+1)$
$= (8x+4)(x+2) + 4x^2 + 4x + 1$
$= 8x^2 + 16x + 4x + 8 + 4x^2 + 4x + 1$

$= 12x^2 + 24x + 9$
$= 3(4x^2 + 8x + 3)$
$f'(3) = 3(4(9) + 24 + 3)$
$= 3(36 + 27)$
$= 189$

7. Differentiate.

a. $f(x) = \dfrac{3\sin x}{\sqrt{x}} = \dfrac{3\sin x}{x^{\frac{1}{2}}} =$

$f'(x) = \dfrac{\sqrt{x}(3\cos x) - (3\sin x)(\frac{1}{2}x^{-\frac{1}{2}})}{x}$

$= \dfrac{3\sqrt{x}\cos x - \frac{3}{2}x^{-\frac{1}{2}}\sin x}{x}$

5

b. $q = -x^2 - 2x$

$q' = -2x - 2 = \boxed{-2(x+1)}$ ✓

c. $v = 3x^5 - 4e^2$

$v' = 15x^4$

5

d. $f(x) = e^{\pi}$

$f' = 0$

4

bottom . d top − top(d bottom)
$\overline{\text{bottom}^2}$

8. Suppose that the amount A, in mg, of a drug left in the body t hours after administration is described by $A(t) = \dfrac{50}{t+2}$.
Use calculus to find the rate of change of the amount of the drug in the body, at t=2hrs.

$A'(t) = \dfrac{(t+2)(0) - (50)(1)}{(t+2)^2} = \dfrac{-50}{t^2+4t+4}$ ✓

$A'(2) = \dfrac{-50}{4+8+4} = \dfrac{-50}{16} = \boxed{-\dfrac{25}{8}}$ mg/hr

9. Differentiate.

a. $f(x) = 3\sec x + 4\cot x$

$f'(x) = 3\sec x \tan x + 4(-\csc^2 x)$
$= 3\sec x \tan x - 4\csc^2 x$

5

b. $y = x \cdot \csc x$

$y' = 1(\csc x) + (-\csc x \cot x)(x)$
$= \csc x - x\csc x \cot x$

5

c. $f(d) = \cos^2 d$ $= (\cos d)^2$

$f'(d) = (-\sin d)^2$

$\cancel{\boxed{-2}}$ $\boxed{-2\sin d \cdot \cos d}$

7-24

all better 7-24

2

d. $f(t) = \tan(\sin t^2)$ $= \tan[(\sin t)^2]$

$f'(t) =$

$\sec^2[(\cos t)^2]$?

7-24

$\boxed{\sec^2(\sin t^2) \cdot (-2\cos t)}$ or $\sec^2/(\cos^2 t)$

$\boxed{\sec^2(\sin t^2) \cdot \cos t^2 \cdot 2t}$

10. Find $\dfrac{dy}{dx}$ if $2x^3 + 2y^3 - 9xy = 0$, and find the equation of the line tangent to this equation at (2,1)

$6x^2 + 6y^2\frac{dy}{dx} - 9\frac{dy}{dx} = 0$

$2x^2 + 2y^2\frac{dy}{dx} - 3\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} - 3\frac{dy}{dx} = -2x^2$

$\dfrac{dy}{dx} = \dfrac{-2x^2}{(2y-3)}$ $\dfrac{-2(4)}{2 \cdot 3} = \dfrac{-8}{-1} = \boxed{8 = m}$

$y - 1 = 8(x-2)$
$y = 8x - 16 + 1$
$\boxed{y = 8x - 15}$

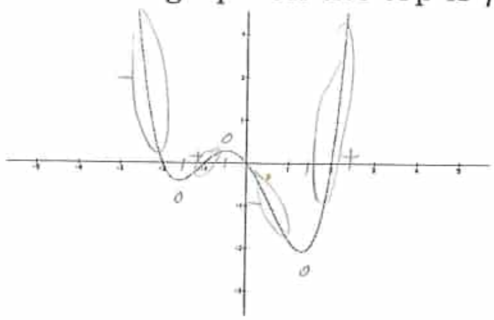
$6x^2 + 6y^2\frac{dy}{dx} - 9x\frac{dy}{dx} = 0$

$2x^2 + 2y^2\frac{dy}{dx}$

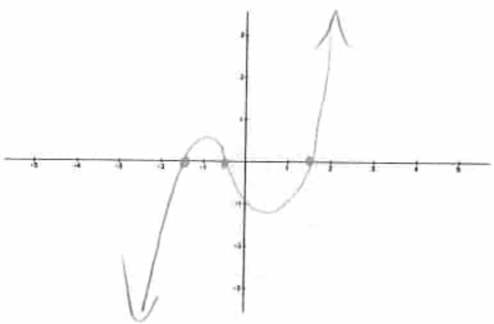
5

20.5

11. If the graph on the top is f, sketch a graph of f' on the bottom.



easy 2-24



12. A particle moves in a straight line with equation of motion of $s = 8t^3 + 6t^2 - 72t$. (s is in meters and t is in seconds)

a. Find $v(t)$ and $a(t)$. $v(t) = s'(t) = 24t^2 + 12t - 72$

$a(t) = v'(t) = 48t + 12$

5

b. What is the displacement of the particle when its acceleration is equal to 0m/s²?

displacement = |pos|

$48t + 12 = 0$

$48t = -12$

$t = \frac{-12}{48} = -\frac{1}{4}$

$s = 8(-\frac{1}{4})^3 + 6(\frac{1}{16}) - 72(-\frac{1}{4})$

$= \frac{-8}{64} + \frac{6}{16} - \frac{72}{4}$

$= \frac{1}{8} + \frac{3}{8} - 18 \quad = \frac{1}{2} - 18 = -17\frac{1}{2}$

c. If you decided to call Mr. Hake " the third derivative of s ", what would you be calling him?

the jerk ~~of s~~ ?

5

18

easy 7-24

13. Boyle's Law states that when a sample of gas is compressed at a constant temperature, the pressure P and Volume V satisfy the equation, $PV = C$, where C is a constant. Suppose that at a certain instant the volume is 400cm^3 and the pressure is 80 kPa, and the pressure is increasing at a rate of , 15kPa/min. At what rate is the volume decreasing at this instant.

$V = 400\,cm^3 \quad P = 80\,kPa + 15\,kPa/min$

$V = \frac{C}{P}$

$V' = \frac{P(C') - C(P')}{P^2} \quad * C' = 0 *$

$80\,kPa\,(400\,cm^3) = C$ or 3200

$V' = \frac{80(0) - \cancel{N}(P')}{80^2}$

$80\,(400) = 3200$ for c

$V' = \frac{-80V(P')}{80^2}$

$95\,(360) = 34200$

$65\,(440) = 28600$

avg = 31400 for c

$V' = -\frac{400\,(P')}{80}$

$= -5(P')$

$= -5\,(15)$

$\boxed{= -40\,cm^3/\text{minute}}$

-75

or rate of decrease = $40\,cm^3/min$

close enough to make sense

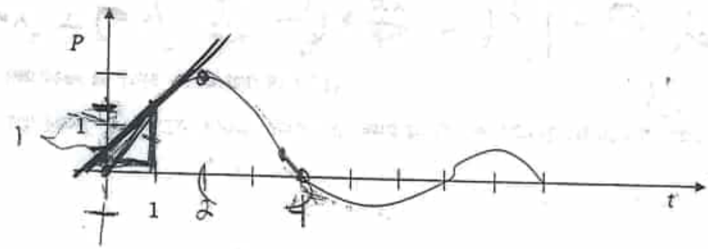
4

Test#2 MTH-210

3.1-3.8

Show Your Work

1. The following is a position function $P(t)$ (in km) of an object moving along a line



a. Use the graph to ***estimate*** the velocity at t=1 sec

$m = v(1) = \frac{1.2km - .2km}{1s - 0} = \frac{1\,km}{1\,sec} \approx \frac{1km}{sec}$

b. State ***a*** time interval over which the object's velocity is negative.

on $(2,4)$

2. **Use the *definition* of the derivative (!)** to find $f'(x)$ if $f(x) = 4x^2 - 5x + 2$.

$f'(x) = \lim_{h\to 0} \frac{4(x+h)^2 - 5(x+h) + 2 - [4x^2 - 5x + 2]}{h} = \lim_{h\to 0} \frac{4x^2 + 8xh + 4h^2 - 5x - 5h + 2 - 4x^2 + 5x - 2}{h}$

$= \lim_{h\to 0} \frac{8xh + 4h^2 - 5h}{h} = \lim_{h\to 0} 8x + 4h - 5 = 8x + 4(0) - 5$

$\boxed{f'(x) = 8x - 5}$

3. Given $f(x) = \sqrt{1-3x}$,

a. Find and state the domain of f without using a calculator.

$1 - 3x \geq 0$

$1 \geq 3x$

$\frac{1}{3} \geq x$

$D_f: (-\infty, \frac{1}{3}]$

3b. Find the derivative of f.

$f'(x)=\frac{1}{2}(1-3x)^{-\frac{1}{2}}(-3)=-\frac{3}{2\sqrt{1-3x}}$

3c. Find and state the domain of f' without using a calculator.

$1-3x>0$ $\quad x<\frac{1}{3}$ $\quad D_{f'}:(-\infty,+\frac{1}{3})$

$1>3x$

4. Over what interval is $f(x)=\sqrt{1-3x}$ differentiable?

$(-\infty,\frac{1}{3})$

5. Over what interval is $f(x)=\sqrt{1-3x}$ continuous?

$(-\infty,\frac{1}{3}]$

6. If $f(x)=(2x+1)^2(x+2)$, find $f'(3)$.

$f'(x)=2(2x+1)^1(2)\cdot(x+2)+1(2x+1)^2$

$f'(3)=2(6+1)(2)\cdot(5)+(6+1)^2=28\cdot5+49=189$

7. Differentiate.

a. $f(x)=\frac{3\sin x}{\sqrt{x}}$

$\boxed{f'(x)=\frac{\sqrt{x}(3\cos x)-3\sin x(\frac{1}{2}x^{-\frac{1}{2}})}{x}}$

b. $q=-x^2-2x$

$\boxed{q'=-2x-2}$

c. $v=3x^5-4e^2$

$v'=15x^4-0$

$\boxed{v'=15x^4}$

d. $f(x)=e^{\pi}$

$\boxed{f'(x)=0}$

13. Boyle's Law states that when a sample of gas is compressed at a constant temperature, the pressure P and Volume V satisfy the equation, $PV = C$, where C is a $\boxed{\text{constant.}}$ Suppose that at a certain instant the volume is 400cm³ and the pressure is 80 kPa, and the pressure is increasing at a rate of , 15kPa/min. At what rate is the volume decreasing at this instant.

$$PV = C$$

$$\frac{dP}{dt} \cdot V + P \cdot \frac{dV}{dt} = 0$$

$$15(400) + 80\left(\frac{dV}{dt}\right) = 0$$

$$80\frac{dV}{dt} = -6000$$

$$\frac{dV}{dt} = -\frac{6000}{80} = \frac{-600}{8} = \frac{-300}{4}$$

$$= (-)75\,cm^3/min$$

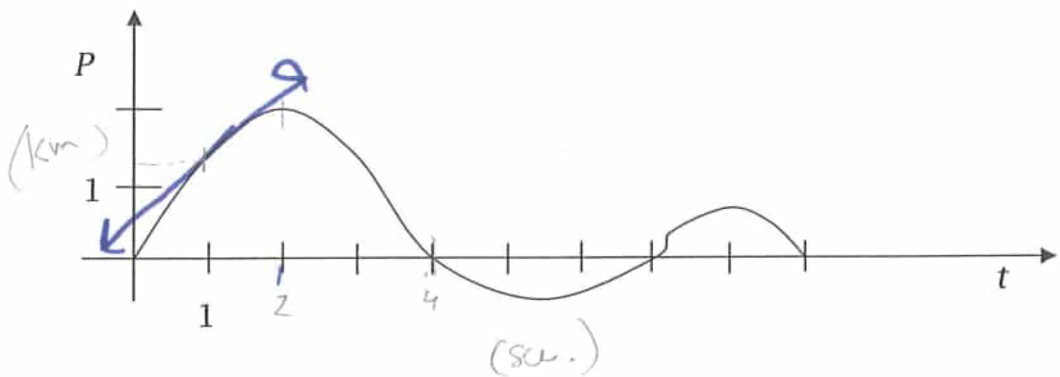
David Malawey

Test#2 MTH-210

3.1-3.8

Show Your Work

1. The following is a position function $P(t)$ (in km) of an object moving along a line



14
41
20.5
18
4

$\frac{97.5}{110} = 89\%$

B

a. Use the graph to ***estimate*** the velocity at t=1 sec

3

$P'(1) \approx 1 \frac{km}{s}$ $V = \frac{d}{dt} P(t) = \text{slope} = 1$

b. State ***a*** time interval over which the object's velocity is negative.

3

$2 < t < 4$

$(2, 4)$

2. **Use the *definition* of the derivative (!)** to find $f'(x)$ if $f(x) = 4x^2 - 5x + 2$.

$f'(x) = \lim_{h \to 0} \frac{4(x+h)^2 - 5(x+h) + 2 - [4x^2 - 5x + 2]}{h} = \lim_{h \to 0} \frac{4x^2 + 8xh + 4h^2 - 5x - 5h + 2 - 4x^2 + 5x - 2}{h}$

$= \lim_{h \to 0} \frac{8xh + 4h^2 - 5h}{h} = \lim_{h \to 0} 8x + 4h - 5 = \boxed{8x - 5}$

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

5

7-24 $= \lim \frac{4(x+h)^2 - 5(x+h) + 2 - (4x^2 - 5x + 2)}{h}$ simplify

3. Given $f(x) = \sqrt{1 - 3x}$, $\frac{1}{2}(1-3x)^{-\frac{1}{2}}(-3)$ 7-24

a. Find and state the domain of f without using a calculator.

3

$1 - 3x \geq 0$
$-3x \geq -1$
$x \leq \frac{-1}{-3}$
$x \leq \frac{1}{3}$

$D(f): [\frac{1}{3}, +\infty)$

$(-\infty, \frac{1}{3}]$

$1 - 3x \geq 0$
$1 \geq 3x$
$\frac{1}{3} \geq x$

14

3b. Find the derivative of f. $f(x)=\sqrt{1-3x} = (1-3x)^{\frac{1}{2}}$

3.5

$f'(x) = \frac{1}{2}(1-3x)^{-\frac{1}{2}}(-3) = -\frac{3}{2}(1-3x)^{-\frac{1}{2}}$

$\frac{3\cdot 1}{2\sqrt{1-3x}}$ $\frac{3}{2\sqrt{1-3x}}$

3c. Find and state the domain of f' without using a calculator.

$x \geq \frac{1}{3}$ & $x > \frac{1}{3}$ ← b/c $2(0)=0$

2.5 $-3x > 0$ $1 > 3x$ $\frac{1}{3} > x$ $x < \frac{1}{3}$ $\therefore D(f') = (\frac{1}{3}, +\infty)$ $(-\infty, \frac{1}{3})$

$f'(x)$: $2\sqrt{1-3x} \neq 0$ $\sqrt{1-3x} \geq 0$ $3x \leq 1$ $x \leq \frac{1}{3}$ $\boxed{(-\infty, \frac{1}{3})}$ 7-24

4. Over what interval is $f(x)=\sqrt{1-3x}$ differentiable?

5 but wrong! $(\frac{1}{3}, +\infty)$ $-\infty, \frac{1}{3}$

5. Over what interval is $f(x)=\sqrt{1-3x}$ continuous?

5 but wrong! $[\frac{1}{3}, +\infty)$

6. If $f(x)=(2x+1)^2(x+2)$, find $f'(3)$.

$f'(x) = 2(2x+1)(2)[x+2] + [1](2x+1)^2$
$= 4(2x+1)(x+2) + (2x+1)(2x+1)$
$= (8x+4)(x+2) + 4x^2 + 4x + 1$
$= 8x^2 + 16x + 4x + 8 + 4x^2 + 4x + 1$
$= 12x^2 + 24x + 9$
$= 3(4x^2 + 8x + 3)$
$f'(3) = 3(4(9) + 24 + 3)$
$= 3(36 + 27)$
$= 189$

7. Differentiate.

a. $f(x)=\frac{3\sin x}{\sqrt{x}}$ $\frac{3\sin x}{x^{\frac{1}{2}}}$

$f'(x) = \frac{\sqrt{x}(3\cos x) - (3\sin x)(\frac{1}{2}x^{-\frac{1}{2}})}{x}$

$= \frac{3\sqrt{x}\cos x - \frac{3}{2}x^{-\frac{1}{2}}\sin x}{x}$

5

b. $q=-x^2-2x$

$q' = -2x - 2 = \boxed{-2(x+1)}$ ✓

c. $v=3x^5-4e^2$

$v' = 15x^4$

5

d. $f(x)=e^{\pi}$

$f' = 0$

4

bottom . d top − top(d bottom)

bottom²

8. Suppose that the amount A, in mg, of a drug left in the body t hours after administration is described by $A(t)=\dfrac{50}{t+2}$.

Use calculus to find the rate of change of the amount of the drug in the body, at t=2hrs.

$A'(t) = \dfrac{(t+2)(0)-(50)(1)}{(t+2)^2} = \dfrac{-50}{t^2+4t+4}$ ✓

$A'(2) = \dfrac{-50}{4+8+4} = \dfrac{-50}{16} = \boxed{-\dfrac{25}{8}}$ mg/hr

9. Differentiate.

a. $f(x)=3\sec x+4\cot x$

$f'(x) = 3\sec x\tan x + 4(-\csc^2 x)$

$= 3\sec x\tan x - 4\csc^2 x$

5

b. $y=x\cdot\csc x$

$y' = 1(\csc x) + (-\csc x\cot x)(x)$

$= \csc x - x\csc x\cot x$

5

c. $f(d)=\cos^2 d$ $= (\cos d)^2$

$f'(d) = (-\sin d)^2$

$\boxed{\cancel{-2}}$ $\boxed{-2\sin d\cdot\cos d}$

7-24

all better 7-24

2

d. $f(t)=\tan(\sin t^2)$ $= \tan[(\sin t)^2]$

$f'(t) =$

$\sec^2[(\cos t)^2]$?

7-24

$\boxed{\sec^2(\sin t^2)\cdot(-2\cos t)}$ or $\sec^2(\cos^2 t)$

$\boxed{\sec^2(\sin t^2)\cdot\cos t^2\cdot 2t}$

10. Find $\dfrac{dy}{dx}$ if $2x^3+2y^3-9xy=0$, and find the equation of the line tangent to this equation at (2,1)

$6x^2+6y^2\frac{dy}{dx} - 9\frac{dy}{dx} = 0$

$2x^2+2y^2\frac{dy}{dx} - 3\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} - 3\frac{dy}{dx} = -2x^2$

$\dfrac{dy}{dx} = \dfrac{-2x^2}{(2y-3)}$ $\dfrac{-2(4)}{2\cdot 3} = \dfrac{-8}{-1} = \boxed{8 = m}$

$y-1 = 8(x-2)$

$y = 8x - 16 + 1$

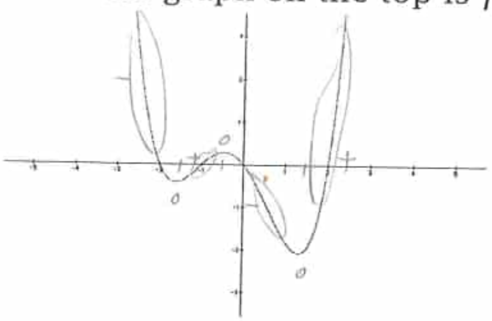
$y = 8x - 15$

$6x^2+6y^2\frac{dy}{dx} - 9x\frac{dy}{dx} = 0$

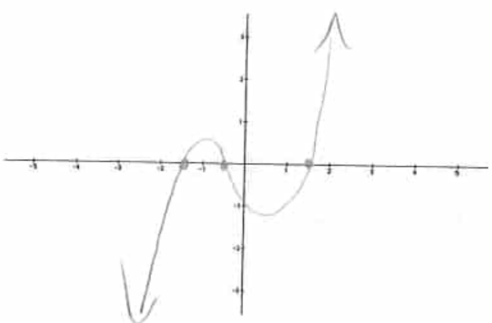
$2x^2+2y^2\frac{dy}{dx}$

20.5

11. If the graph on the top is f, sketch a graph of f' on the bottom.



easy 2-24



12. A particle moves in a straight line with equation of motion of $s = 8t^3 + 6t^2 - 72t$. (s is in meters and t is in seconds)

a. Find $v(t)$ and $a(t)$. $v(t) = s'(t) = 24t^2 + 12t - 72$

$a(t) = v'(t) = 48t + 12$

5

b. What is the displacement of the particle when its acceleration is equal to $0 m/s^2$?

displacement = |pos|

$48t + 12 = 0$

$48t = -12$

$t = \frac{-12}{48} = -\frac{1}{4}$

$s = 8(-\frac{1}{4})^3 + 6(\frac{1}{16}) - 72(-\frac{1}{4})$

$= -\frac{8}{64} + \frac{6}{16} - \frac{72}{4}$

$= \frac{1}{8} + \frac{3}{8} - 18 \quad = \frac{1}{2} - 18 = -17\frac{1}{2}$!

c. If you decided to call Mr. Hake " the third derivative of s ", what would you be calling him?

the jerk ~~of s~~ ?

5

18

easy 7-24

13. Boyle's Law states that when a sample of gas is compressed at a constant temperature, the pressure P and Volume V satisfy the equation, $PV = C$, where C is a constant. Suppose that at a certain instant the volume is 400cm^3 and the pressure is 80 kPa, and the pressure is increasing at a rate of , 15kPa/min. At what rate is the volume decreasing at this instant.

$V = 400 cm^3$ $\quad P = 80\ kPa + 15\ kPa/min$

$V = \frac{C}{P}$

$V' = \frac{P(C') - C(P')}{P^2}$ $\quad * C' = 0 *$

$80\ kPa\ (400\ cm^3) = C$ or 3200

$V' = \frac{80(0) - \cancel{N}(P')}{80^2}$

$80(400) = 3200$ for C

$V' = -\frac{80V(P')}{80^2}$

$95(360) = 34200$

$65(440) = 28600$

$\Big\}$ avg = 31400 for C

$V' = -\frac{400(P')}{80}$

$= -5(P')$

close enough to make sense

$= -5(15)$

$\boxed{= -40\ cm^3/minute}$

-75

or rate of decrease = $40 cm^3/min$

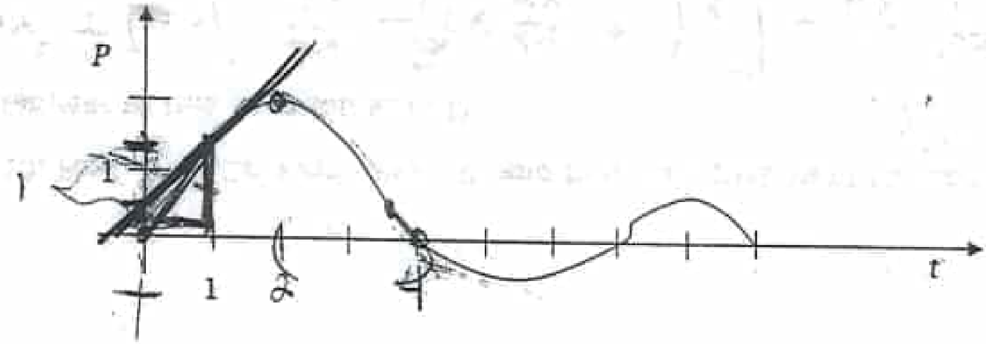
4

Test#2 MTH-210

3.1-3.8

Show Your Work

1. The following is a position function $P(t)$ (in km) of an object moving along a line



a. Use the graph to ***estimate*** the velocity at t=1 sec

$m = v(1) = \frac{1.2km - .2km}{1s - 0s} = \frac{1 km}{1 sec} \approx \frac{1 km}{sec}$

b. State *a* time interval over which the object's velocity is negative.

on $(2,4)$

2. **Use the *definition* of the derivative (!)** to find $f'(x)$ if $f(x) = 4x^2 - 5x + 2$.

$f'(x) = \lim_{h\to 0} \frac{4(x+h)^2 - 5(x+h) + 2 - [4x^2 - 5x + 2]}{h} = \lim_{h\to 0} \frac{4x^2 + 8xh + 4h^2 - 5x - 5h + 2 - 4x^2 + 5x - 2}{h}$

$= \lim_{h\to 0} \frac{8xh + 4h^2 - 5h}{h} = \lim_{h\to 0} 8x + 4h - 5 = 8x + 4(0) - 5$

$\boxed{f'(x) = 8x - 5}$

3. Given $f(x) = \sqrt{1-3x}$,

a. Find and state the domain of f without using a calculator.

$1 - 3x \geq 0$

$1 \geq 3x$

$\frac{1}{3} \geq x$

$D_f: (-\infty, \frac{1}{3}]$

3b. Find the derivative of f.

$f'(x)=\frac{1}{2}(1-3x)^{-\frac{1}{2}}(-3)=-\frac{3}{2\sqrt{1-3x}}$

3c. Find and state the domain of f' without using a calculator.

$1-3x>0$ $\quad x<\frac{1}{3}$ $\quad D_{f'}:(-\infty,+\frac{1}{3})$

$1>3x$

4. Over what interval is $f(x)=\sqrt{1-3x}$ differentiable?

$(-\infty,\frac{1}{3})$

5. Over what interval is $f(x)=\sqrt{1-3x}$ continuous?

$(-\infty,\frac{1}{3}]$

6. If $f(x)=(2x+1)^2(x+2)$, find $f'(3)$.

$f'(x)=2(2x+1)^1(2)\cdot(x+2)+1(2x+1)^2$

$f'(3)=2(6+1)(2)\cdot(5)+(6+1)^2=28\cdot5+49=189$

7. Differentiate.

a. $f(x)=\frac{3\sin x}{\sqrt{x}}$

$\boxed{f'(x)=\frac{\sqrt{x}(3\cos x)-3\sin x(\frac{1}{2}x^{-\frac{1}{2}})}{x}}$

b. $q=-x^2-2x$

$\boxed{q'=-2x-2}$

c. $v=3x^5-4e^2$

$v'=15x^4-0$

$\boxed{v'=15x^4}$

d. $f(x)=e^{\pi}$

$\boxed{f'(x)=0}$

13. Boyle's Law states that when a sample of gas is compressed at a constant temperature, the pressure P and Volume V satisfy the equation, $PV = C$, where C is a $\boxed{\text{constant.}}$ Suppose that at a certain instant the volume is 400cm³ and the pressure is 80 kPa, and the pressure is increasing at a rate of , 15kPa/min. At what rate is the volume decreasing at this instant.

$$PV = C$$

$$\frac{dP}{dt} \cdot V + P \cdot \frac{dV}{dt} = 0$$

$$15(400) + 80\left(\frac{dV}{dt}\right) = 0$$

$$80 \frac{dV}{dt} = -6000$$

$$\frac{dV}{dt} = \frac{-6000}{80} = \frac{-600}{8} = \frac{-300}{4}$$

$$\boxed{-75 \, cm^3/min}$$

David Makuwey 7/7/08

Test#3 MTH-210
3.8-4.7
Show Your Work

1. Use calculus and find the critical numbers of the following function.

$f(x) = 5x^2 - 2\sqrt{x}$ = $5x^2 - 2x^{1/2}$

$f'(x) = 10x - x^{-1/2}$ = $\frac{10x}{-\sqrt{x}}$ $\frac{10x^{3/2}-1}{\sqrt{x}}$

$x = 0$

cind ?

7.5
14
18
7.5
$\frac{47}{65} = 72\%$
C

2. Use calculus. On what interval is $f(x) = \left(1-\frac{1}{x}\right)^2$ concave downward?

$f'(x) = 2(1-\frac{1}{x})(x^{-2})$ = $\frac{2-\frac{2}{x}}{x^2}$ = 0 @ x = 4 $\frac{2-2x^{-1}}{x^2}$

$f''(x) = \frac{x^2(2x)^{-2} - (2-\frac{2}{x})(2x)}{x^4}$ x = 0 IP. $f''(1) = \frac{(+)-(-)}{(+)} = (+)$

7-24

5 $f''(x)$ is negative ✓ $(-\infty, 0)$

$f''(-1) = \frac{\frac{1}{-2^2} - 0}{(+)} = (-)$

3. Use calculus. Find and state the intervals of increase and decrease for $f(x) = x^4 - 2x^3 - 9x^2$

$f'(x) = 4x^3 - 6x^2 - 18x = 0$, $2x(2x^2-3x-9) = 0$ $2x(2x+3)(x-3) = 0$

$f''(x) = 12x^2 - 12x - 18$

$x = 0$ $2x = -3$ $x = \frac{-3}{2}$

$x = 3$

$f'(-2) = ?$

$f'(-1) = -4 - 6 + 18 = (+)$

oops, I did too much work

$f'(1) = (+)(+)(-) = (-)$

$f'(-2) = (-)(-) = (+)$

$f'(4) = (+)(+)$

increase: ✓ $(-\infty, \frac{3}{2})$ & ✗ $(3, \infty)$

decrease ? $(-\frac{3}{2}, 3)$

7.5

4. Use calculus. Find and state the horizontal asymptote of the following function.

$y = \frac{5}{2}$

$$f(x) = \frac{5x^4 - 2x^3 + 9x^2}{2x^4 - 17x^2 + 1}$$

$\lim_{x\to\infty} \frac{5 - \frac{2}{x} + \frac{9}{x^2}}{2 - \frac{17}{x^2} + \frac{1}{x^4}} = \frac{5-0+0}{2-0+0}$

$-2x^3 - 51.5x^3 \quad -2.5$

$= \text{lim's of those} +++ \quad \boxed{y = 5/2}$

$\frac{-2x^3 - 51.5x^2 - 2.5}{2x^4 - 17x^2 + 1}$

5. Evaluate the following limit.

$$\lim_{x\to\infty}\left(\sqrt{x^2+x} - x\right)$$

$|\sqrt{x^2+x}| > |x|$

$\lim_{x\to\infty} = \infty$

#4 7-24

$\frac{x^2(5x^2-2x+9)}{2x^4-17x+1} \quad \lim_{x\to\infty} \frac{5 - \frac{2}{x} - \frac{9}{x^2}}{2 - \frac{17}{x^2} + \frac{1}{x^4}} = \frac{5-0-0}{2-0+0}$

$= \boxed{\frac{5}{2}}$ easy

6. (2 full questions) Given $f(x) = \sqrt{9-x^2}$,

i) What does Rolle's Theorem say about this function on the closed interval $[-2,2]$? $f(-2) = \sqrt{9-4} \quad f(2) = \sqrt{9-4}$ diff? cont.?

there $\exists\, c \in (-2,2)$ such that $f'(c) = 0$

ii) What does the Mean Value Theorem say about his function on the closed interval $[1,3]$? diff? cont?

$\exists\, c \in (1,3)$ such that $f'(c) = \frac{f(3) - f(1)}{3-1}$

7. Use calculus. Find and state the inflection point of $f(x) = 4x^3 - 5x^2 + 2x$

$f'(x) = 12x^2 - 10x + 2$

$f''(x) = 24x - 10 \quad = 0 \quad @ \quad 24x = 10$

$x = \frac{10}{24} = \frac{5}{12}$

$\boxed{\left(\frac{5}{12}, .25463\right)}$

14

8. (worth 4 questions) Use calculus.

Given $f(x) = \sqrt{2}x - 2\sin x$ restricted to $[-\pi, \pi]$ $= (2x)^{1/2} - 2\sin x$

i. Find its critical numbers

ii. Find the intervals of increase/decrease.

iii. Find the critical numbers of the first derivative.

iv. Find intervals of concavity.

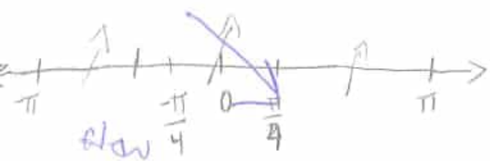
v. Find local maxima and local minima.

vi. Find inflection points.

vii. Sketch a graph.

.707

$f'(x) = \sqrt{2} - 2\cos x = 0$ $\quad -2\cos x = \sqrt{2}$ $\quad \cos x = \frac{\sqrt{2}}{2}$ $\quad$ ✓ $x = \frac{\pi}{4}, -\frac{\pi}{4}$ $\quad$ i]



increase $(-\pi, \pi)$

$f''(x) = 2\sin x = 0$ $\quad \sin x = 0$, ✓ $x = 0$, 3 $\quad$ iv] CD = $(-\pi, 0)$

CU = $(0, \pi)$

$\sin -\frac{\pi}{2} = (-)$

$\sin \pi/2 = (+)$

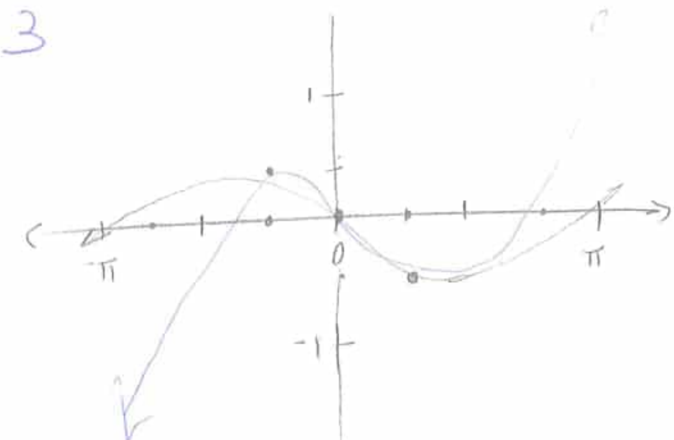
✓/

$f(\frac{\pi}{4}) = \frac{\sqrt{2}\,\pi}{4} - 2(\frac{\sqrt{2}}{2}) = \frac{\pi}{2\sqrt{2}} - \sqrt{2}$ local min

$f(-\frac{\pi}{4}) = \frac{-\sqrt{2}\pi}{4} - 2(-\frac{\sqrt{2}}{2})$ ✓ $-\frac{\pi}{2\sqrt{2}} + \sqrt{2}$ local max

I.P. @ $x = (0, 0)$

3



18

9. A firework explodes in the shape of a sphere. When its radius is 2ft, the **rate of change** of the radius is 3ft/second. Find the **rate of change** in the volume of the explosion at this time.(3.8)

$4\pi r^2$

$r = 2$ $\quad f(r) = 3ft/s$ $\quad f(V) = 4\pi r^2$ $\quad f(V(r)) = 4\pi(3)^2$

$4(9)\pi = \boxed{36\pi \frac{ft^3}{s}}$

.5

10. A plane flying horizontally at an altitude of 1km and with a speed of 500km/h, passes directly over a radar station on the ground. Find the **RATE** at which the distance from the plane to the station is increasing when this distance is 2km from the station.(3.8)

/h

$\sqrt{3}$,

1km

x

✓ $2^2 = 1^2 + x^2$

not always $2^2 = (4)$ $\quad 1 + x^2$

$3 = x$

$x = \sqrt{3}$ km

$f(x) = \sqrt{(x + 500km)^2 + (1km)^2}$

$f(x) = \sqrt{x^2 + 1000x + 2500 + 1}$

$= \sqrt{x^2 + 1000x + 2501}$

distance

$f'(x) = \frac{1}{2}(x^2 + 1000x + 2501)^{-1/2}$

$\frac{1}{2\sqrt{x^2 + 1000x + 2501}}$

Δ dist.

rate $= \frac{1}{2\sqrt{3km + 1000(\sqrt{3}km) + 2501}}$ km/h²

.5

?

11. A running track 400 meters long is to be constructed with two parallel straight-aways of length *s* joining two semicircular curves of radius *r*. What are the dimensions of the track that will enclose the **largest** infield area?(4.7)

s

2r

C/2

s

$r = 18.96$

$s = 140.44$

$2s + C = 400$

$C = 2\pi r$

✓ $2s + 2\pi r = 400$

$2\pi r = 400 - 2s$

$\pi r = 200 - s$

$r = \frac{200 - s}{\pi}$

$2s = 400 - 2\pi r$

$s = 200 - \pi r$

$A = 2sr + \pi r^2$

$= s\left(\frac{200-s}{\pi}\right) + \pi\left(\frac{200-s}{\pi}\right)^2$

$= r(200 - \pi r) + \pi(200 - \pi r)^2$

$= 200r - \pi r^2 + \pi(40000 - 400\pi r + \pi^2 r^2)$

$= -\pi r^2 + 200r + 40000\pi - 400\pi^2 r + \pi^3 r^2$

$A' = 2(\pi^3 - \pi)r + 200 - 400\pi = 0$

$= (\pi^3 - \pi)r = 200\pi - 100$

$r = \frac{200\pi - 100}{(\pi^3 - \pi)}$ $\quad \frac{528.319}{27.865}$

$r = 18.96$

7.5

Test#3 MTH-210

3.8-4.7

Show Your Work

1. Use calculus and find the critical numbers of the following function.

$D_f: [0, +\infty)$

$f(x) = 5x^2 - 2\sqrt{x}$

$f'(x) = 10x - 2\cdot\frac{1}{2}x^{-1/2} = 10x - \frac{1}{\sqrt{x}} = \frac{10x^{3/2} - 1}{\sqrt{x}}$

$\boxed{C = 0 \text{ since } 0 \text{ makes } f' \text{ undefined}}$

and $10\sqrt{x^3} - 1 = 0$ $\quad \sqrt{x^3} = \frac{1}{10}$ $\quad \frac{1}{100} = x^3$ $\quad x = \boxed{\frac{1}{\sqrt[3]{100}} = C}$

$\boxed{\text{Concave down on } (\tfrac{3}{2}, +\infty)}$

Concave up on $(-\infty, 0) \cup (0, \tfrac{3}{2})$

$f''(1) = \frac{+\,+}{+} = +$ $\quad f''(2) = \frac{+\,-}{+} = (-)$

$f''(-1) = \frac{+\,+}{+} = +$

2. Use calculus. On what interval is $f(x) = \left(1 - \frac{1}{x}\right)^2$ concave downward?

$f'(x) = 2\left(1 - \frac{1}{x}\right)\left(+\frac{1}{x^2}\right)$

$= \frac{2}{x^2} - \frac{2}{x^3} = \frac{2x - 2}{x^3} = f'(x)$

$f''(x) = \frac{x^3(2) - (2x-2)(3x^2)}{x^6} = \frac{2x^3 - 6x^3 + 6x^2}{x^6}$

$= \frac{2(x - 3x + 3)}{x^4} = \frac{2(-2x+3)}{x^4}$

$x = 0$ not in $D_{f'}$

$C = \frac{3}{2}$

$0 \quad \frac{3}{2}$

3. Use calculus. Find and state the intervals of increase and decrease for $f(x) = x^4 - 2x^3 - 9x^2$

$f'(x) = 4x^3 - 6x^2 - 18x$

$0 = 2x(2x^2 - 3x - 9)$

$0 = 2x(2x + 3)(x - 3)$

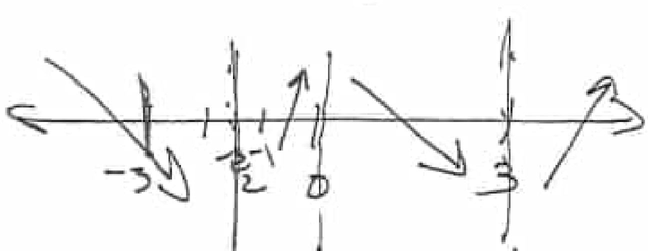
$x = 0$ or $-\frac{3}{2} = x$ or $x = 3$

$f'(-3) = - - - = (-)$

$f'(-1) = - + - = (+)$

$f'(1) = + + - = (-)$

$f'(4) = + + + = (+)$



Increasing on: $(-\frac{3}{2}, 0) \cup (3, +\infty)$

Decreasing on $(-\infty, -\frac{3}{2}) \cup (0, 3)$

4. $\boxed{\text{Use calculus.}}$ Find and state the horizontal asymptote of the following function.

$f(x)=\dfrac{5x^4-2x^3+9x^2}{2x^4-17x^2+1}$

$$\lim_{x\to\infty}\frac{5x^4-2x^3+9x^2}{2x^4-17x^2+1}\cdot\frac{\frac{1}{x^4}}{\frac{1}{x^4}}=\lim_{x\to\infty}\frac{5-\frac{2}{x}+\frac{9}{x^2}}{2-\frac{17}{x^2}+\frac{1}{x^4}}$$

$$=\frac{5-0+0}{2-0+0}=\frac{5}{2}$$

5. Evaluate the following limit.

$$\lim_{x\to\infty}\left(\sqrt{x^2+x}-x\right)\cdot\frac{\sqrt{x^2+x}+x}{\sqrt{x^2+x}+x}=\lim_{x\to\infty}\frac{x^2+x-x^2}{\sqrt{x^2+x}+x}=\lim_{x\to\infty}\frac{x}{\sqrt{x^2+x}+x}\cdot\frac{\frac{1}{x}}{\frac{1}{\sqrt{x^2}}}$$

$$=\lim_{x\to\infty}\frac{1}{\sqrt{1+\frac{1}{x}}+1}=\frac{1}{\sqrt{1+0}+1}=\boxed{\frac{1}{2}}$$

6. (2 full questions) Given $f(x)=\sqrt{9-x^2}$. $f'(x)=\frac{1}{2}(9-x^2)^{-1/2}(-2x)=\frac{-x}{\sqrt{9-x^2}}$

i) What does Rolle's Theorem say about this function on the closed interval $[-2,2]$? $f(-2)=f(2)=\sqrt{5}$

F is continuous on $[-2,2]$ Therefore THERE IS a number

f is diff on $(-2,2)$ c in $(-2,2)$ such that $f'(c)=0$

ii) What does the Mean Value Theorem say about his function on the closed interval $[1,3]$?

~~[illegible]~~ f is cont. on $[1,3]$ (root fcn)

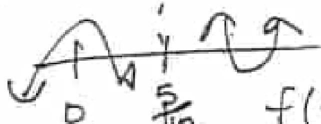
f is diff on $(1,3)$ Therefore THERE IS a number c in $(1,3)$

such that $f'(c)=\dfrac{f(3)-f(1)}{2}$

$f'(c)=\frac{0-\sqrt{8}}{2}=-\sqrt{2}$

7. Use calculus. Find and state the inflection point of $f(x)=4x^3-5x^2+2x$ $\left(\frac{5}{12},\frac{55}{216}\right)$

$f'(x)=12x^2-10x+2$



$f\left(\frac{5}{12}\right)=4\left(\frac{5}{12}\right)^3-5\left(\frac{5}{12}\right)^2+2\left(\frac{5}{12}\right)$

$f''(x)=24x-10$

$=4\left(\frac{125}{1728}\right)-\frac{125}{144}+\frac{5}{6}$

$f''=24x-10$ $f''(0)=-10$ $\boxed{\frac{55}{216}}$

$\frac{500-1500+5(288)}{}$

David Malawey
7/17/08

Test#4 MTH-210

4.7-5.5

Show Your Work

1. A large closed shipping container with a square base is to be made from 1000 ft² of fiberboard. Find the dimensions of the container with the greatest volume.

$A = 2x^2 + 4xh$

$A = 2x^2 + 4x\left(\frac{500-x^2}{2x}\right) = 2x^2 + 2(500-x^2)$

$1000 = 2x^2 + 4xh$

$500 = x^2 + 2xh$

$\frac{500-x^2}{2x} = h =$

2

$6x^2 = 1000 \quad x^2 = \frac{1000}{6}$

How? $x = \sqrt{\frac{1000}{6}}$ because I know a cube has most volume & I ran out of time

all side dimensions = x

2. Find the most general antiderivative of each of the following.

a. $f(x) = 12x^5 - \frac{4}{\sqrt{x}} = 12x^5 - 4x^{-1/2}$

$F(x) = 2x^6 - 8x^{1/2} + C$

5

b. $f(x) = sec^2 x - 3\sqrt[3]{x^2}$ $\quad \frac{d}{dx} \tan x = \sec^2 x$

$\sec^2 x - 3x^{2/3}$

$F(x) = \tan x - \frac{9}{5}x^{5/3} + C$

5

22
12.5
25
17
76.5 / 83 = 92%
A

7-24

3. Given $f'(x) = \sqrt{x}$ and $f(0) = 0$, find $f(1)$.

$x^{1/2}$

$f(x) = \frac{2}{3}x^{3/2} + C \quad C = 0$

$f(1) = \frac{2}{3}(1^{3/2}) = \frac{2}{3} \cdot 1 = \boxed{\frac{2}{3}}$

5

4. Find the position function $s(t)$, if $a(t) = 3t$, $v(2) = 0$, and $s(2) = 1$.

$a(t) = 3t$

$v(t) = \frac{3}{2}t^2 + C \quad v(2) = \frac{3}{2}(4) + C = 6 + C, \quad C = -6$

$s(t) = \frac{1}{2}t^3 - 6t + d \quad s(2) = \frac{1}{2}(8) - 12 + d = 4 - 12 + d = -8 + d$

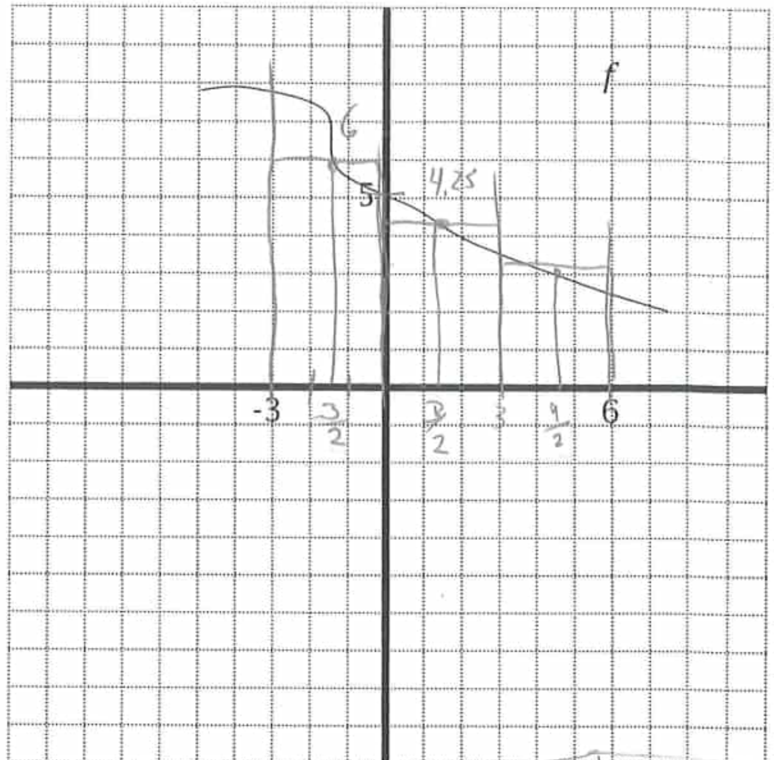
$d = 9, \quad \boxed{s(t) = \frac{1}{2}t^3 - 6t + 9}$

5

22

middle value of 3 sections 7-24

5. Find M_3 to estimate the area between f and the x-axis from x=-3 and x=6.



$\Delta x = \frac{6-(-3)}{3} = 3$

$M = f\left(\frac{-3+3}{2}\right) + f\left(\frac{0+3}{2}\right) + f\left(\frac{3+6}{2}\right)$

$M_3 = 3(6 + 4.25 + 3)$

$= 3(13.25) = \boxed{39.75 \text{ units}^2}$

5

7-24 6. Use $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$ to find $\int_2^3 5x\,dx$

$\Delta x = \frac{3-2}{n} = \frac{1}{n}$ $x_i = x_0 + i\Delta x$ $2 + \frac{i}{n}$

$= \lim_{n\to\infty} \sum_{i=1}^{n} 5\left(2+\frac{i}{n}\right)\frac{1}{n} = \lim_{n\to\infty} \frac{5}{n}\sum_{i=1}^{n}\left(2+\frac{i}{n}\right) = \lim_{n\to\infty} \frac{5}{n}\sum_{i=1}^{n} 2 + \frac{5}{n}\sum_{i=1}^{n}\frac{i}{n}$

$= \lim_{n\to\infty} \frac{5}{n}(2n) + \frac{5}{n^2}\sum_{i=1}^{n} i = \lim_{n\to\infty} 10 + \frac{5}{n^2}\cdot\frac{n(n+1)}{2} = \lim_{n\to\infty} 10 + \frac{5n+5}{2n}$

$\frac{\frac{1}{n}}{\frac{1}{n}}$

5

$= 10 + \frac{5}{2} = \frac{25}{2}$

$\lim_{n\to\infty} \sum_{i=1}^{n} 5\left(2+\frac{i}{n}\right)\Delta x$ $\Delta x = \frac{3-2}{n} = \frac{1}{n}$

$x_i = 2 + i\cdot\frac{1}{n}$

$5\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n}\left(2+\frac{i}{n}\right)$

$\boxed{7\text{-}24}$

$5\lim_{n\to\infty} \frac{2n}{n} + \frac{1}{n^2}\sum_{i=1}^{n} i$

$5\lim_{n\to\infty} \frac{2n}{n} + \frac{1}{n^2}\cdot\frac{(n)(n+1)}{2}$

$5\lim_{n\to\infty} 2 + \frac{n+1}{2n}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$

$= 5\left(2+\frac{1}{2}\right) = \boxed{12.5}$

7. Find the derivative of g.

$g(x) = \int_0^{2x} 3t\,dt$

2.5

$\frac{d}{dx}\int_0^{2x} 3t\,dt = 3x$

$3(2x)\cdot 2$

7-24 $\frac{d}{dx}\int_0^{2x} 3t\,dt =$

12.5

8. Evaluate the integral.

a. $\int_{-2}^{0} x^2 - 4x - 9\,dx$

$\frac{1}{3}x^3 - 2x^2 - 9x \Big]_{-2}^{0} = \left(\frac{1}{3}(0)^3 - 2(0)^3 - 9(0)\right) - \left(\frac{1}{3}(-2^3) - 2(-2^2) - 9(-2)\right)$

5

$= -\left(\frac{1}{3}\cdot\frac{-8}{1} - 2\cdot 4 - (-18)\right) = -\left(-\frac{8}{3} - 8 + 18\right) = -\left(-\frac{8}{3} + \frac{30}{3}\right) = -\frac{22}{3}$

b. $\int_{0}^{\pi/3} \csc x \cot x\,dx$

$\frac{d}{dx}\csc x = -\csc x \cot x$

$\frac{y}{x}$ $\frac{x}{y}$ sin cos tan sec cot csc

$-\csc x$

5!

$= -\csc\left(\frac{\pi}{3}\right) - \left[-\csc(0)\right]$

$-\frac{2}{\sqrt{3}} - \left(-\frac{1}{0}\right)$ ~~not possible~~ $\boxed{\text{not cont}}$!

$\csc = \frac{1}{\sin}$ | $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$; $\sin 0 = 0$

c. $\int_{1}^{32} x^{\frac{1}{5}}\,dx$

$\frac{5}{6}x^{\frac{6}{5}}\Big]_{1}^{32}$

$\frac{5}{6}32^{6/5} - \frac{5}{6}(1)$

$53\frac{1}{3} - \frac{5}{6} = \boxed{52.5}$

5

9. Find the most general integral.

d. $\int 3\sin\theta + \sqrt[4]{\theta}\,d\theta = \int 3\sin\theta + \theta^{\frac{1}{4}}$

$= -3\cos\theta + \frac{4}{5}\theta^{\frac{5}{4}} + C$

5

e. $\int 12\csc^2\theta - \sec\theta\tan\theta\,d\theta$

$\frac{d}{dx}\cot = -\csc$, $\frac{d}{dx}\sec = \sec\tan$

5

$-12\cot\theta - \sec\theta + C$

$\boxed{-12\cot\theta - \sec\theta + C}$

25

f. $\int \frac{4x+1}{\sqrt{x}}\,dx$ $= \cancel{\int (4x+1)(x^{\frac{1}{2}})\,dx}$ $\int \frac{4x}{\sqrt{x}}\,dx + \int \frac{1}{\sqrt{x}}\,dx = \int 4x^{\frac{1}{2}}dx + \int x^{-\frac{1}{2}}dx$

$= \boxed{\frac{8}{3}x^{3/2} + 2x^{\frac{1}{2}} + C}$

5

10. Integrate using u substitution.

a. $\int_{-1}^{0} (4x+5)^{16}\,dx$ $\quad u = 4x+5 \quad du = 4\,dx \quad \frac{du}{4} = dx$

$= \int_{-1}^{0} u^{16} \frac{du}{4} = \frac{1}{4}\int_{-1}^{0} u^{16}\,du = \frac{1}{4}\left(\frac{1}{17}u^{17}\right)\Big| = \frac{1}{68}u^{17}\Big|_{-1}^{0} = \boxed{\frac{(4x+5)^{17}}{68}\Big|_{-1}^{0}}$

3

b. $\int \frac{x^3}{\sqrt{x^2+1}}\,dx$ $\quad \cancel{u = x^2+1 \quad du = 2x \quad \frac{du}{2} = x\,dx}$ $\quad u = x^2+1 \quad du = \frac{1}{2}(x^2+1)^{3/2}(2x)$

$du = \frac{1}{3}x^3 + x = x(x^2+1)^{3/2}$

$3du = x^3 + 3x \quad \frac{x}{\sqrt{x^2+1}^3}$

$\cancel{= \int \frac{x^2}{\sqrt{u}} \cdot \frac{du}{2} = \frac{1}{2}\int \frac{x^2}{\sqrt{u}}\,du}$

$= \frac{1}{3}\int \frac{du^3}{\sqrt{u}}$

$\int \frac{x^2}{\sqrt{u}} \frac{du}{2} \quad \frac{1}{2}\left(\frac{x^2}{\frac{2}{3}u^{3/2}}\right) = \frac{1}{2}\left(\frac{x^2}{\frac{2}{3}(x^2+1)^{3/2}}\right)$

2

11. The velocity of a particle moving along a line is $v(t) = t^3 - t$ (in meters/second).

a. Find the displacement of the particle during the time interval $0 \le t \le 2$.

$d(t) = \frac{1}{4}t^4 - \frac{1}{2}t^2\Big|^2 + C$ m/s $\quad$ C is irrellevant

$\frac{1}{4}16 - 2 + C$

$C\,m/s \le d \le 2 + C$ m/s

2 m/s

$\frac{1}{4}t^4 - \frac{1}{2}t^2\Big|_0^2$

$\frac{1}{4}(16) - \frac{1}{2}(4)$

5

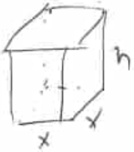
b. Find the distance traveled by the particle during the time interval $0 \le t \le 2$.

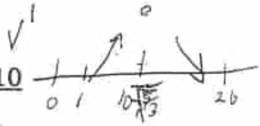
dist travelled $= \cancel{4 - 2 = 2}$ $\quad \cancel{8 - 2 = 6}$

no it's 2 m

2

17





Test#4 MTH-210

4.7-5.5

Show Your Work

1. A large closed shipping container with a square base is to be made from 1000 ft² of fiberboard. Find the dimensions of the container with the greatest volume.

$h = \frac{1000-2x^2}{4x}$ $\quad h = \frac{500-x^2}{2x}$

$SA = 2x^2 + 4xh = 1000$

$V = x^2 h$

$V(x) = x^2\left(\frac{500-x^2}{2x}\right) = 250x - \frac{x^3}{2}$

$V'(x) = 250 - \frac{3}{2}x^2$

$\boxed{h = \frac{500-\frac{500}{3}}{20\sqrt{\frac{5}{3}}}\text{ ft}}$

$\frac{3}{2}x^2 = 250 \quad x^2 = \frac{500}{3} \quad x = \pm\sqrt{\frac{500}{3}} = \boxed{10\sqrt{\frac{5}{3}}\text{ ft} = x}$

2. Find the most general antiderivative of each of the following.

a. $f(x) = 12x^5 - \frac{4}{\sqrt{x}}$ $= 12x^5 - 4x^{-\frac{1}{2}}$

$F(x) = 2x^6 - \frac{4}{\frac{1}{2}}x^{\frac{1}{2}}$

$\boxed{F(x) = 2x^6 - 8x^{\frac{1}{2}} + C}$

b. $f(x) = \sec^2 x - 3\sqrt[3]{x^2}$ $= \sec^2 x - 3x^{\frac{2}{3}}$

$F(x) = \tan x - \frac{3}{\frac{5}{3}}x^{\frac{5}{3}}$

$\boxed{F(x) = \tan x - \frac{9}{5}x^{\frac{5}{3}} + C}$

3. Given $f'(x) = \sqrt{x}$ and $f(0) = 0$, find $f(1)$.

$f' = x^{\frac{1}{2}}$

$f(x) = \frac{1}{\frac{3}{2}}x^{\frac{3}{2}} = \frac{2}{3}x^{\frac{3}{2}} + C$

$f(1) = \frac{2}{3}(1)^{\frac{3}{2}}$

$f(0) = 0 = \frac{2}{3}(0)^{\frac{3}{2}} + C \quad C = 0$

$\boxed{= \frac{2}{3}}$

4. Find the position function $s(t)$, if $a(t) = 3t$, $v(2) = 0$, and $s(2) = 1$.

$v(t) = \frac{3}{2}t^2 + C$

$s(2) = 1 = \frac{1}{2}(2)^3 + 6(2) + D$

$s(t) = \frac{1}{2}t^3 + Ct + D$

$1 = 4 - 12 + D$

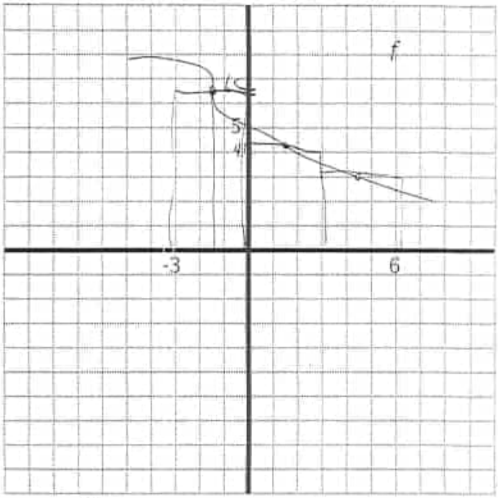
$9 = D$

$v(2) = 0 = \frac{3}{2}(4) + C$

$C = -6$

$\boxed{s = \frac{1}{2}t^3 - 6t + 9}$

5. Find M_3 to estimate the area between f and the x-axis from x=-3 and x=6. $\Delta x = \frac{9}{3} = 3$



$M_3 = \sum_{i=1}^{3} \bar{x}_i \Delta x$

$= 3[6.5 + 4.2 + 3]$

$3(13.7)$

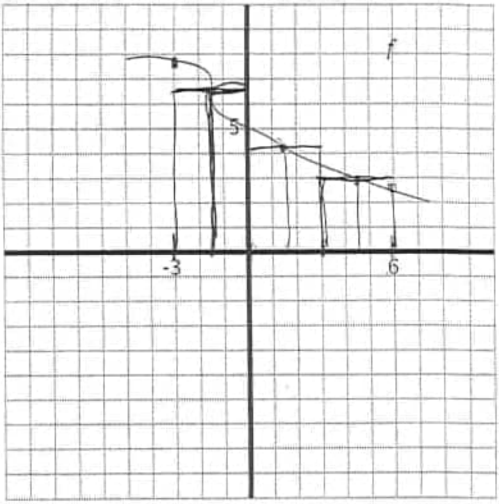
$= 39 + 2.1$

$\boxed{40.1 \text{ sq units}}$

6. Use $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$ to find $\int_2^3 5x\,dx$ $\quad \Delta x =$

7. Find the derivative of g.
$g(x) = \int_0^{2x} 3t\,dt$

5. Find M_3 to estimate the area between f and the x-axis from x=-3 and x=6.



6. Use $\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$ to find $\int_2^3 5x\,dx$ $\quad \Delta x = \frac{1}{n} \quad x_i = 2+\frac{i}{n}$

$\frac{45}{2} - \frac{20}{2} = \frac{25}{2}$

$= \frac{25}{2}$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} 5\left(2+\frac{i}{n}\right)\frac{1}{n} = 5\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\left(2+\frac{i}{n}\right)$$

$$= 5\lim_{n\to\infty}\frac{1}{n}\left(\sum_{i=1}^{n}2 + \frac{1}{n}\sum_{i=1}^{n} i\right) = 5\lim_{n\to\infty}\frac{1}{n}\left(2n+\frac{1}{n}\left(\frac{n(n+1)}{2}\right)\right)$$

$$= 5\lim_{n\to\infty} 2+\frac{n+1}{2n} = 5\left(2+\frac{1}{2}\right)$$

$$= 10+\frac{5}{2}$$

7. Find the derivative of g.

$g(x) = \int_0^{2x} 3t\,dt$

$$\frac{d}{dx}(g(x)) = \frac{d}{dx}\int_0^{2x} 3t\,dt = 3(2x)\,(2) \quad 3(t)\quad dt$$

$$\boxed{= 12x}$$

$$\frac{d}{dx}(g(x)) = \int_0^{2x} 3t\,dt = 3(2x) - 3(0) \cdot 2-0 = 12x$$

8. Evaluate the integral.

a. $\int_{-2}^{0} x^2 - 4x - 9\,dx$

$\frac{1}{3}x^3 - 2x^2 - 9x \Big|_{-2}^{0}$

$= \frac{1}{3}(0) - 2(0) - 9(0) - \left[\frac{1}{3}(-8) - 2(4) - 9(-2)\right]$

$= \frac{8}{3} + 8 - 18 = \frac{8}{3} - 10 = \frac{8-30}{3}$

$\boxed{-\frac{22}{3}}$

b. $\int_{0}^{\pi/3} \csc x \cot x\,dx$

not continuous on $[0, \frac{\pi}{3}]$

$= -\csc x \Big|_{0}^{\pi/3}$

$= -\csc \frac{\pi}{3} + \csc 0 = \frac{1}{\frac{\sqrt{3}}{2}}$ — Undefined

Does not exist
not defined on
$[0, \pi/3]$

c. $\int_{1}^{32} x^{\frac{1}{5}}\,dx$

$= \frac{1}{\frac{6}{5}} x^{\frac{6}{5}}$

$= \frac{5}{6} x^{6/5} \Big|_{1}^{32} = \frac{5}{6}(32)^{6/5} - \frac{5}{6}(1)^{6/5} = \frac{5}{6}(64) - \frac{5}{6} = 63\left(\frac{5}{6}\right)$

$= 21\left(\frac{5}{2}\right) = \boxed{\frac{105}{2}}$

9. Find the most general integral.

d. $\int 3\sin\theta + \sqrt[4]{\theta}\,d\theta = \int 3\sin\theta + \theta^{1/4}\,d\theta$

$= -3\cos\theta + \frac{1}{\frac{5}{4}}\theta^{\frac{5}{4}} + C$

$\boxed{= -3\cos\theta + \frac{4}{5}\theta^{\frac{5}{4}} + C}$

e. $\int 12\csc^2\theta - \sec\theta\tan\theta\,d\theta$

$= 12\int \csc^2\theta - \int \sec\theta\tan\theta\,d\theta$

$\boxed{= -12\cot\theta - \sec\theta + C}$

f. $\int \frac{4x+1}{\sqrt{x}} dx$ $= \int \frac{4x}{\sqrt{x}} + \frac{1}{\sqrt{x}}\, dx = 4\int \sqrt{x} + \int x^{-\frac{1}{2}}$

$= 4\,\frac{2}{3}\,x^{\frac{3}{2}} + 2x^{\frac{1}{2}} = \boxed{\frac{8}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C}$

9. Integrate using u substitution.

a. $\int_{-1}^{0} (4x+5)^{16}\, dx$ $U = 4x+5 \quad du = 4\,dx \quad \frac{du}{4} = dx$

$\frac{1}{4}\int_{1}^{5} u^{16}\, du = \frac{1}{4}\,\frac{1}{17}\,u^{17}\Big|_{1}^{5} = \boxed{\left[\frac{1}{68}(5)^{17} - \frac{1}{68}(1)^{17}\right]}$

b. $\int \frac{x^3}{\sqrt{x^2+1}} dx$ $U = x^2+1 \quad du = 2x\,dx \quad \frac{du}{2} = x\,dx$

$U - 1 = x^2$

$= \int \frac{x^2 \cdot x \cdot dx}{\sqrt{x^2+1}} = \int \frac{(U-1)\left(\frac{du}{2}\right)}{\sqrt{U}} = \frac{1}{2}\int \frac{U-1}{\sqrt{U}}\, du$

$= \frac{1}{2}\int U^{\frac{1}{2}} - U^{-\frac{1}{2}}\, dU = \frac{1}{2}\left[\frac{2}{3}U^{\frac{3}{2}} - 2U^{\frac{1}{2}}\right] = \boxed{\frac{1}{3}(x^2+1)^{\frac{3}{2}} - \sqrt{x^2+1} + C}$

10. The velocity of a particle moving along a line is $v(t) = t^3 - t$ (in meters/second).

a. Find the displacement of the particle during the time interval $0 \le t \le 2$.

b. Find the distance traveled by the particle during the time interval $0 \le t \le 2$.

f. $\int \frac{4x+1}{\sqrt{x}}\,dx$

10. Integrate using u substitution.

a. $\int_{-1}^{0} (4x+5)^{16}\,dx$

b. $\int \frac{x^3}{\sqrt{x^2+1}}\,dx$

11. The velocity of a particle moving along a line is $v(t) = t^3 - t$ (in meters/second).

a. Find the displacement of the particle during the time interval $0 \le t \le 2$.

$$\int_0^2 t^3 - t\,dt = \frac{1}{4}t^4 - \frac{1}{2}t^2 \Big|_0^2 = \frac{1}{4}(16) - \frac{1}{2}(4) - [0-0]$$

$$= 4 - 2 = \boxed{2\text{ m}}$$

b. Find the distance traveled by the particle during the time interval $0 \le t \le 2$.

$$= -\left[\frac{1}{4} - \frac{1}{2}\right] + 4 - 2 - \frac{1}{4} + \frac{1}{2}$$

$$\int_0^2 |t^3 - t|\,dt = -\int_0^1 t^3 - t\,dt + \int_1^2 t^3 - t\,dt \quad \boxed{2.5\text{ m}}$$

$$= -\left(\frac{1}{4}t^4 - \frac{1}{2}t^2\right)\Big|_0^1 + \left(\frac{1}{4}t^4 - \frac{1}{2}t^2\right)\Big|_1^2 = -\left[\frac{1}{4} - \frac{1}{2}\right] + \left[\frac{1}{4}(16) - \frac{1}{2}(4)\right] - \left[\frac{1}{4} - \right.$$

David Malawey 6/10/08

7-24 ✓

.00025

Quiz#2 MTH210

2.1-2.4

1. The point $P\left(1,\frac{1}{2}\right)$ lies on the curve $y=\frac{x}{1+x}$. If Q is the point $\left(x,\frac{x}{1+x}\right)$, use your calculator to find the slope of the secant line that goes through P and Q when x=0.999 and when x=1.001.

M_{PQ} when $x = 0.999 = 0.25$

M_{PQ} when $x = 1.001 = 0.25$

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\frac{(\frac{1}{2} - .999}{(1 - \quad)}$ 7/24/03

$x = .999$	$y = .49975$
1.001	$.50025$

rounded too much

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-2.5 \times 10^{-4}}{-.001}$

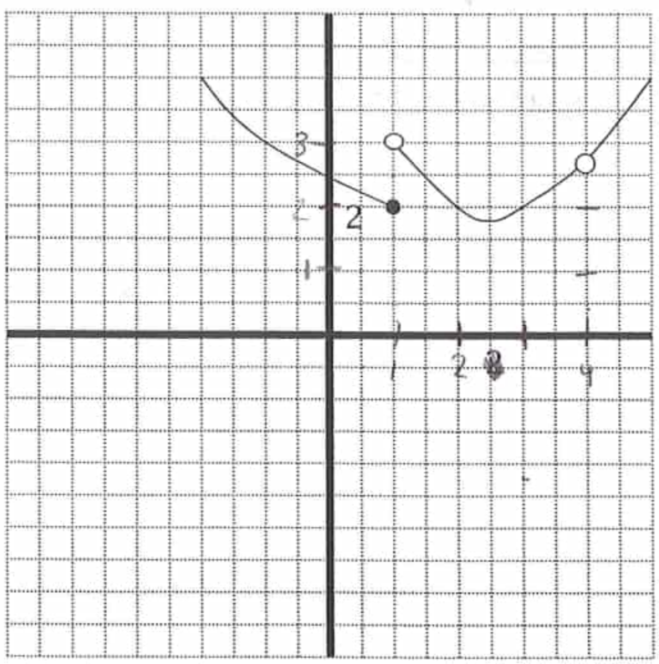
$\frac{.00025}{.001}$

2

2. State the value of each quantity if it exists.

a. $\lim_{x\to1^-} f(x)$

✓ ~~≈ 4.5~~ ~~4~~ 2



b. $\lim_{x\to1^+} f(x)$

✓ 3

c. $\lim_{x\to1} f(x)$

✓ DNE

2

d. $\lim_{x\to4} f(x)$

✓ ≈ 2.75

5/8

7/24 $D(f)$: $(-\infty,2)(2,\infty)$  negative slope from $x \to 2^-$ & Positive slope from $x \to 2^+$

3. Evaluate the limit if it exists.

$$\lim_{x\to2}\frac{x^2-x+6}{x-2} = \cancel{\lim_{x\to2}(x+2)(x}\quad \lim_{x\to2} \frac{x^2-x+6}{x-2}\cdot\frac{(x+2)}{(x+2)} = x^2-4$$

1.9

Power on top is bigger vertical asymptote at $x=2$

$$= \lim_{x\to2} x^2 - \lim_{x\to2} x + \lim_{x\to2} 6 \qquad \lim_{x\to2}\frac{4-2+6}{2}$$

Does not exist

$$\lim_{x\to2} 8 - \lim_{x\to2} 2 \lim \ldots$$

5

4. Evaluate the limit if it exists.

$$\lim_{x\to7}\frac{\sqrt{x+2}-3}{x-7} \cdot \frac{(\sqrt{x+2}+3)}{(\sqrt{x+2}+3)} \quad \frac{x+2-9}{x-7(\sqrt{x+2}+3)} \quad \frac{\cancel{x-7}}{\cancel{x-7}(\sqrt{x+2}+3)} = \frac{1}{\sqrt{x+2}+3}$$

$$\frac{1}{\sqrt{9}+3} = \frac{1}{6}$$

= .166 ?

Calculator guess...

$$\frac{x+7}{x+7}\cdot\frac{\sqrt{x+2}-3}{x-7} = \frac{x\sqrt{x+2}-3x+7\sqrt{x+2}-21}{x^2-49}\cdot\frac{\sqrt{x-2}+3}{\sqrt{x-2}+3}$$

5

?

$$\frac{\sqrt{x+2}-3}{x-7} \qquad \overline{x^2-49}$$

7/24 $$\lim_{x\to7}\frac{\sqrt{x+2}-3}{x-7} = \frac{(x+2-9)}{(x-7)(\sqrt{x+2}+3)} = \frac{1}{\sqrt{x+2}+3} = \frac{1}{3+3} = \frac{1}{6}$$

Quiz#2 MTH210

2.1-2.4

1. The point $P\left(1,\frac{1}{2}\right)$ lies on the curve $y=\frac{x}{1+x}$. If Q is the point $\left(x,\frac{x}{1+x}\right)$, use your calculator to find the slope of the <u>secant</u> line that goes through P and Q when x=0.999 and when x=1.001.

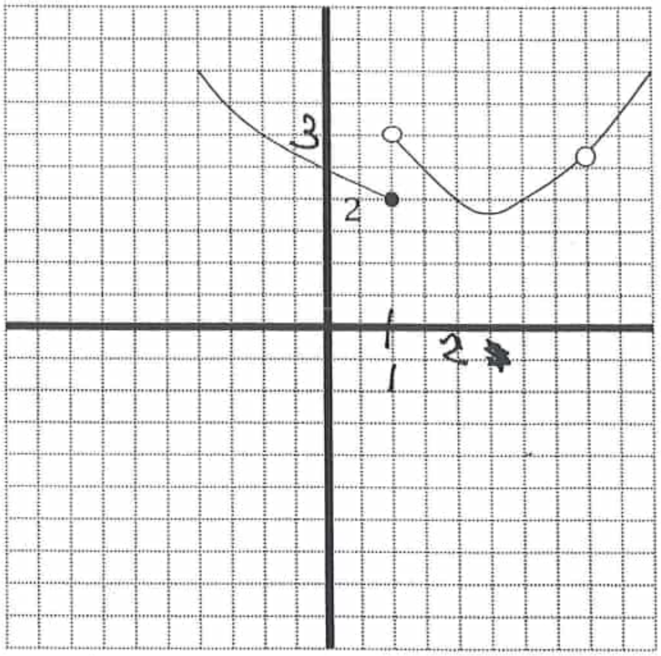
$$m_{PQ}=\frac{\frac{x}{1+x}-\frac{1}{2}}{x-1}$$

$$m_{PQ_1}=\frac{.4997499749937-\frac{1}{2}}{.999-1}=\boxed{.25125063}$$

$$m_{PQ_2}=\frac{.500249875062-\frac{1}{2}}{1.001-1}=\boxed{.2498750621}$$

2. State the value of each quantity if it exists.

a. $\lim\limits_{x\to 1^-} f(x)=2$



b. $\lim\limits_{x\to 1^+} f(x)=3$

c. $\lim\limits_{x\to 1} f(x)=$ DNE

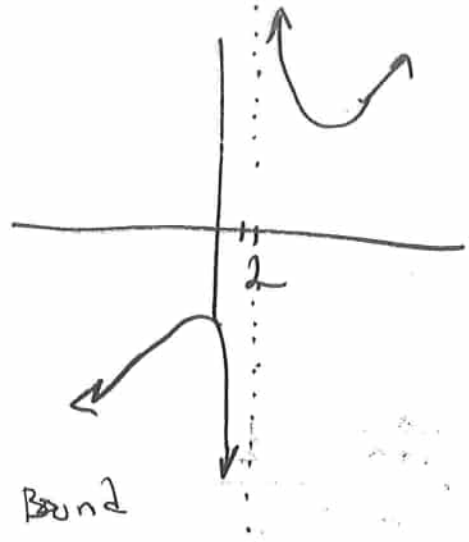
d. $\lim\limits_{x\to 4} f(x)\approx 2.7$

3. Evaluate the limit if it exists.

$$\lim_{x\to 2}\frac{x^2 - x + 6}{x - 2}$$

Does not factor

$$= \lim_{x\to 2}\frac{(x^2 - x + 6)}{x - 2} = \text{DNE}$$



inc and dec w/o Bound

4. Evaluate the limit if it exists.

$$\lim_{x\to 7}\frac{\sqrt{x+2} - 3}{x - 7}$$

$$= \lim_{x\to 7}\frac{\sqrt{x+2} - 3}{x - 7}\cdot\frac{\sqrt{x+2} + 3}{\sqrt{x+2} + 3} = \lim_{x\to 7}\frac{x + 2 - 9}{(x-7)(\sqrt{x+2} + 3)}$$

$$= \lim_{x\to 7}\frac{\cancel{x-7}}{(\cancel{x-7})(\sqrt{x+2} + 3)} = \lim_{x\to 7}\frac{1}{\sqrt{x+2} + 3} = \frac{1}{\sqrt{7+2} + 3}$$

$$= \frac{1}{6}$$

David Malawey

Quiz#1 MTH-220

1.1-1.4

1. Evaluate the difference quotient for the given function. Simplify your answer.

$f(x)=4+3x-x^2$, $\dfrac{f(3+h)-f(3)}{h}$

$f(3+h) = \dfrac{4+(9+3h)-(9+h^2+6h)-(4+9-9)}{h}$ $-3h$ $\dfrac{-h^2-6h}{h}$ $\boxed{-h-6}$

1

2. Find the domain of the function and sketch the function without the use of a calculator. Show your work. Justify your answer.

$f(t)=t^2-6t$

~~4-12~~ ~~9-18~~ 16-24

1+6 25-30

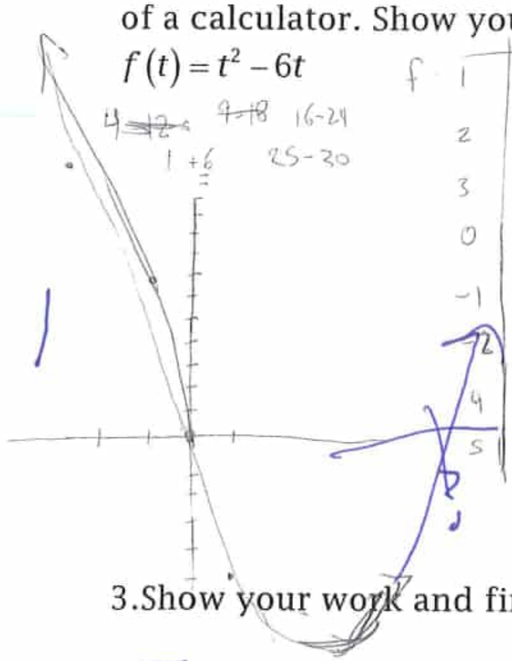
t	f
1	-5
2	-8
3	-9
0	0
-1	7
-2	16
4	-8
5	-5

Range

$D_f = [-9, \infty)$

stops decreasing

keeps increasing



1

3. Show your work and find $f\circ g\circ h$ if $f(x)=x+1$, $g(x)=2x$, $h(x)=x-1$.

$g\circ h(x) = 2(x-1)$

$f\circ g\circ h(x)\ (2(x-1))+1 = 2x-2+1 = \boxed{2x-1}$

–

1.5

5/10

4. A spherical balloon with radius r inches has volume $V(r)=\frac{4}{3}\pi r^3$. Find a function that represents the amount of air required to inflate the balloon from a radius of r inches to a radius of $r+1$ inches.

$$V(r+1) = \frac{4}{3}(r+1)^3 = \frac{4}{3}(r^2+2r+1)(r+1)$$

$$= \frac{4}{3}(r^3+r^2+2r^2+2r+r+1)$$

$$= \frac{4}{3}(r^3+3r^2+3r+1)$$

$$V(r+1) = \tfrac{4}{3}r^3+4r^2+4r+\frac{4}{3} \quad \cancel{\text{or } 4(r^2+3r+3)+\tfrac{4}{3}}$$

.5

c is constant

5. What do all of the members of the family of linear functions $f(x)=c-x$ have in common? Sketch 3 members of the family on the same Cartesian coordinate system.

c is 1

$f(1) = c-1$

$f(2) = c-2$

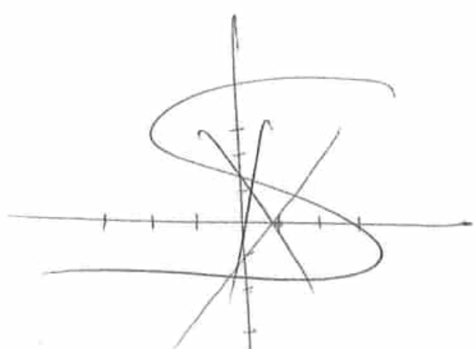
$f(x) = -x+c$

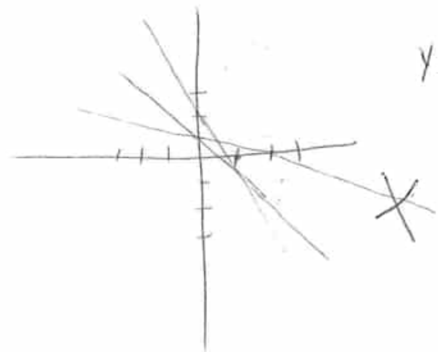
$f(4) = -4x+1$

$y = -4x+1$

$y = 2-1x$

1

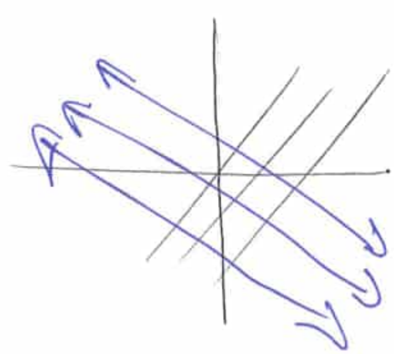




same slope ✓

$m = -1$

f(1)



Quiz#1 MTH-220

1.1-1.4

1. Evaluate the difference quotient for the given function. Simplify your answer.

$f(x) = 4 + 3x - x^2$, $\dfrac{f(3+h) - f(3)}{h}$

$$D(x) = \frac{4 + 3(3+h) - (3+h)^2 - [4 + 3(3) - 3^2]}{h} = \frac{4+9+3h-9-6h-h^2-4-9+9}{h}$$

$$= \frac{3h - 6h - h^2}{h} = \frac{h(-3-h)}{h} = -3-h$$

2. Find the domain of the function and sketch the function without the use of a calculator. Show your work. Justify your answer.

$f(t) = t^2 - 6t$ D_f : $\mathbb{R}$ (polynomial)

$= t(t-6)$

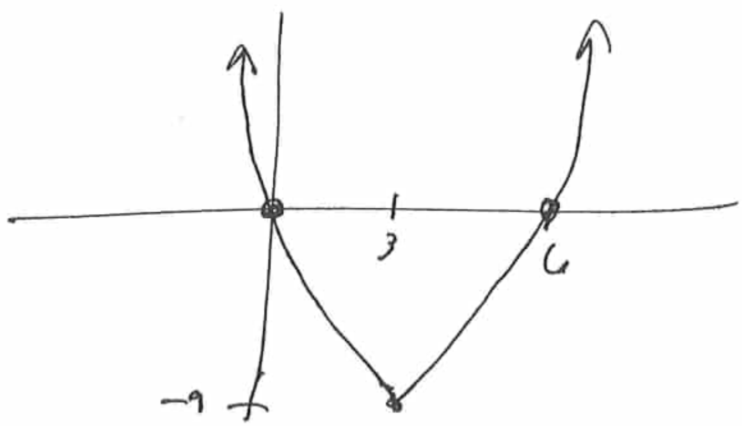
$t=0$ $t=6$ ← zeros

x ints $(0,0)(6,0)$

$h = \dfrac{-(-6)}{2(1)} = 3$

$k = 9 - 18 = -9$

$(3,-9)$: V

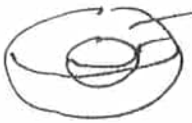


3. Show your work and find $f \circ g \circ h$ if $f(x) = x+1$, $g(x) = 2x$, $h(x) = x-1$.

$f(g(h(x))) = f(g(x-1)) = f(2(x-1)) = 2x-2+1$

$= 2x-1$

4. A spherical balloon with radius r inches has volume $V(r) = \frac{4}{3}\pi r^3$. Find a function that represents the amount of air required to inflate the balloon from a radius of r inches to a radius of $r+1$ inches.



Volume of shell between $= V(r+1) - V(r)$

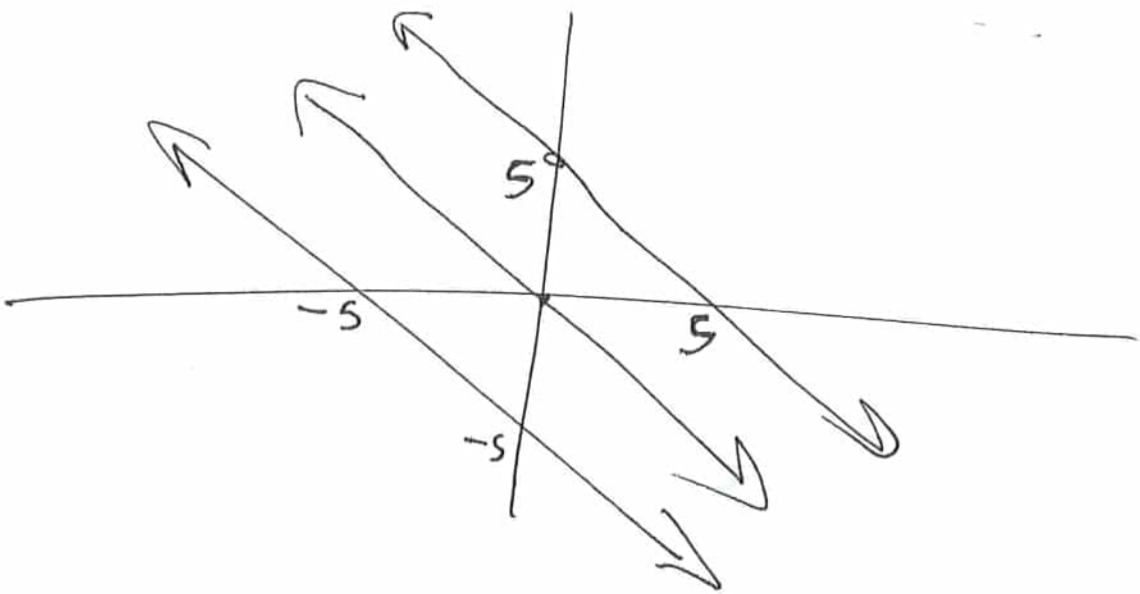
$$\boxed{= \frac{4}{3}\pi(r+1)^3 - \frac{4}{3}\pi r^3}$$

$$\frac{4}{3}\pi(r^3+3r^2+3r+1) - \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}(3r^2\pi) + \frac{4}{3}3r\pi + \frac{4}{3}\pi = \boxed{4\pi r^2 + 4r\pi + \frac{4}{3}\pi}$$

5. What do all of the members of the family of linear functions $f(x) = c - x$ have in common? Sketch 3 members of the family on the same Cartesian coordinate system.

$f(x) = -x + c$ has same slope $= -1$



David Makinney

7/15/08

Quiz#8 MTH210

5.2-5.4

Read Directions

1. Express this limit as a definite integral on the given interval. Do not evaluate. Do not evaluate.

$\lim_{n\to\infty}\sum_{i=1}^{n}\sqrt{2x_i^* + (x_i^*)^2}\ \Delta x,$ [2,6] Do not evaluate, read directions.

$= \int_2^6 \sqrt{2x + x^2}\ dx$

1

2. Evaluate the integral using the fundamental theorem of calculus part 2.

a. $\int_1^4 (5 - 2t + 3t^2)\ dt$ $= t^3 - t^2 + 5t \Big|_1^4 = 64 - 16 + 20 - (1 - 1 + 5(1))$ $\begin{array}{r} 68 \\ - 5 \\ \hline \end{array}$ $= \boxed{63}$

2

b. $\int_1^9 \frac{x-1}{\sqrt{x}}\ dx$ $= \int_1^9 x - 1 (x^{-1/2})\ dx = \int_1^9 x^{1/2} - x^{-1/2}\ dx = \frac{2}{3}x^{3/2} - 2x^{1/2} \Big|_1^9$

$= \frac{2}{3}(\sqrt{9}^3) - 2(\sqrt{9}) - \left(\frac{2}{3} 1 - 2(\sqrt{1})\right) = \frac{2}{3} \cdot 27 - 6 - (-1\frac{1}{3}) = 12 + 1\frac{1}{3} = 13\frac{1}{3}$

2

$\frac{40}{3}$

3. Find the general indefinite integral.

2

$\int\left(x^4 - \frac{1}{2}x^3 + \frac{1}{4}x - 2\right) dx$

$= \frac{1}{5}x^5 - \frac{1}{8}x^4 + \frac{1}{8}x^2 - 2x + C$

7/8

KBT

Quiz#8 MTH210

5.2-5.4

Read Directions

1. Express this limit as a definite integral on the given interval. Do not evaluate. Do not evaluate.

$\lim_{n\to\infty}\sum_{i=1}^{n}\sqrt{2x_i^* + (x_i^*)^2}\,\Delta x,$ $\quad[2,6]\quad$ Do not evaluate, read directions.

$$= \int_2^6 \sqrt{2x+x^2}\,dx$$

2. Evaluate the integral using the fundamental theorem of calculus part 2.

a. $\int_1^4 \left(5-2t+3t^2\right) dt$

$$= 5t - t^2 + t^3 \Big|_1^4$$

$$= 5(4) - 16 + 64 - [5-1+1]$$

$$= 20 - 16 + 64 - 5 = 84 - 21 = \boxed{63}$$

b. $\int_1^9 \frac{x-1}{\sqrt{x}}\,dx = \int_1^9 \left(\sqrt{x} - x^{-\frac12}\right)dx$

$$= \frac{1}{\frac32}x^{\frac32} - \frac{1}{\frac12}x^{\frac12}\Big|_1^9$$

$$= \frac23 x^{\frac32} - 2x^{\frac12}\Big|_1^9 = \frac23(9)^{\frac32} - 2(9)^{\frac12} - \left[\frac23(1)^{\frac32} - 2(1)^{\frac12}\right]$$

$$= \frac23(27) - 6 - \frac23 + 2 = 18 - 6 + 2 - \frac23$$

$$= 14 - \frac23 = \frac{42-2}{3}$$

$$\boxed{\frac{40}{3}}$$

3. Find the general indefinite integral.

$\int\left(x^4 - \frac12 x^3 + \frac14 x - 2\right) dx$

$$\boxed{= \frac15 x^5 - \frac18 x^4 + \frac18 x^2 - 2x + C}$$

David Malawey

100%
− Stupid Mistakes
= 6/10

Quiz#5 MTH210

4.1,4.2

1. Sketch the graph of f by hand and use your sketch to find the absolute and local maximum and minimum values of f. (State what they are and where they occur).

$f(x) = 1 - \sqrt{x}$

abs max $= f(0) = 1$ of f at $x = 0$

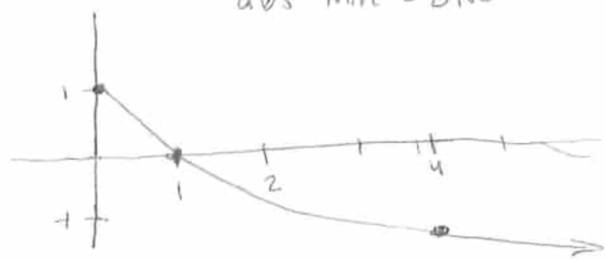
abs min = DNE

local max $\boxed{DNE}$

local min DNE

✓

1.5



2. Find the critical numbers of the function.

a. $s(t) = 3t^4 + 4t^3 - 6t^2$ $\quad D_{s(t)}: \mathbb{R}$

$s'(t) = 12t^3 + 12t^2 - 12t \quad D_{s(t)} = \mathbb{R}$

$= (12t)(t^2 + t - 1)$

$(12t)(t^2+t-1) = 0$

$t^2 + t - 1 = 0$

~~$t^2 + t - 1$~~

~~$t(t+1) = 1$~~

$\dfrac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2}$

$c = \dfrac{-1 \pm \sqrt{5}}{2} = -\dfrac{1}{2} \pm \dfrac{\sqrt{5}}{2}$

$\dfrac{-1 \pm \sqrt{5}}{2}$

2

Critical #'s: $c = 0,$

✓

b. $F(x) = x^{4/5}(x-4)^2 \quad D: \mathbb{R}$

$f'(x) = \frac{4}{5}x^{-1/5}(x-4)^2 + 2(x-4)(x)(x^{4/5})$

$f'(x) = \dfrac{\frac{4}{5}(x^2 - 8x + 16)}{\sqrt[5]{x}} + \dfrac{2x - 8(x)(x^{4/5})}{1}$

$= \dfrac{4x^2 - 32x + 64}{5\sqrt[5]{x}} + \dfrac{2x^2 - 8x\,(x^{4/5})}{1} = 0$

$4x^2 - 32x + 64 = -(2x^2 - 8x\,(x^{4/5}))(5x^{1/5})$

$4x^2 - 32x + 64 = -2x^2 + 8x - 5x + 3x$

$6x^2 - 35x + 64 = 0$

1

?

Critical #'s: $0, \dfrac{35 \pm \sqrt{2 \cdot 6 \ldots}}{2 \cdot 6}$

3. Find the absolute maximum and minimum values on the interval.

$f(x) = 3x^2 - 12x + 5 \qquad [0,3]$

1

$f'(x) = 6x - 12$

$6x - 12 = 0$

$6x = 12$

$x = \frac{12}{6} = 2$

$3(4) - 12(4) + 5 = 12 - 48 + 5 = -31$ abs. min @ $x = 2$

?

$12 - 24 + 5 = -12 + 5 = \boxed{-7 \text{ abs min @ } x = 2}$

6/10

4. Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all the numbers c that satisfy the the conclusion of the mean value theorem.

$f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$ $\quad [0,1]$ $\quad$ Domain includes $[0,1]$

$f'(x) = \frac{1}{3}x^{\frac{2}{3}} = \frac{1}{3\sqrt[3]{x}}$ $\quad$ Domain includes $(0,1)$

?

$f(0) = 0$

$f(1) = 1$

Stupid Mistakes

.5 $\quad$ mean slope = 1

$\frac{1}{3}x^{\frac{-1}{3}} = 1$

$\frac{1}{3\sqrt[3]{x}} = 1$

$1 = 3\sqrt[3]{x}$

$\frac{1}{3} = \sqrt[3]{x}$

$\boxed{\sqrt[3]{\frac{1}{3}} = x} = c$

$4(x^2 - 8x + 16)$

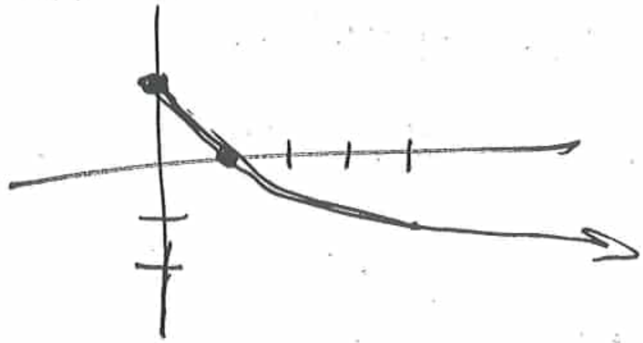
$(x + 4)(x - 4)$

Quiz#5 MTH210

4.1,4.2

1. Sketch the graph of f by hand and use your sketch to find the absolute and local maximum and minimum values of f. (State what they are and where they occur).

$f(x)=1-\sqrt{x}$



Local max
ABS max of 1 at x = 0

2. Find the critical numbers of the function.

a. $s(t)=3t^4+4t^3-6t^2$ $D_s: \mathbb{R}$

$s'(t) = 12t^3+12t^2-12t$

$12t(t^2+t-1)$

$12t=0$ or $t=\frac{-1\pm\sqrt{1-4(1)(-1)}}{2}$

$t \; \frac{-1\pm\sqrt{5}}{2}$

$c=0$ and $c=\frac{-1+\sqrt{5}}{2}$

$c=\frac{-1-\sqrt{5}}{2}$

b. $F(x)=x^{4/5}(x-4)^2$ $D_F: \mathbb{R}$

$F'(x)=\frac{4}{5}x^{-\frac{1}{5}}(x-4)^2+2(x-4)\cdot x^{\frac{4}{5}}$

$=\frac{4(x-4)^2}{5\sqrt[5]{x}}+\frac{2(x-4)\cdot\sqrt[5]{x^4}\cdot 5\sqrt[5]{x}}{5\sqrt[5]{x}}$

$=\frac{4(x-4)^2+2(x-4)5x}{5\sqrt[5]{x}}$

$=\frac{4x^2-32x+64+10x^2-40x}{5\sqrt[5]{x}}$

$=\frac{14x^2-72x+64}{5\sqrt[5]{x}}=\frac{2(7x^2-36x+32)}{5\sqrt[5]{x}}$

$\frac{2(7x-8)(x-4)}{5\sqrt[5]{x}}$

$c=x=\frac{8}{7}$ $c=4$

$c=0$

3.Find the absolute maximum and minimum values on the interval.

$f(x)=3x^2-12x+5$ $[0,3]$ f is continuous on $[0,3]$ and diff. on $(0,3)$

$f(0)=5$

$f(3)=27-36+5=-4$

$f'(x)=6x-12$ $f(2)=12-24+5=-7$

$0=6x-12$

$2=x$

Max of 5 at x = 0
min of -7 at x = 2

4. Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all the numbers c that satisfy the the conclusion of the mean value theorem.

$f(x) = \sqrt[3]{x}$ $[0,1]$

Mean Value Thm.

$\frac{f(1) - f(0)}{1-0} = \frac{1-0}{1-0} = 1$

f(x) is cont on [0,1] [root fun]

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3x^{2/3}}$ is Diff on (0,1)

$f'(x) = 1 = \frac{1}{3}x^{-\frac{2}{3}}$ $x \neq 0$ $m = \frac{1-0}{1-0} = 1$

$x^{2/3} = \frac{1}{3}$

$x^2 = \frac{1}{27}$

$\boxed{x = \frac{1}{\sqrt{27}}}$

David Malawey 7/9/08

Quiz#7 MTH210

4.9-5.1

1. Find f.

$f''(x) = 24x^2 + 2x + 10, \qquad f(1) = 5, \qquad f'(1) = -3$

✓ $f'(x) = 8x^3 + x^2 + 10x + C \qquad f'(1) = -3 = 8 + 1 + 10 + C \qquad -3 - 19 = C = -22$ ✓

$f(x) = 2x^4 + \frac{1}{3}x^3 + 5x^2 - 22x + D$

$f(1) = 5 = \underbrace{2 + \frac{1}{3} + 5 - 22}_{-14\,2/3} + D \qquad D = 5 + 14\,2/3 = 19\,2/3$

2

$f(x) = 2x^4 + \frac{1}{3}x^3 + 5x^2 - 22x + 19\,2/3$

2. A particle is moving with the given data. Find the position function of the particle.

$a(t) = t - 2, \qquad s(0) = 1, \qquad v(0) = 3$

$V(t) = \frac{1}{2}t^2 - 2t + C \qquad V(0) = 3 = 0 + 0 + C, \quad C = 3$

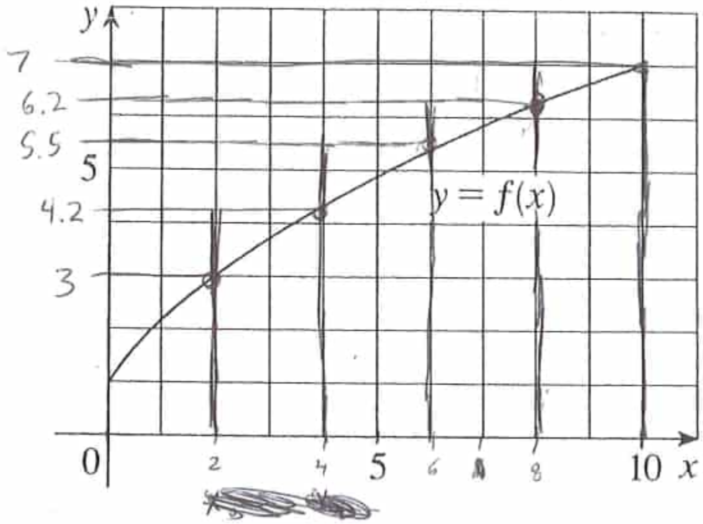
$s(t) = \frac{1}{6}t^3 - t^2 + 3t + D \qquad s(0) = 1 = \frac{1}{6}(0) - 0 + 0 + D, \quad D = 1$

$s(t) = \frac{1}{6}t^3 - t^2 + 3t + 1$

2

6/6

3. By reading the values from the graph of f, use five rectangles to find an upper estimate for the area under the graph from x=0 to x=10



$\Delta x = \left(\frac{10-0}{5}\right) = 2$

upper L

$A \approx R_5 = 2(3 + 4.2 + 5.5 + 6.2 + 7)$

2

$= 2(25.9)$

$= 51.8 \text{ units}^2$

.9
25

Quiz#7 MTH210

4.9-5.1

1. Find f.

$f''(x) = 24x^2 + 2x + 10, \quad f(1) = 5, \quad f'(1) = -3$

$f'(x) = \frac{24}{3}x^3 + \frac{2}{2}x^2 + 10x + C = 8x^3 + x^2 + 10x + C$

$f(x) = 2x^4 + \frac{1}{3}x^3 + 5x^2 + Cx + D$

$-3 = 8 + 1 + 10 + C \quad (f'(1) = -3)$

$\boxed{f(x) = 2x^4 + \frac{1}{3}x^3 + 5x^2 - 22x + \frac{59}{3}}$

$-3 = 19 + C \quad C = -22$

$5 = 2 + \frac{1}{3} + 5 - 22 + D \quad D = 20 - \frac{1}{3} = \frac{60}{3} - \frac{1}{3} = \frac{59}{3}$

2. A particle is moving with the given data. Find the position function of the particle.

$a(t) = t - 2, \quad s(0) = 1, \quad v(0) = 3$

$v(t) = \frac{1}{2}t^2 - 2t + C$

$s(t) = \frac{1}{6}t^3 - t^2 + Ct + D$

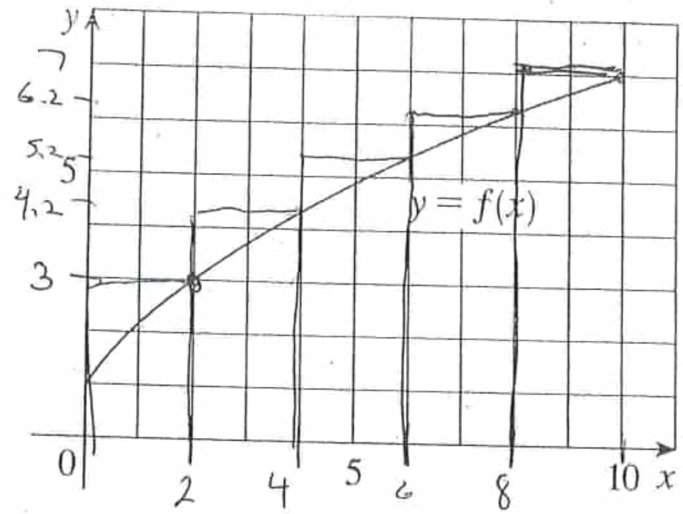
$v(0) = 3 = \frac{1}{2}(0)^2 - 2(0) + C$

$3 = C$

$s(0) = \frac{1}{6}(0) - 1(0) + 3(0) + \boxed{D = 1}$

$\boxed{s(t) = \frac{1}{6}t^3 - t^2 + 3t + 1}$

3. By reading the values from the graph of f, use five rectangles to find an upper estimate for the area under the graph from $x=0$ to $x=10$



$\Delta x = \frac{10-0}{5} = \frac{10}{5} = \boxed{2}$

$x_i = 0 + i(2)$

$R_5 = 2[3 + 4.2 + 5.2 + 6.2 + 7]$

$= 2[7.2 + 11.4 + 7]$

$= 2[18.6 + 7]$

$= 2[25.6] = \boxed{51.2 \text{ sq units}}$

David Malawey

Quiz#4 MTH210

3.5-3.7

1. Find the derivative of the function.

a. $g(t)=\frac{1}{(t^4+1)^3}$ $= (t^4+1)^{-3}$

$g'(t) = -3(t^4+1)^{-4} \cdot (4t^3)$

b. $y=\sin\sqrt{1+x^2}$ $= \sin(1+x^2)^{1/2}$

$y' = \cos(1+x^2)^{\frac{1}{2}} \cdot \left(\frac{1}{2}(1+x^2)^{-1/2}\right) \cdot (2x)$

2

2

2. The displacement of a particle on a vibrating string is given by the equation $s(t)=10+\frac{1}{4}\sin(10\pi t)$, where s is measured in centimeters and t in seconds. Find the velocity of the particle after t seconds.

$s'(t) = v(t) = \frac{1}{4}\cos(10\pi t) \cdot [10\pi]$

$= \frac{10}{4}\pi\cos(10\pi t) = \frac{5}{2}\pi\cos(10\pi t)$

2

3. Find $\frac{dy}{dx}$ by implicit differentiation.

$x^2+xy-y^2=4$

$2x + x\frac{dy}{dx} + 1(y) - 2y\frac{dy}{dx} = 0 \qquad x\frac{dy}{dx} - 2y\frac{dy}{dx} = -2x - y$

$\boxed{\frac{dy}{dx} = \frac{-2x-y}{x-2y}}$

2

4. Use implicit differentiation to find an equation of the line tangent to the curve $x^2+xy+y^2=3$ at the point $(1,1)$.

$2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$

$x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - y$

$\frac{dy}{dx}(x+2y) = -2x - y$

$\boxed{\frac{dy}{dx} = \frac{-2x-y}{x+2y}}$

y'

$2x + \frac{x(-2x-y)}{x+2y} + y + \frac{2y(-2x-y)}{x+2y}$

$2x \quad \frac{-2x^2-y}{x+2y} + 2y - \left(\frac{4xy+2y^2}{x+2y}\right)$

$2 + \left(\frac{-2-1}{3}\right) + 1 - \left(\frac{4+2}{3}\right)$

$2 - 1 + 1 - \frac{5}{3} \quad m = -11/3$

$y - 1 = -11/3(x-1)$

$y = 1 - 11/3 - 11/3x$

$y = -\frac{11}{3}x - \frac{8}{3}$

1

13/14

5. A particle moves according to a law of motion $s(t) = cos\left(\frac{\pi t}{4}\right)$, $t \geq 0$, where t is measured in seconds and s is in feet.

a. Find the velocity at time t.

$s'(t) = v(t) = -\sin\left(\frac{\pi t}{4}\right) \cdot \left(\frac{\pi}{4}\right) = -\frac{\pi}{4} \sin\left(\frac{\pi t}{4}\right)$

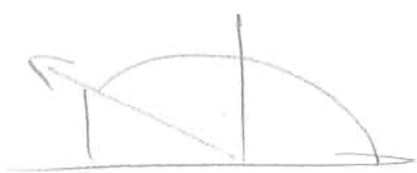
2

b. What is the velocity after 3seconds?

$v(3) = -\frac{\pi}{4} \sin \frac{3\pi}{4} = -\frac{\pi}{4} \cdot \frac{\sqrt{2}}{2} = \boxed{-\frac{1}{8} \sqrt{2}\,\pi}$

2

Sin 45 Q 2



KEY

Quiz#4 MTH210

3.5-3.7

1. Find the derivative of the function.

a. $g(t) = \frac{1}{(t^4+1)^3}$ $= (t^4+1)^{-3}$

$\boxed{g'(t) = -3(t^4+1)^{-4} \cdot 4t^3}$

$= \frac{-12t^3}{(t^4+1)^4}$

b. $y = \sin\sqrt{1+x^2}$

$\boxed{y' = \cos(\sqrt{1+x^2}) \cdot \frac{1}{2}(1+x^2)^{-1/2} \cdot 2x}$

$= \frac{x\cos(\sqrt{1+x^2})}{\sqrt{1+x^2}}$

2. The displacement of a particle on a vibrating string is given by the equation $s(t) = 10 + \frac{1}{4}\sin(10\pi t)$, where s is measured in centimeters and t in seconds. Find the velocity of the particle after t seconds.

$s'(t) = \boxed{v(t) = \frac{1}{4}\cos(10\pi t) \cdot 10\pi}$

$\boxed{v(t) = \frac{5\pi}{2}\cos(10\pi t)}$

3. Find $\frac{dy}{dx}$ by implicit differentiation.

$x^2 + xy - y^2 = 4$

$2x + y + x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(x - 2y) = -2x - y$

$\boxed{\frac{dy}{dx} = \frac{-2x-y}{x-2y}}$

4. Use implicit differentiation to find an equation of the line tangent to the curve $x^2 + xy + y^2 = 3$ at the point $(1,1)$.

$2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(x + 2y) = -y - 2x$

$\frac{dy}{dx} = \frac{-y-2x}{x+2y}$

$y'(1) = \frac{-1-2(1)}{1+2(1)} = \frac{-3}{3} = -1$

$y - 1 = -1(x-1)$

$\boxed{y = -x + 2}$

5. A particle moves according to a law of motion $s(t) = cos\left(\frac{\pi t}{4}\right)$, $t \geq 0$, where t is measured in seconds and s is in feet.

a. Find the velocity at time t.

$s'(t) = v(t) = -\sin\left(\frac{\pi t}{4}\right) \cdot \frac{\pi}{4}$

b. What is the velocity after 3seconds?

$v(3) = \frac{\pi}{4}\left(-\sin\frac{3\pi}{4}\right)$

$= \frac{\pi}{4}\left(-\frac{\sqrt{2}}{2}\right) = \cancel{\frac{\pi\sqrt{2}}{8}}$

$-\frac{\pi\sqrt{2}}{8}\ \frac{ft}{s}$

David Malavey

$$\frac{(a+h)-1}{(a+h)-2}\cdot\frac{(a+h)+2}{(a+h)+2} = \frac{(a+h)^2+(a+h)-2}{(a+h)^2-4}\cdot\frac{(a+h)+1}{(a+h)+1} =$$

$$\frac{(a+h)^3+2(a+h)^2-(a+h)-2}{(a+h)^3-4(a+h)+(a+h)^2-4} \qquad \frac{(a+h)^3+(a+h)^2-2(a+h)+(a+h)^2+(a+h)-2}{(a+h)^3-4(a+h)+(a+h)^2-4}$$

On back

Quiz#3 MTH210

3.1-3.4

1. Use Equation #2 to find the slope of the line tangent to the curve, $y=\frac{x-1}{x-2}$, at the point, $\overset{(a,\,f(a))}{(3,2)}$.

$$y' = \lim_{h\to 0} = \frac{\frac{(a+h)-1}{(a+h)-2}-\left(\frac{3-1}{3-2}\right)}{h} = \lim_{h\to0}\frac{\boxed{\frac{(a+h)-1}{(a+h)-2}}-2}{h} = \lim_{h\to0}\frac{\frac{3+h-1}{3+h-2}-2}{h} = \lim_{h\to0}\frac{\frac{2+h}{1+h}-2}{h}$$

/

$$\lim_{h\to0}\frac{\frac{2+h(1+h)}{(1+h)(1-h)}-2}{h} = \frac{\frac{2+h-2h-h^2}{1-h^2}-2}{h} = \frac{\frac{-h^2-h+2}{-h^2+1}\cdot\frac{(2-h)}{(2-h)}}{h} = \frac{\frac{-2h^2-2h+4+h^3+h^2-2h}{-2h^2+h^3+2-h}}{h}$$

$$= \frac{-h^2+h^3-4h+4}{-2h^2+h^3-h+2}$$

2. This limit represents the derivative of some function f at some number $x=a$. **State** f and a for this limit.

/.

$$\lim_{h\to0}\frac{\cos(\pi+h)+1}{h}$$

$f=\cos x$ $\qquad$ $a=\pi$

$$f'(a)=\lim_{h\to0}\frac{f(a+h)-f(a)}{h}$$

$f(a)=1$ $\quad f(a+h)=\cos(\pi+h)$

3. Find the derivative using the definition of the derivative. State the domain of the function and the domain of the derivative.

/

$f(t)=5t-9t^2$

$$=\lim_{h\to0}\frac{5(t+h)-9(t+h)^2-(5t-9t^2)}{h} = \lim_{h\to0}\frac{5t+5h-9(t^2+2th+h^2)-5t+9t^2}{h}$$

$$=\lim_{h\to0}\frac{-18th-9h^2+5h}{h} = \lim_{h\to0}-18t-9h+5 = -18t+5$$

4. Differentiate. (use differentiation formulas)

2

a. $Y(t)=6t^{-9}$

$$Y'(t)=-9(6)t^{(-9-1)} = -54t^{-10}$$

5

b. $y=\frac{r^2}{1+\sqrt{r}}$

$$y'=\frac{2r}{1+\frac{1}{2}r^{\frac{1}{2}-1}} = \frac{2r}{1+\frac{1}{2}r^{-1/2}}$$

$$=\frac{2r}{1}+\frac{2r}{\frac{1}{2}r^{-1/2}} = 2r + r^{-1/2}$$

$$2r+\frac{1}{r^2}$$

6/12

5. Differentiate.

$g(t) = t^3 \cos t$

$$g'(t) = \frac{3t^2(-\sin t)}{?} = \boxed{-3t\sin t}$$

-5

1331

$$\frac{(a+h)^3 + 2(a+h)^2 - a - h - 2}{(a+h)^3 - 4a - 4h + a + h^2 - 4}$$

$$= \frac{a^3 + 3a^2h + 3h^2a + h^3 + 2a^2 + 4ah + 2h^2 - a - h - 2}{a^3 + 3a^2h + 3h^2a + h^3 - 4a - 4h + a + h^2 - 4} =$$

h

$$\frac{\frac{(a+h)-1}{(a+h)-2} - 2}{h} = \frac{\frac{3+h-1}{3+h-2}}{} = \frac{\frac{2+h}{1+h} - \frac{2(1+h)}{1+h}}{h}$$

$$= \frac{\frac{2+h-2h-2}{1+h}}{h}$$

Quiz#3 MTH210

3.1-3.4

1. Use Equation #2 to find the slope of the line tangent to the curve, $y=\frac{x-1}{x-2}$, at the point, $(3,2)$.

$\lim_{h\to 0} \frac{\left(\frac{3+h-1}{3+h-2}\right)-2}{h} = \lim_{h\to 0} \frac{3+h-1-2[3+h-2]}{h[3+h-2]} = \lim_{h\to 0} \frac{3+h-1-6-2h+4}{h[3+h-2]}$

$= \lim_{h\to 0} \frac{-h}{h[h+1]} = \lim_{h\to 0} \frac{-1}{h+1} = -1$

2. This limit represents the derivative of some function f at some number x=a. **State** f and a for this limit.

$\lim_{h\to 0}\frac{\cos(\pi+h)+1}{h}$ $f= \cos x$ $a= \pi$

3. Find the derivative using the definition of the derivative. State the domain of the function and the domain of the derivative.

$f(t)=5t-9t^2$

$\lim_{h\to 0} \frac{5(t+h)-9(t^2+2th+h^2)-[5t-9t^2]}{h} = \lim_{h\to 0} \frac{5t+5h-9t^2-18th-9h^2-5t+9t^2}{h}$

$= \lim_{h\to 0} \frac{5h-18th-9h^2}{h} = \lim_{h\to 0} 5-18t-9h = 5-18t$

$D_f := D_{f'} = \mathbb{R}$

4. Differentiate. (use differentiation formulas)

a. $Y(t)=6t^{-9}$

$Y'(t) = -54t^{-10}$

b. $y=\frac{r^2}{1+\sqrt{r}}$

$y' = \frac{(1+\sqrt{r})(2r)-r^2\left(\frac{1}{2}r^{-\frac{1}{2}}\right)}{(1+\sqrt{r})^2}$

5. Differentiate.

$g(t) = t^3 \cos t$

$g'(t) = 3t^2 \cos t - t^3 \sin t$

Notes 6/5/08

if c is an element of R

If $c \in \mathbb{R}$ and $\lim_{x\to a} f(x)$ exists and $\lim_{x\to a} g(x)$ exists. (meaning that they are not infinite or undefined)

Then the following is true:

lim of sum of 2 func

1. $\lim_{x\to a}\left[f(x)+g(x)\right]=\lim_{x\to a} f(x)+\lim_{x\to a} g(x)$ $= L+M$

diff 2 func

2. $\lim_{x\to a}\left[f(x)-g(x)\right]=\lim_{x\to a} f(x)-\lim_{x\to a} g(x)$ $= L-M$

* 3. $\lim_{x\to a}\left[cf(x)\right]=c\lim_{x\to a} f(x)$ $= cL$

4. $\lim_{x\to a}\left[f(x)\cdot g(x)\right]=\lim_{x\to a} f(x)\cdot\lim_{x\to a} g(x)$

quotients

5. $\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right]=\frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ *IF!* $\lim_{x\to a} g(x)\neq 0$

constant func.

6. $\lim_{x\to a} c = c$

p84 1d $\lim_{x\to 2}\frac{3f(x)}{g(x)}=$

7. $\lim_{x\to a}\left[f(x)\right]^n=\left[\lim_{x\to a} f(x)\right]^n$

8. $\lim_{x\to a} x = a$

9. $\lim_{x\to a} x^n = a^n$

10. $\lim_{x\to a}\sqrt[n]{x}=\sqrt[n]{a}$

11. $\lim_{x\to a}\sqrt[n]{f(x)}=\sqrt[n]{\lim_{x\to a} f(x)}$

12. If f is a polynomial $\lim_{x\to a} f(x)=f(a)$

Evaluate using limit laws and or techniques:

$$\lim_{x\to2}\left(\frac{2x^2+1}{x^2+6x-4}\right) = \frac{\lim_{x\to2}(2x^2+1)}{\lim_{x\to2}(x^2+6x-4)} = \frac{\lim_{x\to2}2x^2+\lim_{x\to2}1}{\lim_{x\to2}x^2+\lim_{x\to2}6x-\lim_{x\to2}4}$$

$$\frac{2\lim_{x\to2}x^2+\lim_{x\to2}1}{\lim_{x\to2}x^2+6\lim_{x\to2}x-\lim_{x\to2}4} = \frac{2(4)+1}{4+6(2)-4} = \frac{9}{12} = \boxed{\frac{3}{4}}$$

Can't use $x = -4$

$$\lim_{x\to-4}\frac{x^2+5x+4}{x^2+3x-4} \quad \lim_{x\to-4}\frac{\cancel{(x+4)}(x+1)}{(x+4)(x-1)} \quad \lim_{x\to-4}\frac{x+1}{x-1} \quad \frac{\lim_{x\to-4}x+\lim_{x\to-4}1}{\lim_{x\to-4}x-\lim_{x\to-4}1} = \frac{-4+1}{-4-1} = \frac{-3}{-5} = \frac{3}{5}$$

1
1 1
1 2 1
1 3 3 1

use pascal's triangle

$$\lim_{h\to0}\frac{(2+h)^3-8}{h} = 8+3(2^2h)+3(h^2 2)+h^3-8$$

$$= \frac{12h + 6h^2 + h^3}{h}$$

$$\lim_{h\to0} 12+6h+h^2 = \lim_{h\to0}12 + 6\lim_{h\to0}h + \lim_{h\to0}h^2$$

$$= 12+6(0)+0 = 12$$

Theorem: $\lim_{x\to a} f(x) = L$ If and only if $\lim_{x\to a^-} f(x) = L = \lim_{x\to a^+} f(x)$

Show: $\lim_{x\to0}|x| = 0 \leftarrow$ true

$$\lim_{x\to0^+}|x| = \lim_{x\to0^+}x = 0$$

$$\lim_{x\to0^-}|x| = \lim_{x\to0^-}-x = -\left[\lim_{x\to0^-}x\right] = 0$$

$\lim_{x\to0}\frac{|x|}{x}$ = D.N.E.

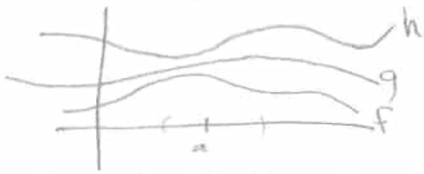
$$\left.\begin{aligned}\lim_{x\to0^+}\frac{|x|}{x} &= \lim_{x\to0^+}\frac{x}{x} = \lim_{x\to0^+}1 = 1\\ \lim_{x\to0^-}\frac{|x|}{x} &= \lim_{x\to0^-}\frac{-x}{x} = \lim_{x\to0^-}-1 = -1\end{aligned}\right\}\text{not the same}$$

DNE

Absolute value function: $f(x) = |x| = \begin{cases} x & x\geq0 \\ -x & x<0\end{cases}$

The squeeze theorem. p 83

If g is between f and h near x=a , and the limit as x approaches a of f and h is L, then the limit of g as x approaches a is also L.



$\lim_{x \to a} f(x) = L$
AND
$\lim_{x \to a} h(x) = L$
$\Big\} \lim_{x \to a} g(x) = L$

Example 11: show that $\lim_{x \to 0} x^2 \sin\frac{1}{x} = 0$

I know -1 is always $\leq \sin\frac{1}{x} \leq 1$

$-x^2 \leq x^2 \sin\frac{1}{x} \leq x^2$

$\lim_{x \to 0} -x^2 = 0$

AND

$\lim_{x \to 0} x^2 = 0$

$0 \leq \lim_{x \to 0} \left(x^2 \sin\left(\frac{1}{x}\right)\right) \leq 0$

True by squeeze theorem