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ME160 - DYNAMICS

Dynamics

GRAPH NOTEBOOK

100 SHEETS

4X4 QUAD RULED

Norcom, Inc. Griffin, GA 30224
Item # 77105
MADE IN USA

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Help session 7pm-9pm

Date	Day	Lecture	Topic	Reading	Homework
12-Jan	Tue	1	Introduction & Kinematics	12.2-3	12-10,15,20,30,47
14-Jan	Thu	2	Curvilinear motion	12.4-6	12-75,76,87,102,110
19-Jan	Tue	3	Curvilinear motion	12.7-8	12-118,130,132,159,170
21-Jan	Thu	4	Dependent & relative motion	12.9-10	12-195,203,214,220,230
26-Jan	Tue	5	Newton's laws	13.1-4	13-5,20,33,35,43
28-Jan	Thu	6	Equations of motion	13.5-6	13-59,74,82,102,107
2-Feb	Tue	7	Work & energy	14.1-3	~~14-15,23,34,39,41~~
4-Feb	Thu	8	Power & energy conservation	14-4-6	14-61,67,83,99,106
9-Feb	Tue	9	Impulse & momentum	15.1-3	15-5,19,31,42,46
11-Feb	Thu	10	Impact & angular momentum	15.4-7	15-59,69,86,91,107
16-Feb	Tue		ADAMS		
18-Feb	Thu		**Exam 1**		
23-Feb	Tue	11	Translation, rotation & absolute motion	16.1-4	16-3,7,30,42,50,52
25-Feb	Thu	12	Relative velocity	16.5	16-55,67,68,73
2-Mar	Tue	13	Instantaneous center	16.6	16-89,93,98,101,104,107
4-Mar	Thu	14	Relative acceleration	16.7	16-110,114,131 redo these
9-Mar	Tue	15	Rotating axes	16.8	16-139,140,157,159
11-Mar	Thu		**Spring recess**		
16-Mar	Tue	16	Moment of inertia, translation & rotation	17.1-4	17-14,38,55,63,90
18-Mar	Thu	17	General plane motion	17.5	17-103,109,111,114,123
23-Mar	Tue	18	Work & energy	18.1-5	18-15,26,44,56,67
25- Mar	Thu	19	Impulse & momentum	19.1-3	19-10,23,31,34,37
30- Mar	Tue		**Spring break**		
1-Apr	Thu		**Spring break**		
6-Apr	Tue	20	Impact	19.4	19-44,48,54,55
8-Apr	Thu	21	3-D kinematics	20.1-2	20-3,6,10
13-Apr	Tue		**Exam 2**		
15-Apr	Thu	22	3-D kinematics	20.1-2	20-11,13,14
20-Apr	Tue	23	General motion	20.3	20-22,23,26
22-Apr	Thu	24	Rotating axes	20.4	20-42,47,51
27-Apr	Tue	25	3-D kinetics	21.1-3	21-14,27,35,38
29-Apr	Thu	26	3-D kinetics	21.4	21-32,37
4-May	Tue	27	3-D kinetics	21.4	21-44,46,59
6-May	Thu	28	3-D kinetics	21.4	21-50,51

Homework	(.2) (96.84)	
ADAMS	(.1) (99)	
Hour exams	(.2) (95)	
	(.2) (75)	63.268
Final	(.3) ( )	

26.73
30.00

.89 needed

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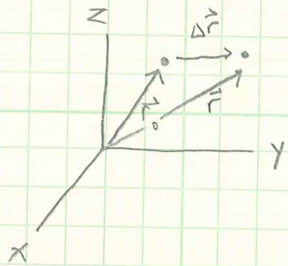
94

12.2) Rectilinear Kinematics

1.) review



position $\vec{r}$

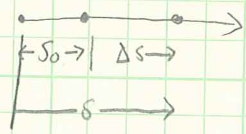
displacement $\Delta\vec{r} = \vec{r} - \vec{r}_0$

Velocity $\vec{v} = \frac{d\Delta\vec{r}}{dt} = \frac{d\vec{r}}{dt}$

accell $\vec{a} = \frac{d\vec{v}}{dt}$

2) rectilinear motion
scalars

position s



displacement $\Delta s = s - s_0$

velocity $v = \frac{ds}{dt}$

acelleration $a = \frac{dv}{dt}$

$$dt = \frac{ds}{v} = \frac{dv}{a}$$

$v = \frac{ds}{dt}$ $\quad$ integrate $\quad$ $s = s_0 + \int_0^t v\,dt$

$a = dv/dt$ $\quad\Longrightarrow\quad$ $v = v_0 + \int_0^t a\,dt$

$v\,dv = a\,ds$ $\quad$ $v^2 = v_0^2 + 2\int_{s_0}^{s} a\,ds$

$\Downarrow$

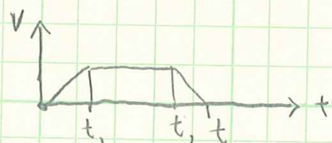
$a = a_c = \text{const}$

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$v = v_0 + a_c t$$

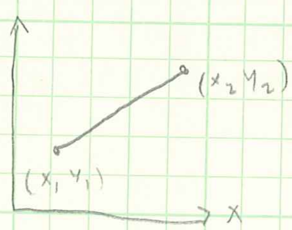
$$v^2 = v_0^2 + 2a_c(s - s_0)$$

3) erratic motion



• use basic equations for each time period

1.



$$y = y_1 + \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Notes 1-1

$v = \frac{ds}{dt}$

$a = \frac{dv}{dt}$

$a\,ds = v\,dv$

$\Rightarrow$

$s = s_0 + \int_0^t v\,dt$

$v = v_0 + \int_0^t a\,dt$

$v^2 = v_0^2 + 2\int_{s_0}^{s} a\,ds$

$a = a_c$ $\Rightarrow$

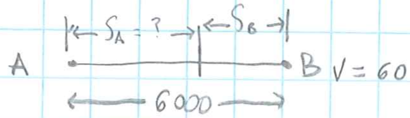
1) $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

2) $v = v_0 + a_c t$

3) $v^2 = v_0 + 2a_c(s - s_0)$

Problem

12-10) ~~Notes~~



A has 2 periods: ① $v_0 = 0$ $V = 80$ $a = 6$

② $V = 80$

$S_{A_1} + S_{A_2} + S_B = 6000$

(1) $S_B = V_B(t) = 60t$

(2) $S_{A_1} \Leftarrow$ Eq. 3

(3) S_{A_2} Eq (1) $\Rightarrow$ $S_{A_2} = S_{A_1} + 80(t - t_1)$

find t_1, use eq. 2

Problem

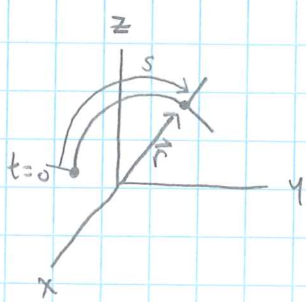
12-20) $v = -4s^2$ $s_0 = 2$ find $v(t)$, $a(t)$

$v = ds\ dt$ $-4s^2 = ds/dt$

$dt = -\frac{ds}{4s^2} \Rightarrow \int_0^t dt = -\int_2^s \frac{ds}{4s^2} \Rightarrow s(t) \Rightarrow v(t) = \frac{ds(t)}{dt}$

3D curve for path

<u>12.4)</u> curvilinear motion



velocity is tangent to the curve

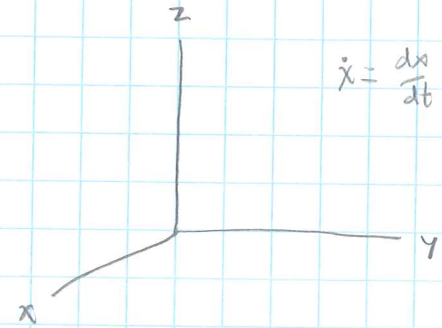
$\bar{v} = \frac{d\vec{r}}{dt}$

$v = \frac{ds}{dt}$ $\bar{a} = \frac{dv}{dt}$

coord systems

- x - y - z
- r - θ - z
- n - t - b

12.5 Cartesian coord's.



$\dot{x} = \frac{dx}{dt}$

$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

$\vec{v} = \frac{d\vec{r}}{dt} = \dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k}$

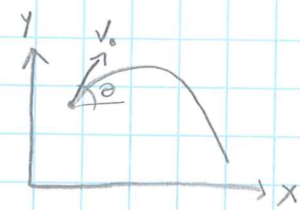
$\vec{a} = \ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\hat{k}$

$r = \sqrt{x^2+y^2+z^2}$

$v = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$

$a = \sqrt{\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2}$

12.6 Motion of projectile



X: $a_x = 0$

$v_x = \text{const.}$

$x = x_0 + v_0 \cos\theta\, t$

$v_x = v_0 \cos\theta$

y: $a_y = -g = -9.81\ m/s^2 = -32.2\ ft/s^2$

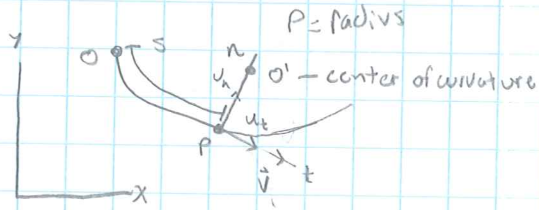
$y = y_0 + v_0 \sin\theta\, t - \frac{1}{2} g t^2$

$v_y = v_0 \sin\theta - gt$

$v_y^2 = (v_0 \sin\theta)^2 - 2g(y-y_0)$

12.7 Normal and tangential coordinates review

$\vec{v}$ is tangent to s-path $v = \frac{ds}{dt}$



2-D MOTION

1. 2-D t - tangential $\vec{u}_t$
   n - normal $\vec{u}_n$

$\vec{v} = \dot{s}\vec{u}_t$

$\vec{a} = \ddot{s}\vec{u}_t + \dot{s}\frac{d\vec{u}_t}{dt}$

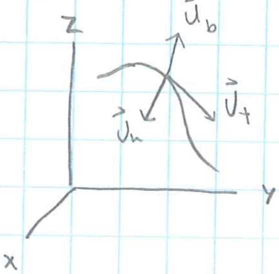
$= a_t\vec{u}_t + a_n\vec{u}_n$

$= \dot{v}\vec{u}_t + \frac{v^2}{\rho}\vec{u}_n$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

3D Motion

2. 3-D binormal $u_b = \vec{u}_t \times \vec{u}_n$
   no motion in u_b direction

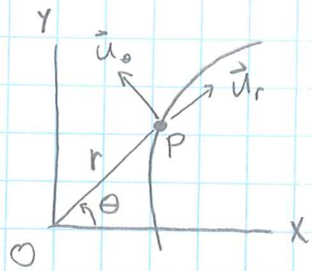


12.8

1. Polar (2D)

r – radial, $\vec{u}_r$

θ – transverse coordinate, $+\circlearrowleft$ u_θ

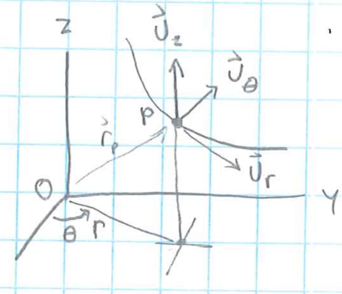


$\vec{r} = r\vec{u}_r$

$\vec{v} = \frac{d\vec{r}}{dt} = \dot{r}\vec{u}_r + r\frac{d\vec{u}_r}{dt}$

$\vec{v} = \dot{r}\vec{u}_r + r\dot{\theta}\vec{u}_\theta$

$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{u}_\theta$



$\vec{r}_p = r\vec{u}_r + z\vec{u}_z$

$\vec{v} = \dot{r}\vec{u}_r + r\dot{\theta}\vec{u}_\theta + \dot{z}\vec{u}_z$

$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{u}_\theta + \ddot{z}\vec{u}_z$

12-130

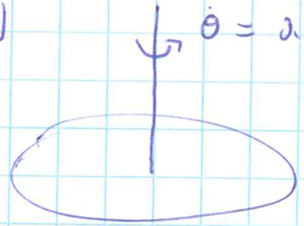
find $a_B \leftarrow \begin{cases} a_t \\ a_n = \frac{v_B^2}{\rho} \end{cases}$

v_B ? $v_B^2 = v_0^2 + 2\int_0^{40} \underbrace{(6 - .06s)}_{a_t}\, ds$

in t

12-170

$\dot{\theta} = 0.2$ rad/s



$\dot{\theta} = 0.2 =$ const.

$\ddot{r} = 0.5$ m/s² = const

$r_0 = 0$ $v_0 = 0$

$s = s_0 + v_0 t + \frac{1}{2} \ldots$

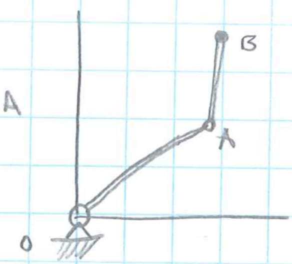
$r = r_0 + v_0$

Notes 1-21

12.1 Relative Motion Analysis

- using translating axes

'robotic arm'



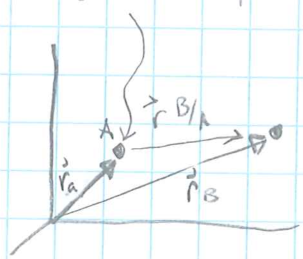
motion of B = motion of A + motion of B w.r.t. A

$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$

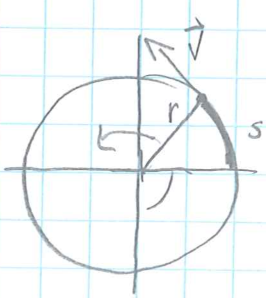
$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$

$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

new axes set up at A, x', y', z'



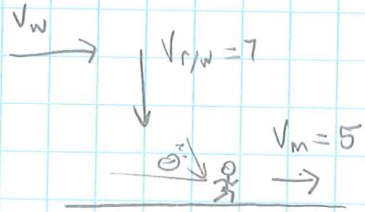
review, circ. motion



$S = r\theta$

$V = \omega r$

Notes 1-21 continued



12-230 given: $V_{r/w} = 7$

Find θ ← $V_{r/m} = V_r - V_m$

$V_r = V_w + V_{r/w}$

$V_r = V_m + V_{r/m}$

$V_W + V_{r/w} = V_m + V_{r/m}$

?? 7 5 $V_{r/m}$

D → ↓ → θ

* Review *

$a\,ds = v\,dv$

$a = \frac{dv}{dt}$

$v = \frac{ds}{dt}$

$S = s_0 + v_0 t + \frac{1}{2} a_c t^2$

Notes 1-26-10

Kinetics

13.1 - 13.3 — Newtons second law

$\boxed{\Sigma \vec{F} = m\vec{a}}$

For a system: $\vec{F}_i + \vec{f}_i = m_i \vec{a}_i$

$\Sigma \vec{F}_i + \underbrace{\Sigma f_i}_{\vec{0}} = \Sigma m_i \vec{a}_i$ $\quad m_i \bullet$ — F_i : external, f_i : internal

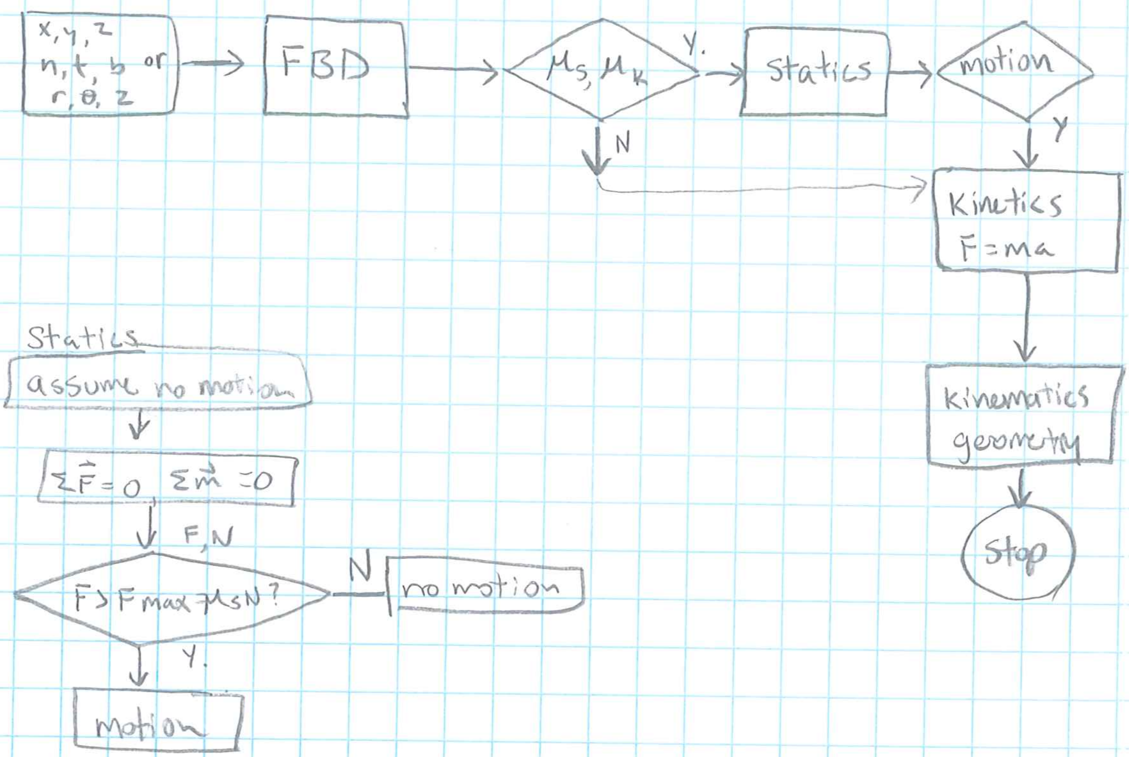
$\Sigma \vec{F}_i = m_i \vec{a}_i$

G: center of mass $\quad \vec{r}_A = \dfrac{\Sigma m_i \vec{r}_i}{\Sigma m_i} = \dfrac{\Sigma m_i \vec{r}_i}{m} \quad m\vec{r}_G = \Sigma m_i \vec{r}_i$

$m\vec{a}_G = \Sigma m_i \vec{a}_i = \Sigma \vec{F}_i \qquad \Sigma F_i = m\vec{a}_G$

13.4 Cartesian system

$\Sigma \vec{F} = m\vec{a} \Rightarrow$ ① $\Sigma F_x = ma_x$, ② $\Sigma F_y = ma_y$, ③ $\Sigma F_z = ma_z$



Notes 1-28

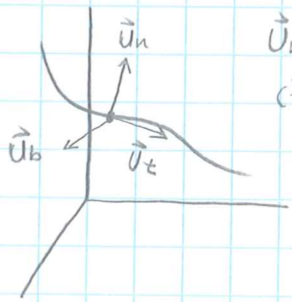
13.5) n-t-b coords 1) review

$\vec{v} = v\vec{u}_t$

$\vec{a} = \dot{v}\vec{u}_t = \frac{v^2}{\rho}\vec{u}_n$

$a_n = \frac{v^2}{\rho}$ $a_t = \dot{v}$

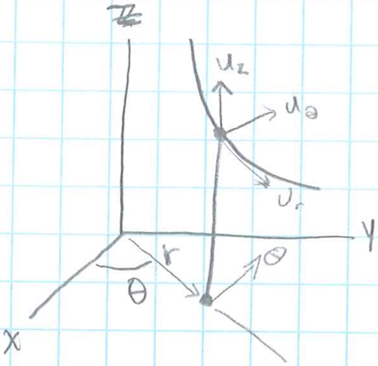
$a_b = 0$



$\vec{U}_n$ towards ctr. of curvature

2) EOM

$$\Sigma\vec{F} = m\vec{a} \Rightarrow \begin{cases} \Sigma F_t = ma_t = m\dot{v} \\ \Sigma F_n = ma_n = m\frac{v^2}{\rho} \\ \Sigma F_b = 0 \end{cases}$$



13.6) r-θ-z coords

1. review

$\vec{a} = a_r\vec{u}_r + a_\theta\vec{u}_\theta + a_z\vec{u}_z$

$a_r = \ddot{r} - r\dot{\theta}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$a_z = \ddot{z}$

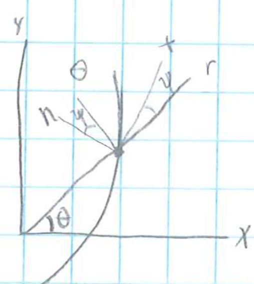
θ vector is perp. to r vector

$\Sigma F_r = m(\ddot{r} - r\dot{\theta}^2)$

$\Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

$\Sigma F_z = m(\ddot{z})$

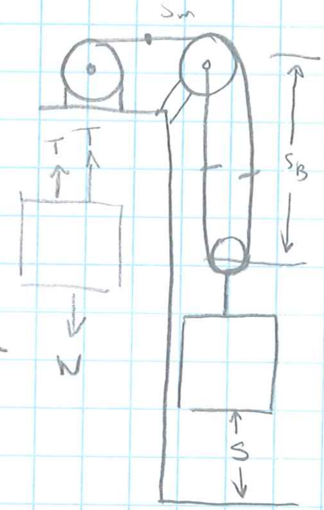
$$\tan\psi = \frac{r}{\frac{dr}{d\theta}}$$



Notes for 2-4-10

2. given

$T = 30\,lb$
$W = 50\,lb$
$S = 10\,ft$
$V_0 = 0$
$S_0 = 0$
$P_i = ?$
$\varepsilon = 1$, ideal
$P_o = P_i$



$\Sigma F_y = ma_y$

$2T - W = ma_y \Rightarrow 2(30) - 50 = \frac{50}{32.2} a$

$a = 6.44\ ft/s^2$

$V^2 = V_0^2 + 2a(S - S_0)$

$V_B = 11.349\ ft/s$

$2S_B + S_m = const$ $\quad V_B$ is negative

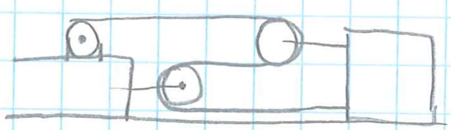
$2V_B + V_m = 0 \qquad V_m = -2V_B = 22.698\ ft/s$

$(P_i)\ ideal = TV_m = 30(22.698) = 680.94\ ft.lb/s$

$P_i = \frac{P(ideal)}{\varepsilon} = 895.97\ ft\,lb/s = 1.63\,hp$

14-61) $P_i = 2.05\,hp$

14.67)



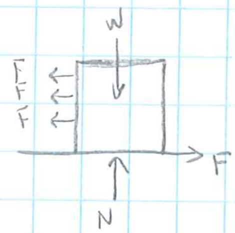
$m, \mu_s, \mu_k \quad F = 8t^2 + 20$

Find P_o when $t = 5s$

statics: $N = W \leftarrow y$

$x: 3F - \mu_s N = 0$

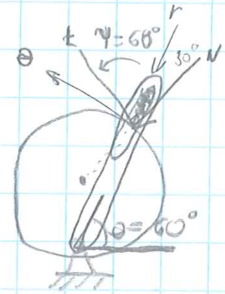
$\Downarrow t = 3.9867s$



EOM $x: 3F - \mu_k N = ma \Rightarrow a(t) \Rightarrow V(t) \int_{3.9867}$

13-107 $r(\theta)$, $\dot{\theta}$, $\ddot{\theta}$, θ given

Find N, F



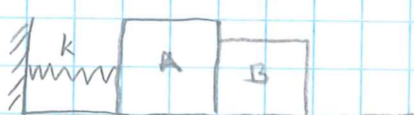
$\tan\psi = \frac{r}{dr/d\theta} = \frac{.6\sin\theta}{.6\cos\theta} = \tan\theta$

$\psi = \theta = 60°$

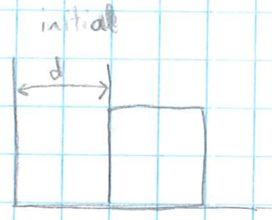
$\Sigma F_r = ma_r \quad N - F\sin 30° = ma_r$

13-43

given d



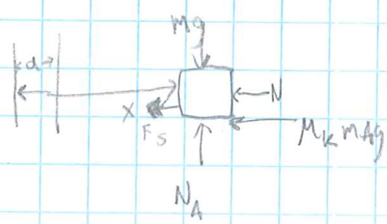
show $d > 2\mu_k g(m_A + m_B)$ for separation



A: $\Sigma F_x = ma$

$-k(x-d) - N - \mu_k m_A g$

$= m_A a$



B: $\Sigma_o M$ a N

$N = 0$, $V > 0$

$\Downarrow$

x

$\int a \Rightarrow V > 0$

Notes

14.1-3 Principle of work & Energy

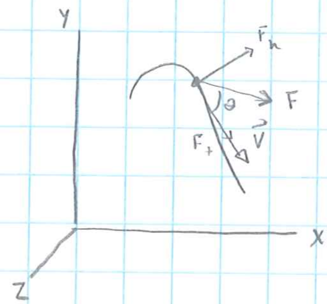
$\Sigma F_t = m a_t$

$a_t ds = v dv \quad v = \frac{ds}{dt} \quad a_t = \frac{dv}{dt} \Rightarrow dt = \frac{ds}{v} = \frac{dv}{a_t}$

$\Sigma F_t = m \frac{v dv}{ds} \qquad \Sigma F_t ds = m v dv \qquad \boxed{\Sigma \int_{s_1}^{s_2} F_t ds = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2}$

• new concepts

1 work $U_{12} = \int_1^2 F_t ds$
$= \int_1^2 F \cos\theta \, ds$
$= \int_1^2 \vec{F} \cdot d\vec{r}$

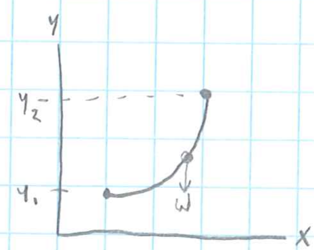


2 kinetic energy $T = \frac{1}{2} m v^2$

W & E

$\boxed{\frac{1}{2} m v_1^2 + \Sigma U_{12} = \frac{1}{2} m v_2^2}$

1) calculate $U_{12} = -\int_{y_1}^{y_2} w \, dy$
weight ①
$= -w(y_2 - y_1)$

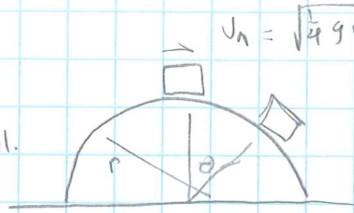


② $U_{12} = \int_{x_1}^{x_2} F_s dx$
$= \int_{x_1}^{x_2} kx \, dx$
$= \frac{1}{2} k (x_2^2 - x_1^2)$

2. when $\vec{v}, \vec{F}, s$

3. how many unknowns? 1

4. System? $\Sigma \frac{1}{2} m_i (v_i)_1^2 + \Sigma U_{12} = \Sigma \frac{1}{2} m_i (v_i)_2^2$



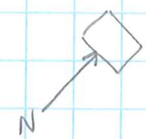
given V_A Find θ when the box leaves cyl.

$V_B^2 = r g \cos\theta$

$\frac{1}{2} m V_A^2 + \Sigma U_{12} = \frac{1}{2} m V_B^2$

$\frac{1}{2} m (\frac{1}{4} g r) + wh = \frac{1}{2} m (r g \cos\theta)$

$h \downarrow (r - r\cos\theta)$



$\Sigma F_n = m a_n$

$mg\cos\theta - \cancel{N}^{0} = m \frac{v^2}{r}$

14.5-6 Notes — 2-4-10 —

$T_1 + \Sigma {}_1U_2 = T_2$

$T = \frac{1}{2} m v^2$

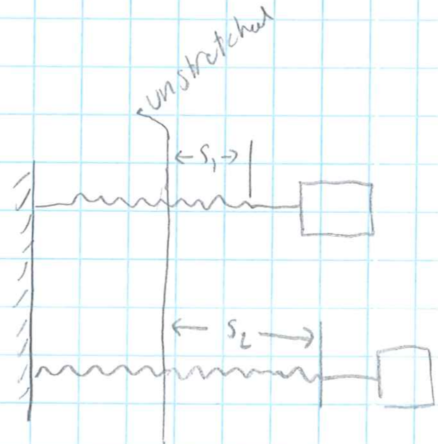
constant F: ${}_1U_2 = F_t (s_2 - s_1)$

1. Potential Energy (P.E)

(1) Gravitational P.E. $V_g = mgy$

$U_{12} = mg(y_2 - y_1)$

(2) Elastic P.E $V_e = \frac{1}{2} k s^2$



$U_{12} = -\frac{1}{2} k (s_2^2 - s_1^2)$

$= \frac{1}{2} k s_1^2 - \frac{1}{2} k s_2^2$

$= (V_e)_1 - (V_e)_2$

(3) conservative forces mg & F_s U_{12} is independent of the path of a particle

2. Conservation of (mechanical) energy

NOTES 2-4-10

2. Conservation of energy

$$T_1 + \sum U_{12}^{cons.} + \sum U_{12}^{non\text{-}con} = T_2$$

If.... Proof here....

$$\boxed{T_1 + V_1 = T_2 + V_2}$$ if only mg & F_s do work

T ↑ motion, V ↑ potential

$$Energy = \frac{1}{2}mv^2 + mgy + \frac{1}{2}ks^2$$

14.4) Power

$$P = \frac{du}{dt} = \frac{d\vec{F}\cdot d\vec{r}}{dt} = \vec{F}\cdot\vec{v}$$

F = const FV

SI: watt = 1 J/s

English: 1 hp = 550 ft·lb/s

1 hp = 746 w

Efficiency

$$P_i \longrightarrow \boxed{\text{System}} \longrightarrow P_o \qquad P_o < P_i$$

$$\varepsilon = \frac{P_o}{P_i} < 1$$

$\boxed{\text{working load F, speed V}}$ ↓ $\boxed{P_o = FV}$ ↓ $\boxed{P_i = P_o/\varepsilon}$ ↓ $\boxed{\text{Select a motor}}$

If no energy loss

$\varepsilon = 1 \qquad (P_i)_{ideal} = P_o$

$$\varepsilon = \frac{(P_i)_{ideal}}{P_i}$$

Mustang

300 3,270 lb

Miata

130 2293 lb

Notes 2-9-10

15 1-3

1. $\Sigma F = m\vec{a}$ replace $\vec{a}$ with $\vec{v}$ & t

$\Sigma F = m\frac{d\vec{v}}{dt}$ $\quad \Sigma\vec{F}\,dt = m\,dv$

$\Sigma\int_{t_1}^{t_2}\vec{F}\,dt = m\vec{V}_2 - m\vec{V}_1$

~ New concepts: ~

(1) Impulse $\vec{I} = \int_{t_1}^{t_2} \vec{F}\,dt \overset{\vec{F}=const}{=\!=\!=} \vec{F}(t_2 - t_1)$

(2) Linear momentum $\vec{L} = m\vec{V}$

~ New Principle: ~

Q's:

$$\boxed{m\vec{V}_1 + \Sigma\int_{t_1}^{t_2}\vec{F}\,dt = m\vec{V}_2}$$

(1) when do we use it? $\vec{F}, \vec{V}, t$

(2) how many eq's? 3 x: $m\vec{V}_{x1} + \Sigma\int \vec{F}_x\,dt = m\vec{V}_{x2}$

y. - - -

z. - - -

(3) System? yes $\Sigma m_i(\vec{V}_i) + \Sigma\int_{t_1}^{t_2}\vec{F}\,dt = \Sigma m_i(\vec{V}_{2i})$

(4) cons. of momentum If: $\Sigma\int_{t_1}^{t_2}\vec{F}\,dt = 0$

Then: $\Sigma m_i(\vec{V}_i)_1 = \Sigma m_i(\vec{V}_i)_2$

Center of system G $\quad \Sigma m_i\,\vec{r}_G = \Sigma m_i\vec{r}_i$

$\Sigma m_i\,\vec{V}_G = \Sigma m_i\vec{V}_i = const$

$V_G = constant \quad (V_G)_1 = (V_G)_2$

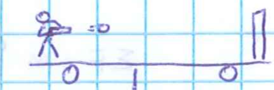
Notes 2-9-10

15.5

$(V_A)_2 = 1.27 \text{ m/s} \uparrow$

$(V_B)_2 = 1.27 \text{ m/s} \downarrow$

46) $W_m = 150 \text{ lb} \quad W_c = 600 \text{ lb}$



$W_B = .2 \text{ lb}$

$V_{B/c} = 300 \text{ ft/s}$

1) Before firing

2) after firing

3) bullet imbedded

(1) $(V_c)_2$? $(V_c)_3$?

$\Sigma m_i (V_i)_1 = \Sigma m_i (V_i)_2$

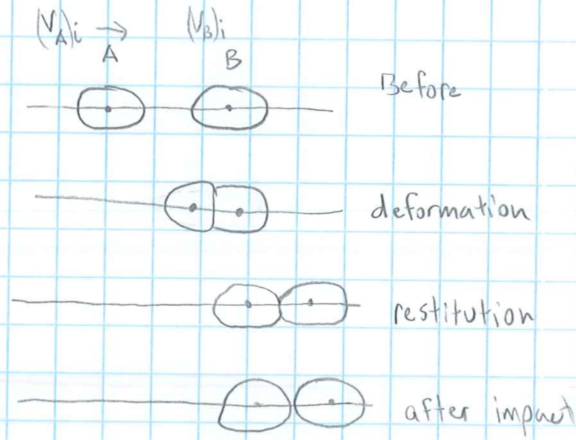
$0 + 0 = (m_B v_B + m_m m_c)(V_c)_2$ (1)

$V_{B/c} = \vec{V}_B - \vec{V}_c \qquad x: 3000 = (V_B)_2 - (V_c)_2$

$x: 3000 = (V_B)_2 - (V_c)_2$

(2) $\Sigma m_i V_i)_2$

15.4 Impact 1) central impact



$$\Sigma m_i (\vec{V_i})_1 + \Sigma \int_{t_1}^{t_2} \vec{F} \, t \, dt = \Sigma m (\vec{V_i})_2$$

(1) $m_A (V_A)_1 + m_B (V_B)_1 = m_A (V_A)_2 + m_B (V_B)_2$

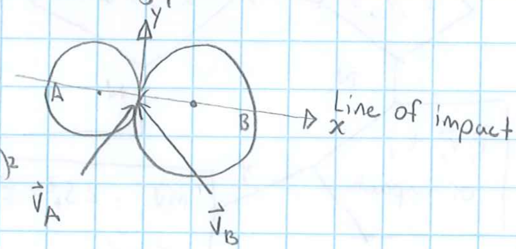
– the coefficient of restitution

(2) $$e = \frac{(V_B)_2 - (V_A)_2}{(V_A)_1 - (V_B)_1}$$

special case: $e = 1$ elastic impact (NO ENERGY LOSS]

$e = 0$ plastic impact [max energy loss]

—— oblique impact ——



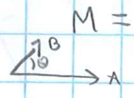
x: $m_A (V_{Ax})_1 + (m_B V_{Bx})_1 = (m_A V_{Ax})_2 + (m_B V_{Bx})_2$

$$e = \frac{(V_{Bx})_2 - (V_{Ax})_2}{(V_{Ax})_1 - (V_{Bx})_1}$$

y: $(V_{Ay})_2 = (V_{Ay})_1$, $(V_{By})_2 = (V_{By})_1$

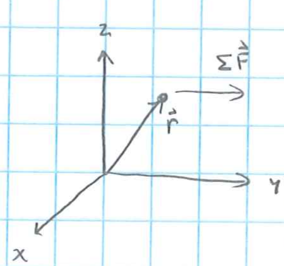
Review 1. Moment of $\vec{F}$ $M = r \times \vec{F}$

2. $\vec{C} = \vec{A} \times \vec{B}$



$C = AB \sin\theta$

15.5-7) Angular momentum $\Sigma \vec{F} = m\vec{a}$



$$\Sigma \vec{M}_o = \vec{r} \times \Sigma \vec{F} = \vec{r} \times m\vec{a}$$
$$= r \times (m \frac{dv}{dt})$$

$$\frac{d}{dt}(\vec{r} \times m\vec{v}) = \underbrace{\frac{d\vec{r}}{dt}}_{\vec{v}} \times m\vec{v} + \vec{r} \times m \frac{dv}{dt}$$

$\vec{v} \times m\vec{v} = 0$

$$\Sigma \vec{M}_o = \frac{d}{dt}(\vec{r} \times m\vec{v})$$

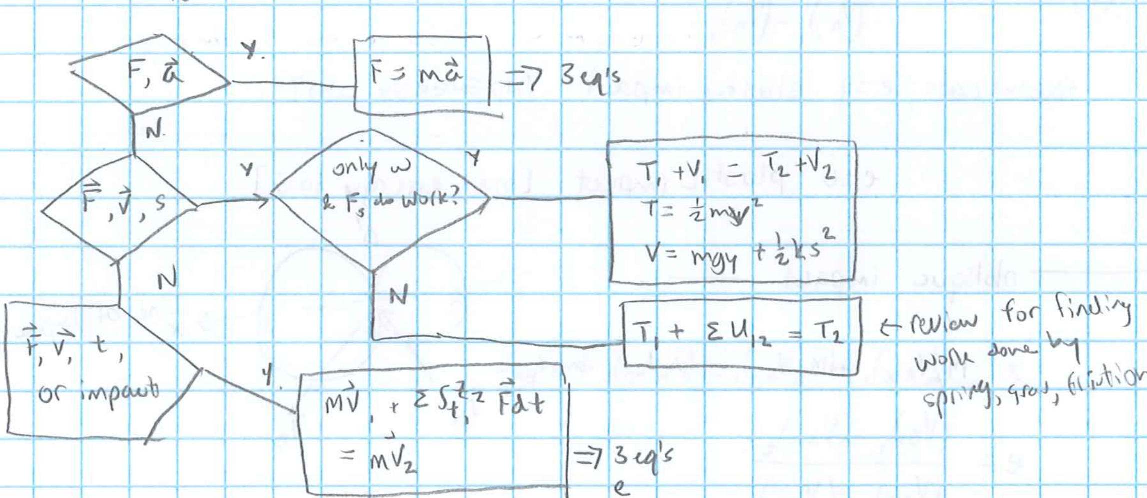
$$\Sigma M_o dt = d(\vec{r} \times m\vec{v}) \qquad \Sigma \int_{t_1}^{t_2} M_o \, dt = \vec{r}_2 \times m\vec{V}_2 - \vec{r}_1 \times m\vec{V}_1$$

Angular momentum $\vec{L} = \vec{r} \times m\vec{v}$ Angular Impulse $\int_{t_1}^{t_2} \vec{M} dt$

$$\vec{r}_1 \times m\vec{V}_1 + \Sigma \int_1^2 M dt = \vec{r}_2 \times m\vec{V}_2$$

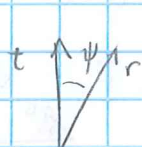
Exam 1:

2. Kinetics



$F = \mu_k N$

$F_{max} = \mu_s N$



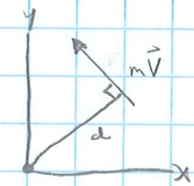
Notes 2-11

Angular momentum, 3D

$$\vec{L} = \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ mV_x & mV_y & mV_z \end{vmatrix}$$

2D

$L = d\,mV$



Notes 2-16

Exam 1: chap 12-15

- 6 problems
  - 3 kinetic
  - 3 kinematic (1 for each principal)

1) Kinetics Basic eq.

$v = \frac{ds}{dt}$ $\quad s = s_0 + \int_0^t v\,dt$ $\quad a = \text{const}$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$a = \frac{dv}{dt}$ $\Rightarrow$ $v = v_0 + \int_0^t a\,dt$ $\Rightarrow$ $v = v_0 + a_c t$

$a\,ds = v\,dv$ $\quad v^2 = v_0^2 + \int_{s_0}^{s} a\,ds$ $\quad v^2 = v_0^2 + 2a_c(s - s_0)$

$\Downarrow$ projectile motion

$x: V_x = V_{x_0}$

$x = x_0 + (V_x)_0 t$

$y = y_0 + (V_y)_0 t - \frac{1}{2} g t^2$

$V_y^2 = V_{0y}^2 - 2g(y - y_0)$

(2) coord. systems $x-y-z$ $\quad n-t-b$

$\vec{a} = a_t \vec{u}_t + a_n \vec{u}_n$

$a_t = \frac{dV}{dt}$ $\quad a_n = \frac{V^2}{\rho}$

$r-\theta-z$ $\quad \vec{r} =$

$\vec{v} =$

$\vec{a} =$

3) abs. motion analysis

a) S_A, S_B, . . .

b) relate S_A, S_B w/ constant length

c) time derivatives

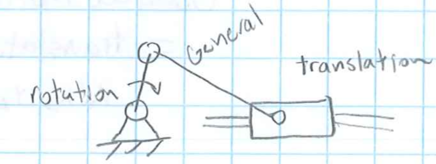
4) relative motion:

$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$

$\vec{a}_B = \ldots$

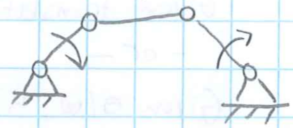
16.1 -3 Rigid body motion

4-bar linkage



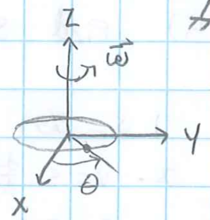
Rigid bodies

- 2D
- 3D
  - Translation,
  - Rotation
  - general



3) Rotation

$$\omega = \frac{d\theta}{dt}$$



$$\alpha = \frac{d\omega}{dt}$$

$\alpha \, d\theta = \omega \, d\omega \Rightarrow$

$$\theta = \theta_0 + \int_0^t \omega \, dt$$
$$\omega = \omega_0 + \int_0^t \alpha \, dt$$
$$\omega^2 = \omega_0^2 + 2\int_{\theta_1}^{\theta_2} \alpha \, d\theta$$

α is const $\Rightarrow$

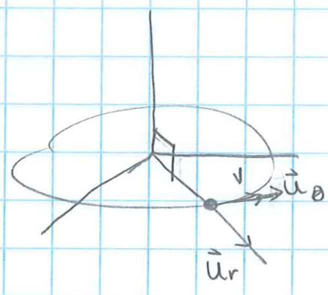
$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$$
$$\omega = \omega_0 + \alpha_c t$$
$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta_2 - \theta_1)$$

$$V = \dot{r}\vec{u}_r + r\dot{\theta}\vec{u}_\theta$$
$$= r\omega \vec{u}_\theta$$

$$\boxed{V = \vec{\omega} \times \vec{r}}$$

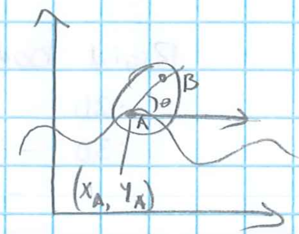
MEMORIZE

$$\boxed{\vec{a} = \vec{\alpha} \times \vec{r} - \omega^2 \vec{r}}$$



16.4 Absolute motion Analysis

General motion
= translation + rotation
$A(x_A, y_A)$ $\qquad \theta$



Given translation, $s(v, a)$ find rotation $\theta(\omega, \alpha)$
— or —
Given $\theta(\omega, \alpha)$ Find $s(v, a)$

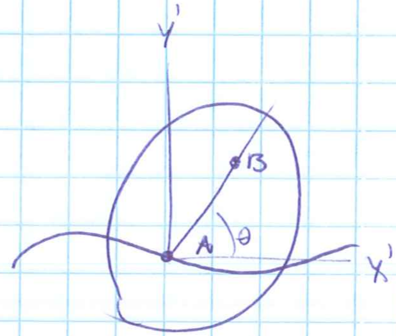
Solution : 1. $s = s(\theta)$ or $\theta = \theta(s)$

2. $v = \dot{s} \frac{ds}{d\theta}\omega$

$a = \ddot{s} \frac{d^2s}{d\theta^2}\omega^2 + \frac{d^2s}{d\theta^2}\alpha$

16.5 Relative Velocity Analasys

1. Review $\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$



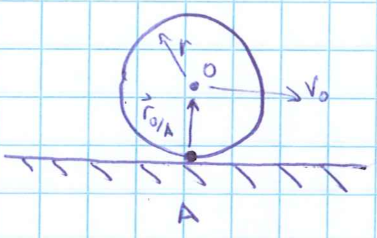
rotation $\vec{v} = \omega \times r$

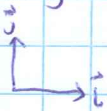
$$\vec{V}_B = V_A + \vec{\omega} \times \vec{r}_{B/A}$$

$\omega r_{B/A}$

$I r_{B/A}$

Specal case: rolling without slipping

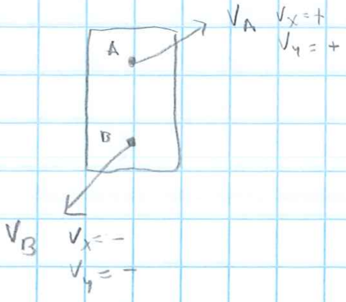




$$V_o = V_A + \vec{\omega} \times \vec{r}_{o/A}$$
$$= \vec{0} + (-\omega)\vec{k} \times (\vec{r}\,\vec{j})$$
$$= \boxed{\omega \vec{r}\,\vec{i}}$$

Notes
3-2-10

16.6 Instantaneous Center of zero velocity



1) review rotation: $\vec{V} = \vec{\omega} \times \vec{r}$

$V_B = V_A + V_{B/A}$

2) IC if $A = IC$, $\vec{V}_A = \vec{V}_{IC} = 0$

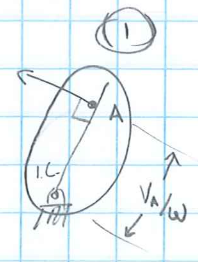
$\vec{V}_B = \vec{\omega} \times \vec{r}_{B/IC}$

$V_B = \omega r_{B/IC}$

IC is on the line through B & $\perp$ $\vec{V}_B$

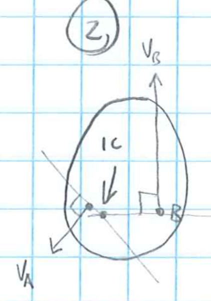
3) Location of the IC

1) given V_A and ω



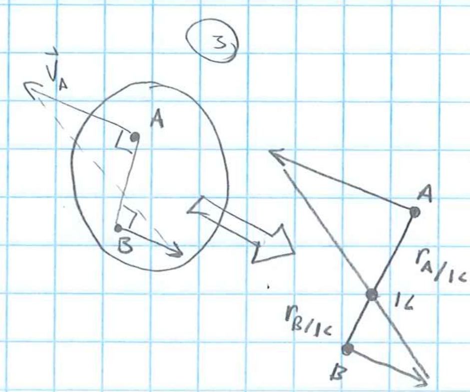
2) given directions of V_A & V_B

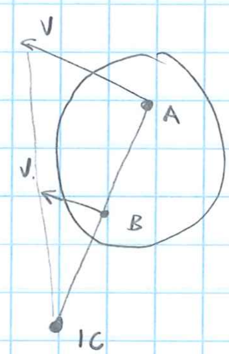
$V_A \nparallel V_B$



3) given $V_A \parallel V_B$

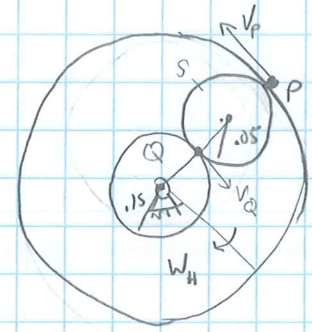
$$\frac{V_A}{r_A} = \frac{V_B}{r_B}$$





Notes

16-98



$\omega_H = 5$ rad/s

$\omega_R = 20$ $\quad V_P = \omega_R(0.25) = 20(.25) = 5$

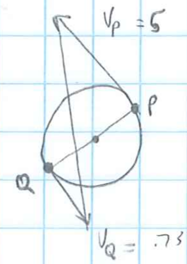
$V_Q = \omega_H(.15) = 5(.15) = \boxed{.75}$

$V_P = \omega_H r_{P/IC}$ $\quad 5 = \omega_S(.1 - x)$

$V_A = \omega_H r_{A/IC} = \omega_S(.05 - x)$

$V_A = \omega_{OA} r_{A/O}$

$\frac{5}{.1 - x} = \frac{.75}{x}$



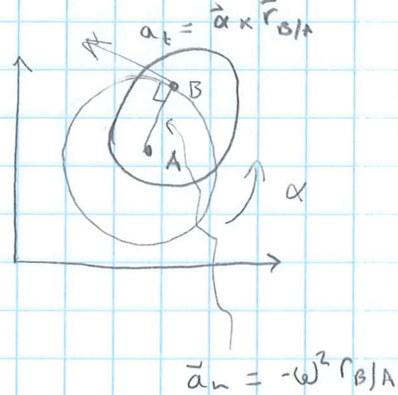
16.6 Accel analysis

1) review $\quad \vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

$\vec{V} = \vec{\omega} \times \vec{r}$ $\quad \vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{V}$

$= \vec{\alpha} \times \vec{r} - \omega^2 \vec{r}$

$= \vec{a}_t + \vec{a}_n$



2) analysis $\quad \vec{a}_B = \vec{a}_A + \vec{\alpha} \times r_{B/A} - \omega^2 \vec{r}_{B/A}$

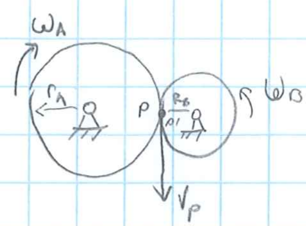
Mag.	$\alpha r_{B/A}$	$\omega^2 r_{B/A}$
Dir.	$\perp r_{B/A}$	$B \to A$

3) case study - contacting bodies w/o slipping

$V_P = V_{P'}$

$r_A \omega_A = r_B \omega_B$ $\quad \alpha_A r_A = \alpha_B r_B$

$a^n_P \neq a^n_{P'}$ $\quad$ ✳ play w/ this



Reading Notes
for 3-9-10 Homework

Kinematic Equations

$$V_B = V_A + \Omega \times r_{B/A} + (V_{B/A})_{xyz}$$

$$a_B = a_A + \dot{\Omega} \times r_{B/A} + \Omega \times (\Omega \times r_{B/A}) + 2\Omega (V_{B/A})_{xyz} + (a_{B/A})_{xyz}$$

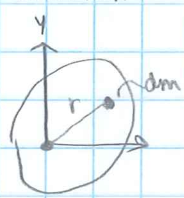
- Ω = rotation of A & B added together

Notes 3-15-10

1 INTRO

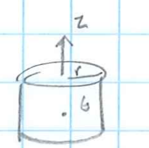
$$\Sigma \vec{F} = m \vec{a}_G \qquad G: \text{center of mass}$$
$$\Sigma \vec{M}_G = I_G \vec{\alpha}$$

2 DEFINITION



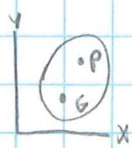
$$I = I_z = \int_M r^2 \, dm$$

Ex:



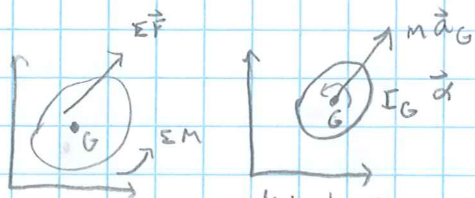
$I_c = \frac{1}{2} m r^2$

3 Parallel AXIS THEOREM



$$I_P = I_G + m d^2$$

17.2 Eq's of Motion

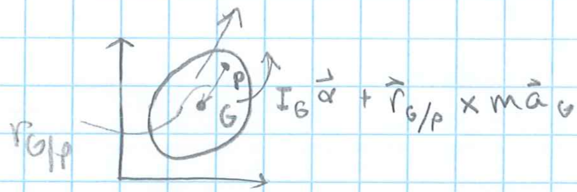


FBD   Kinetic diagram

$$\Sigma\vec{F} = m\vec{a}_G \quad (x,y,z) \qquad (1)$$

$$\Sigma\vec{M}_G = I_G\vec{\alpha} \quad (z) \qquad (2)$$

$$\Sigma\vec{M}_P = I_G\vec{\alpha} + r_{G/P}\times m\vec{a}_G \qquad (2)'$$



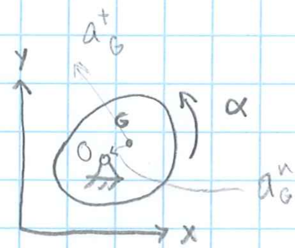
KD

17.3 TRANSLATION

$$\Sigma F = m\vec{a}_G \quad (1)$$
$$\Sigma\vec{M}_G = \vec{0} \quad (2)$$
$$\Sigma\vec{M}_P = \vec{r}_{G/P}\times m\vec{a}_G \quad (2)'$$

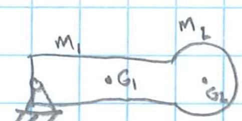
17.3 rotation

$$\Sigma F_t = ma_G^t = mr\alpha \qquad (1)$$
$$\Sigma F_n = ma_G^n = mr\omega^2$$
$$\Sigma M_O = I_G\alpha + ma_G^t r$$
$$= \underbrace{(I_G + ma_G^t)}_{=I_O}\alpha$$
$$\boxed{\Sigma M_O = I_O\alpha} \quad (2)''$$



for an assy of m_1 & m_2

$$\Sigma M_O = [(I_1)_O + (I_2)_O]\alpha$$



17.5 General Motion

1. EOM

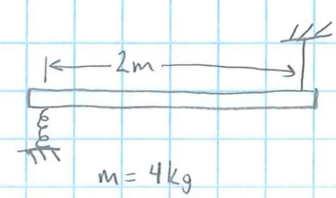
$\Sigma \vec{F} = m\vec{a}_G$ (1)

$\Sigma \vec{M}_G = I_G \vec{\alpha}$ (2)

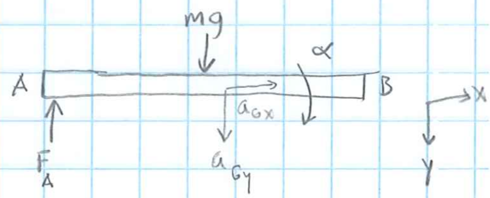
$\Sigma \vec{M}_P = I_G \vec{\alpha} + r_{G/P} \times m\vec{a}_G$ (2)'

$\Sigma \vec{M}_O = I_O \vec{\alpha}$ (2)''

$\uparrow$ fixed



example 17-63



$\Sigma M_A = I_A \alpha$

WRONG Eq ⇨ $mg\frac{L}{2} = \frac{1}{3}mL^2 \alpha \Rightarrow \alpha$

$\Sigma M_A = I_G \vec{\alpha} + \vec{r}_{G/P} \times m\vec{a}_G$

$\Sigma F_x = m(a_G)_x$?

$0 = m(a_G)_x$

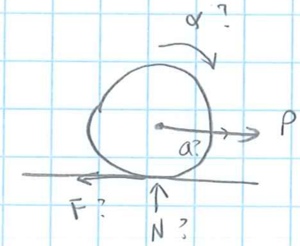
$\Sigma F_y = m(a_G)_y$

$mg - F_A = m(a_G)_y$ $\qquad F_A = \frac{1}{2}mg$ (hasn't changed since rope was cut)

Using center: $\quad F_A \frac{L}{2} = \frac{1}{12}mL^2 \alpha$

Notes 3-18

2. Frictional rolling



Given P, μ_s, μ_k

Find α

→ assume no slipping

$\Sigma F_x = m(a_G)_x = P - F = ma$ (1)

$\Sigma F_y = m(a_G)_y = N - mg = 0$ (2)

$\Sigma M_G = I_G \alpha \quad Fr = I_G \alpha$ (3)

$\boxed{a = r\alpha \ (4)}$

↓ α, a, N, F

↓

$F > F_{max} = \mu_s N$

↓ Y → slipping

$\boxed{(1), (2), (3)}$

↓

$\boxed{F = \mu_k N \ (4)}$ α, a, N, F

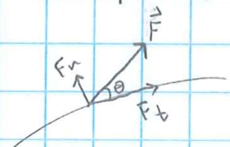
Stop

Notes Ch 18 Work & Energy

Note 3-23

18-44) $V_4 = 21.0$ ft/s

1) Review particles



$T_1 + \Sigma U_{12} = T_2$

$T = \frac{1}{2}mv^2$

$U_{12} = \int_1^2 F\cos\theta\, ds \overset{F=const}{=} F\cos\theta(s_2 - s_1)$

V = pot. energy

T = kinetic energy

U = work

$\boxed{\begin{array}{l}\text{if only } W \text{ \& } F_s \text{ do work} \\ T_1 + V_1 = T_2 + V_2\end{array}}$

$V = V_g + V_e$

$\boxed{V = mgy + \frac{1}{2}ks^2}$

2) →

next page

2) Rigid bodies in 2d motion

$T_1 + \Sigma U_{12} = T_2$

$T = \frac{1}{2} m v_G^2 + \underline{\frac{1}{2} I_G \omega^2}$

For M

$U_{12} = \int_{\theta_1}^{\theta_2} M d\theta \overset{M=const}{=} M(\theta_2 - \theta_1)$

$I_G = m k_G^2$

if only rotation about a fixed point:

$T = \frac{1}{2} I_O \omega^2$

V = pot. energy
T = kinetic energy
U = work

Notes 3-2

19.1-3 Momentum & impulse

1) Review - Particles

Linear momentum: $\vec{L} = m\vec{v}$

Angular momentum: $\vec{H}_o = \vec{r} \times m\vec{v}$

principle: $m\vec{v}_1 + \Sigma \int_{t_1}^{t_2} \vec{F} dt = m\vec{v}_2 \Rightarrow$ 3 eq's

$\vec{H}_1 + \Sigma \int_{t_1}^{t_2} M dt = \vec{H}_2 \Rightarrow$ 3 eq's

2) Rigid bodies w/ 2-D motion

$m(\vec{V}_G)_1 + \Sigma \int_{t_2}^{t_1} \vec{F} dt = m(\vec{V}_G)_2 \Rightarrow x, y$, 2 eq's

$(\vec{H}_{G(O)})_1 + \Sigma \int_{t_2}^{t_2} \vec{M}_{G(O)} dt = (\vec{H}_{G(O)})_2 \Rightarrow z$, 1 eq.

fixed

$\vec{H}_{G(O)} = I_{G(O)} \vec{\omega}$

$\vec{H}_P = I_G \vec{\omega} + \vec{r}_{G/P} \times m\vec{V}_G$

or $H_P = I_G \omega + d m V_G$

USE MOMENTUM - IMPULSE WHEN GIVEN TIME

Notes 4-6-16

19.4 Eccentric Impact

1.) review for a system

$$\Sigma m_i (\vec{V}_{Gi})_1 + \Sigma \int_{t_1}^{t_2} F\,dt = \Sigma m_i (\vec{V}_{Gi})_2 \quad (1)$$
$$(\Sigma \vec{H}_{oi})_1 + \Sigma \int_{t_1}^{t_2} \vec{M}_o\,dt = (\Sigma \vec{H}_{oi})_2 \quad (2)$$

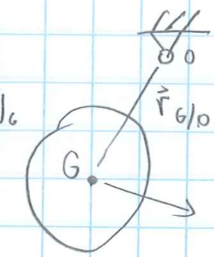
$$\vec{H}_o = I_o \vec{\omega} \qquad H_o = I_o \omega$$

d is perpendic. distance from $\vec{V}_G$ to O

When O is off the body: $\vec{H}_o = I_G \vec{\omega} + \vec{r}_{G/O} \times m\vec{V}_G$

or

$$H_o = I_G \omega + m V_G d$$

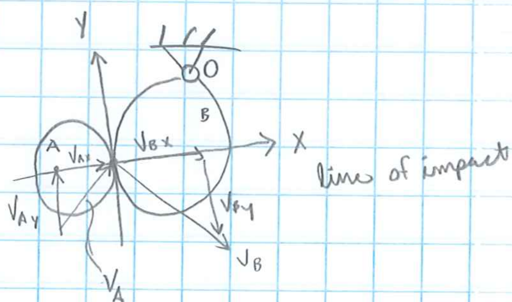


2) Eccentric Impact

From (2), $\Sigma \int_{t_1}^{t_2} M_o dt = 0$

$$\Sigma (\vec{H}_{io})_1 = \Sigma (\vec{H}_{io})_2 \quad (1)$$

$$e = \frac{(V_{Bx})_2 - (V_{Ax})_2}{(V_{Ax})_1 - (V_{Bx})_1}$$ } this eq'n is for the contact point in x-axis (2)



Review, Exam 2 — Notes 4-6-10

kinematics: 3
- Absolute motion: 1
- relative, 2pts, 1body: 1 — Ch 16
- relative, 2pts, 2 bodies: 1 ~ no accellerations

kinetics: 2
- Ch 17 planar kinetics, rigid body F & a
- Ch 18 rigid body work & Energy
- Ch 19 no impact Impulse & momentum rigid body

2D kinetics

forces & accelerations → Y → EOM
$\Sigma \vec{F} = m\vec{a}_G \Rightarrow$ 2eqs
$\Sigma \vec{M}_G = I_G\alpha \Rightarrow$ 1eq. Used M_{G_o} for fixed point

forces & accelerations → N → $\vec{F}, \vec{V}, s$

$\vec{F}, \vec{V}, s$ → Y → Only gravity & spring forces do work

Only gravity & spring forces do work → Y →
$T_1 + V_1 = T_2 + V_2$
$\Rightarrow$ 1eq

Only gravity & spring forces do work → N →
$T_1 + \Sigma U_{12} = T_2$
1eq.

$T = \frac{1}{2}MV_G^2 + \frac{1}{2}I_G\omega^2$

rotation $\frac{1}{2}I_o\omega^2$

trans $\frac{1}{2}mV_G^2$

$V = mgy_G + \frac{1}{2}ks^2$

$\vec{F}, \vec{V}, s$ → N → $\vec{F}, \vec{V}, t$ →

$\Sigma m(\vec{V}_G)_1 + \Sigma\int_{t_1}^{t_2}\vec{F}\,dt = m(\vec{V}_G)_2$ 2eq's

$(H_{G(o)})_1 + \Sigma\int_{t_1}^{t_2} M_{G(o)}\,dt = (H_{G(o)})_2$ 1eq

$H_G = I_G\omega$

$H_o = I_G\omega$ o on the body

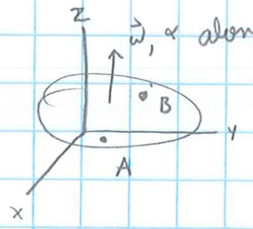
$H_o = I_G\omega + mV_Gd$ o off the body

sliding $F = M_kN$

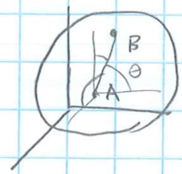
rolling w/o slip ω = $V = r\omega$, $a = r\alpha$

20.1 3-D kinematics

1. Overview 2D



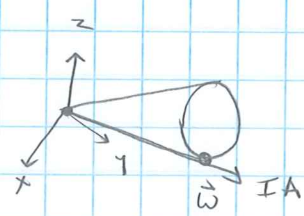
3-D



X (1) D.O.F : 3 X_A, Y_A, θ
(2) EOM's : 3 ΣF_x ΣF_y, ΣM_z
(3) $\vec{\omega}$ & $\vec{\alpha}$

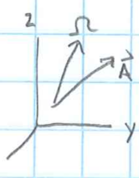
(1) EOF : 6, X_A, Y_A, Z_A $\theta_x, \theta_y, \theta_z$
(2) EOM : 6, ΣF_x ΣF_y ΣF_z ΣM_x, ΣM_y, ΣM_z
(3) $\vec{\omega}$ & $\vec{\alpha}$ not along z
(4) rotating axes

2. Rotation about a fixed point O
(1) Euler's theorem: 3d rotation about O
$\Leftrightarrow$ 2d rotation about an axis through O
instantaneous axis (IA)

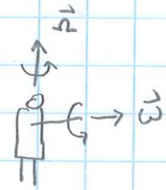


3. time derivative of a rotating vector

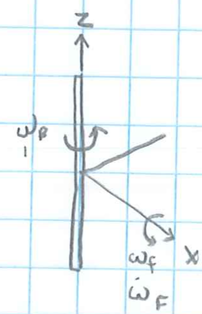
$\dot{\vec{A}} = ?$
$= (\dot{\vec{A}}_{rel}) + \vec{\Omega} \times \vec{A}$
$\vec{A}$ when $\vec{\Omega} = 0$
Angular velocity of A



$\dot{\vec{\omega}} = (\dot{\vec{\omega}})_{rel} + \vec{\Omega} \times \vec{\omega}$



1) Rotating vector



2-d body does not rotate

$\vec{\omega}_F = .1i$ rad/s

$\dot{\omega}_F \quad .2i$ rad/s²

3D body rotates with

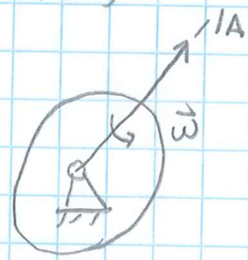
$\vec{\omega}_B = .3\vec{k}$ rad/s

when Ω is 0 — angular velocity of $\vec{A}$

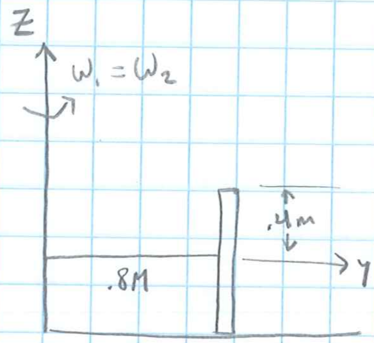
$\vec{A}$ is rotating with $\vec{\Omega}$: $\boxed{\dot{\vec{A}} = (\dot{\vec{A}})_{rel} + \Omega \times \vec{A}}$

$\dot{\vec{\omega}}_F = (\dot{\vec{\omega}}_F)_{rel} + \vec{\omega}_B \times \omega_F = (.2\vec{i}) + (.3\vec{k}) \times (.1\vec{i}) =$

2) 3D rotation



Example 3D rot.

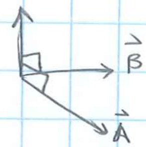


Notes 4-20

Review

$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$

$\vec{v} = \vec{\omega} \times \vec{r}$



2) $\vec{C} = \vec{A} \times \vec{B}$

$$= \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{bmatrix}$$

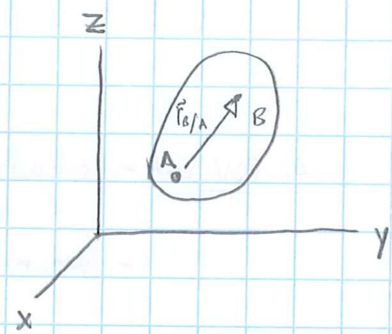
$\vec{C} \perp \vec{A}$

$\vec{C} \perp \vec{B}$

3) $\vec{A} \perp \vec{B}$ $\quad \vec{A} \cdot \vec{B} = \|A\| \cdot \|B\| \cos\theta = 0$

$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

4) $\vec{A} \cdot \vec{A} = \|A\|^2$



Gen. Motion $\quad V_B = V_A + V_{B/A} \leftarrow$ 3D rotation about A

$$\boxed{V_B = V_A + \vec{\omega} \times \vec{r}_{B/A}}$$

$$\boxed{a_B = a_A + \vec{\alpha} \times \vec{r}_{B/A} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A})}$$

without finding ω : $\quad V_B - V_A = \vec{\omega} \times \vec{r}_{B/A}$

$\rightarrow (V_B - V_A) \perp \vec{r}_{B/A} \rightarrow \boxed{(\vec{V}_B - \vec{V}_A) \cdot \vec{r}_{B/A} = 0}$

$(\vec{a}_B - \vec{a}_A) \cdot \vec{r}_{B/A} + (\vec{V}_B - \vec{V}_A) \cdot (\vec{V}_B - \vec{V}_A) = 0$

$$\boxed{(\vec{a}_B - \vec{a}_A) \cdot (\vec{r}_{B/A}) = -\|\vec{V}_B - \vec{V}_A\|^2}$$

Notes 4-23

$V_B - V_A + \omega \times r_{B/A}$

$a_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A})$

Now, $\vec{\omega}$, $\vec{\alpha}$
20-26

$$\begin{cases} (\vec{V}_B - \vec{V}_A) \cdot \vec{r}_{B/A} = 0 \\ (\vec{a}_B - \vec{a}_A) \cdot \vec{r}_{B/A} = -\|\vec{V}_B - \vec{V}_A\|^2 \end{cases}$$

$(V_B - V_A) \cdot \vec{r}_{B/A} = 0$ ← this eqn is derived from

$V_B = V_A + \vec{\omega} \times r_{B/A}$

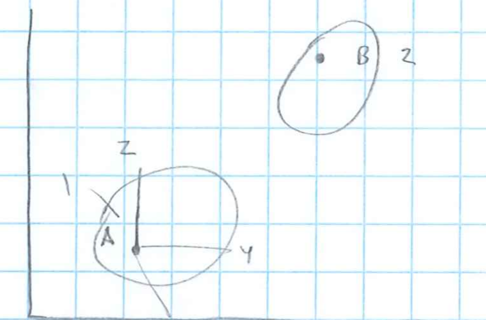
• cannot use these together without a 4th eqn

20-4 Gen Motion 3-D | 2pts, 2 bodies

1. Motion analysis

$\vec{V}_B = \vec{V}_A + \vec{\Omega} \times \vec{r}_{B/A} + (\vec{V}_{B/A})_{rel}$

$\uparrow$ ω_1 $\qquad$ $\uparrow$ V_B when $\Omega = 0$



$\vec{a}_B = \vec{a}_A + \dot{\vec{\Omega}} \times \vec{r}_{B/A} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_{B/A}) + 2\vec{\Omega} \times (V_{B/A})_{rel} + (a_{B/A})_{rel}$

20.5)

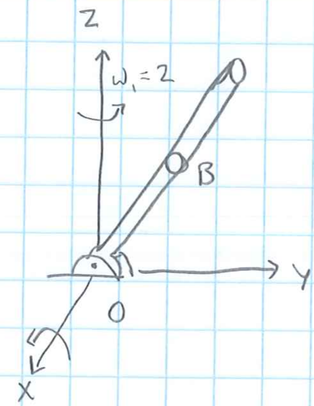
attach xyz to OB

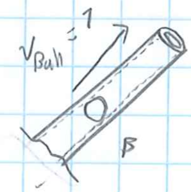
$\vec{\Omega} = \omega_1 + \omega_2 = 2k + 5i$

$\dot{\Omega} = \dot{\omega}_1 + \dot{\omega}_2 = 0 + (0 + \vec{\omega}_1 \times \vec{\omega}_2)$

$V_B = V_o + \Omega \times r_{B/o} + (V_{B/o})_{rel}$

$\quad = 0 + \Omega \times r_{B/o} + \dot{r}(\cos 30 j + \sin 30 k)$

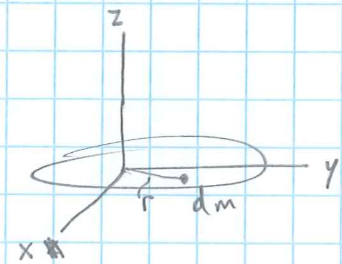




3D Kinematics

1) review $\Sigma (M_G)_z = (I_G)_z \alpha$

$I_z = \int_m r^2 dm$

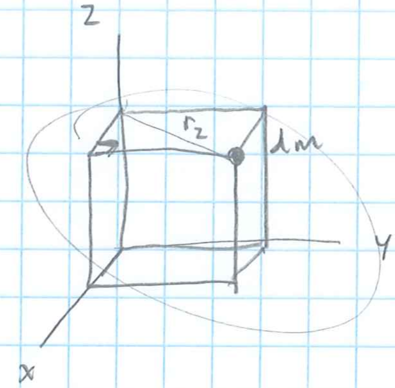


2) 3-D

$I_{xx} = \int_m r_x^2 dm = \int_m (y^2+z^2) dm$

$I_{yy} = \int_m r_y^2 dm = \int_m (x^2+z^2) dm$

$I_{zz} = \int_m r_z^2 dm = \int_m (x^2+y^2) dm$



$$\left.\begin{aligned} I_{xy} &= \int_m xy\, dm = I_{yx} \\ I_{yz} &= \int_m yz\, dm = I_{yz} \\ I_{xz} &= \int_m xz\, dm = I_{xz} \end{aligned}\right\} \text{products of intertia}$$

Inertia tensor

$$[I] = \begin{pmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{pmatrix}$$

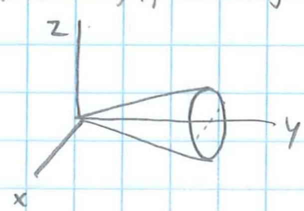
if body is symmetric to plane ij (i,j,k) $(i,j,k) = x,y,z$ $i \neq j \neq k$

then $I_{ik} = I_{jk} = 0$ Symmetric to x-y plane

$i = x$ $j = y$ $k = z$ $I_{xz} = I_{yz} = 0$

symmetric to yz plane $i = y$ $j = z$, $k = x$

$I_{xy} = I_{zx} = 0$



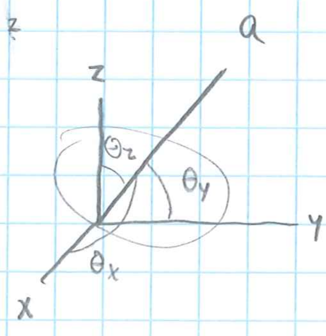
If $I_{xy} = I_{yz} = I_{xz} = 0$

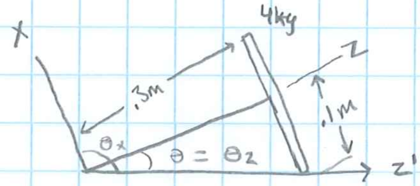
x, y, z : principal axes I_{xx}, I_{yy}, I_{zz} principal moments I_x, I_y, I_z

arbitrary axis:

$$I_{aa} = I_x U_x^2 + I_y U_y^2 + I_z U_z^2 - 2 I_{xy} U_x U_y - 2 I_{yz} U_y U_z - 2 I_{xz} U_x U_z$$

where $U_x = \cos\theta_x$ $U_y = \cos\theta_y$ $U_z = \cos\theta_z$





- body is symmetric to xz & zy planes

$$I_{aa} = I_x u_x^2 + I_z u_z^2 + I_y u_y^2$$

$$I_x = \frac{1}{4}(4)(.1)^2 + 4(.3)^2 \text{ disk parallel axis theorem}$$
$$+ \frac{1}{3}(1.5)(.3)^2 \text{ rod} = .415 \text{ kg}\cdot\text{m}^2 = I_y$$
$$I_z = \frac{1}{2}(4)(.1)^2 \leftarrow \text{disk} + 0 = .02 \text{ kg}\cdot\text{m} \qquad \theta = \tan^{-1}\left(\frac{.1}{.3}\right) = 18.43^\circ$$
$$\theta_x = 90 + 18.43 = 108.43 \qquad \theta_y = 90^\circ \qquad \theta_z = 18.43^\circ$$
$$I_{z'z'} = .415\cos^2(108.43) + .415\cos^2 90 + .02\cos^2 18.43 = .0595 \text{ kg}\cdot\text{m}^2$$

21.2 Momentum & Impulse

1. 2-D = $\vec{L} = m\vec{V}_G$
$$\vec{H}_{G(0)} = I_{G_0}\vec{\omega}$$
$$\vec{H}_p = I_G\vec{\omega} + \vec{r}_{G/P} \times m\vec{V}_G$$

2) 3D $\vec{L} = m\vec{V}_G \quad H_{G(0)} = [I]_G\vec{\omega}$

$$\begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix} = \begin{pmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{pmatrix}\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} I_{xx}\omega_x - I_{xy}\omega_y - \\ \cdots \\ \end{pmatrix}$$

Eq 21-10

For principal axes
$$\begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix} = \begin{pmatrix} I_x\omega_x \\ I_y\omega_y \\ I_z\omega_z \end{pmatrix}$$

$$m(\vec{V}_G)_1 + \Sigma\int_{t_1}^{t_2} F\,dt = m(\vec{V}_G)_2 \qquad (\vec{H}_{G(0)})_1 + \Sigma\int_{t_1}^{t_2} M_{G(0)}\,dt = (\vec{H}_{G(0)})_2$$

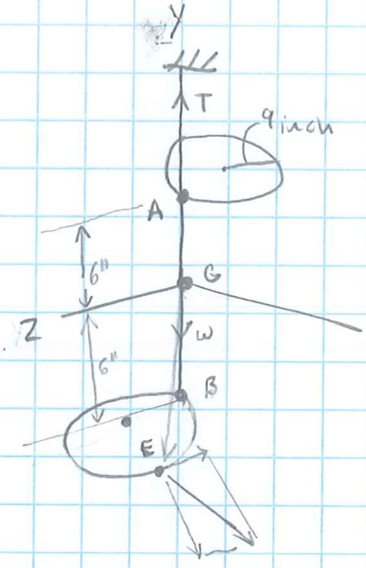
(2) Given

$I_{xx} = I_{yy} = 3.6 \text{ slug-ft}^2$

$I_{zz} = 1.8 \ldots \qquad I_{yz} = 1.2 \ldots$

$I_{xy} = I_{xz} = 0 \qquad m = 4.6 \text{ slug}$

$F\Delta t = -4j - 3\vec{k}$ lb·s



Find $\vec{\omega}$, $\vec{V}_G$ after impact

$r_{E/G} = (9i - 6j + 9k)$ in

$(.75i - .5j + .75k)$ ft

$(H_G)_1 + \Sigma \int_0^t M_G dt = (\vec{H}_G)_2$

$\vec{0} + \underbrace{\vec{r}_{E/G} \times \vec{F}_{\Delta t}}_{*} =$

$$* = \begin{bmatrix} i & j & k \\ .75 & -0.5 & .75 \\ 0 & -4 & -3 \end{bmatrix} = 4.5i + 2.25j - 3k$$

$$[I]\vec{w} = \begin{pmatrix} 3.6 & 0 & 0 \\ 0 & 3.6 & -1.2 \\ 0 & -1.2 & 1.8 \end{pmatrix} \begin{pmatrix} w_x \\ w_y \\ w_z \end{pmatrix} = \begin{pmatrix} 4.5 \\ 2.25 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} [I]^{-1} \begin{pmatrix} 4.5 \\ 2.25 \\ -3 \end{pmatrix} = \begin{pmatrix} 1.25 \\ .5179 \\ -2.6786 \end{pmatrix} \text{rad/s}$$

$\vec{0} + (-4j - 3\vec{k}) + T\Delta t\, j =$

$4.6((V_G)_x i + (V_G)_y j + (V_G)_z k)$

$i: (V_G)_x = 0 \qquad k: \; -3 = 4.6(V_G)_z \qquad (V_G)_z = -.6522$ ft/s

$(V_G)_y = (V_A)_y^0 + (V_{G/A})_y^0 = 0$

Momentum & Impulse

$m(\vec{V}_G)_1 + \Sigma \int_1^2 F dt = m(\vec{V}_G)_2$ (3 eqs)

$(\vec{H}_{G(O)})_1 + \Sigma \int_{t_1}^{t_2} M_{G(O)} dt = (\vec{H}_{G(O)})_2$ (3 eqs)

$H_{G(O)} = I_x \omega_x \hat{i} + I_y \omega_y j + I_z \omega_z k$ ← Principal axes
if the body is symmetric to two of x-y x-z yz planes

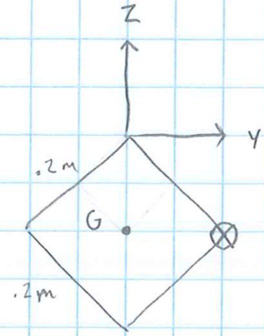
21.3 Work & Energy

1. Principle $T_1 + \Sigma U_{12} = T_2$ $T_1 + V_1 = T_2 + V_2$ if only F_s & mg do work
$U_{12} = \int F \cos\theta ds$

2) K.E. particles $T = \frac{1}{2} m v^2 + \frac{1}{2} I_G \omega^2$ O: fixed $= \frac{1}{2} I_O \omega^2 = \frac{1}{2} H_O \omega$
3D bodies: $T = \frac{1}{2} m V_G^2 + \frac{1}{2} \vec{H}_G \vec{\omega} = \frac{1}{2} \vec{H}_O \cdot \vec{\omega}$

$\vec{H}_{G(O)} = [I]\vec{\omega}$ principal axes $T = \frac{1}{2} m V_G^2 + \frac{1}{2} I_x \omega_x^2 + \frac{1}{2} I_y \omega_y^2 + \frac{1}{2} I_z \omega_z^2$

21-35) given $m = 4kg$ $\vec{I} = -60i$ N·s



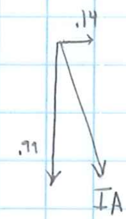
symmetric in yz plane & xz plane

Dv gives $(I_G)_y = .01333 \ kg \cdot m^2 = (I_G)_z$

Find IA, I_o

$$(\vec{H}_o) + \Sigma \int_{t_1}^{t_2} \vec{M}_o dt = (\vec{H}_o)_2$$

$$\vec{0} + \vec{r}_{B/o} \times \vec{I} = I_{ox} \omega_x i + I_{oy}\omega_y j + I_{oz}\omega_z k$$



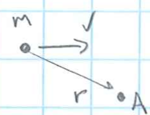
$(0, .2\sin 45, -.2\cos 45) \times (60i)$ $\quad I_{oy} = I_{Gy} + md^2 = .01333 + 4(.2\cos 45)^2$

$I_{oz} = I_{Gz} + \cancel{md_z^2}$ $\quad j: \omega_y \quad k\,\omega_z \quad u_{IA} = \dfrac{\vec{\omega}}{\|\vec{\omega}\|} = .14j - .99k$

$(V_G)_2 = \vec{\omega} \times \vec{r}_{G/o}$

$0 - 60i + I_o = m(\vec{V}_G)$

Angular momentum H_A, particle,



$H_A = \vec{r} \times m\vec{V}$

Notes @ help session

body,

$H_G = I_G \vec{\omega}$

$H_A = I_G \vec{\omega} + \vec{r} \times mV_G$

I = linear impulse $\quad \int \vec{M} dt = \vec{r} \times \vec{I}$

Lec 27 EOM's

H_G = angular momentum $= \frac{d}{dt}(I\omega)$ 2-D

$\Sigma \vec{M}_{G(O)} = \dot{\vec{H}}_{G(O)}$ $H_{G(O)} = [I]\vec{\omega}$ $\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}\begin{bmatrix}\omega_x \\ \omega_y \\ \omega_z\end{bmatrix} = H_x$

$\dot{\vec{A}} = (\dot{\vec{A}})_{rel} + \bar{\Omega} \times \bar{A}$

attach xyz to body $\Rightarrow \vec{\Omega} = \vec{\omega}$ rotational motion eq (21-24)

Notes 5-6-10

3D EOM's

$\Sigma M_x = I_x \dot{\omega}_x - (I_y - I_z)\,\omega_y \omega_z$ Eq. 21.25

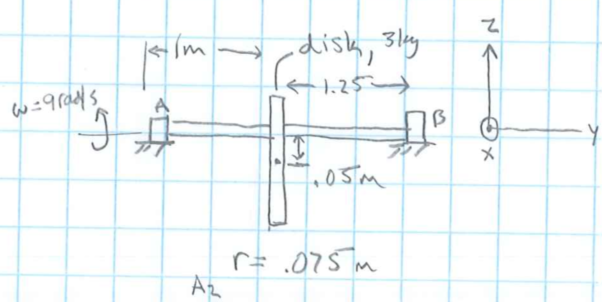
$\Sigma M_y = \ldots$

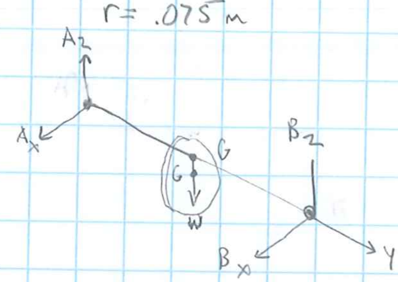
$\Sigma M_z = \ldots$

Homework 21 - 44, 46, 59)

44)

reactions @ A & B = ?





$\omega_x = 0 \quad \omega_y = -9 \quad \omega_z = 0$

For G $\quad \Sigma M_x = \cancel{I_x \dot{\omega}_x}^{0} - (I_y - I_z)\cancel{\omega_y \omega_z}^{0}$

$\Sigma M_x = 0 \quad B_z(1.25) \quad - A_z(1) = 0$

$\Sigma M_z = I_z \dot{\omega}_z - (I_x - I_y)\omega_x \omega_y = 0$

$A_x(1) - B_x(1.25) = 0$ (2)

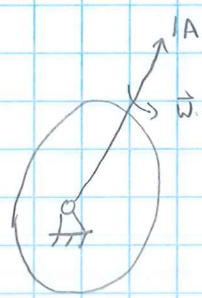
$\Sigma F_x = m(a_G)_x = 0 \quad \Rightarrow A_x + B_x = 0$ (3)

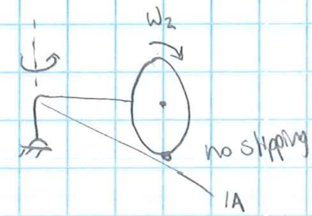
$\Sigma F_z = (m a_G)_z = m(a_G)_z = m(a_G)_n$

$A_z + B_z - mg = m\omega_y^2 r$

$A_z + B_z - 3(9.81) = 3(9)^2(.05)$ (4)

— Final Review —





3-D Kinematics   No 20.4

1) 3-D rotation

$\vec{\omega}$ is along IA

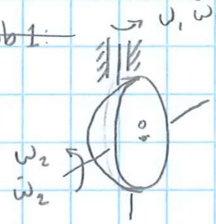
where $\vec{v} = \vec{0}$

$\vec{v} = \vec{\omega} \times \vec{r}$

$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$

$= \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$

~~Prob 1:~~



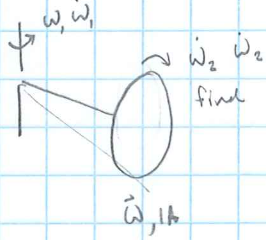
ω_1 is used here as ω about fixed axis

Prob 1: given $\omega_1\ \omega_2\quad \dot{\omega}_1\ \dot{\omega}_2$

$\omega = \omega_1 + \omega_2$

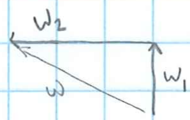
$\vec{v} = \vec{\omega} \times \vec{r} \qquad a = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$

$\vec{\alpha} = \dot{\vec{\omega}}_1 + \dot{\vec{\omega}}_2 \quad = \dot{\vec{\omega}}_1 + \left((\dot{\vec{\omega}}_2)_{rel} + \vec{\omega}_1 \times \vec{\omega}_2\right)$



Prob 2: given $\omega_1\ \dot{\omega}_1$

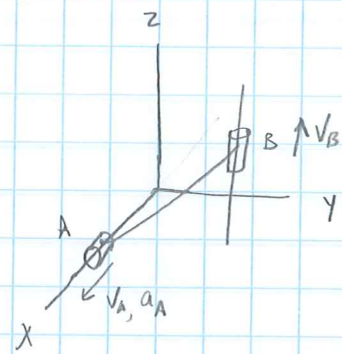
	$\omega =$	$\omega_1 +$	ω_2
M	?	✓	?
D	✓	✓	✓



— Final Review —

2) 3D general motion

$$\vec{V}_B = \vec{V}_A + \vec{\omega} \times \vec{r}$$
$$a_B = \ldots$$



prob 1: given $\vec{V}_A, \vec{a}_A$ find $\vec{V}_B, \vec{a}_B$

$$(V_B - V_A) \cdot \vec{r}_{B/A} = 0$$

$$V_B \vec{U}_B \qquad a_B \vec{U}_B$$

$$(\vec{a}_B - \vec{a}_A) \cdot \vec{r}_{B/A} = -\| \vec{V}_B - \vec{V}_A \|^2$$

prob 2: given V_A, a_A Find $V_B, a_B, \vec{\omega}, \vec{\alpha}$

$$\vec{V}_B = \vec{V}_A + \vec{\omega} \times \vec{r}_{B/A} \rightarrow \text{eq's } (1)(2)(3)$$

$V_B \vec{U}_B$ $\quad \omega_x \vec{i} + \omega_y \vec{j} + \omega_z \vec{k}$

$$\dot{\vec{\omega}} \cdot \dot{r}_{B/A} = 0 \qquad \text{eq. } (4)$$

— Final Review —

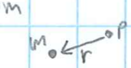
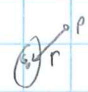
Kinetics

1. principles $\vec{F}, \vec{a} \rightarrow$ EOM's
   $\vec{F}, \vec{v}, s \longrightarrow$ W & E
   $\vec{F}, \vec{v}, t$ or impact $\longrightarrow$ M & I

2. EOMS
   Translational $\Sigma \vec{F} = m a_G \Rightarrow$ 3 eq's
   Rotational • particles: $\Sigma \vec{M} = \vec{0}$
   • 2D Bodies $\Sigma M_{G(O)} = I_{G(O)} \alpha$
   • 3D Bodies $\begin{cases} \text{eq. } 21-24 \\ \text{eq. } 21-25 \leftarrow \text{principal axes} \end{cases}$

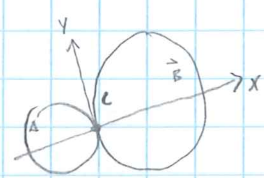
3. W & E
   particles $T = \frac{1}{2}mv^2$ $\quad T_1 + \Sigma_1 U_2 = T_2$
   2-D bodies $T = \frac{1}{2}mV_G^2 + \frac{1}{2}I_G\omega^2 = \frac{1}{2}I_O\omega^2$
   3-D bodies (principal axes) $T = \frac{1}{2}mV_G^2 + \frac{1}{2}(I_G)_x\omega_x^2 + \frac{1}{2}(I_G)_y\omega_y^2 + \frac{1}{2}(I_G)_z\omega_z^2$

   If only mg & Fs do work $T_1 + V_1 = T_2 + V_2$
   $V = mgy_G + \frac{1}{2}ks^2$

4. M & I $\quad$ H = angular momentum
   $\vec{H}$ • particles : $H_P = \vec{r} \times m\vec{v}$ 
   • 2d Bodies : $H_G = I_G\omega$ $\quad \vec{H}_P = I_G\omega + \vec{r} \times m\vec{V}_G$ 
   • 3d Bodies : $H_G = [I_G]\vec{\omega} = \begin{bmatrix} I_{xx} & -I_{xy} & \cdots \\ & \ddots & \\ & & \ddots \end{bmatrix}_G \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}$

Impact

$$e = \frac{(V_{Cx}^B)_2 - (V_{Cx}^A)_2}{(V_{Cx}^A)_1 - (V_{Cx}^B)_1}$$



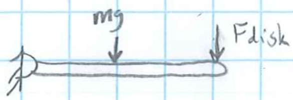
$e \geq 0$

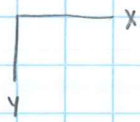
Y: no change in V

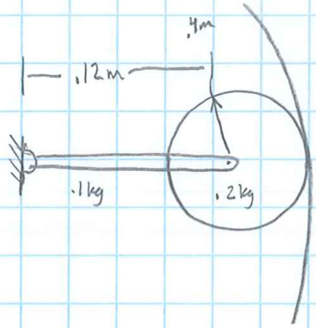
Rework Exam 2 problem 5

disk $I_G = \frac{1}{2} m r^2$

bar $I_G = \frac{1}{12} m l^2$







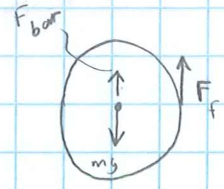
$\circlearrowright \sum M_{bar} = I\alpha_{bar}$

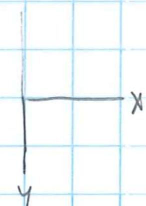
$= (I_G + md^2)\alpha_{bar}$

$= \left(\frac{1}{12}ml^2 + md^2\right)\alpha_b$

$(.1)(9.81)(.06) + F(.12) = \frac{(.1)(.12)^2}{12} + (.06)^2(.1) = .00048\,\alpha_b$ ✓

$.05886 + .12 F_d = .00048\,\alpha_b$ (1) ✓





$a_{bar} = \omega_{disk}(.04) = \omega_b(.12)$

$\omega_d = 3\,\omega_b$

$\alpha_d = 3\alpha_b$ (4)

$a_d = \alpha_{disk}(.04)$

$\downarrow + \sum F_{disk} = m a_d$

$-F_f - F_{bar} + (.2)(9.81) = (.2)a_d$

$F_f - F_b + 1.962 = .2 a_d +$

$-F_b - F_f + 1.962 = .008\alpha_d$ (2) ✓

$\circlearrowleft + \sum M_d = I_d \alpha_d$

$F_f(.04) = \frac{1}{2}(.2)(.04)^2 \alpha_d$

$.04 F_f = .00016\,\alpha_d$

$F_f = .004\,\alpha_d$ (3) ✓

$F: \quad \alpha_b \quad \alpha_d \quad F_f$