# ME/AE 160 Dynamics

## Spring 2010

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| **Instructor** | Dr. Xiaoping Du<br>Office: Toomey Hall 290D<br>Tel: 573-341-7249<br>E-mail: dux@mst.edu |
| **Lecture** | TuTh 12:30PM - 1:45PM, Toomey Hall 295 |
| **Homework** | Late submittal is not accepted. |
| **Website** | Blackboard |
| **Make-ups** | Missed exams result in zero points. If it is absolutely impossible for you to be present for the exam due to illness, emergency, or other reasons, you must notify me as soon as you are aware of it. |
| **Attendance** | Each lecture introduces significant new materials. If class is missed, you are responsible for obtaining announced information, handouts, and notes from other students or from me. If you miss more than 2 or 3 class sessions and do not make them up, you will certainly get into difficulties. |
| **Reading** | Reading text materials for each day's class is expected prior to attending class. |
| **Text** | Engineering Mechanics – Dynamics, 12th edition, by Russell Hibbeler, Prentice Hall, 2010 |
| **Grade** | Homework 20%<br>Computer assignments (ADAMS) 10%<br>Hour exams (2) 40%<br>Common final 30%<br>TOTAL 100% |

| Date | Day | Lecture | Topic | Reading | Homework |
|---|---|---|---|---|---|
| 12-Jan | Tue | 1 | Introduction & Kinematics | 12.2-3 | 12-10,15,20,30,47 |
| 14-Jan | Thu | 2 | Curvilinear motion | 12.4-6 | 12-75,76,87,102,110 |
| 19-Jan | Tue | 3 | Curvilinear motion | 12.7-8 | 12-118,130,132,159,170 |
| 21-Jan | Thu | 4 | Dependent & relative motion | 12.9-10 | 12-195,203,214,220,230 |
| 26-Jan | Tue | 5 | Newton's laws | 13.1-4 | 13-5,20,33,35,43 |
| 28-Jan | Thu | 6 | Equations of motion | 13.5-6 | 13-59,74,82,102,107 |
| 2-Feb | Tue | 7 | Work & energy | 14.1-3 | 14-15,23,34,39,41 |
| 4-Feb | Thu | 8 | Power & energy conservation | 14-4-6 | 14-61,67,83,99,106 |
| 9-Feb | Tue | 9 | Impulse & momentum | 15.1-3 | 15-5,19,31,42,46 |
| 11-Feb | Thu | 10 | Impact & angular momentum | 15.4-7 | 15-59,69,86,91,107 |
| 16-Feb | Tue | | ADAMS | | |
| 18-Feb | Thu | | **Exam 1** | | |
| 23-Feb | Tue | 11 | Translation, rotation & absolute motion | 16.1-4 | 16-3,7,30,42,50,52 |
| 25-Feb | Thu | 12 | Relative velocity | 16.5 | 16-55,67,68,73 |
| 2-Mar | Tue | 13 | Instantaneous center | 16.6 | 16-89,93,98,101,104,107 |
| 4-Mar | Thu | 14 | Relative acceleration | 16.7 | 16-110,114,131 |
| 9-Mar | Tue | 15 | Rotating axes | 16.8 | 16-139,140,157,159 |
| 11-Mar | Thu | | **Spring recess** | | |
| 16-Mar | Tue | 16 | Moment of inertia, translation & rotation | 17.1-4 | 17-14,38,55,63,90 |
| 18-Mar | Thu | 17 | General plane motion | 17.5 | 17-103,109,111,114,123 |
| 23-Mar | Tue | 18 | Work & energy | 18.1-5 | 18-15,26,44,56,67 |
| 25- Mar | Thu | 19 | Impulse & momentum | 19.1-3 | 19-10,23,31,34,37 |
| 30- Mar | Tue | | **Spring break** | | |
| 1-Apr | Thu | | **Spring break** | | |
| 6-Apr | Tue | 20 | Impact | 19.4 | 19-44,48,54,55 |
| 8-Apr | Thu | 21 | 3-D kinematics | 20.1-2 | 20-3,6,10 |
| 13-Apr | Tue | | **Exam 2** | | |
| 15-Apr | Thu | 22 | 3-D kinematics | 20.1-2 | 20-11,13,14 |
| 20-Apr | Tue | 23 | General motion | 20.3 | 20-22,23,26 ✓ |
| 22-Apr | Thu | 24 | Rotating axes | 20.4 | 20-42,47,51 |
| 27-Apr | Tue | 25 | 3-D kinetics | 21.1-3 | 21-14,27,35,38 |
| 29-Apr | Thu | 26 | 3-D kinetics | 21.4 | 21-32,37 |
| 4-May | Tue | 27 | 3-D kinetics | 21.4 | 21-44,46,59 |
| 6-May | Thu | 28 | 3-D kinetics | 21.4 | 21-50,51 |

email Dr. Du to meet for a quick lecture

Exams

1 95

2 75

HOMEWOR

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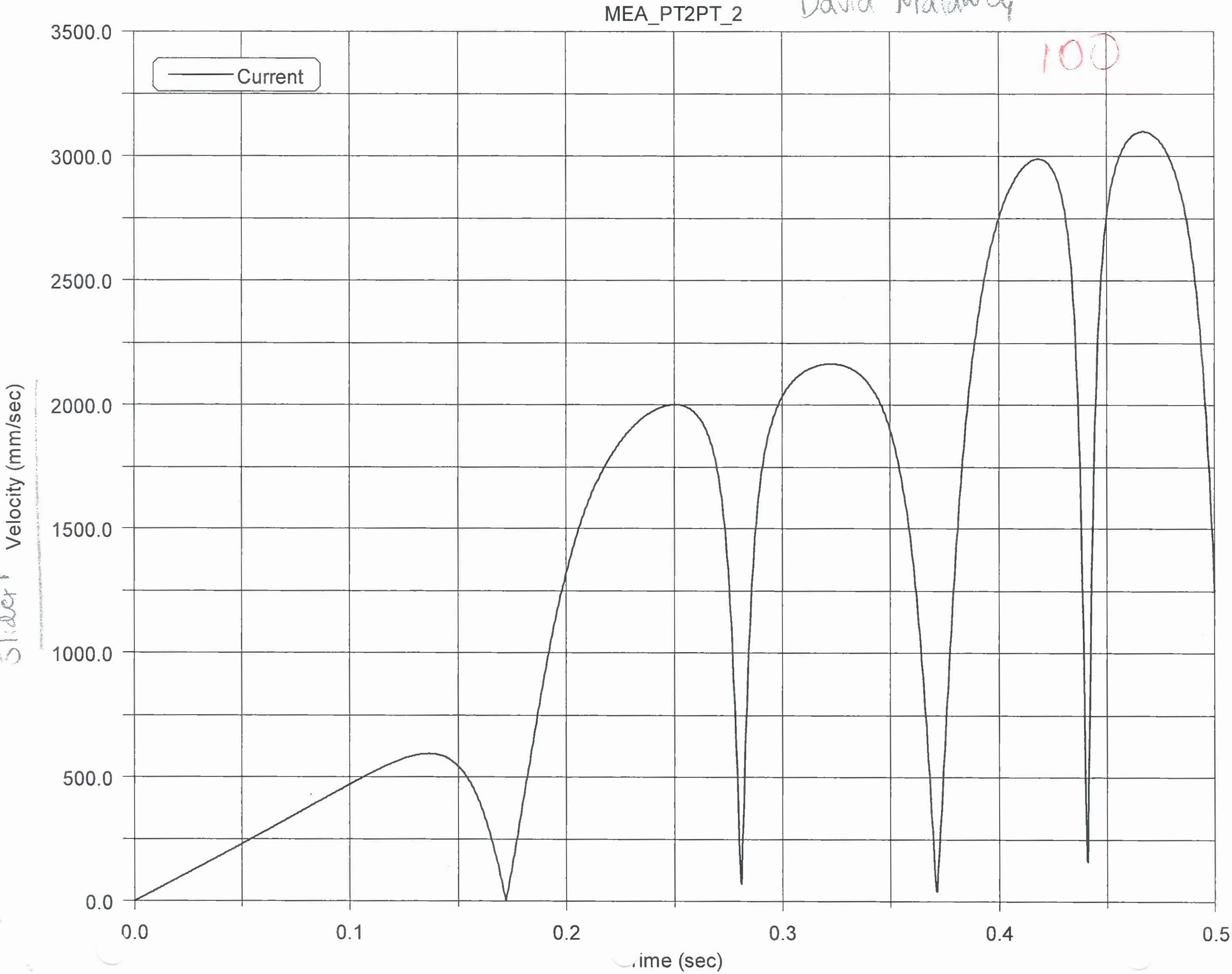
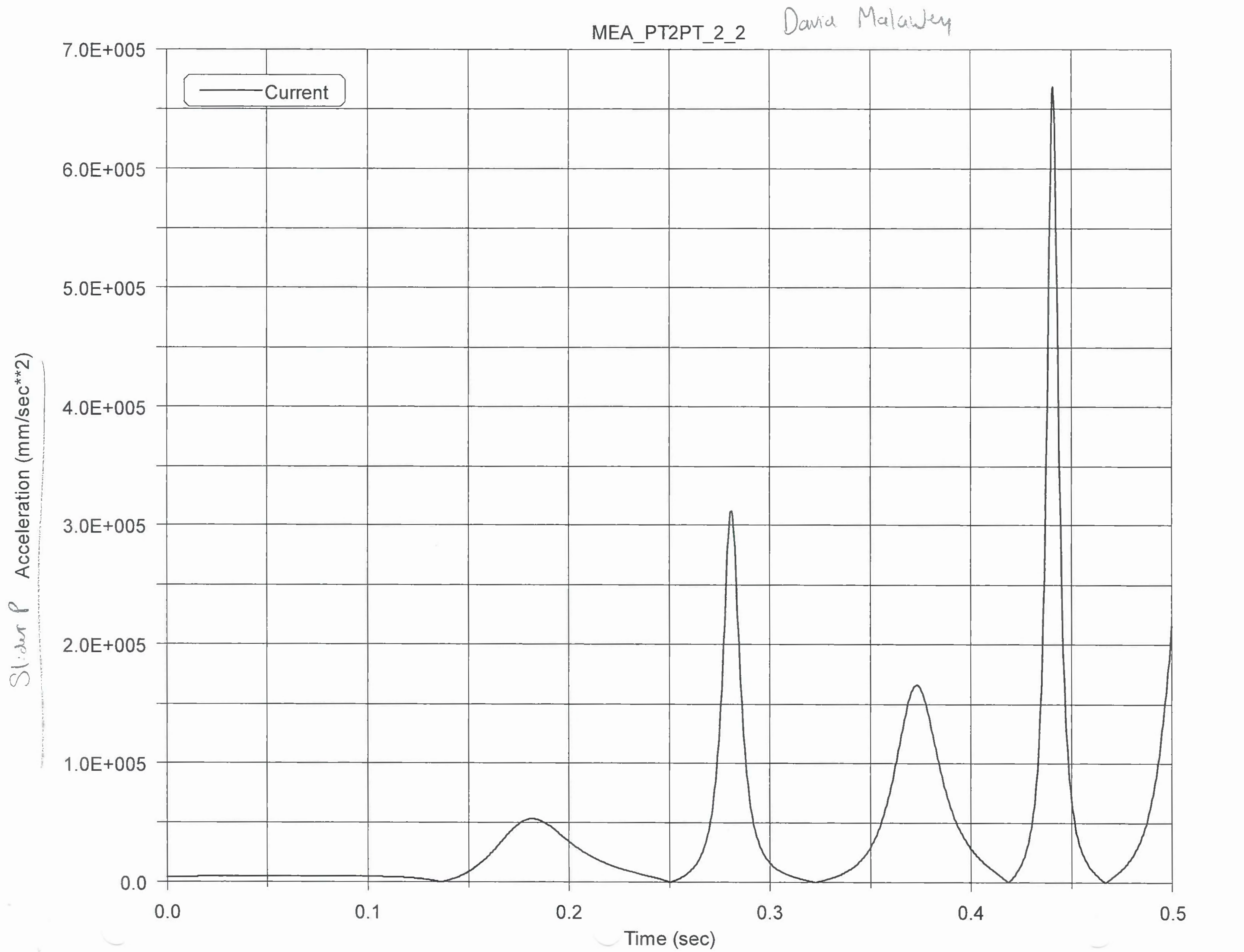
100

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David Malawey
Current
Slider P
Acceleration (mm/sec**2)
7.0E+005
6.0E+005
5.0E+005
4.0E+005
3.0E+005
2.0E+005
1.0E+005
0.0
0.0
0.1
0.2
0.3
0.4
0.5
Time (sec)

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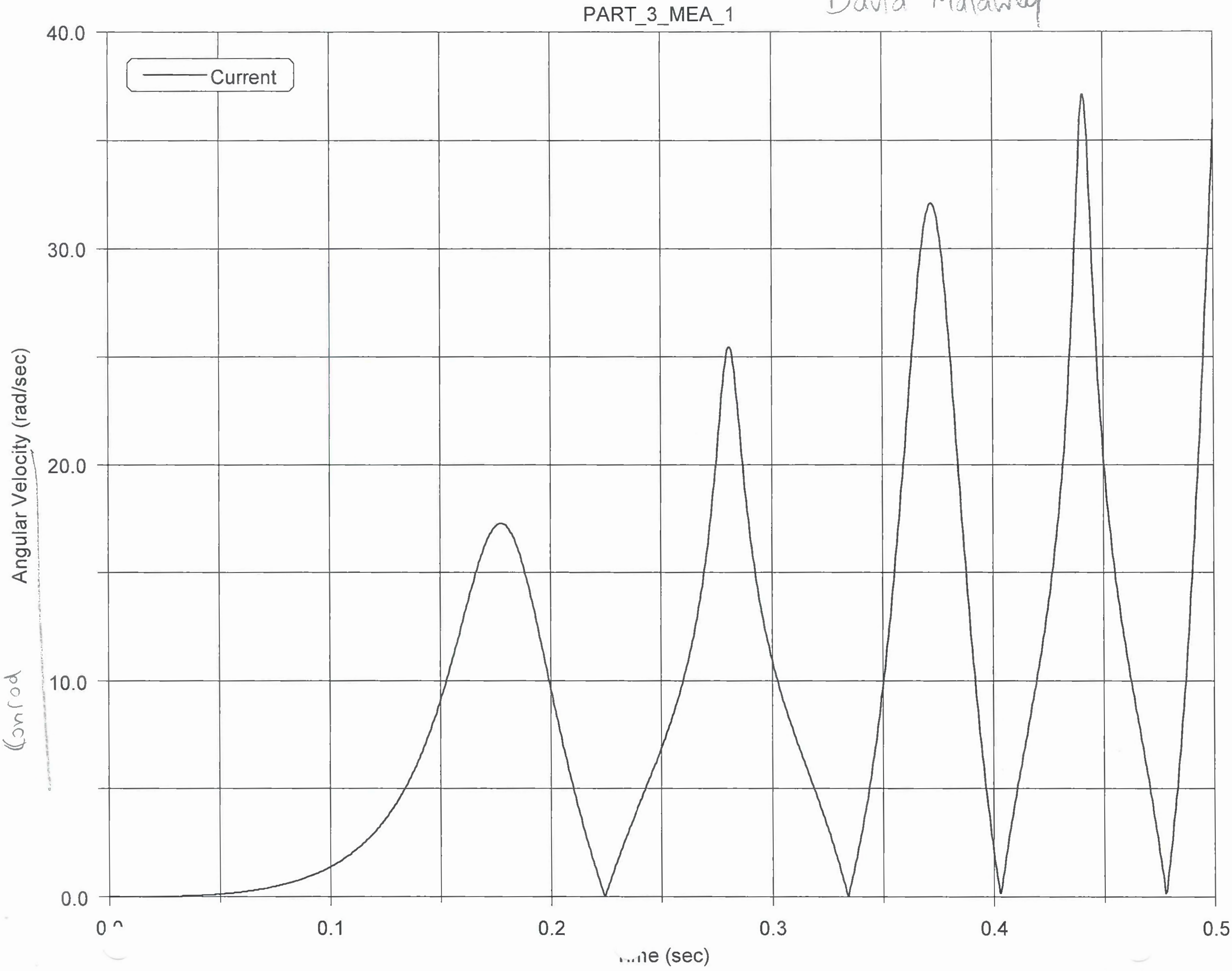
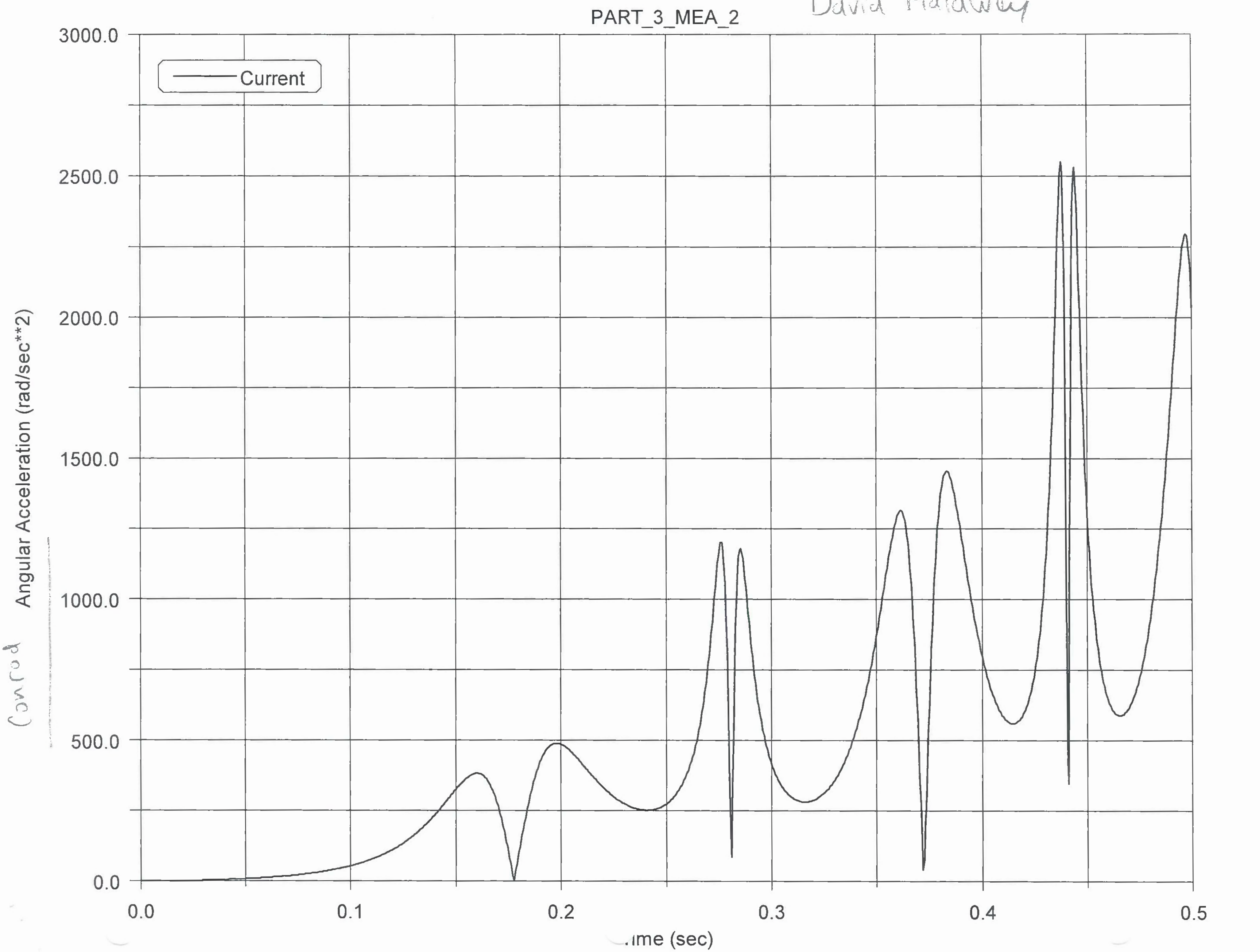
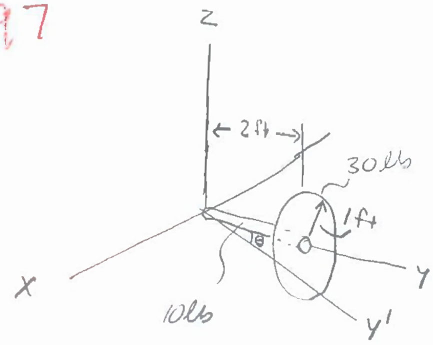


David Malaway



21.1-3 | 14, 27, 35, 38 D. Malawey

97




$\theta = 26.565°$

14) $I_{y'} = ?$

$I_{aa} = I_x u_x^2 + I_z u_z^2 + I_y u_y^2$

$I_z = \frac{1}{4}\left(\frac{30}{32.2}\right)(1)^2 + \left(\frac{30}{32.2}\right)(2)^2 + \frac{1}{3}\left(\frac{10}{32.2}\right)(2)^2$

$= 4.3737$

$I_y = \frac{1}{2}\left(\frac{30}{32.2}\right)(1)^2 + 0 = .4658$

$I_x = 4.3737$

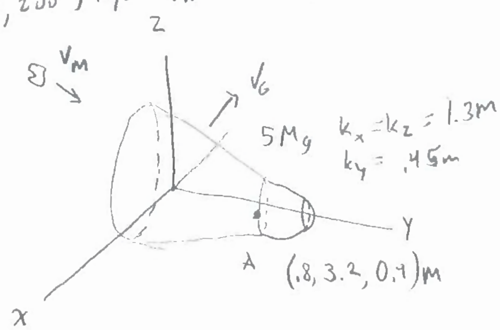
$u = (0, \cos 26.565, -\sin 26.565) = (0, .8944, -.4472)$ why negative?

$I_{y'} = 0 + (4.3737)(-.4472)^2 + (.8944)^2(.4658)$ Symmetric, $I_{xy} = I_{yz} = I_{xz} = 0$

$= \boxed{1.25 \text{ slug}\cdot\text{ft}^2}$

27) $\omega = ?$

$V_G = (0, 400, 200)$ m/s $V_m = (-300, 200, -150)$ m/s




$(H_G)_1 = (H_G)_2$

$r_{G/A} \times m_m V_m = I_G \omega = (.8, 3.2, .9) \times (.8)(-300, 200, -150)$

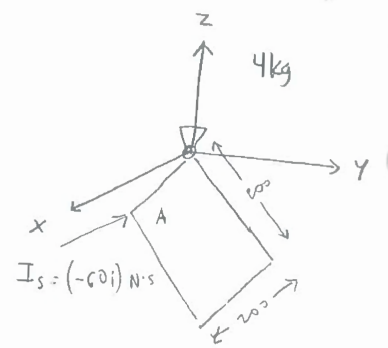
$= 5000(1.3)^2 \omega_x i + 5000(.45)^2 \omega_y i + 5000(1.3)^2 \omega_z k$

$-528i - 120j + 896k = 8450\omega_x i + 1012.8\omega_y j + 8450\omega_z k$

$\omega_x = -.0625$
$\omega_y = -.1185$
$\omega_z = .1060$
} $\omega$, rad/s

21-35)

instantaneous axis of rotation = ?

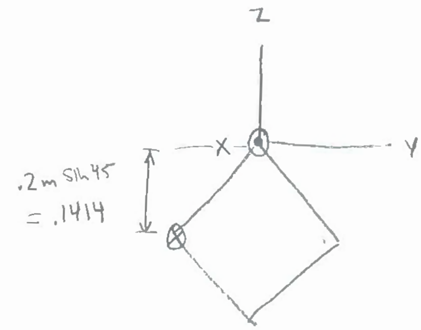



$(I_G)_y$ (given) $= .01333 \ kg \cdot m^3 = (I_G)_z$

$(H_o)_1 + \Sigma \int_{t_1}^{t_2} \vec{M}_o \, dt = (\vec{H}_o)_2$

$\cancel{I_G \vec{\omega}}^{\,0} + \cancel{r \times m v_G}^{\,0} + \vec{r} \times I = I_G \vec{\omega}_2 + r \times m V_{G\,2}$

$= I_{ox} \omega_x i + I_{oy} \omega_y j + I_{oz} \omega_z k$




$(0, -.1414, -.1414) \times (-60i) = (0, \ 8.485, \ -8.485)$

$-8.485 = -I_{oy} \omega_y = I_{Gz} \omega_z$

$\omega_y = +90.936$

$\omega_z = -636.56$

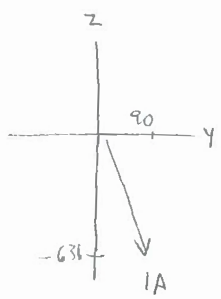
$I_{oy} = I_{Gy} + md^2$

$= .01333 + 4(.1414)^2$

$= .09331$

$I_{oz} = I_{Gz} + \cancel{md^2}\ 0$

$= .01333$




-3

$\boxed{U_{IA} = .1414 j - .990 k}$

$I_o = 8.57 i$ N·s

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21.38)

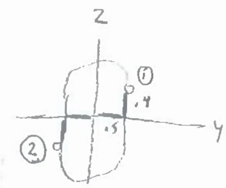
$\omega$ immeditely after impulse

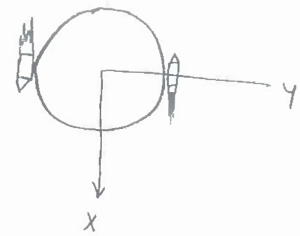
M = 200kg

radii gyration $k_x = k_y = .400\,m$

$k_z = .250\,m$

$I_{(each\ jet)} = 1000\ N\cdot s$




$(H_o)_1 + \Sigma \int_{t_1}^{t_2} M_o\,dt = (H_o)_2$

$0 + r \times I = I_{ox}\omega_x i + I_{oy}\omega_y + I_{oz}\omega_z k$

$r_1 \times I_1 + r_2 \times I_2 = (0, -800, 1000)$

$r_1 = (0, .5, .4)$ $\quad I_1 = -(1000\,N\cdot s)\,i$

$r_2 = (0, -.5, -.5)$ $\quad I_2 = (1000\,N\cdot s)\,i$

$\omega_y = \frac{-900}{32} = -28.125$ rad/s

$\omega_z = \frac{1000}{12.5} = 80$ rad/s

$\omega_x = 0$

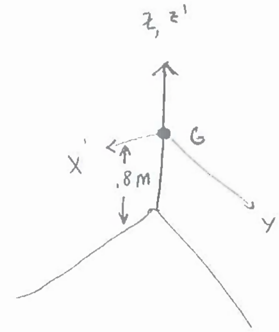
$I_{oy} = I_{Gy} + \cancel{md^2}^{\,0}$

$= mk^2 = 200(.4)^2 = 32$

$I_{oz} = 200(.25)^2 = 12.5$

$$\boxed{\omega = [0, -28.125, 80]\ rad/s}$$

21– 32, 37 D. Malawey




32) $ke = ?$

200kg

$k_{z'} = .3m$

$k_{x'} = k_{y'} = .5m$

$\omega_{x',y',z'} = [600, -300, 1250]$ rad/s

$V_G = (-250, +200, +120)$ m/s

$= 341.906$

$\frac{1}{2} m V_G^2 = \frac{1}{2}(200)(341.91)^2$

$I_x = mk_x^2 = 50$

$I_y = 50$

$I_z = 18$

$\frac{1}{2} I \omega^2 = .5\left(50(600)^2 + 50(300)^2 + 18(1250)^2\right)$

$ke = 3.70027\ E7 = \boxed{37.0\ MJ}$

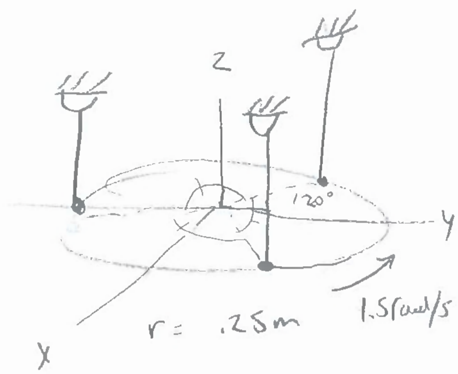
37) $T_1 = \frac{1}{2} I_G \omega^2 = \frac{1}{2}\left[\frac{1}{2}(10)(.25^2)\right](1.5)^2 = .3516\ J$

$T_1 + V_1 = T_2 + V_2$ $V = mgy$

$.3516\ J + 0 = 0 + 9.81(10kg)h$

$h = .003584\ m$

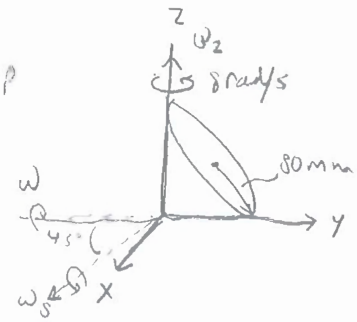
$\boxed{h = 3.58mm}$



20 - 11, 13, 14 David Malawey 4-18-10

100

roll w/o slip
$\omega_{cone} = ?$
$\alpha_{cone} = ?$




11) $\omega = \omega_s + \omega_{\cancel{z}}$

$\omega_j = -\omega_s \sin 45° j - \omega_s \sin 45° k + 8k$

$8 = \omega_s \sin 45$ $\omega_s = 11.31 \text{ rad/s}$

$-\omega = 11.13 \cos 45$ $\omega = 8 \text{ rad/s}$

$\boxed{\omega = -8.0 j \text{ rad/s}}$

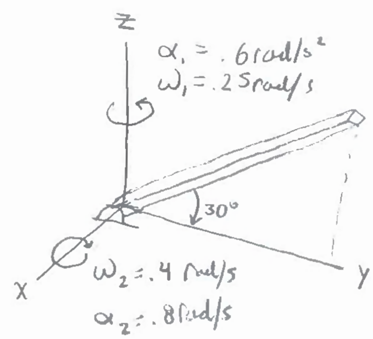
$\Omega = \omega_z = 8k \text{ rad/s}$

$\alpha = \dot{\omega} = \dot{\omega}_{rel} + \omega_z \times \omega = 0 + 8k \times (-8.0 j) = \boxed{64 i \text{ rad/s}}$

$V_A = \omega \times r_A = -8j \times (.16 \sin 45° k) = \boxed{-.905 i \text{ m/s}}$

$a_A = \alpha \times r_A + \omega \times (\omega \times r_A) = 64i \times (.16 \sin 45 k) + 8j \times (-.905 i)$

$\boxed{a_A = [-7.24 j - 7.24 k] \text{ m/s}^2}$




13) $\omega = \omega_1 + \omega_2 = [.25k - .4i]$

$\Omega = .25k \text{ rad/s}$

$\dot{\omega} = \dot{\omega}_{12} + \Omega \times \omega = (-.8i + .6k) + (.25k) \times (.4i + .25k)$

$= [-.8i - .1j + .6k] \text{ rad/s}$

$r_A = 40 \cos 30° j + 40 \sin 30° k = (34.64 j + 20k) \text{ ft}$

$V_A = \omega \times r_A = (1 - .4i + .25k) \times (34.64 j + 20k)$

$= \boxed{[-8.66 i + 8j - 13.9 k] \text{ ft/s}}$

$a_A = \alpha \cdot r_A + \omega \times V_A = (-.8i - .1j + .6k) \times (34.64 j + 20k) + (-.4i + .25k) \times (-8.66i + 8j - 13.9 j)$

$\boxed{a_A = [-24.8 i + 8.29 j - 30.9 k] \text{ ft/s}^2}$

20-14)

$V_p = \omega_{DE} \times r_c = 10k \times (-.015j) = 1.5i$ m/s

$\omega_B = (\omega_B)_y j + (\omega_B)_z k$

$r_{FP} = [-.15j + .15k]$ m

$V_p = \omega_B \times r_{FP}$

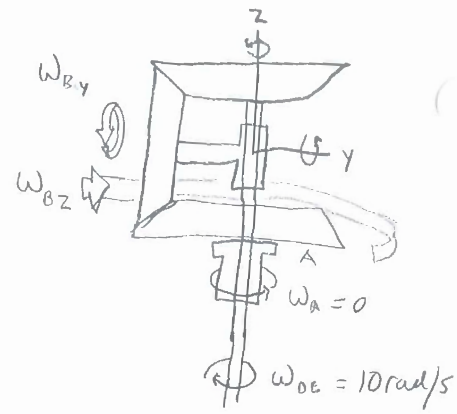
$1.5i = [(\omega_B)_y j + (\omega_B)_z k] \times -.15j + .15k$

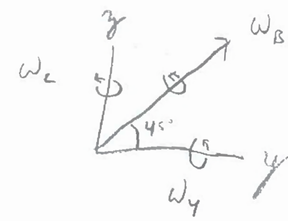
$1.5i = .15(\omega_B)_y + .15(\omega_B)_z$

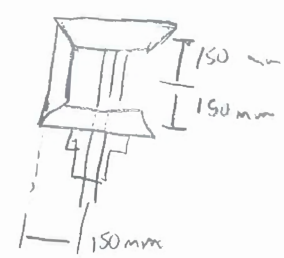
$\omega_{By} + \omega_{Bz} = 10$

$\frac{(\omega_B)_z}{(\omega_B)_y} = \tan 45°$  $\omega_{Bz} = \omega_{By} = 5$ rad/s

$\boxed{\omega_B = 5j + 5k \text{ rad/s}}$



21.1-3 14, 27, 35, 38 D. Malawey

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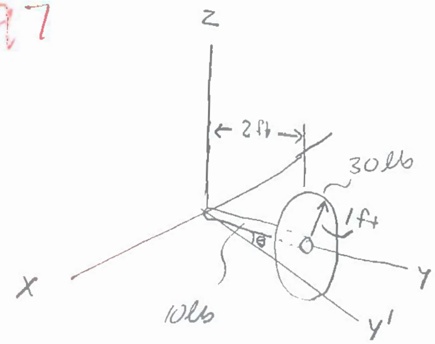



14) $I_{y'} = ?$

$I_{aa} = I_x u_x^2 + I_z u_z^2 + I_y u_y^2$

$I_z = \frac{1}{4}\left(\frac{30}{32.2}\right)(1)^2 + \left(\frac{30}{32.2}\right)(2)^2 + \frac{1}{3}\left(\frac{10}{32.2}\right)(2)^2$

$= 4.3737$

$\theta = 26.565°$

$I_y = \frac{1}{2}\left(\frac{30}{32.2}\right)(1)^2 + 0 = .4658$

$I_x = 4.3737$ why negative?

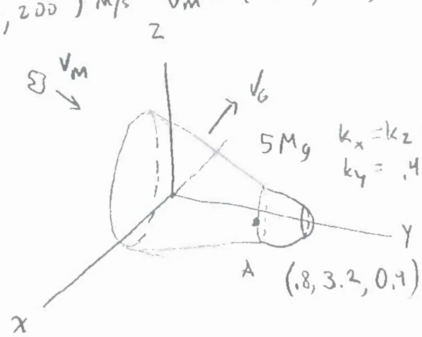
$u = (0, \cos 26.565, -\sin 26.565) = (0, .8944, -.4472)$

$I_{y'} = 0 + (4.3737)(-.4472)^2 + (.8944)^2(.4658)$ symmetric, $I_{xy} = I_{yz} = I_{xz} = 0$

$= \boxed{1.25 \text{ slug}\cdot\text{ft}^2}$

$V_G = (0, 400, 200)$ m/s $V_m = (-300, 200, -150)$ m/s

27) $\omega = ?$




$(H_G)_1 = (H_G)_2$

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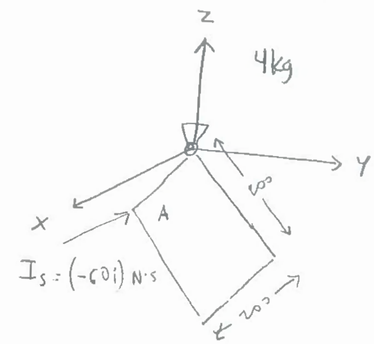
$-528i - 120j + 896k = 8450\omega_x i + 1012.8\omega_y j + 8450\omega_z k$

$\left.\begin{aligned} \omega_x &= -.0625 \\ \omega_y &= -.1185 \\ \omega_z &= .1060 \end{aligned}\right\} \omega, \text{ rad/s}$

21-35)

instantaneous axis of rotation = ?

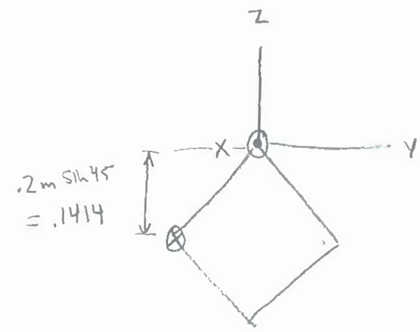



$(I_G)_y$ (given) = .01333 kg·m$^3$ = $(I_G)_z$

$(H_o)_1 + \Sigma \int_{t_1}^{t_2} \vec{M}_o\, dt = (\vec{H}_o)_2$

$\cancel{I_G \vec{\omega}} + \cancel{r \times mV_G} + \vec{r} \times I = I_G \vec{\omega}_2 + r \times mV_{G\,2}$

$= I_{ox}\omega_x i + I_{oy}\omega_y j + I_{oz}\omega_z k$




$(0, -.1414, -.1414) \times (-60i) = (0,\ 8.485,\ -8.485)$

$-8.485 = -I_{oy}\omega_y = I_{Gz}\omega_z$

$\omega_y = +90.936$

$\omega_z = -636.56$

$I_{oy} = I_{Gy} + md^2$

$= .01333 + 4(.1414)^2$

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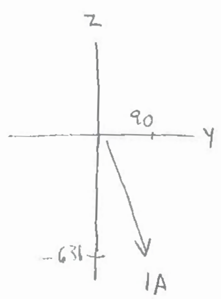
$I_{oz} = I_{Gz} + \cancel{md^2}\ 0$

$= .01333$

-3

$I_o = 8.57 i$ Ns

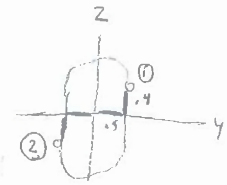



$\boxed{U_{IA} = .1414 j - .990 k}$

David Malaway

21.38)

$\omega$ immediately after impulse

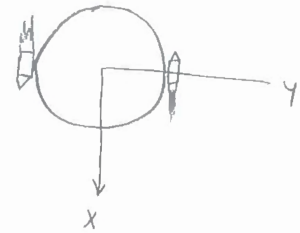



$M = 200\,kg$

radii gyration $k_x = k_y = .400\,m$

$k_z = .250\,m$

$I_{(each\ jet)} = 1000\ N\cdot s$




$$(H_o)_1 + \Sigma \int_{t_1}^{t_2} M_o\,dt = (H_o)_2$$

$$0 + r \times I = I_{Ox}\omega_x i + I_{Oy}\omega_y j + I_{Oz}\omega_z k$$

$$r_1 \times I_1 + r_2 \times I_2 = (0, -800, 1000)$$

$r_1 = (0, .5, .4)$ $\quad I_1 = -(1000\,N\cdot s)\,i$

$r_2 = (0, -.5, -.5)$ $\quad I_2 = (1000\,N\cdot s)\,i$

$$\omega_y = \frac{-900}{32} = -28.125\ rad/s$$

$$\omega_z = \frac{1000}{12.5} = 80\ rad/s$$

$$\omega_x = 0$$

$$I_{Oy} = I_{Gy} + \cancel{md^2}^{\,0}$$

$$= mk^2 = 200(.4)^2 = 32$$

$$I_{Oz} = 200(.25)^2 = 12.5$$

$$\boxed{\omega = [0, -28.125, 80]\ rad/s}$$

21- 32, 37 D. Malawey

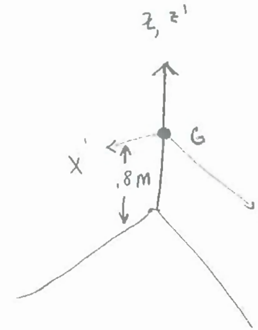
32) $ke = ?$

200 kg

$k_{z'} = .3m$

$k_{x'} = k_{y'} = .5m$

$\omega_{x',y',z'} = [600, -300, 1250]$ rad/s




$V_G = (-250, +200, +120)$ m/s

$= 341.906$

$\frac{1}{2} m V_G^2 = \frac{1}{2}(200)(341.91)^2$

$I_x = mk_x^2 = 50$

$I_y = 50$

$I_z = 18$

$\frac{1}{2} I \omega^2 = .5\left(50(600)^2 + 50(300)^2 + 18(1250)^2\right)$

$ke = 3.70027\ E7 = \boxed{37.0\ MJ}$

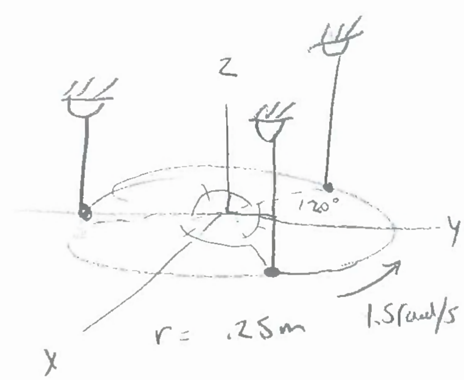
37) $T_1 = \frac{1}{2} I_G \omega^2 = \frac{1}{2}\left[\frac{1}{2}(10)(.25^2)\right](1.5)^2 = .3516\ J$

$T_1 + V_1 = T_2 + V_2$ $V = mgy$

$.3516\ J + 0 = 0 + 9.81(10kg)h$

$h = .003584\ m$

$\boxed{h = 3.58\ mm}$



20 - 11, 13, 14 David Malawey 4-18-10

100

11) $\omega = \omega_s + \omega_z$

$\omega_j = -\omega_s \sin 45° j - \omega_s \sin 45 k + 8k$

$8 = \omega_s \sin 45 \quad \omega_s = 11.31\ rad/s$

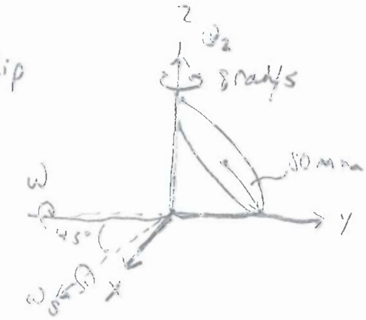
$-\omega = 11.13 \cos 45 \quad \omega = 8\ rad/s$

$\boxed{\omega = -8.0 j\ rad/s}$

roll w/o slip

$\omega_{cone} = ?$

$\alpha_{cone} = ?$




$\Omega = \omega_z = 8k\ rad/s$

$\alpha = \dot\omega = \dot\omega_{rel} + \omega_z \times \omega = 0 + 8k \times (-8.0j) = \boxed{64i\ rad/s}$

$V_A = \omega \times r_A = -8j \times (.16 \sin 45\ k) = \boxed{-.905 i\ m/s}$

$a_A = \alpha \times r_A + \omega \times (\omega \times r_A) = 64i \times (.16 \sin 45\ k) + 8j \times (-.905 i)$

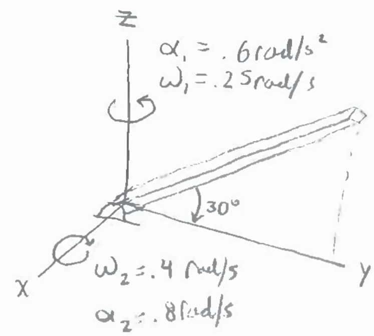
$\boxed{a_A = [-7.24 j - 7.24 k]\ m/s^2}$

13) $\omega = \omega_1 + \omega_2 = [.25k - .4i]$

$\Omega = .25k\ rad/s$

$\dot\omega = \dot\omega_{12} + \Omega \times \omega = (-.8i + .6k) + (.25k) \times (.4i + .25k)$

$= [-.8i - .1j + .6k]\ rad/s$




$r_A = 40 \cos 30° j + 40 \sin 30° k = (34.64 j + 20k)\ ft$

$V_A = \omega \times r_A = (-.4i + .25k) \times (34.64j + 20k)$

$= \boxed{[-8.66 i + 8j - 13.9 k]\ ft/s}$

$a_A = \alpha \cdot r_A + \omega \times V_A = (-.8i - .1j + .6k) \times (34.64j + 20k) + (-.4i + .25k) \times (-8.66i + 8j - 13.9j)$

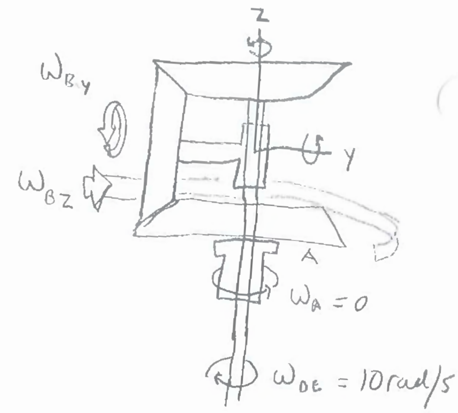
$\boxed{a_A = [-24.8i + 8.29j - 30.9k]\ ft/s^2}$

20-14)

$$V_P = \omega_{DE} \times r_C = 10k \times (-.015j) = 1.5i \text{ m/s}$$

$$\omega_B = (\omega_B)_y j + (\omega_B)_z k$$

$$r_{FP} = [-.15j + .15k] \text{ m}$$

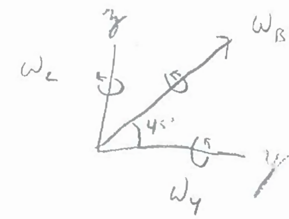



$$V_P = \omega_B \times r_{FP}$$

$$1.5i = [(\omega_B)_y j + (\omega_B)_z k] \times -.15j + .15k$$

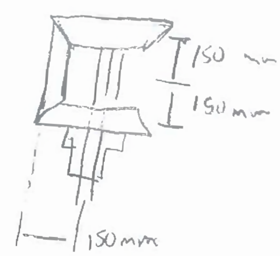
$$1.5i = .15(\omega_B)_y + .15(\omega_B)_z$$

$$\omega_{By} + \omega_{Bz} = 10$$




$$\frac{(\omega_B)_z}{(\omega_B)_y} = \tan 45° \quad \omega_{Bz} = \omega_{By} = 5 \text{ rad/s}$$

$$\boxed{\omega_B = 5j + 5k \text{ rad/s}}$$



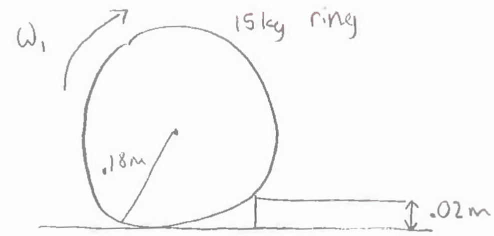
David Malawey

Dynamics HW 4-6-10

19| 44, 48, 54, 55

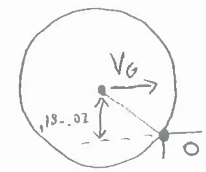
100

44) $\omega_1$ minimum to roll over the step




$$H_{O_1} = H_{O_2}$$

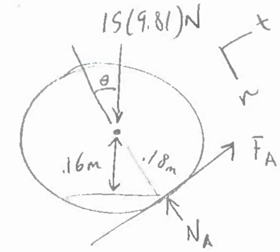
$$H_O = I_G \omega + m V_G d \qquad \text{Thin ring } I_G = mr^2$$




$$H_{O_1} = (15\text{ kg})(.18)^2 \omega_1 + 15\,\omega_1 (.18)(.18-.2) = .918\,\omega_1$$

$$H_{O_2} = (15)(.18)^2 \omega_2 + + 15(.18)^2 \omega_2 = .972\,\omega_2$$

$$\omega_2 = .944\,\omega_1$$



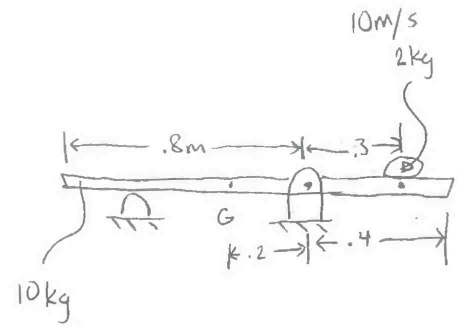

$$\Sigma F_n = m(a_G)_n \qquad a_n = \frac{v^2}{r} = \frac{\omega^2 r^2}{r} = \omega^2 r$$

$$15(9.81)\cos\theta - \cancel{N_A}^{\,0} = 15\,\omega_2^2 (.18) \qquad \cos\theta = .16/.18$$

$$\omega_2^2 = 48.44$$

$$\omega_2 = 6.960 \text{ rad/s} \qquad \boxed{\omega_1 = 7.37 \text{ rad/s}}$$

48) $H_{O_1} = \Sigma H_{O_2}$




① $\cancel{I_G\omega}^{\,0} + m V_G d = 2(10\text{m/s})(.3)$

② $I_{G\,rod} = \frac{1}{12} m l^2 = \frac{1}{12}(10)(1.2)^2 = 1.2 \text{ kg}\cdot\text{m}^2$

$$2(10)(.3) = (1.2)\omega_2 + 10(.2\omega)(.2) + (2)(.3\omega_2)(.3)$$

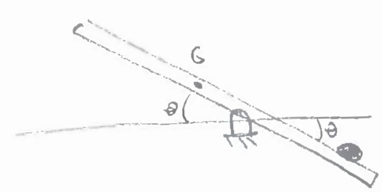
$$6 = 1.78\,\omega_2 \qquad \boxed{\omega_2 = 3.371 \text{ rad/s}}$$


$$T_2 + \cancel{V_2}^{\,0} = \cancel{T_3}^{\,0} + V_3$$

$$V_3 = 10(9.81)(.2\sin\theta) + 2(9.8)(-.3\sin\theta) = 13.734\sin\theta$$

$$T_2 = \frac{1}{2} I_{G\,rod}\,\omega_2^2 + \frac{1}{2} m_{rod} (V_{G\,rod})_2^2 + \frac{1}{2} m_D (V_D)_2^2$$

$$= \frac{1}{2}(1.2)(3.371)^2 + \frac{1}{2}\,10(.2\cdot 3.371)^2 + \frac{1}{2}(2)(.3\cdot 3.371)^2 = 10.11\text{ J}$$



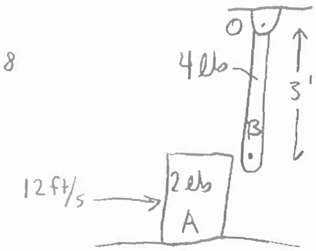

$$10.11 = 13.734\sin\theta$$

$$\boxed{\theta = 47.4^\circ}$$

19- 54)

$(V_A)_2 = ?$

$e = 0.8$




$$0.8 = \frac{V_{B2} - V_{A2}}{V_{A1} - V_{B1}} = \frac{V_{B2} - V_{A2}}{12\ ft/s}$$

$V_{B2} - V_{A2} = 9.6\ ft/s$

$H_{O1} = H_{O2}$

$H = I_G \omega + m V_G d$ $\qquad \omega_2 = \frac{V_{B2}}{3}$

$[M_A V_{A1}] r_A = I_B \omega_2 + M_A V_{A2} r_A$

$\cancel{\frac{2}{32.2}}(12)(3) = \frac{1}{3}\left(\frac{4}{32.2}\right) 3^2 \omega \cancel{+ \left(\frac{4}{32.2}\right)(1.5\omega)(1.5)} + \cancel{\left(\frac{2}{32.2}\right)} V_{A2}(3)$

$72 = 12\omega + \cancel{9\omega} + 6 V_{A2}$

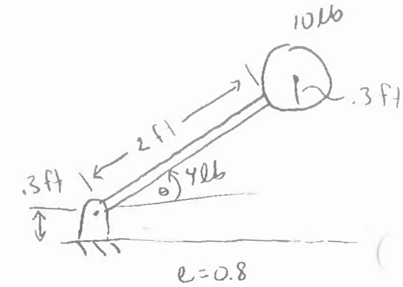
$24 = 4\omega_2 + 2V_{A2}$

$24 = \frac{4}{3} V_{B2} + 2V_{A2}$ $\qquad \boxed{V_{A2} = 3.36 \rightarrow}$

$V_{B2} = 12.96 \rightarrow$

55)

$\cancel{T_1}^0 + V_1 = T_2 + \cancel{V_2}^0$

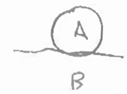



$10(2.3) + 4(1) = \frac{1}{2} I \omega_2^2$

$I = \frac{1}{3}\left(\frac{4}{32.2}\right)(2)^2 + \frac{2}{5}\left(\frac{10}{32.2}\right)(.3)^2 + \left(\frac{10}{32.2}\right)(2.3)^2 = 1.8197$

$\frac{1}{2}(1.8197)\omega_2^2 = 27 \qquad \omega_2 = 5.4475\ rad/s$

$V_{ball} = 2.3(5.4475) = 12.529\ ft/s$




$$0.8 = \frac{\cancel{V_{B2}}^0 - V_{A2}}{V_{A1} - \cancel{V_{B1}}^0} = \frac{-V_{A2}}{V_{A1}} \qquad V_{A2} = -0.8 V_{A1} = -0.8(12.529)$$

$\cancel{T_3}^0 + V_3 = T_2 + \cancel{V_2}^0$

$10(2.3\sin\theta) + 4(\sin\theta) = \frac{1}{2}(1.8197)\underbrace{\left[\frac{.8(12.59)}{2.3}\right]^2}$ $\qquad \left(\frac{.8 V_2}{r}\right)^2 = \left(\frac{V_3}{r}\right)^2 = \omega_3^2$

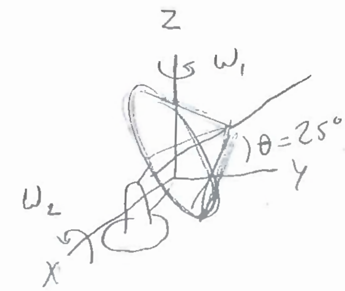
$27\sin\theta = \cancel{64/2}$

$\sin\theta = .6462$

$\boxed{\theta = 40°}$

Homework 4-13-10

20|- 3, 6, 10




3) $\omega_1 = 6 \text{ rad/s}$ $\quad \dot{\omega}_1 = 3 \text{ rad/s}^2$

$\omega_2 = 2 \text{ rad/s}$ $\quad \dot{\omega}_2 = 1.5 \text{ rad/s}^2$

$\omega = \omega_1 + \omega_2 = [2i + 6k] \text{ rad/s}$

$\dot{\omega}_2 = (\dot{\omega}_2)_{xyz} + \omega_1 \times \omega_2 = 1.5i + 6k \times 2i = [1.5i + 12j] \text{ rad/s}^2$

$\dot{\omega}_1 = (\dot{\omega}_1)_{xyz} + 0 \times \omega_1 = [3k] \text{ rad/s}^2$

$r_A = [1.4\cos 25° j + 1.4 \sin 25° k] \text{ m}$

$= [1.2688j + .5917k] \text{ m}$

$\alpha = \dot{\omega}_1 + \dot{\omega}_2 = [1.5i + 12j + 3k] \text{ rad/s}^2$

$a_A = \alpha \times r_A + \omega \times (\omega \times r_A)$

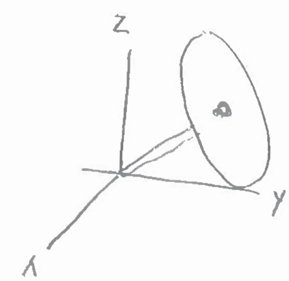
$= (1.3i + 12j + 3k) \times (1.2688j + .5917k) + (2i + 6k) \times [(2i + 6k) \times (1.2688j + .5917k)]$

$= \boxed{[10.4i - 51.6j - .463k] \text{ m/s}^2}$

$v_A = \omega \times r_A = (2i + 6k) \times (1.2688j + .5917k)$

$\boxed{= [-7.61i - 1.18j + 2.54k] \text{ m/s}}$

20-6)

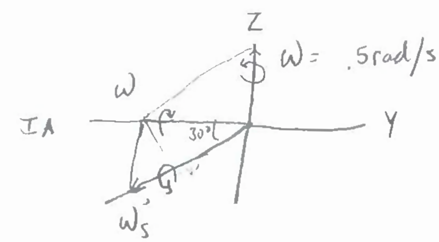



$$\omega = \omega_s + \omega_z$$

$$-\omega j = -\omega_s \cos 30° j - \omega_s \sin 30° k + .5k$$

$k \quad 0 = -\omega_s \sin 30° + .5 \quad \omega_s = 1.00 \text{ rad/s}$

$j \quad -\omega = -1.00 \cos 30° \quad \omega = .8660 \text{ rad/s}$




$\omega = -.8660 j$ rad/s

$$\Omega = \left\{ -\frac{.3}{\sin 30°}(\cos 30°) j - \frac{.3}{\sin 30}(\sin 30°) k \right\} \text{rad/s}^2$$

$$= [-.5196 j - .3k] \text{ rad/s}$$

$\Omega = \omega_z = .5k$ rad/s $(\dot{\omega}_z)_{xyz}$

$$(\dot{\omega})_{xyz} = \dot{\omega}_z + (\dot{\omega})_{xyz} = -.5196 \text{ rad/s}^2$$

$$\alpha = \dot{\omega} = (\dot{\omega})_{xyz} + \omega_z \times \omega$$

$$= -.5196 j + .5k \times (-.8660 j)$$

$$= (0.433 i - .5196 j) \text{ rad/s}^2$$

$r_A = [(0.3 - .3\cos 60) j + .3 \sin 60 k]$ m

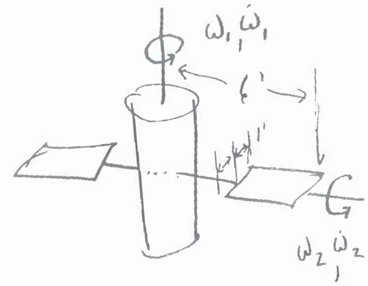
$= (.15 j + .2598 k)$ m

$$V_A = \omega \times r_A$$

$$= \boxed{(-.225 i) \text{ m/s}}$$

$$a_A = \alpha \times r_A + \omega \times (\omega \times r_A)$$

$$= \boxed{[-.135 i - .1125 j - .130 k] \text{ m/s}^2}$$




20 - 10)

$\omega = \omega_1 + \omega_2 = [6j + 15k]$

$\alpha = \dot{\omega} = \omega_1 \times \omega_2$

$\Omega = \omega_1 = 15k \quad \omega_2$ in y axis


$\dot{\omega}_2 = (\dot{\omega}_2)_{xyz} + \omega_1 \times \omega_2 = 1.5j + (15k \times 6j) = (-90i + 1.5j)$ rad/s$^2$

$\dot{\omega}_1 = (\dot{\omega}_1)_{xyz} + \omega_1 \times \omega_1 = (3k)$ rad/s$^2$

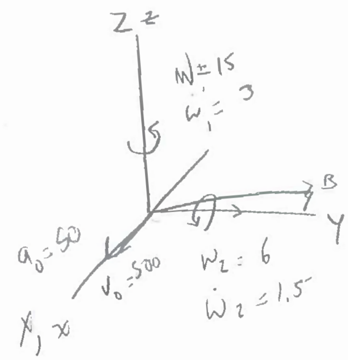
$\alpha = 3k + (-90i + 1.5j)$

$= -90i + 1.5j + 3k]$ rad/s$^2$

$\theta = 90, \quad R_{B/O} = (-1i + 6j)$ ft

$v_B = v_o + \omega \times r_{B/O} = 500i + (6j + 15k) \times (-1i + 6j)$

$= \boxed{[470i - 15j + 6k] \text{ ft/s}}$




$a_B = a_o + \alpha \times r_{B/O} + \omega \times (\omega \times R_{B/O})$

$= 50i + (-90i + 1.5j + 3k) \times (-1i + 6j) + (6j + 15k) \times [(6j + 15k) \times (-1i + 6j)]$

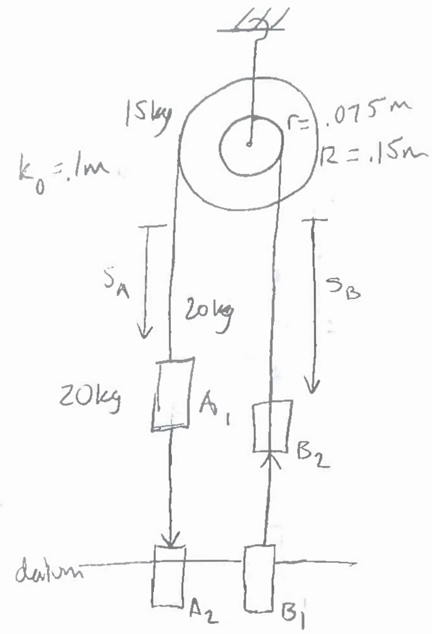
$= [293i - 1353j + 1.5k]$ ft/s$^2$

David Malawey HW 4-5-10

18-15) $V_{cyl}$ after A moves down 2m ?

$V_A = 2V_B$ (1) $S_A + 2S_B = \text{Const}$

100




$T_1^{\circ} + V_1 = T_2 + V_2$

$V_{1A} + V_{1B} = V_{2A} + V_{2B} + U_{2A} + U_{2B} + U_{2P}$

$mg(2m) + 0 = 0 + mg(1m) + \frac{1}{2}mv_A^2 + \frac{1}{2}Mv_B^2 + \frac{1}{2}I_G\omega^2$

$mg(1) = \frac{1}{2}mV_A^2 + \frac{1}{2}m\left(\frac{V_A^2}{4}\right) + \frac{1}{2}I_G\frac{V_A^2}{R^2}$

$V_A^2\left(\frac{1}{2}m + \frac{1}{8}m + \frac{I_G}{2R^2}\right) = mg$

$V_A^2 = mg \Big/ \left(\frac{5}{8}M + \frac{I_G}{2R^2}\right)$

$V_A^2 = \dfrac{20(9.81)}{\left(\frac{5}{8}\right)(20) + \dfrac{(15)(.1m)^2}{2(.15)^2}}$

$\omega = \dfrac{V_A}{R}$

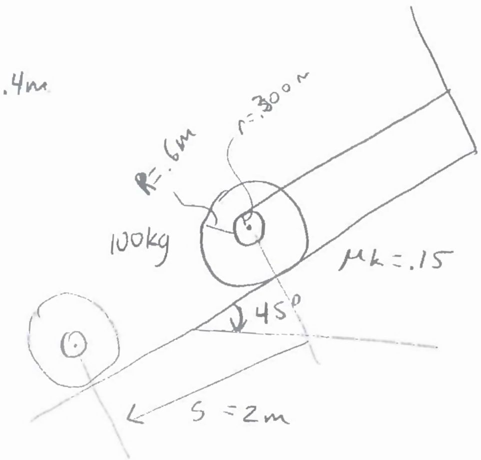
$I_G = \frac{1}{2}mr^2$

$V_B = \dfrac{V_A}{2}$

$$\boxed{\begin{array}{l} V_A = 3.521 \text{ m/s} \downarrow \\ V_B = 1.761 \text{ m/s} \uparrow \end{array}}$$

David Malawey

18-26) $\omega = ?$ after $s = 2m$



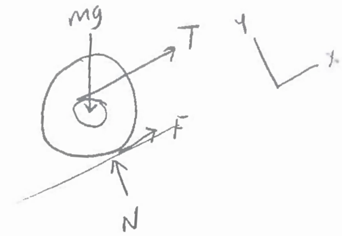

$\Sigma F_y = 0$

$mg \sin 45 = N = 693.672$

$F_f = 104.051$

$\circlearrowright \Sigma M_G = I_G \vec{\alpha}$ $\quad I_G = \frac{1}{2} m k_o^2$

$T(.3) - F(.6) = \frac{1}{2}(100)(.4m)^2 \vec{\alpha}$




$T_1 + \Sigma U_{12} = T_2$

$(100)(9.81)(2 \sin 45) + 0 - \int_1^2 w\,ds = 0 + \frac{1}{2} m v^2 + \frac{1}{2} I_G \omega^2$

pot kinet. work pot kin kin.

$1387 - \int_1^2 F_f\,ds = \frac{1}{2} m (0.3w)^2 + \frac{1}{2}(100)(.4)^2 \omega^2$

$\omega = \frac{V}{r}, \quad V = \omega r$

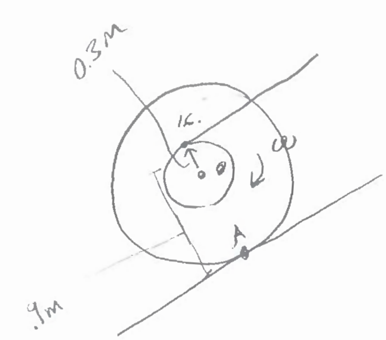
$1387 - 104.05(6m) = 4.5\,\omega^2 + 8\omega^2$

$762.7 = 12.5\,\omega^2$

$\boxed{\omega = 7.811 \text{ rad/s}}$

$S = 2$

$\Delta \text{slip} = 2\left(\frac{.6m}{.3m}\right) =$




contact point A travels

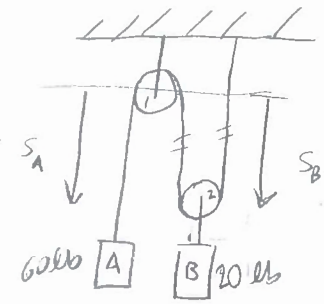
$s_A = \left(\frac{r_{A/IC}}{r_{O/IC}}\right) s_o = \left(\frac{.9}{.3}\right) 2m = 6m$

David Malaway

18) $\underline{44}$

$r = 0.5 m$

B rises 5', $V_A = ?$
A falls 10'




$\cancel{T_1 + V_1 = T_2 + V_2}$

$\cancel{m_A g y_A + m_B g y_B = m_A g y_A + m_B g y_B + \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 + \frac{1}{2} I \omega_1^2 + \frac{1}{2} I \omega_2^2}$

$\cancel{60(10) = 25(5) + \frac{1}{2}\left(\frac{60}{32.2}\right) V_A^2 + \frac{1}{2}\left(\frac{20}{32.2}\right)\left(\frac{V_A}{2}\right)^2 + .01941 \frac{V_a^2}{4} + .01941 V_A^2}$

$\cancel{600 = 125 + V_A^2 [.9316 + .0776 + .00243 + .009705]}$

$S_A + 2 S_B = \text{const}$

$V_B = -.5 V_A = \frac{V_A}{2}$

$\omega_1 = V_A / R = \frac{V_A}{2}$

$\omega_2 = V_B / R = \frac{V_A}{2R} = V_A$

$I = \frac{1}{2}\left(\frac{5}{32.2}\right)(.5)^2 = .01941$

$V_1 = 60 lb(10') + \cancel{20 lb(0)} = 600$

$V_2 = \cancel{60 lb}(0) + 20 lb(5') + 5(5') = 125$

$V_2 - V_1 = -475$

$T_1 + V_1 = T_2 + V_2$

$\cancel{T_1}^{0} = T_2 - \cancel{500}\ 475$

$T_2 = \cancel{500}\ 475 = \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2 + \frac{1}{2} m_D V_B^2 + \frac{1}{2} I_G \omega_D^2 + \frac{1}{2} I_G \omega_E^2$

$\frac{1}{2}\left(\frac{60}{32.2}\right) V_A^2 + \frac{1}{2}\left(\frac{20}{32.2}\right)(-.5 v_A)^2 + \frac{1}{2}\left(\frac{5}{32.2}\right)(.5 v_A)^2 + \frac{1}{2}(.01941 V_A^2) + \frac{1}{2}(.01941)(2 v_A)^2$

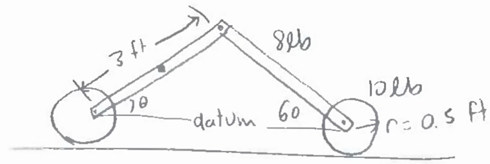
$475 = 1.0773 v_A^2$

$\boxed{V_A = 21.6 \text{ ft/s}}$

David Malawey

18 - 15, 26, 44, 56, 67

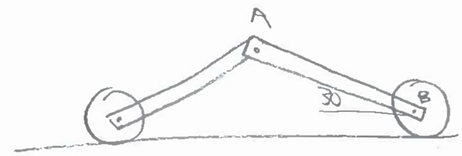
67) conservation of energy




no slipping

$$\cancel{\Sigma T_1}^{0} + \Sigma V_1 = \Sigma T_2 + \Sigma V_2$$

$$2(8lb)(1.5 \sin 60) = \Sigma T_2 + 2(8lb)(1.5 \sin 30^\circ)$$




$T_2$:

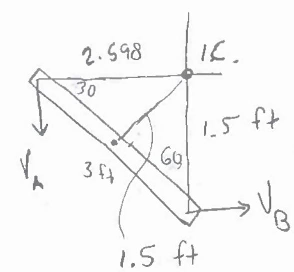
rod: $\frac{1}{2} m V_G^2 + I_G \omega^2$

$$2\left[\underbrace{\tfrac{1}{2}\left(\tfrac{8}{32}\right) V_G^2}_{\text{rod}} + \underbrace{\left(\tfrac{1}{2}\right)\tfrac{1}{12}\left(\tfrac{8}{32}\right)(3\,ft)^2 \omega_{AB}^2}_{\text{rod}} + \underbrace{\tfrac{1}{2}\left(\tfrac{10}{32.2}\right) V_B^2}_{\text{disc.}} + \underbrace{\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)\left(\tfrac{10}{32.2}\right)(.5)^2\left(\tfrac{V_B}{.5}\right)^2}_{\text{disc}}\right]$$

$$V_{disc} = V_B = \omega r_{B/IC}$$

$$V_B = \omega_{AB}(1.5)$$

$$V_G = \omega_{AB}(1.5)$$




$$I_G = \frac{1}{2}\left(\frac{10}{32.2}\right)(.5)^2$$

$$T_2 = 2\left[.125(1.5)^2 \omega_{AB}^2 + .09375(\omega_{AB}^2) + .3474\,\omega_{AB}^2 + .1747\,\omega_{AB}^2\right.$$

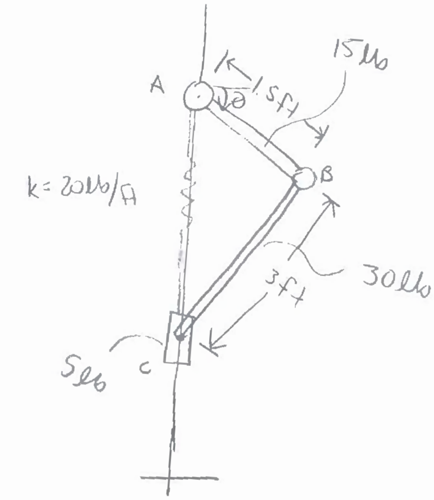
$$T_2 = 2\omega_{AB}^2(.8991) + 2(8)(1.5 \sin 30) = 2(8)(1.5 \sin 60)$$

$$\boxed{\omega_{AB} = 2.21025}$$

David Malawey

18 - 56)

$$\cancel{\Sigma T_1}^{\,0} + \Sigma V_1 = \Sigma T_2 + \Sigma V_2 \qquad \theta_1 = 0^\circ \quad \theta_2 = 90^\circ$$




$$mgy_1 \quad \cancel{\tfrac{1}{2}ks^2}^{\,0} = mgy_2 + \tfrac{1}{2}ks^2 + \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

$$mg(y_2 - y_1) = 15(.75) + 30(1.7) + 5(1.9) = \boxed{71.75}$$

$$71.75 = \tfrac{1}{2}20(1.9)^2 + \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

$$35.65 = \tfrac{1}{2}mV^2 + \tfrac{1}{2}I\omega^2$$

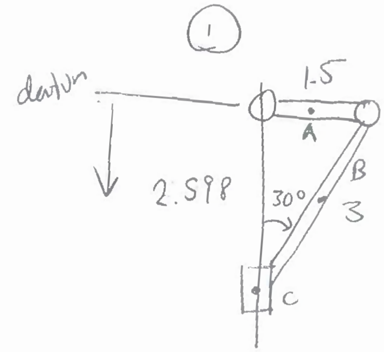
$$= \tfrac{1}{2}I_{AB}\omega_{AB}^2 + \tfrac{1}{2}M_{BC}V_G^2 + \tfrac{1}{2}I_{BC}\omega_{BC}^2$$

$$= \tfrac{1}{2}(.3494)\omega_{AB}^2 + \tfrac{1}{2}\left(\tfrac{30}{32.2}\right)(.75\omega_{AB})^2 + \tfrac{1}{2}(.6988)(.5\omega_{AB})^2$$

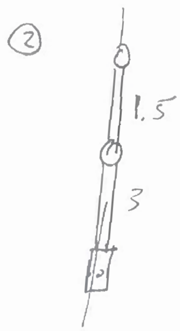
$$35.65 = .5241\,\omega_{AB}^2$$

$$\boxed{\begin{aligned}\omega_{AB} &= 8.248 \text{ rad/s}\\ \omega_{BC} &= 4.124 \text{ rad/s}\end{aligned}}$$

1




2




| | | | $-Y_1$ $-Y_2$ |
|---|---|---|---|
| $Y_A$ | 0 | $-.75$ | $-.75$ |
| $Y_B$ | 1.30 | $-3.0$ | $-1.70$ |
| $Y_\Sigma$ | 2.60 | $-4.5$ | $-1.9$ |

AB| $\quad V_g = \omega(.75) \qquad B:\ V_B = \omega(1.5)$

BC| $\quad \omega_{BC} = \dfrac{V_B}{r_{B/IC}} = \dfrac{\omega(1.5)}{3} = .5\,\omega_{AB\,2}$

$$V_{g2} = \omega_{BC}\, r_{G2/IC} = 0.5\,\omega_{AB\,2}(1.5)$$

$$= .75\,\omega_{AB\,2}$$

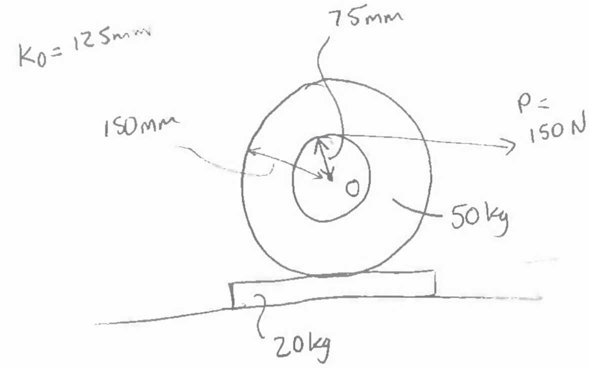
$$I_{AB} = \tfrac{1}{3}\left(\tfrac{15}{32.2}\right)(1.5)^2 = .3494 \text{ slug/ft}^2$$

$$I_{BC} = \tfrac{1}{12}\left(\tfrac{30}{32.2}\right)(3)^2 = .6988 \text{ slug/ft}^2$$

David Malawey

Dynamics HW 4-5-10

19- 10, 23, 31, 34, 37




10) $I_o = mk_o = 50(.125) = .7813 \text{ kg·m}^2$

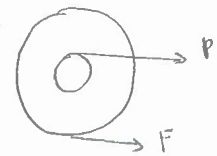
$t = 4 \text{ sec.}$

$\circlearrowleft \quad \cancel{I_o\omega_1}^{0} + \Sigma \int_{t_1}^{t_2} M_o dt = I_o \omega_2$

$F(4)(.15) - 150(4)(.075) = -.7813\,\omega_A$

$F = 75 - 1.302\,\omega_A$

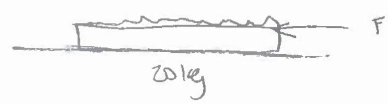
Fixed axis o. $v_p = \omega_A r_p = \omega_A(.15)$




$mv_1 + \Sigma \int_{t_1}^{t_2} F_x dt = mv_2$

$0 + F(4) = 20\,\omega_A(.15)$

$F = .75\,\omega_A$



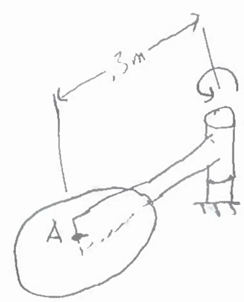

$.75\,\omega_A = 75 - 1.302\,\omega_A$

$\boxed{\omega_A = 36.5 \text{ rad/s}}$

$V = 36.5(.15) = \boxed{5.48 \text{ m/s}}$

23) $M = 5t^2 \quad t = 3s$

disc unlocked. $V_A = r_{OA}\omega = .3\omega$




$\circlearrowleft + \quad \cancel{(H_o)_1}^{0} + \Sigma \int_1^2 M_o dt = (H_o)_2$

$\int_0^{3s} 5t^2 dt = 25[\omega .3]\,0.3$

$\frac{5}{3}t^3 \Big|_0^3 = \ldots \boxed{\omega = 20 \text{ rad/s}}$

Angular momentum $\vec{H}_o = \vec{r} \times m\vec{v}$

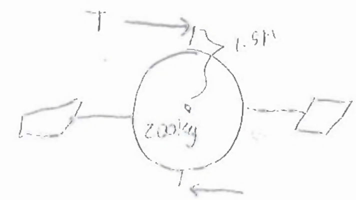
19:31) $k_z = 1.25\,m$ $\quad (M)(k^2)$

$I_z = 200(1.25)^2$

$= 312.5$

$\omega_0 = \{1500k\}$ rev/min

$T = 5e^{-.1t}$ kN $\qquad t = 0-5$




$I_z\omega_1 + \Sigma \int_1^2 M_z\,dt = I_z\omega_2$

$\omega_1 = 1500 \frac{rev}{min}\left(2\pi\,rad/rev\right)\left(\frac{1\,min}{60\,sec}\right) = 50\pi$ rad/s

$312.5(50\pi) - 2\int_0^5 \left(5000e^{-.1t}(1.5)\,dt\right) = 312.5\omega_2$

$\text{"} \quad -15000 \frac{e^{-.1t}}{.1}\Big|_0^5 = \text{"}$

$\boxed{\omega_2 = -31800 \text{ rad/s}}$

19-34) $\omega_r = 5$ rad/s

$\omega_t = 3$ rad/s


rod: $(I_r)_z = \frac{1}{12}ml^2 = 2\,kg\cdot m^2$ $\qquad$ man & turntable : $\frac{1}{2}(5)(.3)^2 + 75k_z^2$

$= .225 + 75k_z^2$

$(H_z)_1 = (H_z)_2$

$0 = 2(\omega_r) - (0.225 + 75k_z^2)(3)$

$\omega_r = \omega_m + \omega_{r/m}$

$\omega_r = -3 + 5 = 2$ rad/s

$2(2) = (.225 + 75k_z^2)\,3$

$\boxed{k_z = .122\,m}$

David Maidway

19-37)

3 rad/s

$$I_z = m k^2 + 2 m_w k_w^2$$

$$= \frac{160}{32.2}(0.55)^2 + 2\left(\frac{5}{32.2}\right)(2.5)^2$$

$$= 3.44 \text{ slug}\cdot\text{ft}^2$$

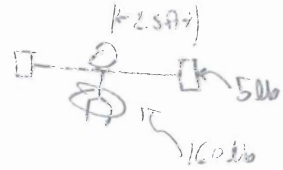
$$I_{z_2} = \frac{160}{32.2}(.55)^2 + 2\left(\frac{5}{32.2}\right)(.3)^2 = 1.531 \text{ slug}\cdot\text{ft}^2$$

$$H_{z_1} = H_{z_2}$$

$$(3.44)\,3\text{ rad/s} = 1.531\,\omega_2$$

$$\boxed{\omega_2 = 6.75 \text{ rad/s}}$$

①




$k_z = .55$ ft

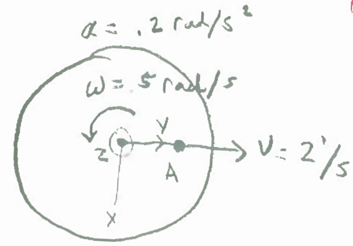
②

David Malawey Homework 3/15

16)-139, 140, 157, 159

100

139) find $V_A$ & $a_A$




$$V_A = V_0 + \Omega \times r_{A/O} + (V_{A/O})_{xyz}$$

$$V_A = 0 + (.5k)\times(5j) + 2j$$

$$\boxed{V_A = (-2.5i + 2.00j)\ ft/s}$$

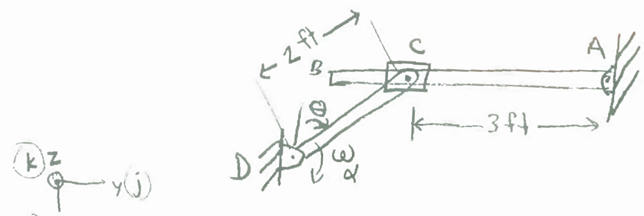
$$a_A = a_0 + \dot{\Omega}\times r_{A/O} + \Omega\times(\Omega\times r_{A/O}) + 2\Omega\times(V_{A/O})_{xyz} + (a_{A/O})_{xyz}$$

$$a_A = 0 + (0.2k)\times(5j) + (.5k)\times(.5k\times 5j) + 2(.5k)\times(2j) + 3j$$

$$a_A = -1i - 1.25 - 2i + 3j$$

$$\boxed{a_A = (-3.00i + 1.75j)\ ft/s^2}$$

140) $\alpha_{AB} = ?$ $\omega_{AB} = ?$ $\Omega_{CD} = -4k$




$\theta = 45°$

$\omega = 4 rad/s$ $\alpha = 2 rad/s^2$

$$V_c = 0 + -4k \times (2\sin 45 i + 2\cos 45 j)$$

$$= 5.6569i + -5.6569j \ ft/s$$

$$a_c = 0 + -2k\times(2\sin 45 i + 2\cos 45 j) - 4k\times(5.6569i - 5.6569j)$$

$$= [-19.799i - 25.46j]\ ft/s^2$$

$$V_c = V_A + \Omega\times r_{c/A} + (V_{c/A})_{xyz}$$

$$5.657i - 5.657j = 0 + \Omega_{AB}\times(-3i) + (V_{c/A})_{xyz}\, i$$

$$(\qquad) = V_{c/A\,xyz}\, i - 3\omega_{AB}\, j$$

$(V_{c/A})_{xyz} = 5.657\ ft/s$ $\boxed{\omega_{AB} = 1.89 rad/s}$

$$a_c = a_A$$

$$-19.80i - 25.46j = 0 + \alpha_{AB}k\times -3i + 1.89k\times[1.89k\times -3i] + 2(1.89k)\times 5.6569i + (a_{c/A})_{xyz}\, i$$

$$-19.80i - 25.46j = -3\alpha_{AB}j + 10.72i + 21.3831j + (a_{c/A})_{xyz}\, i$$

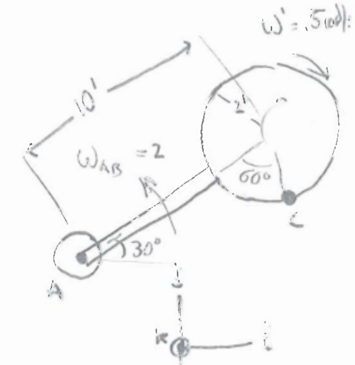
$$-3\alpha_{AB} + 21.3831 = -25.46$$

$$\boxed{\alpha_{AB} = 15.61\ rad/s}$$

16) 157).

$\alpha_{AB} = 1\,rad/s^2$

$\omega' = -.6k\ rad/s$




$r_{B/A} = 10\cos 30 i + 10 \sin 30 j$

$V_B = 0 + 2k \times (8.66i + 5j) = -10.0i + 17.32j$ ft/s

$a_B = 1k \times (8.66i + 5j) + 2k \times (-10i + 17.32j)$

$a_B = (-39.64i - 11.34j)$ ft/s

$V_c = (-10i + 17.32j) + 1.5k \times (-2j) + 0$

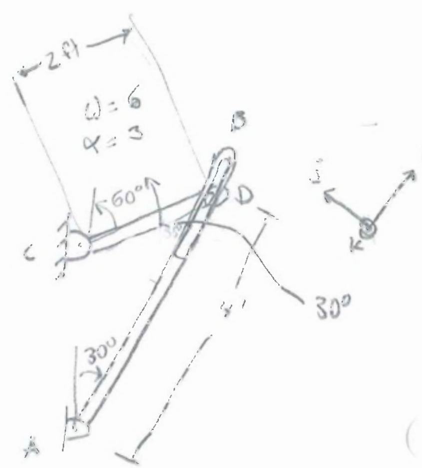
$\dot{\Omega} = 1 - .6 = .4k \qquad \Omega = 2 - .5 = 1.5k$

$\boxed{V_c = -7i + 17.32j}$

$a_c = (-39.64i - 11.34j) + .4k \times (-2j) + 1.5k \times (1.5k \times -2j) + 0 + 0$

$a_c = -39.64i - 11.34j \qquad .8i + 4.5k$

$\boxed{a_c = -38.8i - 6.84j \ ft/s^2}$




159) $r_{D/C} = 1.732i + 1j$

$r_{A/B} = 2i + 3.464j$

$V_D = 6k \times 2\cos 30i - 2\sin 30j$

$= 6i + 10.39j$ f/s

$a_D = 3k \times (2\cos 30i - 2\sin 30j) - 6^2(2\cos 30i + 2\sin 30j)$

$= [-59.35i + 41.20j]$ f/s²

$V_D = V_A + \omega_{AB} \times r_{D/A} + (V_{rel})_{xyz}$

$6i + 10.39j = 0 + (\omega_{AB} k) \times (4i) + (V_{rel})_{xyz} i$

$6i + 10.39j = (V_{rel})_{xyz} i + 4\omega_{AB} j$

$4\omega_{AB} = 10.39 \quad \boxed{\omega_{AB} = 2.598 \ rad/s}$

$a_D = a_A + \dot{\omega}_{AB} \times r_{D/A} + \omega_{AB} \times (\omega_{AB} \times r_{AB}) + 2\omega_{AB} \times (V_{rel})_{xyz} + (a_{rel})_{xyz}$

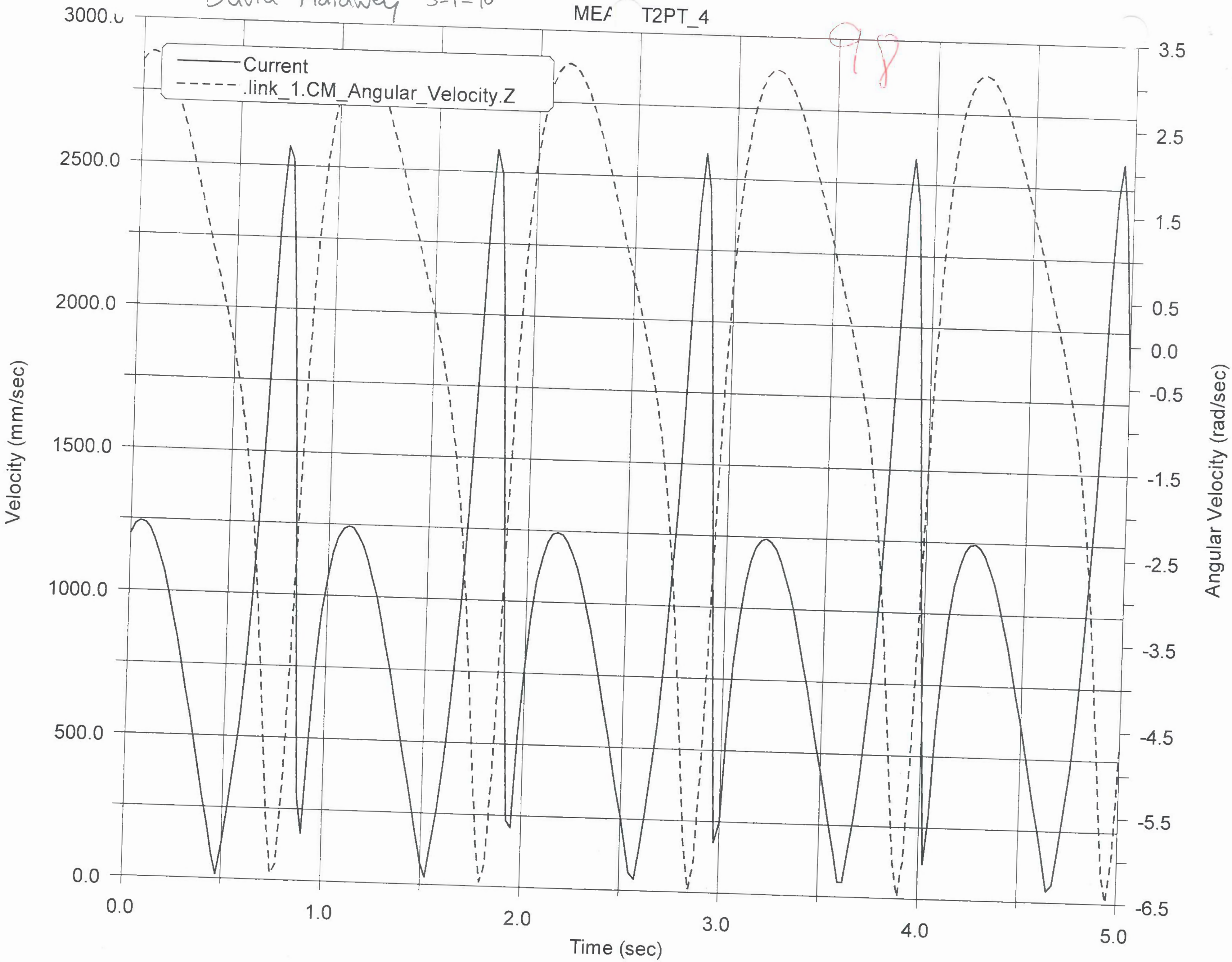
$-59.35i + 41.20j = 0 + (\alpha_{AB} k) \times 4i + 2.59k \times [2.598k \times 4i] + 2(2.598k) \times (6i) + (a_{rel})_{xyz} i$

$-59.35i + 41.20j = ((a_{rel})_{xyz} - 27)i + (4\alpha_{AB} + 31.18)j$

$\boxed{\alpha_{AB} = 2.5 \ rad/s^2}$

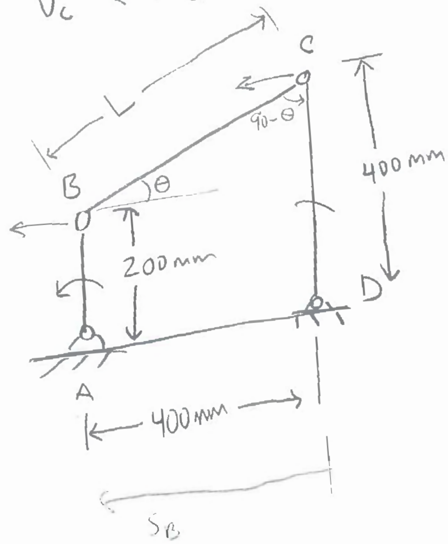
David Malawey 3-1-10

98



Dynamics - ADAMS By hand - calculations

$\omega_{CD}$ & $V_c$ & $a_c$




$\omega_{AB} = 6 rad/s$

$V_B = \omega_{AB} r_{B/A} = .2(6) = 1.2 m/s \hat{i}$

$S_B = L\cos\theta$

$\dot{S}_B = -L\sin\theta\dot{\theta}$

$V_B = 1.2 = -L\sin\theta\dot{\theta}$

$1.2 = -.200\dot{\theta}$

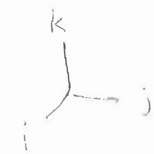
$\dot{\theta} = -6 rad/s$

BC $V_c = V_B + \omega r_{C/B}$

$= -1.2\hat{i} + \omega_{CB} \times (.4m\, i + .2j)$

$V_c = (-1.2 + .4\omega_{CB})i + .2\omega_{CB} j$

i j +k




CD $V_c = \vec{\omega}_{CD} \times \vec{r}_{C/D} = \vec{\omega}_{CD}k \times (.4j)$

$V_c = -.4\hat{i}\,\omega_{CD}$

$\hat{i} = -1.2 + .4\omega_{CB} = .4\omega_{CD}$

$\hat{j}$ $-.2\omega_{CB} = -1.2\omega_{CD}$

$\boxed{\omega_{CB} = 0}$

$-1.2 + .4(0)\omega_{CD} = .2\omega_{CD}$

$-1.2 = .4\omega_{CD}$

$\boxed{\omega_{CD} = 3 rad/s}$

$V_c = ?$ -2

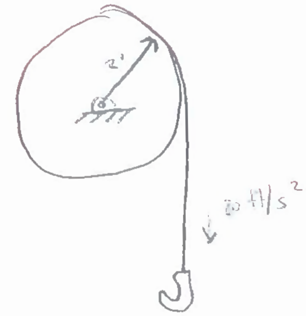
David Malawey 2-24-10 Dynamics HW 96

16 - 3, 7, 30, 42, 50, 52

3) find $\omega$ after 10 rev & $\dot\omega$

$\dot\omega = \frac{a}{r} = \frac{20 \text{ ft/s}^2}{4\pi}\left(\frac{2\pi \text{ rad}}{\text{rev}}\right)$

$\boxed{\dot\omega = 10 \text{ rad/s}^2}$




$\omega^2 = \omega_0^2 + 2\alpha_c(\theta_2 - \theta_1)$ $\theta_2 = 20\pi$ rad -4

$\omega^2 = 2(10)(20\pi)$ $\theta$ calculations?

$\omega^2 = 400\pi$ $\boxed{\omega = 35.45 \text{ rad/s}}$

7) $r_A = .5$ in $r_B = 1.2$ in $t = 1.5$ s shaft $\alpha = 400t^3$ rad/s²

$\omega_A = 0 + \int_0^{1.5} 400t^3\,dt = \frac{400t^4}{4}\Big|_0^{1.5} = 506.25$

$\omega_B = \frac{.5}{1.2}$ $\boxed{\omega_A = 210.94 \text{ rad/s}}$

30) $\omega_0 = 20$ rev/min $\alpha = 30$ rev/min² $t = 3$ sec

~~$\frac{40\pi}{60} = \frac{2\pi}{3}$ rad/s $\alpha = \frac{\pi}{2}$ rad/s²~~

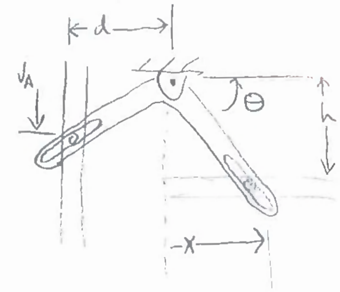
$\omega = 20 + 30\left(\frac{3}{60}\right) = 21.5$ rev/min $\left(\frac{125}{20}\right)\left(\frac{175}{30}\right)$ $\boxed{\omega = 783.85 \text{ rev/min}}$

? — 42) $\theta$ decreases, x increases

in terms of $\theta$?

$\frac{h}{x} = \frac{y}{d} = \tan\theta$ $V_A = \frac{dy}{dt} =$

$x = \left(\frac{h}{d}\right)y$ $\dot x = \left(\frac{h}{d}\right)\dot y = \boxed{\frac{h}{d}V_A = V_B}$

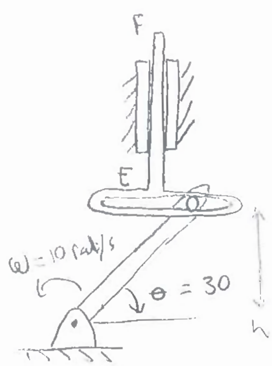



50) v & a of EF $\omega = 10$ rad/s

$V_{EF} = \frac{dh}{dt}$ $h = 3\sin\theta$

$\frac{dh}{dt} = \frac{~~3\sin\theta\, d\theta~~}{dt}$ ~~$= 3\sin 30\,(10) = 15$ ft/sec~~

$= 3\cos\theta\,\dot\theta = 3\cos 30(10) = \boxed{25.98 \text{ ft/sec}}$ ↑

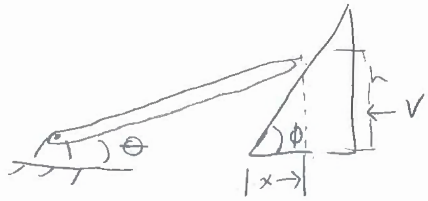


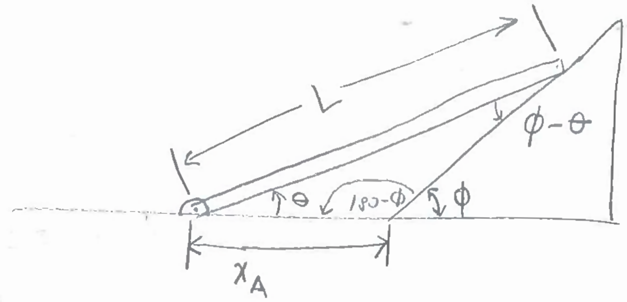

$\dot v = -3\sin\theta\,\dot\theta^2 + \cancel{\ddot\theta\, 3\cos\theta}$ 0

$= -3\sin 30°\,(10^2)$

$= -300(.5) = \boxed{-150 \text{ ft/s}^2}$ ↓

16 - 52)




$$\frac{x_A}{\sin(\phi-\theta)} = \frac{L}{\sin(180-\phi)}$$

$$x_A = \frac{L\sin(\phi-\theta)}{\sin(180-\phi)}$$

$$x_A = \frac{L\sin(\phi-\theta)}{\sin\phi}$$

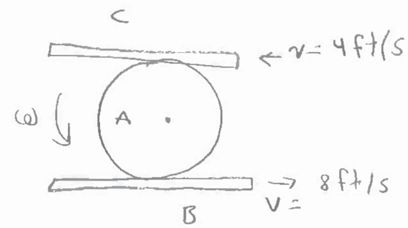
$$\dot{x}_A = \frac{L\cos(\phi-\theta)(-\dot{\theta})}{\sin\phi} = v_A$$

$$\dot{\theta} = \frac{v\sin\phi}{L\cos(\phi-\theta)}$$

Dynamics HW 2-25-10

16 - 55, 67, 68, 73

55)




$V_C = V_B + V_{C/B}$

$-4 = 8 + V_{C/B}$

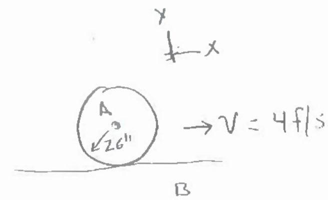
$V_{C/B} = -12$

$\frac{-12 \text{ ft/s}}{(.3 \cdot 2)} = \boxed{20 \text{ rad/s}}$

$V_A = V_B + V_{A/B}$

$V_A = 8 - 20(.3) = \boxed{2 \text{ ft/s} \rightarrow}$

67)

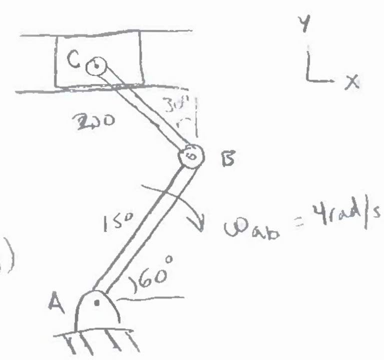



$3 \text{ rad/s} \left(\frac{26}{12''}\right) = 6.5 \text{ ft/s}$

$V_A = V_B + V_{A/B} = 4 - 6.5 =$

$\boxed{= -2.5 \text{ ft/s}}$

68)




$V_B = V_{B/A} = \omega_{AB} r_{AB} = 4(.15\text{m}) = .6 \text{ m/s}$

link BC: $V_B = .6\cos 30 i - .6\sin 30 j$ m/s  $\quad V_C = V_C \hat{i}$, $\omega = \omega_{BC} \hat{k}$

$r_{C/B} = \{-.2\sin 30° \hat{i} + .2\cos 30° \hat{j}\}$ m

$V_C = V_B + \omega \times r_{C/B}$

$V_C i = (.6\cos 30 i - .6\sin 30° j) + (\omega_{BC} k) \times (-.2\sin 30° i + .2\cos 30° j)$

$V_C i = (.5196 - .1732\,\omega_{BC}) i + (.3 + .1\,\omega_{BC}) j$

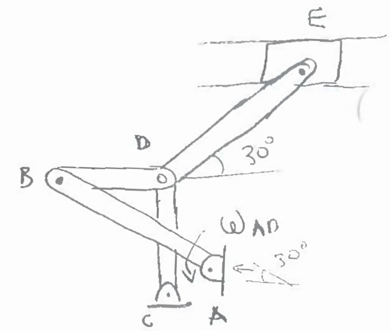
$0 = .3 + .1\omega_{BC} \qquad \omega_{BC} = -3 \text{ rad/s}$

$V_C = .5196 - .1732(-3) = \boxed{1.04 \text{ m/s} \rightarrow}$

73) → over

73)

AB: $V_B = \omega_{AB} r_{AB} = 4(2) = 8\ ft/s$




BD: $V_B = (-8\cos 60° i - 8\sin 60° j)\ ft/s$ $\quad V_D = -V_D i$ $\quad \omega_{BD} = \omega_{BD} k$

$r_{D/B} = (1i)\ ft$

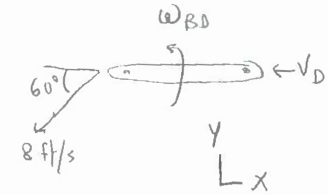
$V_D = V_B + \omega_{BD} \times r_{D/B}$

$-V_D i = (-8\cos 60° i - 8\sin 60° j) + (\omega_{BD} k) \times (1i)$

$-V_D i = -8\cos 60° i + (\omega_{BD} - 8\sin 60°) j$

$-V_D = -8\cos 60° \quad V_D = 4\ ft/s$

$0 = \omega_{BD} - 8\sin 60° \quad \omega_{BD} = 6.928\ rad/s$




DE:

$V_D = -4i\ ft/s \quad \omega_{DE} = \omega_{DE} k \quad V_E = -V_E i$

$r_{E/D} = (2\cos 30° i + 2\sin 30° j)\ ft$

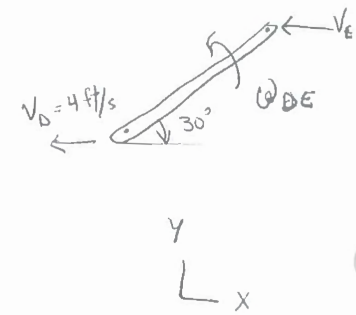
$V_E = V_D + \omega_{DE} \times r_{E/D}$

$-V_E i = -4i + (\omega_{DE} k) \times (2\cos 30° i + 2\sin 30° j)$

$-V_E i = (-4 - 2\sin 30\ \omega_{DE}) i + (2\cos 30°\ \omega_{DE}) j$

$0 = 2\cos 30°\ \omega_{DE} \quad \omega_{DE} = 0$

$-V_E = -4 - 2\sin 30°(0)$ $\quad \boxed{V_E = 4\ ft/s \leftarrow}$

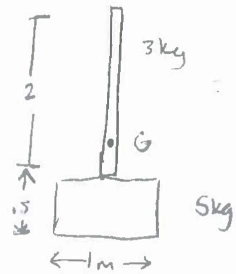


David Malawey (1)

17- 14, 38, 55, 63, 90

97

17- 14)

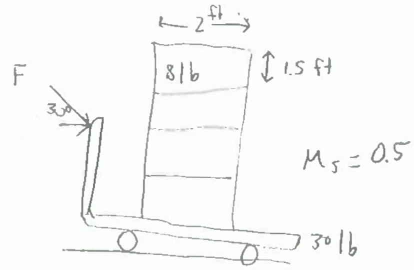



$\bar{y} = \frac{5(.25) + (3)(1.5)}{8} = .7188$ m ~~from b~~ 1.781 m from b

$I = \frac{1}{12}(3)(2)^2 + 3(.781)^2 + \frac{1}{12}5(.5^2+1^2) + 5(.469)^2$

$= 4.451$ kg$\cdot m^2$

17 - 38) ~~assume slipping~~

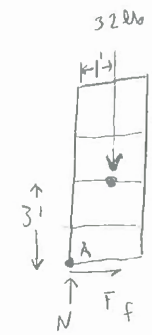



$\circlearrowleft \Sigma M_A = \Sigma (M_k)_A$

$-32(1) = \left(\frac{32}{32.2}\right) a_G (3) \qquad a_G = 10.73$ ft/s$^2$

$\Sigma F_y = m(a_G)_y \qquad N_a - 32 = 0 \qquad N_a = 32$ lb

$\Sigma F_x = m(a_G)_x \qquad \left(\frac{32}{32.2}\right)(10.73) = F_f = 10.67$ lb


$F_{f\,max} = \mu_s N_A = (0.5)(32 \text{ lb}) = 16$ lb $\qquad F_f < F_{f\,max}$ slipping will not occur

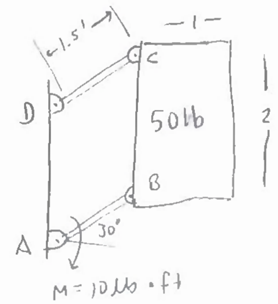
$\Sigma F_x = m(a_G)_x \rightarrow F\cos 30 = \left(\frac{32+30}{32.2}\right)(10.73)$

$\boxed{F = 23.9 \text{ lb}}$

David Malaway 3-22-10

17-55)

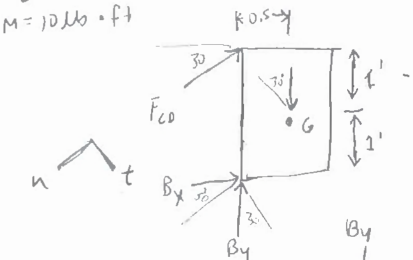
$$(a_G)_n = \omega^2 r = 2^2(1.5) = 6\ \text{ft/s}^2$$




$$\Sigma F_n = m(a_G)_n \quad -F_{CD} + B_x \cos 30° - B_y \sin 30° + 50 \sin 30° = \left(\frac{50}{32.2}\right)(6)$$

$$\Sigma F_t = m(a_G)_t \quad B_x \sin 30 - B_y \cos 30° + 50 \cos 30° = \frac{50}{32.2}(a_G)_t$$

$$\circlearrowleft + \Sigma M_G = 0 \quad B_x(1.5 \sin 30°) - B_y(1.5 \cos 30°) - 10 = 0$$

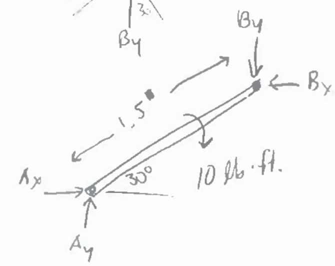



$B_x = 8.975$ lb

$B_y = -2.516$ lb

$\boxed{F_{CD} = 9.169\ \text{lb}}$

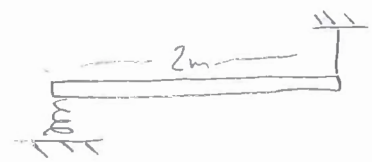
$\boxed{(a_G)_t = 32.18\ \text{ft/s}^2}$




17-63)

$$\Sigma M_A = I_G \vec{\alpha} + \vec{r}_{G/P} \times m\vec{a}_G \qquad a_G = ?$$

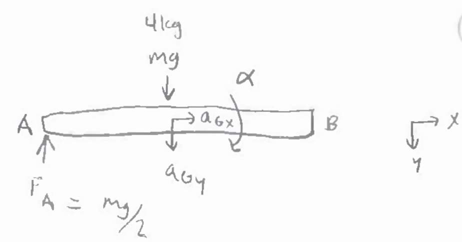
$$\Sigma F_x = M(a_G)_x \qquad \alpha_{AB} = ?$$




$$\underline{0 = a_{Gx}}$$

$$\Sigma F_y = M(a_G)_y \qquad mg - \frac{mg}{2} = m(a_{Gy})$$

$$\frac{\cancel{m}g}{2\cancel{m}} = \boxed{a_{Gy} = 4.905\ \text{m/s}^2}$$



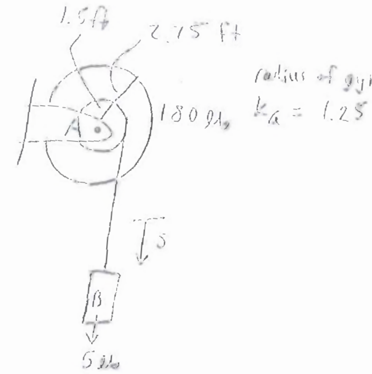

Using center $\quad F_A \frac{L}{2} = \frac{1}{12} m L^2 \alpha$

$$3\frac{\cancel{m}g\cancel{L}}{L\cancel{4}} = \frac{1}{\cancel{12}}\cancel{m}L^{\cancel{2}}\alpha$$

$$\alpha = \frac{3g}{L} = \frac{3(9.81)}{2} = \boxed{\alpha = 14.715\ \text{rad/s}}$$

David Malawey (2)

17-90) $\omega = ?$ @ 3 sec.




System

$\curvearrowright + \Sigma M_A = \Sigma (M_k)_A$

$5(1.5) = \frac{180}{32.2}(1.25)^2 \alpha + \frac{5}{32.2}(1.5\alpha)(1.5)$

$\alpha = 0.8256 \text{ rad/s}$

$(\curvearrowright +)$ $\omega = \omega_0 + \alpha_c t$

$\omega = 0 + .8256(3)$

$\boxed{\omega = 2.48 \text{ rad/s}}$

Spool $\curvearrowright + \Sigma M_A = I_c \alpha$

$T(1.5) = \left(\frac{180}{32.2}\right)(1.25)^2 \alpha$

Weight

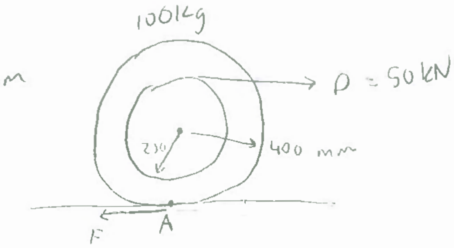
$+\downarrow$ $\Sigma F_y = M(a_G)_y$

$5 - T = \left(\frac{5}{32.2}\right)(1.5\alpha)$

$\alpha = .8256 \text{ rad/s}^2$

David Malawey ③

17-103, 109, 111, 114, 123




$k_G = .3m$

$\mu_s = .2$

$\mu_k = .15$

103) $\alpha = ?$

assume no slip

— $\Sigma F_x = m(a_G)_x$

$P + F = 100kg(a_G)_x$

$50 + F = 100 a_G$ (1)

— $\Sigma F_y = m(a_G)_y$

$-Mg + N = 0$ (2)

— $\Sigma M_G = I_G \alpha$ $+\circlearrowright$

$50(.250) - F(.4) = 100(.3)^2 \alpha$ (3)

$I_G = mk_g^2$

— $a = R\alpha$

$a = 0.4\alpha$ (4)

$a_G = .52$ $m/s^2$

$N = 981N$ $F = 2.0N$

$\alpha = 1.3$ $rad/s^2$

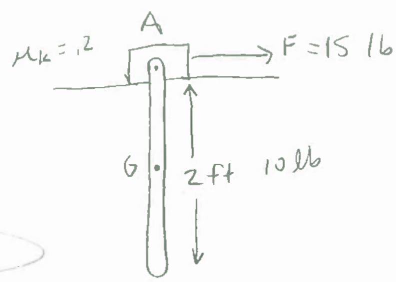
Check

$F_{max} = \mu_s N$

$= .2(981) = 196.2$

$2 < 196.2$ ✓

109.)




$I_G = \frac{1}{12} mL^2 = \frac{1}{12}\left(\frac{10}{32.2}\right)(2)^2 = .1035$ slug ft$^2$

$\omega = 0$

$(a_G)_n = 0$

$\Sigma F_n = m(a_G)_n$ $10 - N = 0$ $N = 10 lb$

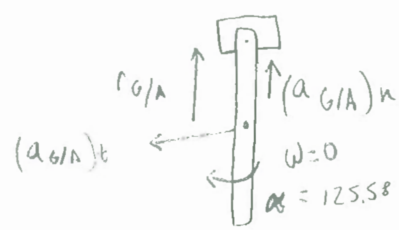
$\Sigma F_t = m(a_G)_t$ $15 - .2(10) = \frac{10}{32} a_{Gt}$ $a_G = 41.86$ $ft/s^2$

$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A$ $0 = \left(\frac{10}{32}\right)(41.86)(1) - .1035\alpha$ $\boxed{\alpha = 125.58 \ rad/s^2}$

$a_G = a_A + a_{(G/A)t} + (a_{G/A})_n$

$41.86 = a_A - 125.58(1) + 0$

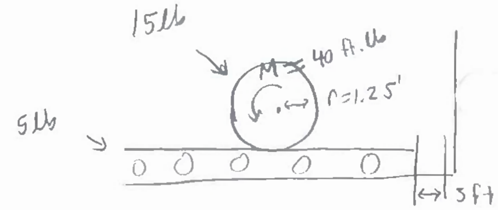
$\boxed{a_A = 167 \ ft/s^2}$



17 – 111, 114, 123

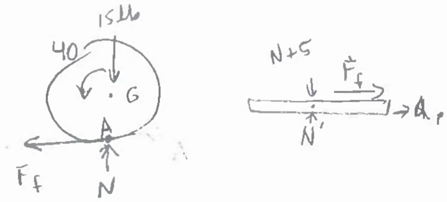
(111) $\alpha = ?$

$t$ to strike wall




$I_G = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{15}{32.2}\right)(1.25)^2 = .3639 \text{ slug}\cdot\text{ft}^2$

$\curvearrowright \Sigma M_A = \Sigma (M_k)_A \qquad -40 = -\left(\frac{15}{32.2}\right) a_G (1.25) - .3639\,\alpha$




$\Sigma F_x = m(a_G)_x \qquad F_f = \left(\frac{15}{32.2}\right) a_G$

$\Sigma F_x = m(a)_x \qquad F_f = \left(\frac{5}{32.2}\right) a_p$

$a_G = a_A + \alpha \times r_{G/A} - \omega^2 (1.25j)$

$-a_G i = [(a_A)_x - 1.25\alpha]\, i + [(a_A)_y - 1.25\omega^2]\, j$

$a_G = 1.25\alpha - (a_A)_x$

$a_p = (a_A)_x$ — no slip —

$a_G = 1.25\,\alpha - a_p$

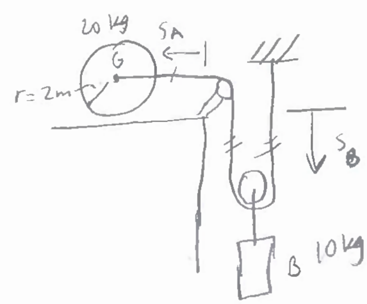
$\boxed{\alpha = 73.27 \text{ rad/s}^2} \qquad a_p = 68.69 \text{ ft/s}^2$

$s = s_0 + v_0 t + \frac{1}{2} a_p t^2$

$3 = 0 + 0(t) + \frac{1}{2}(68.69)\, t^2$

$\boxed{t = .296 \text{ s}}$

---




114) Disk $\curvearrowright + \Sigma M_{IC} = \Sigma (M_k)_{IC} \qquad T(0.2) = -\left[\frac{1}{2} 20(.2)^2 + 20(.2)^2\right]\alpha$

Block $+\downarrow \Sigma F_y = m(a_G)_y \qquad 10(9.81) - 2T = 10 a_B$

$2 s_B + s_A = l$

$2 a_B = -a_B \qquad a_A = .2\,\alpha \qquad a_B = -.1\,\alpha$

How?

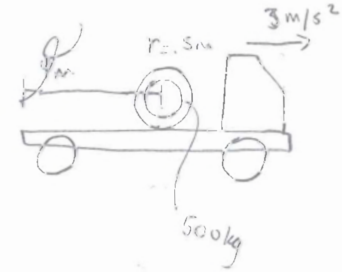
$$\boxed{\begin{aligned} a_B &= .755 \text{ m/s}^2 \downarrow \\ \alpha &= -7.55 \text{ rad/s}^2 \\ T &= 45.3 \text{ N} \end{aligned}}$$

-3

David Malaway (4)

17-123)

$I_G$ (culvert) $= mr^2 = 500(.5^2) = 125\ kg \cdot m^2$



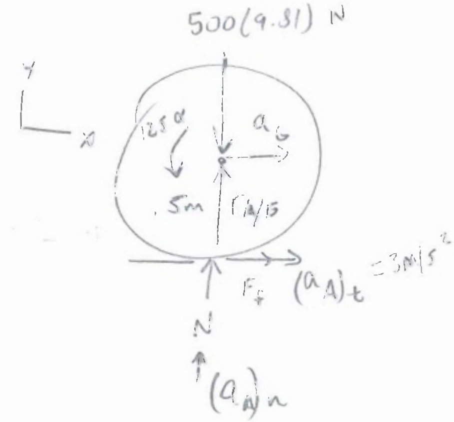

$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A \qquad 0 = 125\alpha - 500 a_G (0.5) \qquad (1)$

$(a_A)_t = 3 m/s^2$ * no slip *

$a_G = a_A + \alpha \times r_{G/A} - \omega^2 r_{G/A}$

$a_G i = 3i + (a_A)_n j + (\alpha k \times .5j) - \omega^2 (0.5j)$

$a_G i = (3 - .5\alpha) i + [(a_A)_n - 0.5\omega^2] j$



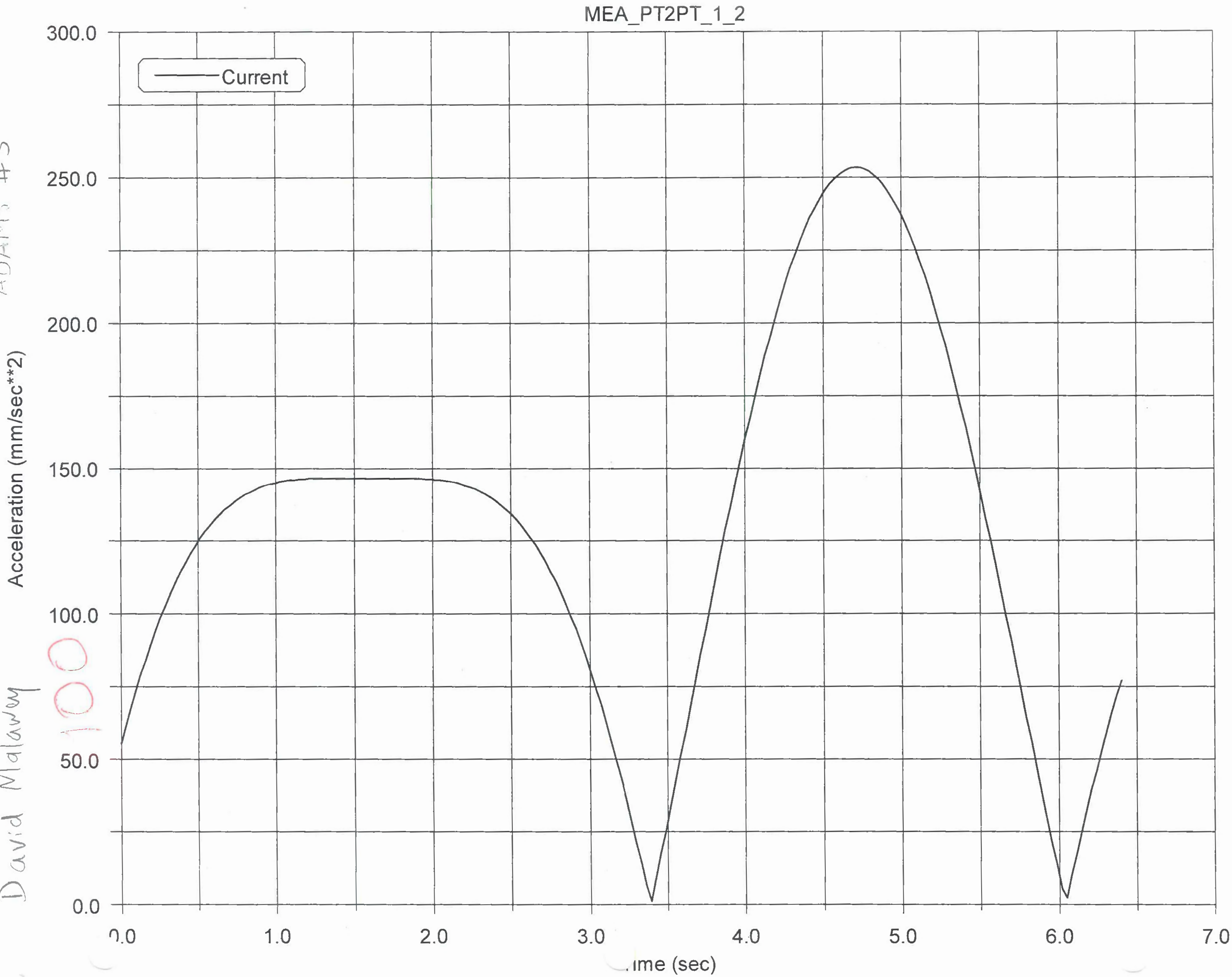

$i: \quad a_G = 3 - .5\alpha \quad (2) \qquad a_G = 1.5 m/s \rightarrow$

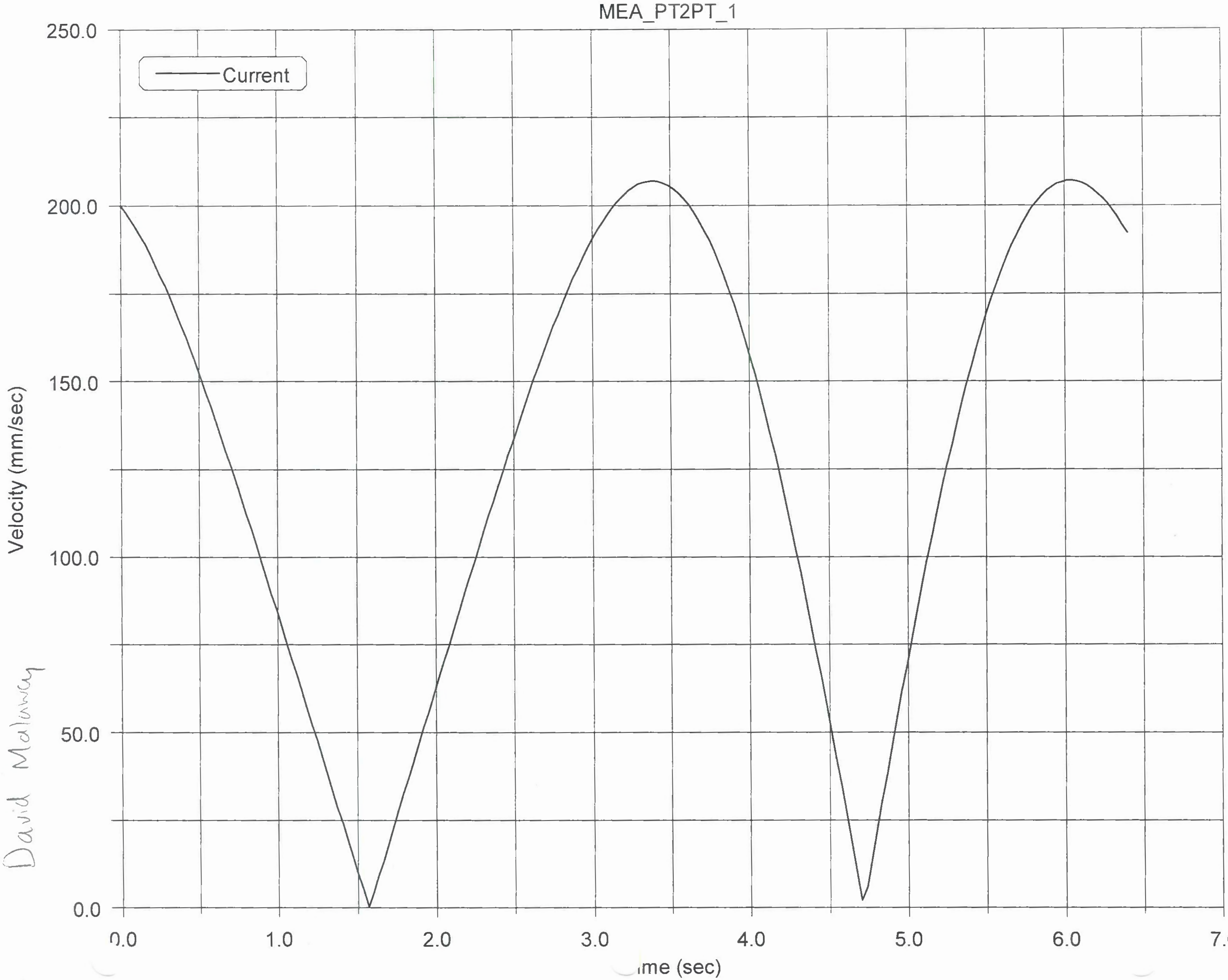
$\boxed{j \qquad \alpha = 3 \text{ rad/s}}$

ADAMS #3

David Malaway

100

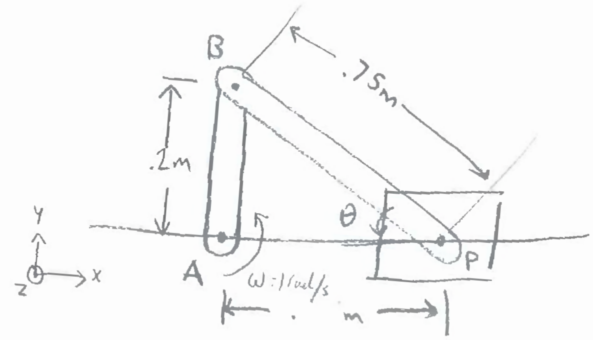



MEA_PT2PT_1
Current
Velocity (mm/sec)
250.0
200.0
150.0
100.0
50.0
0.0
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
ime (sec)
David Maloney

David Malawey

ADAMS HW#3

$V_p = ?$ $a_p = ?$ for $t_o$




$V_B = \cancel{V_A} + \Omega \times r_{B/A} + \cancel{(V_{B/A})_{xyz}}$

$= 1k \times .2j$

$= \underline{-.2i}$

$\leftarrow a_p$

$\leftarrow V_p$

$a_B = 0 + \cancel{\dot{\Omega} \times r_{B/A}} + \Omega \times (\Omega \times r_{B/A}) + 0 + 0$

$= 1k \times -.2i$

$= -.2j$

$r_{P/B} = -.75\cos\theta\, i - .75\sin\theta\, j$

$\omega_{BP} = 0$

$V_P = V_B + \Omega \times r_{P/B} + (V_{P/B})_{xyz}$ $\boxed{V_P = V_B = .2\ m/s}$ ←

$\dot{\theta} = 0$ $r_{ic} = \infty$

$a_P = a_B + \alpha_{BP} \times r_{P/B} - \omega_{BP}^2 r_{P/B}$

$-.2j + \alpha_{BP} k \times (-.75\cos 15.46 i - .75\sin 15.46° j) - 0$

$(a_P)_i = -.2_j - .7229 j\, \alpha_{BP} + .1999 i\, \alpha_{BP}$

i : $a_P = .1999\, \alpha_{BP}$ $\boxed{a_P = .0553\ m/s^2}$ →

j : $0 = -.2 - .7229\, \alpha_{BP}$ $\alpha_{BP} = .276$

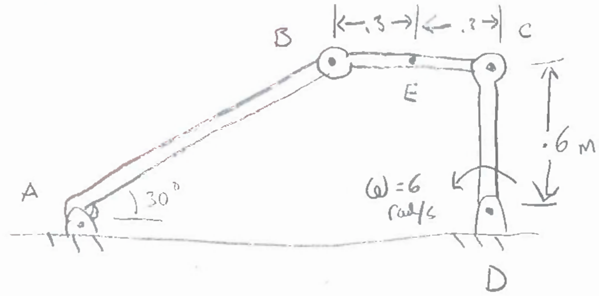
David Malawey Dynamics 3/2/10

16- 89, 93, 101, 104, 107

80

16-98 not there

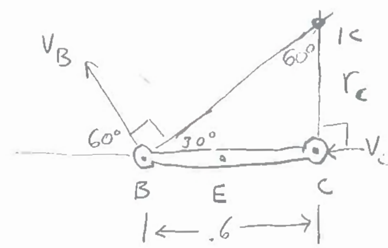
89) find $V_E$ & $\omega_{AB}$




$V_c = \omega_{DC} r_{CD} = .6(6) = \boxed{3.6 \text{ m/s}}$

$\omega_{BC} = \dfrac{V_c}{r_{c/IC}}$

$V_B = \omega r_{B/IC}$




$\omega_{BC} = \dfrac{V_c}{r_{c/IC}} = \dfrac{3.6}{.3464} = 10.39 \text{ rad/s}$

$\dfrac{V_E}{r_E} = \dfrac{V_c}{r_c}$ $\boxed{V_E = 4.768 \text{ m/s}}$

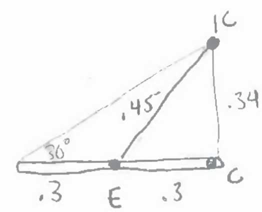
$V_A = 7.206$ m/s

$\omega_{AB} = V/r$

$\boxed{\omega_{AB} = 6.005 \text{ rad/s}}$ +↺

$r \sin 60 = .6 \sin 30$

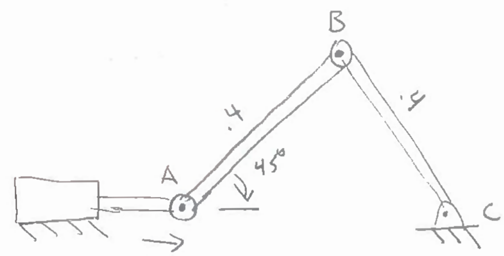
$r_{c/IC} = .3464$




$\sin 30\ AB = \sin 90(.6)$

$AB = 1.2$

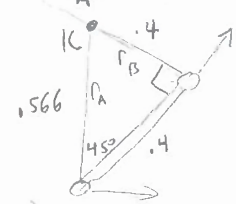
93) $\omega_{BC} = ?$



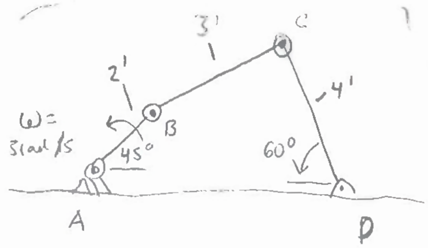

$\dfrac{V_A}{r_A} = \dfrac{V_B}{r_B}$ $V_B = 2.12$ m/s

$\omega_{BC} = \dfrac{V_B}{r_{BC}} = \boxed{5.3 \text{ rad/s}}$ ✓

$V_A = 3$ m/s




101) $\omega_{CD} = ?$



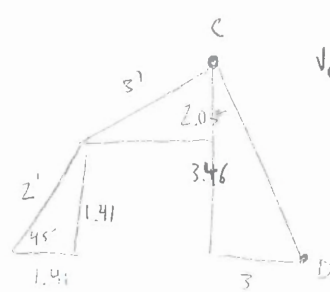

$\dfrac{r_{B/IC}}{\sin 103.1°} = \dfrac{3}{\sin 75°}$ $r_B = 3.025'$

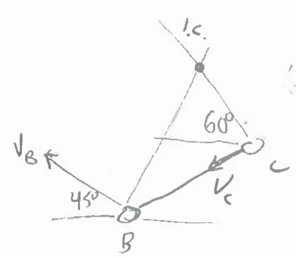
$\dfrac{r_c}{\sin 1.898} = \dfrac{3}{\sin 75}$ $r_c = .1029'$

$\omega_{BC} = \dfrac{V_B}{r_{B/IC}} = \dfrac{6.00}{3.025} = 1.983$

$4\omega_{CD} = 1.983(.1029)$

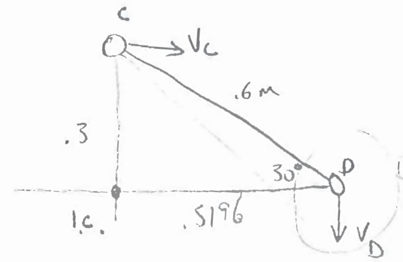
$\boxed{\omega_{CD} = .0510 \text{ rad/s}}$



104, 107

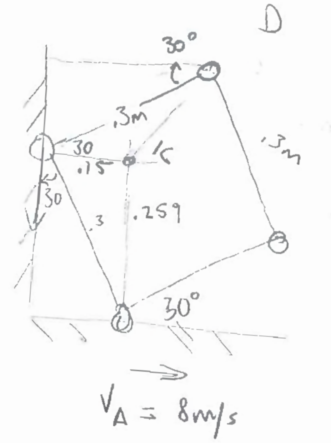
104) $V_c = 10\,rad/s\,(.15m) = 1.5\,m/s$




$\frac{V_D}{.5196} = \frac{V_c}{.3}$ $V_D = 2.598\ m/s$

$\omega_B = \frac{V_D}{r_{BD}} = \frac{2.6\ m/s}{.1m} = \boxed{26\ rad/s}$

107) $V_D = ?$

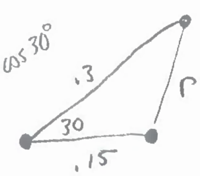



$\frac{V_A}{r_{A/IC}} = \frac{V_D}{r_{D/IC}}$

$\frac{8}{.259} = \frac{V_D}{.186}$

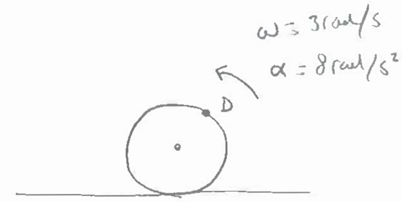
$\boxed{V_D = 5.745\ m/s}$

$r^2 = .3^2 + .15^2 - 2(.3)(.15)\cos 30°$



David Malawey homework 3-8-10

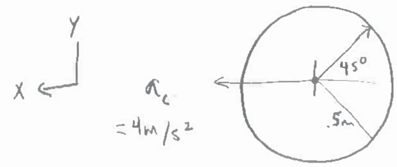
16) 110, 114, 131




110) $a_c = .5(8) = 4\,m/s^2$

$a_D = A_c + A_{D/c}$

$= 4i + \begin{bmatrix} 3^2(.5) \\ \nearrow 45 \end{bmatrix} + \begin{bmatrix} 8(.5) \\ \searrow 45° \end{bmatrix}$

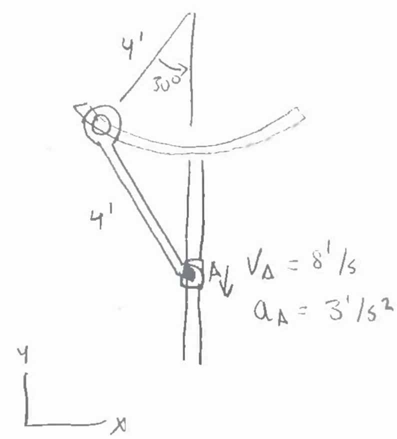



$(a_D)_x = -4 - 4.5\sin 45 - 4\cos 45 = -10.01\,m/s^2$

$(a_D)_y = 0 - 4.5\cos 45° + 4\sin 45 = -.3536\,m/s^2$

$\theta = \tan^{-1}\left(\frac{.3536}{10.01}\right) = \boxed{2.02°}$

$a_D = \sqrt{10.01^2 + .3536^2} = \boxed{10.02\,m/s^2}$




114) $\boxed{\omega = 8/4 = 2\,rad/s}$

$v_B = 4(2) = 8\,ft/s$

$(a_B)_n = \frac{8^2}{4} = 16\,ft/s$

$a_B = a_A + a_{B/A}$

$\underset{\nearrow 30°}{16} + \underset{\searrow 30°}{(a_B)_t} = \underset{\downarrow}{3} + \underset{\nearrow 60°}{\alpha(4)} + \underset{\searrow 60°}{2^2(4)}$

$+X:\quad 16\sin 30° + (a_B)_t\cos 30° = 0 + \alpha(4)\sin 60 + 16\cos 60°$

$+Y:\quad 16\cos 30° - (a_B)_t\sin 30 = -3 + \alpha(4)\cos 60 - 16\sin 60°$

$\boxed{\alpha = 7.68\,rad/s^2}$ $(a_B)_t = 30.7\,ft/s^2$

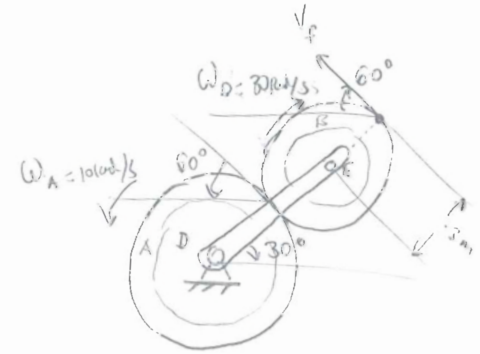
16-131)

$v_E = \omega_{DE} r_E = 6(.5) = 3 m/s$

$v_F = 10(.3) = 3 m/s$

$r_{E/IC} = r_{F/IC} = 0.1 m$

$\omega_B = \dfrac{v_E}{r_{E/IC}} = \dfrac{3}{.1} = \boxed{30\ rad/s}$





$a_E = \alpha_{DE} \times r_E - \omega_{DE}^2 r_E$

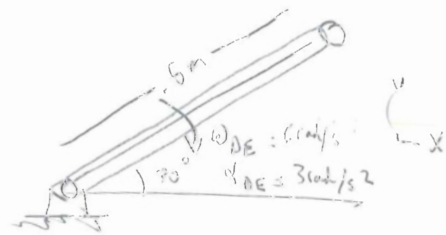
$= (-3k) \times (.5\cos 30° i + .5\sin 30° j) - 6^2(.5\cos 30 i + .5\sin 30 j)$

$= [-14.84 i - 10.30 j]\ m/s^2$

$a_F = a_E + \alpha_B \times r_{F/E} - \omega_B^2 r_{F/E}$

$a_F\cos 30° i + a_F \sin 30° j = -14.84 i - 10.30 j) + (-\alpha_B k) \times (.2\cos 30 i - .2\sin 30 j)$
$- 30^2(-.2\cos 30 i + .2\sin 30 j)$

$.8660 a_F i + .5 a_F j = (141.05 - .1\alpha_B) i + (79.70 + .1732\alpha_B) j$





$.866 a_F = 141.05 - .1\alpha_B$

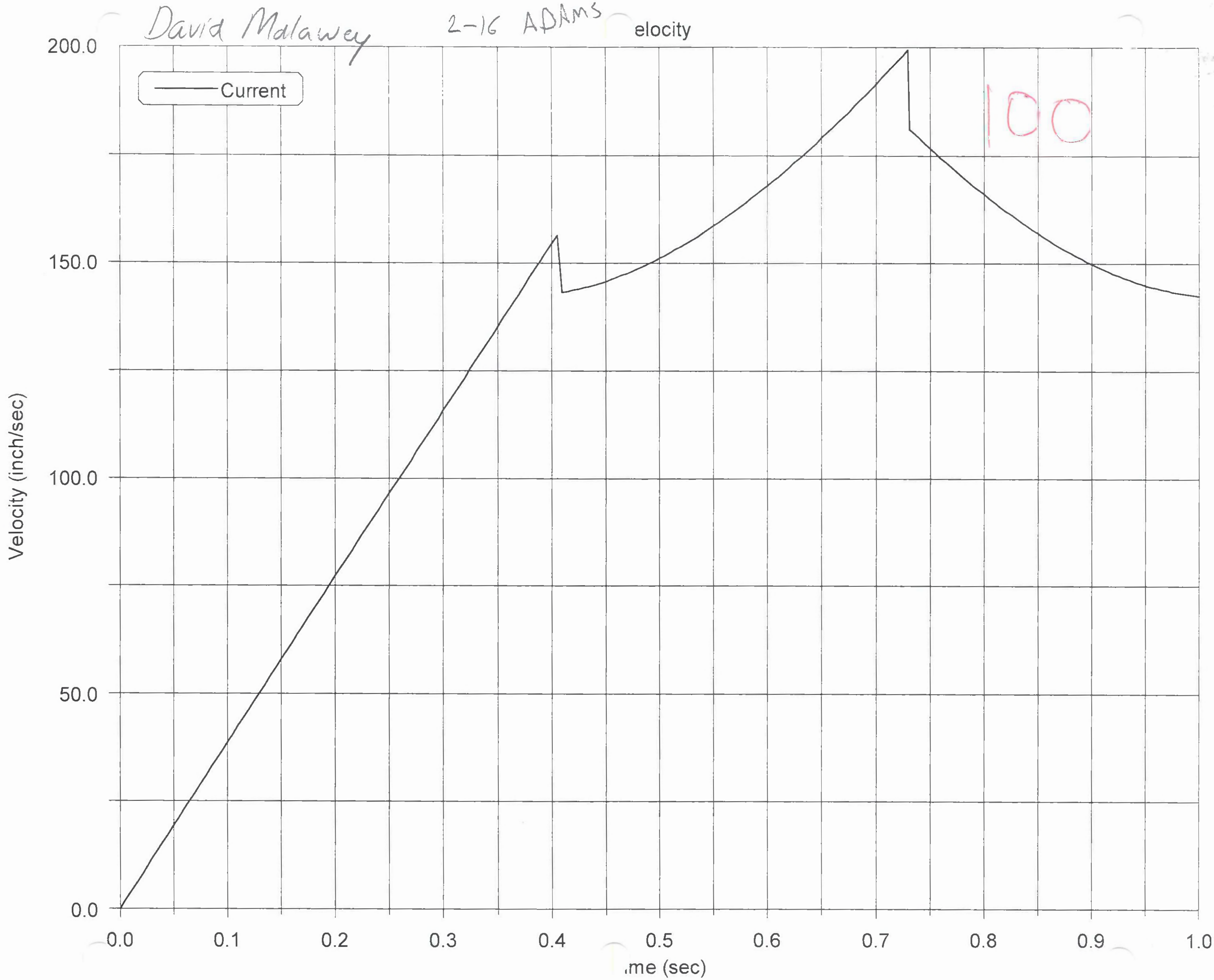
$.5 a_F = 79.70 + .1732\alpha_B$

$\boxed{a_F = 162 m/s^2}$

$\boxed{\alpha_B = 7.5 rad/s^2}$

~~16-139, 159 are not there~~

David Malawey 2-16 ADAMS



David Malaway

100

$\boxed{15}$ - 5, 19, 31, 42, 46

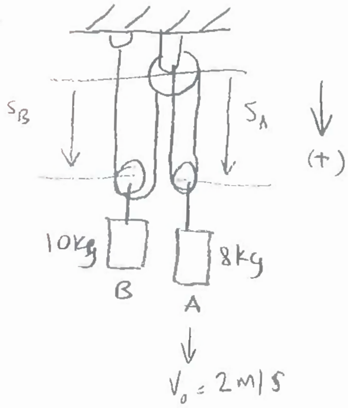
5) $\Sigma m\vec{V}_1 + \Sigma\int_{t_1}^{t_2} F\,t\,dt = \Sigma m V_2$

$t_2 = 3s$

$2S_A + 2S_B = \text{const}$

$V_A = -V_B$

~~$8(2m/s) + 10(-2m/s)$~~ $+$ ~~$9.81(8)(3) + (9.81)(10)(3s) = 8(V_2) + 10(-V_2)$~~




A: $+8(2m/s) + (-2T + mg)(3\,sec) = 8(V_2)$

$16 - 6T + 235.44 = 8V_2$

$251.44 - 6T = 8V_{A2}$

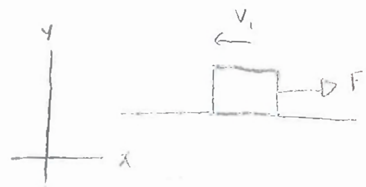

B: $-10(2m/s)(-2T + mg)(3) = 10V_2$

$-20 - 2T + 294.3 = 10V_2$

$274.3 - 6T = 10V_2$

$\boxed{V_2 = 1.27\ m/s}$ $\boxed{B\downarrow, A\uparrow}$ $\boxed{T = 43.6\ N}$

19) 30 lb $V_1 = 6\ ft/s$ $F = 25\cos(\frac{\pi}{10}t)$ (lbs) $t_2 = 15s$




x: $m\vec{V}_1 + \int_{t_1}^{t_2} F\,t\,dt = mV_2$

$\frac{30}{32.2}(-6) + \int_0^{15} 25\cos(\frac{\pi}{10}t)\,dt = \frac{30}{32.2}V_2$

$-5.59 + 25\left[\sin(\frac{\pi}{10}t)\right]_0^{15} \frac{10}{\pi} = .932V_2$

$-85.17 = .932V_2$ ← * radians

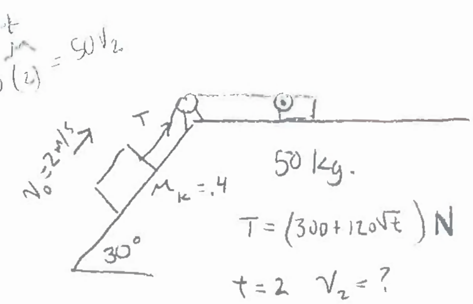
$\boxed{V_2 = -91.38\ ft/s}$

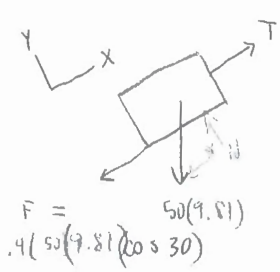
31) $MV_1 + \Sigma\int_{t_1}^{t_2} F\,t\,dt = mV_2$

$50(2) + \int_0^2 (300 + 120\sqrt{t})\,dt - \underbrace{50(9.81)\cos 30}_{mg_y}\ \underbrace{(.4)}_{\mu_x}\underbrace{(2)}_{t} - \underbrace{50(9.81)\sin 30}_{mg_x}\ \underbrace{(2)}_{t} = 50V_2$

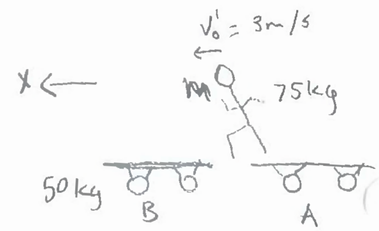
$100 + 826.27 - 339.83 - 490.5 = 50V_2$

$\boxed{V_2 = 1.92\ m/s}$



42, 46




5 - 42) a) System: cart A & boy $\quad \Sigma m_i v_i + \int \Sigma F dt = \Sigma m_2 v_2$

$$75(0) + 0(50) = 75(v_m) + 50(-v_A)$$

$$v_A = -\frac{3}{2} v_m$$

$$v_{M/A} + v_A = v_m$$

$$3 + v_A = v_m$$

$$3 - \frac{3}{2} v_m = v_m$$

$$v_m = 1.2\ m/s \quad \boxed{v_A = -1.8\ m/s} \rightarrow$$

b) $\cancel{50(-1.8)} + 1.2(75) + 0(50) = \cancel{50(-1.8)} + (50+75)(v_f)$

$$\boxed{v_f = .72\ m/s} \leftarrow$$


5-46) a) $v_{B/c} = 3000$

$v_{B/c} + v_c = v_B$ ✱ can use weight for proportions

$$3000 + v_{c_2} = v_{B_2}$$

$$.2 v_{B_2} + v_{c_2}(750) = \cancel{0(750)} + \cancel{0(0.2)}$$

$$v_{B_2} = -3750\ v_{c_2}$$

$$v_{c_2} = -.7998\ ft/s$$

b) external forces $= 0 \qquad v_1 = 0 \quad \boxed{v_2 = 0}$

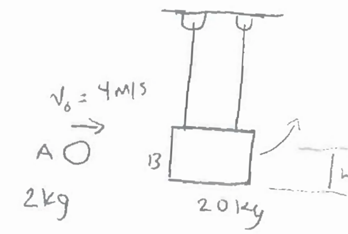
$$0(.2 + 150 + 6000) = v_2(.2 + 150 + 6000)$$

David Molawey

15) 59, 69, 86, 91, 107

59)

$4(2) + 0 = V_{A_2}(2) + V_{B_2}(20)$ $\quad e = 0.8$




$V_{A_2} + 10V_{B_2} = 4$

$0.8 = \dfrac{(V_B)_2 - (V_{A_2})}{(V_A)_1 - (V_B)_1}$ $\quad .8 = \dfrac{(V_B)_2 - (V_A)_2}{4}$

$V_{B_2} - V_{A_2} = 3.2$

Block:

$T_1 + V_1 = T_2 + V_2$

$10V_{B_2} + V_{B_2} - 3.2 = 4$

$\frac{1}{2}(.6545)^2(20) = 20(9.81)h$

$11V_{B_2} = 7.2$

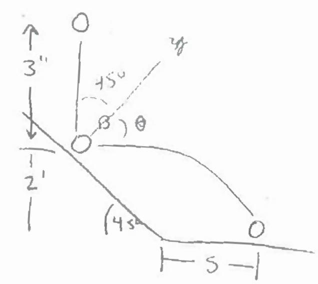
$h = .0218$ m

$\underline{V_{B2} = .6545 \text{ m/s}}$ $\quad \underline{V_{A_2} = -2.545}$

$\boxed{h = 21.8 \text{ mm}}$

69)

fall $\begin{cases} \frac{1}{2}m_B V_B^2 = mgh \\ \frac{1}{2}\frac{2}{32.2}V_B^2 = 2(3') \end{cases}$ $\quad V_B = 13.876$ ft/s $\quad e = .8$, $w = 2$ lb




impact $\quad .8 = \dfrac{V_{By2} - 0}{0 - V_{By1}} = \dfrac{V_{B_2}\cos\theta}{-13.88\sin 45}$ $\quad -7.852 = V_{B_2}\cos\theta$

$mV_{x1} = mV_{x2}$

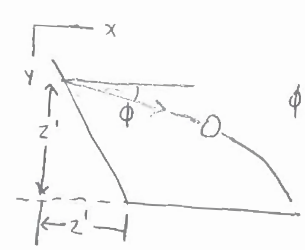
$13.88\sin 45 = V_{B_2}\sin\theta = 9.815$

$\dfrac{\sin\theta}{\cos\theta} = \dfrac{9.815}{7.852} \Rightarrow \boxed{\theta = 51.34°}$ $\quad V_{B2} = 12.570$ ft/s $\quad V_{B2x} = 12.493$ ft/s

$Y: \; v^2 = v_0^2 + 2a(s - s_0)$

$V_t = 12.493$

$v^2 = (-12.57\sin 6.34)^2 + 2(9.81)(2)$




$\phi = 6.34°$

$V_f = 11.26$ ft/s

$2' + S = V_x(t)$

$V = V_0 + a_c t$

$S = 12.493t - 2'$

$11.26 = -1.39 + 32.2(t)$

$\boxed{S = 1.90 \text{ ft}}$

$t = \cancel{.393 s}$ $\quad .3119$ s

$V_f = \sqrt{11.26^2 + 12.493^2} = \cancel{16.82}$ $\boxed{16.9 \text{ ft/s}}$

15) 86, 91, 107

86)

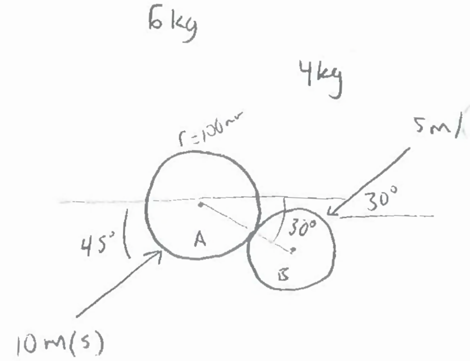
x) $m_A(V_{Ax})_1 + m_B(V_{Bx})_1 = m_A(V_{Ax})_2 + m_B(V_{Bx})_2$

$6(10\sin 15) + 4\cdot 5(-\sin 30) = 6(V_{Ax})_2 + 4(V_{Bx})_2$

$-6V_A\cos\phi_A + 4V_B\cos\phi_B = 5.529$

y) $V_{Ay_1} = V_{Ay_2}$

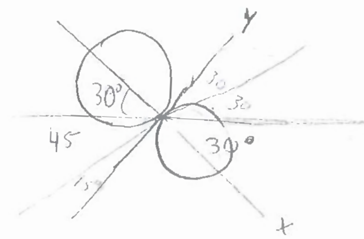
$10\cos 15^\circ = V_{Ay2} = 9.66\text{ m/s}$ $\quad V_{By2} = 5\cos 30 = 4.33\text{ m/s}$



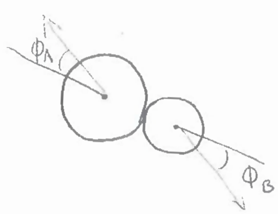

$V_{A2}\sin\phi_A = 9.66\text{ m/s}$ $\quad V_{B_2}\sin\phi_B = 4.33\text{ m/s}$

e = 0.6

$$.6 = \frac{\overbrace{V_{B_2} - V_{A_2}}^{x}}{V_{A_1} - V_{B_1}} = \frac{V_{B_2}\cos\phi_B - V_{A_2}\cos\phi_A}{+10(\sin 15) \overset{(-)}{-} 5\sin 30} \Rightarrow V_{B_2}\cos\phi_B + V_{A_2}\cos\phi_A = 3.053$$




$V_{Ax} = .668 \quad V_{Bx} = 2.385$




$\boxed{V_{A_2} = 9.68\text{ m/s}}$ $\boxed{V_{B_2} = 4.93\text{ m/s}}$

$\phi_A = 86.04^\circ$ $\quad$ ~~$\phi_B = 61.15^\circ$~~ Not needed

91)

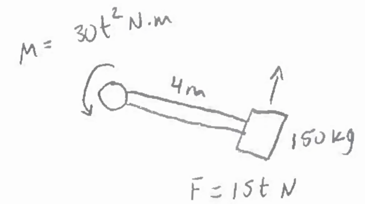
$\vec{r}_1 \times m\vec{V}_1 + \Sigma\int_1^2 M\,dt = \vec{r}_2 \times m\vec{V}_2$

$0 + \int_0^5 30t^2\,dt + \int_0^5 15t(4)\,dt = 150\,V(4)$

$10t^3\big|_0^5 + 30t^2\big|_0^5 = 600\,V$

$1250 + 750 = 600\,V$

$\boxed{V = 3.33\text{ m/s}}$




t = 5 s
V = ?

107)

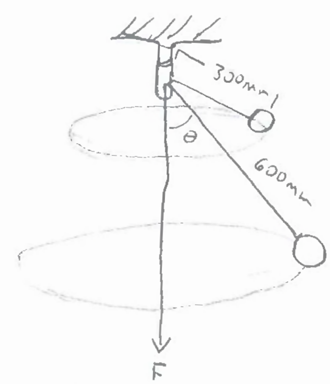
$\Sigma F_n = ma_n \quad F\sin\theta = 2\text{kg}\left(\frac{V^2}{l\sin\theta}\right) \quad l\sin\theta = r$

$\Sigma F_b = 0 \Rightarrow 2(9.81) = F\cos\theta$

$\frac{\cancel{F}}{\cancel{F}}\left(\frac{\sin\theta}{\cos\theta}\right) = \frac{\cancel{2}V^2}{l\sin\theta}\ \frac{1}{\cancel{2}(9.81)} \Rightarrow \frac{\sin^2\theta}{\cos\theta} = \frac{V^2}{9.81\,l} = \frac{1-\cos^2\theta}{\cos\theta}$

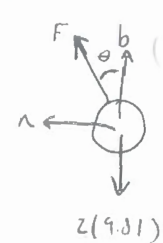
$\frac{1-\cos^2\theta}{\cos\theta} = \frac{1.5^2}{9.81(.6)} \quad 1-\cos^2\theta = .382\cos\theta \quad \cos\theta = .827$

$\boxed{\theta = 34.21^\circ}$




$V_A = 1.5\text{ m/s}$

$r_1\cancel{m}V_1 = r_2\cancel{m}V_2 \Rightarrow .337(1.5) = .3\sin\theta_2 V_2 \quad V_2\sin\theta_2 = 1.685$



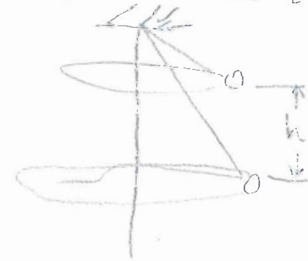
107) continued...

$$\frac{1-\cos^2\theta_2}{\cos\theta_2} = \frac{V_2^2}{9.81(.3)} = \frac{V_2^2}{2.943}$$

$V_2 \sin\theta_2 = 1.685$

$$2.943\,\frac{\cancel{V_2^2}\sin\theta_2^2}{\cancel{V_2^2}} = (1.685)^2\,\frac{\cos\theta_2}{1-\cos^2\theta}$$

$$.9647 = \frac{\sin^2\theta(1-\cos^2\theta)}{\cos\theta_2} = \frac{\sin^4\theta_2}{\cos\theta_2}$$




calculator $\Rightarrow$ $\theta_2 = 57.84$

$V_2 \sin 57.84 = 1.685$

$$\boxed{V_2 = 1.99\ m/s}$$

$.6\cos 34.21° - .3\cos 57.86° =$

$.3366$

$T_1 + \Sigma U_{1-2} = T_2$

$\frac{1}{2}mV_1^2 + U_F + (-Wh) = \frac{1}{2}mV_2^2$

$\frac{1}{2}(2)(1.5)^2 + U_F - 2(9.81)(.3366) = \frac{1}{2}(2)(1.99)^2$

$$\boxed{U_F = 8.314\ N\cdot m}$$

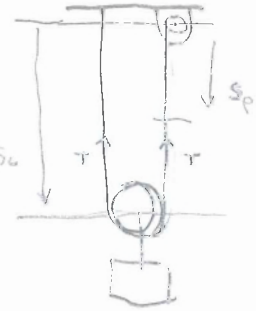
David Malawey Homework

14 | 61, 67, 83, ~~89~~ 99, 106

100

61) $S = 10'$ $P = ?$ @ 10 ft

$V_f = 12$ ft/s $\epsilon = .65$ efficiency




$W = 50$ lbs

$\Sigma F_y = ma$ $2T - 50 = \frac{50}{32.2} a_y$

$(+\uparrow)$ $v^2 = v_0^2 + 2a_c(s - s_0)$

$144 = 0 + 2a_c(10)$

$\frac{14.4}{2} = \boxed{a_c = 7.2 \text{ ft/s}^2}$

$2T = 50 + \frac{50}{32.2}(7.2)$

$\boxed{T = 30.59 \text{ lb}}$

$S_c + (S_c - S_p) = l$

$2V_c = V_p$

$2(12) = V_p = 24$ ft/s

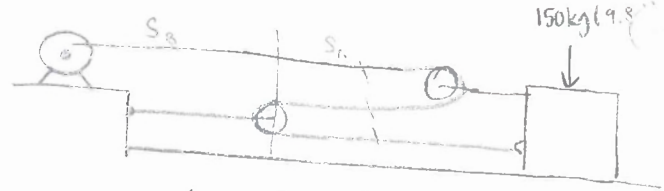
$P_o = F(V) = 30.59(24) = 734.16$ (Power out) ft.lb

$P_i = \frac{734.2}{.65} = \boxed{1129.5 \text{ ft.lb}}$ 540 ft·lb/s = 1 hp

$\boxed{P_i = 2.05 \text{ hp}}$

67) $F = (8t^2 + 20)$N $3S_A + S_B = l$

$P_o$ @ $t = 5s$ ? $V_A = 3V_B$




$\mu_s = .3$ $\mu_k = 0.2$

$\Sigma F = Ma$

$3T - mg\mu = 150\text{kg}(a_x)$

$3(8t^2 + 20) - 1472(.2) = 150 a_x$

$\Sigma F_x = 0 \Rightarrow .3(1471.5) - 3(8t^2 + 20) = 0$ $t = 3.987$

$t = 5\text{sec} \Rightarrow$ $24t^2 + 60 - 294.4 = 150 a_x$

$a_x = .16t^2 - 1.562$ m/s²

$dv = a\,dt$ $\int_{3.99}^{5} (.16t^2 - 1.562)\,dt = V$

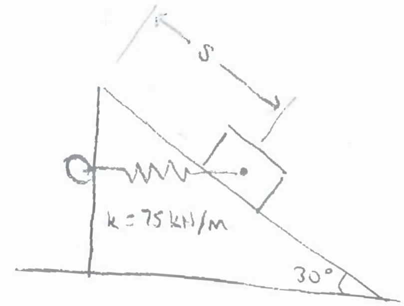
$1.704 = V$

$P = F \cdot V = 1.704(660)$

$= 1125 \text{ W} = \boxed{1.125 \text{kW}}$

14) 83, 99, 106

83) $S_0 = 1m$




T = motion
V = potential

$\cancel{T_1} + \cancel{V_1} = T_2 + V_2$
0 0

$T_2 = -V_2$ when $s = 3$, spring = 2.60

~~$\frac{1}{2}ks = -mgy + \frac{1}{2}mv^2$~~ $mgy = \frac{1}{2}ks^2 + \frac{1}{2}mv^2$

$-\frac{1}{2}(75kN/m)(1.73\ m)^2 = -(9.8)(15kg)(3\sin 30 - 1\sin 30) + \frac{1}{2}(15)v^2$

$112.23 = -147.15 + 7.5v^2$

$v^2 = 4.656$

$\boxed{V = 2.16\ m/s}$

99) $y_1 = 6ft$ $s_1 = 7ft$ Unstretched $s = 4ft$ $k = 50lb/ft$
$y_f = 0ft$ $s_f = 5ft$ $mg = 20lb$

$mgy_1 + \frac{1}{2}ks_1^2 + \cancel{V_0}^{0} = \cancel{mgy_2}^{0} + \frac{1}{2}ks_2^2 + \frac{1}{2}mv_2^2$

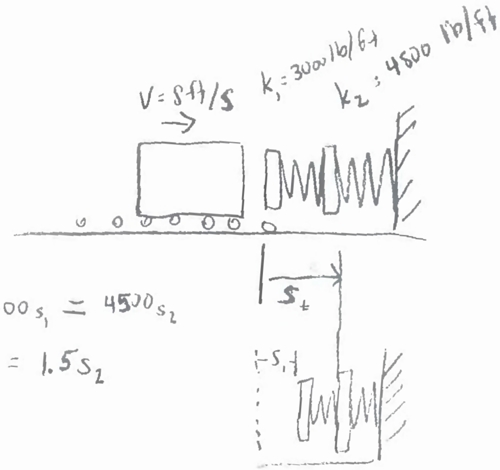
$20(6) + \frac{1}{2}50(9) = \frac{1}{2}50(1^2) + \frac{1}{2}\left(\frac{20}{32.2}\right)v^2$

$345 = 25 + .311v^2$

$v^2 = 1029$

$32.1 = v$

106)




$\cancel{\frac{1}{2}}mv^2 = \cancel{\frac{1}{2}}k_1s_1^2 + \cancel{\frac{1}{2}}k_1s_2$

$\frac{1500(64)}{32.2} = k_1s_1^2 + k_2s_2^2$

$2981 = 3000s_1^2 + 4500s_2^2$ $F_s = 3000s_1 = 4500s_2$

$s_1 = 1.5s_2$

$= 3000(1.5s_2)^2 + 4500s_2^2$

$2981 = 11250s_2^2$

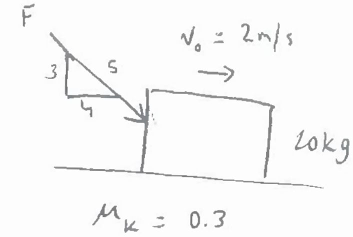
$S_2 = .5148\ ft$ $S_1 = .7722\ ft$

$S_t = S_1 + S_2 = \boxed{1.29\ ft}$

David Maloway Dynamics HW 2-3-10

14 | 15, 23, 34, 39, 41

15)




$S_0 = 0m$

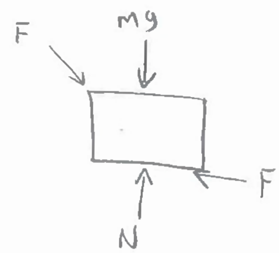
$S_2 = 3m$

$F = 50s^2$

$v(3m) = ?$

$N = mg + \frac{3}{5}F$

$N = 196N + 30s^2$




T = energy U = work

$T_1 + \Sigma_1 U_2 = T_2$

$\frac{1}{2}mv^2 + \int_0^3 F_x\,dx - \mu_s N_{const}\,dx - \mu_s \int_0^3 N\,dx = \frac{1}{2}mv^2$

$\frac{1}{2}(20)(2m/s)^2 + \frac{4}{5}(50)\int_0^3 s^2\,ds - .3(196)(3) - .3\int_0^3 30s^2\,ds = \frac{1}{2}(20)v_f^2$

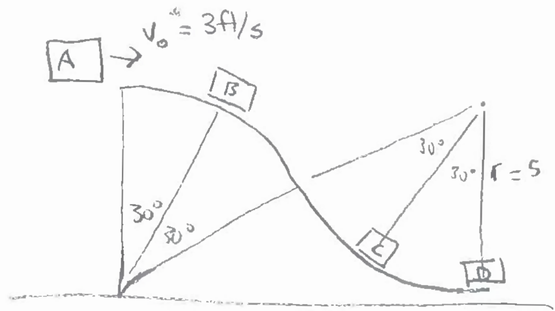
$40 + 40\frac{s^3}{3}\Big|_0^3 - 176.4 - .3\left(\frac{30s^3}{3}\right)\Big|_0^3 = 10v_f^2$

$40 + 360 - 176.4 - 81 = 10v_f^2$

$V_f = \sqrt{14.26} = \boxed{3.78\ m/s}$

23) W = 50 lb

m = 1.553 slug




$T_1 + \Sigma_1 U_2 = T_2$

$\frac{1}{2}mv_0^2 + (5 - 5\cos 30)mg = \frac{1}{2}mv_B^2$

$\frac{1}{2}(1.553)3^2 + 33.49 = \frac{1}{2}1.553\,V_B^2$

$V_B = \sqrt{\frac{40.48}{.7765}}$ $\boxed{V_B = 7.22\ ft/s}$

$6.9 + (5\cos 30)mg = .7765\,V_C^2$

$\boxed{V_C = 16.97\ ft/s}$

$\frac{6.9 + 5mg}{.7765} = V_D^2$

$\boxed{V_d = 18.19\ ft/s}$

$+\swarrow\ \Sigma F_n = ma_n \Rightarrow -N_B + Mg\cos 30° = m\frac{V_B^2}{r}$

$N_B = 50\cos 30° - 1.55\frac{7.22^2}{5}$

$\boxed{N_B = 27.14\ lb}$

$N_C - mg\cos 30 = m\frac{V_C^2}{r}$

$N_C = +50\cos 30 + m\frac{V_C^2}{r}$

$\boxed{N_C = 132.58}$

$N_D - mg = m\frac{V_D^2}{R}$

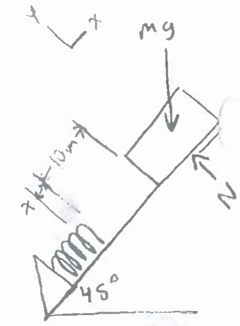
$N_D = 50 + 1.55\frac{18.19^2}{5}$

$N_D = 152.57$

David Malawey
Homework, Dynamics

14) 34, 39, 41

34) $m = 100kg$ $\mu_k = 0.25$ find $v$ when $x = 1.5m$ $k = 2kN/m$




$\Sigma F_y = 0$

$Mg \sin 45 = \boxed{N = 693.7}$

$T_1 + \Sigma_1 U_2 = T_2$

$\frac{1}{2} m v_0^2 - \int F ds + mg\,dh + \text{energy(spring)} = \frac{1}{2} m v_f^2$

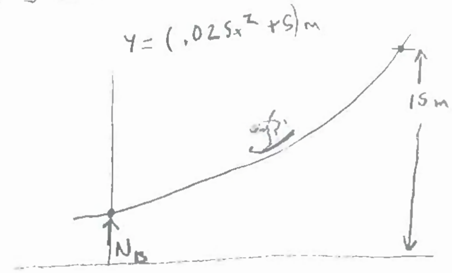
$0 - (11.5)(.25)(693.7) + 100(9.81)(11.5 \sin 45) - \frac{1}{2}(2000)(1.5)^2 = \frac{1}{2}(100) v_f^2$

$-1994 + 7977 - 2250 = 50 v_f^2$

$v_f^2 = 74.66$ $\boxed{v_f = 8.64 \text{ m/s}}$

39) $T_1 + \Sigma_1 U_2 = T_2$

$m = 60kg$
$v_A = 5m/s$
$v_B = ?$
$N_B$




$\frac{(60kg)5m/s^2}{2} + 10m(9.81)(60kg) = \frac{1}{2} v_B^2 (60kg)$

$750 + 5886 = 30 v_B^2$

$\boxed{v_B = 14.87 \text{ m/s}}$

$\rho_B = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2x}{dy^2}} = \frac{\left[1 + (.05x)^2\right]^{3/2}}{.05}\Bigg|_{x=0}$ $\rho = 20m$

$\Sigma F_N = m a_n \Rightarrow F_N - mg = m\frac{v_2^2}{r} \Rightarrow F_N = 60\frac{(14.87^2)}{20} + 60(9.81)$

Slope is zero,

$F_N = 1251.95 N$

$\boxed{F_N = 1.252 kN}$

41) $\Sigma F_n = m a_n$

$+mg\cos\theta - N = m\frac{v^2}{r}$

$\cancel{m}g\cos\theta - 0 = \cancel{m}\frac{\left(\frac{9}{4} - 2\cos\theta\right)\cancel{g}r}{r}$

$\cos\theta = \frac{9}{4} - 2\cos\theta$

$3\cos\theta = 9/4$

$\cos\theta = 3/4$

$\theta = \cos^{-1}(3/4)$

$\boxed{\theta = 41.41°}$

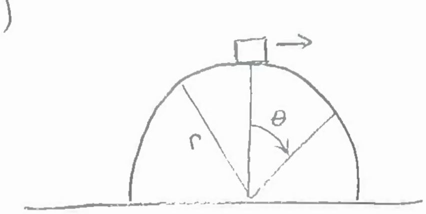
$\frac{m}{2} v_f^2 = \frac{m}{2} v_0^2 + r(1-\cos\theta)mg$

$v_f^2 = \frac{2}{\cancel{m}}\left(\frac{\cancel{m}}{2} v_0^2 + r(1-\cos\theta)\cancel{m}g\right)$

$v_f^2 = v_0^2 + 2rg(1-\cos\theta)$

$v_f^2 = \frac{1}{4}gr + 2(1-\cos\theta)gr$

$v_f^2 = \left(\frac{9}{4} - 2\cos\theta\right)gr$

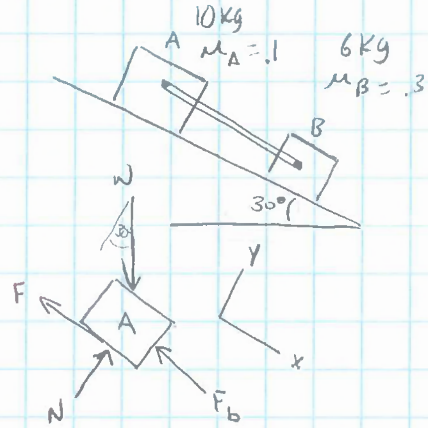



$v_0 = \sqrt{.25gr}$
find $\theta$ s.t. $F_N = 0$

David Malawey

13 - 5, 20, 33, 35, 43

98




5) $\Sigma F_x A = m a_x$

$g m_A \sin 30 - F - F_b = m a_x$

$9.8(10)\sin 30) - .10(9.8(10)\cos 30) - F_b = 10 a_x$

$40.51 - F_b = 10 a_x$

$\Sigma F_x(B) = m a_b$

$M_b g \sin 30 - F + F_b = m_b a_x$

$6(9.8)\sin 30 - .3(9.8)(6)\cos 30 + F_b = 6 a_x$

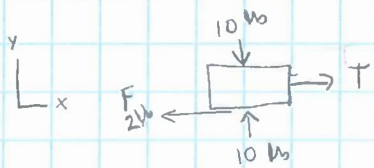
$14.12 + F_b = 6 a_x$

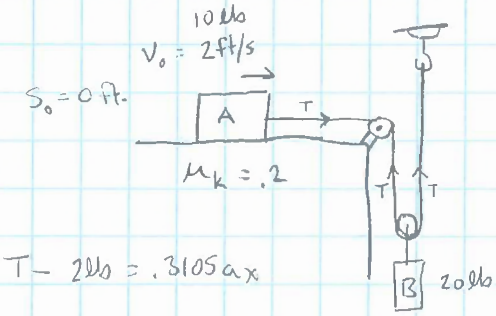
$a_x = 2.35 + \frac{F_b}{6}$

$40.51 - F_b = 23.5 + 1.67 F_b$

$2.67 F_b = 16.98$

$\boxed{F_b = 6.36 \text{ N}}$

20) 


$\Sigma F_{xA} = m a_x$

$T - F_k = T - 2lb = \frac{10 lb}{32.2} a_A$ | $T - 2lb = .3105 a_x$

$a_A = 2 a_B$

$\Sigma F_{yB} = -20 lb + 2T = \frac{20 lb\, a_B}{32.2}$

$-20 + 2T = .3106 a_A$ $\quad T - 2 = .3106 a_x$

$3T - 22 = 0 \quad 3T = 22 \quad \boxed{T = 7.333} \quad \boxed{a_A = 17.171}$

$v^2 = v_o^2 + 2a_c(s - s_o)$

$v^2 = 4 \text{ ft}/s^2 + 34.34(4) \Rightarrow 141.36^{1/2} = \boxed{v = 11.89 \text{ ft/s}}$

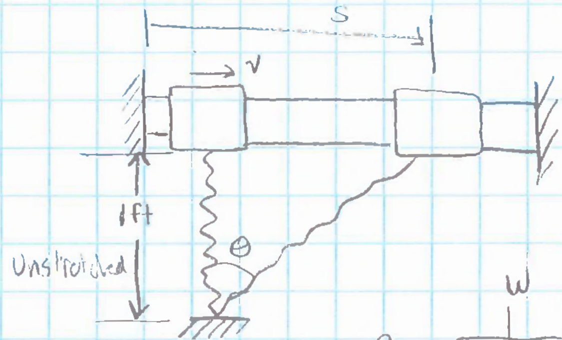
Notes 1-26

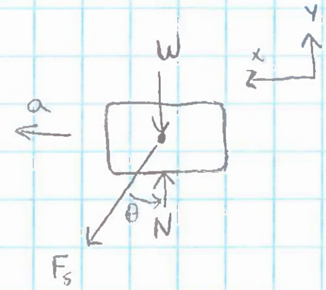
13-33)

$W = 2\text{lb}$ $S_0 = 0$

$V_0 = 15\text{ ft/s}$ $k = 4\text{lb/ft}$

Find V @ s = 1 ft




$$\Sigma F_x = Ma$$

$$F_s \sin\theta = Ma$$

$$k\left(\sqrt{1^2+s^2} - 1\right)\frac{s}{\sqrt{1^2+s^2}} = ma$$

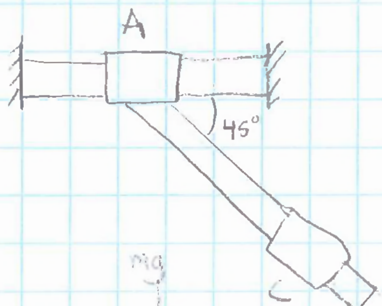
$$V^2 = V_0^2 + 2\int_0^s a\,ds = (-15)^2 + 2\int_0^1 \frac{k}{m}\left(\sqrt{1+s^2} - 1\right)\left(\frac{s}{\sqrt{1^2+s^2}}\right)ds$$

$$\int_0 a(s) \quad = 2s^2 - 4\sqrt{1+s^2}\,\Big]_0^1 \qquad \boxed{V = 14.6\text{ ft/s}}$$

35) given $m = 2\text{kg}$

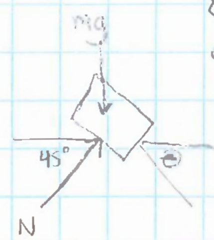
$a_A = 2\text{m/s}^2$

case a and b? find $\vec{a}_c$ −2




$$\Sigma F = m\vec{a}_c$$

$$-mg\vec{j} + N(\cos 45\,\vec{i} + \sin 45\,\vec{j}) = ma_c$$




$$\vec{a}_c = \vec{a}_A + \vec{a}_{c/A}$$

$$m\left[2\vec{i} + a_{c/A}\left(\cos 45^\circ i - \sin 45^\circ \vec{j}\right)\right]$$

$$-mg\vec{j} + N(\cos 45\,i + \sin 45\,j) = m\left[2i + a_{c/A}\left(\cos 45^\circ i - \sin 45^\circ j\right)\right]$$

$$a_{c/A} = 8.351\text{ m/s}^2$$

$$a_c = \sqrt{3.905^2 + -5.905^2} = \boxed{7.08\text{ m/s}^2}$$

$$\theta = \tan^{-1}\left(\frac{5.905}{3.905}\right) = \boxed{56.5^\circ}$$

David Malawey Homework

13 - ( ~~33~~, ~~35~~, 43)

A: $\Sigma F_x = m a_x$

$$-k(x-d) - N - (\mu_k)(M_A g) = M_A a_x$$

B: $\Sigma F_x = m a_x$

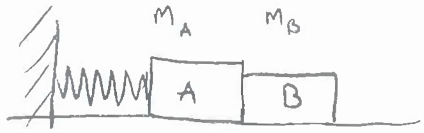
$$\cancel{N}_0 - \mu_k (m_B g) = \cancel{m_B} a_x$$

$$a_x = -\mu_k g \quad \text{if } N = 0$$

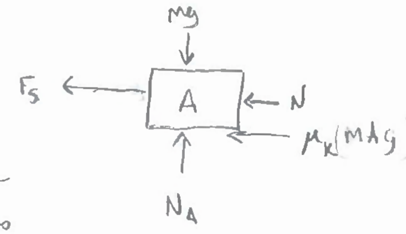
$$a = \frac{k(d-x) - \mu_k g (m_A + m_B)}{(m_A + m_B)}$$

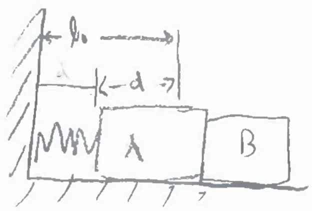
$$a = \frac{k(d-x)}{(m_A + m_B)} - \mu_k g$$

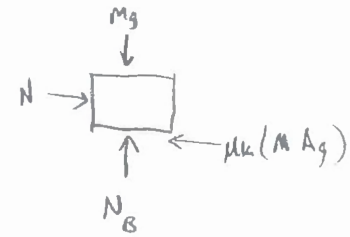
$$N = \frac{k m_B (d-x)}{(m_A + m_B)} \quad \text{if } N = 0,\ x = d$$




k, unstretched length $l_0$




$d > 2\mu_k g (M_A + m_B)/k$

for sep. to occur

dist before separation ?


$$v\,dv = a\,dx$$

$$\int_0^v v\,dv = \int_0^d \left[ \frac{k(d-x)}{(m_A + m_B)} - \mu_k g \right] dx$$

$$\frac{1}{2} v^2 = \frac{k}{(m_A + m_B)} \left[ (d)x - \frac{1}{2} x^2 - \mu_k g x \right]_0^d$$

$$v = \sqrt{\frac{k d^2 - 2\mu_k g (m_A + m_B) d}{(m_A + m_B)}}$$

$$V > 0, \Rightarrow k d^2 - 2\mu_k g (m_A + m_B) d > 0$$

$$k d > 2 \mu_k g (m_a + m_B)$$

$$\boxed{d > \frac{2\mu_k g}{k} (m_a + m_B)}$$

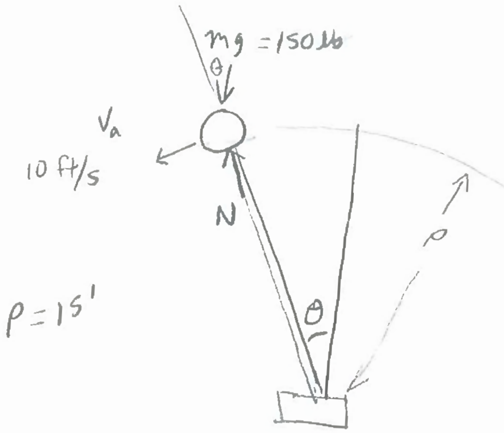
David Malawey Homework

13 - 59, 74, 82, 102, 107

59)

$$w \cos \theta = \frac{w}{g} \frac{v^2}{r}$$

$$\Sigma F_n = m a_n$$



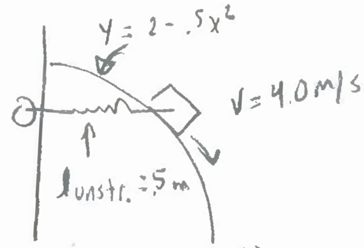

$$150 \cos \theta = \frac{150 \text{ lb}}{32.2 \text{ ft/s}^2} \left( \frac{(10 \text{ ft/s})^2}{15} \right)$$

$$\cos \theta = .20704$$

$$\theta = 78.05^\circ$$

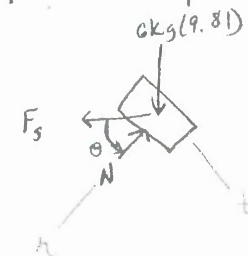
74)

$$y = 2 - 0.5x^2 \qquad k =$$




$$\frac{dy}{dx} = \tan \theta = -x \Big|_{x=1} = -1 \quad \theta = 45^\circ$$

$$\frac{d^2y}{dx^2} = -1$$




$$\rho = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left[1 + (-1)^2\right]^{3/2}}{|-1|} = 2.8284 \text{ m}$$

$$F_s = kx = 10(1 - 0.5)$$

$$\Sigma F_n = m a_n \qquad 6(9.81)\cos 45 - N + 5 \cos 45^\circ = 6\left(\frac{4^2}{2.8284}\right)$$

$$\boxed{N = 11.2 \text{ N}}$$

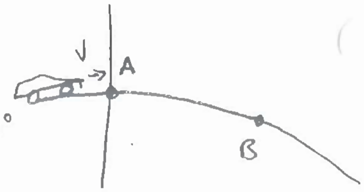
$$\Sigma F_t = m a_t \Rightarrow 6(9.81) \sin 45^\circ - 5 \sin 45^\circ = 6 a_t$$

$$\boxed{a_t = 6.35 \text{ m/s}^2}$$

David Maloney

13) 82, 102, 107

82) $\frac{dy}{dx} = -.01x \quad \frac{d^2y}{dx^2} = -.01$



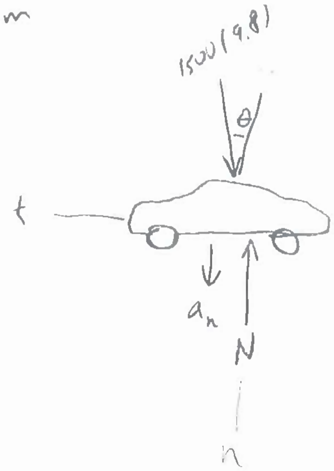

$y = 25 - \frac{1}{200}x^2$

$\theta_A = \tan^{-1}\left(\frac{dy}{dx}\right)\Big|_{x=0m} \quad \theta_B = \tan^{-1}(-.01(25)) = -14.04°$

$= \tan^{-1}(0) = 0°$

$\rho_A = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}\Bigg|_{x=0m} = \frac{(1+0^2)^{3/2}}{0.01} = 100m$

$\rho_B = \Big|_{x=25m} \left[\frac{1+(25)^2}{0.01}\right]^{3/2} = 109.52m$




$\Sigma F = ma_n \quad 1500(9.81)\cos\theta - N = 1500\left(\frac{v^2}{\rho}\right)$

$N = 14715\cos\theta - \frac{1500v^2}{\rho}$

$N = N_A = 0 \quad \theta = \theta_A = 0 \quad \rho = \rho_A = 100m$

$0 = 14715\cos 0° - \frac{1500v^2}{100} \quad \boxed{v = 31.32 m/s}$

B: $\theta = \theta_B = 14.04° \quad \rho = \rho_B = 109.52m$

$N_B = 14715\cos 14.04° - \frac{1500(31.32^2)}{109.52}$

$\boxed{N_B = 839.74N}$

David Malawey H/W 2-1-10

13 – 102, 107

Force

$r$? $\theta$? $z$?

$M = 20\,kg$ $\theta = 120°$

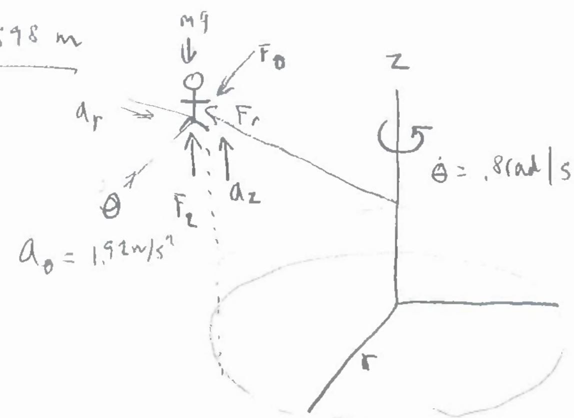
102) $\dot\theta = 0.8$ rad/s $r = (3\sin\theta + 5)$ m $z = (3\cos\theta)$ m

$\theta = 120 \Rightarrow r = (3\sin\theta + 5)\big|_{\theta=120} = 3\sin 120 + 5 = \underline{7.598\ m}$

$\dot r = 3\cos\theta\dot\theta\big|_{\theta=120} = 3\cos 120°(.8) = \underline{-1.2\,m/s}$

$\ddot r = 3\cos\theta\ddot\theta - \sin\theta\dot\theta^2\big|_{\theta=120}$

$= 3[\cos 120°(0) - \sin 120°(.8^2)] = \underline{1.663\,m/s}$




$a_r = \ddot r - r\dot\theta^2 = 1.633 - 7.598(.8^2) = 6.526\,m/s^2$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 7.598(0) + 2(-1.2)(.8) = -1.92\,m/s^2$

$z = 3\cos\theta$ m $\dot z = -3\sin\theta\dot\theta$ m/s

$a_z = \ddot z = -3(\sin\theta\ddot\theta + \cos\theta\dot\theta^2)\big|_{\theta=120°} = -3[\sin 120°(0) + \cos 120°(.8^2)]$

$= 0.96\,m/s^2$

$\Sigma F_r = ma_r$ $F_r = 20(-6.526) = \boxed{-131\,N}$

$\ldots\ \theta$ $F_\theta = 20(1.92) = \boxed{-38.4\,N}$

$\ldots\ z$ $F_z - 20(9.81) = 20(0.96)$ $\boxed{F_z = 215\,N}$

David Malawey

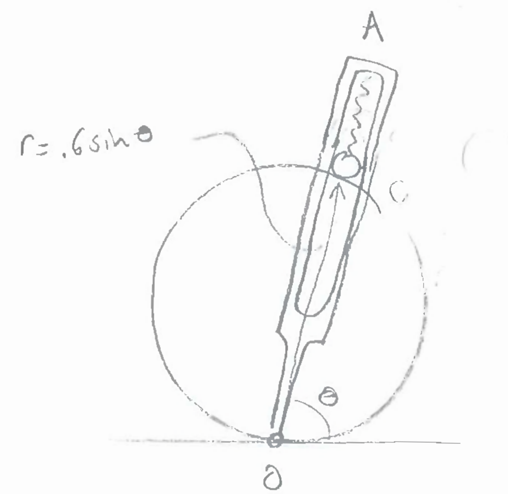
HW 107)

1.5kg cylinder C

$\dot{\theta} = 3 \text{ rad/s}$

Force from smooth slot

$K = 100 \text{ N/m}$

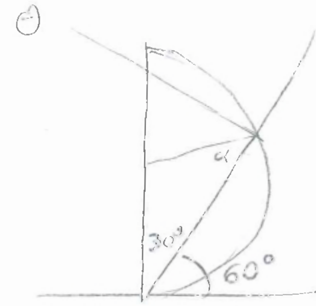



$\theta = 60°$ : $r = .6\sin\theta \big|_{\theta=60°} = .6\sin 60° = .5196\text{ m}$

$\dot{r} = .6\cos\theta\,\dot{\theta} \big|_{\theta=60} = \quad = .9\text{ m/s}$

$\ddot{r} = (.6)\cos\theta\,\ddot{\theta} - \sin\theta(\dot{\theta})^2 \big|_{\theta=60}$

$= .6[\cos 60(0) - \sin 60°(3^2)] = -4.677\text{ m/s}$

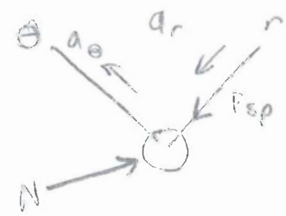



$a_r = \ddot{r} - r\dot{\theta}^2 = -4.677 - .5196(3^2) = -9.353\text{ m/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = .5196(0) + 2(.9)(3) = 5.4\text{ m/s}^2$

$\Sigma F_r = ma_r \quad N\cos 30 - 21.96 = 1.5(-9.353)$

$\boxed{N = 9.159\text{ N}}$




$\Sigma F_\theta = ma_\theta \quad F_{OA} - 9.159\sin 30° = 1.5(5.4)$

$\boxed{F_{OA} = 12.68\text{ N}}$

13.33)

$$v^2 = 15^2 + 2\int_0^1 \frac{k}{m}\left(\sqrt{1+s^2}-1\right)\left(\frac{s}{\sqrt{1+s^2}}\right) ds$$

$$= 15^2 + \frac{2k}{m}\left[\int_0^1 s\,ds + \int_0^1\left(\frac{-s}{\sqrt{1+s^2}}\right)ds\right]$$

$$= 225 + \frac{2k}{m}\left[\frac{s^2}{2}\Big|_0^1 + -\sqrt{s^2+1}\Big|_0^1\right]$$

$$= 225 + \frac{2k}{m}\left[\frac{1}{2} - \sqrt{2} + 1\right]$$

$$= 225 + \frac{2k}{m}\left[\frac{3}{2} - \sqrt{2}\right]$$

$$= 225 + \frac{k}{m}\left[3 - 2\sqrt{2}\right]$$

$$= 225 + \frac{4}{(2/32.2)}[.17157]$$

$$= 225 + 11.281$$

$$\boxed{v = 14.62 \text{ ft/s}}$$

13-35)

$-mg\vec{j} + N\sin 45\hat{j} = m\,a_{c/A} - \sin 45\hat{j}$

$-19.62 + N\,.707 = 2\,a_{c/A} - .707$

$N(.707) = 2\,a_{c/A} + 18.913$

$N = 2.83\,a_{c/A} + 26.75$

$N\cos 45\,\vec{i} = m\,2\vec{i} + a_{c/A}\cos 45\,\vec{i}$

$N(.707) = 4 + a_{c/A}(.707)$

$a_{c/A} = N - 5.66$

$a_{c/A} = 2.83\,a_{c/A} + 26.75 - 5.66$

$-1.83\,a_{c/A} = + 21.09$

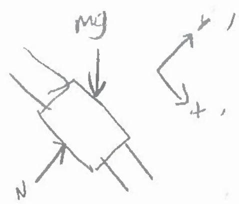
$\boxed{a_{c/A} = -11.52}$

$N = \boxed{-5.86}$

$\vec{a}_c = 2kg\,[\,2i - 11.52\,(.707\,i - .707\,j\,)]$

$\vec{a}_c = 4i - 16.29\,i + 16.29\,j$

$\vec{a}_c = 12.29\,i + 16.29\,j$




a) $a_c$ <fixed shaft>

$\Sigma F_x = m\,a_x$

$mg\sin 45 = m\,a_x$

$9.8(\cancel{2})(.707) = \cancel{2}\,a_x$

$\boxed{m/s^2\ 6.936 = a_x}$

b) $a_c$ < $\rightarrow v_{shaft} = $ const>

$\boxed{a_x = 6.936\ m/s^2 \text{ still}}$

c) collar A has $a = -2\,m/s^2$

$\Sigma F_c = m\,a_c$

$\Sigma F_{c_{x'}} = m(a_A + a_{c/A})$

$2(9.81)(\sin 45) = m(a_{c/A}) + m(-2\cos 45)$

$9.81(.707) = a_{c/A} - 2\,(.707)$

$a_{c/A} = 8.35\ m/s^2$

$a_c = a_A + a_{c/A}$

$a_c = 2\,m/s\,\hat{i} + 8.35\,m/s^2(\cos 45\hat{i} + \sin 45\hat{j})$

$a_i = -3.903\,\hat{i} + 5.903\,\hat{j}$

$\boxed{a_c = 7.07\ m/s^2}$ along $x'$ axis

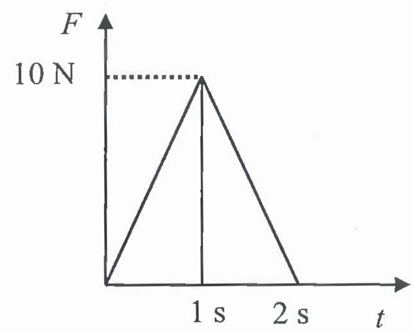
## Lecture 9: Impulse and Momentum

### Concept questions

1. Which parameter is not involved in the principle of impulse and momentum equation?

| | | | |
|---|---|---|---|
| A) | Velocity | B) | Force |
| C) | Time | D) | Acceleration |

2. Calculate the impulse due to the force.




| | | | |
|---|---|---|---|
| A) | 20 kg·m/s | B) | 10 kg·m/s |
| C) | 5 N·s | D) | 15 N·s |

3. The internal impulses acting on a system of particles always

A) equal the external impulses. B) sum to zero.
C) equal the impulse of weight. D) None of the above.

4. If the conservation of linear momentum holds for a system of particles, one can conclude that

A) the velocities of all particles are zero
B) the velocities remain constant
C) the velocity of the mass center is zero
D) the velocity of the mass center is constant

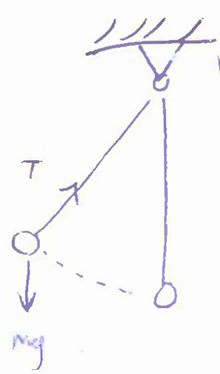
5. If the momentum of a system of particles is conserved, the energy is also conserved. (true or false)

if only $mg$ & $F_s$ do work

$T + V = \text{const}$

if $\Sigma \int F\,dt = \vec{0}$

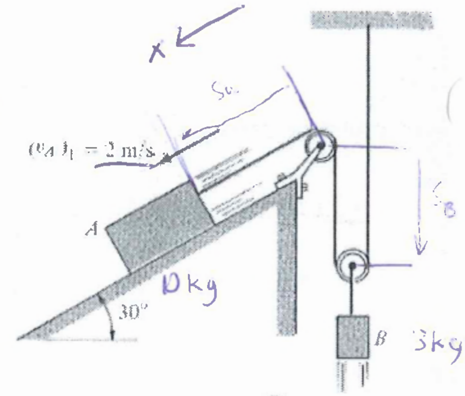
$\Sigma m_i \vec{v}_i = \text{const}$




$W(T)$ is zero

$\int_{t_1}^{t_2} T\,dt \neq 0$

**Example 1** Blocks $A$ and $B$ have masses $m_A = 10$ kg and $m_B = 3$ kg, respectively. If block $A$ is moving down the inclined plane with an initial velocity $(v_A)_1 = 2$ m/s, determine the velocity of $B$ when $t = 2$ s. ? Assume that the inclined plane is smooth. Neglect the mass of the pulleys and cord.




$s_A + 2s_B = \text{Const}$

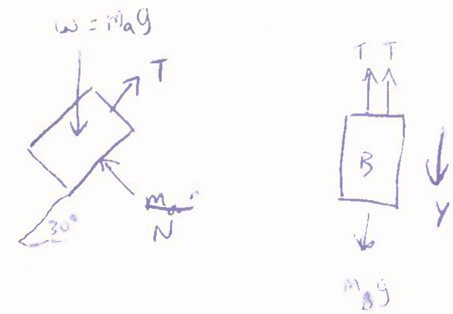
$v_A = -2v_B \Rightarrow v_B = -1$ m/s

$(v_B)_2 = -0.5(v_A)_2$

Kinetics

A: $m_A(v_{Ax})_1 + \Sigma\int F_x\,dt = m(v_x)_2$

$(10)2 + 10(9.81)\sin 30^\circ(2) - T(2) = 10(v_A)_2$




B: $m_B(v_y)_1 + \Sigma\int F_y\,dt = m(v_y)_2$

$3(-1) - 2T(2) + 3(9.81)(2) = 3(v_y)_2$

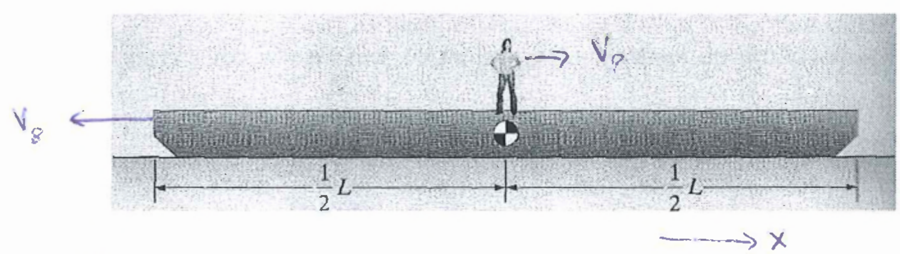
$(v_B)_2 = -4.19$ m/s

**Example 2** A person of mass $m_p$ stands at the center of a stationary barge of mass $m_b$. Neglect horizontal forces exerted on the barge by the water.

(a) If the person starts running to the right with velocity $v_p$ relative to the bank, what is the resulting velocity of the barge relative to the bank? (b) If the person stops when he reaches the right-hand end of the bar, what are his position and the barge's position relative to their original positions?

(1) $v_B$

(2)



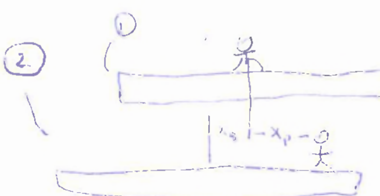

$\Sigma m_i v_i = \Sigma m_i (v_i)_2$ negative

$0 = m_p V_p + m_B V_B$

$V_B = \frac{m_p}{m_B} V_p$

(2) $x_B + x_p = \frac{1}{2}L$ (1)

$x_B = \frac{m_p}{m_B} x_p$ (2)



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12 – 118, 130, 132, 159, 170

118) $\rho = 50\,m$ $\quad v = (0.2t^2)\ m/s$ $\quad v(3)?\ \&\ a(3)?$

$\boxed{v(3) = 1.8\,m/s}$

$a_t = 0.4t\ m/s$ $\quad a_t(3) = 1.2\,m/s^2$

$a_n = \frac{v^2}{\rho} = \frac{(1.8)^2 m/s}{50\,m} = .0648\,m/s^2$

$\boxed{\vec{a}(3) = 1.2\,m/s^2\ \vec{u}_t + .0648\,m/s^2\ \vec{u}_n}$

$\boxed{a = 1.20\,m/s^2}$

130) $a_t = (6 - .06s)\ m/s^2$ $\quad$ find $a(B)$ $\quad S_B = 40\,m$

$6 - 0.6(40) = -18\,m/s^2$

$\int_0^v v\,dv = \int_0^s (6 - .06s)\,ds = 6s - .03s^2$

$\frac{1}{2}v^2 = 6s - .03s^2$

$v^2 = 12s - .06s^2$

$v = \sqrt{12s - .06s^2}$

$\boxed{v(40) = 19.60\,m/s}$

$y = 1/100\,x^2$

$S_B = \frac{1600}{100} =$

$\frac{dy}{dx} = \frac{2}{100}x = \frac{1}{50}x$

$\frac{d^2y}{dx^2} = \frac{1}{50}$

$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\frac{1}{50}x\right)^2\right]^{3/2}}{\frac{1}{50}}\right|_{x=38}$

$\rho = 79.30\,m$

$a_t = 3.6\,m/s^2$

$a_n = \frac{v^2}{\rho} = 4.844\,m/s^2$

$\boxed{a(40) = 6.034\,m/s^2}$

132) $v_0 = 30\,m/s$ $\quad a_t = \left(-\frac{1}{8}t\right) m/s^2$ $\quad t = 15\,s$ $\quad$ find $a_c$

$dv = a_t\,dt$ $\quad \int_{30\,m/s}^{v} dv = \int_0^t -\frac{1}{8}t\,dt$ $\quad v - 30 = -\frac{1}{16}t^2$

$v = \left(30 - \frac{1}{16}t^2\right) m/s$ $\quad \underline{v(15) = 15.94\,m/s}$

$\frac{ds}{dt} = v$ $\quad v\,dt = ds$ $\quad \int_0^s ds = \int_0^t v\,dt = \int_0^t \left(30 - \frac{1}{16}t^2\right)dt$

$s = \left(30t - \frac{1}{48}t^3\right) m$ $\quad S_c = 379.7$

$S_{BC} = S_c - S_B = 379.7 - 100 = 279.7\,m$ $\quad \rho = \frac{S_{BC}}{\theta} = \underline{356.11\,m}$

$a_t(c) = \dot{v} = 1.875\,m/s^2$

$a_n(c) = \frac{v_c^2}{\rho} = .713\,m/s^2$

$\boxed{a(c) = 2.01\,m/s^2}$

David Malaway Homework

12-159, 170

159) $r = (t^3 + 4t - 4)\,m \quad \theta = t^{3/2}$ rad $\quad$ t seconds

$v(2)$? $\quad a(2)$?

$v_t = \frac{dr}{dt} = 3t^2 + 4 \quad v_t(2) = 16\,m/s \qquad r(2) = 8 + 8 - 4 = 12\,m$

$\dot\theta = \frac{d\theta}{dt} = \frac{3}{2}t^{1/2} \quad \dot\theta(2) = \frac{3\sqrt2}{2}$ rad/s $= 2.121$ rad/s

$v_n = \omega r = \frac{3\sqrt2}{2}(12\,m) = 18\sqrt2 = 25.46 \qquad v = \sqrt{v_t^2 + v_n^2} = \boxed{30.07\,m/s}$

$a_r = \ddot r - r\dot\theta^2 = 12 - 12(2.121)^2 = -42.0\,m/s^2$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 12m\left(\frac{3}{4}t^{-1/2}\right) + 2(16)(2.121) = 74.24$

$\boxed{a = 85.31\,m/s^2}$

170) $\ddot r = .5\,m/s^2 \qquad \dot\theta = .2$ rad/s $\qquad t = 3s \quad r_0 = 0$

$r = \frac{1}{2}\ddot r t^2 \quad r(3) = \quad = \underline{2.25\,m}$

$\dot r(3) = \ddot r t = .5(3) = 1.5\,m/s$

$v_\theta = r\dot\theta = 2.25\,m(.2)$ rad/s $= \boxed{.45\,m/s} \quad \boxed{v_r = 1.5\,m/s}$

$a_r = \ddot r - r\dot\theta^2 = .5 - 2.25(.2)^2 = \boxed{.41\,m/s^2}$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 2.25(0) + 2(1.5\,m/s^2)(.2) = \boxed{0.6\,m/s}$

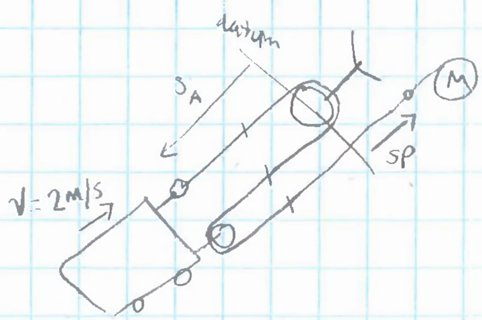
12-195, 203, 214, 220, 230

195) $3S_A + S_p = $ Constant

$3V_A + V_p = 0$

$V_p = 3V_A = 3(2\,m/s)$

$\boxed{V_p = 6\,m/s}$




203) $2S_c + S_D = $ const

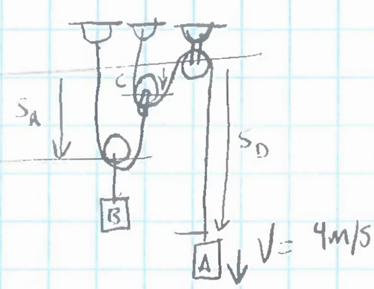
$S_A + S_A - S_c = $ Const $\Rightarrow \quad 2S_A - S_c = C$

$2V_c + V_D = 0 \qquad 2V_c = -4\,m/s \qquad V_c = -2\,m/s$

$2V_A - V_c = 0$

$2V_A - (-2\,m/s) = 0$

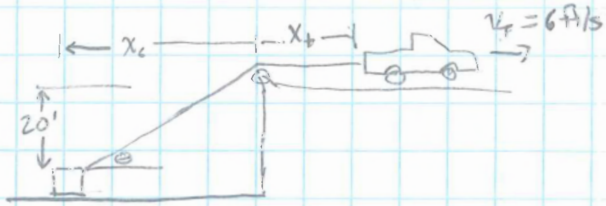
$2V_A = 2\,m/s \qquad V_A = 1\,m/s \qquad \boxed{V_B = 1\,m/s}$



David Malaway

214, 220, 230

214)




$$\sqrt{x_c^2 + 400} + x_T = 100'$$

$$\tfrac{1}{2}(x_c^2+400)^{-1/2} \cdot 2x_c\dot{x}_c + \dot{x}_T = 0$$

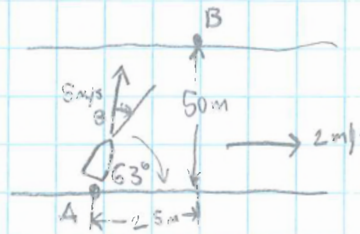
$\dot{x}_T = v_T = 6$ ft/s $\qquad v_c = \dot{x}_c$

$\cot\theta = \frac{adj}{opp} = \frac{x_c}{20'} \qquad x_c = 20\cot(\theta) \quad \rightarrow \quad \tfrac{1}{2}(20^2\cot^2\theta + 400)^{-1/2} \cdot 2$

$\frac{(20\cot\theta)v_c}{(400+400\cot^2\theta)^{1/2}} = -6 \qquad 1+\cot^2\theta = \csc^2\theta \qquad \frac{(20\cot\theta)v_c}{(400(\csc^2\theta))^{1/2}} = -6$

$\frac{\cancel{20}\cot\theta\, v_c}{\cancel{20}\csc\theta} = -6 \qquad \frac{opp/adj}{adj/hyp} \Rightarrow \cos\theta\, v_c = -6 \qquad \boxed{v_c = -6\sec\theta = 6\sec\theta \text{ ft/s}}$

220. $v_{m/w} = 5 \qquad v_w = 2$




$V_m = V_w + V_{m/w}$

| | $V_m$ | $V_w$ | $V_{m/w}$ |
|---|---|---|---|
| M | | 2 | 5 |
| D | 26° | $\rightarrow$ | $\theta$ |

$$V_M\left(\frac{25}{55.9}\hat{i} + \frac{50}{55.9}\hat{j}\right) = 2\hat{i} + 5(\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$$

$i: \; V_M(.447) = 2 + 5\sin\theta$

$j: \; V_M(.894) = 5\cos\theta \qquad \theta = \cos^{-1}\left(\frac{V_M \cdot .894}{5}\right)$

$V_M = 5.5928\cos\theta$

$2.5\cos\theta = 2 + 5\sin\theta$

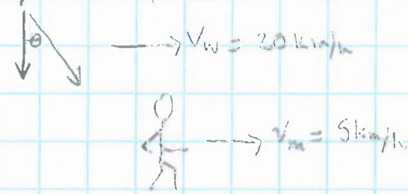
$-2 = 2.5\cos\theta + 5\sin\theta \qquad V_M = 5.56$ m/s $\quad \theta = 84.4°$

$\frac{-2}{2.5} = \cos\theta + 2\sin\theta$

David Maldwey Homework

12-230)

$V_{r/w} = 7$ km/h down




find direction drops fall w/respect to man

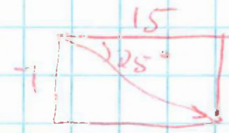
$V_r = V_w + V_{r/w}$

$V_r = 20 \frac{km}{h} \hat{i} - 7 \frac{km}{h} \hat{j}$

$V_r = V_m + V_{r/m}$ $\quad V_{r/m} = V_r - V_m = 20\hat{i} - 7\hat{j} - [5\hat{i}]$

$V_{r/m} = (15\hat{i} - 7\hat{j})$ km/h

$\theta = 64.98$




$\theta = 25°$

-3

Homework David Malawey

12-10)


A $V_0 = 0$ $a_c = 6$ $V_1 = 80$

94

$S_B = V_B(t) = 60t$

$S_{A_1}$ $V^2 = V_0 + 2a_c(S - S_0)$

$S_{A_1} = \frac{V^2 - V_0}{2a_c} = \frac{80^2 - 0}{12} = \boxed{533 \text{ ft}}$

$S_{A_2} = S_{A_1} + V(t) + 0 = 533 + 80(t - t_1)$

$t_1: S = \cancel{S_0}^{0} + \cancel{V_0}^{0} t + \frac{1}{2} a_c t^2$ -6

$533 = \frac{1}{2}(6)t^2$

$\boxed{t_1 = 13.3 s}$

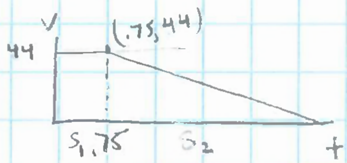
$6000 = 533 + 80(t - 13.3) + 60t$

$\boxed{t = 31.45}$ $t = 31.45(60)$

$S_{A_2} = 1985$

12-15)

a)




$S_1$ $V_0 = 44$ ft/s $t_1 = 0.75 s$ $S_1 = 44(.75) = 33$ ft.

$S_2$ $V_0 = 44$ $V^2 = V_0 + 2a_c(S - S_0)$

$0 = 44^2 + 2(-2)(S)$ 484

a) $\boxed{528 \text{ ft}}$ b) $S_1 = 44(3) = 132$, $132 + 484 = \boxed{616 \text{ ft}}$

12-20) $V = (-4s^2)$ m/s $S = 2m$ @ $t = 0$

$\frac{ds}{dt} = -4s^2$ $\underbrace{\int_2^s s^{-2} ds = \int_0^t -4dt}$ $-s^{-1}\Big|_2^s = -4t\Big|_0^t$

$-s^{-1} + .5 = -4t$

$s^{-2} ds = -4dt$

$\frac{1}{s} = 4t + .5$ $S = \frac{2}{8t+1}$ $V = -4\left(\frac{2}{8t+1}\right)^2 = \boxed{-\frac{16}{(8t+1)^2} \text{ m/s}}$

$a = \frac{dv}{dt} = \frac{256}{(8t+1)^3}$

30, 47

David Malawey

12-30) $v = v_0 - ks$ $s=0$ $t=0$ $k$ is constant

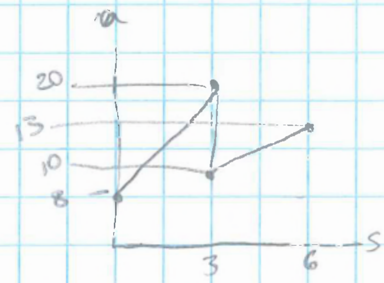
$s(t)$ & $a(t)$

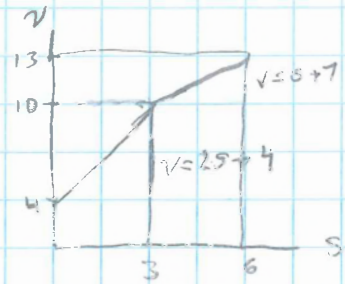
$v = \frac{ds}{dt}$ $dt = \frac{ds}{v}$ $\int_0^t dt = \int_0^s \frac{1}{v_0 - ks} ds$ $t|_0^t = -\frac{1}{k} \ln(v_0 - ks) |_0^s$

$t = -\frac{1}{k} \ln(v_0 - ks) + \frac{1}{k} \ln(v_0) = \frac{1}{k} \ln\left(\frac{v_0}{v_0 - ks}\right)$ $e^{tk} = \frac{v_0}{v_0 - ks}$

$v_0 - ks = \frac{v_0}{e^{tk}}$ $s = \frac{v_0 e^{-tk} - v_0}{-k}$ $\boxed{s = \frac{v_0}{k}(1 - e^{-tk})}$

47)




for $0 \le s \le 3$ m $a = v \frac{dv}{ds}$ $a = (2s+4)(2) = (4s+8)$ m/s$^2$

$s=0 \Rightarrow a = 8$ $s=3 \Rightarrow a = 20$

$3 \le s < 6$ m $a = (s+7)(1)$ $3 \Rightarrow a = 10$ $6 \Rightarrow a = 13$

David Malawey

Homework 12 – 75, 76, 87, 102, 110

75) $x^2 + y^2 = r^2$ $\quad V_y = 2r\cos 2t$ $\quad V_x = \pm 2r\sin 2t$ $\quad V = \sqrt{\quad}$

$dy = V_y\,dt$ $\quad \int_0^y dy = \int_0^t 2r\cos 2t\,dt = 2r\,\frac{1}{2}\sin 2t\,\Big|_0^t = r\sin 2t$

$y = r\sin 2t$ $\quad x = r\cos 2t$

accelleration: $a_x = \dot{v}_x = \frac{d}{dt}(\pm 2r\sin 2t) = \boxed{\pm 4r\cos 2t}$

$a_y = \dot{v}_y = \frac{d}{dt}(2r\cos 2t) = \boxed{-4r\cos 2t}$

76) $y = (0.05x^2)m$ $\quad V_x = -3m/s$ $\quad a_x = -1.5m/s^2$ @ $x = 5m$

$y = 1.25$ $\quad \dot{y} = 0.1x\dot{x}$ $\quad V_y = 0.1xV_x$

$V_y = .1(5m)(-3m/s) = \boxed{-1.5m/s}$ $\quad a_y = 0.1x\,a_x + 0.1\,V_xV_x$

$a_y = 0.1(xa_x + V_x^2) = 0.1(5(-1.5) + 9) = 0.1(1.5) = \boxed{.15m/s^2}$

87) $V_{ox} = V_A\cos 30°$ $\quad V_{oy} = V_a\sin 30$ $\quad a_x = 0$ $\quad a_y = -9.81\,m/s^2$

$x_o = 0$ $\quad y_o = 1$ $\quad y = y_o + V_y t + \frac{1}{2}a_y t^2$

$5m = V_x(t) \Rightarrow t = \frac{5}{V_a\cos 30°}$

$-1 = V_a\sin 30(t) + \frac{1}{2}(-9.81\,m/s^2)\,t^2$

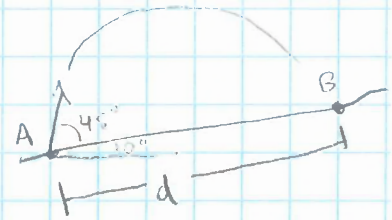
$0 = 4.905t^2 - V_a\sin 30t - 1$

$4.905t^2 - \frac{5}{\cancel{t}\cos 30}(\sin 30\,\cancel{t}) - 1 = 0 \Rightarrow 4.905t^2 = 3.887$ $\quad \boxed{t = 0.890}$

$V_a = \frac{5}{t\cos 30°} \Rightarrow \boxed{V_a = 6.49m/s}$

David Malaway

12 – 102, 110




102) $V_a = 80\ ft/s$

$S_x = 0 + 80\ f/s(t)\cos 55$

$d\cos 10° = 45.89\ t$

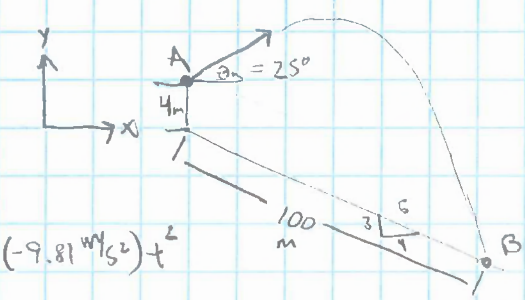
$S_y = 0 + 80\ ft/s\ \sin 55\ t + \frac{1}{2}(-32.2)t^2$

$d\sin 10 = 65.53t + \frac{1}{2}(-32.2)t^2$

$d = 166\ ft$

$t = 3.568\ ft$

110) $s = v_0 t$




$x: \quad 100\left(\frac{4}{5}\right)^m = V_a \cos 25° t$

$y: \quad s = s_0 + V_0 t + \frac{1}{2}a_c t^2$

$-4 - 100\left(\frac{3}{5}\right) = 0 + V_a \sin 25\ t + \frac{1}{2}(-9.81\ m/s^2)t^2$

$$\boxed{\begin{aligned} V_a &= 19.4 \\ t &= 4.54 \end{aligned}}$$

LECNOTES

# Lecture 27 3-D Kinetics: EOMS

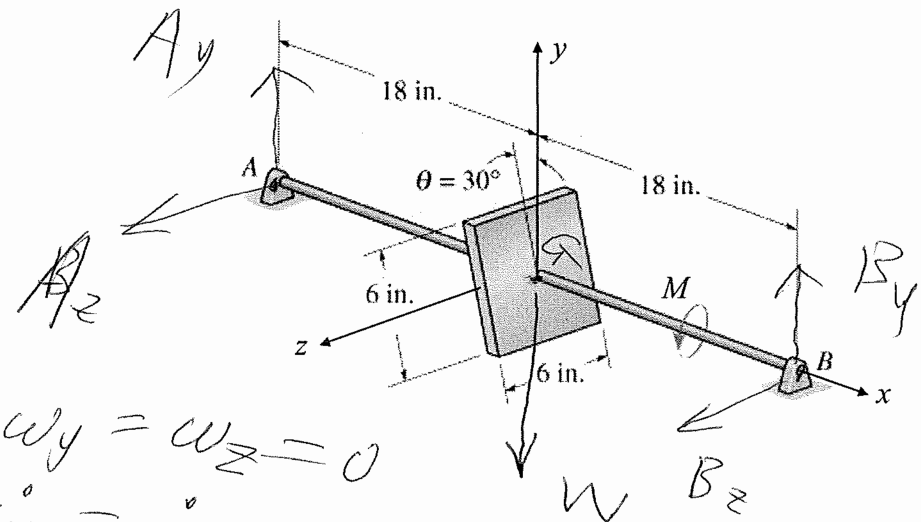
## Concept questions

1. The translational motion of a rigid body in $x$-axis is governed by only the sum of external forces in $x$-axis. (true or false)

2. The rotational motion of a rigid body in $x$-axis is governed by only the sum of external moments in $x$-axis. (true or false)

3. If the $x$-$y$-$z$ rotating axes are attached to the rigid body under consideration, then the angular acceleration of the body observed from the ground is equal to that observed from the rotating $x$-$y$-$z$ axes. (true or false)

**Example** The 30-lb square plate shown is mounted on a shaft of negligible mass, at an angle of 30° to what is currently the vertical. With respect to the *xyz* coordinate axes shown, the plate has inertia about its center of mass properties a $I_{xx} = 0.0340$ slug-ft$^2$, $I_{xy} = -0.0084$ slug-ft$^2$, $I_{xz} = 0$. In the configuration shown, the shaft has angular velocity $\omega = 6$ rad/s and is subject to a driving couple $\mathbf{M} = 10\mathbf{i}$ lb-ft. Find the angular acceleration of the shaft and the *y*- and *z*- components of the reaction at *B* at the instant.




$\omega_x = 6 \quad \omega_y = \omega_z = 0$

$\dot{\omega}_y = \dot{\omega}_z = 0$

For $A$: $\Sigma M_x = I_{xx}\dot{\omega}_x$

$M = I_{xx}\dot{\omega}_x \qquad 10 = 0.034\,\dot{\omega}_x$

$\downarrow$

$\dot{\omega}_x = \ldots$

$\Sigma M_y = I \ldots$

$-B_z(3) = \ldots$

$\downarrow$

$B_z$

$\Sigma M_z = \ldots$

## Lecture 24: Inertia Properties & Momentum & Impulse

### Concept questions

1. A moment of inertia is always nonnegative. (T or F)

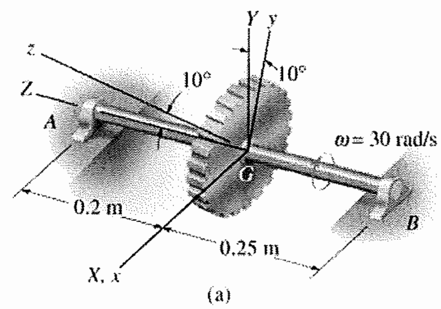
2. A product of inertia is
   A) $> 0$ B) $<0$ C) $=0$ D) $>0$, $<0$, or $= 0$

3. A product of inertia about a principal axis is equal to zero. (T or F)

4. The angular momentum of a rigid body undergoing 3-D general motion about $x$ axis is equal to $I_{xx}\omega_x$. (T or F)

5. If the sum of external moments about the center of mass in one axis is zero, then the angular momentum about the center of mass in that axis is conserved. (T or F)

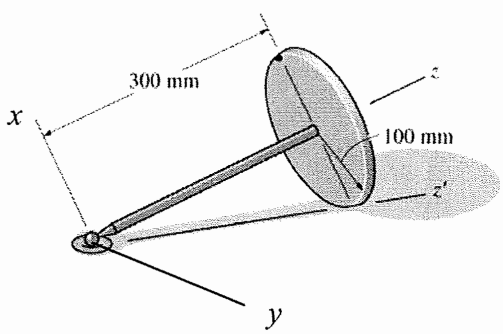
6. The angular momentum about the center of mass $G$ is
   A) $I_{ZZ}\omega$ B) $I_z\omega$ C) $(I_{ZZ}+I_z)\omega$ D) None of the above




**Example 1** Determine the moment of inertia of both the 1.5-kg rod and 4-kg disk about $z'$ axis.

**Solution**

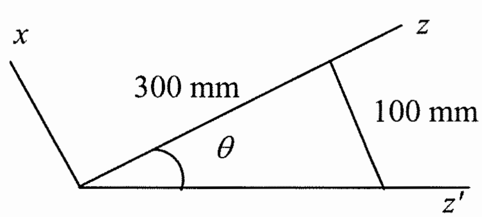
Since the body is symmetric to the $x$-$z$ and $y$-$z$ planes, $I_{xy} = I_{yz} = I_{xz} = 0$ .




$$I_{xx} = I_{yy} = \left[\frac{1}{4}(4)(0.1)^2 + 4(0.3)^2\right] + \frac{1}{3}(1.5)(0.3)^2 = 0.415 \text{ kg}\cdot\text{m}^2$$

$$I_{zz} = \frac{1}{2}(4)(0.1)^2 + 0 = 0.02 \text{ kg}\cdot\text{m}^2$$

$$\theta = \tan^{-1}\left(\frac{100}{300}\right) = 18.43^\circ$$




$$\theta_x = 90^\circ + 18.43^\circ = 108.43^\circ,\ \theta_y = 90^\circ,\ \theta_z = 18.43^\circ$$

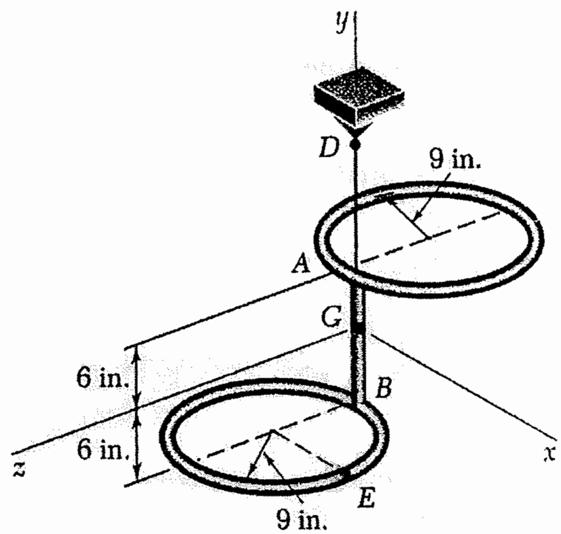
$$u_x = \cos\theta_x = \cos 108.43^\circ = 0.9487$$

$$u_y = \cos\theta_y = \cos 90^\circ = 0$$

$$u_z = \cos\theta_z = \cos 108.43^\circ = -0.3162$$

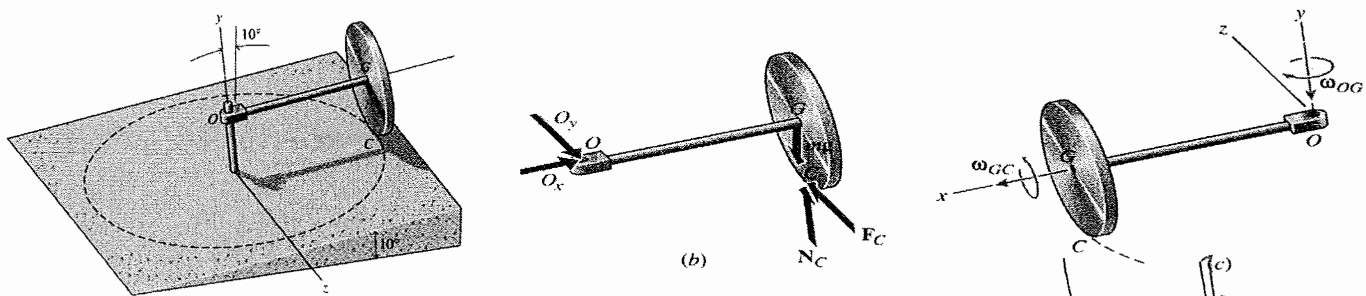
$$I_{z'} = I_{xx}u_x^2 + I_{yy}u_y^2 + I_{zz}u_z^2 - 2I_{xy}u_xu_y - 2I_{yz}u_yu_z - 2I_{xz}u_xu_z = 0.0595 \text{ kg}\cdot\text{m}^2$$

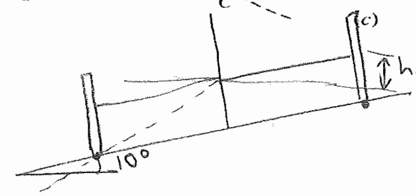
**Example** 2 The assembly shown has total mass 4.6 slug and inertia properties $I_{xx} = I_{yy} = 3.6$ slug-ft$^2$, $I_{zz} = 1.8$ slug-ft$^2$, $I_{yz} = -1.2$ slug-ft$^2$, $I_{xy} = I_{xz} = 0$. It is hanging at rest when it is struck at $E$ with an impulse $\mathbf{F}\Delta t = \{-4\mathbf{j} - 3\mathbf{k}\}$ lb·sec. Find the $x$, $y$, $z$ components of its angular velocity $\omega$ and of the velocity $v_G$ of its center of mass, immediately after impact.



## Lecture 26 3-D Kinetics: Work & Energy

**Example 1** The 5 kg, thin, homogeneous disk has a diameter of 200 mm and rotates freely on the 300 mm long axis $OG$. As the disk rolls without slipping on the inclined surface, the axle rotates freely about point $O$. If the system is released from rest in the position shown (with the disk at its highest position on the inclined surface), determine the angular velocity of the disk $\omega_{GC}$ when the disk is at its lowest position along the inclined surface. The mass of the bar $OG$ is negligible.




$\cancel{T_1}^{0} + V_1 = T_2 + V_2$

$V_1 = mgh = 5(9.81)\sin 10°$
$= 2.55\ J$
$V_2 = -2.55\ J$

IA $\vec{\omega}$

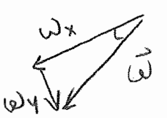
$T_2 = \frac{1}{2} I_{ox}\,\omega_x^2 + \frac{1}{2} I_{oy}\,\omega_y^2 + 0$

$I_{ox} = \frac{1}{2} mr^2 = \frac{1}{2}(5)(.1)^2 = .025\ kg\cdot m^2$

$I_{oy} = I_{Gy} + md^2 = \frac{1}{4} mr^2 + md^2$
$= \frac{1}{4} 5(.1)^2 + 5(.3)^2$

ke: $0 + 2.55 = \left[\frac{1}{2}(.025)\omega_x^2 + \frac{1}{2}(.4625)\,\omega_y^2\right] - 2.555$

$\omega = \omega_x + \omega_y$




$\frac{\omega_y}{\omega_x} = \frac{.1}{.3}$ (2)

$\omega_x = 11.57\ rad/s$
$\omega_y = 3.86\ rad/s$

$\boxed{\omega = -11.57\,i - 3.86\,j}$

## Lecture 23: General Motion - Two Points on Different Bodies like 22-47 !

A motor and attached rob $AB$ have the angular motion as shown. A collar $C$ on the rod is located 0.25 m from $A$ and is moving downward along the rod with a velocity of 3 m/s and an acceleration of 2 m/s$^2$. Determine the velocity and acceleration of $C$ at this instant.

Motion of each body

| | Absolute motion | Relative motion |
|---|---|---|
| Motor | rotation, $\omega_p$ | |
| Rod $AB$ | general | rotation |
| Collar $C$ | general curvilinear | |

$V_A = \omega \times r \qquad a_A = \alpha_p \times r_{A/o} + \omega_p \times V_A$

attach rotating x-y-z to platform

$\vec{\Omega} = \vec{\omega}_p = 5k, \quad \dot{\vec{\Omega}} = 2k$

1. motion of A $V_A = 5k \times 2i = 10j$ m/s

$a_A = 2k \times 2i + 5k \times 10j = -50i + 4j$ m/s$^2$

. Motion of C to A, Gen.

$(V_{C/A})_{rel} = \dot{r}\vec{u}_r + r\dot{\theta}\vec{u}_\theta = 3(-k) + .25(3)(j)$

$(a_{C/A})_{rel} = (\ddot{r} - r\dot{\theta}^2)u_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{u}_\theta$

$= [2 - .25(3)^2](-k) + [.25(1) + 2(3)(3)](j)$

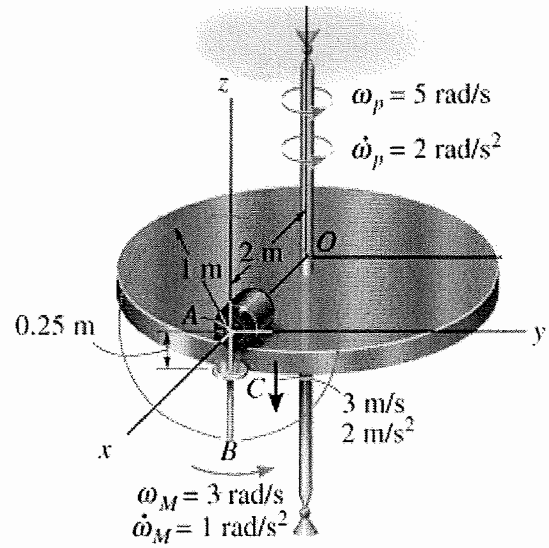
$\boxed{= 18.25j + 0.25k \text{ m/s}^2}$

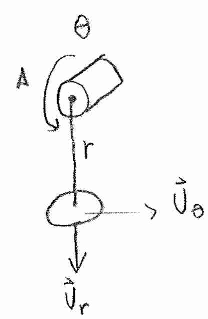
3) motion of c

$V_c = 10j + (5k \times -.25k) + (.75j - 3k)$

$\boxed{= 10.8j - 3\vec{k} \text{ m/s}}$

$\boxed{a_c = -57.5i + 22.2j + .25k \text{ m/s}^2}$

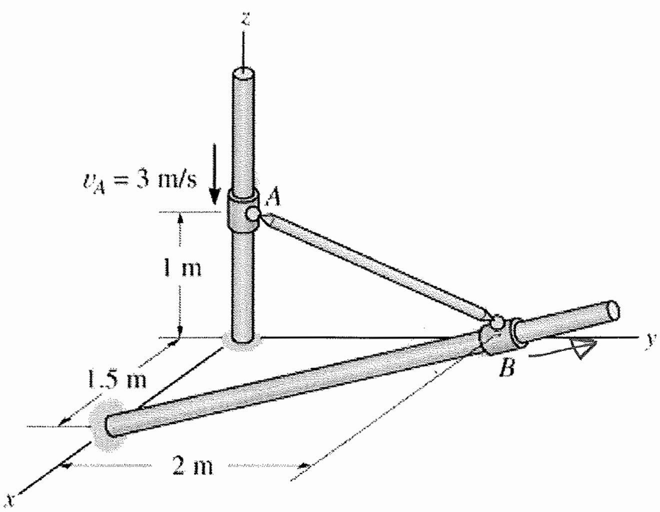


**Example** The collar at $A$ has an acceleration of $\mathbf{a}_A = \{-2\mathbf{k}\}$ m/s$^2$ and velocity of $\mathbf{v}_A = \{-3\ \mathbf{k}\}$ m/s at the instant.

**Type 1**: Determine the magnitude of the velocity and acceleration of the collar at $B$, and the angular velocity and angular acceleration of rod $AB$.

**Type 2**: Determine the magnitude of the velocity of the collar at $B$.




$\vec{V}_B = ?\quad \vec{a}_B = ?$

$(V_B - V_A)\cdot(r_{B/A}) = 0$

$\vec{V}_B = \dfrac{-1.5i + 2j + 0k}{\sqrt{1.5^2 + 2^2}} \Rightarrow [-.6V_B i + .8V_B j + 3k]\cdot[0i + 2j - k] = 0$

$= 0 + .8V_B(2) + (3k)(-1) = 0 \Rightarrow \boxed{V_B = 1.875\text{ m/s} \text{ plug into vector}}$

Eq. for a

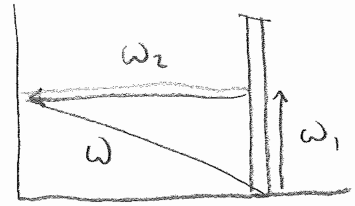
$(a_B - a_A)\cdot\vec{r}_{B/A} = -\|\vec{V}_B - \vec{V}_A\|^2$

$\vec{a}_B - \vec{a}_A = -.6a_B i + .8a_B j + 2k$

$V_B - V_A = [-1.125i + 1.5j + 3k] \Rightarrow \|V_B - V_A\| = 3.537$

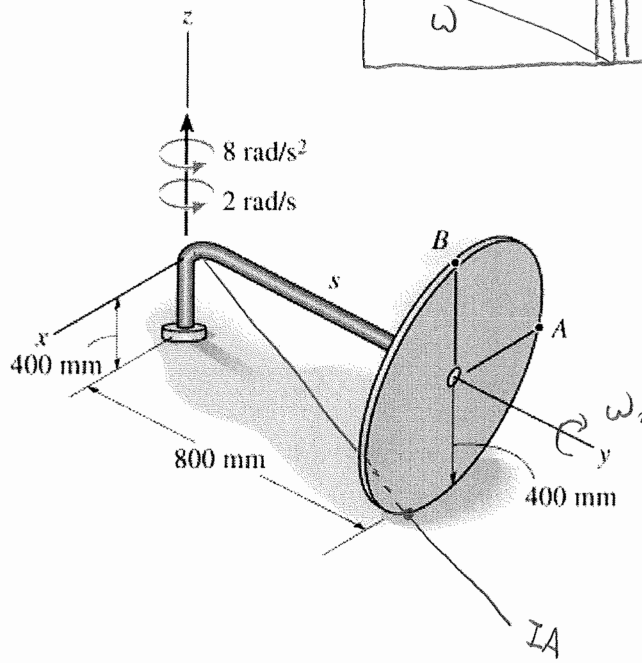
$[-.6a_B i + .8a_B j + 2k]\cdot[2j - k] = -(3.537)^2$

# Lecture 22: 3-D Kinematics (2)




## Example

The disk $B$ is free to rotate on the shaft S. If the shaft is turning about $z$ axis at $\omega_z = 2$ rad/s, while increasing at $8$ rad/s$^2$, determine the velocity and acceleration of point $A$ at the instant shown.




$V_A,\ a_A = ?$

$\vec{\omega}_1 = \vec{\omega}_1 + \vec{\omega}_2$

$\frac{\omega_2}{\omega_1} = \frac{.8}{.4} =$ $\quad \omega_2 = 2\omega_1 = 2(2) = 4$ rad/s

$\dot{\omega}_2 = (\dot{\omega}_2)_{rel} = 2\dot{\omega}_1 = 2(8) = 16$ rad/s$^2$

$= 2\vec{k}\ \overset{(-)}{\cancel{+}}4j$

$\vec{r}_A = -.4m\, i + .8j$

$V_A = \omega \times r = \ldots -1.6i - .8j - 1.6k$ m/s

$\alpha = \dot{\vec{\omega}}_1 + \dot{\vec{\omega}}_2 = 8k + 16(-j) + \underline{(2k \times -4j)} = 8i - 16j + 8k$ rad/s$^2$

$= \dot{\vec{\omega}}_1 + \left[(\dot{\vec{\omega}}_2)_{rel} + (\vec{\omega}_1 \times \vec{\omega}_2)\right]$

$a_A = \alpha_A \times \vec{r}_A + \vec{\omega} \times \vec{v}_A = \ldots = \left[-1.6i - 6.4j - 6.4k\right]$ m/s$^2$

## Lecture 21: 3-D Kinematics

### Concept questions

1. If a rigid body rotates about a fixed point, any position of the body is obtainable by a single rotation about some axis through the fixed point. (T or F)

2. If a rigid body rotates about a fixed point, the direction of its angular velocity is the same as that of its angular acceleration. (T or $\boxed{\text{F})}$

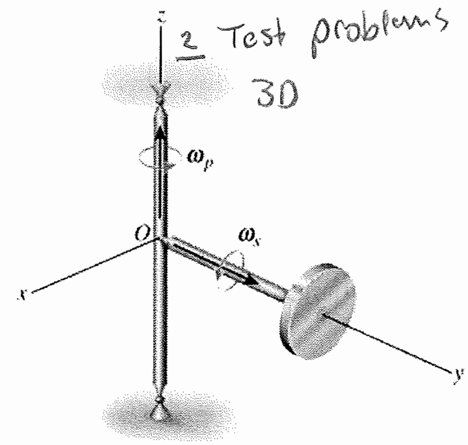
3. The angular velocity of the disk is
   a. $\omega_p \mathbf{k}$
   b. $\omega_s \mathbf{j}$
   c. $\boxed{\omega_s \mathbf{j} + \omega_p \mathbf{k}}$
   d. None of the above

$\vec{\omega}_{disk} = \vec{\omega}_{bar} + \omega_{disk/bar}$

" All the 3D problems are pretty much the same. "

2 Test problems on 3D

4. If $\omega_p$ and $\omega_s$ are constant, then the angular acceleration of the disk is zero. (T or $\boxed{\text{F})}$




5. If the angular acceleration of the disk is not zero, how do you calculate it?

$\dot{\vec{\omega}} = \dot{\vec{\omega}}_p + \dot{\vec{\omega}}_s$

$\dot{\vec{\omega}}_p = \dot{\omega}_p \vec{k}$

$\dot{\vec{\omega}}_s = (\dot{\vec{\omega}}_s)_{rel} + \vec{\omega}_p \times \vec{\omega}_s$

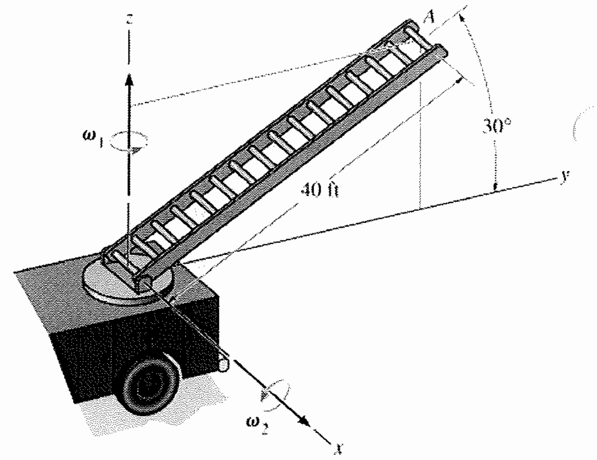
$= \dot{\omega}_s \vec{j} + (\omega_p \vec{k}) \times (\omega_s \vec{j}) = -\omega_p \omega_s \vec{i} + \omega_s \vec{j}$

$\alpha = \dot{\vec{\omega}} = \omega_s \omega_p \vec{i} + \dot{\omega}_s \vec{j} + \dot{\omega}_p \vec{k}$

these terms are zero for constant $\omega_s$ & $\omega_p$

**Example**

The ladder of the fire truck rotates around the $z$ axis with an angular velocity of $\omega_1 = 0.15$ rad/s, which is increasing at 0.2 rad/s$^2$. At the same instant it is rotating upwards at $\omega_2 = 0.6$ rad/s while increasing at 0.4 rad/s$^2$. Determine the vecoloty and acceleration of point $A$ located at the top of the ladder at this instant.




$\omega_1 = .15$ rad/s

$\dot{\omega}_1 = .2$ rad/s$^2$

$\omega_2 = (\omega_2)_{rel} = .6$ rad/s, $\dot{\omega}_2 = (\dot{\omega}_2)_{rel} = .4$ rad/s

$d_{A/O}$ is constant $\Rightarrow$ pure rotation

Find $V_A$, $a_A$

$\vec{\omega} = \vec{\omega}_1 + \vec{\omega}_2$

$= .15\vec{k} + .6i$ rad/s

$\vec{r}_A = 40(\cos 30 j + \sin 30° k)$

$= 34.64 j + 20 k$

$V_A = \vec{\omega} \times \vec{r}_A = (.15k + .6i) \times (34.64j + 20k)$

$= (-5.20i - 12j + 20.8\vec{k}$ ft/s

$\alpha = \dot{\omega}_1 + \dot{\omega}_2 = .2k + \dot{\vec{\omega}}_2$

do NOT omit this equation $\rightarrow$ $\dot{\vec{\omega}}_2 = (\dot{\vec{\omega}}_2)_{rel} + (\vec{\omega}_1 \times \vec{\omega}_2)$

$= .4i + .15k \times .6i = .\vec{09}j + .\vec{4k}\ i$ rad/s$^2$

$a_A = \vec{\alpha} \times r_A + \vec{\omega} \times \vec{V}_A$

$= (.4\vec{i} + .09\vec{j} + .2\vec{k}) \times (34.04j + 20k) + (.15k, +.6i) \times (-5.20i - 12j + 20.08\vec{k})$

$= -3.33i - 21.3j + 6.66k$ ft/s$^2$

4-5-16

# Lecture 20: Eccentric Impact

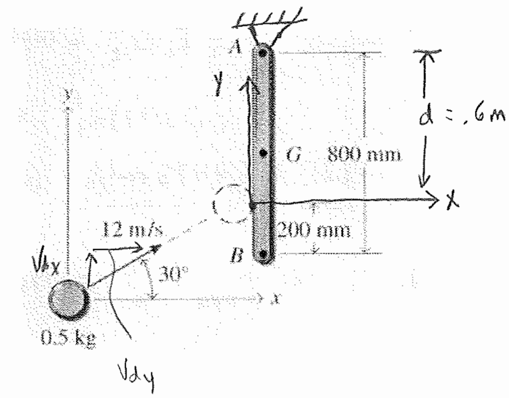
**Concept questions**

1. In addition to using the principle of momentum and impulse, we can also use the principle of work and energy to solve impact problems. (true or false)
2. For eccentric impact, $e = \dfrac{(v_{BG})_2 - (v_{AG})_2}{(v_{AG})_1 - (v_{BG})_{\cancel{2}\,1}}$. (true or false)
3. The coefficient of restitution is

   A) ≥0 B) <0 C) =0 D) >0 or <0

**Example 1** A 1.5 kg uniform rod 800 mm long is at rest on a frictionless horizontal surface when it is struck by a 0.5 kg disk as shown. The rod is pinned at point *A*. If the disk strikes the rod 200 mm from the end and the coefficient of restitution is $\boxed{0.4}$, determine the velocity of the disk and the velocity of the mass center of the rod after the collision.

e




find $(V_d)_2$, $(\omega_r)_2$

$H_o = I_G\,\omega + mV_G\,d$

$(\Sigma H_A)_1 = (\Sigma H_A)_2$

$m_d (V_{dx})_1 d + 0 = m_d (V_{dx})_2 d + I_A (\omega_r)_2$

$.5\,(12)\cos 30^\circ\,(0.6) = .5\,(V_{dx})_2\,(0.6) + \frac{1}{3}(1.5)(.8)^2\,\omega_r$ (1)

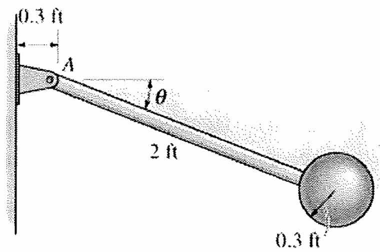
$e = \dfrac{(V_{rx})_2 - (V_{dx})_2}{(V_{dx})_1 - (V_{rx})_1}$ $\qquad 0.4 = \dfrac{(.6\omega_r)_2 - (V_{dx})_2}{12\cos 30 - 0}$

$(V_{dx})_2 = 1.08\ m/s \rightarrow$

$(\omega_r)_2 = 8.73\ rad/s$

$(V_{dy})_2 = (V_{dy})_1 = 12 \sin 30 = 6.0\ m/s$

**Example 2** (19-52) The pendulum consists of a 10-lb solid ball and 4-lb rod. If it is released from rest when $\theta_1 = 0°$, determine the angle $\theta_2$ after the ball strikes the wall, rebounds, and the pendulum swings up to the point of momentary rest. Take $e = 0.6$.



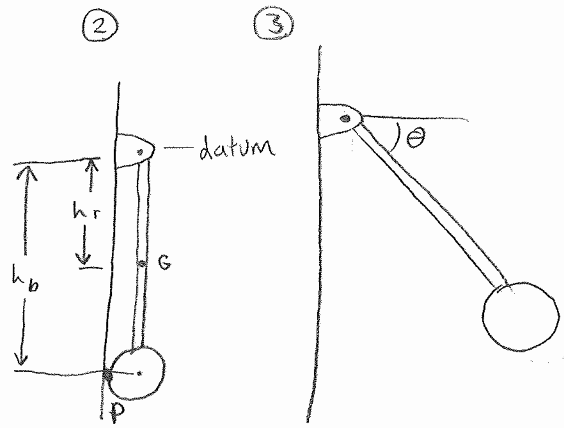

1

moment of inertia about point A

rod: $\frac{1}{3}ml^2 = \frac{1}{3}\frac{4}{32.2}(2)^2$

ball: $\frac{2}{5}\frac{10}{32.2}(.3)^2 + \frac{10}{32.2}(2.3)^2$

$I_A = \boxed{1.8197}$




1) pos. 1 to 2

$T_1 + V_1 = T_2 + V_2$

$0 + 0 = \frac{1}{2} I_A \omega_2^2 - W_r h_r - W_b h_b$

$= \frac{1}{2}(1.8197)\omega_2^2 - 4(1) - 10(2.3)$

$\omega_2 = 5.4475$ rad/s before impact

2) $(V_P)_2 = \omega_2 r_{P/A} = 5.4475(2.3) = 12.53$ ft/s

$e = \frac{(V_P)_{2'} - 0}{0 - (V_P)_2} = 0.6$ ← 12.53 $\quad (V_P)_{2'} = 7.518$

$\omega_{2'} = \frac{(V_P)_{2'}}{r} = \frac{7.518}{2.3} = 3.2685$ rad/s

3) Pos 2 to 3

$T_2 + V_2 = T_3 + V_3$

$\frac{1}{2} I_A \omega_{2'}^2 - W_b h_b - W_r h_r = 0 - W_b h_{b3} - W_r h_{r3}$

$\frac{1}{2}(1.8197)(3.269)^2 - 10(2.3) - 4(1) = 0 - 10(2.3\sin\theta) - 4(1)\sin\theta$

$\theta = 39.8°$

## Lecture 19: Impulse and Momentum

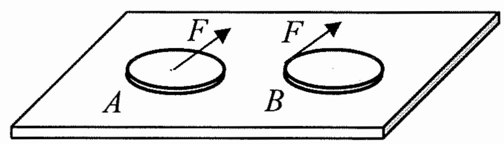
**Concept questions**

1. The angular momentum of a rotating rigid body about its center of mass $G$ is
   A) $mv_G$ B) $I_G v_G$ C) $m\omega$ $\boxed{\text{D) } I_G\omega}$

2. If a rigid body rotates about a fixed axis passing through its center of mass, the body's linear momentum is
   $\boxed{\text{A) a constant}}$ $\boxed{\text{B) zero}}$ $\boxed{\text{C) } mv_G}$ D) $I_G\omega$
   √ √ √

3. If a force does no work, it does not need to be shown in the impulse and momentum equation. (T or F)

4. Two identical disks are placed on a smooth table. They are subjected to an external force $F$. At any moment after they start to move from rest, their linear momenta satisfy



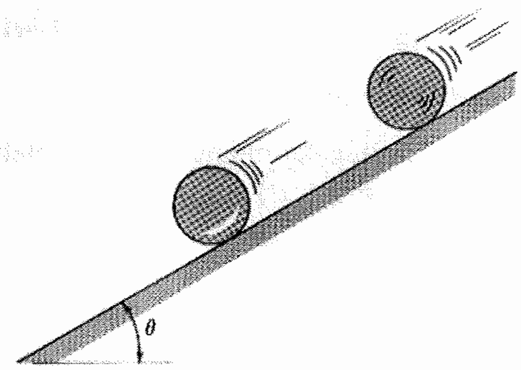

   A) $L_A > L_B$ B) $L_A < L_B$ $\boxed{\text{C) } L_A = L_B}$ D) $L_A = L_B = 0$
   √
   $\vec{L} = m\vec{v}$

5. For the above question, the angular momenta about the centers of mass satisfy
   A) $H_A > H_B$ $\boxed{\text{B) } H_A < H_B}$ C) $H_A = H_B$ D) $H_A = H_B = 0$

   $H_A = 0$ $H_B > 0$

   $H_{G(0)} = I_{G(0)} \vec{\omega}$

**Example 1** A sphere and cylinder are released from rest on the ramp at $t$=0. If each has a mass $m$ and a radius $r$, determine their angular velocities at time $t$. Assume no slipping occurs.



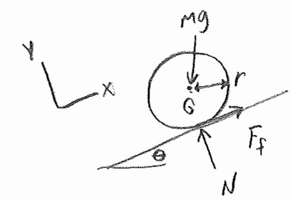

$V = ?\ @\ t_2$

$$m(V_{Gx}) + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(V_{Gx})_2$$

$$0 + mg\sin\theta\, t - Ft = mV \quad (1)$$

$$(\vec{H}_G)_1 + \Sigma\int_{t_1}^{t_2} \vec{M}_G\,dt = (\vec{H}_G)_2$$

$$0 + Frt = I_G\,\omega \quad (2)$$



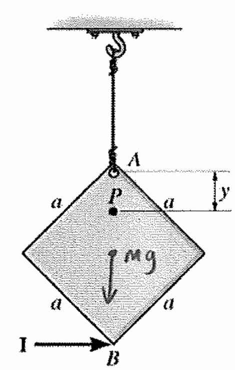

$$V = \omega r \quad (3)$$

$$\omega = \frac{mgr\sin\theta}{I_G + mr^2}\,t$$

$$I_{G\,cyl} = \frac{1}{2}mr^2$$

$$I_{G\,sphere} = \frac{2}{5}mr^2$$

**Example 2** (19-30) The square plate has a mass $m$ and is suspended at its corner $A$ by a cord. If it receives a horizontal impact at corner $B$, determine the location of $y$ of the point $P$ about which the plate appears to rotate during the impact.




Find location of $I_C$

$$m(V_{Gx})_1 + \Sigma\int_{t_1}^{t_2} F\,dt = m(V_{Gx})_2$$

$$0 + I = m(V_{Gx}) = mV \quad (1)$$

$$(H_G)_1 + \int_{t_1}^{t_2} M_G\,dt = (H_G)_2$$

$$0 + Ia\sin 45 = I_G\,\omega = \frac{m}{12}(a^2 + a^2)\,\omega \quad (2)$$

$V = I/m \quad \omega = 6I/\sqrt{2}\,am$

Find point P $\quad V_G = r\omega = y\,\omega, \quad y = \dfrac{V}{\omega}$

## Lecture 18: Work & Energy

### Concept questions

1. When calculating work done by forces, the work of an internal force does not have to be considered because
   - A) internal forces do not exist.
   - B) the forces act in equal but opposite collinear pairs.
   - C) the body is at rest initially.
   - D) the body can deform.

2. Kinetic energy due to rotation is defined as
   - A) $\frac{1}{2}mv_G^2$
   - B) $\frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$
   - C) $\boxed{\frac{1}{2}I_G\omega^2}$
   - D) $I_G\omega^2$

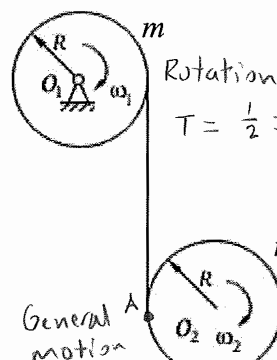
3. The kinetic energy of a rigid body with general motion is $\frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2$, where $A$ is an arbitrary point on the body. (true or $\boxed{\text{false}}$)

   must use center for general motion

4. Two disks are interconnected with an inextensible cable without mass. Their angular velocities are $\omega_1$ and $\omega_2$ respectively. The kinetic energy of the system is

   $I_{G\,disk} = \frac{1}{2}mr^2$

   - A) $T = \frac{1}{2}\left(\frac{1}{2}mR^2\right)\omega_1^{\;2} + \frac{1}{2}m(R\omega_2)^2$
   - B) $T = \frac{1}{2}\left(\frac{1}{2}mR^2\right)\omega_1^{\;2} + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\omega_2^{\;2}$
   - C) $T = \frac{1}{2}\left(\frac{1}{2}mR^2\right)\omega_1^{\;2} + \frac{1}{2}m(R\omega_2)^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\omega_2^{\;2}$
   - $\boxed{\text{D)}}$ $T = \frac{1}{2}\left(\frac{1}{2}mR^2\right)\omega_1^{\;2} + \frac{1}{2}m(R\omega_1 + R\omega_2)^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\omega_2^{\;2}$




Rotation, $T = \frac{1}{2}I_{O_1}\omega^2$

General motion

$T = \frac{1}{2}mV_{O_2}^2 + \frac{1}{2}I_{O}\omega_2^2$

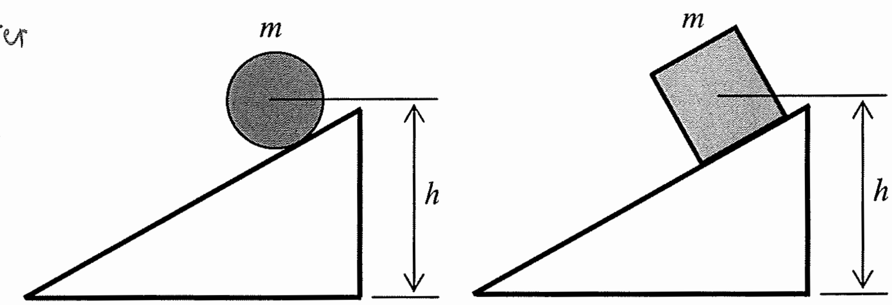
$V_{O_2} = V_A + V_{O_2/A} = \omega_1 R + \omega_2 R$

5. The elastic potential energy is defined as
   A) $\frac{1}{2}ks^2$ B) $-\frac{1}{2}ks^2$ C) $\frac{1}{2}kv^2$ D) None of the above.

6. The location of the datum for the gravitational potential energy does not affect the result of using conservation of energy. $\boxed{\text{(ture}}$ or false)

7. A disk rolls down on a smooth plane without slipping, and a block also moves down on the same plane. Are their mass centers' velocities same after the mass centers descend a vertial distance $h$?

   no, block is faster

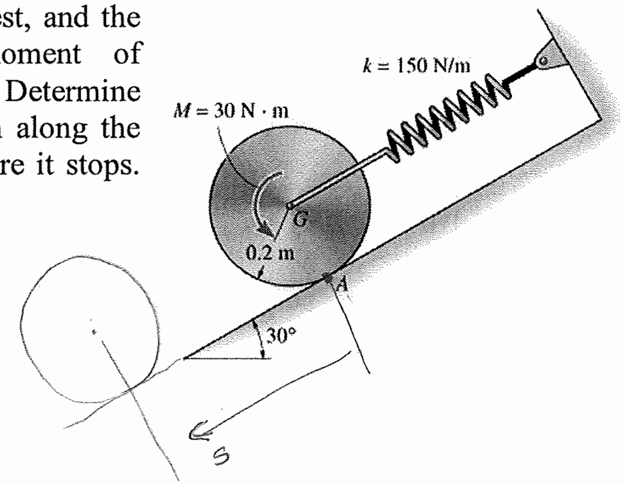



Disc $T_1 + V_1 = T_2 + V_2$

$0 + mgh = \frac{1}{2}mv^2 + \frac{1}{2}I_G\omega^2$

Block $0 + mgh = \frac{1}{2}mv^2$

**Example 1 (18-23)** The 20-kg disk is originally at rest, and the spring holds it in equilibrium. A couple moment of $M = 30\ \text{N}\cdot\text{m}$ is then applied to the disk as shown. Determine how far the center of mass of the disk travels down along the incline, measured from the equilibrium position, before it stops. The disk rolls without slipping.




General motion

$(F_0)_{spring} = ?$

Sum moments about A

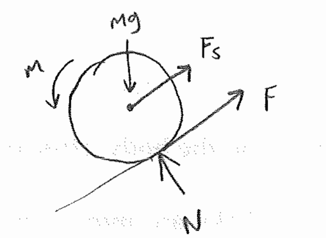
$\Sigma M_A = 0 \quad +\circlearrowright$

$mg(0.2)\sin 30 - F_s(0.2) = 0$

$F_s = 98.1\ \text{N}$

$S_1 = \frac{F_s}{k} = \frac{98.1}{150} = 0.654\ \text{m}$

$S_2 = S + S_1$




$\cancel{T_1}^{\,0} + \Sigma U_{12} = \cancel{T_2}^{\,0}$

$U_{12}^{mg} = mg\Delta y_G = 20(9.81)\,s\sin 30 = 98.1\,s$

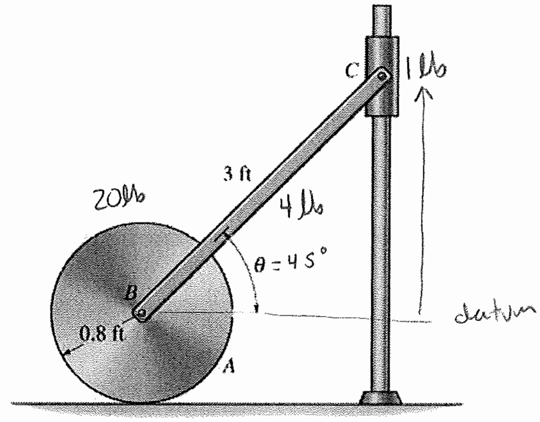
$U_{12}^{M} = M(\theta_2 - \theta_1) = 30\,\frac{s}{0.2} = 150\,s$

(this one should be negative) $U_{12}^{F_s} = \frac{1}{2}k(s_2^2 - s_1^2) = \frac{1}{2}(150)\left[(s + 0.654)^2 - 0.654^2\right] = 98.1 + 75s^2$

$U_{12}^{F} = 0$

$\boxed{S = 2.0\text{m}}$

**Example 2 (18-28)** The system consists of a 20-lb disk $A$, a 4-lb slender rod $BC$, and a 1-lb smooth collar $C$. If the disk rolls without slipping, determine the velocity of the collar at the instant the rod become horizontal, i.e. $\theta = 0°$. The system is released from rest when $\theta = 45°$




$$\cancel{\Sigma T_1}^{0} + \Sigma V_1 = \Sigma T_2 + \cancel{\Sigma V_2}^{0}$$

$$V_1 = 1(3\sin 45) + 4(1.5\sin 45) = 6.3626 \text{ ft·lb}$$

$$\Sigma T_2 = \frac{1}{2} m_c v_c^2 + \underbrace{\frac{1}{2} m_G v_G^2 + \frac{1}{2} I_G \omega_R^2}_{\text{rod}} + \underbrace{\cancel{\frac{1}{2} m_B v_B^2}^{0} + \cancel{\frac{1}{2} I_B \omega_B^2}^{0}}_{\text{disk}}$$

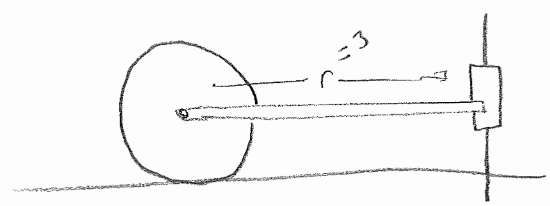
Evaluate motion; pt. B is IC

$v_B = 0 \quad \omega_B = 0$

$v_c = \omega_r(3) = 3\omega_r$

$v_G = \omega_r(1.5) = 1.5\omega_r$

$$\boxed{v_c = 13.25 \text{ ft/s} \downarrow}$$



## Lecture 17: General Plane Motion

**Concept questions**

1. The sum of the moments of external forces and couples taken about an arbitrary point $P$ on a rigid body is equal to $I_p\alpha$. (true or $\boxed{\text{false}}$)

   $I_p \alpha + r_{G/P} \times m\vec{a}_G$

2. Select the equation that best represents the "no-slip" assumption for a disk rolling on ground.

   A) $F_f = \mu_s N$ B) $F_f = \mu_k N$

   C) $a_G = r\alpha$ D) None of the above

3. A slender 100 kg beam is suspended by a cable. The moment equation about point A is

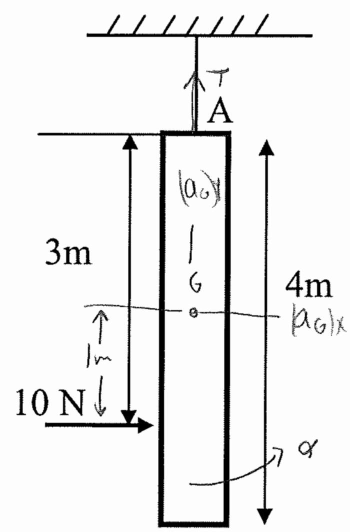
   A) $3(10) = 1/12(100)(4^2)\,\alpha$

   B) $3(10) = 1/3(100)(4^2)\,\alpha$

   C) $3(10) = 1/12(100)(4^2)\,\alpha + (100\,a_{Gx})(2)$

   D) $3(10) = 1/12(100)(4^2)\,\alpha - (100\,a_{Gx})(2)$

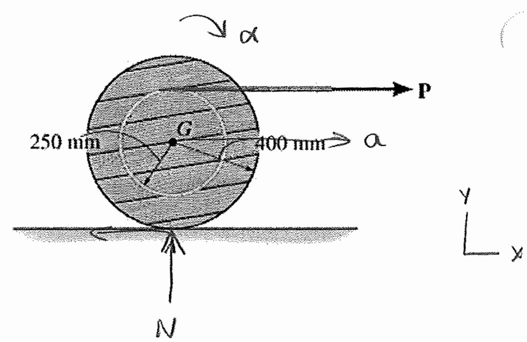
   *Hint:* $I_G = \frac{1}{12}ml^2$ and $I_A = \frac{1}{3}ml^2$




Goodyear wants me to interview in Topeka!

**Example 1**

The spool has a mass of 100 kg and a radius of gyration $k_g = 0.3$ m. If the coefficients of static and kinetic friction at $A$ are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively, determine the angular acceleration of the spool if $P = 600$ N.




$\alpha = ?$

→ assume no slipping

$\Sigma F_x = m(a_G)_x \qquad P + F = ma$

$600 + F = 100a \quad (1)$

$\Sigma F_y = m(a_G)_y \qquad N - mg = 0 \quad (2)$

$\Sigma M_G = I_G\alpha \quad +\circlearrowright: \; Pr - F_r = I_G\alpha \qquad 600(.25) - F(0.4) = 100(.3)^2 \quad (3)$

$k_g = 0.3\,m \qquad I_G = mk_g^2$

$a = R\alpha$

$a = 0.4\alpha \quad (4)$

$\alpha = 15.6$ rad/s

$a = 6.24\,m/s^2$

$N = 981.0N$

$F = 24.0N$

check assumption

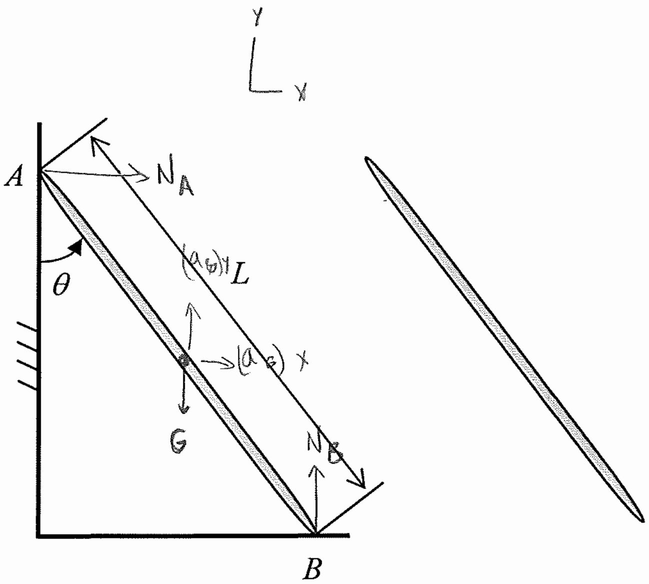
$F_{max} = \mu_s N$

$= 0.2(981) = 196.2$ N

**Example 2**

The slender bar of mass $m$ slides on the smooth floor and wall. Determine its angular acceleration as a function of $\theta$ when it is released and allowed to slide downward.

1) Draw free body diagram
2) Draw kinetic diagram
3) Apply EOM




Free Body Diagram

Kinetic Diagram

(1) $\sum F_x = m(a_G)_x :\ N_A = m(a_G)_x$

(2) $\sum F_y = m(a_G)_y :\ N_B - mg = m(a_G)_y$

$\sum M_G = I_G \alpha :\ -N_A \frac{L}{2}\cos\theta + N_B \frac{L}{2}\sin\theta = \frac{1}{12} m L^2 \alpha$

4) Kinematics

use A as base pt.

use x

$\vec{a}_G = \vec{a}_A + \vec{\alpha} \times \vec{r}_{G/A} - \omega^2 \vec{r}_{G/A}$

$(\vec{a}_G)_x + (\vec{a}_G)_y = a_A j + (\alpha k) \times \left[\frac{L}{2}(\sin\theta i - \cos\theta j)\right]$

$a_G = \frac{L\alpha}{2}\cos\theta i + \left(a_A + \frac{L\alpha}{2}\sin\theta\right) j$

$(a_G)_x = \frac{L\alpha}{2}\cos\theta$

$a_G = a_B + \alpha \times \vec{r}_{G/B}$

$(a_G)_x + (a_G)_y = a_B i + (\alpha k) \times \left[-\cos\theta i + \sin\theta j\right]\left(\frac{L}{2}\right)$

$= \left(a_B + -\frac{L\alpha}{2}\sin\theta\right) i - \frac{L\alpha}{2}\cos\theta j$

$(a_G)_y = -\frac{L\alpha}{2}\cos\theta$

$\alpha = \frac{3}{2}\frac{g}{L}\sin\theta$

**Concept questions**

1. Moment of inertia is a measure of the resistance of a body to
   A) translational motion. B) deformation.
   C) angular acceleration. D) impulsive motion.

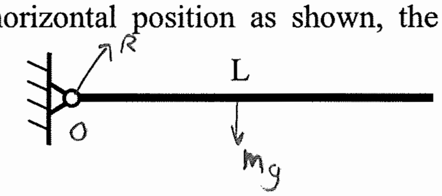
2. Moment of inertia is always
   A) a negative quantity.
   B) a positive quantity.
   C) an integer value.
   D) zero about an axis perpendicular to the plane of motion.

3. The moment of inertia of any body about its center of mass is always
   A) maximum. B) minimum.
   C) zero. D) None of the above.

4. The rotational EOM about the mass center of a rigid body indicates that the sum of moments due to the external loads equals _____________.
   A) $I_G \alpha$ B) $m\mathbf{a}_G$ C) $I_G \alpha + m\,\mathbf{a}_G$ D) None of the above.

5. The sum of moments acting at an arbitrary point on a rigid body is directly proportional to the angular acceleration of the body. (true or false)

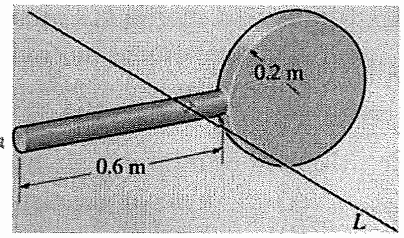
6. If a rigid bar of length L is released from rest in the horizontal position as shown, the magnitude of its angular acceleration at the moment is
   A) 3g/(2L) B) 3g/L C) 12g/L D) 0




$\sum M_O = I_O \alpha$

$mg\,\frac{L}{2} = \left(\frac{1}{12} mL^2 + m\left(\frac{L}{2}\right)^2\right) \alpha$

**Example 1** The object in the figure consists of a slender 3-kg bar welded to a thin, circular 2-kg disk. Determine the moment of inertia of the object about the axis $L$ through its center of mass.

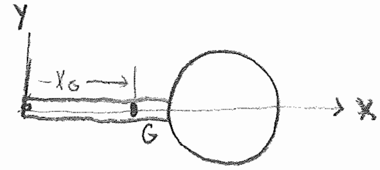



$I_G = ?$ $\quad x_G = \frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{3(.3) + 2(0.8)}{3+2} = .5\text{ m}$

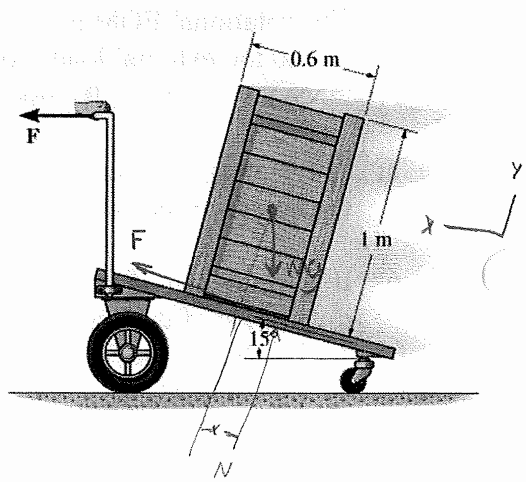
$(I_{bar})_G = \frac{1}{12} m L^2 = \frac{1}{12}(3)(.6)^2 + 3(.2)^2 = 0.210 \text{ kg}\cdot\text{m}^2$

$(I_{disc})_G = \frac{1}{2} m_2 r^2 + m_2 d_2^2 = \frac{1}{2}(2)(.2)^2 + 2(.3)^2 = .220 \text{ kg}\cdot\text{m}^2$

$I_G = .21 + .22 = .430 \text{ kg}\cdot\text{m}^2$




**Example 2** The crate has a mass of 50kg and rests on the cart having an inclined surface. Determine the maximum acceleration that will not cause the crate to slide or tip over. The coefficient of static friction between the crate and the cart is 0.5.




Tipping over $x = .3\text{ m}$

Sliding $F = \mu_s N$

assume sliding

$\Sigma F_x = m(a_{Gx}) = F - mg\sin 15 = ma\cos 15$

$F - 50(9.81)\sin 15 = 50a\cos 15$

$\Sigma F_y = m(a_G)_y = N - mg\cos 15° = -ma\sin 15$

$N - 50(9.81) = -50a\sin 15°$

$\Sigma M_G = I_G\alpha = 0 \qquad -F(.5) + Nx = 0 \ (3) \qquad F = \mu_s N = 0.5N \ (4)$

$N = 447.81\text{ N} \qquad a = 2.01\text{ m/s}^2 \qquad x = .25\text{ m} < 0.3$, does not tip

no rotation

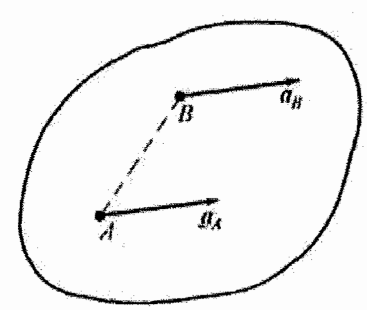
## Lecture 14: Relative Acceleration

### Concept questions

1. After the IC has been found at $A$, the acceleration of point $B$ on a rigid body can be calculated by $\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A} = \mathbf{a}_{B/A}$. (true or false)

2. If the acceleration of point $A$ is equal to that of point $B$, then their velocities are also equal to each other. (true or false)

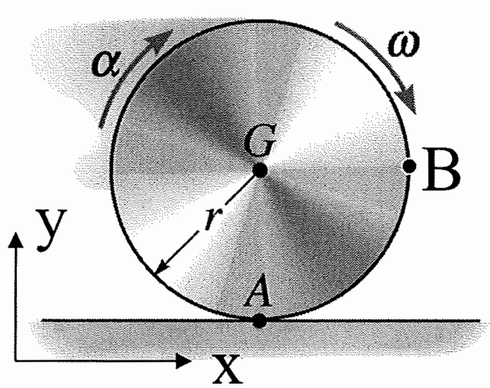
translational




3. If the acceleration of point $A$ is equal to that of point $B$, then the acceleration of any point on the rigid body is the same. (true or false)

4. If a ball rolls without slipping, select the tangential and normal components of the relative acceleration of point $A$ with respect to $G$.
   A) $\alpha r\,\mathbf{i} + \omega^2 r\,\mathbf{j}$ B) $-\alpha r\,\mathbf{i} + \omega^2 r\,\mathbf{j}$
   C) $\omega^2 r\,\mathbf{i} - \alpha r\,\mathbf{j}$ D) Zero.

5. What are the tangential and normal components of the relative acceleration of point $B$ with respect to $G$.
   A) $-\omega^2 r\,\mathbf{i} - \alpha r\,\mathbf{j}$ B) $-\alpha r\,\mathbf{i} + \omega^2 r\,\mathbf{j}$
   C) $\omega^2 r\,\mathbf{i} - \alpha r\,\mathbf{j}$ D) Zero.



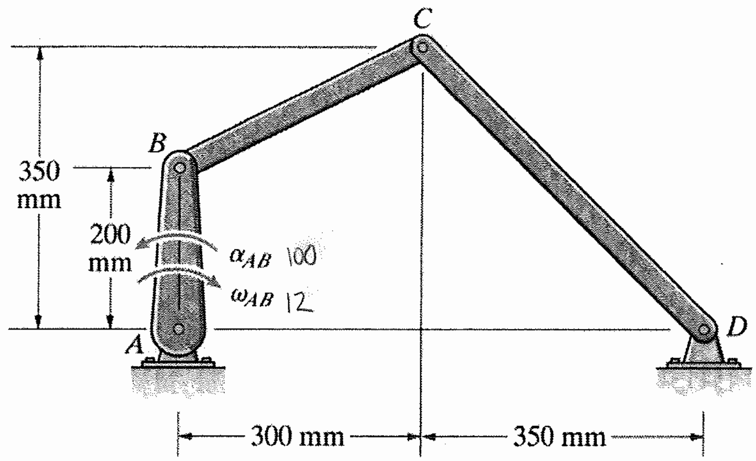

4) $\vec{a}_{A/G} = ?$ $V_G = \omega r$ $a_G = \alpha r$

$= \vec{\alpha} \times \vec{r}_{A/G} - \omega^2 \vec{r}_{A/G}$

$= (-\alpha \vec{k}) \times (-r\hat{j}) - \omega^2(-r\hat{j}) = -\alpha r\hat{i} + \omega^2 r\hat{j}$

crank

**Given**: $\omega_{AB} = 12$ rad/s, $\alpha_{AB} = 100$ rad/s$^2$
**Find**: $\alpha_{BC}, \alpha_{CD}$




Found: $\omega_{BC} = 5.33$ rad

$\omega_{CD} = 4.57$ rad/s

AB: $\vec{a}_B = \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega^2_{AB} \vec{r}_{B/A}$

$(100\,k) \times (.2\,j) - (144)(.2\,j)$

$\boxed{\vec{a}_B = -20\,i - 28.8\,\vec{j} \quad m/s^2}$

BC: $\vec{a}_C = \vec{a}_B + \vec{\alpha}_{BC} \times \vec{r}_{C/B} - \omega^2_{BC} \vec{r}_{C/B}$

$= -20i - 28.8j + (\alpha_{BC}\,\vec{k}) \times (.3\vec{i} + .15\vec{j}) - (5.33)^2 (.3i + .15j)$

(1) $= (-28.523 - .15\,\alpha_{BC})\,i + (-33.061 + 0.3\,\alpha_{BC})\,\hat{j}$

CD: $\vec{a}_C = \alpha_{CD} \times \vec{r}_{CD} - \omega^2_{CD} \vec{r}_{C/D} = \alpha_{CD}\,\hat{k} \times (-.35\,i + 0.35\,j) - 4.5^2 (-.35\,i + .35\,j)$

(2) $= (-.35\,\alpha_{CD} + 7.31)\,i + (-.35\,\alpha_{CD} - 7.31)\,j$

$i$: $-28.523 - .15\,\alpha_{BC} = -.35\,\alpha_{CD} + 7.31$

$j$: $-33.061 + .3\,\alpha_{BC} = -.35\,\alpha_{CD} - 7.31$

$\alpha_{BC} = -22.43$ rad/s$^2$

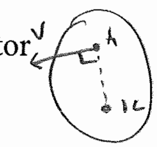
$\alpha_{CD} = 92.8$ rad/s$^2$

# Lecture 13: Instantaneous Center of Zero Velocity

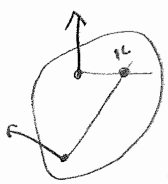
**Concept questions**

1. The method of instantaneous center can be used to determine the __________ of any point on a rigid body
   A) velocity  B) acceleration
   C) velocity and acceleration  D) force

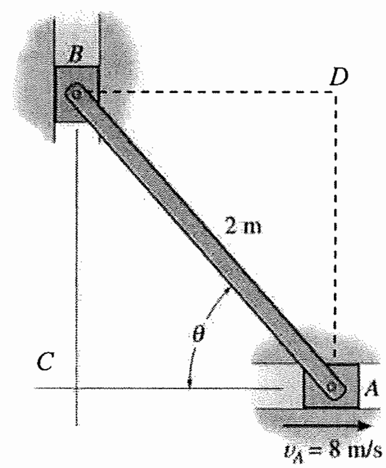
2. The velocity of any point on a rigid body is __________ to the relative position vector extending from the IC to the point.
   A) always parallel  B) always perpendicular
   C) in the opposite direction  D) in the same direction



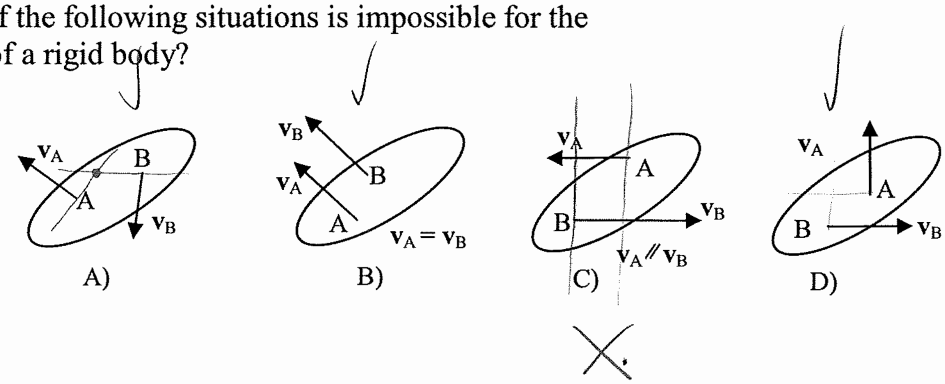

3. When the direction of velocities of two points on a body are perpendicular to each other, the IC is located at
   A) infinity.
   B) one of the two points.
   C) the midpoint of the line connecting the two points.
   D) None of the above.



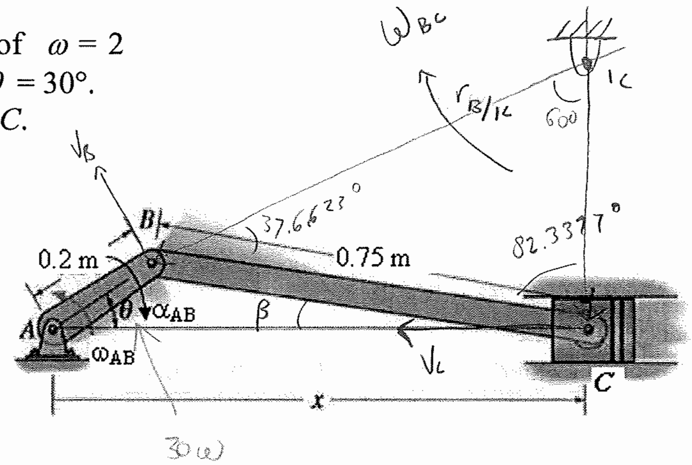

4. Point $A$ on the rod has a velocity of 8 m/s to the right. Where is the IC for the rod?
   A) Point $A$.
   B) Point $B$.
   C) ~~Point $C$.~~
   D) Point $D$.




5. Which of the following situations is impossible for the motion of a rigid body?



**Example 1**
**Given**: Crank $AB$ rotates at an angular velocity of $\omega = 2$ rad/s and angular acceleration $\alpha = 1$ rad/s$^2$ when $\theta = 30°$.
**Find**: The velocity and acceleration of the slider $C$.




AB. $V_B = \omega_{AB} r_{B/A} = .2(.02) = .4$ m/s

BC: $V_B = \omega_{BC} r_{B/IC}$

$\triangle ABC \quad \frac{.75}{\sin 30} = \frac{.2}{\sin\beta} \Rightarrow \beta = 7.1123°$

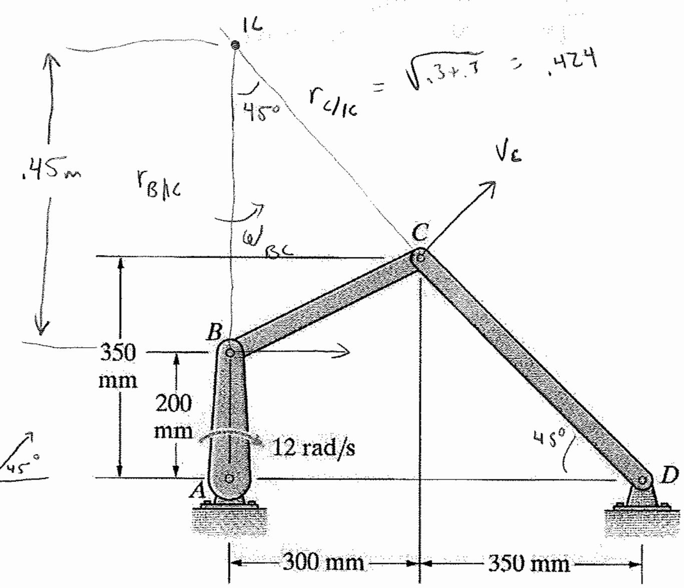
$\triangle B\,IC\,C \quad \frac{r_{B/IC}}{\sin 82.34} = \frac{r_{C/IC}}{\sin 37.66} = \frac{.75}{\sin 60°}$

$r_{B/IC} = .8583 \qquad r_{C/IC} = 0.5291$

$\omega_{BC} = 0.466$ rad/s

$V = \omega r \Rightarrow V_C = r_{C/IC}\,\omega_{BC} = 0.466(.5291) = \boxed{.2466 \text{ m/s}}$

**Example 2**
**Given**: $\omega_{AB} = 12$ rad/s
**Find**: $\mathbf{v}_C$, $\boldsymbol{\omega}_{BC}$, and $\boldsymbol{\omega}_{CD}$




$V_B = \omega_{AB} r_{B/A} = 12(.2) = 2.4$ m/s

BC

$\omega_{BC} = \frac{V_B}{r_{B/IC}} \quad \frac{2.4}{.45} = 5.33$ rad/s

$V_C = \omega_{BC} r_{C/IC} = (.424)(.533) = 2.26$ m/s $\measuredangle 45°$

$\omega_{CD} = \frac{V_C}{r_{CD}} = \frac{2.26}{\sqrt{.35^2+.35^2}}$

$\omega_{CD} = 4.57$ rad/s

# Lecture 12: Relative Velocity

## Concept questions

1. When a relative-motion analysis involving two sets of coordinate axes is used, the x' - y' coordinate system will
   - A) be attached to the selected point for analysis.
   - B) rotate with the body.
   - C) not be allowed to translate with respect to the fixed frame.
   - D) None of the above.

2. If *A* and *B* are two different points on the same rigid body, then $\mathbf{v}_{B/A} = -\mathbf{v}_{A/B}$ (true or false)

   $V_{B/A} = \omega \times r_{B/A}$  $V_{A/B} = \omega \times r_{A/B}$  omega is same, r is opposite

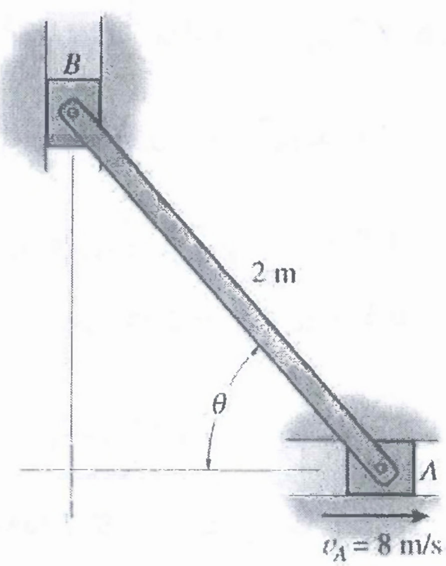
3. In the relative velocity equation, $\mathbf{v}_{B/A}$ is
   - √ A) the relative velocity of *B* with respect to *A*.
   - √ B) due to the rotational motion.
   - √ C) $\boldsymbol{\omega} \times \mathbf{r}_{B/A}$ .
   - D) All of the above.

4. If we know the velocities of two different points *A* and *B* on a rigid body, we can also know the velocity of any other point on the rigid body. (true or false)

**Example 1**

**Given**: Two slider blocks are connected by a rod of length 2 m. Also, $v_A = 8$ m/s.

**Find**: The angular velocity of the rod *AB*, ω, and the velocity of *A* when $\theta = 60°$.




$V_A + \vec{\omega} \times \vec{r}_{B/A}$

$V_B = 8i + \omega\vec{k} \times 2(-\cos 60 i + \sin 60 j)$

$= (8 - 2\omega \sin 60)\vec{i} \quad (-2\omega \cos 60)\vec{j}$

$i_B: \quad 0 = 8 - 2\omega \sin 60 \quad (1)$

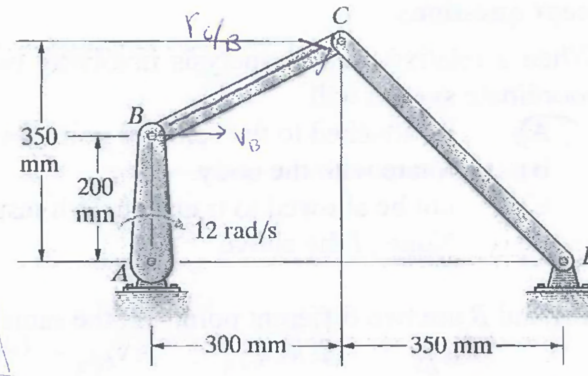
$j_B: \quad V_B = -2\omega \cos 60 \quad (2)$

$V_B = -4.62$ m/s

$\omega = 4.62$ rad/s

**Example 2**

**Given**: $\omega_{AB} = 12$ rad/s
**Find**: $\mathbf{v}_C$, $\boldsymbol{\omega}_{BC}$, and $\boldsymbol{\omega}_{CD}$




AB: $\vec{V}_B = \vec{\omega}_{AB} \times \vec{r}_{B/A}$

$= (-12\hat{k}) \times (.2\hat{j}) = \boxed{2.4\hat{i} \text{ m/s}}$

BC: $\vec{V}_C = V_B + \vec{\omega}_{BC} \times r_{C/B}$

$= 2.4i + \omega_{BC}\hat{k} \times (.3i + .15j)$

$\left.\begin{aligned} &= (2.4 - .15\omega_{BC})\hat{i} \\ &+ (.3\omega_{BC})\hat{j} \end{aligned}\right\} (1)$

$) \; 650 + 200 = \sqrt{2(350)^2} + \sqrt{150^2 + 300^2}$

$) \; 850 = 830.4$

$V_C = \vec{\omega}_{CD} \times \vec{r}_{C/D} \quad (\omega_{CD}\hat{k}) \times (-.35i + .35j)$

$= -.35\,\omega_{CD}\hat{i} - 0.35\,\omega_{CD}\hat{j} \; \} (2)$

i: $2.4 - .15\,\omega_{BC} = -0.35\,\omega_{CD}$

j: $0.3\,\omega_{BC} = -0.35\,\omega_{CD}$

$\omega_{BC} = 5.33$ rad/s

$\omega_{CD} = -4.57$ rad/s

$V_C = -0.35\,\omega_{CD}(\hat{i} + \hat{j})$

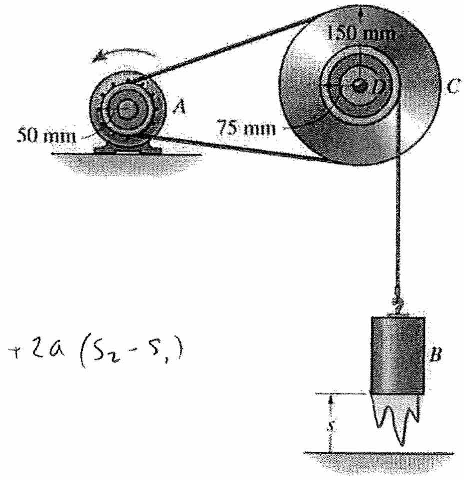
$= -.35(-4.57)(\hat{i} + \hat{j})$

$V_C = \sqrt{1.6^2 + 1.6^2} = 2.26$ m/s

**Example 1**

**Given**: Starting from rest when $s = 0$, pulley $A$ ($r_A = 50$ mm) is given a constant angular acceleration, $a_A = 6$ rad/s$^2$. Pulley $C$ ($r_C = 150$ mm) has an inner hub $D$ ($r_D = 75$ mm) which is fixed to $C$ and turns with it.

**Find**: The speed of block $B$ when it has risen $s = 6$ m.




$$a_t = \alpha_A r_A = \alpha_B r_B$$

$$6(.005) = \alpha_B (.150)$$

$$\alpha_B = 2 \text{ rad/s}^2$$

$$a_B = a_t = \alpha_B r_c$$

$$= 2(.075) = .15 \text{ m/s}^2 \uparrow$$

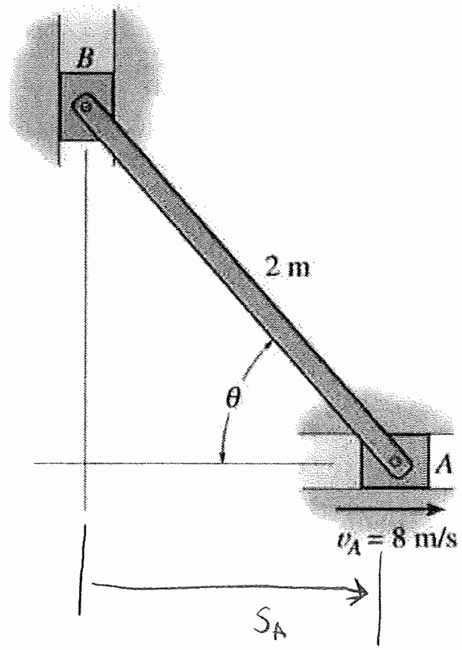
$$V_{B_0} = 0$$

$$V_B^2 = (V_B)_0^2 + 2a(s_2 - s_1)$$

**Example 2**

**Given**: Two slider blocks are connected by a rod of length 2 m. Also, $v_A = 8$ m/s and $a_A = 0$.

**Find**: Angular velocity, ω, and angular acceleration, **a**, of the rod when $\theta = 60°$.




$$S_A = 2\cos\theta$$

$$V_A = \dot{S}_A = -2\sin\theta\,\omega$$

$$8 = -2\sin 60°\,\omega \Rightarrow \omega = -4.62 \text{ rad/s}$$

$$a_A = \dot{V}_A = -2\cos\theta\,\omega^2 - 2\sin\theta\,\alpha$$

$$0 = -2\cos 60°\,(4.62)^2 - 2\sin 60\,\alpha$$

$$\alpha = -18.10 \text{ rad/s}$$

## Lecture 11: Translation, Rotation & Absolute Motion

### Concept questions

1. If a rigid body is in translation only, the velocity at points $A$ and $B$ on the rigid body ______.
   A) are usually different
   B) are always the same
   C) depend on their position
   D) depend on their relative position

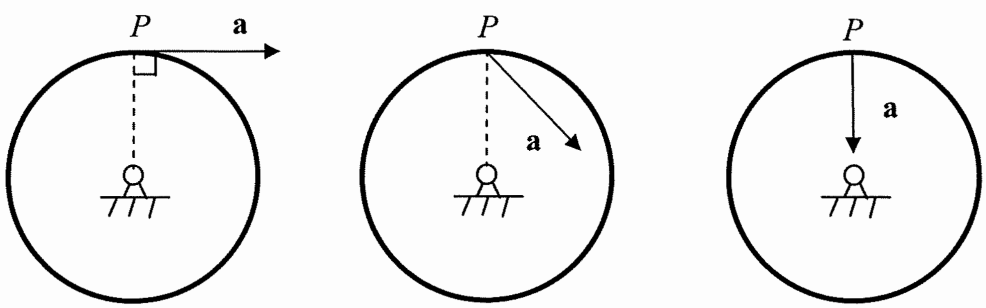
2. If the path of a point on a rigid body is a curve, then the rigid body cannot undergo a translational motion. (true or $\boxed{\text{false}}$)

3. If a rigid body is rotating with angular velocity $\boldsymbol{\omega}$ about a fixed axis, the velocity at point $P$ is __A__.

   A) $\boldsymbol{\omega} \times \mathbf{r}_P$  B) $\mathbf{r}_P \times \boldsymbol{\omega}$  C) $\dfrac{d\boldsymbol{\omega}}{dt}$  D) None of the above

4. If a rigid body rotates about a fixed axis with a constant angular velocity, then the acceleration of a point on the body is zero. (true or $\boxed{\text{false}}$)

5. The disk rotates about the axis that passes its center. The acceleration of point $P$ is shown below. Which of the following statements corresponds to Figs. (a), (b), and (c)?
   A) $(a)$ $\alpha = 0, \omega \neq 0$, $(b)$ $\alpha \neq 0, \omega = 0$, and $(c)$ $\alpha = 0, \omega \neq 0$
   B) $(a)$ $\alpha \neq 0, \omega = 0$, $(b)$ $\alpha \neq 0, \omega \neq 0$, and $(c)$ $\alpha \neq 0, \omega = 0$
   C) $(a)$ $\alpha \neq 0, \omega = 0$, $(b)$ $\alpha \neq 0, \omega \neq 0$, and $(c)$ $\alpha = 0, \omega \neq 0$
   D) $(a)$ $\alpha \neq 0, \omega = 0$, $(b)$ $\alpha = 0, \omega \neq 0$, and $(c)$ $\alpha \neq 0, \omega \neq 0$




$(a)$  $(b)$  $(c)$

6. A body is subjected to general plane motion undergoes a/an
   A) translation.
   B) rotation.
   C) simultaneous translation and rotation.
   D) out of plane movement.

7. The motion of a rigid body can be completely determined by the translation of a fixed point $A$ on the body and the rotational motion of another fixed point $B$ on the body relative to $A$. (true or false)

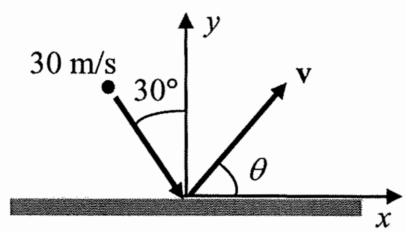
# Lecture 10: Impact & Angular Momentum

## Concept questions

1. Two balls impact with a coefficient of restitution of 0.79. It is possible that one of the balls leaves the impact with a kinetic energy greater than before the impact. (true or false)

2. Under what condition is the energy lost maximum during a collision?
   A) $e = 1.0$ B) $e = 0.0$
   C) $e = -1.0$ D) Collision is non-elastic

3. A particle strikes the smooth surface with a velocity of 30 m/s. If $e = 0.8$, $(v_x)_2$ is _____ after the collision.
   A) zero B) equal to $(v_x)_1$
   C) less than $(v_x)_1$ D) greater than $(v_x)_1$



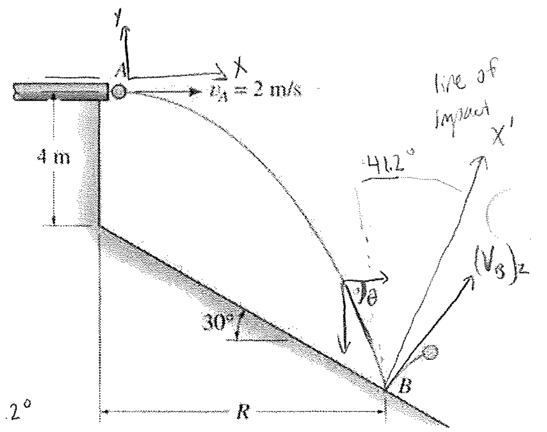

4. Angular momentum is defined as
   A) the moment of angular impulse. B) the time rate of change of linear momentum.
   C) the moment of linear momentum. D) the time rate of change of resultant force.

5. If the angular momentum of a particle is conserved about a point $O$, we can conclude that
   A) linear impulses acting on the particle are zero.
   B) the sum of the moments about $O$ is constant.
   C) the sum of the forces are zero.
   D) the sum of angular impulses about $O$ is zero.

6. If the linear momentum of a system is conserved, the angular momentum is therefore conserved as well. (true or false)

**Example 1 Given**: A 0.5 kg ball is ejected from the tube at $A$ with a horizontal velocity $v_A = 2$ m/s. The coefficient of restitution at $B$ is $e = 0.6$.

**Find**: The horizontal distance $R$ where the ball strikes the smooth inclined plane and the speed at which it bounces from the plane.

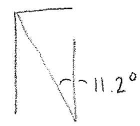



$x = x_0 + (V_x)_0 t$

$R = 0 + 2t$

$y = 0 + (V_y)t + \frac{1}{2} g t^2$

$-(4 + R\tan 30°) = 0 + 0 - \frac{1}{2}(9.81)\,t^2 \Rightarrow t = 1.028\,s \quad R = 2.06\,m$




$\tan\theta = \dfrac{10.09}{2}, \quad \theta = 78.8°$

plug into eq.(2) B: ball A: ground

$(V_{Bx})_1 = 2\,m/s$

$(V_{By})_1 = (V_y)_0 - gt$

$=$ $= 10.09\,m/s \downarrow$ $t$

$x' : \; e = \dfrac{(V_{Bx})_2 - 0}{0 - (-10.09\cos 41.2°)}$

$(V_{Bx})_2 = 4.64\,m/s \uparrow$

$(V_{By})_2 = (V_{By})_1 = 10.09 \sin 41.2°$

$= 6.77\,m/s$

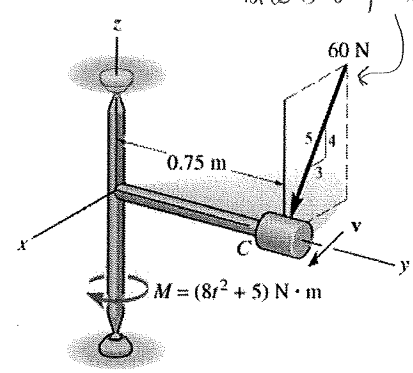
$(V_x)_2 = 8.21$

$\vec{L} = \vec{r} \times m\vec{v}$

$L = d\,mv$

**Example 2** The small cylinder $C$ has a mass of 10 kg and is attached to the end of a rod whose mass may be neglected. If the frame is subjected to a couple $M = (8t^2 + 5)$ N·m, where $t$ is in seconds, and the cylinder is subjected to a force of 60 N, which is always directed as shown, determine the speed of the cylinder when $t$ = 2 s. The cylinder has a speed of $v_0 = 2$ m/s when $t = 0$.

z moment of F force is only $F_x$ component




$(L_z)_1 + \sum \int_{t_1}^{t_2} M_z \, dt = (L_z)_2$

$-rmV_1 + \int_0^2 \left[-(8t^2 + 5) - 60\left(\frac{3}{5}\right)(r)\right] dt = -rmV_2$

$V_2 = -13.4\,m/s$

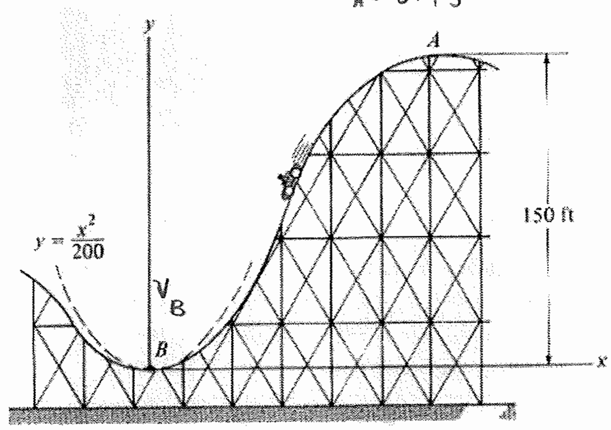
**Lecture 8: Power and Conservation Systems**

**Concept questions**

1. The potential energy of a spring is
   A) always negative. B) always positive.
   C) positive or negative. D) equal to $ks$.
2. When the potential energy of a conservative system increases, the kinetic energy
   A) always decreases. B) always increases.
   C) could decrease or increase. D) does not change.
3. If the work done by a conservative force on a particle as it moves between two positions is –10 ft-lb, the change in its potential energy is
   A) 0 ft-lb. B) -10 ft-lb. C) +10 ft-lb. D) none of the above
4. The location of the datum for the gravitational potential energy does not affect the result of using conservation of energy. (ture or false)

$V_A = 5 ft/s$

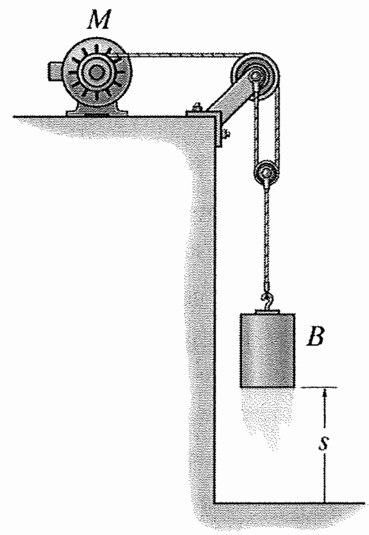
**Example 1** If the roller-coaster car has a speed $v_A = 5$ ft/s when it is at $A$ and coasts freely down the track, determine the speed $v_B$ it attains when it reaches point $B$. Also determine the normal force a 150-lb passenger exerts on the car when it is at $B$. At this point the track follows a path defined by $y = x^2/200$. Neglect the effects of friction, the mass of the wheels, and the size of the car.




$\frac{1}{2} m$

**Example 2**

The 50-lb load is hoisted by the pulley system and motor *M*. If the motor exerts a constant force of 30 lb in the cable, determine the power that the motor must supply if the load has been hoisted $s$ = 10 ft starting form rest. The pulley-cable system has an efficiency $\varepsilon = 0.76$.



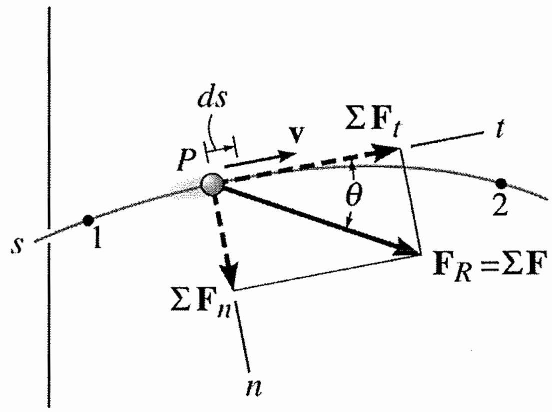
# Lecture 7: The Principle of Work and Energy

## Concept questions

1. If a particle moves from 1 to 2, the work done on the particle by the force, $\mathbf{F}_R$, will be

   A) $\int_{s_1}^{s_2} \Sigma F_t ds$ B) $-\int_{s_1}^{s_2} \Sigma F_t ds$

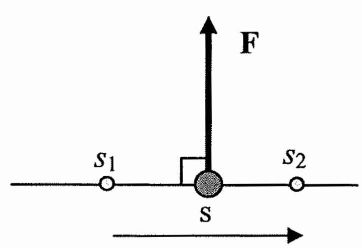
   C) $\int_{s_1}^{s_2} \Sigma F_n ds$ D) $-\int_{s_1}^{s_2} \Sigma F_n ds$




2. What is the work done by the force **F**?

   A) $Fs$ B) $-Fs$

   C) Zero D) None of the above




3. Kinetic energy results from ________.

   A) displacement B) velocity

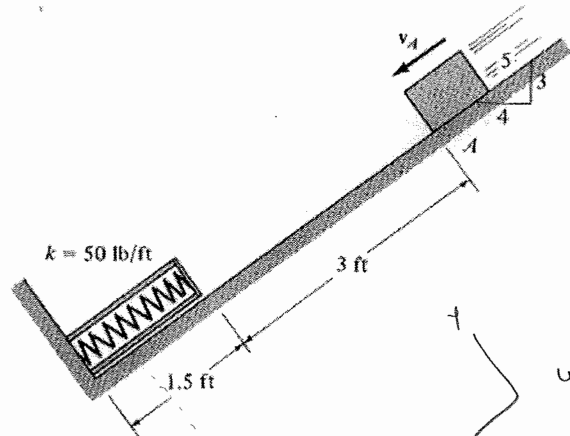
   C) gravity D) friction

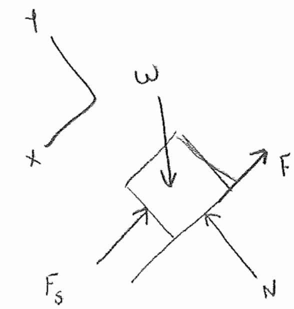
4. If the velocity of a particle at time $t_1$ and the work done by the external forces from time $t_1$ to time $t_2$ are known, it is possible to know the velocity of the particle at time $t_2$. (true or false)

   known: only speed, not direction

5. Work done by a friction force can be positive. (true or false)

**Example 1** The spring has a stiffness $k = 50$ lb/ft and an unstretched length of 2ft. As shown, it is confined by the plate and wall using cables so that its length is 1.5 ft. A 4-lb block is given a speed $v_a$ when it is at $A$, and it slides down the incline having a coefficient of friction $\mu_k = 0.2$. If it strikes the plate and pushes it forward 0.25 ft before stopping, determine its speed at $A$. Neglect the mass of the plate and spring.




$K = 50$ lb/ft

$W = 4$ lb $\quad \mu_k = .2 \quad V_2 = 0$

find $V_1$

$\Sigma F_y = 0 \qquad N - W\cos\theta = 0$

$F = \mu_k N = \mu_k W\cos\theta = .2(4)\,4/5 = .64$ lb

$x \qquad \frac{1}{2}mv_1^2 + \Sigma U_{12} = \frac{1}{2}\cancel{m}_v V_2^2$

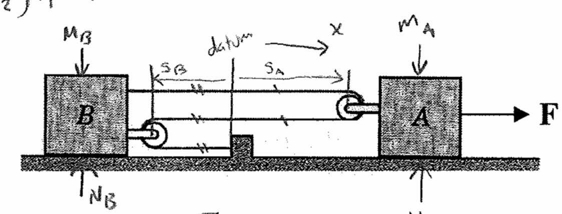
$U_{12} = W(y_2 - y_1) = wh = 4(3.25)\sin\theta = 4(3.25)\frac{3}{5} = +7.8$ lb·ft

$F: U_{12} = -Fs = -.64(3.25) = -2.08$ ft.lb

$F_s: U_{12} = \frac{1}{2}k(\ell_2^2 - \ell_1^2) = \frac{1}{2}(50)(.75^2 - .5^2)$

$U_{12} = -7.8125$ ft.lb $\quad \frac{1}{2}\left(\frac{4}{32.2}\right)V_1^2 + 7.8 - 2.08 - 7.8125 = 0 \qquad V_1 = 5.8$

**Example 2** (system of particles) Determine the force **F** that must be applied to block $A$ to make block $B$ attain a speed of 8 m/s after moving 4 m to the right from rest. The masses of the blocks are $m_A = 20$ kg, $m_B = 20$ kg. Neglect friction.




$(V_B)_2 = 8$ m/s $\quad \Delta S_B = 4$ m $\qquad$ Find F $\qquad \Sigma T_1 + \Sigma U_{12} = \Sigma T_2$

$V_B = \frac{2}{3}V_A \qquad\qquad 0 + F\Delta S_A = \frac{1}{2}M_A(V_A)_2^2 + \frac{1}{2}M_B(V_B)_2^2$

$2S_A + 3S_B = \text{const} \qquad ① \; F(6) = \frac{1}{2}(20)(12)^2 + \frac{1}{2}(20)(8)^2$

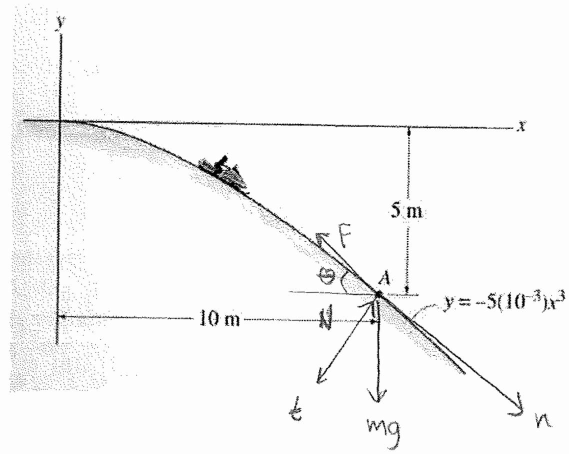
$2V_A + 3V_B = 0 \qquad F = 346.6$ N

$V_A = -3/2(V_B)_2 = -\frac{3}{2}(-8) = 12$ m/s

$\Delta S_A = -\frac{3}{2}\Delta S_B = -\frac{3}{2}(4) = 6$ m

14.-41 $\theta = 41.41°$

**Example 1** A $\boxed{200 \text{ kg}}$ snowmobile travels down the hill. When it is at point $\boxed{A,}$ it is traveling at the speed of $\boxed{4 \text{ m/s}}$ and increasing its speed at $\boxed{2 \text{ m/s}^2}$. Determine the resultant normal force and resultant frictional force exerted on the tracks at point $A$. Find $N, F$




$\Sigma F_t = ma_t$

$mg\sin\theta - F = ma_t$

$200(9.81)\sin\theta - F = 200(2)$

$\tan\theta = \frac{dy}{dx} = -15(10^{-3})x^2$

$-15(10^{-3})(10^2) = -1.5$

$\theta = 56.31°$

$F = 1232.0 \text{ N}$

$\Sigma F_n = ma_n$

$-mg\cos\theta + N = m\frac{v^2}{\rho}$

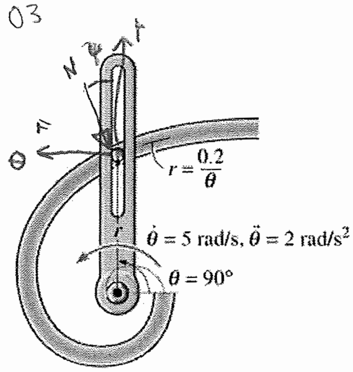
$-200(9.8)\cos 56.31° + N = 200\frac{4^2}{\rho}(2)$

curvature $\rightarrow$ $\frac{d^2y}{dx^2} = -30(10^{-3})x = -30(10^{-3})(10) = -03$

p56

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = 1953 \text{ m}$$

**Example 2** The arm is rotating at a rate of $\boxed{\dot{\theta} = 5 \text{ rad/s}}$ when $\boxed{\ddot{\theta} = 2 \text{ rad/s}^2}$ and $\boxed{\theta = 90°}$. Determine the normal force it must exert on the 0.5 kg particle if the particle is confined to move along the slotted path by the horizontal hyperbolic spiral $\boxed{r\theta = 0.2 \text{ m}}$.




Find F

$r = .2\theta^{-1}$

$\frac{dr}{d\theta} = -.2\theta^{-2}$

$\tan\psi = \frac{r}{dr/d\theta} = \frac{0.2\theta^{-1}}{-.2\theta^{-2}} = -\theta = -\frac{\pi}{2}$

$r = .2\theta^{-1} = .2\left(\frac{\pi}{2}\right)^{-1} = .12732 \text{ m}$

$\dot{r} = \frac{dr}{dt} = \frac{dr}{d\theta}\left(\frac{d\theta}{dt}\right) = -.2\theta^{-2} = -.2\left(\frac{\pi}{2}\right)^{-2}(5) = -.40528 \text{ m/s}$

$\ddot{r} = -.2(-2\theta^{-3}\dot{\theta}^2 + \theta^{-2}\ddot{\theta})$

$= -.765 \text{ m/s}^2$

$a_r = \ddot{r} - r\dot{\theta} = -.765 \text{ m/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = -3.7982 \text{ m/s}^2$

$\Sigma F_r = ma_r$

$-N\cos\psi = ma_r$

$-N\cos 57.52° = .5(-.7651)$

$N = 0.453 \text{ N}$

$\Sigma F_\theta = ma_\theta$

$F - N\sin 57.52 = ma_\theta$

$F = 1.66 \text{ N}$

## Lecture 6: EOM – n-t-b and r-θ-z Coordinates

### Concept questions

1. Friction forces always act in the ___________ direction of the relative motion.

   A) radial  B) tangential  C) transverse  D) None of the above

2. A particle is moving in space. If the resultant external forces in $\mathbf{u}_r$ and $\mathbf{u}_\theta$ directions are zero, then the acceleration of the particle is zero. (true or false)  $\ddot{z} \neq 0$

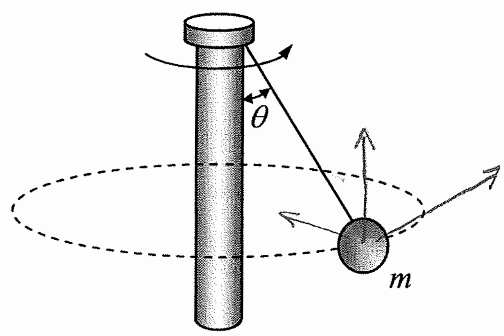
3. For problem 2, if the resultant external forces in $\mathbf{u}_t$ and $\mathbf{u}_n$ directions are zero, then the acceleration of the particle is zero. (true or false)  $a_b = 0$

   $\Sigma F_b$ is already zero

4. The small ball rotates around the vertical pole in a horizontal circular path. What is the normal acceleration of the ball?

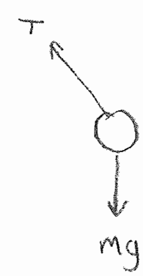
   A) $g\sin\theta$  B) $g\cos\theta$

   C) $g\tan\theta$  D) $g/\tan\theta$




$$\Sigma F_n = m a_n$$

$$m a_n = T\sin\theta$$




$$\Sigma F_z = 0 = T\cos\theta - mg = 0$$

$$T = \frac{mg}{\cos\theta}$$

$$a_n = \frac{T\sin\theta}{m} = \frac{\frac{mg}{\cos\theta}(\sin\theta)}{m}$$

$$a_n = g(\tan\theta)$$

## Lecture 5: Newton's Second Law

### Concept questions

1. Newton's second law can be written in mathematical form as $\sum \mathbf{F} = m\mathbf{a}$. Within the summation of forces **F**, ________ are (is) not included.

   A) external forces    B) weight

   C) internal forces    D) All of the above.

2. The equation of motion for a system of *n*-particles can be written as $\sum \mathbf{F}_i = \sum m_i \mathbf{a}_i = m\mathbf{a}_G$, where $\mathbf{a}_G$ indicates ________.

   A) summation of each particle's acceleration

   B) acceleration of the center of mass of the system

   C) acceleration of the largest particle

   D) None of the above.

3. A 10 kg particle is subject to forces $\mathbf{F}_1$= (3 **i** + 5 **j**) N and $\mathbf{F}_2$= (-7 **i** + 9 **j**) N. Determine the acceleration of the particle.

   A) (-0.4 **i** + 1.4 **j**) m/s$^2$    B) (-4 **i** + 14 **j**) m/s$^2$

   C) (-12.9 **i** + 45 **j**) m/s$^2$    D) (13 **i** + 4 **j**) m/s$^2$

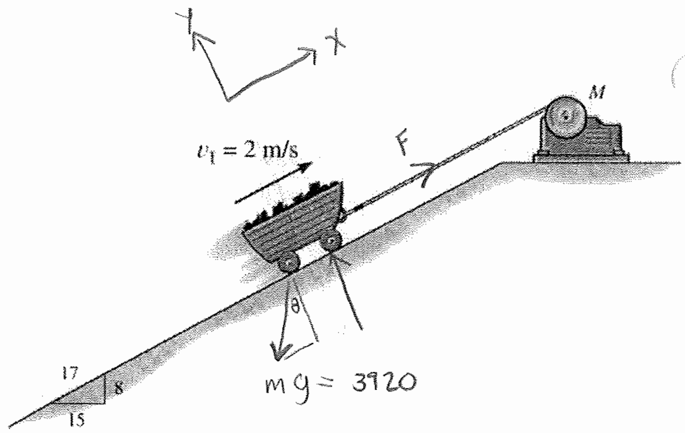
4. In a dynamic problem, the friction force acting on a particle is always

   A) $\mu_s$N    B) greater than $\mu_s$N    C) $\mu_k$N    D) none of the above

**Example 1**

**Given**: The $\boxed{400 \text{ kg}}$ mine car is hoisted up the incline. The force in the cable is $\boxed{F = 3200t^2 \text{ N}}$. The car has an initial velocity of $\boxed{v_0 = 2 \text{ m/s}}$ at $t = 0$.

**Find**: The velocity when $t = 2$ s.




$F(t) \rightarrow a(t) \rightarrow v$

Using flowchart

$x: \Sigma F_x = m a_x$

$F - mg\sin\theta = m a_x$

$3200t^2 - 400(9.81)\frac{8}{17} = 400a$

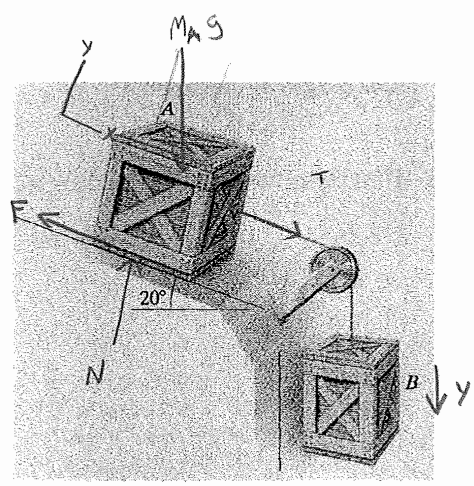
$a = 8t^2 - 4.616$

$v = v_0 + \int_0^t a\,dt$

$= 2 + \int_0^2 (8t^2 - 4.616)\,dt$

$= 14.1$ m/s

**Example 2** Two crates are released from rest. Their masses are $\boxed{m_A = 40 \text{ kg}}$ and $\boxed{m_B = 30 \text{ kg}}$, and the coefficients of friction between crate *A* and the inclined surface are $\boxed{\mu_s = 0.2}$ and $\boxed{\mu_k = 0.15}$. What is the acceleration of the crates?




find a. assume static

A: $F_x = 0$ $\quad T - F + M g \sin 20° = 0$

$\Sigma F_y = N - M_A g \cos 20° = 0$

B: $\Sigma F_y = 0 \quad M_B g - T = 0$

$F = M_B g + M_A g \sin 20$

$= 30(9.81) + 40(9.81)(\sin 20)$

$= 429.0$ N

$N = M_A g \cos 20°$

$= 40(9.81)\cos 20° = 368.74$ N

$F_{max} = \mu_s N = .2(368.74) = 73.74$ N

$F > F_{max}$ impossible

motion → kinetics

$\Sigma F_x = M_A a$ (1)

$T - F + M_A g \sin 20° = M_A a$ (2)

$M_B g - T = M_B a$ (3)

$F = \mu_k N$ (4)

$a = 5.33$ m/s$^2$

Lecture 4: Absolute Dependent and Relative Motion

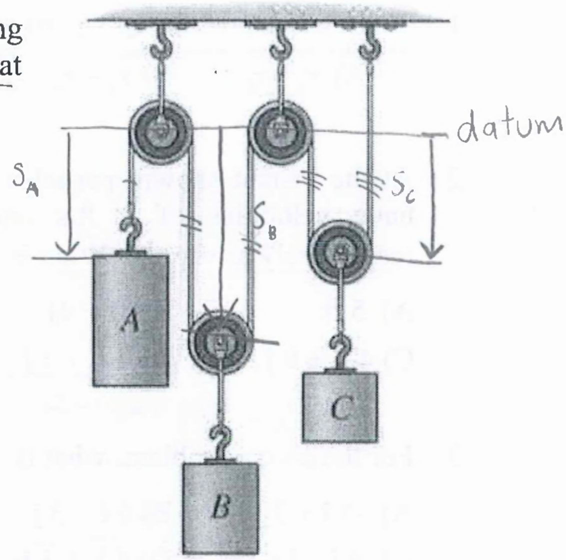
## Part 1: Absolute dependent motion analysis

### Example 1: Pulley and cable system (1 cable)

**Given**: In the pulley and cable system, block *A* is moving downward with a speed of $\boxed{4\ \text{ft/s}}$ while block *C* is moving up at $\boxed{3\ \text{ft/s}}$

C A

**Find**: The speed of block *B*.




$V_B$

1. position coordinates datum → point
2. constant cable length

$S_A + 2S_B + 2S_C = \text{const}$

3. differentiation.

$V_a + 2V_B + 2V_c = 0$

$V_B = -\frac{1}{2}(V_a + 2V_c)$

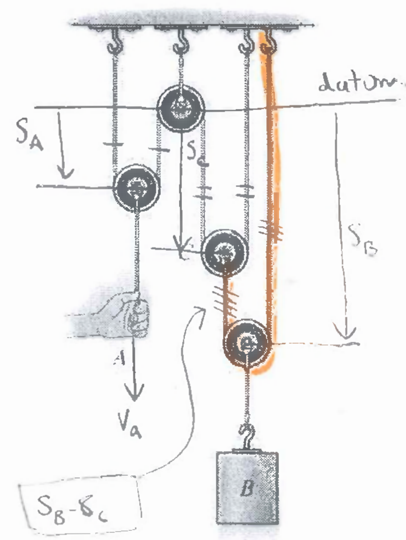
$= -\frac{1}{2}(4 + 2(-3))$

$= 1$ ft/s positive ⇒ down

### Example 2: Pulley and cable system (2 cables)

In the figure on the right, the cord at *A* is pulled down with a speed of 8 ft/s. Determine the speed of block *B*.

find $V_B$




(2) $2S_A + 2S_c = \text{Const}$

$(S_B - S_c) + S_B = \text{const}$

(3) $V_A + V_c = 0 \Rightarrow 8 + V_c = 0 \Rightarrow V_B = -4$ ft/s

$2V_B - V_c = 0 \quad 2V_B - V_c = 0$

**Part 2: Relative motion analysis**

**Concept questions**

1. The relative position of a particle $B$ with respect to particle $A$ is defined as

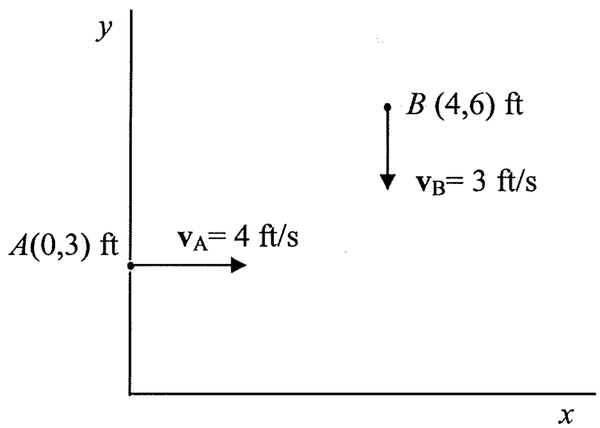
   A) $r_A + r_B$ B) $r_A - r_B$ C) $r_B - r_A$ D) $r_B + r_A$

2. At the instant shown, particles $A$ and $B$ have velocities of 4 ft/s and 3 ft/s respectively as shown. What is $\mathbf{r}_{B/A}$?

   A) 5 ft B) $3\mathbf{i} + 4\mathbf{j}$

   C) $4\,\mathbf{i} + 6\,\mathbf{j}$ D) $4\,\mathbf{i} + 3\,\mathbf{j}$

   $r_B - r_A = (4i + 6j) - 3j$




3. For the above problem, what is $\mathbf{v}_{A/B}$?

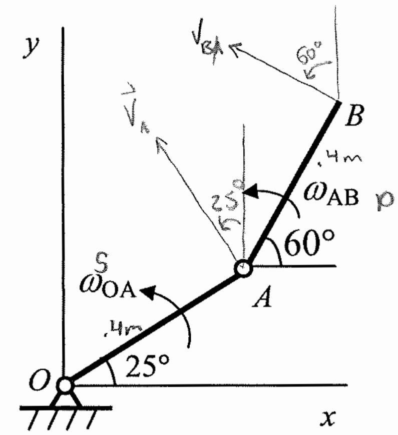
   A) $-4\,\mathbf{i} + 3\,\mathbf{j}$ B) $4\,\mathbf{i} - 3\,\mathbf{j}$

   C) $-4\,\mathbf{i} - 3\,\mathbf{j}$ D) $4\,\mathbf{i} + 3\,\mathbf{j}$

   $v_A - v_B = 4i - (-3j)$

4. If two particles move with the same speed in opposite directions, their relative velocity is zero. (true or false)

**Example 1** Each bar is 0.4 m long and rotates in the *x-y* plan. They are connected by a pin at *A*. Relative to the reference frame shown, bar *OA* has a counterclockwise angular velocity of 5 rad/s and bar *AB* has a counterclockwise angular velocity of 10 rad/s. What is the velocity of point *B* relative to the reference frame at the instant shown?



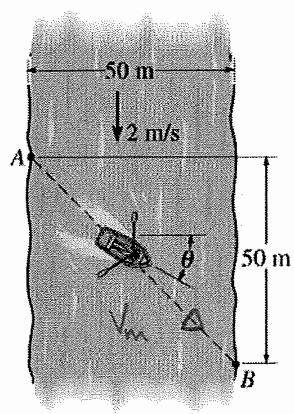

$$\vec{V}_B = \underset{\text{rotation}}{\vec{V}_A} + \underset{\text{rotation}}{\vec{V}_{B/A}}$$

$$= 5(.4)(-\sin 25^\circ i + \cos 25^\circ j)$$

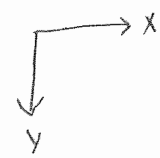
$$+ 10(.4)(-\sin 60^\circ i + \cos 60^\circ j)$$

$$\Rightarrow -4.31 j + 3.81 i \text{ m/s}$$

**Example 2** A man can row a boat at 5 m/s in still water. He wishes to cross a 50 m wide river to point *B*, 50 m downstream. If the river flows with a velocity of 2 m/s, determine the speed of the boat and the time needed to make the crossing.




$V_{m/w} = 5$ m/s $\quad V_w$ 2m/s $\quad$ find $\vec{V}_m$, t




$$V_M = V_W + V_{M/W}$$

M $\quad$ ? $\quad$ 2 $\quad$ 5

D $\quad$ 45 $\quad$ ↓ $\quad$ $\theta$

$$V_M(\cos 45 i + \sin 45 j) = 2j + 5(\cos\theta i + \sin\theta j)$$

$i$: $V_M \cos 45 = 5\cos\theta i \quad (1)$

$j$: $V_M \sin 45 = 2 + 5\sin\theta \quad (2)$

$V_M = 6.21$ m/s $\qquad t = \dfrac{\sqrt{50^2 + 50^2}}{6.21} = 11.397$ s

$\theta = 28.57$

## Lecture 3: Rectilinear Kinematics

### Concept questions

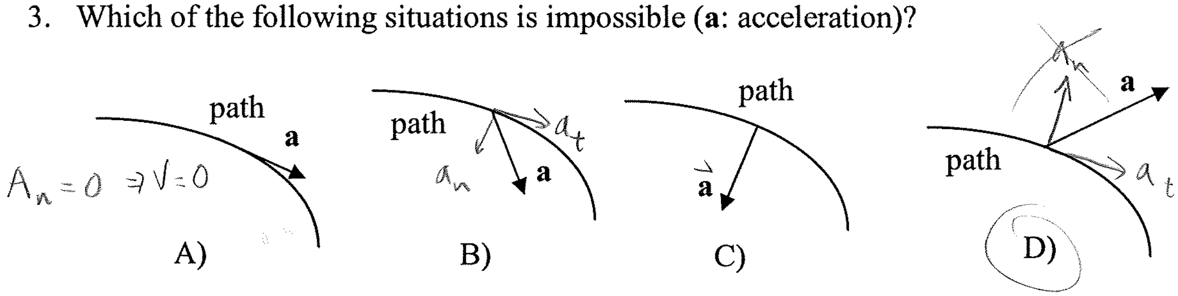
1. If a particle moves along a curve with a constant speed, then its tangential component of acceleration is

   A) positive. B) negative. C) zero. D) constant.

2. The directions of the tangential acceleration and velocity are always

   A) perpendicular to each other. B) collinear.
   C) in the same direction. D) in opposite directions.

3. Which of the following situations is impossible (**a**: acceleration)?



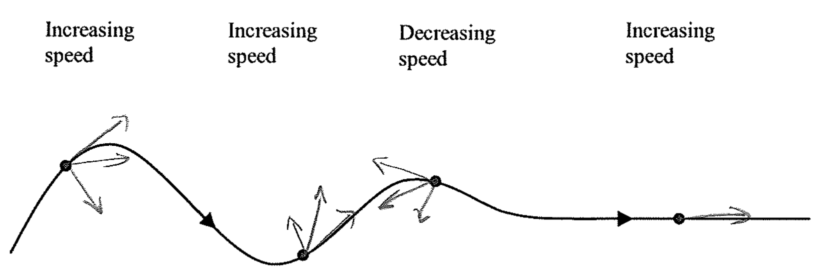

4. If the magnitude of the velocity of a particle is constant, then the acceleration of the particle is zero. (true or false)

5. Suppose initially a particle moves with a nonzero velocity. In what motion will the particle be under the following conditions?

   A) $a_t = 0,\ a_n = 0$:________ B) $a_t \neq 0,\ a_n \neq 0$:________
   C) $a_t = 0,\ a_n \neq 0$:________ D) $a_t \neq 0,\ a_n = 0$: ________

   1) Rectilinear motion with constant speed.
   2) Rectilinear motion with varying speed.
   3) Curvilinear motion with constant speed.
   4) Curvilinear motion with varying speed.

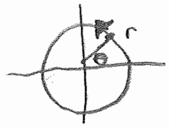
6. Draw the directions of the acceleration of the particle shown below.




7. The speed of a particle in a cylindrical coordinate system is

   A) $\dot{r}$ B) $r\dot{\theta}$ C) $\sqrt{(\dot{r})^2 + (r\dot{\theta})^2}$ D) $\sqrt{(\dot{r})^2 + (r\dot{\theta})^2 + (\dot{z})^2}$

r= constant



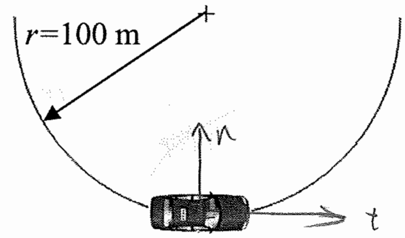

8. If $\dot{r} = 0$, $\dot{\theta} \neq 0$, and $r|_{t=0} \neq 0$ for a particle, then the particle is

A) not moving.  B) moving in a circular path.

C) moving on a straight line.  D) moving with a constant velocity.

**Example**

Starting from rest, the car travels around the circular path, $\boxed{r = 100 \text{ m},}$ at a speed $\boxed{v = 0.5t^2}$ m/s, where $t$ is in seconds. Determine the magnitudes of the car's velocity and acceleration at the instant $\boxed{t = 5 \text{ s.}}$




$$\left.\begin{aligned} a_n &= \frac{v^2}{\rho} \\ a_t &= \dot{v} \end{aligned}\right\} \Rightarrow a$$

$$v = 0.5(5)^2 = 12.5 \text{ m/s}$$

$$a_n = \frac{v^2}{\rho} = \frac{12.5^2}{100} = 1.56 \text{ m/s}^2$$

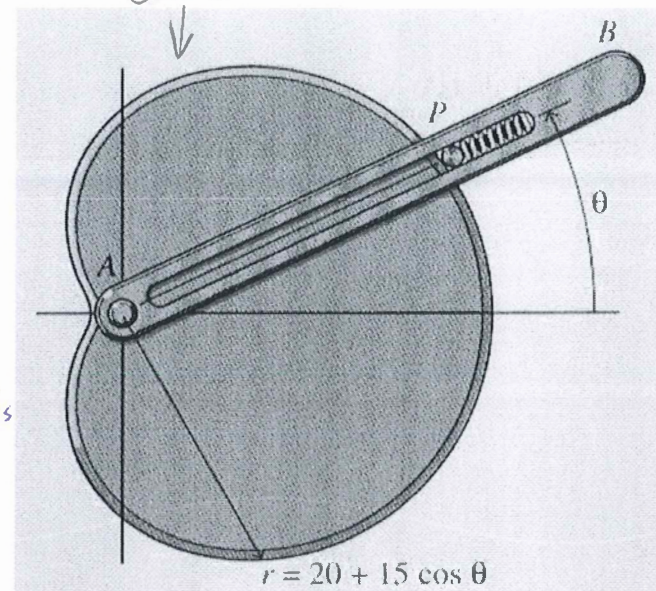
$$a_t = \dot{v} = t = 5 \text{ m/s}^2$$

$$a = \sqrt{1.56^2 + 5m^2} = 5.24 \text{ m/s}^2$$

**Example 2**

A cam lobe has a shape given by $\boxed{r = 20 + 15\cos\theta \text{ mm.}}$ Pin $P$ slides in a slot along arm $AB$ and is held against the cam by a spring. The arm $AB$ rotates counterclockwise about $A$ at a rate of $\boxed{30 \text{ rev/min.}}$ Given that $\theta = 0$ at $t = 0$. Determine the velocity and acceleration of the pin at $\boxed{t = 0.75 \text{ s.}}$

cam




find $\dot{\theta}$, $\dot{r}$, $\ddot{r}$

$\dot{\theta} = 30\,(2\pi)/60$ rad/s $= \pi$ rad/s

$\theta = \theta_0 + \int_0^t \dot{\theta}\,dt = 0 + \pi t \Big|_0^{0.75}$

$= \pi(.75) = 135°$

$\ddot{\theta} = 0$

$r = 20 + 15\cos 135° = 9.39$ mm

$\dot{r} = \frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} = -15\sin\theta(\dot{\theta}) =$

$= 15\sin 35°(\pi) = -33.32$ mm/s

$\ddot{r} = \frac{d\dot{r}}{dt} = -15\left(\cos\theta\,\dot{\theta}^2 + \cancel{\sin\theta\,\ddot{\theta}}\right)$ 0

$= -15\cos 135°\,(\pi)$

$= 104.68$ mm/s$^2$

$V_r = \dot{r} = -33.32$ mm/s

$V_\theta = r\dot{\theta} = 29.50$

$a_r = \ddot{r} - r\dot{\theta} = 11.97$ mm/s$^2$

$a = 209.7$ mm/s$^2$

$a_\theta = -209.4$ mm/s$^2$

12-130

# Lecture 1: Rectilinear Kinematics

## Concept questions

1. When $v$-$t$ graph is given, it is possible to determine $s$-$t$ graph using

   A) $a = \dfrac{dv}{dt}$ B) $v = \dfrac{ds}{dt}$ C) $v = \dfrac{da}{dt}$ D) $a = \dfrac{d^2s}{dt^2}$

2. The area under $a$-$t$ diagram represents the change in _______ during a time interval.

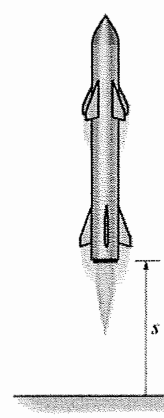
   A) velocity B) position C) acceleration D) none of the above

3. When $a = a(v)$ is given, it is possible to determine $v = v(t)$ graph using

   A) $a = \dfrac{dv}{dt}$ B) $v = \dfrac{ds}{dt}$ C) $v = \dfrac{da}{dt}$ D) $a = \dfrac{d^2s}{dt^2}$

$$a(v) = \frac{dv}{dt}$$
$$dt = \frac{dv}{a(v)}$$
$$t - t_0 = \int_{v_0}^{v} \frac{dv}{a(v)}$$

**Example 1** The acceleration of a rocket traveling upward is given by $a = (6 + 0.02s)\ \text{m/s}^2$, where $s$ is in meters. Determine the time needed for the rocket to reach an altitude of $s = 100$ m. Initially, $v = 0$ and $s = 0$ when $t = 0$.




find t@ s=100

$$a(s)\,ds = v\,dv \Rightarrow v(s)$$
$$v(s) = \frac{ds}{dt} \Big\} \Rightarrow s(t)$$

$$\int_0^s (6 + .02s)\,ds = \int_0^v v\,dv$$

$$6s + .01s^2 = \frac{1}{2}v^2$$

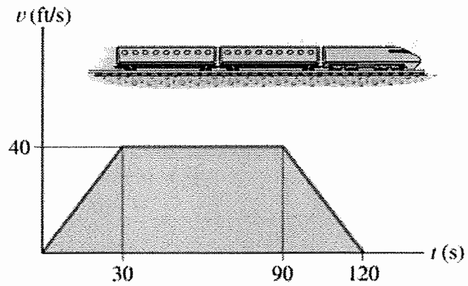
$$v = \sqrt{12s + .02s^2}$$

$$\frac{ds}{v} = dt \xrightarrow{\text{integrate}} \int_0^{100} \frac{ds}{\sqrt{12s + .02s^2}} = t$$

table will give $\dfrac{1}{\sqrt{.02}} \ln\left[\sqrt{12s + .02s^2} + s\sqrt{.02} + \dfrac{12}{2\sqrt{.02}}\right]_0^{100}$

$= 5.625\ s$

**Example 2** The *v-t* graph for the motion of a train as it moves from station A to station B is shown. Draw the *a-t* graph and *s-t* graph and determine the average speed and the distance between the two stations.



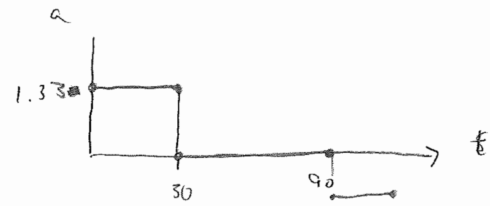

Find a-t, s-t

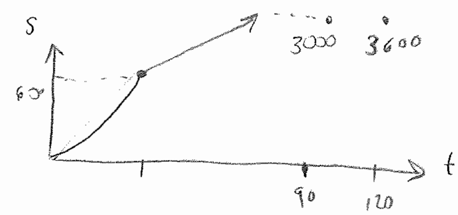
for $0 \le t \le 30$

$$V = \frac{40}{30} t$$

$$a = 4/3 = 1.33 \text{ ft/s}^2$$

$$s = s_0 + \int_0^t \frac{4}{3} t \, dt = \frac{2}{3} t^2$$




for $30 \le t < 90$

$$V = 40 \text{ ft/s}$$

$$a = 0$$

$$s = s(30) + \int_{30}^{t} 40 \, dt = 600 + 40(t-30)$$

$$s(90) = 600 + 40(90-30)$$

$$= 3000 \text{ ft}$$

$90 < t \le 120$

$$V = 40 + \frac{0-40}{120-90}(t-90) = 160 - \frac{4}{3}t$$

$$a = \frac{dv}{dt} = -1.33 \text{ ft/s}^2$$

$$s = s(90) + \int_{90}^{t} v\,dt = s(90) + \int_{90}^{t} \left(160 - \frac{4}{3}t\right) dt = -6000 + 160t - \frac{2}{3}t^2$$

$$\bar{V} = \frac{3600}{120} = 30 \text{ ft/s}$$

## Lecture 2: Rectilinear Kinematics

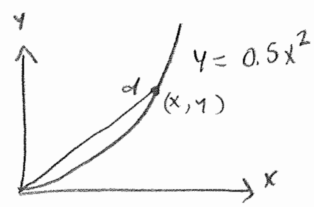
**Concept questions**

1. The direction of the velocity of a particle is always tangent to the path of the particle. (true or false)
2. The direction of the acceleration of a particle is always tangent to the path of the particle. (true or false)
3. If $x = x(t)$, $y = y(t)$, and $z = z(t)$ are given, it is possible to find the position and acceleration of a particle at any instant of time. (true or false)
4. If $\dot{x} = \dot{x}(t)$, $\dot{y} = \dot{y}(t)$, and $\dot{z} = \dot{z}(t)$ are given, we can certainly determine the displacement and acceleration of a particle at any instant of time. (true or false)
5. The position of a particle is given as $\mathbf{r} = (4t^2\mathbf{i} - 2\mathbf{j})$ m. What is the particle's acceleration?

   A) $(4\mathbf{i} + 8)$ m/s$^2$ B) $(8\mathbf{i} - 16\mathbf{j})$ m/s$^2$ C) $8\mathbf{i}$ m/s$^2$ D) $8\mathbf{i}$ m/s

**Example 1**

A particle travels along the path defined by the parabola $y = 0.5x^2$. If the component of velocity along $x$ axis is $v_x = (5t)$ ft/s, where $t$ is in seconds, determine the particle's distance from the origin $O$ and the magnitude of its acceleration when $t = 1$ s. When $t = 0$, $x = 0$, $y = 0$.




$V_x = V_x(t) = 5t$

$x_0, y_0 = 0$

find $d\sqrt{x^2+y^2}$ and a when $t = 1s$

$\dot{x} = 5t \Rightarrow x(t) \Rightarrow y(t)$

$y(x)$

$x = x_0 + \int_0^t \dot{x}\,dt = 0 + \int 5t\,dt = 2.5t^2$

$y = 0.5x^2 = 0.5(2.5t^2)^2 = \frac{25}{8}t^4 = \frac{25}{8}(1)^4 = \frac{25}{8}$ ft

$d = \sqrt{x+y} = 4.0$ ft.

$\ddot{x} = \frac{d\dot{x}}{dt} = \frac{d}{dt}5t = 5$ ft/s$^2$

$\dot{y} = \frac{25}{2}t^3 = \cancel{12.5(1)^3}$

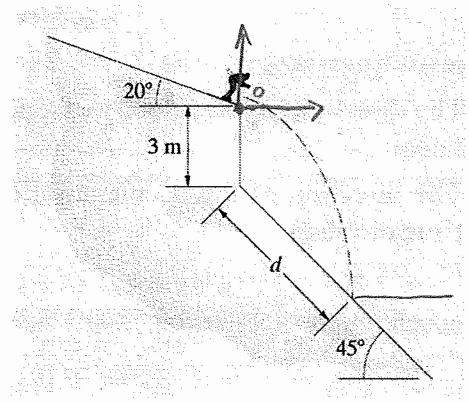
$\ddot{y} = \frac{75}{2}t^2 = \frac{75}{2}(1)^2 = \frac{75}{2}$ ft/s$^2$

$a = \sqrt{\ddot{y}^2 + \ddot{x}^2} = 37.8$ ft/s$^2$

$\vec{a} = \ddot{x}\vec{i} + \ddot{y}\vec{j}$

**Example 2**

The skier leaves the 20° surface at 10 m/s. Determine the distance *d* to the point where he lands.




$X_0 = 0$ $\quad \dot{x} = 10\,m/s \cos 20$

$Y_0 = 0$ $\quad \dot{y} = -10\,m/s \sin 20$

$x = X_0 + (V_0)_x t$

$= 0 + 10 \cos 20° t$

$y = 0 - 10 \sin 20° t - \frac{1}{2}(9.81) t^2$

$\tan(-45°) = \frac{y - (-3)}{x - 0} = -1$

$x = 15.0\,m$

$y = -18.0\,m$

$t = 1.60\,s$

$d = \sqrt{15^2 + (15-3)^2}$

$= 21.3\,m$

EXAMS

# Dynamics Exam 2

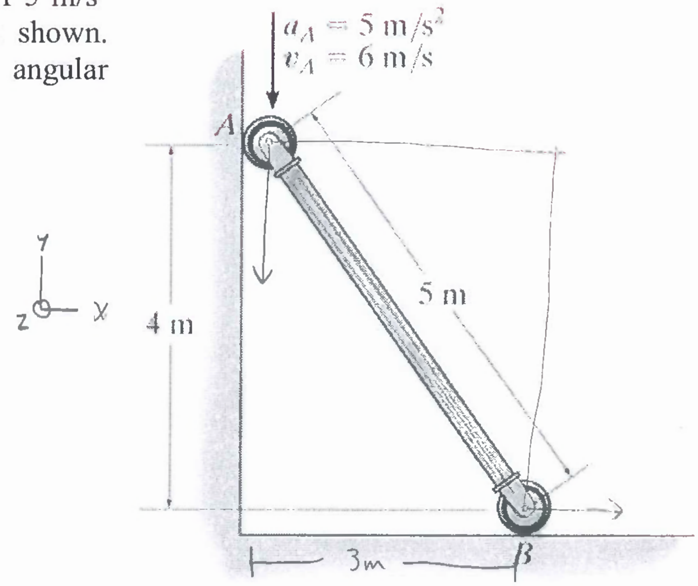
Spring 2009

Name (please print) David Malawey

Malawey

75/100

1. Point $A$ on rod $AB$ has an acceleration of 5 m/s$^2$ and a velocity of 6 m/s at the instant shown. Determine the angular velocity and angular acceleration of the rod at this instant.
**(20 points)**




$\omega = ?\ \alpha = ?$

$V_A = \omega\, r_{A/IC}$

$6\ m/s\ \hat{j} = \omega(3m)$

$\boxed{\omega = \frac{6}{3} = 2\ rad/s}$

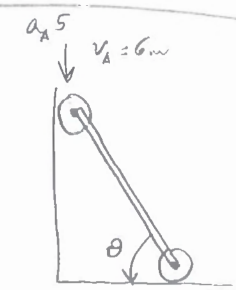
$\alpha_A = \omega\, r_{A/IC}$

$5 = \omega(3)$

$\boxed{\alpha_A = \frac{5}{3}\ rad/s}$

−5

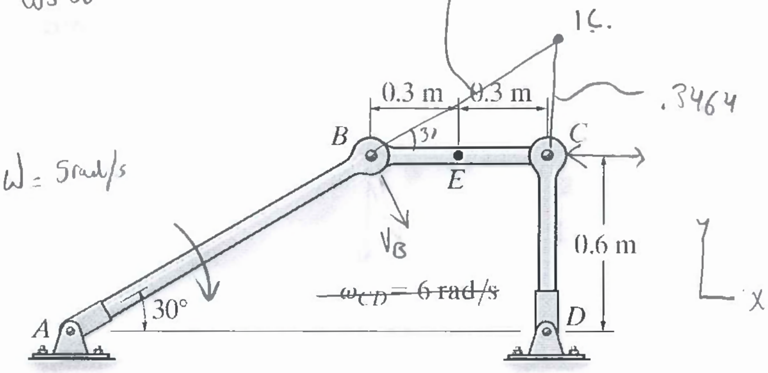
redo: $\ddot{s} = a_A = -5\sin\theta\,\omega^2 +$



2. If link $AB$ has an angular velocity of $\omega_{AB} = 5$ rad/s clockwise, determine the velocity of point $C$ at the instant shown. **(20 points)**

$\cos 30 = \frac{.6}{r_{B/IC}}$

$r_{B/IC} = .69282$




$V_C = V_B + \omega \times r_{C/B}$

$\omega = 5$ rad/s

$V_B = \omega r = 1.2(5) = 6$ m/s

$\omega_{BC} = \frac{V_B}{r_{B/IC}} = \frac{6}{.69282} = 8.66$

$\sin 30 = \frac{.6}{r_{AB}} = .5$

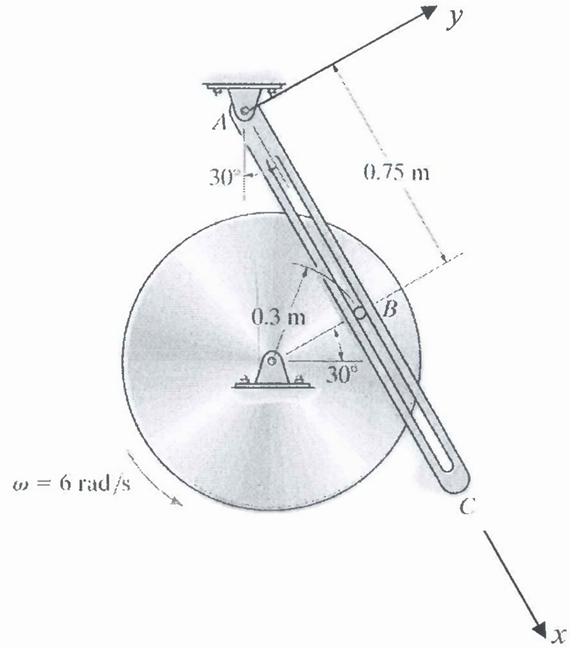
$r_{AB} = 1.2$

$\cancel{V_C = \omega r = 6(0.6) = \boxed{3.6 \text{ m/s}}}$

$V_C = \omega_{BC} r_{C/IC} = 8.66(.3464) = 2.9998$

$\boxed{V_C = (3.0 \text{ m/s } i)}$

3. The disk rotates with $\omega = 6$ rad/s as shown. Determine the angular velocity of link $AC$ at the instant. Use the coordinates given. **(20 points)**




$V_B = r\omega = .3(6)$

$r \times \omega = 6k \times .3j$

$\vec{V}_B = -3.6\, i$

$\omega_{AC} = \dfrac{(V_B)_n}{r_{B/A}} = \dfrac{0}{.75}$

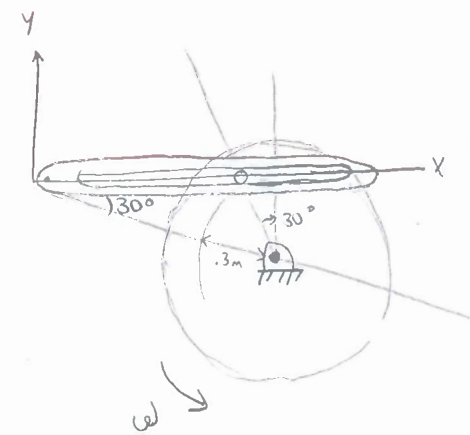
$\boxed{\omega_{AC} = 0}$

$-10$

redo: $V_B = \cancel{V_0}^{\,0} + \vec{\Omega} \times \vec{r}_{B/0} + \cancel{(\vec{V}_{B/A})_{rel}}^{\,0}$

$V_B = 6\vec{k} \times (.3\cos 30\, j - .3\sin 30\, i)$

$= (-1.559, -.9, 0)$




$\omega_{AB} \times r_{B/A} = V_B$

$(\omega_i\ \ \omega_j\ \ \omega_k) \times (.75, 0, 0) = (-1.559, -.9, 0)$

$\begin{vmatrix} i & j & k \\ \omega_i & \omega_j & \omega_k \\ .75 & 0 & 0 \end{vmatrix} = 0\, i \; [-\omega_i(0) + \omega_k(.75)]\, j \; [\omega_i(0) - \omega_j(.75)]\, k = 0$

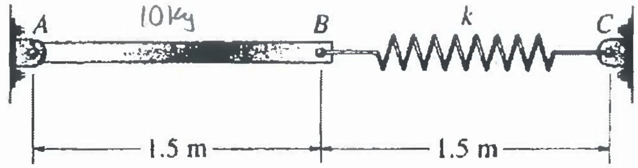
$\omega_i = 0 \quad \omega_j = 0 \quad \omega_k(.75) = -.9$

$\boxed{\omega_k = -1.2 \text{ rad/s}}$

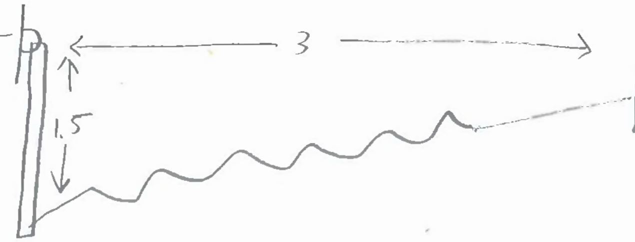
diagram is completely wrong.

$V_G = \omega(.75)$

4. When the slender 10-kg bar is horizontal it is at rest and the spring is unstretched. The spring constant is $k = 30$ N/m. Determine the angular velocity of the bar when it has rotated downward 90°. $I_G = \frac{1}{12}ml^2$. **(20 points)**




datum




$I = I_G + md^2$

$I_A = \frac{1}{12}ml^2 + \frac{1}{2}(10)(.75\omega)(.75)$

$\frac{1}{2} m \quad \frac{L}{2} \quad \frac{1}{2}$

$S = \sqrt{1.5^2 + 3^2} - 1.5 = 1.854$ m

$T_1 + V_1 = T_2 + V_2$

$I_A$

$0 + 0 = mgy + \frac{1}{2}ks^2 + \frac{1}{2}I_G\omega^2$

$0 = (98.1)(-.75) + \frac{1}{2}\,30\,\text{N/m}\,(1.854\text{m}) + \cancel{\frac{1}{2}\frac{1}{12}(10)(1.5)}\ \frac{1}{2}\left[\frac{1}{3}(10)(1.5)^2\right]\omega$ *

$73.575 = 27.81 + 3.75\omega$ *

$45.765 = 3.75\omega$ *

$\boxed{\omega = 12.204}$ rad/s

−4

$P_e = mgh \qquad P_e = \frac{1}{2}ks^2 \qquad K_e = \frac{1}{2}I_G\omega^2$

Rework: $0 = (10)9.81(-.75) + \frac{1}{2}(30)(1.854)^2 + \frac{1}{2}\left[\frac{1}{3}(10)(1.5)^2\right]\omega^2$

$7.5$

$22.01 = 3.75\omega^2$

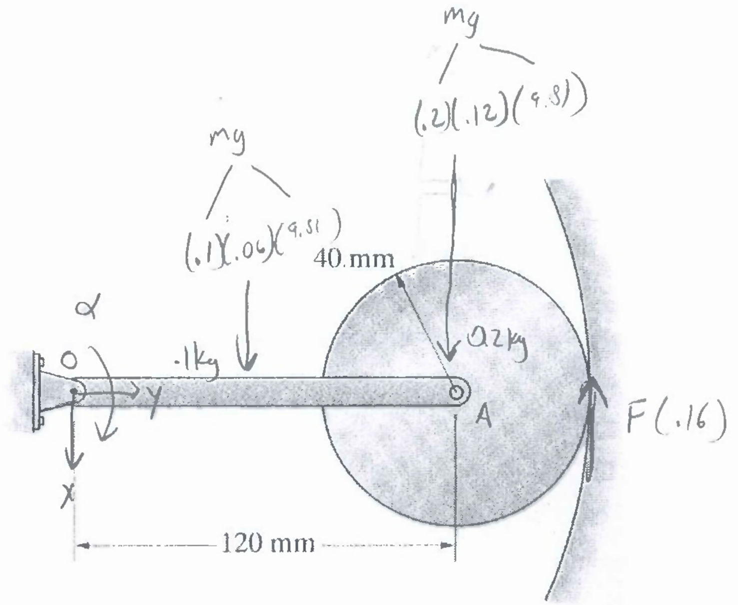
$\boxed{\omega = 2.42 \text{ rad/s}}$

forgot to square omega before

forgot to square "S"

5. The 0.1 kg slender bar and 0.2-kg cylindrical disk are released from rest with the bar horizontal. The disk rolls without slipping on the curved surface. Determine the angular acceleration of the bar. For the bar, $I_G = \frac{1}{12}ml^2$; for the disk, $I_G = \frac{1}{2}mr^2$. **(20 points)**

$\alpha = ?$




$\cancel{m(v_{Gx})_1 + \Sigma \int}$

$+\circlearrowleft \Sigma M_O = I_O \alpha$

$9.81\,[(.1)(.06) + (.2)(.12)] - F(.16) = .00352\,\alpha$

$(.2943) - .16F = .00352\,\alpha$

$I = I_G + md^2$

$= \frac{1}{12}ml^2 + md^2 + \frac{1}{2}mr^2 + md^2$

$= \underbrace{\frac{1}{12}(.1)(.12)^2 + (.1)(.06)^2}_{I\,bar = .00048} + \underbrace{\frac{1}{2}(.2)(.04)^2 + (.2)(.12)^2}_{I_{disk} = .00304}$

$= .00352$ (about pt. O)

$\alpha_O(.120) = a_A \quad (3)$

$\alpha_0 \quad a_A \quad F$ } Simultaneous solve (1)(2)(3)

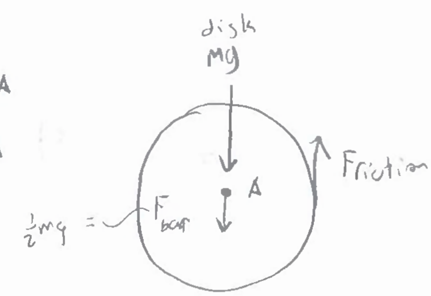
$\alpha = 66.25$

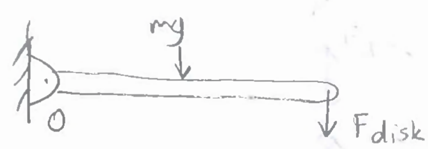
$\Sigma F_{A_x} = m_A a_{A_x}$

$(.2)(9.81) + \frac{1}{2}(.1)(9.81) - F = a_A$

$1.962 + .4905 - F = a_A$

$2.4525 - F = a_A \quad (2)$




6

$+\circlearrowright \Sigma M_O = I_O \alpha_O$

$mg(.06) + (mg)_{disc}.12 - (F)_{disc}(.12) = .00048\,\alpha_0$

$(.1)(9.81) + (.2)(9.81)(.12) - F(.12) = .00048\,\alpha_0$

$1.2164 - .12F = .00048\,\alpha_0 \quad (1)$

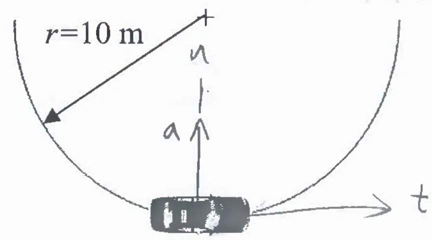
ME/AE 160, Exam One, Spring 2010 2-18-10 95

Name (please print) David Malawey

Malawey

1. The car travels around the circular path with the radius of r = 10 m, at a constant speed v = 8 m/s, determine the magnitude of the acceleration of the car? (13 points) **(18 points)**

$a = ?\ m/s^2$




$r = 10\ m$ $v = 8\ m/s$

$\dot{r} = 0$

$\ddot{r} = 0$

$$a_n = \frac{v^2}{\rho} = \frac{64}{10} = \boxed{6.4\ m/s^2}$$

$a_t = 0$ (const. speed)

---

Rework 5/9/10

$v = \cancel{10}\ 8\ m/s$ const

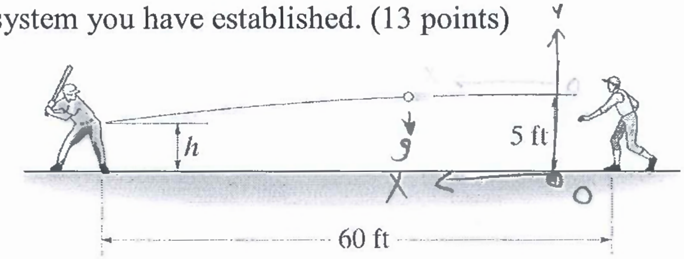
$r = 10\ m$

$$a = \frac{v^2}{r} = \frac{8^2}{10} = 6.4\ m/s \text{ towards ctr}$$

2. The pitcher throws the baseball **horizontally** with a speed of 200 ft/s from a height of 5 ft. If the batter is 60 ft away, determine the time for the ball to arrive at the batter and the height $h$ at which it passes the batter. Include the following in your solution: **(18 points)**

   a. Draw the coordinate system, showing the origin and positive x and y axes in the figure below. (5 points)

   b. Solve the problem with the coordinate system you have established. (13 points)




$V_x = 200\ ft/s$

$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

x: $60 = 0 + 200\,t + \cancel{\frac{1}{2} a_c t^2}$

$\boxed{t = 0.3\ sec}$

$V_{y_0} = 0\ ft/s$

y $h = 5ft + 0\,t + \frac{1}{2}(-32.2\ m/s^2)\,t^2$

$\boxed{h = 3.551\ ft}$

---

Rework 5-10-10

$t = \frac{d\ (m)}{v\ (m/s)} = \frac{60\ ft}{200\ ft/s} = \boxed{0.3\,s}$

$h_2$ $s_2 = (s_0) + (v_0 t) + \frac{1}{2}(a\,t^2)$

$= 5 + 0 + \frac{-32.2\ ft/s^2\,(0.3\ s)^2}{2}$

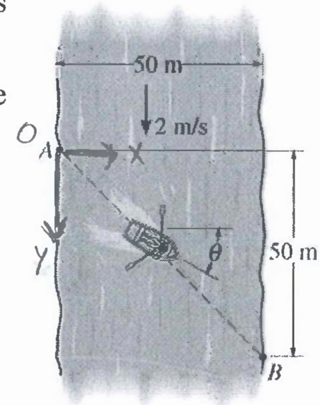
$s_2 = 5 - \frac{2.898}{2}$

$\boxed{h_2 = 3.55\ ft}$

3. A man can row a boat at $\boxed{4\ \text{m/s}}$ in still water. He wishes to cross a 50 m wide river from point $A$ to point $B$, 50 m downstream. If the river flows with a velocity of 2 m/s, determine the speed of the man. **(18 points)**

   a. Draw the coordinate system, showing the positive x and y axes in the figure below. (3 points)

   b. Solve the problem with the coordinate system you have established. (15 points)




$V_B = V_W + V_{B/W}$

$Y:\ V_B = 2\text{m/s} + 4\text{m/s}(\sin\theta)$

$X:\ V_B = 0 + 4\text{m/s}(\cos\theta)$

Since both sides $= 50\text{m}$, $V_{Bx} = V_{By}$

no accel!: $r_x = r_y \Rightarrow V_x = V_y$

$2\text{m/s} + 4\text{m/s}(\sin\theta) = 4\cos\theta$

$1 + 2\sin\theta = 2\cos\theta$

$\theta = 24.295°$

$V_{man} = \sqrt{V_{Bx}^2 + V_{By}^2} = \boxed{5.16\ \text{m/s}}$

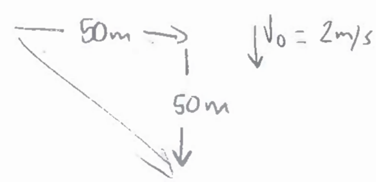
$V_{Bx} = 3.646$

$V_{By} = $ "

---

$V_{rel} = 4\text{m/s}\sin\theta\, y + 4\cos\theta\, x$

$V_b = V_r + V_{b/r} = 2i + 4\sin\theta\, y + 4\cos\theta\, x$




$V_{bx} = V_{by}$ $\quad 2 + 4\sin\theta = 4\cos\theta$

$\frac{1}{2} = \cos\theta - \sin\theta$

$\theta = 24.295°$

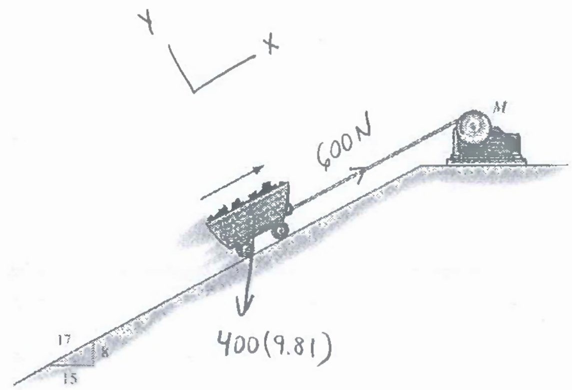
$V = \sqrt{3.645_y + 3.645}$

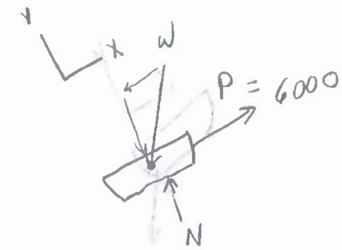
$\boxed{V = 5.16\ \text{m/s}}$

4. The 400 kg mine car is hoisted up the incline. The force in the cable is 6000 N. Determine the acceleration of the car. Neglect the friction between the car and the incline. **(18 points)**

   a. Draw the coordinate system and free body diagram. (5 points)

   b. Solve the problem with the coordinate system you have established. (13 points)




F = 0

$X: \; \Sigma \vec{F} = m\vec{a}$

$P - W\left(\frac{8}{17}\right) = 400(a)$

$6000 - 1847 = 400\, a_x$ $\quad \boxed{a_x = 10.38 \text{ m/s}^2}$

$Y: \; \Sigma \vec{F} = m\vec{a} = 0$ — No accel in y direction

$-W\left(\frac{15}{17}\right) + N = 0$

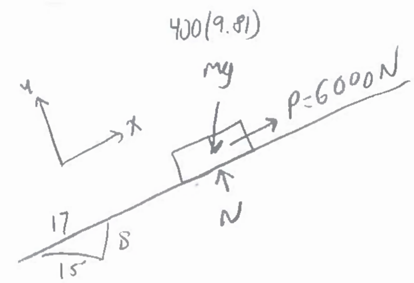
---

rework 5-9-10

$\Sigma F_x = m a_x$

$-400(9.81)(8/17) + 6000 = a_x(400)$

$\boxed{a_x = 10.38 \text{ m/s}^2}$

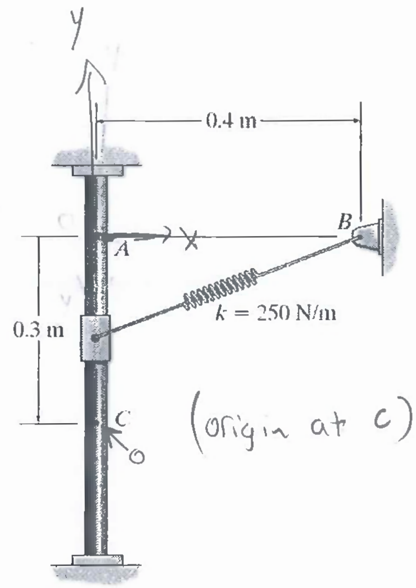


5. The vertical guide is smooth and the 10-kg collar is released from rest at $A$. Determine the speed of the collar when it is at position $C$. The spring has an unstretched length of 400 mm. **(18 points)**

$V_c = ?$

a. Draw the coordinate system, showing the positive x and y axes in the figure. (3 points)

b. Solve the problem with the coordinate system you have established. (15 points)




$\frac{1}{2}mv^2 + mgy + \frac{1}{2}ks^2$

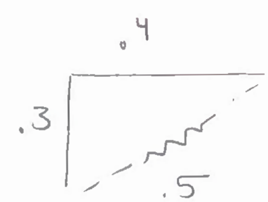
$0 + 10(9.81)(0.3) + \frac{1}{2}(250)\cancel{(.4m - .4m)} = \frac{1}{2}(10)v^2 + 0 + \frac{1}{2}(250)(.5-.4)^2$

$29.43 = 5v^2 + 124.9$

$v^2 = -\frac{95.47}{5}$

$\boxed{V_c = 4.37\ m/s}$ down, & up, later

−2




rework

$T_1 + V_1 = T_2 + V_2$

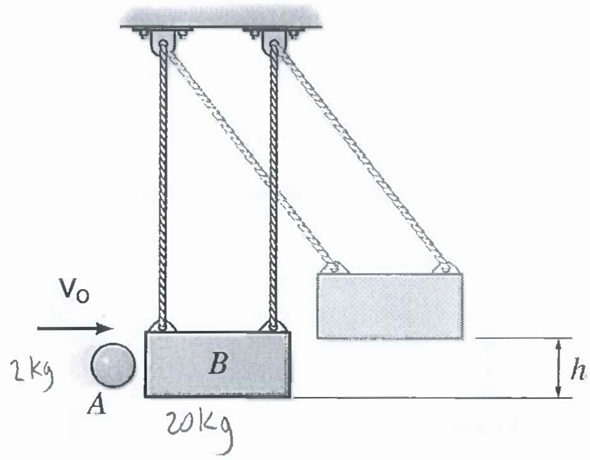
$mgy + \frac{1}{2}mv^2 + \frac{1}{2}ks^2$ " " "

$(0.3)(9.81)(10) + 0 + 0 = 0 + \frac{1}{2}(10)v^2 + \frac{1}{2}(250)(.5-.4)^2$

$29.43 = 5v^2 + 1.25$

$\boxed{V = 2.37\ m/s}$

6. The 2-kg ball is thrown at the suspended 20-kg block with an initial velocity $v_0$. The block is at rest before impact. The coefficient of restitution between the ball and the block is $\boxed{e = 0.8.}$ After impact, if the block can swing by $\boxed{h = 21.8\text{ mm}}$ before it momentarily stops, determine $v_0$. **(10 points)**




21.8mm = .0218m

x: $m_1 v_1 + m_2 v_1 = m_1 v_2 + m_2 v_2$

$2V_0 + 0 = 2V_{A2} + 20 V_{B2}$

$V_0 = V_{A2} + 10 V_{B2}$ (1)

$0.8 = \dfrac{V_{B2} - V_{A2}}{V_{A1} - V_{B1}} = \dfrac{V_{B2} - V_{A2}}{V_0}$

Energy B:

$\frac{1}{2}\cancel{20kg}\, V_{B2}^2 = \cancel{20kg}(9.81)(.0218)$

$V_{B2} = 0.428$ m/s

$V_0 = V_{A2} + 4.28$

$0.8 = \dfrac{.428 - V_{A2}}{V_0}$

$(.8)V_0 = .428 - V_{A2}$

$\boxed{V_0 = 2.616 \text{ m/s}}$

$V_{A2} = -1.664$ m/s

−3

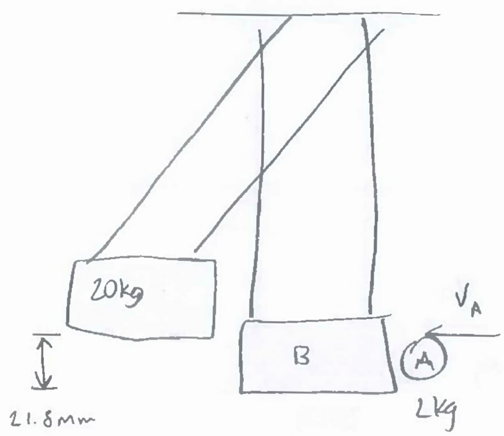
Check:

$2(2.616) = 2(-1.664) + 20(.428)$

$5.232 = -3.328 + 8.56$ ✓

Check ✓

rework prob 6.




impact

$$e = 0.8 = \frac{V_{b2} - V_{A2}}{V_{A1} - \cancel{V_{B1}}^{\,0}} = \frac{.654 - V_{A2}}{V_{A1}}$$

$$m_a V_{A1} + m_b V_{b1} = m_A V_{A2} + m_b V_{b2}$$

$$2(V_{A1}) + 0 = 2 V_{A2} + 20 V_{b2}$$

$$V_{A1} = V_{A2} + 10(.654)$$

Swing: $T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2} m v^2 = mgh$$

$$\frac{1}{2}\cancel{(20)} v^2 = \cancel{20}(9.81)(.0218)$$

$$V_2 = .654 \text{ m/s}$$

$$V_{A1} = V_{A2} + 6.54 \qquad (1)$$

$$.8 V_{A1} = .654 - V_{A2} \qquad (2)$$

$$\boxed{V_{A1} = 4.00 \text{ m/s}}$$

$$V_{A2} = -2.54 \text{ m/s}$$

# Formulas

# FORMULAS FOR DYNAMICS EXAM 1

**Motion:**

* always convert to ft, m, kN, g
* check calculator for degrees/radians

$v = \frac{ds}{dt} \quad a = \frac{dv}{dt} \quad a\,ds = v\,dv$

$s = s_0 + \int_0^t v\,dt$ $\qquad a = a_c \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$v = v_0 + \int_0^t a\,dt \qquad \Rightarrow \qquad v = v_0 + a_c t$

$v^2 = v_0^2 + 2\int_{s_0}^{s} a\,ds \qquad v^2 = v_0^2 + 2a_c(s - s_0)$

**Relative Motion**

$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$

x, y, z

**Radius of curvature**

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

spring force

$F_s = -ks$

$\Delta$ speed $\Leftarrow a_t$

**r-θ-z coords**

$\vec{a} = a_r \vec{u}_r + a_\theta \vec{u}_\theta + a_z \vec{u}_z$

$a_r = \ddot{r} - r\dot{\theta}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$a_z = \ddot{z}$

$a = \sqrt{a_r^2 + a_\theta^2 + a_z^2}$

**Energy** – $1/2\, mv^2 + mgy + \frac{1}{2} ks^2$

motion potential

$T_1 + V_1 = T_2 + V_2$

**n-t-b coords**

$\vec{a} = \dot{v}\vec{u}_t + \frac{v^2}{\rho}\vec{u}_n$

$a_n = \frac{v^2}{\rho} \quad a_t = \dot{v} \quad a_b = 0$

normal vector (+) inside c. of curve

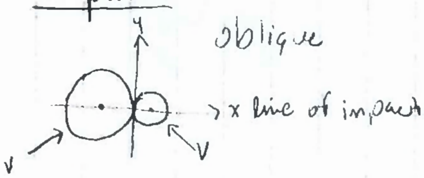
**Efficiency** $\varepsilon = \frac{P_o}{P_i} < 1$ $\qquad 1\,hp = 550$ ft·lb/s

$1\,hp = 745.7$ W $\qquad 1\,W = 1$ N·m/s

$m\vec{v}_1 + \Sigma\int_{t_1}^{t_2} \vec{F}\,dt = m\vec{v}_2$

**Impact:**

oblique




$x: \; m_A(v_{Ax})_1 + m_B(v_{Bx})_1 = m_A(v_{Ax})_2 + m_B(v_{Bx})_2$

$$e = \frac{(v_{Bx})_2 - (v_{Ax})_2}{(v_{Ax})_1 - (v_{Bx})_1}$$

$y: \; (v_{Ay})_2 = (v_{Ay})_1, \; (v_{By})_2 = (v_{By})_1$

$v = \sqrt{v_r^2 + v_\theta^2}$

$v_r = \dot{r}$

$v_\theta = r\dot{\theta}$

**Gravity**

32.2 ft/s²

9.81 m/s²

Review, EXAM 2

2D Kinematics

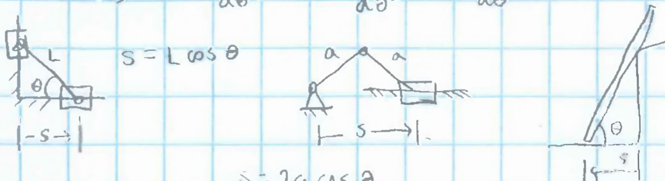
Simple geometry?

Y

Absolute motion

$s = s(\theta) \quad \dot{s} = \frac{ds}{d\theta}\dot{\theta} \quad \ddot{s} = \frac{d^2s}{d\theta^2}\dot{\theta}^2 + \frac{ds}{d\theta}\ddot{\theta}$

$s = L\cos\theta$




$s = 2a\cos\theta$

N

2 points on same body?

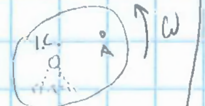
Y

2nd problem

only $\vec{v}$?

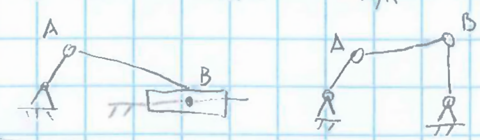
I.C.

$v_A = \omega r_{A/IC}$

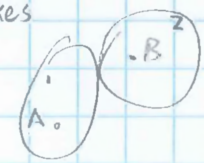



$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$

$a_B = a_A + \vec{\alpha} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A}$




Rotating axes



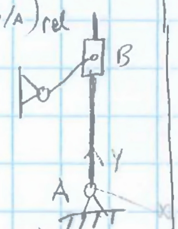

$v_B = v_A + \Omega \times r_{B/A} + (v_{B/A})_{rel}$

sliding contact

$v_B$ = same as above

$= 0 + \omega_{AB} \times r_{B/A}$

$+ (v_{B/A})_{rel}$




— 16 —

$\vec{v} = \vec{\omega} \times \vec{r}$

$\vec{a} = \vec{\alpha} \times \vec{r} - \omega^2 \vec{r}$

abs. motion

$v = \dot{s} \frac{ds}{d\theta} \omega$

$a = \ddot{s} \frac{d^2 s}{d\theta^2} \omega^2 + \frac{d^2 s}{d\theta^2} \alpha$

rolling w/o slip : $v_0 = \omega r$

$\cancel{a = \alpha r}$

rel.

$\vec{v}_B = v_A + \vec{\omega} \times r_{B/A}$

I.C. $v_B = \omega r_{B/IC}$ I.C. is on line through B & $\perp \vec{v}_B$

$a_B = a_A + \alpha \times r_{B/A} - \omega^2 r_{B/A}$

rotating axes: $a_B = a_A + \dot{\Omega} \times r_{B/A} + \Omega \times (\Omega \times r_{B/A}) + 2\Omega \times (v_{B/A})_{xyz} + (a_{B/A})_{xyz}$

— 17 —

$I = I_G + md^2$ d = dist from axis to point

$\Sigma F_x = m(a_G)_x$ rectilinear trans

$\Sigma F_n = m(a_G)_n$ curvilinear trans

$\left.\begin{array}{l} \Sigma F_n = m(a_G)_n = m\omega^2 r_G \\ \Sigma F_t = m(a_G)_t = m\alpha r_G \\ \Sigma M_G = I_G \alpha \text{ or } \Sigma M_0 = I_0 \alpha \end{array}\right\}$ Rotation about fixed Point

$k_G$ = rad. (gyration) $I_G = m k_G^2$

— 18 — Kinetic Energy

$\left.T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2 \right\}$ rotation about fixed Ax.

OR

$T = \frac{1}{2} I_0 \omega^2$

$I_{IC}$ for General plane motion

Work & Force

$V, U = \frac{1}{2} k s^2 + mgy$

$U_M = M(\theta_2 - \theta_1)$

conservation:

$T_1 + V_1 = T_2 + V_2$ (U done by non-cons.) forces

— 19 —

off body $H_A = I_G \omega + (m v_G) d$

on body $H_G = I_G \omega$

$L = m v_G$

$m(v_{Gx})_1 + \Sigma \int_{t_1}^{t_2} F_x \, dt = m(v_{Gx})_2$

$I_G = mk^2$

$I_G \omega_1 + \Sigma \int_{t_1}^{t_2} M_G \, dt = I_G \omega_2$

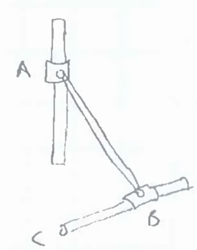
$e = \frac{(v_{Bx})_2 - (v_{Ax})_2}{(v_{Ax})_1 - (v_{Bx})_1}$

$\dot{\vec{\omega}}_2 = (\dot{\vec{\omega}}_2)_{rel} + \vec{\omega}_1 \times \vec{\omega}_2$ (two axes) $\omega = \omega_1 + \omega_2$

$\vec{\alpha} = \dot{\vec{\omega}}_1 + \dot{\vec{\omega}}_2$

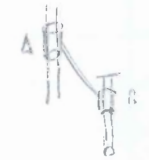
$\vec{a}_p = \vec{\alpha} \times r_{p/0} + \vec{\omega} \times \vec{v}_p$

$\vec{v}_B = v_B \frac{\vec{r}_{C/B}}{|r_{C/B}|}$




$(\vec{v}_B - \vec{v}_A) \cdot r_{B/A} = 0$

$v_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$




Y prod. $\begin{vmatrix} i & j & k \\ \omega_x & \omega_y & \omega_z \\ a & b & c \end{vmatrix}$

## — Final Review — Formulas

### Kinetics

1. principles
   - $\vec{F}, \vec{a} \rightarrow$ EOM's
   - $\vec{F}, \vec{v}, s \longrightarrow$ W & E
   - $\vec{F}, \vec{v}, t$ or impact $\longrightarrow$ M & I

2. EOMS
   - Translational $\Sigma \vec{F} = m a_G \Rightarrow$ 3 eq's
   - Rotational
     - particles: $\Sigma \vec{M} = \vec{0}$
     - 2D Bodies $\Sigma M_{G(0)} = I_{G(0)} \alpha$
     - 3D Bodies $\begin{cases} \text{eq. 21-24} \\ \text{eq. 21-25} \leftarrow \text{principal axes} \end{cases}$

3. W & E

   particles $T = \frac{1}{2} m v^2$ $\qquad \Sigma T_1 + \Sigma U_2 = T_2 \qquad U_M = M(\theta_2 - \theta_1)$

   2-D bodies $T = \frac{1}{2} m V_G^2 + \frac{1}{2} I_G \omega^2 = \frac{1}{2} I_0 \omega^2$

   3-D bodies (principal axes) $T = \frac{1}{2} m V_G^2 + \frac{1}{2}(I_G)_x \omega_x^2 + \frac{1}{2}(I_G)_y \omega_y^2 + \frac{1}{2}(I_G)_z \omega_z^2$

   If only mg & Fs do work $T_1 + V_1 = T_2 + V_2 \qquad W = \int_0^t F\,dt$

   $V = m g y_G + \frac{1}{2} k s^2 \qquad m v_1 + \Sigma \int_0^t F\,dt = m v_2$

4. M & I $\qquad$ H = angular momentum

   $\vec{H}$ · particles : $H_P = \vec{r} \times m \vec{v}$ $\qquad m \bullet \xleftarrow{r} \bullet P$

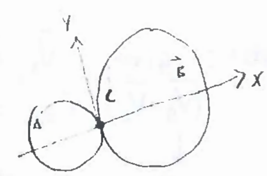
   · 2d Bodies : $H_G = I_G \omega \qquad \vec{H}_P = I_G \omega + \vec{r} \times m \vec{V}_G$ $\qquad$ off body $\quad H_A = I_G \omega + (m v_G) d$

   · 3d Bodies : $H_G = [I_G] \vec{\omega} = \begin{pmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{xz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{pmatrix}_G \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}$

   for principal axes $H_x = I_x \omega_x$, etc.

Impact

$$e = \frac{(V_{C x}^B)_2 - (V_{C x}^A)_2}{(V_{C x}^A)_1 - (V_{C x}^B)_1}$$




$e \geq 0$

Y: no change in V

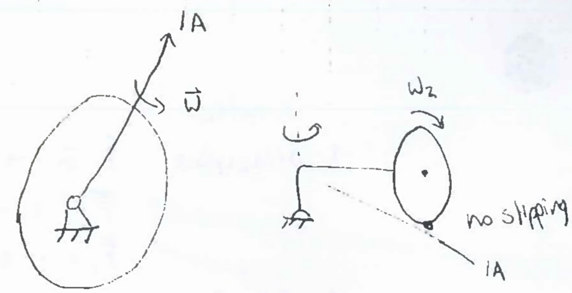
$$\Sigma M_x = I_{xx} \dot{\omega}_x - (I_{yy} - I_{zz}) \omega_y \omega_z - I_{xy}(\dot{\omega}_y - \omega_z \omega_x) - I_{yz}(\omega_y^2 - \omega_z^2) - I_{zx}(\dot{\omega}_z + \omega_x \omega_y)$$

$$\Sigma M_y = I_{yy} \dot{\omega}_y - (I_{zz} - I_{xx}) \omega_z \omega_x - I_{yz}(\dot{\omega}_z - \omega_x \omega_y) - I_{zx}(\omega_z^2 - \omega_x^2) - I_{xy}(\dot{\omega}_x + \omega_y \omega_z)$$

$$\Sigma M_z = I_{zz} \dot{\omega}_z - (I_{xx} - I_{yy}) \omega_x \omega_y \underbrace{- I_{zx}(\dot{\omega}_x - \omega_y \omega_z) - I_{xy}(\omega_x^2 - \omega_y^2) - I_{yz}(\dot{\omega}_y + \omega_z \omega_x)}_{\text{terms drop out if xyz axes are chosen as principal axes of inertia}}$$

(21-24, 25)

# Final Review




3-D Kinematics   NO 20.4

1) 3-D rotation

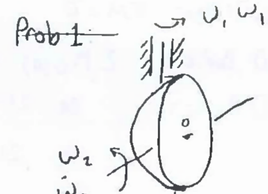
$\vec{\omega}$ is along IA

where $\vec{v} = \vec{0}$

$\vec{v} = \vec{\omega} \times \vec{r}$

$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$

$= \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$



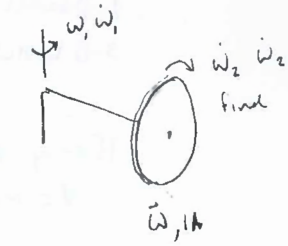

Prob 1  given $\omega_1\ \omega_2\ \ \dot{\omega}_1\ \dot{\omega}_2$

$\omega = \omega_1 + \omega_2$

$\vec{v} = \vec{\omega} \times \vec{r} \qquad a = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$

$\vec{\alpha} = \dot{\vec{\omega}}_1 + \dot{\vec{\omega}}_2 \quad = \dot{\vec{\omega}}_1 + ((\dot{\vec{\omega}}_2)_{rel} + \vec{\omega}_1 \times \vec{\omega}_2)$

$\omega_1$ is used here as $\omega$ about fixed axis,

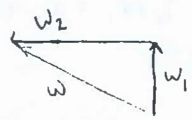



Prob 2:  given $\omega_1\ \dot{\omega}_1$

$\omega = \omega_1 + \omega_2$

M ? ✓ ?

D ✓ ✓ ✓

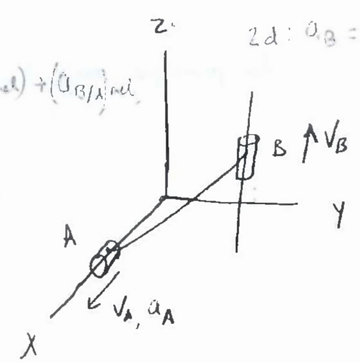



2) 3D general motion

$V_B = \vec{V}_A + \vec{\omega} \times \vec{r}_{B/A} + (V_{B/A})_{rel}$

2d: $a_B = a_A + \alpha \times r_{B/A} - \omega^2 r_{B/A}$

rotating axes  $a_B = \vec{a}_A + \alpha \times \vec{r}_{B/A} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A}) + 2(\omega \times v_{B/A\,rel}) + (a_{B/A})_{rel}$




prob1: given $\vec{V}_A,\ \vec{a}_A$  find $\vec{V}_B,\ \vec{a}_B$

$(\vec{V}_B - \vec{V}_A) \cdot \vec{r}_{B/A} = 0$

↓

$V_B \vec{U}_B \qquad a_B \vec{U}_B \qquad U_B = \dfrac{\vec{r}_{B/A}}{|r_{B/A}|}$

$(\vec{a}_B - \vec{a}_A) \cdot \vec{r}_{B/A} = -\|\vec{V}_B - \vec{V}_A\|^2$

I.C.

$V_B = \omega r_{B/IC}$

prob 2: given $V_A,\ a_A$  Find $V_B,\ a_B,\ \vec{\omega},\ \vec{\alpha}$

$\vec{V}_B = \vec{V}_A + \vec{\omega} \times \vec{r}_{B/A} \Rightarrow$ eqs (1)(2)(3)

$V_B \vec{U}_B \qquad \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}$

$\vec{\omega} \cdot \vec{r}_{B/A} = 0$   eq. (4)

- Find moments of inertia in plane of resulting motion & current motion
- find vector of final rotation, (U of axis)
- angular momentum: (H)

set ~~ang~~ ang. momentum final = ang momentum initial

$H_{os_2} = H_{z_2} \cdot U_{s/o}$

# FORMULAS FOR DYNAMICS

**Motion:** * always convert to ft, m, kN,

$v = \frac{ds}{dt}$ $a = \frac{dv}{dt}$ $a\,ds = v\,dv$

$s = s_0 + \int_0^t v\,dt$ $a = a_c$ $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$v = v_0 + \int_0^t a\,dt$ $\Rightarrow$ $v = v_0 + a_c t$

$v^2 = v_0^2 + 2\int_{s_0}^{s} a\,ds$ $v^2 = v_0 + 2a_c(s - s_0)$

**Relative motion**

$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$

**Radius of curvature**

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

spring force

$F_s = -ks$

$\Delta$ speed $\Leftarrow a_t$

**r-θ-z coords**

$\vec{a} = a_r \vec{u}_r + a_\theta \vec{u}_\theta + a_z \vec{u}_z$

$a_r = \ddot{r} - r\dot{\theta}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$a_z = \ddot{z}$

**Energy** – $1/2\, mv^2 + mgy + \frac{1}{2} ks^2$

motion potential

$T_1 + V_1 = T_2 + V_2$

**n-t-b coords**

$\vec{a} = \dot{v} u_t + \frac{v^2}{\rho} \vec{u}_n$

$a_n = \frac{v^2}{\rho}$ $a_t = \dot{v}$ $a_b = 0$

normal vector (+) inside c. of curve

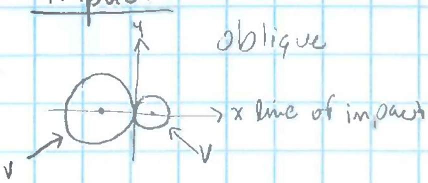
**Efficiency** $\varepsilon = \frac{P_o}{P_i} < 1$ $1hp = 550\ \text{ft}\cdot\text{lb}/s$

$1hp = 745.7\ W$ $1W = 1\ N\cdot m/s$

$m\vec{v}_1 + \Sigma\int_{t_1}^{t_2} \vec{F}\,dt = m\vec{v}_2$

**Impact:**

oblique




$x:\ m_A(v_{Ax})_1 + m_B(v_{Bx})_1 = m_A(v_{Ax})_2 + m_B(v_{Bx})_2$

$$e = \frac{(v_{Bx})_2 - (v_{Ax})_2}{(v_{Ax})_1 - (v_{Bx})_1}$$

$y:\ (v_{Ay})_2 = (v_{Ay})_1,\ (v_{By})_2 = (v_{By})_1$