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Machine Design

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Machine Design I
ME 208 Fall 2010

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Office hours: 3:30-4:30pm on Mondays and Wednesdays (except as indicted in my weekly schedule posted outside of my office or http://web.mst.edu/~liou) or by appointment. E-mail communication is also encouraged.

This course is also supported partially through Blackboard Instructional Software at the following site: http://blackboard.mst.edu

Prerequisites: IDE 110 (C or better), ME 161(preceded or accompanied), ME 153, MET 121

Text: Shigley's Mechanical Engineering Design, 9th Edition, Budynas and Nisbett 8th is still ok probably - Notes based on 8th

Grading:

Exam 1 Sep 30	20%		90 - 100	A
Exam 2	20%		80 - 89	B
Exam 3	20%		70 - 79	C
Comprehensive Final	20%		60 - 69	D
Quizzes	20%		< 60	F
	100%			

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Do not expect a curve on grades. It is my responsibility to help you learn the material, but it is your responsibility to do it. If you need to make a certain grade, make the necessary effort from the beginning to do so. Expect adherence to the grading scale shown above.

Exams: open-book

All exams are made to reflect the standard grading scale shown above. Your grade is *not* determined by comparison to how everyone else did. It is determined by comparison of your demonstrated understanding of the material to the understanding which I expect you to have. The final exam is comprehensive. However if a student demonstrated excellent learning outcome, I may waive his/her final exam if he/she has : 1) grade before final exam of 90+% (not curved, all quizzes count); and 2) kept good class notes (will need to turn in at the end). Here is a sample formula I will use (# of quizzes may change) to find out where you stand for your ME 208 class before final exam, assuming 18 quizzes:

Your overall percentage = (3 Exams)/300 *80 + (18 Quizzes)/180*20

If your percentage is 90+ (including all quizzes; say 18 quizzes), you do not need to take the final exam. Please bring your class note to me and give it to me after class if you expect to be 90+ before the final exam. If your score is 89.99-, you will need to take the final.

top 19 quizzes will be used

Homework: Homework problems have been assigned regularly. The intent of the homework is not to grade your understanding of the material, but to provide a stimulus to you to understand the material. You will be given the solutions. You are encouraged to work together to help each other learn. You do not need to turn in your homework, but be expect the related materials in quizzes and exams.

Quizzes: Short quizzes (5 minutes or less) should be expected, usually at the end of each class period. The quizzes will generally focus on key points of suggested before-class readings and recently covered material. Quizzes which are missed can not be made up. A small number of your lowest quiz scores will be dropped (approximately 15%.)

Attendance: Attendance in class will not generally be checked. However, any quizzes which are missed can not be made up. Make-up exams will generally only be administered if prior arrangements have been made.

Course Content for Machine Design ME 208 Fall 2010

Tentative Date	Topic	Sections	
Aug 24	Introduction, Design Considerations	*1.1-8, 13-15*	
	States of Stress Review	*3.1-2, 4-7, 9-12*	
26	Material Properties	*2.1-21*	
	Strength vs. Stress, Factor of safety	1.9-12, 5.1	
31	Static Failure Theories (Ductile Materials)	5.3-7	HW#1 Due
Sep 2	Static Failure Theories (Brittle Materials)	5.8-10	
7			Out of town
9			Out of town
14	Deflection, Strain energy, Castigliano	*4.1-4*, 4.7-8	HW#2 Due
16	Castigliano with Curved Members	4.9	
21	Stress Concentrations	3.13, 5.2, 6.10	HW#3 Due
23	Fatigue Failure (S-N Diagram)	6.1-4, 6.7-8	
28	Fatigue Failure (Endurance Limit Factors)	6.9-10	
Sep 29	Review		Time: 3:30-4:30pm
30	EXAM 1		
Oct. 5	Fatigue Failure (Fluctuating Loading)	6.11-12	HW#4 Due
7	Fatigue Failure (Combination Loading)	6.14	
12	Fatigue Failure example		
14	Gears (Fundamentals and Standards)	13.1-7, 13.12, *13.8*	HW#5 Due
	Gears (Types, Force)	13.9-11, 13.14-17	
19	Gears (AGMA Strengths and Stresses)	14.3-19	
21	Gear Example		
26	Rolling Bearings (Types, Life, Mounting)	11.1-5, 11.11-12	HW#6 Due
Oct 27	Review		Time: 3:30-4:30pm
28	EXAM 2		
Nov. 2	Rolling Bearings (Combined loads, Ball)	11.6,8	
4	Shaft Design (Layout, Stress)	7.1-4	
9	Shaft Design (Deflection)	7.5	
11	Fits and Tolerances		HW#7 Due
16	Bolted Joints (Standards, Stiffness)	8.1,8.3-6	
18	Bolted Joints (Joint Mechanics)	8.7-9	
	Bolted Joints (Fatigue)	8.11	
23	Holiday		
25	Holiday		
Nov 29	Review		Time: 3:30-4:30pm
30	EXAM 3		
Dec. 2	Welding	9.1-7	
7	Welding	9.1-7	
9	Review and Class Evaluations		HW#8 Due
Dec. 14	Final Exam		10:30am-12:30pm

Note: Section numbers in italics will not necessarily be covered in class, but you are responsible for understanding it. Most of it is review from previous classes.

How to get an A, 208

- keep good class notes & turn them in at the end.

- I share this class with: Scott Kapshandy, N. Tourville,

- Quizzes are 10pts, every day? Liou expects to drop 4 quizzes

- Exams are related to homework, open book, closed notes
    - 1 sheet of paper 1 side eqns allowed no solutions, attach to exam & turn in.
- Be able to find tables & etc in the book
    - Open book exam, post-it the book
- Previous exams on blackboard

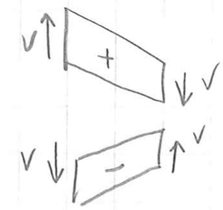
ME 208 Machine Design Frank Liou Notes 8-24-10

Intro & review

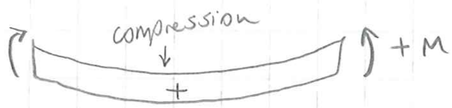
§ "section"

1. beam sign convention p95 §(3-2)

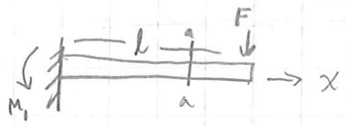
"+" internal shear force to cause a clockwise rotation of beam segment



"+" internal moment to cause compression in top fibre of segment



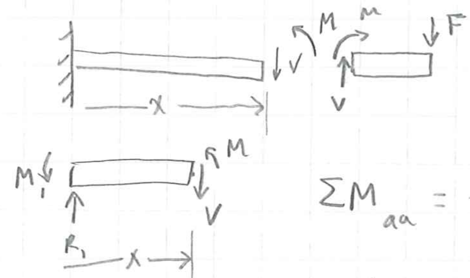
eg.



1) find boundary condition

$F = R_1$ $-M_1 + Fl = 0$ $\therefore$ $M_1 = Fl$

2) Find unknowns M, V

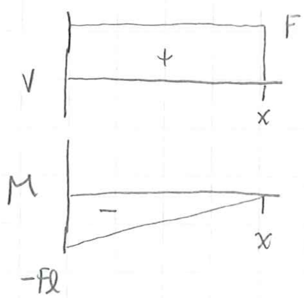


$F = V$

$$\Sigma M_{aa} = -R_1 x + M_1 + M$$

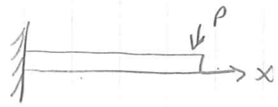
$$M = -M_1 + R_1 x = -Fl + R_1 x$$

$$\boxed{= -Fl + Fx}$$



Notes 8-24

2- Beam deflection p-150 § 4-3



$M = -Pl + Px = EI \frac{d^2y}{dx^2}$ $\quad EI\frac{dy}{dx} = -Plx + \frac{Px^2}{2} + C_1$

@ $x=0$ $\frac{dy}{dx}=0 \Rightarrow C_1 = 0$ $\quad EIY = -\frac{Plx^2}{2} + \frac{Px^3}{6} + C_2$

@ $x=0$ $y=0 \Rightarrow C_2=0 \Rightarrow EIy \; -\frac{Plx^2}{2} + \frac{Px^3}{6}$

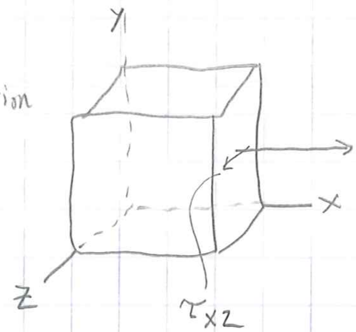
$\therefore Y_{max} = \frac{Pl^3}{3EI}$ (Table A-9)

"dont sell the book, use it as your design library"

3 Cartesian Stress components p. 80 (S. 3-5)

+ shear; positive force, pos. direction or neg. force in neg direction

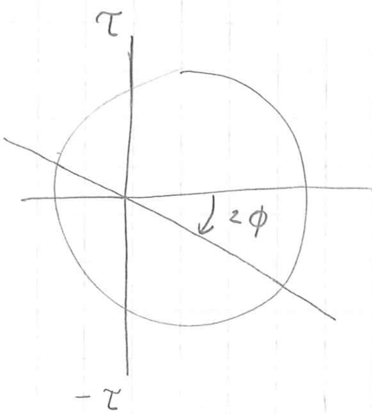
fig 3-8 all positive



σ_x means tension in x

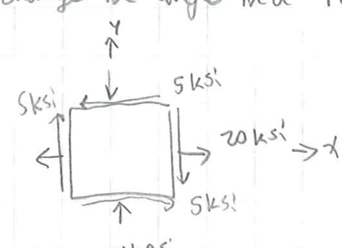
4. Mohr's Circle P.80 S. 3-6

- to extract principal stresses in a 2d stress state (for combined loads)



a change in angle of ϕ in the element orientation corresponding to to 2ϕ change in angle in a Mohr's Circle

Example



$\tau_{xy} = -5ksi$ or 5ksi C.W

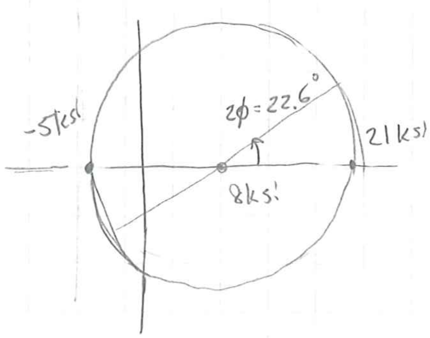
$\sigma_{avg.} = \frac{\sigma_x + \sigma_y}{2} = \frac{20-4}{2} = 8ksi$

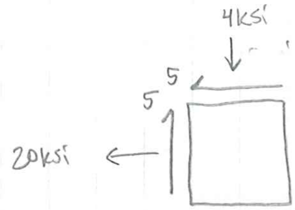
$2\phi = \tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) = 22.6°$

$\phi = 11.3$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= 13$ ksi

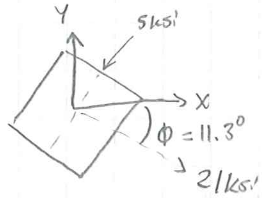




continued diagram from last pg.

$$2\phi = \tan^{-1}\frac{2\times(5)}{20+4} = -22.6^\circ \text{ or } 22.6^\circ \text{ C.W.}$$

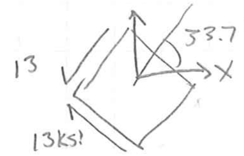
$\phi = 11.3^\circ$



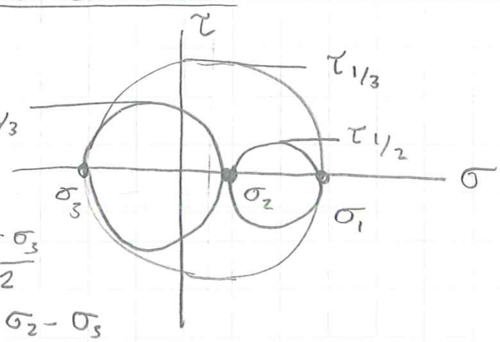
principal stresses

$$2\theta = 90 - 2\phi = 67.4$$

$$\theta = 33.7^\circ$$



General 3d stress



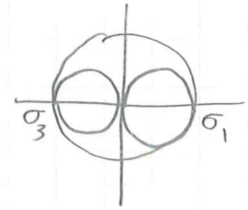
$$\tau_{1/3} = \frac{\sigma_1 - \sigma_3}{2}$$

$$\tau_{2/3} = \frac{\sigma_2 - \sigma_3}{2}$$

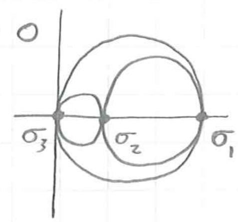
$$\tau_{1/2} = \frac{\sigma_1 - \sigma_2}{2}$$

plane stress another principal stress $= 0$

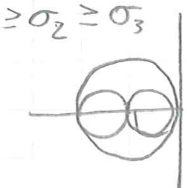
$\sigma_1 \geq 0 \geq \sigma_3$



$\sigma_1 \geq \sigma_2 \geq 0$

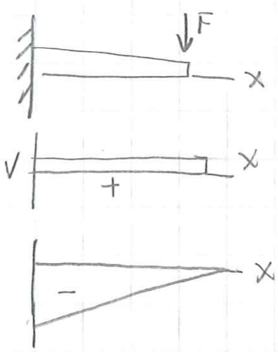


$0 \geq \sigma_2 \geq \sigma_3$



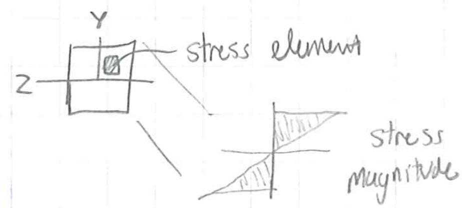
* we are interested in maximum, $\tau_{1/3}$

~5~ beam in bending

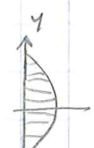


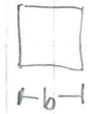
$\sigma = \frac{My}{I}$ this is stress in x direction

cross-section



Shear stress $\tau = \frac{VQ}{Ib}$ $Q = \int_{y_1}^{c} y\,dA$





width of x-section

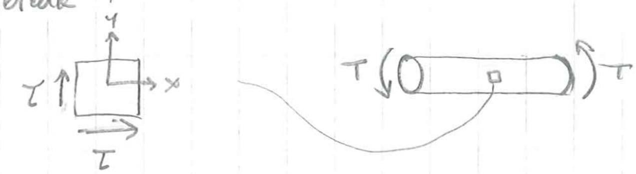
- if the beam length L to depth (H) ratio for the beam (L/H) is greater than 10 the τ is often negligible

Ex. loading T applied to the bar, at what angle would the bar break?

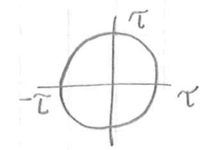
$\sigma_x = 0$

$\sigma_y = 0$

$\tau_{xy} = \tau$



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \pm \tau$$



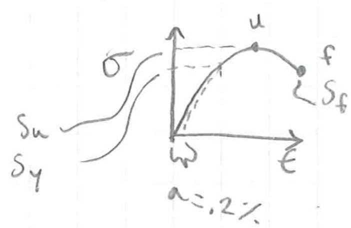
$$2\phi = \tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) = \tan^{-1}\left(\frac{2\tau_{xy}}{0}\right) = \pm 90° \Rightarrow \phi = 45° \pm \qquad \tau_{max} \Rightarrow \theta = 0, 90°$$

☆ ductile material will fail due to shear stress (fail @ 0, 90°)
brittle will fail due to normal stress (@ 45°)

2. Material properties

1. what is ductile material/brittle? $\sigma = \frac{P}{A}$ $\epsilon = \frac{\delta}{L}$

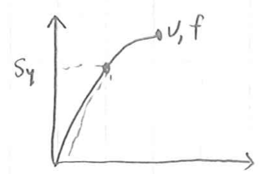
1.1) ductile material



S_u = ultimate strength
S_y = yield strength
S_f = fracture

$\epsilon_f > 5\% \Rightarrow$ ductile material
• most steels are ductile

1.2) $\epsilon_f < 5\%$: cast irons, very high-strength steels



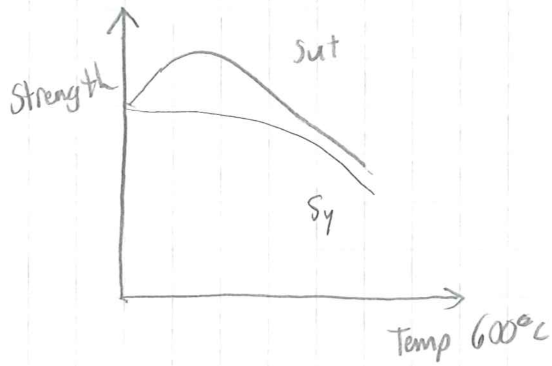
2. hardness H_B: Brinell hardness — for steels, $(200 \le H_B \le 450)$

$$S_u = \begin{cases} .5\,H_B & \text{kpsi} \\ 3.41\,H_B & \text{MPa} \end{cases} \quad (2\text{-}21)$$

for cast iron. $S_u = \begin{cases} .23\,H_B - 12.5\ \text{kpsi} \\ 1.58\,H_B - 86\ \text{MPa} \end{cases}$ (2-22)

3. Temperature effect Fig 2.9

NOTE review Ex. 3-8 & 3-9 before doing the h/w

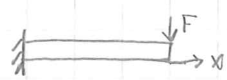
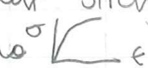


4. Physical constants of materials (E, G, V, P – Table A-5)

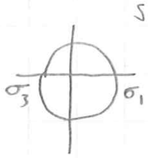
5. S_u, S_y (Table A-20)

Notes 8-31-10

<u>Static Failure theories</u>

1) Static failure: part fails b/c applied stresses exceed material strength
ie. strength < σ
– what kind of stresses cause failure? e.g.  $\sigma_x = \frac{My}{I}$ bending shear
$\tau_{xy} = \frac{VQ}{Ib}$ –under any applied loads there is always a combination of normal and shearing stresses
– ductile materials are limited by their  shear strengths
– brittle materials are limited by their tensile strengths 

2) theories for ductile material
2.1– MSS theory "Failure occurs when maximum shear stress in the part exceeds the shear stress in a tensile test specimen at yield"

 ie, $\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$   $\tau_{max} = \frac{\sigma_1}{2} = \frac{S_y}{2}$

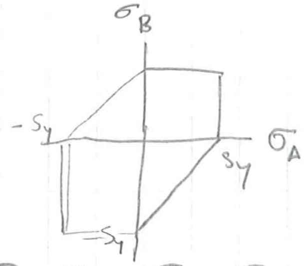
eq. 5-1 $\boxed{\sigma_1 - \sigma_3 \geq S_y}$ factor of safety $\boxed{n = \frac{S_y}{\sigma_1 - \sigma_3}}$ (5-3)

Check plane stress condition (one principal stress = 0) case 1: $\sigma_A \geq \sigma_B \geq 0$
$\therefore$ $\sigma_1 = \sigma_A$ $\sigma_3 = 0$

Case 2: $\sigma_A \geq 0 \geq \sigma_B$ $\sigma_1 = \sigma_A$ $\sigma_2 = 0$ $\sigma_3 = \sigma_B$

Case 2 (5-1) $\sigma_A - \sigma_B \geq S_y$

(5-3) $n \; \dfrac{S_y}{\sigma_A - \sigma_B}$



case 3 $0 \geq \sigma_A \geq \sigma_B \Rightarrow \sigma_1 = 0 \quad \sigma_2 = \sigma_A \quad \sigma_3 = \sigma_B$

(5-1) $\boxed{-\sigma_B \geq S_y}$ (5-3) $n = \dfrac{S_y}{-\sigma_B}$

this theory is a) simple/easy to use b) more conservative

2.2 Distortion Energy (DE) theory "yield occurs when the distortion strain energy exceeds the distortion strain energy for yield in simple tension or compression

$\Rightarrow$ Von Mises stress

$$\sigma' = \sqrt{\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}{2}}$$

assignment - ~~learn~~ know Mhors circle

if $\sigma' \geq S_y$, then fail $\boxed{n = \dfrac{S_y}{\sigma'}}$

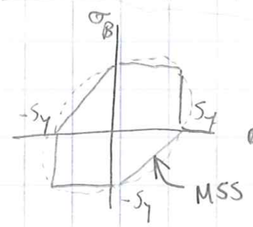
$$\sigma' = \frac{1}{\sqrt{2}}\left[(\sigma_x-\sigma_y)^2 + (\sigma_y-\sigma_z)^2 + (\sigma_z-\sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)\right]^{1/2}$$

in terms of applied stress

plane stress: $\sigma' = (\sigma_A^2 + \sigma_B^2 - \sigma_A\sigma_B)^{1/2}$ ← ellipse!

$\sigma' = [\sigma_x^2 - \sigma_{xy} + \sigma_y^2 + 3\tau_{xy}^2]^{1/2}$

when ($\sigma_z = \tau_{yz} = \tau_{zx} = 0$)

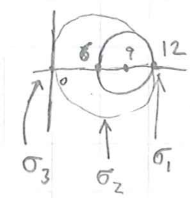


Ex: example: a ductile hot-rolled steel bar $S_y = 50psi$ $\sigma_x = 12$ Kpsi $\sigma_y = 6$kpsi

determine $n = ?$

MSS $n = \dfrac{S_y}{\sigma_1 - \sigma_3}$ DE $n = \dfrac{S_y}{\sigma'}$ $\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{12+6}{2} = 9$

$R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\dfrac{12-6}{2}^2} = 3$



MSS $\Rightarrow n = \dfrac{50}{12-0} = \boxed{4.17}$

$\sigma' = \sqrt{\dfrac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_1-\sigma_3)^2}{2}}$

$\sigma' = (\sigma^2 - \sigma_x\sigma_y + \sigma_y^2 - 3\tau_{xy}^2)^{1/2}$

$= [12^2 - (12)(6) + 6^2]^{1/2} = \sqrt{108}$ $n = \dfrac{S_y}{\sigma'} = \dfrac{50}{10.39} = \boxed{4.81}$

Notes 8-31

2.3 Ductile Columb-Mohr theory (DCM)

MSS: $\sigma_1 - \sigma_3 \geq S_y$ $\quad \frac{\sigma_1}{\sigma_y} - \frac{\sigma_3}{\sigma_y} \geq 1$ $\quad \frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}} \geq 1 \Rightarrow$ failure occurs

(tensile s.) (compression s)

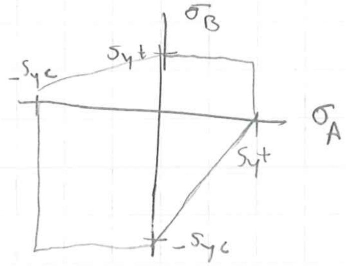
this theory is for materials with $S_t \neq S_c$

Case 1 $\sigma_A \geq \sigma_B \geq 0$ $\quad \sigma_1 = \sigma_A \quad \sigma_3 = 0$ $\quad \frac{\sigma_A}{S_{yt}} \geq 1 \quad n = \frac{S_{yt}}{\sigma_A}$

Case 2: $\sigma_A \geq 0 \geq \sigma_B$ $\quad \sigma_1 = \sigma_A \quad \sigma_3 = \sigma_B \Rightarrow \frac{\sigma_A}{S_{yt}} - \frac{\sigma_B}{S_{yc}} \geq 1$

$$\boxed{\frac{1}{n} = \frac{\sigma_A}{S_{yt}} - \frac{\sigma_B}{S_{yc}}}$$

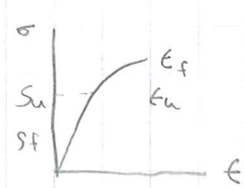
Case 3 $0 \geq \sigma_A \geq \sigma_B$ $\quad \sigma_1 = 0 \quad \sigma_3 = \sigma_B \Rightarrow - \frac{\sigma_B}{S_{yc}} \geq 1 \quad n = -\frac{S_c}{\sigma_B}$



Summary:
1) static failure
2) theories ductile mat.
   2.1 Max Shear stress (MSS) ← more conservative
   (5-4) p 219
   2.2. Distortion Energy, DE ← the ellipse, more accurate than MSS
   (5-5) p 221
   2.3 Ductile coulomb Mohr
   (5.6

3) Brittle materials $\varepsilon = \frac{\delta}{l} < 5\%$.
   3.1 – Maximum normal stress (MNS)
   3.2 – Brittle Coulomb-Mohr (BCM)
   3.3 – Modified Mohr (MM)

In general, brittle materials, Su



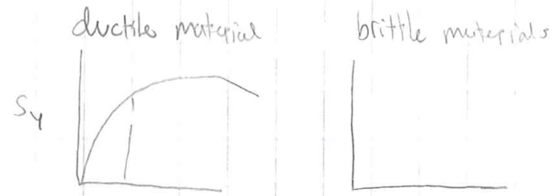
Quiz now, 1:35 pm check B.B. again

Notes 9-2-10

~~3~~4 Example, energy method

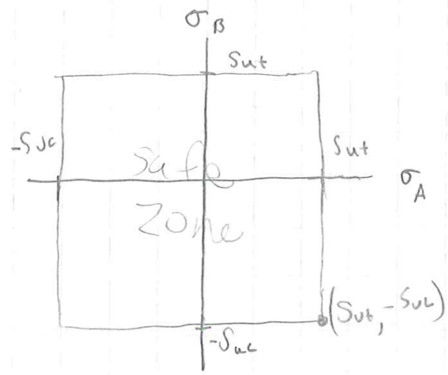
1) spring rates (4-7) p 162

2) energy method p164



3.1) MNS : Failure occurs when one of the 3 principal stresses equals or exceeds the strength

ie, $\sigma_1 > \sigma_2 > \sigma_3$ if $\sigma_1 \geq S_{ut}$ = fail or $\sigma_3 \leq -S_{uc}$

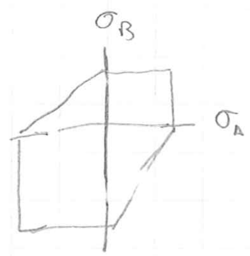


$n = \frac{S_{ut}}{\sigma_A}$ if σ_A & $\sigma_B \geq 0$ or $\sigma_A \geq 0 \geq \sigma_B$ and $\left|\frac{\sigma_B}{\sigma_A}\right| \leq \frac{S_{uc}}{S_{ut}}$

$n = -\frac{S_{uc}}{\sigma_B}$ if $0 \geq \sigma_A \geq \sigma_B$ or $\sigma_A \geq 0 \geq \sigma_B$ and $\left|\frac{\sigma_B}{\sigma_A}\right| > \frac{S_{uc}}{S_{ut}}$

3.2) BCM

$\frac{\sigma_1}{S_{ut}} - \frac{\sigma_3}{S_{uc}} = \frac{1}{n}$



3.3) (1) when $\sigma_A \geq \sigma_B \geq 0$ $n = \frac{S_{ut}}{\sigma_A}$ or when $\sigma_A \geq 0 \geq \sigma_B$ & $\left|\frac{\sigma_B}{\sigma_A}\right| \leq 1$

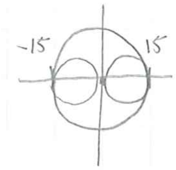
(2) When $0 \geq \sigma_A \geq \sigma_B \Rightarrow n = \frac{-S_{uc}}{\sigma_B}$

(3) when $\sigma_A \geq 0 \geq \sigma_B$ and $\left|\frac{\sigma_B}{\sigma_A}\right| > 1$ $\frac{(S_{uc} - S_{ut})\sigma_A}{S_{uc} S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{n}$

Example p 5-13 Using MM a) $\sigma_A \geq \sigma_B \geq 0$ $n = \frac{S_{ut}}{\sigma_A}$

table A 24 $n = \frac{31}{20} = 1.55$ $S_{uc} = 109$ $S_{ut} = 31$

b) $R = \pm\sqrt{\left(\frac{\cancel{\sigma_x - \sigma_y}}{2}\right)^2 + \tau_{xy}^2} = \pm 15\text{kpsi}$



$\sigma_A = ?$ $\sigma_1 = 15$ $\sigma_2 = 0$ $\sigma_3 = -15$

$\sigma_A = 15$ $\sigma_B = -15$ $\sigma_A \geq 0 \geq \sigma_B$ $\left|\frac{\sigma_B}{\sigma_A}\right| = 1 \Rightarrow n = \frac{S_{ut}}{\sigma_A} = \frac{S_{ut}}{15} = 2.1$

c) $0 \geq \sigma_A \geq \sigma_B$ $\therefore n = \frac{-S_{uc}}{\sigma_B} = \frac{-109}{-80} = 1.36$

d) $\sigma_A \geq 0 \geq \sigma_B$ $\frac{\sigma_B}{\sigma_A} = -\frac{25}{15} > 1$ Use MM, eqn 3 $\frac{(109-31)15}{(109)(31)} - \frac{-25}{109} = \frac{1}{n}$ $n = 1.69$

Notes 9-2-10

know how to draw mohrs circle

Energy Method — used to find deformation (will be covered in 2 lectures)

"very general method"

1. Spring rates $k(x) = \lim_{\Delta x \to 0} \frac{\Delta F}{\Delta x}$ $\Rightarrow$ we assume that they are linear, $k = \frac{F}{x}$

example (1) uniform bar $\delta = \frac{F\ell}{AE}$

$k = \frac{F}{\delta} = \frac{AE}{\ell}$

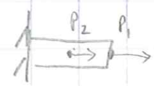
(2) angular deflection of a uniform round bar with Torsion T $\theta = \frac{T\ell}{GJ} \Rightarrow k = \frac{T}{\theta} = \boxed{\frac{GJ}{\ell}}$

2. Energy method for deflection (castgliano's method)

step 1: define strain energy U

step 2: $x_i = \frac{\partial U}{\partial P_i}$ P_i = Force @ pt. of interest

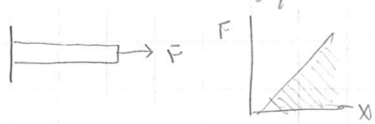
$\theta_i = \frac{\partial U}{\partial M_i}$ → Step 3 plug in loads & solve. – if no loads at pt. we add a dummy load Q, work problem, then set $Q = 0$



Notes 9-14-10

Review Session Sep 29th

– Define strain energy: (1) Bar in tension/compression $U = \frac{Fx}{2} = \frac{F \cdot F}{2k} = \frac{F^2}{2k}$



$k = \frac{AE}{\ell}$ $\boxed{U = \frac{F^2 \ell}{2AE}}$ for uniform bar

(2) Torsion For uniform bar: $U = \frac{T\theta}{2} = \frac{T^2}{2k}$ $k = \frac{GJ}{L}$ $\boxed{U = \frac{T^2 \ell}{2GJ}}$

General form of strain energy $\sigma_x = E\epsilon_x$, $\mu = \frac{U}{V} = \frac{1}{2}\sigma_x \epsilon_x$ $\frac{\sigma_x^2}{2E}$

$\uparrow$ F/A $\uparrow$ δ/ℓ

(1) $\begin{cases} U = \int \frac{\sigma_x^2}{2E} dv \\ \text{if uniform x-section, } U = \int \frac{\sigma_x^2}{2E} A\,dx = \frac{\sigma_x^2 A\ell}{2E} = \frac{F^2 A\ell}{A^2 2E} = \frac{F^2 \ell}{2AE} \end{cases}$

(2) Torsion $U = \int \frac{1}{2}\left(\frac{\tau^2}{G}\right) dv$

(3) Beam in bending

$\sigma_x = \frac{My}{I}$ $U = \int \frac{\sigma_x^2}{2E} dv \ldots \boxed{\int \frac{M^2}{2EI} dx}$

Tension/comp

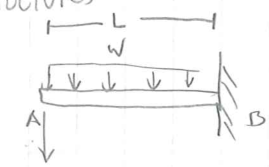
$\delta_i = \frac{\partial U}{\partial F_i} = \int \frac{1}{AE}\left(F \frac{\partial F}{\partial F_i}\right) dx$ F = external force, F_i = F @ pt of interest

Torsion
$$\theta_i = \frac{\partial U}{\partial M_i} = \int \frac{1}{GJ}\left(T \cdot \frac{\partial T}{\partial M_i}\right) dx$$
T = external torque
M_i = moment at pt. (interest)

Bending
$$\delta_i = \frac{\partial U}{\partial F_i} = \int \frac{1}{EI}\left(M \frac{\partial M}{\partial F_i}\right) dx$$
M = external moment
F_i = force @ pt. of interest

3) application in structures

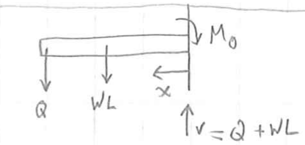
Example



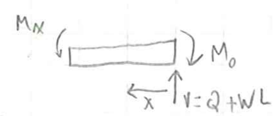
determine the deflection @ A

$$\underline{\delta A} = \frac{\partial U}{\partial Q} = \int_0^L \frac{1}{EI} M\left(\frac{\partial M}{\partial Q}\right) dx$$

Finding M:



$M_0 = QL + WL\left(\frac{L}{2}\right)$

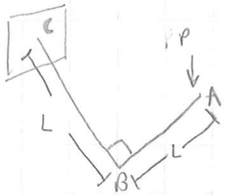


$$M_x - QL - WL^2 - \frac{Wx^2}{2} + (Q + WL)x = 0$$

$$\delta_A = \int_0^L \frac{1}{EI}\left[QL + \frac{WL^2}{2} + \frac{Wx^2}{2} - (Q+WL)x\right](L - x)\, dx$$
$$= \frac{WL^4}{8EI}$$
also can be derived from left end of beam

Assume Q is downwards, δA is down

Ex 2;



find pt. A deflection of member shown

Note: deflection = Σ of deflections due to energy @ various members

AB: $M_{AB} = Px$ $\quad \frac{\partial M_{AB}}{\partial P} = x$

$M_x = Px$

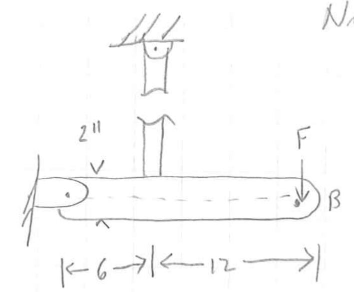
BC: $T_{BC} = \underline{PL}$ $\quad \frac{\partial T_{BC}}{\partial P} = L$

$M_{BC} = Py$ $\quad \frac{\partial M_{BC}}{\partial P} = y$

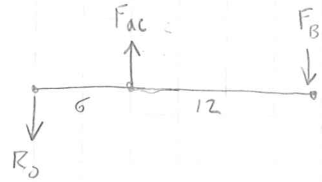
$$\delta_{Total} = \delta_A = \frac{1}{EI}\int_A^B M_{AB}\left(\frac{\partial M_{AB}}{\partial P}\right) dx + \frac{1}{EI}\int_B^C M_{BC}\left(\frac{\partial M_{BC}}{\partial P}\right) dy + \frac{1}{GJ}\int_B^C T_{BC}\left(\frac{\partial T_{BC}}{\partial P}\right) dy$$
$$= \frac{1}{EI}\int_0^L (Px)(x)\, dx + \frac{1}{EI}\int_0^L (Py)(y)(dy) + \frac{1}{GJ}\int_0^L (PL)(L)\, dy = \frac{PL^3}{3EI} + \frac{PL^3}{3EI} + \frac{PL^3}{GJ}$$
$$= PL^3\left(\frac{2}{3EI} + \frac{1}{GJ}\right)$$

Examples

E = 10 Mpsi for both bars
Find deflection @ B

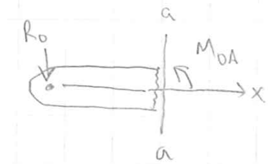


ans: $\delta_{Total,B} = \delta_{OA/B} + \delta_{AB/B} + \delta_{AC/B}$



$F(18) - F_{ac}(6) = 0 \quad F_{ac} = 3F$

$R_o = 2F$



$$\delta_{OA/B} = \int_0^6 \frac{M_{OA}\left(\frac{\partial M_{OA}}{\partial F}\right)}{EI}\,dx$$

$R_o x + M_{OA} = 0$

$M_{OA} = -R_o x$

$= -2F(x) \Rightarrow \frac{\partial M_{OA}}{\partial F} = -2x$

$I = \frac{bh^3}{12} = \frac{\frac{1}{4}(2)^3}{12} = .1667\ in^4$

$$\delta_{OA} = \frac{(-2F_x)(-2x)\,dx}{10\times10^6 \times 0.1667} = \frac{288F}{10\times10^6 \times .1667} = 0.01382$$

$$\delta_{AB/B} = \int_0^{12} \frac{M_{AB}\left(\frac{\partial M_{AB}}{\partial F}\right)}{EI}\,dx$$

$M_{AB} + F_x = 0$

$M_{AB} = -F_x$

$\frac{\partial_{AB}}{\partial f} = -x$

$$\delta_{AB/B} = \int_0^{12} \frac{(-F_x)(-x)}{EI}\,dx$$

$$= \frac{\overset{80}{F}\,12^3}{3EI} = .02764''$$

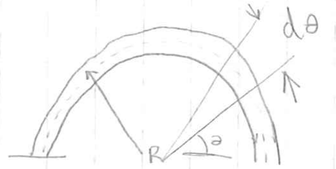
$$\delta_{AC/B} = \int_0^{12} \frac{F_{AC}\left(\frac{\partial F_{AC}}{\partial F}\right)}{AE}\,dx \qquad F_{AC} = 3F \qquad \frac{\partial F_{AC}}{\partial F} = 3 \qquad \delta_{AC/B} = \int_0^{12} \frac{(3F)(3)}{AE}\,dx$$

↑
energy contribution of deflection at ac to pt. B $\quad A = \frac{\pi d^2}{2} = \frac{\pi\left(\frac{1}{2}\right)^2}{2}$

$$\delta_{AC/B} = \frac{\overset{80}{9F}(12)}{\frac{\pi}{4}\left(\frac{1}{2}\right)^2 \times 10\times10^6} = .0044$$

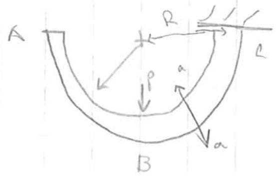
$$\boxed{\delta_{total} = 0.04586}$$

Example



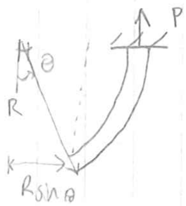
if $R/h > 10$ $\quad U = \int \frac{M^2 R\, d\theta}{2EI}$

P. 4.76



Pt B $\quad M_c - P \cdot R + P(R - R\sin\theta) = 0$

$M = PR\sin\theta$



$M = PR\sin\theta$

$\delta_B = \frac{\partial U}{\partial P} = \frac{1}{EI}\int_0^{\pi/2} M\left(\frac{\partial U}{\partial P}\right) R\, d\theta = \frac{1}{EI}\int_0^{\pi/2} (PR\sin\theta)(R\sin\theta) R\, d\theta$

$= \frac{1}{EI}\int_0^{\pi/2} PR^3 \sin^2\theta\, d\theta = \boxed{\frac{\pi PR^3}{4EI}}$ for pt. b

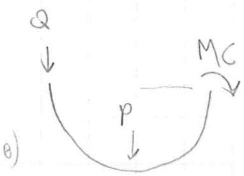
assuming deflection is only in direction of P

Pt. A $\quad M_c - RP - Q2R = 0$

$M_c = PR + 2QR$

$0 = M - M_c + (P+Q)(R - R\sin\theta) \Rightarrow M = PR + 2QR - (PR + QR - PR\sin\theta - QR\sin\theta)$

$M = QR + PR\sin\theta + QR\sin\theta$



$\delta_A = \frac{\partial U}{\partial Q} = \frac{1}{EI}\int_0^{\pi/2} M\left(\frac{\partial M}{\partial \theta}\right) R\, d\theta$

$= \frac{1}{EI}\int_0^{\pi/2} (\overset{0}{QR} + PR\sin\theta + \overset{0}{\cancel{QR\sin\theta}})(R + R\sin\theta)\, d\theta$ $\quad$ because $Q = 0$

$= \frac{PR}{EI}\int_0^{\pi/2} (\sin\theta + \sin^2\theta)\, d\theta$

$= \boxed{\frac{PR^3}{EI}\left(1 + \frac{\pi}{4}\right)}$

Only integrating to $\frac{\pi}{2}$ because Q is zero

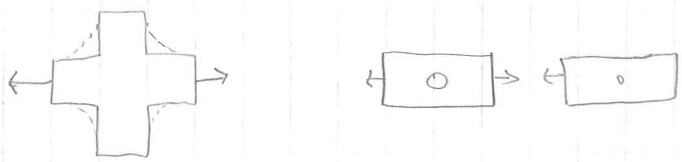
Stress Concentration

Notes 9-15-10

1 Introduction

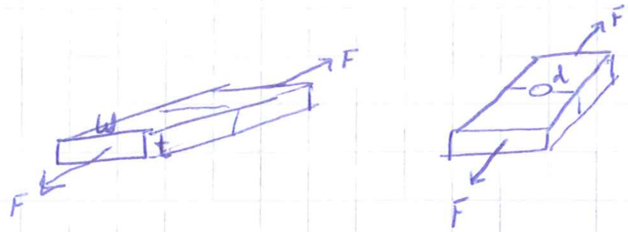
- geometric discontinuities cause an object to experience a local increase in the intensity of a stress field

eg: cracks, sharp corners, holes, changes in cross sections



Notes 9-21-10

Eg

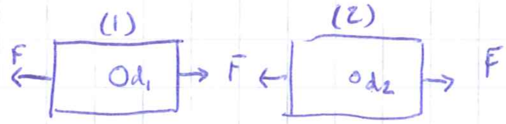


$\sigma = \frac{F}{A} = \frac{F}{wt}$ $\qquad$ $\sigma_o = \frac{F}{A_o} = \frac{F}{(w-d)\,t}$ $\qquad$ stress concentration factor, ~~kt~~ k_t

$\sigma_{max} = k_t\,\sigma_o$ $\quad$ - k_t is for normal stress $\qquad$ $\tau_{max} = \tau_o k_{ts}$

- k_{ts} is for shear stress

See Fig A-15 p. 1026



t is same

σ_o : (1) is larger $\quad k_t$: (2) is larger

A-15-2 Moments (type of load will affect k_t)

A-15-3 $\quad r \downarrow,\ k_t \uparrow$

A-15-4 $\quad$ type of load $\quad r\downarrow\ k_t\uparrow,\ d\downarrow\ k_t\uparrow$

A-15-5 $\quad r\downarrow,\ k_t\uparrow \quad d\downarrow\ k_t\uparrow$ (from curve to curve)

Note: k_{ts}, k_t factor depends on the geometry of the part

- k_t (k_{ts}) is a function of the specific geometric stress raiser in the part (eg. fillet radius, notch, hole, etc.)
- k_t, (k_{ts}) is a function of type of loading applied to the part (axial, bending, torsion)

Homework #1

1) $\Sigma M_x = 0$; $F_A \cancel{\cos 20}(12'') = F_c \cancel{\cos 20}(5'') \Rightarrow F_c = 720 \text{ lbf}$

$\Sigma M_{By'} = -F_A \sin 20(16) + F_{o,z}(36'') - F_c \cos 20\,(10'')$

$\boxed{F_{o,z} = 233.5 \text{ lbf}}$

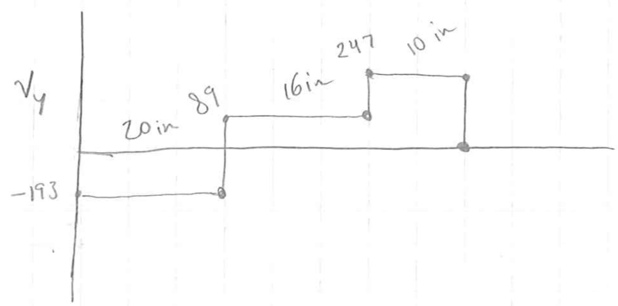
$\Sigma M_{Bz'} = 0 = -F_A \cos 20(16) - F_c \sin 20(10) + F_{oz}(36)$

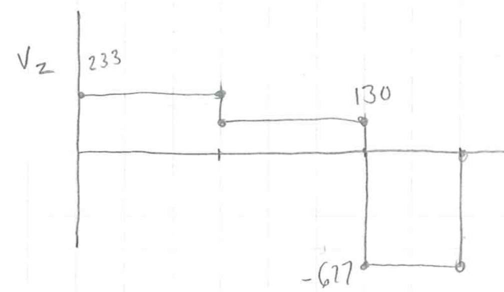
$\boxed{F_{o,y} = -193.7 \text{ lbf}}$

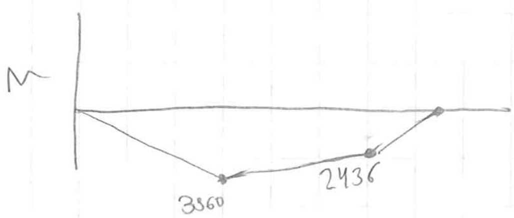
$F_{ay} = 300 \cos 20$ 282
$F_{az} =$ sin 20 −103
$F_{cy} = 720 \sin 20$ −246
$F_{cz} =$ cos 20 677

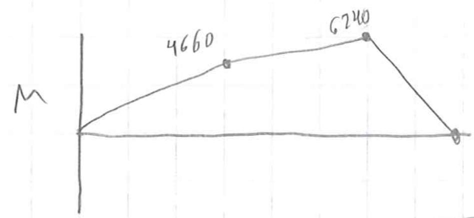
$\Sigma F_y = 0 \quad -193.7 + 300 \cos 20 + B_y - 720 \sin 20$ $\boxed{B_y = 158 \text{ lbf}}$

. . . $\boxed{B_z = -807.5 \text{ lbf}}$









maximum moment at $\boxed{x = 36''}$

$M = \sqrt{6740^2 + 2436^2}$

$\boxed{M = 7167}$

Notes 9-21-10

2. SC vs. materials

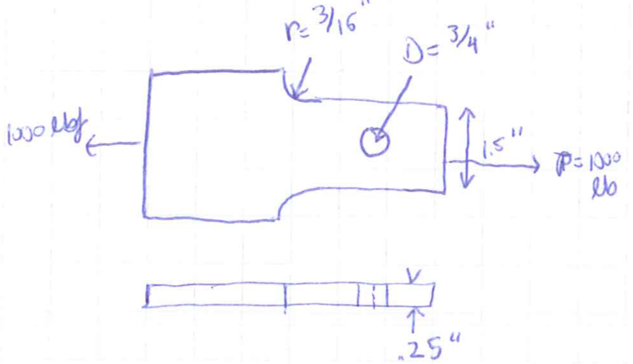
* in static loading for ductile materials, ($\epsilon_f > 0.05$) the S.C. factor is not usually applied because plastic strain has a strengthening effect.

i.e. ductile material, static, $\sigma_{max} = \sigma_0$

- in dynamic loading, SC. needs to be considered.

* Brittle material, $\sigma_{max} = k_t \sigma_0$

Example:



(1) $S_{ut} = 22$ kpsi

(2) actual stress in shoulder, Fig A-15-5

$D/d = ?$ $\quad \dfrac{2.25}{1.5} = 1.5$

$r/d = \dfrac{0.1875}{1.5}$

$k_t = 1.95$

$\sigma_0 = \dfrac{F}{A} = \dfrac{1000}{(1.5)(0.25)} = 2.67$ kpsi

$\sigma_{max} = 5.2$ kpsi

(3) actual stress at hole

a) $D_{hole} = 0.75"$

$w = 1.5"$

$d/w = .75/1.5 = 0.5$

$K_t = 2.19 \quad \sigma_0 = F/A = 1000/(1.5-.75)(0.25) = 5.333$ kpsi

$\sigma_{max} = 2.19\,(5.33)$ kpsi $= 11.68$ kpsi, $>$ 5.2 kpsi

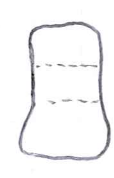
How materials fail

1) mechanical overload (car crash, ball through window)

80% → 2) Fatigue (cyclic loading, or dynamic loading)

3) environmentally assisted problem

4) creep



what is fatigue? mechanical element fails due to fluctuating stresses

- accounts for 80% of structure failures
- normally below S_{ut} or S_y

Notes 9-23-10

Review Session will be ________

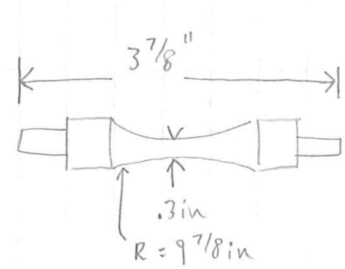
How does fatigue happen
1) microcracks appear due to cyclic plastic deformation
2) grow to real cracks (normal to direction of max stress
3) Continue to grow with cycle, until
4) snap!

Approaches to predict fatigue failure
- both engineering & science
- often science fails, and thus use engineering (experiments)

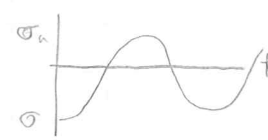
Fatigue life methods
this class → 1) stress life method (§ 6-3, § 6-4) for high cycles ($>10^3$)
2) strain life method (§ 6-5)
3) linear elastic fracture mechanics (§) 6-6

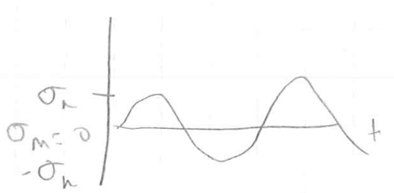
2. fatigue strength

Experiment: - rotating shaft that experiences bending moment
- infinite life: 10^7 cycles (ten million)
$S_f = S_e$ = endurance limit
- Test Samples: a constant bending load is applied & # of cycles of beam required for failure is recorded

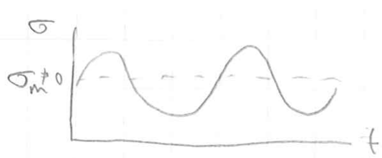


- S_f must always be accompanied by # of cycles
eg. $S_f(N) = 50$ kpsi @ 10^5 cycles. $S_f(N>10^6) = S_e$
- in S-N diagram, beyond the "knee" failure will not occur
- S-N diagram only applicable to alternating load.





$\sigma_m = 0$, alternating load



$\sigma_m \neq 0$. Fluctuating load

Notes 9-23-10

3) Analysis of alternating stresses

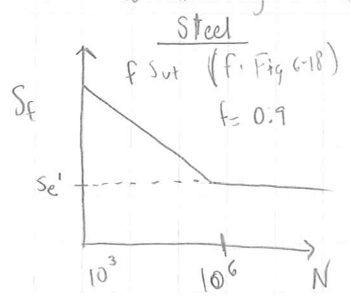
$S_f(N)$ S.F. $n = \frac{S_f(N)}{\sigma_a}$ when $N \to \infty$ (10^7) $S_f(N) = S_e \Rightarrow n = \frac{S_e}{\sigma_a}$

- prime ('): Se': implies strength on a lab specimen in a controlled condition
- variables without ('): Se: implies strength for actual case
- First find Se' or based on estimation, then we modify Se' to find Se

$$S_e' \begin{cases} 0.5\, S_{ut} & S_{ut} \le 200\text{kpsi},\ 1400\text{MPa} \\ 100\text{kpsi} & S_{ut} > 200\text{kpsi} \\ 700\text{MPa} & S_{ut} > 1400\text{MPa} \end{cases}$$

For aluminum & copper alloys, there is no endurance limit (Se')
- they will fail due to repeated loading
- to find equivalent Se' designers use $S_f'(10^8)$

- constructing an estimated S-N diagram



$S_f = aN^b$ $a = \frac{(f\, S_{ut})^2}{S_e}$ (6-14) $b = -\frac{1}{3}\log\left(\frac{f\, S_{ut}}{S_e}\right)$ (6-15)

$N = \left(\frac{\sigma_a}{a}\right)^{1/b}$ (6-16)

Next time - correcting factors

Notes 9-28-10

Exam Covers up-to & including HW #3
In regular room

4) Alternating stresses with correction factors

$S_e = k_a k_b k_c k_d k_e k_f S_e'$

k_a = surface factor

k_b = size factor

k_c = load factor

k_d = temperature factor

k_e = reliability factor

k_f = miscellaneous effects

4.1 Surface Factor, k_a

- the surface of a rotating beam is highly polished
- k_a depends on the quality of the finish of the actual part

$k_a = a\, S_{ut}^{\,b}$ a & b can be found from Table 6-2

4.2 Size factor, k_b -use eq 6-20 p 288

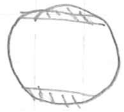
for axial loading, no size effect ($k_b = 1$)

- what if bar is not round?
- what if round bar in bending, not rotating?

⇒ an effective dimension is obtained by equating the volume of material stressed at & above 95% of maximum stress to the same volume in the rotating beam specimen

Rotating beam specimen $A_{.95\sigma} = \frac{\pi}{4}\left[d^2 - (.95d)^2\right]$  $= .0766\, d^2$

$A_{.95\sigma} = .05hb$ $\therefore d_e = .808\sqrt{hb}$

eg.  $d_e = .37d$ Table 6-3

4.3 load factor k_c

$$k_c = \begin{cases} 1 & \text{bending} \\ .85 & \text{axial} \\ .59 & \text{torsion} \end{cases}$$

4.4 Temperature factor Table 6-4 p291

4.5 Reliability factor (6-5 table) only a measure of statistics

4.6 Misc. effects - just a reminder • residual stresses • surface treatment • corrosion, etc.

4.7 Stress Concentration factor p 295 (§ 6-10)

$\sigma = K_f\, \sigma_0$ $\tau = K_{fs}\, \tau_0$ K_f is a reduced value of K_t

σ_0 is the nominal normal stress K_{fs} " " " K_{ts}

τ_f is " shear stress

K_f is called fatigue stress concentration factor

why reduced? some materials are not fully sensitive to the presence of notches, (notch sensitivity, q) use fig. 6-20 to find q p295

6-21 to find q_s

fatigue stress concentration factor

Notes 9-28-10

$q = \dfrac{K_f - 1}{K_t - 1}$ $\quad q_s = \dfrac{K_{fs} - 1}{K_{ts} - 1}$ $\quad$ when $q \to 0,\ K_f \to 1$

when $q \to 1$ $\quad K_f \to K_t$ (sensitive)

procedure

find $K_f \to$ find $\sigma_{max} = K_f \sigma_o$ $\qquad S_e = k_a k_b k_c k_d k_e k_f S_e'$

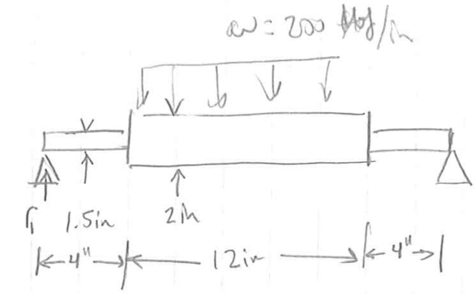
miscellaneous factor (ignore usually)

☆ $\boxed{N > 10^6 \quad n = \dfrac{S_e}{\sigma_{max}}}$ $\qquad 10^3 < N < 10^6 \quad S_f = aN^b,\ n = \dfrac{S_f}{\sigma_{max}}$

Note: difference between K_f & k_f (k_f part of S_e)

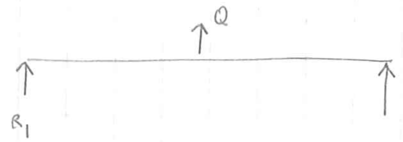
HW #3

2)

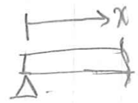


dummy load Q at center

$R_1 = 1200 - \frac{Q}{2}$



Find M
as func. of (x)



$M = ?\quad \overbrace{(x-4)(200)}^{F} \overbrace{\left(\frac{x-4}{2}\right)}^{l}$

Moment about pt. x

$m = \left(1200 - \frac{Q}{2}\right) x$

$$\delta_i = \int \frac{1}{EI}\left(M \frac{\partial M}{\partial F_i}\right) dx = \int \frac{1}{EI_1}\left(M_1 \frac{\partial M_1}{\partial F}\right) dx + \int \frac{1}{EI_2}\left(M_2 \frac{\partial M_2}{\partial F_1}\right) dx$$

7 short answer
2 longer problems

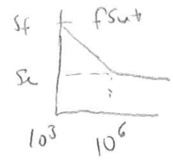
$\boxed{\text{Test 2}}$ material begins here

notes 10-5-10

$n_f = \frac{S_f(N)}{\sigma_{max}} = 1.6$ if $N > 10^6$ $\boxed{n_f = \frac{S_e}{\sigma_{max}}}$ Example from slides

$S_f = aN^b$ $a = \frac{(f\ S_{ut})^2}{S_e}$ $b = -\frac{1}{3}\log\left(\frac{f\ S_{ut}}{S_e}\right)$ life: 1150 rpm (3min)

= 3450 cycles



(T. A-20)

$S_{ut} = 120$ kpsi, $S_y = 66$ $S_e = k_a k_b k_c k_d k_e S_e'$

$S_e' = 0.5(S_{ut})$ (eq 6-8) when $S_{ut} \leq 200$ kpsi $S_e' = 0.5(120\text{ kpsi}) = 60$ kpsi

surface $k_a = a(S_{ut})^b$ (Table 6-2) $a = 2.70$ $b = -.265$ $k_a = 2.7(120)^{-.265} = .759$

size k_b = Initial guess assume $d = 2.0$ inch $k_b = \left(\frac{2.00}{0.3}\right)^{-.107} = .816$

$k_c = 1$; $k_d = 1$; assumed 1, not important $k_e = 1$

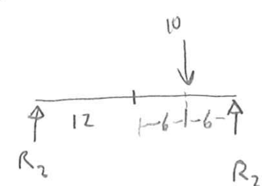
$S_e' = (0.759)(0.816)(60) = 37.2$ kpsi

$a = \frac{(f\ S_{ut})^2}{S_e} = \frac{(0.9 \cdot 120)^2}{37.2\text{ kpsi}}$ $b = -\frac{1}{3}\log\left(\frac{.9 \cdot 120}{37.2}\right) = -.15429$

$= 313.5$

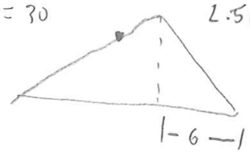
$S_f(N) = aN^b = 313.5(3450)^{-.15429} = 89.2$ kpsi

$n_f = \frac{S_f(N)}{\sigma_{max}} \geq 1.6$? $\sigma_{max} = ?$ $R_1 + R_2 = 10$



$10 \cdot 18 = R_2(24)$ $R_2 = 7.5$ $R_1 = 2.5$

$2.5 \times 12 = 30$ $2.5(18) = 45$



$\frac{45}{\frac{\pi(1.5d)^2}{4}}$ vs $\frac{30}{\frac{\pi(1.0d)^2}{2}}$

$\sigma_0 = \frac{Mc}{I}$ $M = 30$ $c = \frac{d}{2}$ $\frac{I}{c} = \frac{\pi d^4}{64} / \frac{d}{2} = .09817 d^3$

$\sigma_0 = \frac{M}{\frac{I}{c}} = \frac{30}{.09817 d^3} = 38.2$ kpsi (if $d = 2$) see fig A-15.9

$r/d = \frac{0.1d}{d} = 0.1$ $D/d = \frac{1.5d}{d} = 1.5$ $K_t = 1.68$

fig 6

Share this class w Scott k.

fig 6-20 $d = 2$ $r = .1d = 0.2$ $q = 0.87$ ($r = 0.2$, $S_{ut} = 120$)

$K_f = 1 + q(K_t - 1) = 1.0 + 0.87(1.68 - 1) = 1.59$

$\sigma_{max} = \sigma_o(1.59) = (38.2)(1.59) = 60.7$ kpsi

$n_f = \frac{89.2}{60.7} = 1.47$

$\Rightarrow$ may need to increase d to say 2.25"

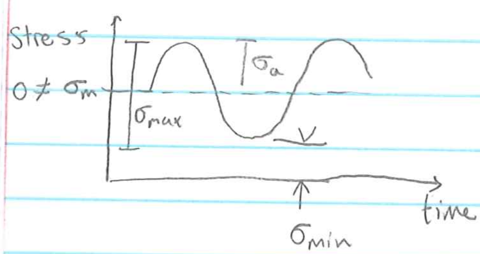
$K_b = 0.91 d^{-.157} = .801$ $S_e = (0.759)(0.801)(60) = 36.40$ kpsi

$\sigma_o = \frac{30}{.09817 d^3} = 26.8$ kpsi

$r = .1d = .225$ $r/d = 0.1$ $q = 0.87$ $K_t = 1.60$ ($r/d = .1$ $D/d = 1.5$)

$K_f = 1.59$ $\sigma_{max} = 42.61$ $n_f = \frac{89.2}{42.61} = 2.1 > 1.6$

5. fluctuating load

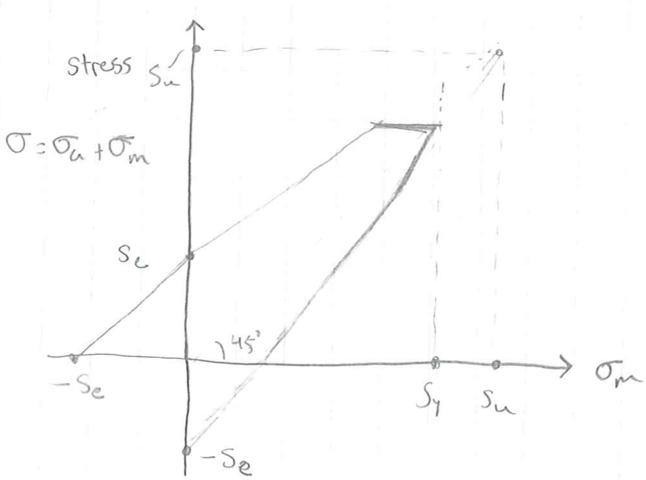


average stress

$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$

$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$

5.1 Modified Goodman Diagram $\boxed{\text{Figure 6-24}}$



σ_m average stress
S_u ultimate strength
σ_a amplitude of fluctuating load
S_e allowable stress after stress factors

Note: when $\sigma_m = 0$, S_e governs (S-N diagram)

$n = \frac{S_e}{\sigma}$; $n = \frac{S_f}{\sigma}$

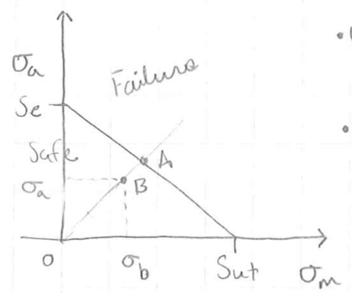
when $\sigma_m \geq S_y$ fail. or when $\sigma_m \pm \sigma_a \geq S_y$ fail

Figure 6.25 – shows that when $\sigma_m < 0$, existence of σ_m has little effect $\Rightarrow$ use S-n diagram

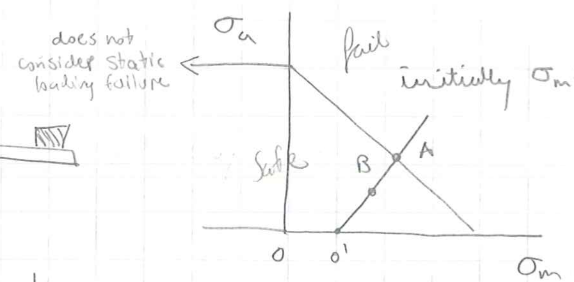
– $\sigma_m > 0$ has great impact

- many theories to use

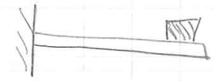
Fig 6.27



- when $\sigma_m = 0$, $n = \dfrac{Se}{\sigma_a}$
- when $\sigma_m > 0$, $n = \dfrac{OA}{OB}$



- if initially $\sigma_m \neq 0$

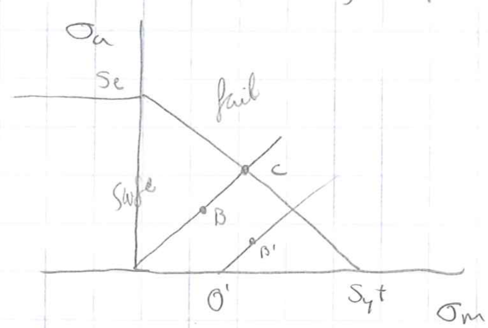


$$n = \frac{O'A}{O'B}$$

- if $\sigma_a = 0$, $\sigma_m = 0$, initially $\quad \dfrac{S_a}{S_e} + \dfrac{S_m}{S_{ut}} = 1$

$$\boxed{\frac{\sigma_a}{Se} + \frac{\sigma_m}{Sut} = \frac{1}{n}}$$

soderberg theory

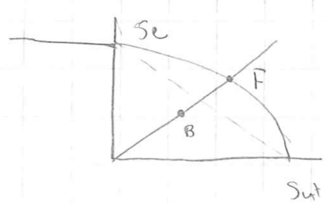


5.2 Soderberg Theory

$$n = \frac{OC}{OB}$$

for finite life $(N < 10^6)$ $S_f = aN^b$ replaces Se

5.3 Gerber theory



$$n = \frac{OF}{OB}$$

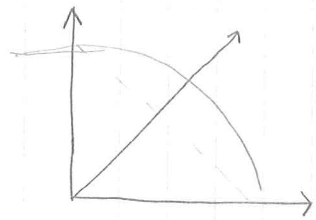
$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1 \quad \text{initially } \sigma_a = \sigma_m = 0$$

$$\boxed{\frac{n\sigma_a}{Se} + \left(\frac{n\sigma_m}{Sut}\right)^2 = 1}$$

Note: Pt B is current operating condition

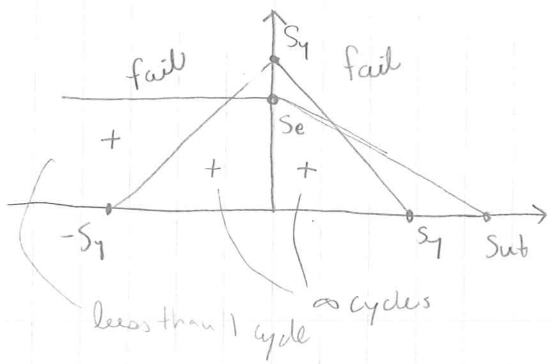
Notes 10-5-2010

5.4 ASME ELLIPTIC



$$n = \frac{OE}{OB} \qquad \left(\frac{n\sigma_a}{S_e}\right)^2 + \left(\frac{n\sigma_m}{S_y}\right)^2 = 1$$

5.5 Yield (Langer) line



need to consider Langer line in fatigue design

$$\sigma_{max} = \sigma_a + \sigma_m = \frac{S_y}{n}$$

$$n = \frac{S_y}{\sigma_{max}}$$ $\Rightarrow$ Langer line is guarding against yield.

5.6 analysis strategy

approach I (1) assume fatigue occurs first then use eqn to determine n or size

(2) follow with a static check. If static failure governs, analysis is repeated using Langer theory 1st

approach II use 6-6, 6-7, 6-8 tables to determine the load line & establish which criterion the load line intersects 1st & use the corresponding eqn

Tables 6-6, 6-7, 6-8 — Notes 10-12-10

summary for fluctuating stresses

(1) For $\sigma_m \le 0$

(1.1) if $\sigma_a > S_f$ (or S_e) Fail, dynamic

(1.2) if $|\sigma_{max}| \ge S_{yc}$ (or S_{uc}) $\rightarrow$ static failure

(2) For $\sigma_m \ge 0$

use criterion (such as goodman) $\frac{\sigma_a}{S_f} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$ if $n<1 \rightarrow$ fail

use yield criterion $\frac{\sigma_a}{S_y} + \frac{\sigma_m}{S_y} = \frac{1}{n}$, $n = \left|\frac{S_y}{\sigma_{max}}\right|$ $n<1 \rightarrow$ failure

not exactly static, but different from dynamic failure.

Ex. in class

Table A-20 $S_{ut} = 64$ kpsi

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} \qquad \sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_0 = \frac{3000}{(0.375)(1-.25)} = \frac{3000(10^{-3})}{.2813} = 10.67 \text{ kpsi}$$

$$\boxed{\sigma_{max} = K_f \sigma_0} \text{ (dynamic condition)}$$

K_t, table A-15-1 $d/w = .25 \Rightarrow K_t = 2.45$ $K_f = 1 + q(K_t - 1)$

$q = 0.8$ (from fig 6-20) $\Rightarrow$ $K_f = 2.16$

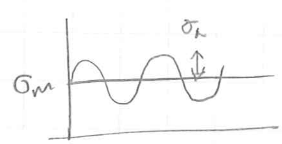
$$\sigma_{max} = 10.67(2.16)$$

$$n_y = \frac{S_y}{|\sigma_{max}|} = \frac{54}{(10.67)(2.16)} = \boxed{2.343} \text{ yield, langer}$$

$$\sigma_a = \left|\frac{F_{max} - F_{min}}{2A}\right| = 2.16\left|\frac{(3-.8)}{2(.2813)}\right| = 8.45 \text{ kpsi}$$

$$\sigma_m = \left|\frac{'' + ''}{''}\right| = 14.6 \text{ kpsi}$$

$\Rightarrow$



initial condition not given, assumed zero

$S_e' = k_a k_b k_c k_d k_e S_e'$ $\quad S_e' = 0.5(64) = 32$ kpsi $\quad k_a = 2.7(64)^{-.265} = .897$

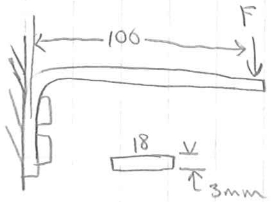
$k_b = 1$ (axial load) $k_c = .85$ (") $k_d = 1$ $k_e = 1$ $\quad S_e = 24.4$ kpsi

$$n_f = \frac{1}{2}\left(\frac{64}{14.6}\right)^2\left(\frac{8.45}{24.4}\right)\left[-1 + \sqrt{1 + \left(\frac{2\,\sigma_m S_e}{S_{ut}\,\sigma_a}\right)^2}\,\right] = \boxed{2.17} < 2.343$$

$$\text{ASME}: \sqrt{\frac{1}{\left(\frac{\sigma_a}{S_e}\right)^2 + \left(\frac{\sigma_m}{S_y}\right)^2}} = 2.28$$

Notes 10-12-10

6-21) inclass example



bending

a) $y = \frac{Fl^3}{3EI}$ $\quad F = \frac{3EIy}{l^3}$ } Formula: p 1013

$I = \ldots 40.5\ mm^4$

$E = 207(10^9)$

$F_{min} = \frac{3(207 \times 10^9)\ 40.5(10^{-12})\ (2\times10^{-3})}{100^3\ (10^{-9})}$

$F_{min} = 50.3N$

$F_{max} = \frac{6}{2}(50.3) = 150.9N$

Fig 3.34 p118 Table 3-4 p121 $\quad \sigma_i = \frac{Mc_i}{Aer_i}$

Notes 10-14-10

Quiz 10-12:

Modified Goodman: - designed for fluctuating loads
- when $\sigma_m < 0$, σ_m has no effect in calculating factor of safety
- when $\sigma_m = 0$, $n_f = \frac{S_e}{\sigma_a}$

example continued.

$\sigma_o = -\frac{Mc_o}{Aer_o}$ $\quad$ Given $r_i = 3mm$ $\quad r_o = 6mm$

From T 3-4: $r_c = r_i + \frac{h}{2}$ $\quad r_n = \frac{h}{\ln(r_o/r_i)}$ $\quad e = r_c - r_n = 4.5 - 4.328 = .1719\ mm$

$\sigma_i = -\frac{Mc_i}{Aer_i} - \frac{F}{A} = \frac{(101.5\ N\cdot mm\ F)(\frac{3}{2} - .1719)\ mm}{(18\cdot3\ mm^2)(.1719\ mm)(3mm)} - \frac{F}{54} = \underline{4{,}859\ F\ MPa}$

* Moment $= F(100\ mm + \frac{3\ mm}{2}) = F(101.5mm)$ $\quad C_i = (\frac{h}{2} - e)$ $\quad C_o = (\frac{h}{2} + e)$ shown in 3-34

$\sigma_o = \frac{Mc_o}{Aer_o} - \frac{F}{A} = \frac{(101.5F)(1.5 + .1719)}{54(.1719)(6mm)} - \frac{F}{54} = \underline{3.028 F MPa}$

plug $F_{min,max}$ into $\sigma_{o,i}$ to get

$\sigma_{i\ min} = \ldots -733.2\ MPa$

$\sigma_{i\ max}\ -244.4\ MPa$

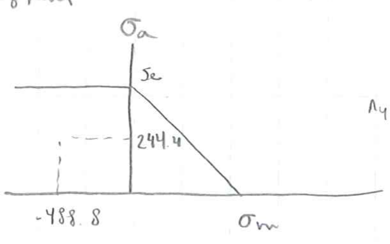
$\sigma_{o\ min}\ 456.8\ MPa$

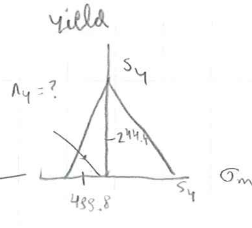
$\sigma_{o\ max}\ 152.3\ MPa$

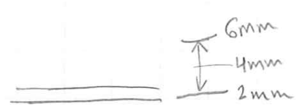
at inner radius $(\sigma_i)_a = \left|\frac{-733.2 + 244.4}{2}\right|\ 244.4$

$(\sigma_i)_m = \frac{-733.2 - 244.4}{2} = -488.8\ MPa$

fatigue







HB

$S_y = 0.9\ S_{ut}$ Eq (2.17) $S_{ut} = 3.41(490) = 1671$ MPa

uses Bhn to find strength

$S_y = 0.9(1671) = 1504$ MPa

$S_e = ?$ eq (6-8) $S_e' = 700$ MPa $(S_{ut} > 1400$ MPa$)$

$S_e = k_a k_b k_c k_d k_e S_e'$ $k_a = 1.58(S_{ut})^{-.085} = 0.841$

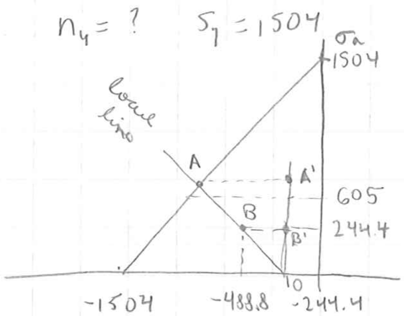
k_b : Table 6-3 $d_e = .808\sqrt{18 \times 3} = 5.938$ mm $k_b = \left(\frac{5.938}{7.62}\right)^{-.107}$

$k_b = -.107$ k_c = bending, $= 1$ $k_d = 1$, ignored $k_e = 1$, ignored

$S_e = (.841)(1.027)(700) = \boxed{605 \text{ MPa}}$

initial conditions NOT $= 0$, CANNOT USE Table 6-7, 6-8, 6-6

$n_y = ?$ $S_y = 1504$



load line $\sigma_a = A\sigma_m + B$ $(\sigma_a = 0$
$\sigma_m = -244)$ $(\sigma_a = 244.4$ $\sigma_m = -488)$
$A = -1$ $B = -244.4$ $\Rightarrow \sigma_a = -\sigma_m = -244.4$

Langer yield line: $\sigma_m = \sigma_a - 1504$

Intersection: $\sigma_m - 1504 = -\sigma_a - 244.4$ $\sigma_a = 629.8$ MPa $\sigma_m = -872.4$ MPa

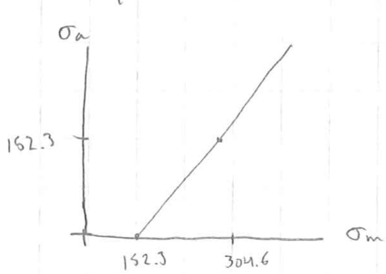
Need to find) $n_y = \frac{OA}{OB} = \frac{OA'}{OB'} = \frac{629.8}{244.4} = 2.58$

$n_f = ?$ $\sigma_m < 0 \searrow$ $\frac{S_e}{\sigma_a} = \frac{605}{244.4} = 2.48 < n_y$

At outer radius $(\sigma_o)_a = \left|\frac{456.9 - 152.3}{2}\right| = 152.3$ MPa

$(\sigma_o)_m = \frac{456.9 + 152.3}{2} = 304.6$ MPa > 0

n_y : Yield line



load line $\sigma_m = 152.3 + \sigma_a$ Langer line: $\sigma_m = 1504 - \sigma_a$

$\sigma_a = 675.9$ MPa $\sigma_m = 828.2$ MPa $n_y = \frac{675.9}{152.3} = \frac{OA'}{OB'} = 4.44$

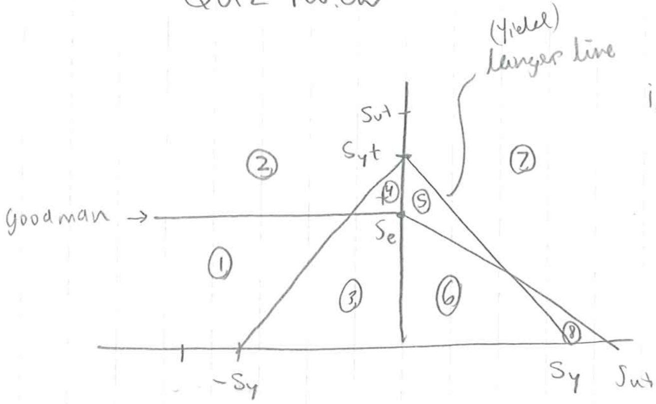
$n_f = ?$ say Gerber $\frac{\sigma_a}{S_e} + \left(\frac{\sigma_m}{S_{ut}}\right)^2 = 1$ $\sigma_a = \left[1 - \left(\frac{\sigma_m}{S_{ut}}\right)^2\right] S_e$

$= \sigma_m - 152.3$ $\sigma_m^2 + 4615.3\,\sigma_m - 3.4951 \times 10^6 = 0$

$\sigma_m = 662.2$ MPa > 0 $\sigma_a = 509.9$ MPa $n_f = \frac{509.9}{152.3} = 3.35$

Quiz review

Notes 10-19-2010



infinite	finite	less than 1 cycle
6	4	7 1
3	5	2
		8

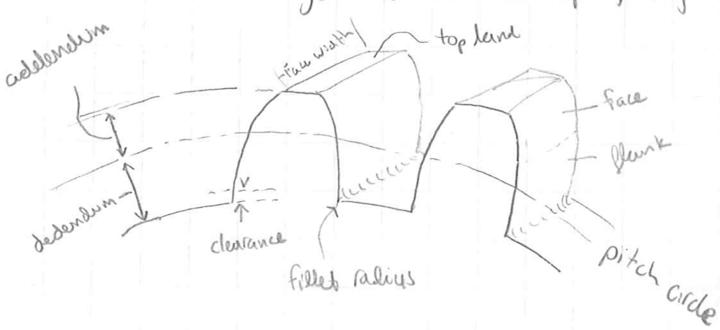
outside yield line ⇒ fails < 1 cycle

Exam 2 - covers fatigue & part of gears

Gears 1) Introduction p 674 (§13-1)
2) Force Analysis p 705 (§ 13-14)

1.1 types of gears - parallel axis gears : spur & helical gears
- non-parallel, coplanar gears: bevel & spiral gears
- non parallel, noncoplanar gears: worm gears

- spur gear : low cost, noisy,
- helical gears : smoother, thrust load, can transmit heavier load, shafts may/may not be parallel
- Bevel gears : power redirection
- worm gears : worm is input, high speed ratio (3 or more), carrying capability is low



circular pitch $p = \frac{\pi d}{N}$ where N = # teeth

module $m = \frac{d}{N}$

diametral pitch $P = \frac{N}{d} = \frac{1}{m}$

- for 2 gears to mesh must have same diametral pitch P & p & m
- addendum $= a = \frac{1}{P}$ • dedendum $b = 1.25/P$ • clearance $= b - a$

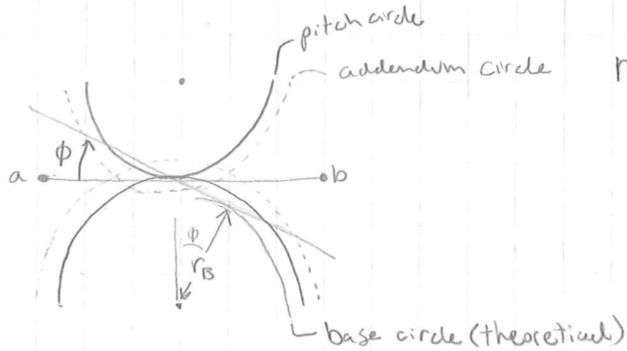
Conjugate action: involute profile is commonly used for gear teeth to produce conj. action (a constant velocity ratio during meshing)

$V = |r_1 \omega_1| = |r_2 \omega_2|$

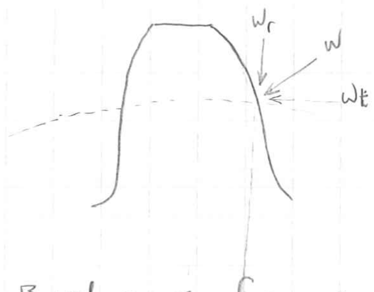
Notes 10-19-2010

involute properties

ϕ is called pressure angle $(20°, 25°, 14.5°)$



r_B = radius of base circle $= r\cos\phi$



W_r always points towards center of circle

W_t does work

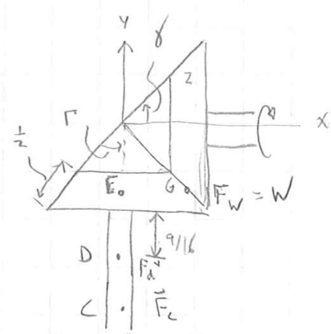
Formulas

$W_t =$

$W_r =$

Bevel gears force analysis

Ex | Problem like 13-43



$10 = P = \frac{N}{d} = \frac{15}{d_2} \Rightarrow d_2 = 1.5 \text{ in.}$ $\quad W_t = 30 \text{ lb}$

$10 = P = \frac{N}{d} \Rightarrow d_3 = \frac{25}{10} = 2.5 \text{ in}$

$\gamma = \tan^{-1}\frac{N_P}{N_G} = \tan^{-1}\frac{15}{25} = 30.96°$

$\Gamma = \tan^{-1}\frac{N_G}{N_P} = \tan^{-1}\left(\frac{25}{15}\right) = 59.04°$

$W^r = W^t \tan\phi\cos\gamma = 30\tan 20°\cos\Gamma$

$= 30\tan 20\cos 59.04$ (gear 3)

$W^r = 5.617 \text{ lbf}$

$W^a = W^t\tan\phi\sin\Gamma = 30\tan 20°\sin 9.04 = 9.363 \text{ lbf}$ $\quad W^t = 30 \text{ lbf}$

$W = (-5.617 \text{ lbf } i - 9.363 j + 30 k)$

Now, $\vec{DG} = ?$ $\quad DE = \frac{9}{16} + \frac{.5}{2}\cos\Gamma = .6916 \text{ in}$ $\quad EG = \frac{d_3}{2} = 2.5/2\, i$ $\quad \boxed{DG = 1.25 i + .6916 j}$

$DC = -\frac{5}{8} j = -.625 j$ $\quad \Sigma M_D = \vec{DG}\times\vec{W} + \vec{DC}\times\vec{F_c} + \vec{T} = 0$

$\vec{DG}\times\vec{W} = (1.25 i + .696 j)\times(-5.617 i - 9.363 j + 30 k) = 20.739 i - 37.5 j - 7.819 k$

$\vec{F_c} = F_c^x i + F_c^z k$ $\quad \vec{D_c}\times\vec{F_c} = -.625 F_c^z i + .625 F_c^x k$ $\quad \vec{T} = T j$

$\Sigma T_i = 0$ $\quad j = 0$ $\quad F_c^z = 33.2$ $\quad F_c^x = 12.51$ $\quad T = 37.5 \text{ lbf}\cdot\text{in}$

$|F_c| = 35.48$ $\quad F_D = -.6893 i + 9.363 j - 63.2\hat{k} \text{ lbf}$ $\quad F_D(\text{thrust}) = 9.363 \text{ lbf}$

$F_D(\text{radial}) = 63.57 \text{ lbf}$

Notes 10-21-10

Quiz review -all correct

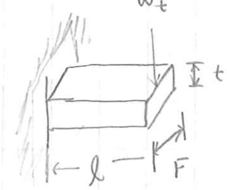
circular pitch p. circle p. diameter, addendum dedendum fig 13-5

3.0 Lewis Bending Equation p734 § 14-1

3.1 AGMA Bending & pitting 745 §14-15

3.0| Lewis Bending Eq.

Simple model, whereas gears have complex geometry



$I = \frac{Ft^3}{12}$

$$\sigma = \frac{Mc}{I} = \frac{W_t\, l\, \frac{t}{2}}{\frac{Ft^3}{12}}$$

$$= \frac{W_t}{F} \frac{1}{t^2/6l} = \frac{W_t}{F} \frac{1}{t^2/4l} \left(\frac{1}{4/6}\right) = \frac{W_t}{F} \frac{p}{\left(\frac{2}{3}\right) x\, p}$$

where $x = \frac{t^2}{4l}$

let $y = \frac{2x}{3p}$ = Lewis form factor $\sigma = \frac{W_t}{Fpy}$ $P = \frac{\pi}{p}$

$$\boxed{\sigma = \frac{W_t P}{FY}}$$

$Y = \pi y$

P diametral pitch

F modified Lewis form factor

3.1|

(1) $\sigma = W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}$ AGMA Bending stress 14-15 eq.

(2) $\sigma_c = C_p \sqrt{W^t K_o K_v K_s \frac{K_m}{d_p F} \frac{C_f}{I}}$ AGMA Pitting stress (US units) eq. 14-16

3.2| AGMA strength

allowable bending stress "this is the strength. but different from mat'l strength"

$$\sigma_{all} = \frac{S_t}{S_F} \frac{Y_N}{K_T K_R}$$ (US units) 14-17

allowable pitting stress

$$\sigma_{c,all} = \frac{S_c}{S_H} \frac{Z_N C_H}{K_T K_R}$$ (14-18) (US. units)

Note: - BOTH bending & pitting must be checked. Whichever is less safe is controlling factor

3.3| Summary for gear analysis & design

Fig. 14-17 Bending

Fig. 14-18 Pitting

Modyfying factors

Notes 10-21-10

K_v = dynamic factor, to account for inaccuracies of tooth

$K_v = \left(\frac{A+\sqrt{V}}{A}\right)^B$ where $A = 50 + 56(1-B)$

$B = .25(12-Q_v)^{2/3}$

Q_v = quality standard AGMA has std.

3 — 7 (commonly used)

8 - 12 (precision quality)

K_o = overload factor

Table in Fig 4-18, refers to level of shock in machinery

K_s = size factor $= 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{.0535}$

where F = Face width Y = Table 14-2 depending on # of teeth

K_m = load distribution factor (use pinion)

$= 1 + C_{mc}(C_{pf} + C_{pm} + C_{ma}\,C_e)$

- $C_{mc} = \begin{cases} 1 & \text{for uncrowned} \\ 0.8 & \text{for crowned teeth} \end{cases}$
- C_{pf} $\begin{cases} F/10d - 0.025 & F \le 1\text{in} \\ F/10d - .0375 + .01025F & 1 < F \le 17\text{in} \\ F/10d - .1109 + .0207F - 0.000228F^2 & 17 < F \le 40\text{in} \end{cases}$ (14-32)
- C_{pm}

Quiz review

b) ✓ point of contact between 2 gear teeth is always on the pressure line

pressure line is tangent to both base circles

* Liou available 3:30 tomorrow to answer questions on material

bending stress eq (14-15)
contact stress eq (14-16)
allowable bending stress (14-17)
allowable contact stress (14-18)

K_m, dynamic factor $\quad k_m = 1 + C_{mc}(C_{pf} C_{pm} + C_{ma} C_e)$ (14-30)

$C_{pm} = 1$ for $\frac{S_1}{S} < 0.175$ $\quad C_{pm} = 1.1$ for $\frac{S_1}{S} \geq .175$

S_1 = offset of gear wrt. centerline of shaft between bearings p760 Fig 14-10

$C_{ma} = A + BF + CF^2$ mesh alignment factor Table 14-9 or $\boxed{\text{Fig 14-11}}$ p.761

$C_e = \begin{cases} .8 \text{ for gearing adjusted at assembly is improved by} \\ \text{lapping or both, for all other conditions} \end{cases}$

K_B = rim thickness factor $\quad m_B = \frac{t_R}{h_t} \quad \begin{cases} m_B \geq 1.2 & k_B = 1 \\ m_B \leq 1.2 & k_B = 1.6 \ln\left(\frac{2.242}{m_B}\right) \end{cases}$

J: geometry factor; Fig 14-6

Example: find bending stress: use fig. 14-17

$d_p = \frac{N_p}{P_d} \quad W_t = \frac{33{,}000\, H^{(hp)}}{V\ (ft/min)} \quad V = \frac{\pi d^{(in)} n^{(rpm)}}{12} \quad d_p = \frac{N_p}{P_d} = \frac{16 \text{ teeth}}{6 \text{ teeth/inch}} = 2.667 \text{ in.}$

$d_G = \frac{N_G}{P_d} = \frac{48}{6} = 8 \text{ inch} \quad V = \frac{\pi d n}{12} = \frac{\pi (2.667 \text{ in})(300 \text{ rev/min})}{12 \text{ in/ft}} = 209.4 \text{ ft/min}$

$W_t = \frac{3300H}{V} = \frac{3300(5hp)}{209.4 \text{ ft/min}} = 787.8 \text{ lbf}$

$K_o = 1 \quad k_v \Rightarrow$ eq (14-27, 28) $\quad Q_v = 6 \quad B = .25(12-6)^{2/3} = .8255$

$A = 50 + 56(1-B) = 59.77 \quad \boxed{k_v = 1.196}$

$K_s = 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{.0535} \quad F = 2 \quad P = 6 \quad$ Table 14-2 $\quad N_p = 16 \quad N_G = 48$

$Y_p = .296 \quad Y_G = .4056 \quad (K_s)_P = 1.088$

$(K_s)_G = 1.097$

Example continued

(14-30)

$K_m = 1 + C_{mc}(C_{pf} C_{pm} + C_{ma} C_e)$ $C_{mc} = 1$ uncrowned teeth (14-31)

$C_{pf} = \frac{F}{10d} - .0375 + .0125 F$ $C_{pf,p} = \frac{2}{10(2.667)} - .0375 + .0125(2) = .0625$

$(C_{pf})_G = \frac{2}{10(8)} - .0375 + .0125(2) = .0125$

$C_{pm} = 1$ C_{ma} = eq 14-34 fig 14-11 → Curve 2 ⇒ $C_{ma} = .15$

$C_e = 0.8$ (accurately & rigidly mounted)

$K_{m,p} = 1 + 1[.0625(1) + 0.15(.8)] = 1.183$

$K_{m,G} = 1 + 1[.0125(1) + 0.15(.8)] = 1.1325$

$K_B = 1$ (not mentioned)

J: $N_p = 16$ $N_G = 48$ Fig 14-6 $J_p = 0.27$ $J_G = .39$

$\sigma_p = 787.8(1)(1.196)(1.083)\left(\frac{6}{2}\right)\frac{1.183(1)}{.27} = 13475$ psi

$\sigma_G = 787.8(1)(1.196)(1.097)\frac{6}{2}\left(\frac{1.1325(1)}{.39}\right) = 9245$ psi

Exam review

Fluctuating load problem steps
look up S_u & S_t

Check:
Quiz 14, I answered this correctly

1 side crib sheet

Quiz review

11-1-2010

√ A) SI cannot directly use eqn
B) P_d NOT pitch diam
C) F Not contact force
√ d) k_o overload
√ e) k_v dynamic

~~10-2~~

11-2-10

3.4) summary for contact stress Fig 14-18

geometry factor

$$I = \begin{cases} \dfrac{\cos\phi_t \sin\phi_t}{2 m_N} \dfrac{m_G}{m_G+1} & \text{external gears} \quad (14\text{-}23) \\[2ex] \dfrac{\cos\phi_t \sin\phi_t}{2 m_N} \dfrac{m_G}{m_G-1} & \text{internal gears} \end{cases}$$

$$m_G = \frac{N_G}{N_P} = \frac{d_G}{d_P}$$

C_p, find using table 14-8
← Elastic coefficient

Example $\sigma_c = ?$ P 14-19

$$m_G = \frac{48}{16} = 3 \qquad I = \frac{\cos 20 \sin 20}{2(1)} \frac{3}{(3-1)} = .1205$$

C_p (Table 14-8) $= 2300 \sqrt{psi}$

$$(\sigma_c)_P = 2300\left[(787.8)(1)(1.196)(1.088)\frac{\overset{K_m}{1.183}}{(2.667)(2)(.1205)}\right]^{1/2} \qquad \sigma_c = 99905 \text{ psi}$$

K_o K_v d_P F

$$(\sigma_c)_G \quad 2300\left[787.8(1)(1.196)(1.097)\frac{1.1325}{2.667(2)}\frac{1}{.1205}\right]^{1/2} \qquad \sigma_{c,G} = 100317$$

K_o K_v K

3) summary for bending strength, (σ_{all}) & contact strength $(\sigma_{c,all})$

$$\sigma_{all} = \frac{S_t}{S_F}\frac{Y_N}{K_T K_R} \quad (14\text{-}17) \qquad \sigma_{c,all} = \frac{S_c}{S_H}\frac{Z_N}{K_T}\frac{C_H}{K_R} \quad (14\text{-}18)$$

S_t based on grade of materials

S_F, S_H = safety factors

if $< 250°F$ ($120°C$) $K_T = 1$
$\geq 250°F$ $K_T > 1$ } book doesnt have this

K_R = reliability factor ↔ S_t & S_c are based on reliability factor of 99%.

C_H = hardness ratio factor Fig 14-12

Y_N & Z_N stress cycle factors to modify gear strength for cycles other than 10^7 cycles

Y_N above 10^7 → use eqn above curve $Y_N = 1.3558 N^{-...}$ fig 14-14

Z_N fig 14-15

bending endurance allowable

Material grade

stress cycle factor

$$\frac{S_t \quad Y_N}{S_F \quad K_T \, K_R}$$

reliability factor (modify from S_t)

S.F

temp factor

contact endurance allowable

material grade

stress cycle factor

$$\frac{S_c \, Z_N \, C_H}{S_H \, K_T \, K_R}$$

hardness ratio (only for gear)

S.F.

reliability factor

Temp Factor

11-2-2010

Example 14-19

$(S_t)_P = (S_t)_G = 77.3(200) + 12800 = 28260$ psi

Fig 14-15 $(S_c)_P = (S_c)_G = 332(200) + 29100 = 93500$ psi

Fig 14-14 $Y_{N_P} = 1.3558(10^8)^{-.0178} = .997$

Y_{NG} " $\left(\frac{10^8}{3}\right)$ " $= .996$

$(Z_N)_P = 1.448(10^8)^{-.023} = .948$ $(Z_N)_G = 1.448 \left(\frac{10^8}{3}\right)^{-.023} = .973$

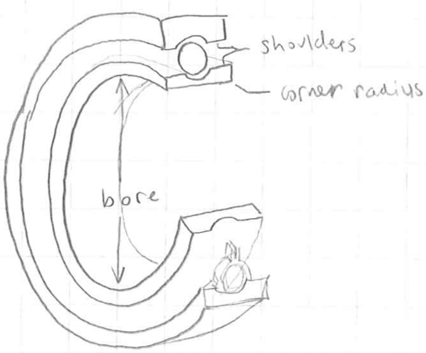
reiteration: $\boxed{\sigma_{all} \quad \frac{S_t}{S_F} \quad \frac{Y_N}{K_T K_R} \qquad \sigma_{c,all} = \frac{S_c Z_N C_H}{S_H K_T K_R}}$

$C_H = 1$ $H_{BP}/H_{BG} = 1$ $K_T = 1$ $K_R = .85$ (Table 14-10)

$S_{F,P} = \ldots 2.41$ | $(S_H)_P = \ldots 1.04$
$S_{F,G} = 3.58$ | $(S_H)_G = \ldots 1.07$

} hardness of pinion & gear should be increased.

Bearings



11-4-2010

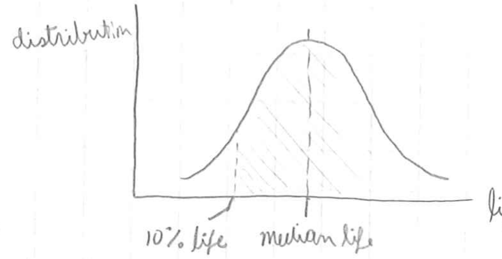
Quiz review B C D E are right 100%

2| Bearing life

- # of revolutions of inner ring until 1st fatigue
- # of hours of use " "

2.1 Rating life = # of revolutions (or hours at const. speed) that 90% of a group of bearings will achieve before failure (10th percentile location)

• also called L_{10} life



2.2 reliability vs bearing life,

$FL^{1/a}$ = constant

F ← load, L ← rating life

$a = 3$ for ball bearings

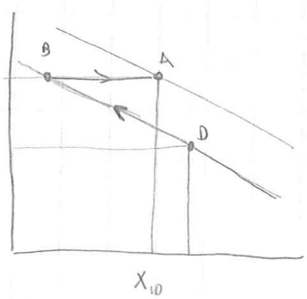
$a = 10/3$ for roller bearings

- a bearing company may choose a rated cycle value of, say, 10^6 revolutions as declared in the catalog

(11-3) $C_{10} = (L_R n_R 60)^{1/a} = F_D(L_D n_D 60)^{1/a}$

C_{10} – catalog rating; L_R – rating life; n_R – rating speed; F_D – design radial load; L_D – designed life; n_D – designed speed

$$C_{10} = F_D\left(\frac{L_D n_D 60}{L_R n_R 60}\right)^{1/a}$$



← fig 11-5

$$R = \exp\left[-\left(\frac{x - x_0}{\theta - x_0}\right)^b\right]$$

x_0 is minimum value of x

$x = \frac{L}{L_{10}}$

θ = characteristic parameter

<u>Manufacturer # 2</u> p608 – example of x & x_0

to find catalog rating, D → B → A

① D → B $F_B x_B^{1/a} = F_D x_D^{1/a}$ $F_B = F_D\left(\frac{x_D}{x_B}\right)^{1/a}$

② B → A $R_D = \exp\left[-\left(\frac{x - x_0}{\theta - x_0}\right)^b\right]$ $x_B = x_0 + (\theta - x_0)\left(\ln \frac{1}{R_D}\right)^{1/b}$

$$C_{10} = F_B = F_D\left[\frac{x_D}{x_0 + (\theta - x_0)\left(\ln \frac{1}{R_D}\right)^{1/b}}\right]^{1/a}$$

$F_B = C_{10}$

As loads are often non-steady, desired load is modified by an application factor, a_f

$R_D = \exp\left[-\left\{ x_D \ldots\right.\right.$ (11-18)

23 | multiple bearings .579

a shaft w/ 2 bearings with individual reliabilities R_a & R_B
the bearing reliability of the shaft is $\boxed{R = R_A \cdot R_B}$

Notes 11-9-10

quiz review - ~~I got~~ ans: A, B, C, E, I gave A, B E $\Rightarrow \boxed{8/10}$

Example - learn to look up bearings in catalogs

$R = R_A R_B = .9 = R_A^2 \quad R_A = 0.95$

Fig 11.5 $\quad x_D = \dfrac{L_D}{L_{10}} = \dfrac{50{,}000\,hr\,(480\,rpm)(60\,min/hr)}{10^6} = 1440$, dimensionless

$F_D = \underset{\text{app. factor}}{1.4}(\underset{\text{load}}{610}) = 854\ lb = 3.8\,kN \qquad$ Table A2 $\quad$ K lbf = 4.45kN

$$C_{10} = F_D \left[\frac{x_D}{x_0 + (\theta - x_0)(\ln 1/R_A)^{1/b}}\right]^{1/a} = 3.8\left[\frac{1440}{.02 + (4.459 - .002)\ln(1/.95)^{1/1.483}}\right]^{1/3}$$

$= 50.4\,kN \quad \Rightarrow$ Table 11-2 $\Rightarrow$ 02-60mm bore

$C_{10} = 55.9\,kN$

(11-18) $\quad R_A = \exp - \left\{\dfrac{x_D\left(\dfrac{a_f F_D}{C_{10}}\right)^a - x_0}{\theta - x_0}\right\}^b = .969 > .95 \quad$ if not, calculation is wrong

For 03, roller bearing

$x_D = 1440 \quad F_D = 1.4(1650) \quad 2310\,lbf = 10.279\,kN$

$$C_{10} = 10.279\left\{\frac{1440}{.02 + (4.459 - .02)(\ln 1/.95)^{1/1.483}}\right\}^{3/10} = 105.2\,kN$$

03 - 60mm bore from table 11-3 $\quad$ 123kN

$$R_B = \exp\left\{-\left[\frac{1440\left(10.28/123\right)^{10/3} - .02}{4.439}\right]^{1.483}\right\} = .977 > .95 \quad ok$$

$R = (\underset{R_a}{.969})(\underset{R_B}{.977}) = .947, > 0.9 \quad$ Ok

3. combined radial & thrust loads
   - some bearings can carry/resist radial and thrust loading
   - for bearing selection, the two loads used to be combined into F_e, equivalent radial load.

A rotation factor V is designed so that $V=1$ when inner ring rotates, $V=1.2$ when outer ring rotates

Fig 11-6 $e = \frac{F_a}{VF_r}$ 2 dimensionless groups are formed & plotted

$\frac{F_e}{VF_r}$ $\frac{F_a}{VF_r}$ F_a = axial load, F_r = radial load

$\frac{F_e}{VF_r} = 1$ if $\frac{F_e}{VF_r} \le e$

$\frac{F_e}{VF_R} = X + Y\frac{F_a}{VF_r}$, if $\frac{F_e}{VF_r} > e$

Table 11-1
C_0 basic static loading
C_{10} basic load rating

Example 02 Series F_{radial} = 8kN F_a = 4kN L_D = 5000 hr
inner ring ω = 900 rpm basic load rating = ? reliability goal = 0.90

$V=1$ $x_D = \frac{L_D}{L_{10}} = \frac{(5000)(900)(60)}{10^6}$ $F_D = 8\text{kN}$

eq [11-6] – $C_{10} = 8\left[\frac{(270)}{.02 + 4.439(\ln 1/.9)^{1/1.483}}\right]^{1/3} = 51.8\text{kN}$

assume Deep groove C_{10}, overestimate = 61.8 kN
$C_0 = 37.5\text{kN}$

$\frac{F_a}{C_0} = \frac{4}{37.5} = .107$ → $e = 0.28 \sim 0.3$

$\frac{F_a}{VF_r} = \frac{4}{8} = 0.5 > e$ axial load is substantial

$X_2 = .56$ Y_2 = interpolate, get 1.46

(11-9) $F_e = X_i V F_r + Y_i F_a$
$= .56(1)(8) + 1.46(4) = 10.32\text{ kN}$

(11-3) $\Rightarrow C_{10} = F_D\left(\frac{L_D n_D 60}{L_R n_R 60}\right)^{1/a}$ $x_D = \frac{L_D}{L_{10}}$
$= 10.32(270)^{1/3}$
$\boxed{C_{10} = 66.7\text{kN}}$

– skipped the rest of notes

Trial #2 Notes 11-11-2010

Quiz review - study C_{10} formula

Shaft design (ch.7.) 1. shaft design

1.1 shaft layout

1.2 shaft design for stress

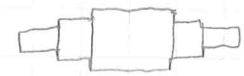
1.3 estimating stress concentration

1) shaft design

- steps (1) consider stress

  (2) determine deflection & slopes

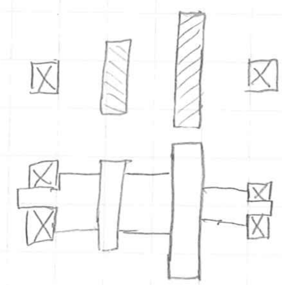
1.1 shaft layout

- the shaft geometry is generally a stepped cylinder eg 

① Lay out the elements to be accomodated by the shaft such as gears, bearings

② find solutions

- better to support load-carrying components between bearings (and close to brngs)
- pulleys & sprockets need to be mounted outboard for ease of install (belts / chain)



- only two bearings should be used in most cases (alignment issues)
- shaft should be kept short (minimize bending)
- some axial space between components is desireable for lubricant flow, disassembly of components
- better to have only one bearing to carry axial load to allow bigger tolerance on shaft length dimensions & prevent binding

1.2 shaft design for stress

$\sigma_{x_i} = \frac{MC}{I}$

- critical locations usually are on the outer surface, at location with large bending moment, where the torque is presant, & where there is stress concentration

$$\sigma_a = K_f \frac{M_a C}{I} \text{ (amplitude)} ; \quad \sigma_m = K_f \frac{M_m C}{I} \quad (7\text{-}1)$$

$$\tau_a = K_{fs} \frac{T_a C}{J} \quad \tau_m = K_{fs} \frac{T_m C}{J} \quad (7\text{-}2)$$

Static analysis

Notes 11-11-2010

Use distortion-energy theory to combine load

DE: $\sigma' = [\sigma_x^2 - \sigma_{xy} + \sigma_y^2 + 3\tau_{xy}^2]^{1/2}$

$\sigma'' = [\sigma_x^2 + 3\tau_{xy}^2]^{1/2}$

$\sigma_a' = \sqrt{\sigma_a^2 + 3\tau_a^2}$ $\sigma_m' = \sqrt{\sigma_m^2 + \tau_m^2}$

for fatigue analysis (eg., Goodman) $\frac{1}{n} = \frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}}$

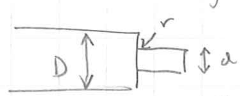
- for a rotating shaft with constant bending & torque $M_m = 0 = \sigma_m$
  $T_a = 0 = \tau_a$

always check for static failure $\sigma'_{max} = [(\sigma_a + \sigma_m)^2 + 3(\tau_a + \tau_m)^2]^{1/2}$

$n_y = \frac{S_y}{\sigma'_{max}}$

1.3 Estimating stress concentration

- shoulders for bearings & support should match catalog recommendation
- Typical bearing $D/d = 1.2 \sim 1.5$



- fillet radius vs bore diameter r/d typically ranging from .02 to .06
- for a std. shoulder fillet, for estimating K_t, assume $r/d = .02$ & $D/d = 1.5$
  $\Rightarrow K_t = 2.7$ for bending
  $K_t = 2.2$ for torsion, Table 7.1
  3.0 for axial

Example 7.3

Note: at table 7.1 $K_f = 1 + q(K_t - 1) \Rightarrow q(d)$ so use K_t to estimate K_f

now plug K_f into $\sigma'_{estimate}$ equation $\Rightarrow$ solve for d., get r based on $\frac{r}{d} = .02$

$q = 0.7$ $q_s = 0.83$ $H_B = S_{ut}/0.49 = 163 < 200$

$K_f = \ldots$

$K_{fs} = \ldots$ $\Rightarrow d = 1.65''$

Notes 11-16-2011

Quiz review   rotating shaft with constant bending & torsion conditions

a) $M_a \neq 0$ b) $\sigma_m = 0$ c) $\sigma_a \neq 0$ d) $T_a = 0$ e) $T_m \neq 0$

I only got this one wrong I think

Example from last class

b) for fatigue analysis (use ASME) $\left(\frac{n_f \sigma_a'}{S_e}\right)^2 + \left(\frac{n_f \sigma_m'}{S_y}\right)^2 = 1$

(1) $\left(\frac{\sigma_a'}{S_e}\right)^2 + \left(\frac{\sigma_m'}{S_y}\right)^2 = \frac{1}{n_f^2}$ $S_e = k_a k_b k_c k_d k_e S_e'$

$S_e' = 0.5\, S_{ut} = 40\text{ kpsi}$ $S_{ut} \leq 200\text{ kpsi}$

$k_a = 2.7(80)^{-0.265} = 0.845$ Assume $d = 2$ to estimate k_b

$k_b = \left(\frac{2}{.3}\right)^{-.107} = .816$ $k_c = 1$, bending $k_d = k_e = 1$

$S_e = (0.845)(0.816)(40) = 27.6\text{ kpsi}$

$\sigma_a' = \sqrt{\left(\frac{32 k_f M_a}{\pi d^3}\right)^2 + 0}$ (7-5) $\sigma_m' = \sqrt{3}\,\frac{16 k_f T_m}{\pi d^3}$ (7-6)

plug these into (1) above, $\Rightarrow \frac{1}{n_f^2} = \frac{1}{2.5^2}$ $k_f =$ $M_a = 4257$ $T_m = 3000$

$\boxed{d = 2.07}$ static $d = 1.65$ fatigue dominates

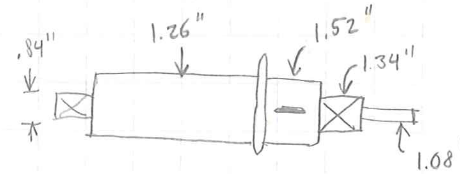
2| Shaft deflection Consideration

- deflection of shaft, both linear & angular, should be checked at gears, bearings
- only gross geometric dimensions need to be included. Local features such as fillets, grooves, keyways, can be neglected for deflection calculation
- once deflection at any point is larger than the allowable deflection at that point, a new diameter can be found using

$d_{new} = d_{old} \left| \frac{n_d\, y_{old}}{y_{all}} \right|^{1/4}$ y_{all} = allowable deflection at that point

n_d = design factor

slope : θ all $d_{new} = d_{old} \left| \frac{n_d\, (dy/dx)_{old}}{(dy/dx)_{new}} \right|^{1/4}$



example: use stress analysis. we found

find deflections & slopes
gear slope = $-.000545$ rad

$\Rightarrow$ gear slope is slightly over $\Rightarrow$ to increase 1.26" diam section

$d_{new} = 1.26 \left| \frac{1.5(.000545)}{0.005} \right|^{1/4} = 1.42''$

3 Limits & Fits § 7.8 p 395 learn definitions

3.1 Capital letters → hole
Lowercase letters → shaft
* definitions Fig 7-20 learn them!

Basic size, to which limits / deviations are assigned (D, d)
deviation difference between a size & the basic size
δF Fundamental deviation upper or lower deviation, whichever is closer to basic size
tolerance max size limit - min. size limit

Hole basis system of fits corresponding to a basic hole size
(H) fundamental deviation
Shaft basis: " " " " shaft size (h)

3.2 IT Grades
- table 7-9 shows the tolerance grade combinations to establish a preferred fit
eg. sliding fit (H6/g6) 16 H7
16 mm basic size 7: IT7 ⇒ .018 mm tolerance
16 g6 16, basic, IT 6 ⇒ .011 mm tolerance

IT: International tolerance #, Table A-11 (SI) Table A-13 (US)

Notes 11-18-2010

Quiz review

√ A crucial location @ pt c.
√ B pt. a, only torsional load
× c k_t can be est. 1.7
× d k_f can be est 2.7
√ e k_{ts} can be est @ 2.2

Notes 11-18-10

3.3) Hole Basis

for shaft: Fundamental deviation, δ_F = upper deviation, δ_u

for hole: Fundamental deviation δ_F = lower deviation, ± 0

ΔD tolerance: = upper deviation

$D_{min} = D \quad D_{max} = D + \Delta D \quad (7\text{-}36)$

for shafts with clearance fits c, d, f, g, h $\quad d_{max} = d + \delta_F \quad d_{min} = \delta + \delta_F - \Delta d \quad (7\text{-}37)$

for shafts with interference fits k, n, p, s, u $\quad d_{min} = d + \delta_f \quad d_{max} = d + \delta_F + \Delta d \quad (7\text{-}38)$

Example: "7-20"

guide pin, nominal size 15mm

find dims for 15mm basic size Locational clearance fit

→ 15 H7/h6 $\quad$ Table A-11 $\quad$ 15 H7 → $\Delta D = .018$ mm

15 H6 → $\Delta d = .011$ mm

Hole $D_{min} = D = 15$mm $\quad D_{max} = D + \Delta D = 15.018$ mm

Shaft: TAble A-12

$d_{max} = d + \delta_f \quad \delta_f = 0 \quad d + \delta_f = 15^{mm} + 0 \quad d_{max} = 15$mm

$d_{min} = d + \delta_f - \Delta d = 15 - 0.011 = 14.989$ mm

Ex "7-21" $\quad$ interference fit, cast iron hub 45mm basic size, medium drive fit

Table 7-9 ⇒ use H7/s6 $\quad$ 45 H7 ⇒ $\Delta D = .025$ mm

45 s6 ⇒ $\Delta d = .016$ mm

hole: $D_{min} = D = 45$mm

$D_{max} = D + \Delta D = 45.025$ mm $\quad \delta_f = +.043$ mm

$d_{min} = d + \delta_f = 45 + .043 = 45.043$ mm

$d_{max} = d + \delta_f + \Delta d = 45.043 + .016$ mm $= 45.059$ mm

Ex - sliding fit

Exam covers $\quad$ Gears → (13, 14)

Bearings (11)

Shaft design (7)

Tolerances (7)

Notes 11-18-2010

1. introduction Fig 8-1

pitch diameter d_p: theoretical,
major diameter, d (nominal diameter)
minor diameter, d_r
- threaded part specs Table 8-1, 8-2, Table A-29

<u>table 8-7</u>

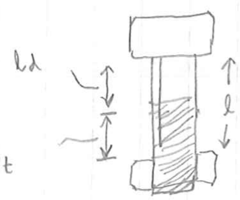
$$L_T = \begin{cases} 2d + \frac{1}{4}in & L \le 6in \\ 2d + \frac{1}{2}in & L > 6in \end{cases}$$

metric

$$L_T \begin{cases} 2d + 6mm & L \le 125 \quad d \le 48mm \\ 2d + 12mm & 125 < L \le 200mm \\ 2d + 25mm & L > 200mm \end{cases}$$

- preload/clamping force: when bolt is tightened, the tension in the bolt

2 Fastener Stiffness (Fig 8-13)



l = grip thickness = $l_d + l_t$

- given d
- $l = l_d + l_t$ $\quad l_d = L - L_T$

model:

$$\frac{1}{k} = \frac{1}{k_d} + \frac{1}{k_t} \qquad k_t = \frac{A_t E}{l_t} \qquad k_d = \frac{A_d E}{l_d}$$

$$k = \frac{k_1 k_2}{k_1 + k_2} \qquad k_{bolt} = \frac{k_t k_d}{k_t + k_d}$$

$A_d = \frac{\pi d^2}{4}$ = area of unthreaded portion

$A_t \Rightarrow$ Table (8-1) or (8-2)

Table 8-7 case 2 $\quad l' = \begin{cases} h + t/2 & t_2 < d \\ h + d/2 & t_2 \ge d \end{cases}$ $\quad h = t + t_1$

washer $\quad$ member 1 thickness

$l_d = L - L_T$

Exam 3 preparation

Ch 7 - shafts 359-400

tolerances

· limits & fits 395-397

shaft stresses 367

fundamental deviations T. A10

design Criteria 369

✓ Ch 11 - bearings 570-604

✓ weibull parameters, x_0, θ, b 608

✓ combined loadings 579

spur & helical

Ch 14 Gears 733-767

spur gear wear 767

Shafts - static analysis

Check for yielding, p370

Notes

12-2-2010

Quiz review

T - hole basis represents a system of fits corresponding to a basic hole size

F - fund. dev. is also IT value

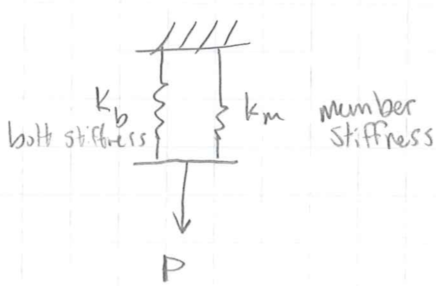
T - 32 H7 means basic hole size of 32

T - 15H7 implies tolerance of .018mm for a hole

F - 15h6 implies a tolerance of .011mm for a hole

(Tolerance Grades chart shown)

analogy for bolts



then we can find how much load is carried by each component

N/A

dream: borrowed Bronco from Andrew
- backed up into restaurant, people upstairs
- called cops
- out of gas

find fatigue factor of safety

get S_{ut}, S_y, H_b

3) Joint member stiffness
- to determine the stiffness of the members in the clamping zone

$$\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} \dots \quad (8\text{-}18)$$

use ultrasonic techniques $\Rightarrow$ eq (8-19)

$25^\circ \le \alpha \le 33^\circ$ for Al, steel use $\alpha = 30^\circ$

eq (8-20)

define: frustum - area under load for the joint members

$\Rightarrow$ if members of the joint have the same E, (same t, thickness) with symmetrical frusta back to back, then $l = 2t$

$$\frac{1}{k_m} = \frac{1}{k} + \frac{1}{k} = \frac{2}{k} \Rightarrow k_m = \frac{k}{2} \quad (8\text{-}22) \Rightarrow (8\text{-}23)$$

a & B come from table (8-8) coeff's of materials to use in eq (8-23)

d is bolt diam

4) Bolt strength
- Proof strength - max load that a bolt can withstand without acquiring a permanent set (.0001")
= proof load / A_t tensile stress area

5) The external load see fig. 8-13

assume the load P stretch δ

$$\delta = \frac{P_b}{k_b} = \frac{P_m}{k_m} = P_M = P_b \frac{k_m}{k_b}$$

$$P = P_b + P_m = P_b + P_b\left(\frac{k_m}{k_b}\right) = P_b\left(\frac{k_b + k_m}{k_b}\right)$$

$$P_b = \frac{k_b}{k_b + k_m} P = CP$$

C - stiffness constant of the joint

$$P_m = \left(\frac{k_m}{k_b + k_m}\right)P = (1 - C)P$$

$$F_b = P_b + F_i = \underbrace{CP}_{\text{tension}} + \underbrace{F_i}_{\text{tension}} \quad (8\text{-}24)$$

$$F_m = P_m - F_i \quad \boxed{F_m < 0}$$

$\boxed{F_m < 0}$ member is in compression

Table 8-12 % of load taken by the members

6) Torque & Bolt tension

$$T = \left[\left(\frac{d_m}{2d}\right)\left(\frac{\tan\lambda + f\sec\alpha}{1 - f\tan\lambda\sec\alpha}\right) + .025 f_c\right] F_i d$$

coefficient of friction for collar

coeff. friction between bolt & members

$T = kF_i d$ (8-26)

7) Tension joint with preload

Bolt: $\sigma_b = \frac{F_b}{A_t} = \frac{CP}{A_t} + \frac{F_i}{A_t}$ load factor n

$$\frac{nCP}{A_t} + \frac{F_i}{A_t} = S_p \text{ (proof strength)}$$

$$n = \frac{S_p A_t - F_i}{CP}$$

⇒ avoid joint separation if $F_m \geq 0$, separation occurs

$F_m = (1-C)P_0 - F_i \geq 0$ separate

or

$F_m = (1-C)\, n_0 P - F_i = 0$

$n_0 = \frac{F_i}{P(1-C)}$ safety factor to avoid joint separation

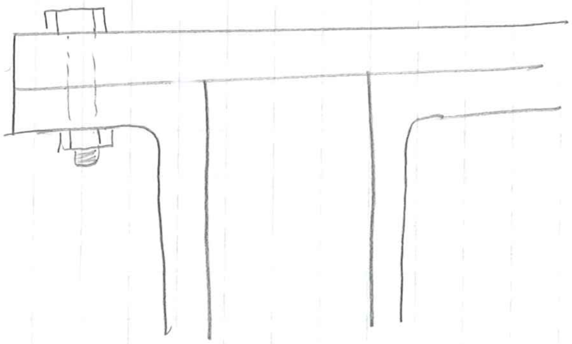
Preload $F_i = \begin{cases} 0.75\, F_p & \text{for non-permanent connection} \\ 0.9\, F_p & \text{for permanent connection} \end{cases}$ (8-30)

$F_p = A_t S_p$ (8-31)

S_p can be obtained from Table 8-9 ~ 8-11

F

Example



$C = \frac{k_b}{(k_b + k_m)}$

$P = \frac{\pi}{4}(.15)^2 (6 \times 10^6 \ N/m^2)(\frac{1}{10 \text{ bolts}})$

$L_T = 2d + 6$ (standard length, threaded) $d \leq 48$ $L \leq 125$

$= 2(12) + 6 = 30 mm$

L_d, unthreaded length $= L - L_T = 60 - 30 = 30 mm$

$l_t = l - L_d = 40 - 30 = 10 \ mm$ $A_d = \frac{\pi (12)^2}{4} = 113 mm^2$ $A_t \Rightarrow$ Table 8-1 p412

Table 8-1 = 84.3 mm = A_t Table 8-11 ⇒ Table 8.8 → medium carbon steel

↳ E = 207 GPa

$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{(113)(84.3)(207)}{113(10) + (84.3)(30)} = 538.9 \ \frac{MN}{m}$

Notes 12-8

Joints - Member stiffness

$n = \frac{S_p A_t - F_i}{CP}$ (8-28)

k_1 20

k_2 20

$k_1 = 4722 \ MN/m$

$k_2 = 2488 \ MN/m$

$\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2}$ $k_m = 1523 \ MN/m$

Final Exam we will have 60 minutes or 70 minutes

2-sided crib sheet allowed

all multiple choice

$C = \frac{k_b}{k_b + k_m} = .261$ Table 8-1 ⇒ $A_t = 84.3 \ mm^2$

Table 8-11 ⇒ $S_p = 600 \ MPa$

$F_i = .75 F_p = .75(600 \ MPa)(84.3) mm^2 = 37.9 kN$

$n = \frac{600 \ MPa (84.3 \ mm) \times 10^{-3} k - 37.9 \ kN}{(.261)(10.6 \ kN)} = \boxed{4.58}$

another check you can do: same example
to avoid joint separation

$$n_0 = \frac{F_i}{P(1-C)} = \frac{37.9\text{ kN}}{10.6\text{ kN}(.739)} = 4.84$$

8) fatigue loading Table 8-16 $\Rightarrow K_f$
Table 8-17 $\Rightarrow S_e$
In most cases, external applied load fluctuates from $0 \sim P$

$F_{max} = F_b = CP + F_i$
$F_{min} = F_i$
$F_a = \frac{F_{max} - F_{min}}{2}$ σ_a, σ_m (8-35, 36)

For goodman intersection point $S_a = \frac{S_e(S_{ut} - \sigma_i)}{S_{ut} + S_e}$ like (8-45)

$$n_f = \frac{S_a}{\sigma_a} = \frac{S_e(S_{ut} - \sigma_i)}{\sigma_a(S_{ut} + S_e)} \quad (8.45)$$

$$n_f = \frac{\frac{S_e(S_{ut} - \sigma_i)}{S_{ut} + S_e}}{\frac{CP}{2A_t}} = \frac{2S_e(S_{ut}A_t - F_i)}{CP(S_{ut} + S_e)} \quad (8.48)$$

$F_i \uparrow$ $n_f \downarrow$

without Preload, $F_i = 0$, $C = 1$

$$n_{f_0} = \frac{2S_e(S_{ut}A_t)}{P(S_{ut} + S_e)} \quad (8-49)$$

when $\frac{n_f}{n_{f_0}}$ $\Rightarrow$ $S_{ut}A_t - F_i \geq C S_{ut} A_t$ see 8-50, upper bound on preload

P-8-31 $P_{max} = 6\text{ MPa} \Rightarrow P = 10.6\text{ kN}$ $P_{min} = 0\text{ MPa}$
$C = 0.261$ $F_i = 37.9\text{ kN}$ $A_t = 84.3\text{ mm}^2$

$\sigma_i = \frac{F_i}{A_t} = 450\text{ MPa}$ $\sigma_m = 16.41 + 450 = 466.41\text{ MPa}$

$\sigma_a = \frac{CP}{2A_t} = 16.41\text{ MPa}$ Goodman: class 8.8 $S_{ut} = 830\text{ MPa}$ Tab. 8-11
$S_e = 129\text{ MPa}$

$$S_a = \frac{S_e(S_{ut} - \sigma_i)}{S_{ut} + S_e} = \frac{129(830 - 450)}{830 + 129}$$

$$n_f = \frac{S_a}{\sigma_a} = \frac{51.12}{16.41 \times 3} = 1.04$$

assume rolled threads ← Tab. 8-16

Final Exam 70 minutes 10:30 Toomey 295

25 problems, 4 points

Only single-answer problems

mostly concepts for previously covered material - study notes

short calculations for bolt problems

Homework 8 prob 1 & 2 will be covered

Exam 1 69/100

Quizzes 1-11 6 x (10/10) Avg .9091
5 x (8/10)
12-15 Avg Avg .825
Avg 1-15 = .8867

Exam 2 ~~88/100~~ 89/100 correction on multiple choice

	Score	Qty
Quizzes	10	(9)
	8	(9)
	7	(1)

Quizzes contributes $.8895(.2) = .1779$ to grade

Exams $\frac{69+89+98}{300}$ contributes $.8533(.6) = .5119$ to grade

grade as of 12/6 = 86.26%

$\Rightarrow$ .6898 towards total overall grade

Cannot get an A
to get a B: Only need .55 on Final exam

to get a 89% in class, need 100.1%
to get 80% in class, need 55.06%