ME 211 Linear Systems

Kumar

Topic: modeling and analysis of dynamic (linear) systems
Fall Semester 2010 @ Missouri S&T

ME 211A: Modeling and Analysis of Dynamic Systems (Fall 2010)

Instructor Dr. Nishant Kumar U. Denver previously
Office 129 Toomey Hall
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Phone 573 – 341 – 5505 projects & exams graded by Kumar
Fax 573 – 341 – 4607 ~~home~~ assignments graded by TA
Email nkwtb@mst.edu
Office Hours Tuesday 4 – 5PM, Friday 11 – 2pm,
Important: I have an open door policy, so you are welcome to stop by my office and if you need appointment, please see me after the class or send me an email.

TA **TBD**
Lecture 10:00 – 10:50 MWF (McNutt Hall 212)
Prerequisite Math 14 (or 8), 15 (or 21), 22, and 204; Physics 24; C or better in ME/AE 160
Textbook *Modeling and Analysis of Dynamic Systems* 3rd Edition by Close, Frederick, and Newell. ISBN: 0471394424 Paperback ok $92.50 used

Course Objectives: The course is designed to introduce students to the basics of modeling and analyzing dynamic systems. A group course project is included for students to model and analyze a dynamic system. The Matlab/Simulink software package is used in this course. Topics covered include: modeling mechanical translational, mechanical rotational, electromechanical, and fluid systems, solutions of linear equations, frequency response of dynamic systems, linearization of nonlinear systems, transfer function formulation, block diagrams, dynamic performance analysis, and simulation.

Must use engineer paper or 8.5x11

Assignments: 6 - 8 homework problems will be assigned each week. These problems provide ample opportunity for learning the topics covered in class. HW's are due in class, a week after they are assigned. There will be several assignments during the semester. You are strongly encouraged to discuss the assignments with other students; however, you **must** turn in your own work. HW's may be turned in ahead of time if you know you will not be in class on a particular day. Late assignments will **not** be accepted for a grade. Work that is not neat will be returned. The lowest grade on HW will be dropped before calculating your final grade. 2pm in office Ok

Blackboard: This is not a web based course but Blackboard will be used as an alternative means of communication between the students and instructor/TA for posting lecture notes, homework solutions, grades, announcements and some useful information. You are advised to log in to your Blackboard account frequently.

Examinations: There will be three in-class midterm exams. The dates will be announced in the class atleast a week before the exam is scheduled. All examinations are closed book and notes. You are allowed the sheet of Laplace Transforms (to be posted on blackboard) and one 8.5 by 11 inch (both side) formula sheet for each examination. The formula sheets may not contain solved problems. Make up examinations will not be provided except in the case of a documented emergency. Permission to miss an examination must be obtained from the instructor prior to examination. Family emergency, University activity and health problems need to have written documentation. Those with unexcused absences will be given a zero grade for this portion of the course; the unexcused absence to the regular date of your exam will not give you the permission to come at the make up time scheduled for that particular test.

Attendance: Attendance is not mandatory. However, each lecture introduces significant new materials. If class is missed, you are responsible for obtaining announced information, handouts and notes from other students or from me. If you miss more than 2 or 3 class sessions and do not make them up, you will certainly get into difficulties.

Cheating: Cheating and/or plagiarism is unethical conduct and will not be tolerated. Anyone found to be cheating or assisting another student in cheating will automatically receive a failing grade for the course and the situation will be brought before the Office of Academic Affairs.

Grading: The following grading scale will be used unless class average requires some adjustment.
A ≥ 90%, 80% ≤ B < 90%, 70% ≤ C < 80%, 60% ≤ D < 70%, F < 60%.
The percent credit for assignments, course projects, and examinations is:

Assignments	20%		
Course Project	10%		
In–Class Examinations (3)	50% (Total)	Top 2 Scores	20%
Final Examination	20%	last score	10%

Reference Books:

1). Introduction to Dynamic Systems Analysis by T. D. Burton, 1994.
2). Feedback control of Dynamic Systems by Franklin, Powell and Emami-Naeini, 2002.
3). MATLAB – The language of technical computing, Version 5, The Math Works, Inc.
4). Fundamentals of Vibrations by Leonard Meirovitch, 2000

Important: Try to understand the material as much as you can during class. Write and draw big. Plan to spend average of 3-5 hours to do each assignment. Learn how to study efficiently. Students are strongly encouraged to visit the instructor during office hours, if they have questions on the homework assignments or any related technical issues. You are welcome to stop by my office during non-office hours.

Exams

FINAL EXAM/ ME 211

- DATE: DECEMBER 16 (THURSDAY); 10.30 am– 12.30 pm
- VENUE: TMH 256
- CLOSED BOOK/ NOTES; LAPLACE TRANSFORM SHEET WILL BE PROVIDED.
- CRIB SHEET (3 A4 SIZE SHEETS ALLOWED (FRONT AND BACK); CAN BRING THE CRIB SHEET USED FOR EXAM 1, 2 & 3; SOLVED EXAMPLE PROBLEMS ARE NOT ALLOWED TO BE COPIED ON THE CRIB SHEET; ONLY EQUATIONS/ FORMULA AND IMPORTANT DIAGRAMS/ FIGURES ALLOWED). MUST ATTACH CRIB SHEET BEFORE YOU TURN IN YOUR TEST.
- PLEASE BRING WRITING PAPER, PEN, PENCIL, ERASER AND CALCULATOR.
- BE ON TIME

EXAM DETAILS:

- COMPREHENSIVE (CHAPTERS 1 – 9; CHAPTERS 4 AND 6 NOT INCLUDED).
- TOTAL 6 PROBLEMS ON THE TEST; YOU HAVE TO DO ANY FIVE; YOU MUST WRITE THE PROBLEM NUMBERS TO BE GRADED, ON THE FRONT SHEET OF THE TEST.
- EACH PROBLEM CARRIES EQUAL POINTS.
- ATTEMPT ALL PROBLEMS. DRAW FREE BODY DIAGRAMS (WHEREVER NECESSARY). PARTIAL CREDIT WILL BE GIVEN. YOUR WORK SHOULD BE LEGIBLE. I WILL ASSIGN A ZERO IF I DON'T UNDERSTAND YOUR WORK.
- PREPARE LECTURE NOTES, HW PROBLEMS AND REVIEW SESSIONS.
- COLLECT YOUR TEST AND KNOW YOUR FINAL GRADES BETWEEN 1 – 4pm ON FRIDAY (12-17). ELSE, YOU MAY CHECK YOUR GRADES ONLINE. I WILL NOT BE IN MY OFFICE BETWEEN DEC-20 TO JAN 07. NOTE: ONLY YOU CAN COLLECT YOUR TEST.

OFFICE HOURS FOR EXAM WEEK:

MONDAY (DEC – 13) TO WEDNESDAY (DEC 15): 9 – 4pm.
THURSDAY (DEC 16): 8.30 AM – 10.20 AM

GRADE EVALUATION POLICY:

Assignments	20%
(LOWEST GRADE ON HW WILL BE DROPPED BEFORE CALCULATING THE FINAL GRADE)	
Course Project	10%
In–Class Examinations (3)	50% (20% + 20% + 10%)
Final Examination	20%

GRADING SCALE: A ≥ 90%, 80% ≤ B < 90%, 70% ≤ C < 80%, 60% ≤ D < 70%, F < 60%.

Final Exam

ME 211/ Modeling and Analysis of Dynamic Systems/ Fall – 2010

Name: David Malawey

Problems to grade: 2, 3, 4, 5, 6

47/50

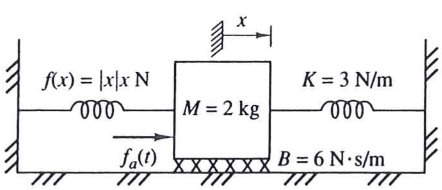
Instructions: **Max. Points: 50**

- *This is a closed book/ notes test.*
- *Attempt* ***any five*** *problems. Each problem carries 10 point. Partial credit will be given.*
- *Before you turn in your test, make sure you have a) written your name; b) arrange the sheets in proper order c) staple the entire work (including the crib sheet) and d) list the five problems you want me to grade.*
- *You must draw free body diagram(s) wherever necessary. Show clearly the solution procedure. The instructor reserves the right to assign a zero if he doesn't understand your messy/ illogical work.*

Problem 1: The translational system shown in the figure has a linear and a nonlinear spring subjected to the applied force $f_a(t)$.

a). Find the nonlinear differential equation of motion for the system.

b). Derive the linearized model when $f_a(t) = 10 + \hat{f}_a(t)$.



Problem 2: A system has the following model:

$\ddot{y} + 7\dot{y} + 12y = 5\dot{u} + 3u$

a). Find the transfer function.

b). Is the system stable? Why?

c). Find $y_U(t)$

Problem 3: Consider the first order linear system described by: $\dot{y} + 0.5y = F(t)$

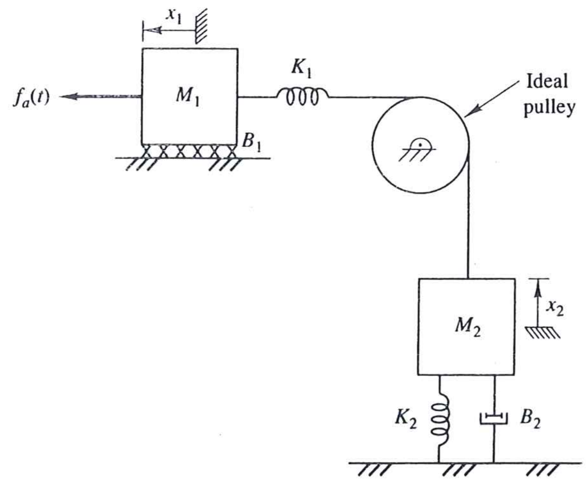
a). Find the response $y(t)$ of the system when $y(0) = 1$ and $F(t) = e^{-t/2}$.

b). Give the value of the time constant.

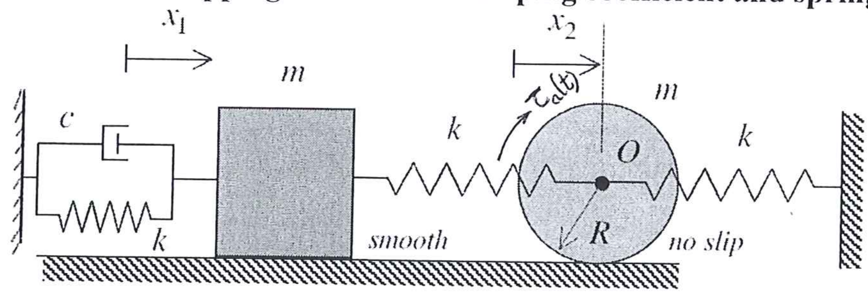
c). Identify the steady state and transient response.

d). Comment on the stability of the system.

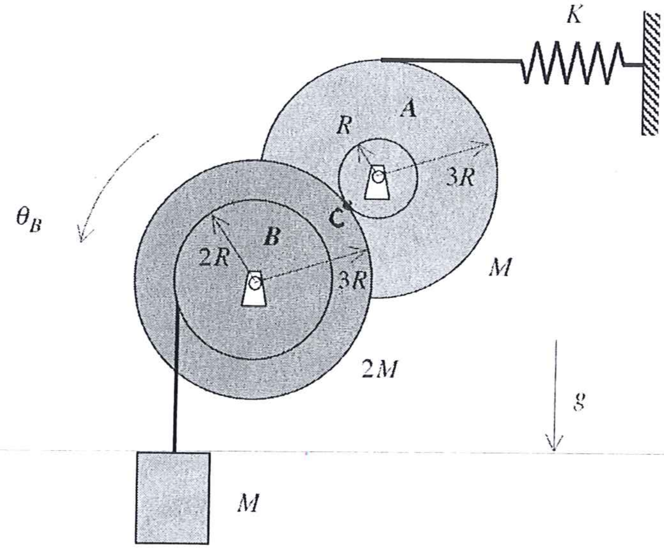
Problem 4: The pulley shown in the figure is ideal. The inputs for the system are $f_a(t)$ and gravity (g).
a). Draw the free body diagram of masses M_1 and M_2 and write the modeling equations for the two masses.
b). Write the state variable equations for the system. Identify the A, B, C and D matrix. Take the output to be the tensile force in spring K_1.



Problem 5: For the system shown in figure, derive the equation of motion for the block and the disk only in terms of coordinate's x_1, x_2, input $\tau_a(t)$ and constants. Assume the surface between the block and ground to be smooth, however significant friction is present between disk and ground. Let J be the moment of inertia of the disk. The disk rolls without slipping. c and k are damping coefficient and spring stiffness respectively.



Problem 6: A gear pair is made up of two gears A and B. Gear A has a mass of M and an outer radius of $3R$, whereas gear B has a mass of $2M$ and an outer radius of $3R$. The two gears are attached to fixed, horizontal shafts passing through their centers and mesh together at point C as shown in the figure. A spring of stiffness K is attached between gear A and wall. A cable is wrapped around an inner radius of gear B with a block of mass M attached to the free end of this cable. Let θ_B describe the rotation of gear B where the spring is unstretched when $\theta_B = 0$. Derive the equation of motion for gear B in terms of the coordinate θ_B. Take J_A and J_B to be the moment of inertia of gears A and B respectively. Gear Ratio, $N = \theta_1/\theta_2 = R_2/R_1$;



David Malaway

P#2

$\ddot{y} + 7\dot{y} + 12y = 5\dot{u} + 3u$

a) $\boxed{H(s) = \dfrac{5s+3}{s^2+7s+12}}$

b) poles $\quad s^2+7s+12=0 \quad (s+3)(s+4)=0$

poles $= -3, -4$

poles have neg. real parts, lie on imaginary axis
$\Rightarrow$ marginally stable

$y_u(t) = \mathcal{L}^{-1}\left(\frac{H(s)}{s}\right)$

$= \mathcal{L}^{-1} \dfrac{5s+3}{s(s+3)(s+4)}$

A B C

$5s+3 = As^2 + 7As + 12A + Bs^2 + 4Bs + Cs^2 + 3Cs$

$12A = 3 \quad \boxed{A = \frac{1}{4}}$

$A + B + C = 0$

$B = -\frac{1}{4} - C = \frac{16}{4} = 4$

$7A + 4B + 3C = 5$

$\frac{7}{4} - 1 - 4C + 3C = 5$

$-1C = 6 - \frac{7}{4}$

$C = \frac{-24+7}{4} = -\frac{17}{4}$

$\mathcal{L}^{-1}\left[\frac{1}{4}\frac{1}{s} + 4\frac{1}{(s+3)} - \frac{17}{4}\frac{1}{(s+4)}\right]$

c) $\boxed{y_u(t) = \frac{1}{4} + 4e^{-3t} - \frac{17}{4}e^{-4t}}$

10

Prob 3

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$\dot{y} + 0.5y = F(t)$

$y(t)$ when $y(0) = 1$ & $F(t) = e^{-t/2}$

b) $\boxed{\tau = 2 \text{, time const}}$

$\mathcal{L} \Rightarrow sY(s) - y(0) + 0.5\,Y(s) = \frac{1}{(s+\frac{1}{2})}$

$Y(s)(s + 0.5) - 1 = \frac{1}{(s+\frac{1}{2})}$

$Y(s) = \frac{\frac{1}{(s+\frac{1}{2})} + 1}{(s+\frac{1}{2})} = \frac{1 + (s+\frac{1}{2})}{(s+\frac{1}{2})^2} = \frac{1}{(s+\frac{1}{2})^2} + \frac{1}{(s+\frac{1}{2})}$

$\mathcal{L}^{-1}\left\{\frac{1}{(s+\frac{1}{2})^2}\right\} = te^{-\frac{1}{2}t}$

$\mathcal{L}^{-1}\left\{\frac{1}{(s+\frac{1}{2})}\right\} = e^{-\frac{1}{2}t}$

a) $\boxed{y(t) = (1+t)e^{-\frac{1}{2}t}}$

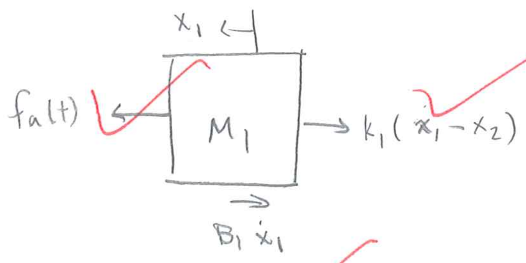
c) $y(t) = e^{-\frac{1}{2}t} + te^{-\frac{1}{2}t}$ $\boxed{y_{ss} = 0 \quad y_{tr} = e^{-\frac{1}{2}t} + te^{-\frac{1}{2}t}}$

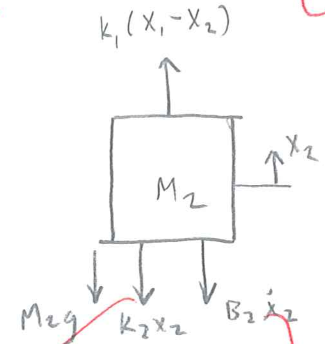
d) $\boxed{\text{System is stable, } \tau > 0}$

10

David Malawey

P4





M_1:

$$M_1\ddot{x}_1 = f_a(t) - B_1\dot{x}_1 - k_1(x_1-x_2)$$

$$\dot{x}_1 = V_1$$

$$\dot{V}_1 = \frac{1}{M_1}\left[f_a(t) - B_1\dot{x}_1 - k_1(x_1-x_2)\right]$$

M_2

$$M_2\ddot{X}_2 = k_1(x_1-x_2) - B_2\dot{x}_2 - k_2x_2 - M_2 g$$

$$\dot{x}_2 = V_2$$

$$\dot{V}_2 = \frac{1}{M_2}\left[k_1x_1 - x_2(k_1+k_2) - \dot{x}_2B_2 - M_2 g\right]$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{V}_1\\ \dot{x}_2\\ \dot{V}_2\end{bmatrix} = \underbrace{\begin{bmatrix}0 & 1 & 0 & 0\\ -\frac{k_1}{M_1} & -\frac{B_1}{M_1} & \frac{k_1}{M_1} & 0\\ 0 & 0 & 0 & 1\\ \frac{k_1}{M_2} & 0 & -\frac{k_1+k_2}{M_2} & -\frac{B_2}{M_2}\end{bmatrix}}_{A}\begin{bmatrix}x_1\\ V_1\\ x_2\\ V_2\end{bmatrix} + \underbrace{\begin{bmatrix}0 & 0\\ \frac{1}{M_1} & 0\\ 0 & 0\\ 0 & -1\end{bmatrix}}_{B}\begin{bmatrix}f_a(t)\\ g\end{bmatrix}$$

Output y = force of k_1 = $k_1(x_1-x_2)$

$$\boxed{\begin{bmatrix}y\end{bmatrix} = \underbrace{\begin{bmatrix}k_1 & 0 & -k_1 & 0\end{bmatrix}}_{C}\begin{bmatrix}x_1\\ V_1\\ x_2\\ V_2\end{bmatrix} + \underbrace{\begin{bmatrix}0 & 0\end{bmatrix}}_{D}\begin{bmatrix}f_a(t)\\ g\end{bmatrix}}$$

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Prob 5)

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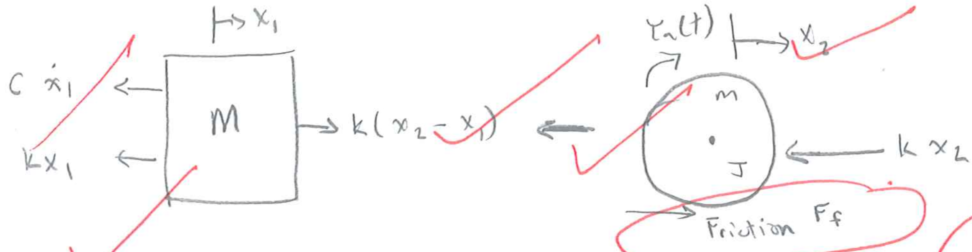

$J = \frac{MR^2}{2}$ not needed

$m\ddot{x}_1 = k(x_2-x_1) - c\dot{x}_1 - kx_1$

$$\boxed{m\ddot{x}_1 = k(x_2-2x_1) - c\dot{x}_1}$$

BLOCK

$\dot{x}_2 = \omega R$ $\quad \ddot{x}_2 = \alpha R$

$\Sigma \tau_o = J_o \alpha$ $\quad \tau_a(t) - F_f R = J\alpha$ (1)

$\Sigma F = m\ddot{x}_2$

$F_f - kx_2 - k(x_2-x_1) = m\ddot{x}_2$ (2)

from(1) $F_f = \frac{1}{R}[\tau_a(t) - J\alpha]$

$F_f = \frac{1}{R}[\tau_a(t) - J\frac{\ddot{x}_2}{R}]$

plug into (2) $\frac{\tau_a(t)}{R} - \frac{J\ddot{x}_2}{R^2} - k(2x_2-x_1) = m\ddot{x}_2$

$\ddot{x}_2[m + \frac{J}{R^2}] = \frac{\tau_a(t)}{R} - k(2x_2-x_1)$

$\ddot{x}_2\left(\frac{3m}{2}\right) = (\qquad)$

$$\boxed{\ddot{x}_2 = \frac{2}{3m}\left[\frac{\tau_a(t)}{R} - k(2x_2-x_1)\right]}$$

DISK

10

Note: I put J in terms of R and m

not needed

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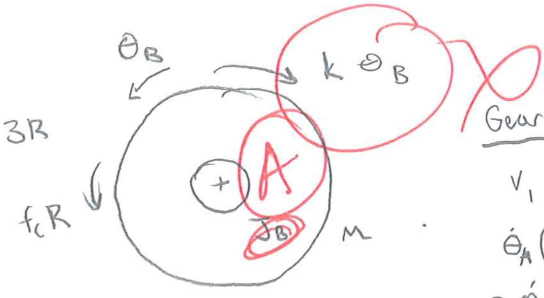
Prob 6



Gear pair:

$V_1 = V_2$

$\dot{\theta}_A (3R) = \dot{\theta}_B (R)$

$3\dot{\theta}_A = \dot{\theta}_B \qquad \ddot{\theta}_A = \frac{1}{3}\ddot{\theta}_B$

$$J_a \ddot{\theta}_A = 2R(2M)g - f_c(3R)$$

$$f_c = \frac{1}{3R}\left[4RMg - J_a \ddot{\theta}_a\right]$$

$$J_B \ddot{\theta}_B = f_c R - k\theta_B$$

$$J_B \ddot{\theta}_B = \frac{4mg}{3} - \frac{J_a \ddot{\theta}_a}{3R} - k\theta_B$$

$$J_B \ddot{\theta}_B = \frac{4}{3}mg - \frac{J_a}{3R}\left(\frac{1}{3}\ddot{\theta}_B\right) - k\theta_B$$

$$\underbrace{N = \frac{\theta_1}{\theta_2} = \frac{R_2}{R_1}}$$

did not use N because all are in terms of scalar "R"

$$\ddot{\theta}_B\left(J_B + \frac{J_a}{9R}\right) = \frac{4}{3}mg - k\theta_B$$

$$\boxed{\ddot{\theta}_B = \frac{\left[\frac{4}{3}mg - k\theta_B\right]}{\left(J_B + \cancel{\frac{J_a}{9R}}\right)}}$$

7

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2nd order D.E

$$a_2\ddot{y} + a_1\dot{y} + a_0 y = b_2\ddot{u} + b_1\dot{u} + b_0 u$$

$$H(s) = \frac{b_2 s^2 + b_1 s + b_0}{a_2 s^2 + a_1 s + a_0}$$

Operating Point

$x \to \bar{x} \quad u \to \bar{u} \quad$ where $\dot{\bar{x}} = \ddot{\bar{x}} = 0$ always

$\bar{x}$ = x-coord of O.P.
$\bar{f}$ = y-coord of O.P.

a) replace output terms $y(t)$ or $x(t)$ by $\bar{y}$ or $\bar{x}$
b) same for input
c) for input terms, $u(t)$, all time-dependent terms are ignored

General form of 3rd order DE

$$a_3\dddot{y} + a_2\ddot{y} + a_1\dot{y} + a_0 y = b_3\dddot{u} + b_2\ddot{u} + b_1\dot{u} + b_0 u$$

$$H(s) = \frac{b_3 s^3 + b_2 s^2 + b_1 s + b_0}{a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

Procedure for linearization:

1) determine O.P.'s
2) identify nonlinear terms, linearize using Taylor S.

Taylor Series: about pt. $\bar{x}$

$$f(x) = f(\bar{x}) + \frac{df}{dx}\Big|_{\bar{x}}(x-\bar{x}) + \frac{1}{2!}\frac{d^2f}{dx^2}\Big|_{\bar{x}}(x-\bar{x})^2 + \dots$$

since $x - \bar{x} = \hat{x}$ (increment. var) $\quad \hat{x}^2 \approx 0$

$$f(x) = f(\bar{x}) + \frac{df}{dx}\Big|_{\bar{x}}\hat{x} + \cancel{\frac{1}{2!}\frac{d^2f}{dx^2}\Big|_{\bar{x}}(\hat{x})^2}^{\,0} + H.O.T$$

3) rewrite all linear terms as $x = \hat{x} + \bar{x}$
4) Solve the linear eqn

- if you get a $|x|x$, choose $x > 0$, ignore $\bar{x}$ values that are $(-)$

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General 1st order system

$$\dot{y} + \frac{y}{\tau} = f(t) \Rightarrow y(t) = \tau A + (y_0 - \tau A)e^{-t/\tau}$$

- if $f(t) =$ const, $f(t)$ is your A
- find response means find $\mathcal{L}(f(t))$

Stable $\tau > 0$ Marginally stable $\tau \to \infty$

unstable $\tau < 0$

τ = time const., measures how quickly sys. reaches steady state

Definition of Laplace:

$$\mathcal{L}[f(t)] = \int_0^\infty f(t)e^{-st}dt$$

Transform derivatives:

$$\mathcal{L}\left[\frac{d^n f(t)}{dt^n}\right] = s^n F(s) - s^{n-1} f(0) + \dots f^{n-1}(0)$$

Int. by Parts

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

- make e^{xt} the V term then $dv = xe^{xt}dt$

matrix, st. var form

$Y = Cq + Du$ outputs

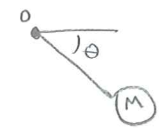
$\dot{q} = Aq + Bu$

st. vars input

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ & & \\ & & \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} + \begin{bmatrix} b \\ b \\ b \end{bmatrix} [fa(t)]$$

H_0 = angular momentum of particle about O

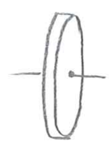
H_0 for mass at a distance



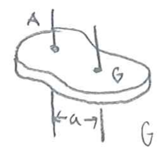
$H_0 = \vec{r} \times \vec{p} = \vec{r} \times mv = ml^2\dot{\theta}$

$M_0 = \dot{H}_0 = ml^2\ddot{\theta} = F d_{perp.} = I\alpha$

Parallel Axis thm



$J = \frac{MR^2}{2}$



$J_{AA'} = J_0 + Ma^2$

G = centroid

Pendulum, moving support

- Create r vector, get $\vec{r}$ in $\hat{i}$ & $\hat{j}$

static equilibrium deflection

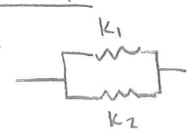
- $\dot{y} \to 0$ $\ddot{y} \to 0$ $y \to y_0$

Gears

$N = \frac{R_2}{R_1} = \frac{\theta_1}{\theta_2}$ $V = r\omega$ $\Sigma T = J\ddot{\theta}$

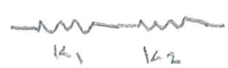
K_{eq} or B_{eq}

parallel



$= k_1 + k_2$

series

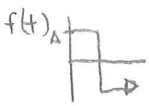


$= \frac{k_1 k_2}{k_1 + k_2}$

State Var form: terms w/x = terms w/out

Y_{tr}, transient response $t \to \infty$, $Y_{tr} \to 0$

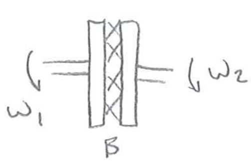
Y_{ss}, steady state, $= \lim_{t\to\infty} f(t)$



$u(t) = \begin{cases} 0 & t \le 0 \\ 1 & t > 1 \end{cases}$ $u(t-t_1) = \begin{cases} 0 & t \le t_1 \\ 1 & t > t_1 \end{cases}$

$$f(t) = Au(t) - 2Au(t-t_1)$$

torque



$\tau = B\Delta\omega$

levers: $\circlearrowleft + \Sigma M_0 = J_0\alpha$ small θ: $\sin\theta = \theta$, $\cos\theta = 1$

$J = \frac{ML^2}{12}$

$J = \frac{ML^2}{3}$

David Malawey

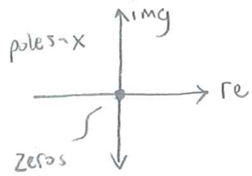
n^{th} order differential eqn $a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \ldots a_1 \frac{dy}{dt} + a_0 y = b_m \frac{d^m u}{dt^m} + \ldots b_1 \frac{du}{dt} + b_0 u$

then $H(s) = \frac{b_m s^m + \ldots b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots a_1 s + a_0} = \frac{Y(s)}{U(s)} = \frac{Y_{zs}}{Fa(s)} = \frac{\text{Output}}{\text{input}}$

Dont need initial conditions to find H(s). It is always taken at zero st.

$P(s)$ = characteristic eqn where $\mathcal{L}[u(t)] = U(s)$, $\mathcal{L}[y(t)] = Y(s)$

stability: $P(s)$, denominator of $H(s)$



1) all poles have neg. real points → stable
2) all " " " " " & 1 or more distinct poles lies on img. axis → → Marginally (critically) stable
3) at least one pole lies in R.H. plane OR has double roots on imaginary axis

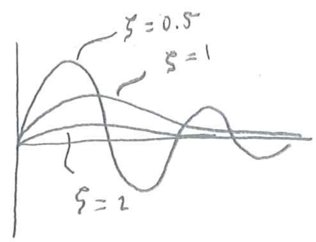
Given: $P(s) = as^2 + bs + c$ · zeros of $p(s)$ ⇒ $s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Damping & natural frequency

Standard form, 2nd order:

$\ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = f(t)$

• stable if $0 < \zeta < 1$



CLASSIFICATION

$0 < \zeta < 1$ underdamped

$\zeta > 1$ overdamped

$\zeta = 1$ critically damped

Cannot take $y(0)$ & $\dot{y}(0) = 0$ for Y_{zi}, only for transfer function

Zero input response, Y_{zi} general form

$a_2\ddot{y} + a_1\dot{y} + a_0 y = \cancel{u(t)}\ 0$ $\quad \mathcal{L}[\quad] = 0$

$Y_{zi} = k_1 s e^{s_1 t} + k_2 e^{s_2 t}$ $\quad s_1$ & s_2 are real & distinct

$Y_{zi} = k_1 e^{s_1 t} + k_2 t e^{s_2 t}$ $\quad s_1 = s_2$

$Y_{zi} = k e^{\alpha t} \cos(\beta t + \phi)$ $\quad s_{1,2} = \alpha \pm i\beta$

initial conditions must be given to find k, ϕ. need $y(0)$ & $\dot{y}(0)$. Set $t=0$ then $y(0) = y_{zi}$

Unit step response $u(t) = 1$

$y_u(t) = \mathcal{L}^{-1}\left[\frac{H(s)}{s}\right]$

for $\frac{1}{s(as^2+bs+c)}$ use $\frac{A}{s} + \frac{Bs+C}{(as^2+bs+c)}$ Also $\frac{1}{(s+c)^2} = \frac{A}{(s+c)} + \frac{B}{(s+c)^2}$

impulse input response $h(t) = \frac{d\, y_u(t)}{dt}$ OR $H(s)$ $h(t) = \mathcal{L}^{-1}(H(s))$

Given $U(s)$ find $Y(t)$

$Y(s) = H(s)\, U(s)$

steady state response $u(t) = $ const. A. $\quad y_{ss} = A H(0)$

$u(t) = B\sin(\omega t + \phi)$ ⇒ $y_{ss}(t) = y_p = BM\sin(\omega t + \phi + \theta)$

FRF, Frequency Response function: $= H(s = j\omega)$

M = magnitude of FRF groups $\sqrt{(\text{real part})^2 + (\text{img part})^2}$

[ignore j's within this prt.

$\arg = \tan^{-1}\left(\frac{\text{img}}{\text{real}}\right)$

$\theta(\omega) = \arg(\text{num}) - \arg(\text{denom})$

21/30

Exam – 2/ Nov-03-2010
ME 211/ Modeling and Analysis of Dynamic Systems/ 10 – 10.50 am/ Fall – 2010

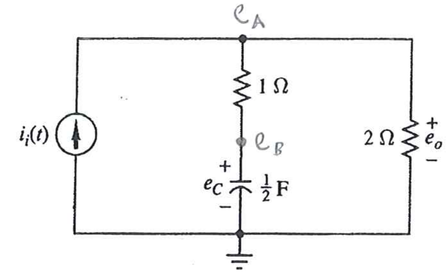
Name: David Malawey

Instructions: **Max. Points: 30**

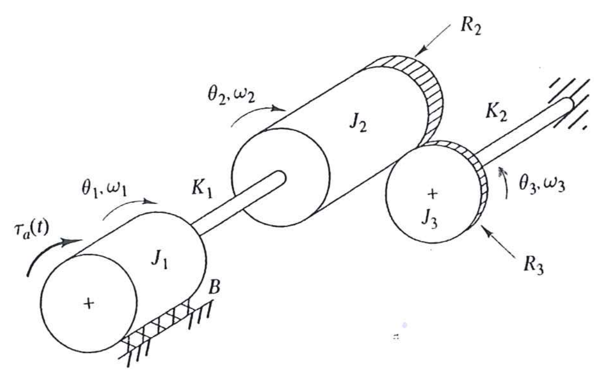
- *This is a closed book/ notes test.*
- *Attempt all problems as partial credit will be given.*
- *Before you turn in your test, make sure you have a) written your name; b) arrange the sheets in proper order and c) staple the entire work (including the cheat sheet).*
- *You must draw free body diagram(s) wherever necessary. Show clearly the solution procedure. The instructor reserves the right to assign a zero if he doesn't understand your messy/ illogical work.*

GOOD LUCK

Problem 1 (10 Points): For the electrical circuit shown in the figure: a). Find the input-output differential equation relating e_0 and $i_i(t)$; b). Determine the time constant (τ).



Problem 2 (10 Points): The input to the rotational system shown in the figure is the applied torque $\tau_a(t)$, applied to the cylinder J_1. The gear ratio $N = R_2/R_3 = \theta_3/\theta_2$. Write the differential equation of motion for J_1 and J_2 involving the variables θ_1, θ_2 and their derivatives.



Problem 3 (10 Points): The transformed response of a system is given by: $Y(s) = \dfrac{10(s+3)}{s(s^2+2s+5)}$; **Find**

a) $y(0)$
b) **Identify the transient and steady state solution.**
c) **Comment on the stability of the system.**

Table of Laplace Transforms

(a and b are constants and n is a constant integer)

$f(t)$	$F(s)$
$\delta(t)$	1
$\delta(t-nT)$	e^{-nTs}
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
$\frac{t^2}{2}$	$\frac{1}{s^3}$
$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{s^n}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\frac{1}{2}t^2e^{-at}$	$\frac{1}{(s+a)^3}$
$\frac{t^{n-1}e^{-at}}{(n-1)!}$	$\frac{1}{(s+a)^n}$
$1-e^{-at}$	$\frac{a}{s(s+a)}$
$\frac{1}{a}\left(at-1+e^{-at}\right)$	$\frac{a}{s^2(s+a)}$
$\sin(bt)$	$\frac{b}{s^2+b^2}$
$\cos(bt)$	$\frac{s}{s^2+b^2}$
$e^{-at}\sin(bt)$	$\frac{b}{(s+a)^2+b^2}$
$e^{-at}\cos(bt)$	$\frac{s+a}{(s+a)^2+b^2}$
$1-e^{-at}\left[\cos(bt)+\frac{a}{b}\sin(bt)\right]$	$\frac{a^2+b^2}{s\left[(s+a)^2+b^2\right]}$
$\dot{f}(t)$	$sF(s)-f(0)$
$\ddot{f}(t)$	$s^2F(s)-sf(0)-\dot{f}(0)$

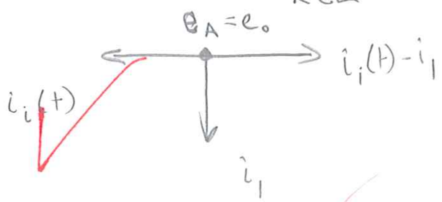
David Malawey

Node A

KCL

DE relating e_o & $i_i(t)$

1)



$$\left.\begin{aligned} i_i(t) - i_1 &= \frac{e_o}{2} \\ i_1 &= \frac{e_o - e_B}{1} \end{aligned}\right\} \quad i_i(t) + \frac{e_o}{2} + \frac{e_o - e_B}{1} = 0$$

$$e_B = i_i(t) + \frac{3e_o}{2} \qquad (1)$$

Node B

e_B

$$i_1 = \frac{e_B - e_A}{1\Omega}$$

$$i_1 = \frac{1}{2}\frac{di_1}{dt}$$

$$\frac{e_B}{2} - \frac{e_A}{2} = \frac{1}{2}\frac{di_1}{dt} \Rightarrow e_B - e_o = \frac{di_1}{dt} \qquad (2)$$

$$i_1 = i_i(t) - \frac{e_o}{2}$$

$$\frac{di_1}{dt} = \dot{i}_i(t) - \frac{\dot{e}_o}{2}$$

$$i_i(t) + \frac{3e_o}{2} - e_o = \frac{di_1}{dt}$$

$$i_i(t) + \frac{e_o}{2} = \dot{i}_i(t) - \frac{\dot{e}_o}{2}$$

$$\boxed{\dot{i}_i(t) = i_i(t) + \frac{\dot{e}_o}{2} + \frac{e_o}{2}}$$

$$-i_i(t) + \dot{i}_i(t) = \frac{\dot{e}_o}{2} + \frac{e_o}{2}$$

$$\dot{e}_o + e_o = +\dot{i}_i(t)\,(2) - i_i(t)\,(2)$$

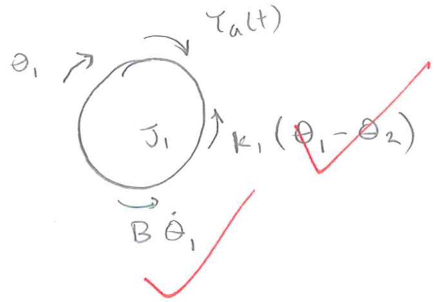
$$\boxed{\tau = 1}$$

7

D. Malawey

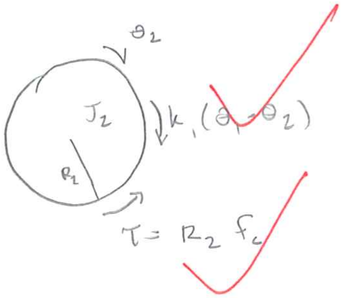
DE for J_1 & J_2

2) $N = \dfrac{R_2}{R_3} = \dfrac{\Theta_3}{\Theta_2}$



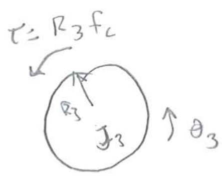
$$\Sigma M_0 = J_1 \ddot{\theta}_1$$

$$\tau_a(t) - k(\theta_1 - \theta_2) - B\dot{\theta}_1 = J_1\ddot{\theta}_1 \qquad (1)$$



$$k_1(\theta_1 - \theta_2) - R_2\left(J_3 \ddot{\theta}_2 R_2\right) = J_2 \ddot{\theta}_2$$

$$k_1(\theta_1 - \theta_2) = J_2\ddot{\theta}_2 + R_2^2 J_3 \ddot{\theta}_2 \qquad (2)$$



$$R_3 f_c = J_3 \ddot{\theta}_3$$

$$f_c = J_3 \frac{\ddot{\theta}_3}{R_3}$$

$$\ddot{\theta}_3 = \ddot{\theta}_2 \frac{R_2}{R_3} \Rightarrow f_c = J_3 \frac{\ddot{\theta}_2}{R_3}\frac{R_2}{R_3}$$

sub (2) into (1)

$$\tau_a(t) - \left[J_2\ddot{\theta}_2 + R_2^2 J_3 \ddot{\theta}_2\right] - B\dot{\theta}_1 = J\ddot{\theta}_1$$

$$\tau_a(t) - \ddot{\theta}_2\left[J_2 + \frac{R_2^2}{R_3^2} J_3\right] - B\dot{\theta}_1 = J\ddot{\theta}_1$$

$$\boxed{\tau_A(t) = J\ddot{\theta}_1 + B\dot{\theta}_1 + \ddot{\theta}_2\left[J_2 + N^2 J_3\right]}$$

10

D. Malawey

3)

$$Y(s) = \frac{10(s+3)}{s(s^2+2s+5)} \qquad (x-(1+2i))(x+1+2i)$$

$$= \frac{A}{s} + \frac{B+C}{(s^2+2s+5)} = \frac{A}{s} + \frac{B}{(s-1-2i)} + \frac{C}{(s+1+2i)}$$

$$\frac{10(s+3)}{s(s^2+2s+5)} \quad s=0 \Rightarrow \quad A = \frac{10(3)}{5} = \frac{30}{5} = 6$$

4

$\mathcal{L}^{-1}\frac{6}{s} = 6$ other 2 terms will have t

$y(0) = 6 + \_\_\_ + \_\_\_$

b) transient solution = part of $y(t)$ that disappears when $t \to \infty$

steady state = $y(t) - Y_{tr}$ = part that remains when $t \to \infty$

probably = 6

???

c) Stable if $\tau > 0$

unstable $\tau < 0$ where τ = time constant

marginally stable if $\tau \to \infty$

$\frac{21}{30}$

Exam – 1

ME 211/ Modeling and Analysis of Dynamic Systems/ Exam – 1/ 10 – 10.50 am/ Fall – 2010

Name: David Malawey

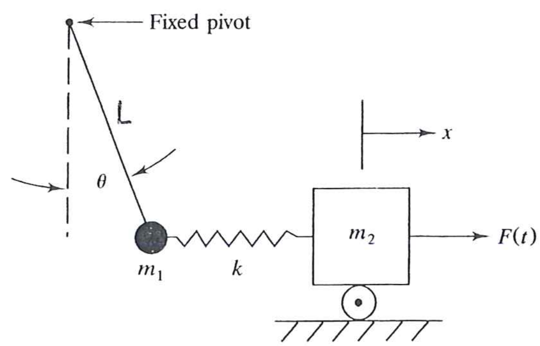
30/30

Instructions: **Max. Points: 30**

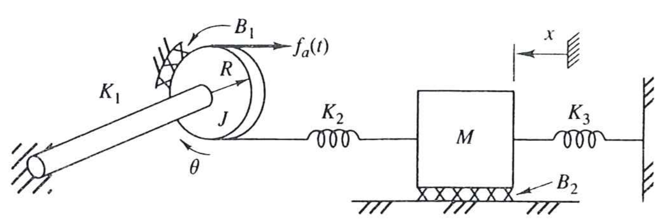
- *This is a closed book/ notes test.*
- *Attempt all problems as partial credit will be given.*
- *Before you turn in your test, make sure you have a) written your name; b) arrange the sheets in proper order and c) staple the entire work (including the cheat sheet).*
- *You must draw free body diagram(s) wherever necessary. Show clearly the solution procedure. The instructor reserves the right to assign a zero if he doesn't understand your messy/ illogical work.*

GOOD LUCK

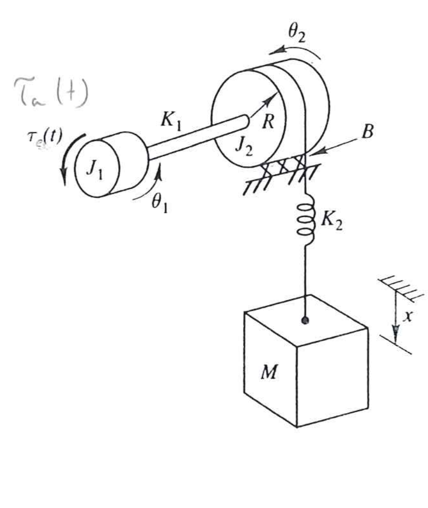
Problem 1 (16 Points): For the system shown in the figure, derive the equation(s) of motion assuming the angle θ to be small. Select a set of state variables and write in matrix form the state variable model where the input is $F(t)$. Let the outputs be the spring force acting on mass m_1 and the velocity of mass m_2. Identify the matrices A, B, C and D.



Problem 2 (8 Points): In the mechanical system shown in figure, the cable is wrapped around the disk and does not slip or stretch. The input force is f_a(t). The springs are undeflected when $\theta = x = 0$. Derive the differential equation(s) of motion for the system.



Problem 3 (6 Points): For the system shown in the figure, the angular displacements (θ_1 and θ_2) of the two drums and the linear displacement (x) of the mass M is shown at any instant of time. Draw the free body diagrams of a) the two drums having moment of inertia J_1 and J_2 respectively and b) the mass M. B is the damping coefficient and K_1, K_2 are stiffness.



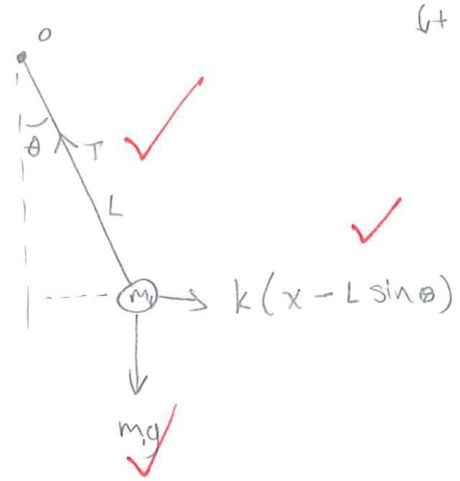
David M.

#1) State vars

Θ, x

Outputs: Spring force on M_1

Velocity of m_2



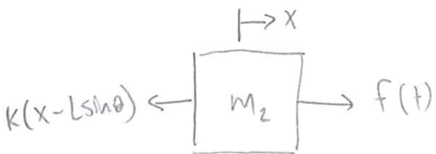
$\circlearrowleft + \Sigma\tau = I_0 \alpha$

$k(x - L\cancel{\sin\theta})(L\cancel{\cos\theta}^{\,1}) - mg(L\cancel{\sin\theta}) = m_1 L^2 \ddot{\theta}$

(θ is small)

$\ddot{\theta} = \frac{1}{m_1 L^2}\left[kxL - kL^2\theta - m_1 g L\theta\right]$

$$\boxed{\begin{aligned}\dot{\theta} &= \omega \\ \dot{\omega} &= \frac{1}{m_1 L}\left[kx - (kL + m_1 g)\theta\right]\end{aligned}}$$



$\Sigma F_x = m a_x$

$m\ddot{x} = f(t) - k(x - L\sin\theta)$

$\ddot{x} = \frac{1}{m}\left[-kx + kL\sin\theta + f(t)\right]$

$$\boxed{\begin{aligned}\dot{x} &= v \\ \dot{v} &= \frac{1}{m_2}\left[-kx + kL\theta + f(t)\right]\end{aligned}}$$

$$\boxed{\begin{bmatrix}\dot{x} \\ \dot{v} \\ \dot{\theta} \\ \dot{\omega}\end{bmatrix} = \underbrace{\begin{bmatrix}0 & 1 & 0 & 0 \\ \frac{-k}{m_2} & 0 & \frac{kL}{m_2} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{m_1 L} & 0 & \frac{-(kL+m_1 g)}{m_1 L} & 0\end{bmatrix}}_{A}\begin{bmatrix}x \\ v \\ \theta \\ \omega\end{bmatrix} + \underbrace{\begin{bmatrix}0 \\ \frac{1}{m_2} \\ 0 \\ 0\end{bmatrix}}_{B} f(t)}$$

outputs $\quad Y_1 = kx - kL\theta$

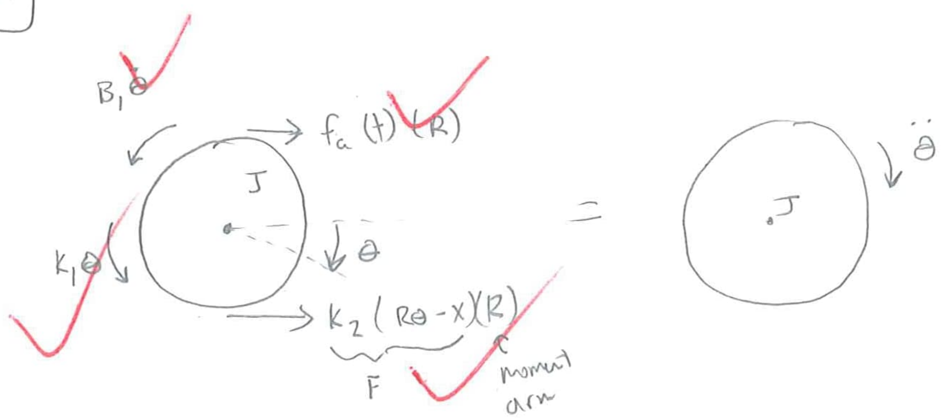
$Y_2 = \dot{x}$

$$\boxed{\begin{bmatrix}Y_1 \\ Y_2\end{bmatrix} = \underbrace{\begin{bmatrix}k & 0 & -kL & 0 \\ 0 & 1 & 0 & 0\end{bmatrix}}_{C}\begin{bmatrix}x \\ v \\ \theta \\ \omega\end{bmatrix} + \underbrace{\begin{bmatrix}0 \\ 0\end{bmatrix}}_{D} f(t)}$$

16

David Malawey

#2)



$\Sigma \tau = J\ddot\theta$

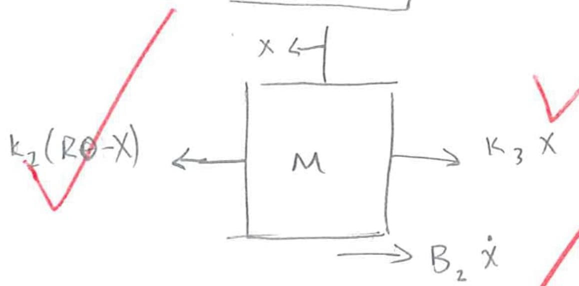
$R\theta$ = arc length that pulls k_2

$$f_a(t)R - B_1\dot\theta - k_1\theta + k_2R^2\theta = J\ddot\theta$$

$$\ddot\theta = \frac{1}{J}\left[-B_1\dot\theta + (k_2R^2 - k_1)\theta + f_a(t)R\right]$$

$$\dot\theta = \frac{d\theta}{dt}$$

$$J = \frac{1}{2} M_{disc} R^2$$



$\Sigma F = m\ddot x$

$$m\ddot x = k_2R\theta - k_2X - k_3X - B_2\dot X$$

$$\ddot x = \frac{1}{m}\left[-B_2\dot x - (k_2+k_3)x - k_2R\theta\right]$$

$$\dot x = dx/dt$$

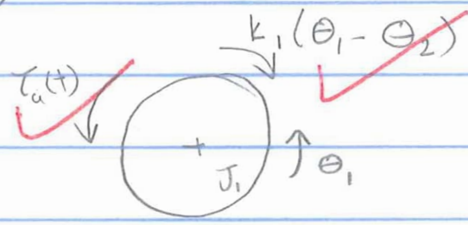
8

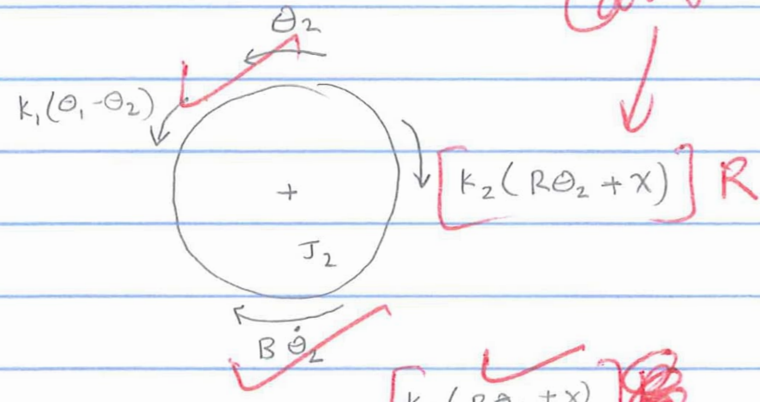
David Malawey

#3

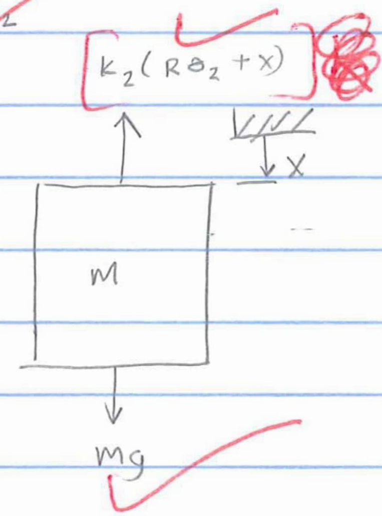
Careful

a)





b)



6

Exam – 3/ Dec-01-2010

ME 211/ Modeling and Analysis of Dynamic Systems/ 10 – 10.50 am/ Fall – 2010

$\frac{25}{30}$

Name: David Malawey

Instructions: **Max. Points: 30**

- *This is a closed book/ notes test.*
- *Attempt all problems as partial credit will be given.*
- *Before you turn in your test, make sure you have a) written your name; b) arrange the sheets in proper order and c) staple the entire work (including the cheat sheet).*
- *Show clearly the solution procedure. The instructor reserves the right to assign a zero if he doesn't understand your messy/ illogical work.*

GOOD LUCK

Problem 1 (10 Points):

The transfer function for a certain system is given: $H(s) = \frac{s^2+2s+2}{s^2+4s+4}$ $\frac{b_m}{a_n}$

a). Plot the pole-zero pattern for the transfer function.
b). Write the general form of the zero input response.
c). Write the general form of systems input-output differential equation.

Problem 2 (10 Points):
For the differential equation $\ddot{y}+5\dot{y}+4y=u(t)$, **find the unit step response** $y_U(t)$ **and the unit impulse response** $h(t)$.

Problem 3 (10 Points):
The unit-step response of a system is $y_U(t) = 4 + e^{-2t}(\cos t - 18\sin t)$ **for t > 0.**
a). Find the damping ratio and natural frequency of the system.
b). Comment on stability of the system.
c). Based on the value of damping ratio, how would you classify the system?
d). Find the steady state response to the input $u(t) = 4 + 3\sin(t + \frac{\pi}{3})$.

$$\mathcal{L}^{-1}\left(\frac{Bs+C}{(s+a)^2+\omega^2}\right) = e^{-at}\left[B\cos\omega t + \left(\frac{C-aB}{\omega}\right)\sin\omega t\right]$$

Table of Laplace Transforms

(a and b are constants and n is a constant integer)

$f(t)$	$F(s)$
$\delta(t)$	1
$\delta(t-nT)$	e^{-nTs}
1	$\frac{1}{s}$
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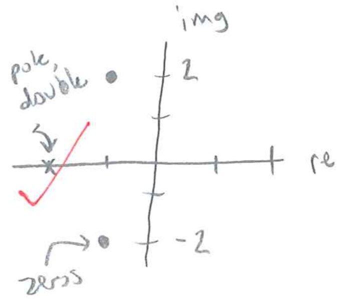
$\frac{t^2}{2}$	$\frac{1}{s^3}$
$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{s^n}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\frac{1}{2}t^2e^{-at}$	$\frac{1}{(s+a)^3}$
$\frac{t^{n-1}e^{-at}}{(n-1)!}$	$\frac{1}{(s+a)^n}$
$1-e^{-at}$	$\frac{a}{s(s+a)}$
$\frac{1}{a}\left(at-1+e^{-at}\right)$	$\frac{a}{s^2(s+a)}$
$\sin(bt)$	$\frac{b}{s^2+b^2}$
$\cos(bt)$	$\frac{s}{s^2+b^2}$
$e^{-at}\sin(bt)$	$\frac{b}{(s+a)^2+b^2}$
$e^{-at}\cos(bt)$	$\frac{s+a}{(s+a)^2+b^2}$
$1-e^{-at}\left[\cos(bt)+\frac{a}{b}\sin(bt)\right]$	$\frac{a^2+b^2}{s\left[(s+a)^2+b^2\right]}$
$\dot{f}(t)$	$sF(s) - f(0)$
$\ddot{f}(t)$	$s^2F(s) - sf(0) - \dot{f}(0)$

1) Poles $s^2 + 4s + 4 = 0$ $(s+2)^2 = 0$ Poles : $s = -2, -2$

Zeros $s^2 + 2s + 2 = 0$ $\dfrac{-2 \pm \sqrt{2^2 - 4(2)}}{2} = -1 \pm \sqrt{-4}$ X

$= -1 \pm 2i$ X

$-1 \pm i$



zero input response: $\ddot{y} + 2\dot{y} + 4y = u(t)^0$

This denotes the zero input differential eqn, not response

general i/o DE $\dfrac{d^2y}{dt^2} + 4\dfrac{dy}{dt} + y = \dfrac{d^2u}{dt^2} + 2\dfrac{du}{dt} + 2u$

$y(t) = K_1 e^{-s_1 t} + K_2 t e^{-s_2 t}$

where $s_1 = s_2$

8

2) $\ddot{y} + 5\dot{y} + 4y = u(t)$ find $y_u(t)$ & $h(t)$

$$H(s) = \frac{\cancel{U(s)}\ 1}{s^2+5s+4}$$

$$y_u(t) = \mathcal{L}^{-1}\left\{\frac{H(s)}{s}\right\} = \frac{1}{s(s^2+5s+4)} = \cancel{\frac{A}{s} + \frac{Bs+C}{s^2+5s+4}}$$

~~$1 = As^2 + 5As + 4A + Bs^2 + Cs$~~

~~$4A = 1 \Rightarrow A = \frac{1}{4}$~~

~~$5A + C = 0 \Rightarrow C = -\frac{5}{4}$~~

~~$A + B = 0 \Rightarrow B = -\frac{1}{4}$~~

$$\mathcal{L}^{-1}\left[\frac{1}{4}\frac{1}{s} \quad \frac{-\frac{1}{4}s - \frac{5}{4}}{s^2+5s+4}\right.$$

$$\frac{A}{s} + \frac{B}{s+4} + \frac{C}{s+1}$$

$1 = As^2 + 5As + 4A + Bs^2 + Bs + Cs^2 + 4Cs$

$A = \boxed{\frac{1}{4}}$

$A + B + C = 0 \Rightarrow B = A - C$

$5A + B + 4C = 0$

$5A + A - C + 4C = 0$

$6A = -3C \qquad C = -\frac{1}{2}A = \boxed{-\frac{1}{8}}$

$\frac{1}{4} - \frac{1}{8} + C = 0 \qquad \boxed{C = -\frac{1}{8}}$

$$\mathcal{L}^{-1}\left\{\frac{1}{4}\frac{1}{s} - \frac{1/8}{s+4} - \frac{1/8}{s+1}\right\}$$

$$\boxed{y_u(t) = \frac{1}{4} - \frac{1}{8}e^{-4t} - \frac{1}{8}e^{-t}}$$

$$h(t) = \frac{d\,y_u(t)}{dt} = -4\left(-\frac{1}{8}e^{-4t}\right) - 1\left(-\frac{1}{8}e^{-t}\right)$$

$$\boxed{h(t) = \frac{1}{2}e^{-4t} + \frac{1}{8}e^{-t}}$$

Math error

9

3) $y_u(t) = 4 + e^{-2t}(\cos t - 18\sin t)$ for $t > 0$

ω_n & $\zeta = ?$

$y_u(t) = \mathcal{L}^{-1}\left[\frac{H(s)}{s}\right]$ $\quad \mathcal{L}[4 + e^{-2t}\cos t - 18e^{-2t}\sin t] = \frac{H(s)}{s}$

$$\frac{H(s)}{s} = \frac{1}{s} + \frac{s+2}{(s+2)^2+1} - 18\frac{1}{(s+2)^2+1}$$

$$\frac{H(s)}{s} = \frac{1}{s} + \frac{s+2-18}{(s+2)^2+1}$$

$$H(s) = 1 + \frac{s^2-16s}{(s+2)^2+1}$$

$$= \frac{(s+2)^2+1+s^2-16s}{(s+2)^2+1}$$

$$H(s) = \frac{2s^2-12s+5}{(s+2)^2+1}$$

25/30

$s^2+4s+4+1+s^2-16s$

$2s^2-12s+5$

lots of math error But you know the method

$P(s) = s^2+2s+5$

gen. form $\ddot{y} + 2\dot{y} + 5y = f(t)$

$\omega_n^2 = 5$ $\quad \boxed{\omega_n = \sqrt{5}}$

$2\zeta\omega_n = 2$ $\quad \boxed{\zeta = \frac{1}{\sqrt{5}}}$

$\zeta : 0 < \zeta < 1$ $\boxed{\text{underdamped}}$

stability:

Poles: $\sqrt{2^2 - 4(1)(5)}$

$$\frac{-2 \pm \sqrt{4-20}}{2}$$

$$-1 \pm \frac{\sqrt{-15}}{2}$$

all poles have neg. real pts,

$\boxed{\text{Stable}}$

find y_{ss}, $u(t) = 4 + 3\sin(t + \frac{\pi}{3})$

$u_1(t) = 4 \Rightarrow y_{ss} = 4\ H(0) = 4\left(\frac{5}{(4+1)}\right) = \boxed{4}$

$u_2(t) = 3\sin(t + \frac{\pi}{3}) \Rightarrow y_{ss} = 3M\sin(t + \frac{\pi}{3})$

8

$B = 3$ $\quad \theta$ $\quad \phi = \frac{\pi}{3}$

$M =$ mag. $H(s=j\omega) =$ FRF $= \frac{2(j\omega)^2 - 12j\omega + 5}{(j\omega)^2 + 4j\omega + 5} = \frac{-2\omega^2 - 12j\omega + 5}{-\omega^2 + 4j\omega + 5}$

$$M = \frac{\sqrt{(2\omega^2+5)^2(-12j\omega)^2}}{\sqrt{(5-\omega)^2 + (4j\omega)^2}}$$

$$\theta = \tan^{-1}\left(\frac{-12j}{5-2\omega^2}\right) - \tan^{-1}\left(\frac{4j\omega}{5-\omega^2}\right)$$

$\boxed{y_{ss} = 4 + 3\sin(t + \frac{\pi}{3})}$

Note: I did not include M and θ because my terms blew up & it does not look right

YES

Homework

— Never turned this in —

1)

9.11) nonlinear System: $\ddot{x} + 2\dot{x} + \dot{x}^3 + \frac{4}{x} = A + B\cos t$

a) solve for operating point conditions on $\bar{x}$ and $\dot{\bar{x}}$ what restrictions must be placed on A value?

$$\cancel{\ddot{x}}^0 + \cancel{2\dot{x}}^0 + \cancel{\dot{x}}^{3\,0} + \frac{4}{\bar{x}} = \bar{u} = A$$

A-value must not be time-dependent $A \neq 0$

$$\boxed{\bar{x} = \frac{4}{A} \qquad \dot{\bar{x}} = 0}$$

b) derive linearized model, expressing all coefficients in terms of A or numbers

$X_0 - \bar{X}_0 = \hat{X}$

nonlinear terms $\dot{x}^3 + 2\dot{x} + \ddot{x}$

$\dot{x}^3$ $f(x) = f(\bar{x}) + \frac{df}{dx}\Big|_{\bar{x}} \hat{x} = \cancel{\dot{\bar{x}}}^{3\,0} + \cancel{\frac{d}{dx}(\dot{x}^3)\Big|_{\bar{x}}}^{0} \hat{x}$ $\quad f(\dot{x}) \approx 0$

$\ddot{x} \to \ddot{\hat{x}} + \cancel{\ddot{\bar{x}}}^0$

$2\dot{x} \to 2(\dot{\hat{x}} + \cancel{\dot{\bar{x}}})^0$

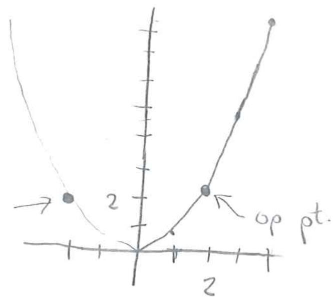
$\frac{4}{x}$ $f(x) = \frac{4}{\bar{x}} + \frac{d}{dx}\left(\frac{4}{x}\right)\Big|_{\bar{x}} \hat{x} = \frac{4}{\bar{x}} + -\frac{4}{\bar{x}^2}(\hat{x}) = \frac{4}{4/A} - \frac{4}{(4/A)^2}\hat{x}$

$= A - \frac{4}{16}A^2\hat{x} = A - \frac{A^2}{4}\hat{x} = A + B\cos t$

$$\boxed{\ddot{\hat{x}} + 2\dot{\hat{x}} - \frac{A^2}{4}\hat{x} = B\cos t}$$

2) $\boxed{9.14}$

$\dot{x} + 0.5x^2 = 2 + A\sin t$ a) sketch nonlinear term $.5x^2$, indicate all possible op. pts.



$0.5\bar{x}^2 = 2$

$\bar{x}^2 = 4$

$\bar{x} = \pm 2$

b) derive linearized model, stability

• for $\bar{x} = 2$ $f(x) = f(\bar{x}) + \left.\frac{df}{dx}\right|_{\bar{x}} (x - \bar{x})$

$= 0.5(\bar{x})^2 + \bar{x}(x - \bar{x}) = 0.5(4) + 2\hat{x}$

$f(x) = 2 + 2\hat{x} + (\dot{\hat{x}} + \cancel{\dot{\bar{x}}}^{0}) = 2 + A\sin t$

$\boxed{2\hat{x} + \dot{\hat{x}} = A\sin t}$ stable

• for $\bar{x} = -2$ $f(x) = 0.5(\bar{x})^2 + \bar{x}(\hat{x}) + (\dot{\hat{x}} + \cancel{\dot{\bar{x}}}^{0}) = 2 + A\sin t$

$= 2 + -2\hat{x} + \dot{\hat{x}} = 2 + A\sin t$

$\boxed{\dot{\hat{x}} - 2\hat{x} = A\sin t}$ unstable

$\tau = -\frac{1}{2} \Rightarrow$ unstable

$\tau < 0$

11-12 notes - stability of 2^{nd} order sys

3) $\underline{9.16|}$ part a only

initial cond.

$x(0) = 2 \quad \dot{x}(0) = 1$

$$\ddot{x} + 3|\dot{x}|\dot{x} + 4x^3 = A + B\sin 2t$$

O.P. $\cancel{\ddot{x}}^0 + 3|\cancel{\dot{x}}|^0\cancel{\dot{x}}^0 + 4x^3 = A$

$$\bar{x} = \left(\frac{A}{4}\right)^{1/3}$$

linearized model for $A = 4$, $\bar{x} = 1$

$\underline{4x^3}|$ $f(x) = 4(\bar{x})^3 + \frac{d}{dx}(4x^3)\Big|_{\bar{x}} \hat{x}$

$$= A + 12\bar{x}^2\hat{x} = 4 + 12\bar{x}^2\hat{x} = 4 + 12\tilde{x} \quad (1)$$

$\underline{3\dot{x}^2}|$ $f(x) = 3\cancel{\dot{\bar{x}}}^{2\,0} + 6\cancel{\dot{\bar{x}}}^0\hat{x}$

$\ddot{x} \rightarrow (\cancel{\ddot{\bar{x}}}^0 + \ddot{\hat{x}}) \quad (2)$

$$\boxed{\cancel{4} + 12\hat{x} + \ddot{\hat{x}} = \cancel{4} + B\sin 2t}$$

for $A = 4$, $\hat{x}(0) = x(0) - \bar{x} = 2 - 1 = 1$

$\dot{\hat{x}}(0) = \dot{x}(0) = 1$

$\Big\}$ how do we get this part?

4) 9.20

$$\dot{\omega} + 2|\omega|\omega + 2\omega = \tau_a(t) \qquad \tau_a(t) = 12 + \hat{\tau}_a(t)$$

O.P. $\cancel{\dot{\bar{\omega}}} + 2|\bar{\omega}|\bar{\omega} + 2\bar{\omega} = 12$

$2\bar{\omega}^2 + 2\bar{\omega} - 12 = 0$

for $\omega > 0$ $\quad \bar{\omega}^2 + \bar{\omega} - 6 = 0$

$(\bar{\omega} + 3)(\bar{\omega} - 2) = 0 \qquad \bar{\omega} = 2, \cancel{-3}$ does not apply

linearized model

nonlinear term $\quad 2\omega^2 \quad f(\omega) = 2\bar{\omega}^2 + 4\bar{\omega}\hat{\omega}$

other terms $\quad \dot{\omega} \to (\cancel{\dot{\bar{\omega}}}^{0} + \dot{\hat{\omega}})$

$2\omega \to 2(\bar{\omega} + \hat{\omega})$

$2\bar{\omega}^2 + 4\bar{\omega}\hat{\omega} + \dot{\hat{\omega}} + 2\bar{\omega} + 2\hat{\omega} =$

$\dot{\hat{\omega}} + 8\hat{\omega} + 2\hat{\omega} + 4 + 8$

$\dot{\hat{\omega}} + 10\hat{\omega} + \cancel{12} = \cancel{12} + \hat{\tau}_a(t)$

$$\boxed{\dot{\hat{\omega}} + 10\hat{\omega} = \hat{\tau}_a(t)}$$

David Malawey

ASSIGNMENT – 11/ ME 211/ MODELING AND ANALYSIS OF DYNAMIC SYSTEMS

DUE: 11/29/2010 (Monday) IN CLASS OR BY 2PM IN MY OFFICE (TMH 129)

Problem 1: Textbook Problem 8.14; [Use values of constants from 8.14/Part b to solve part a (only) of 8.13]. First, solve the problem using Laplace tranform and then verify your answer using the general expression for homogeneous solution for a second order differential equation.

Problem 2: Textbook Problem 8.33 (For part b, solve using the properties of transfer function discussed in class)

Problem 3: Textbook Problem 8.35

Problem 4: Textbook Problem 8.38

Problem 5: Textbook Problem 8.47; Ignore the problem statement and do following;

For each of the transfer function,

a) Plot the pole-zero pattern;

b) Find the frequency response function;

c) Find $M(\omega)$ and $\theta(\omega)$ (Note: M and θ are function of ω)

d) ω_n (natural frequency) and ζ (damping ratio)

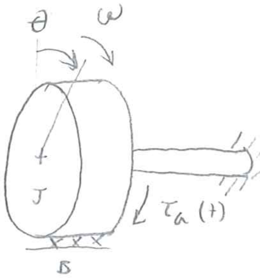
Problem 6: Textbook Problem 8.50 (part-a only, i.e. find A, B and C)

Problem 7: Textbook Problem 8.55

Problem 8: Textbook Problem 8.56

David Malawey
211 assignment 11

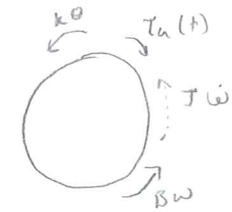
1) 8.13, a) find zero-input response, terms of $\theta(0)$ & $\dot\theta(0)$



$J = 1\,kg\cdot m^2 \quad B = 4\,N\cdot m\cdot s \quad k = 4\,N\cdot m$

$\cancel{\tau_a(t)}^{0} - B\dot\theta - k\theta = J\ddot\theta$

$\ddot\theta = \frac{1}{J}\left[\cancel{\tau_a(t)}^{0} - B\dot\theta - k\theta\right]$



$\theta(0) = \theta_0$
$\dot\theta(0) = \dot\theta_0$

$s^2\theta(s) - s\theta(0) - \dot\theta(0) = \frac{1}{J}\left[\cancel{\tau_a(s)}^{0} - B(s\theta(s) - \theta_0) - k\,\theta(s)\right]$

$s^2\theta(s) - s\theta_0 - \dot\theta_0 = \frac{1}{J}\left[-Bs\theta(s) + B\,\theta_0 - k\theta(s)\right]$

$\cancel{J}\left[s^2\theta(s) - s\theta_0 - \dot\theta_0\right] - \overset{4}{B}s\,\theta(s) - \overset{4}{B}\,\theta_0 - \overset{4}{k}\theta(s) = 0$

$\theta(s)\left[s^2 + 4s + 4\right] + \theta_0\left[-s - 4\right] - \dot\theta_0 = 0$

$\theta(s) = \frac{\dot\theta_0 + 4\theta_0 + s\theta_0}{s^2 + 4s + 4} = \frac{A}{(s+2)} + \frac{B}{(s+2)^2}$

$\dot\theta_0 + 4\theta_0 + s\theta_0 = A(s+2) + B$

$s\theta_0 = As \Rightarrow A = \theta_0$

$\dot\theta_0 + 4\theta_0 = 2A + B$

$B = \dot\theta_0 + 4\theta_0 - 2\theta_0 \Rightarrow B = \dot\theta_0 + 2\theta_0$

$\mathcal{L}^{-1}\left[\theta(s) = \frac{\theta_0}{s+2} + \frac{\dot\theta_0 + 2\theta_0}{(s+2)^2}\right]$

$= \theta(t) = \theta_0 e^{-2t} + (\dot\theta_0 + 2\theta_0)(te^{-2t})$

$\boxed{\theta_{zi}(t) = \theta_0 e^{-2t} + (\dot\theta_0 + 2\theta_0)(te^{-2t})}$ ✓

$\tau_a(t) = J\ddot\theta - B\dot\theta - k\theta$

$= 1\ddot\theta - 4\dot\theta - 4\theta$

$a_1^2 - 4a_2a_0 = 4^2 - 4(1)(4) = 16 - 16 = 0$

$(s+2)^2 = 0 \Rightarrow s_1 = s_2 = -2$

$y_H = k_1e^{s_1t} + k_2te^{s_2t}$

$\theta_{zi} = k_1e^{s_1t} + k_2te^{s_2t}$

$\theta(0) = k_1e^{-2t} + k_2\cancel{te^{-2t}}^{0}$

$\theta_0 = k_1$ ✓

$\dot\theta_0 = -2k_1e^{-2t} + k_2e^{-2t} - 2k_2te^{-2t}$

$\dot\theta_0 = -2k_1 + k_2$

$k_2 = \dot\theta_0 + 2\theta_0$ ✓

$\therefore \boxed{\theta_{zi}(t) = \theta_0e^{-2t} + (\dot\theta_0 + 2\theta_0)te^{-2t}}$ ✓

2) continued

8.33) $\ddot{y} + 4\dot{y} + 25y = 50u(t)$

b) $H(s) = \dfrac{50}{1s^2 + 4s + 25}$

$y_u(t) = \mathcal{L}^{-1}\left(\dfrac{H(s)}{s}\right) = \mathcal{L}^{-1}\ \dfrac{50}{s(s^2+4s+25)} = \dfrac{A}{s} + \dfrac{Bs+C}{(s+2)^2+\sqrt{21}^2}$

$\hookrightarrow = s(s+2)^2 + (\sqrt{25-4})^2$

$50 = As^2 + 4As + 25A + Bs^2 + Cs$

$A = \frac{50}{25} = 2$

$Bs + C = -2s - 8 = (s+2)(-2) - 4$

$As^2 + Bs^2 = 0 \quad 2s^2 = -Bs^2 \quad B = -2$

$4As + Cs = 0 \quad C = -8$

$\mathcal{L}^{-1}\left\{\dfrac{2}{s} - \dfrac{2(s+2)}{(s+2)^2+\sqrt{21}^2} - \dfrac{4}{(s+2)^2+\sqrt{21}^2}\right\} = 2 - 2\left(e^{-2t}\cos\sqrt{21}\,t\right) - \frac{4}{\sqrt{21}}\left(e^{-2t}\sin\sqrt{21}\,t\right)$

$y_u(t) = 2 + e^{-2t}\left(-2\cos\sqrt{21}\,t - \frac{4}{\sqrt{21}}\sin\sqrt{21}\,t\right) =$

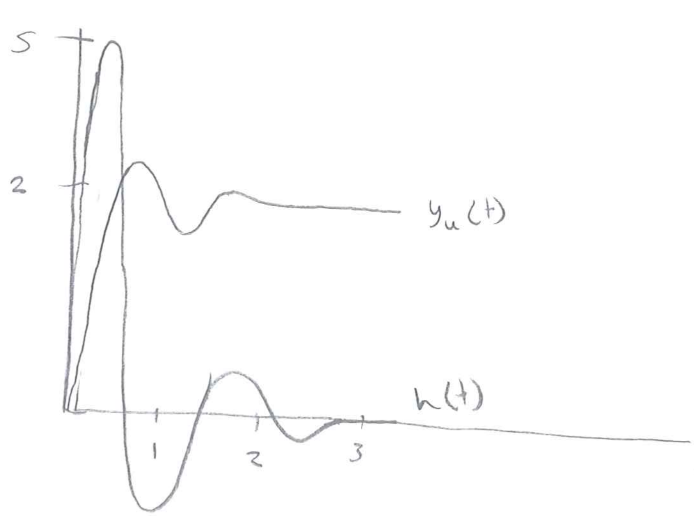
$\boxed{y_u(t)\ \ 2 - e^{-2t}(2\cos 4.583t + .873\sin 4.583t)}$

$h(t) = \mathcal{L}^{-1}(H(s))$

$= \mathcal{L}^{-1}\left[\dfrac{50}{s^2+4s+25}\right] \quad \dfrac{50}{(s+2)^2+(\sqrt{21})^2} = \dfrac{50}{\sqrt{21}}\left(\dfrac{\sqrt{21}}{(s+2)^2+\sqrt{21}^2}\right)$

$h(t) = \frac{50}{\sqrt{21}}\,e^{-2t}\sin\sqrt{21}\,t$

$\boxed{h(t) = 10.91\,e^{-2t}\sin 4.583t}$



2) $\underline{8.33}$

a) $\ddot{y} + 4\dot{y} + 25y = 50\,u(t)$ find ζ and ω_n

$a_2 = \frac{1}{50}$ $\quad \frac{a_0}{a_2} = 25$

$a_0 = \frac{1}{2}$ $\quad \frac{a_1}{a_2} = 4$

$a_1 = \frac{4}{50}$

$$\zeta = \frac{a_1}{2\sqrt{a_0 a_2}} = \frac{\frac{4}{50}}{2\sqrt{\frac{1}{2}\frac{1}{50}}} = \frac{4}{100\sqrt{\frac{1}{100}}} = \frac{4}{10}$$

$\boxed{\zeta = .4}$

$$\omega_n = \sqrt{\frac{a_0}{a_2}} = \sqrt{\frac{\frac{1}{2}}{\frac{1}{50}}} = \sqrt{25} \Rightarrow \boxed{5 = \omega_n}$$

roots complex conjugate, stable syst.

b) Sketch unit step response $y_u(t)$ $\quad y(0) = 0 \quad \dot{y}(0) = 1$

$$a_1^2 - 4a_2 a_0 = 4 - 4(1)(25) \qquad s_{1,2} = \frac{-4 \pm \sqrt{-96}}{2} = -2 \pm \frac{\sqrt{-96}}{2} = -4 \pm$$

$$y_H = e^{\alpha t}[k_1 \cos\beta t + k_2 \sin\beta t] \qquad \alpha = -\frac{4}{2} = -2 \qquad \beta = 4i\sqrt{6}$$

$$y_H = e^{-2t}[k_1 \cos 4i\sqrt{6}t + k_2 \sin 4i\sqrt{6}\,t]$$

$P(s) =$

$$y(0) = (k_1 \cos(0) + k_2 \sin(0)) \Rightarrow y_0 = k_1$$

$$\dot{y}(t) = -2e^{-2t}[k_1 \cos 4i\sqrt{6}t + k_2 \sin 4i\sqrt{6}t] + e^{-2t}[-4i\sqrt{6}k_1 \sin(\ \ t) + 4i\sqrt{6}k_2 \cos(\ \ t)]$$

$$\dot{y}(t) = -2\ k_1 \cos(4i\sqrt{6}t) + 4i\sqrt{6}k_2 \cos(4i\sqrt{6}t)$$

$$\dot{y}(t) = -2k_1 + 4i\sqrt{6}k_2$$

$$\dot{y}(t) = -2y_0 + 4i\sqrt{6}k_2 \Rightarrow k_2 =$$

find/sketch unit impulse response $h(t)$ $\qquad h(t) = \frac{d}{dt}y_u(t)$

3) $\boxed{8.35}$

a) find ζ and ω_n

b) find / sketch response when $u(t) = 2$
& $y(0) = 0$; $\dot{y}(0) = 1$

$\ddot{y} + 2\dot{y} + 4y = 4u(t) \Rightarrow$

$\frac{1}{a_2} = 4 \quad a_2 = \frac{1}{4} \qquad \frac{a_1}{a_2} = 2 \qquad a_1 = \frac{1}{2}$

$\frac{a_0}{a_2} = 4 \Rightarrow a_0 = 1$

$\omega_n = \sqrt{\frac{a_0}{a_2}} = \sqrt{4} = \boxed{2}$ ✓

$\zeta = \frac{a_1}{2\sqrt{a_0 a_2}} = \frac{\frac{1}{2}}{2\sqrt{\frac{1}{4}}} = \frac{\frac{1}{2}}{1} = \boxed{\frac{1}{2}}$ ✓

$\mathcal{L}\left[\ddot{y} + 2\dot{y} + 4y = 8\right] = s^2 Y(s) + \cancel{s\,y(0)}^{0} - \cancel{\dot{y}(0)}^{1} + 2sY(s) - \cancel{2y(0)}^{0} + 4Y(s) - \frac{8}{s} = 0$

$Y(s)\left[s^2 + 2s + 4\right] - 1 - \frac{8}{s} = 0$

$Y(s) = \frac{\left(1 + \frac{8}{s}\right)}{s^2 + 2s + 4} = \frac{s + 8}{s(s^2 + 2s + 4)}$ $\qquad s^2 + 2s + 4 = (s+1)^2 + (\sqrt{2})^2$

$= \frac{A}{s} + \frac{Bs + C}{s^2 + 2s + 4} \Rightarrow s + 8 = As^2 + 2As + 4A + Bs^2 + Cs$

$A + B = 0$

$2A + C = 0$

$A = 2$

$B = -2$

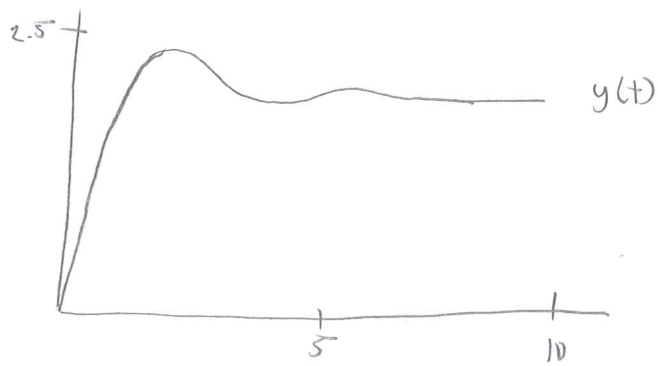
$C = -4$

$-2(s+1) - 2$

$Y(s) = \frac{2}{s} + \left\{ \frac{-(2s+3)}{(s+1)^2 + \sqrt{3}^2} \right\}$

$y(t) = 2 + (-2)(e^{-t}\cos\sqrt{3}t) - \sqrt{3}\,e^{-t}\sin\sqrt{3}t$

$\boxed{y(t) = 2 - e^{-t}(2\cos 1.73t + .577\sin 1.73t)}$ ✓



#4)

8.38) 2 inputs gravity $M_2 g$ & $f_a(t)$ applied to M_1

$$(M_1+M_2)\ddot{x} + (B_1+B_2)\dot{x} + k_2 x = f_a(t) - M_2 g$$

a) solve for x_0 when $f_a(t)=0$

b) find $X(s)$ when $x(0)=x_0$ & $f_a(t)$ = unit step.

c) find $x(t)$ for $M_1 = M_2 = B_1 = B_2 = k_2 = 1$ check answer by finding x_{ss} & $x(0+)$

a) $x = \frac{1}{k_2}\left[-M_2 g - (M_1+M_2)\cancel{\ddot{x}}^{\,0} - (B_1+B_2)\cancel{\dot{x}}^{\,0}\right]$ $\boxed{x_0 = \frac{-M_2 g}{k_2}}$ ✓

b) $(M_1+M_2)\left[s^2 X(s) - s\,x(0) - \cancel{\dot{x}(0)}^{\,0}\right] + (B_1+B_2)\left[s X(s) - x(0)\right] + k_2(X(s)) = \cancel{f_a(s)}^{\,\frac{1}{s}} - \frac{M_2 g}{s}$

$X(s)\left[(m_1+m_2)s^2 + (B_1+B_2)s + k_2\right] + x_0\left(\left(M_1+M_2\right)(-s) - (B_1+B_2)\right) = \frac{1}{s} - \frac{M_2 g}{s}$

$x_0 \leftarrow -\frac{M_2 g}{k_2}$

Let $M = m_1 + m_2$, $B = B_1 + B_2$

$X(s)(Ms^2 + Bs + k_2) + \frac{M_2 g}{k_2}(+Ms + B) = \frac{1}{s} + \frac{M_2 g}{s}$

$X(s) = \frac{1}{(Ms^2+Bs+k_2)}\left[\frac{1}{s} + \frac{M_2 g}{s} - \frac{M_2 g}{k_2}(Ms+B)\right] = \frac{1}{s(Ms^2+Bs+k_2)} - \frac{\frac{M_2 g}{k_2}\cancel{\left(Ms+B+\frac{k_2}{s}\right)(s)}}{\cancel{(Ms^2+Bs+k_2)}(s)}$

$$\boxed{X(s) = \frac{1}{s(Ms^2+Bs+k_2)} - \frac{M_2 g}{s k_2}} ✓ = \frac{1}{s(2s^2+2s+1)} - \frac{g}{s}$$

$\frac{1}{s(2s^2+2s+1)} = \frac{A}{s} + \frac{Bs+C}{2s^2+2s+1}$

$1 = 2As^2 + 2As + A + Bs^2 + Cs$

$A = 1$

$2A + B = 0 \quad B = -2$

$2A + C = 0 \quad C = -2$

$X(s) = \frac{1}{s} + \frac{\cancel{2}s + \cancel{2}}{\cancel{2}s^2 + \cancel{2}s + 1/2}$

$\frac{s+1}{s^2+s+\frac{1}{2}} = \frac{s+\frac{1}{2} + \frac{1}{2}}{(s+\frac{1}{2})^2 + \sqrt{\frac{1}{4}}^{\,2}}$ $\leftarrow \frac{1}{2}$

$\mathcal{L}^{-1}\left[X(s) = \frac{1}{s} + \frac{(s+1)}{s^2+s+\frac{1}{2}} - \frac{g}{s}\right]$

$= 1 - g + e^{-\frac{1}{2}t}\cos(\frac{1}{2}t) + e^{-\frac{1}{2}t}\sin(\frac{1}{2})$

$\boxed{x(t) = 1 - g - e^{-\frac{t}{2}}\left(\cos\frac{t}{2} + \sin\frac{t}{2}\right)}$ ✓

✓ $x_{ss} = \lim_{t\to\infty} x(t) = 1-g$

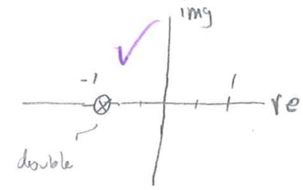
✓ $x(0+) = \frac{-(1)g}{(1)}$

#5) 8.47

a) plot pole-zero pattern b) find frequency-response function, c) find $M(\omega)$ $\theta(\omega)$ d) ω_n & ζ

a) $H(s) = \dfrac{2}{s^2+2s+1}$ Poles: $s^2+2s+1 = (s+1)^2$ double pole @ $s=-1$

zeros: none

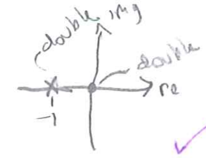


b) FRF: $= H(s=j\omega) = \dfrac{2}{(j\omega)^2+2j\omega+1} = \boxed{\dfrac{2}{-\omega^2+2j\omega+1}}$

c) $M(\omega) - \sqrt{\omega^4+(2\cancel{j}\omega)^2+1^2} = \omega^2+1$ $\boxed{M(\omega) = \dfrac{2}{(\omega^2+1)}}$

$\theta = \arg(\text{num}) - \arg(\text{denom}) = \tan^{-1}\left(\frac{0}{2}\right) - \tan^{-1}\left(\frac{2j\omega}{1-\omega^2}\right) \Rightarrow \boxed{\theta(\omega) = -\tan^{-1}\left(\frac{2\omega}{1-\omega^2}\right)}$

ω_n : $P(s) = s^2+2\omega_n\zeta s+\omega_n^2$ $\boxed{\omega_n = 1}$ $\boxed{\zeta = 1}$



b) $H(s) = \dfrac{2s^2}{s^2+2s+1}$ zeros: double, $s=0$

poles: double $s=-1$

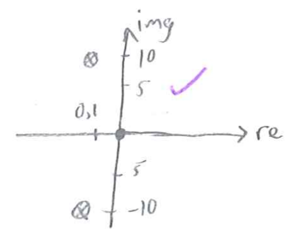
b) FRF $= H(s=j\omega) = \dfrac{2(j\omega)^2}{(j\omega^2)+(2j\omega)+1} = \boxed{\dfrac{-2\omega^2}{-\omega^2+2j\omega+1}}$

c) $M(\omega) = \boxed{\dfrac{2\omega^2}{(\omega^2+1)}}$ X d) $\theta(\omega) = 0 \boxed{- \tan^{-1}\left(\frac{2\omega}{1-\omega^2}\right)}$ X $P(s) = s^2+2\omega_n\zeta s+\omega_n^2 =$

$\boxed{\omega_n = 1}$ $\boxed{\zeta = 1}$

c) $H(s) = \dfrac{s}{s^2+.2s+100}$ zeros : $s=0$

poles: $\dfrac{-b \pm \sqrt{b^2+4ac}}{2a} = \dfrac{-.2 \pm \sqrt{.2^2+4(100)}}{2} = -.1 \pm 10i$



$\boxed{\text{FRF} = \dfrac{j\omega}{-\omega^2+.2j\omega+100}} \rightarrow (100-\omega^2)+.2j\omega$

$\boxed{M(\omega) = \dfrac{\omega}{\sqrt{(100-\omega^2)^2+.04\omega^2}}}$

$\boxed{\theta(\omega) = \tan^{-1}\left(\frac{\omega}{0}\right) - \tan^{-1}\left(\frac{.2\omega}{100-\omega^2}\right)}$

$\boxed{\omega_n = 10}$ $\zeta =$ $0.2 = 2\omega_n\zeta$ $\dfrac{0.1}{10} = \boxed{\zeta = .01}$

−1

#6)

8.50

$u(t) = \underbrace{1}_{u_1} + \underbrace{\sin t}_{u_2} + \underbrace{\sin 10t}_{u_3}$ $y_{ss}(t) = A + B\sin(t+\theta_2) + C\sin(10t+\theta_3)$

find A, B, C $u(t) = B\sin(\omega t+\phi)$ $\rightarrow y_{ss}(t) = y_p = BM\sin(\omega t+\phi+\theta)$

$H(s) = \frac{2}{s^2+2s+1}$

from 8.47 prt a) $M(\omega) = \frac{2}{(\omega^2+1)}$

$u_1 = 1$, constant input, $y_{ss} = (t) = AH(s=0) = A\left(\frac{2}{1}\right)$ let $A=1$ $\Rightarrow y_{ss_1} = 2$ $\boxed{A=2}$

$u_2 = \sin t$ $y_{ss}(t) = BM\sin(\omega t+\phi+\theta)$ $B=1$ $\omega=1$ $\phi=0$ $y_{ss} = M\sin(\omega t+\phi+\theta)$

$M(\omega) = \left(\frac{2}{1^2+1}\right) = 1$ $\Rightarrow \boxed{B=1}$

$u_3 = \sin 10t$ $y_{ss}(t) = BM\sin(\omega t+\phi+\theta)$ $C\sin(10t+\theta_3) \Rightarrow \omega = 10$ $\phi=0$ $B=1$

$M(\omega) = \frac{2}{(10^2+1)} = .0198$ $\Rightarrow \boxed{C = .0198}$

rewriting for practice

input u	steady state response y_{ss}	formula for y_{ss}	ω	M
1	A	$H(s=0)$	NA	NA
$\sin t$	$B\sin(t+\theta_2)$	$M\sin(\omega t+\phi+\theta)$	1	1
$\sin 10t$	$C\sin(10t+\theta_3)$	$M\sin(\omega t+\phi+\theta)$	10	.0198

$M(\omega) = \frac{2}{(\omega^2+1)}$ $H(s) = \frac{2}{s^2+2s+1}$

7) $\boxed{8.55}$ unit step response $y_u(t) = te^{-2t}$ for $t>0$

a) find ζ & ω_n

b) find complete response for $t>0$ when input is $u(t) = tU(t)$

c) fing y_{ss} when $u(t) = 3 + 5\sin(2t + \frac{\pi}{4})$

a) $y_u(t) = \mathcal{L}^{-1}\left[\frac{H(s)}{s}\right]$

$s\,\mathcal{L}[y_u(t)] = H(s) \Rightarrow s\,\frac{1}{(s+2)^2}$ $\quad H(s) = \frac{s}{(s+2)^2}$ $\quad P(s) = s^2+4s+4$

$= s^2 + 2\omega_n\zeta s + \omega_n^2$

$2(2)\zeta = 4$

$$\boxed{\begin{matrix}\omega_n = 2 \\ \zeta = 1\end{matrix}}$$

b) when $u(t) = tU(t)$ $\quad U(s) = \frac{1}{s^2}U(s)$

$Y(s) = H(s)\,U(s) = \frac{s}{(s+2)^2}\left(\frac{1}{s^2}U(s)\right) = \frac{U(s)}{s(s+2)^2}$ $\quad \frac{1}{s(s+2)^2} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$

$1 = As^2 + 4As + 4A + Bs^2 + 2Bs + Cs$

$4A = 1 \Rightarrow A = \frac{1}{4}$

$A + B = 0 \quad B = \frac{-1}{4}$

$4A + 2B + C = 0 \rightarrow 1 - \frac{1}{2} + C = 0 \quad C = -\frac{1}{2}$

$Y(s) = \frac{\frac{1}{4}}{s} - \frac{1/4}{(s+2)} - \frac{1/2}{(s+2)^2}$ take $\mathcal{L}^{-1}$

$y(t) = \frac{1}{4} - \frac{1}{4}(e^{-2t}) - \frac{1}{2}te^{-2t}$

$$\boxed{y(t) = \frac{1}{4} - e^{-2t}\left(\frac{1}{4} + \frac{1}{2}t\right)}$$

c) find y_{ss} $\quad u(t) = \underbrace{3}_{u_1} + \underbrace{5\sin(2t + \frac{\pi}{4})}_{u_2}$ $\quad B = 5 \quad \omega = 2 \quad \phi = \frac{\pi}{4}$

$u_2(t) \quad B\sin(\omega t + \phi)$

get M $\quad$ FRF $= H(s = j\omega) \quad \frac{j\omega}{(j\omega+2)^2} = \frac{j\omega}{-\omega^2 + 4\omega j + 4}$ $\quad M(\omega) = \frac{\omega}{\sqrt{\omega^4 + 4\omega^2 + 16}}$

$M(\omega) = \frac{\omega}{(\omega^2+4)} = \frac{2}{4+4} = \frac{1}{4}$ $\quad \theta = \tan^{-1}\left(\frac{0}{1}\right) - \tan^{-1}\left(\frac{0}{4}\right) = 0$

for u_2: $\quad y_{ss} = BM\sin(\omega t + \phi + \theta)$

$= \frac{5}{4}\sin(2t + \frac{\pi}{4})$

for u_1: $\quad u(t) = A = 3$

$y_{ss}(t) = A\,H(s=0) = 3(0) = 0$

$$\boxed{y_{ss}(t) = \frac{5}{4}\sin\left(2t + \frac{\pi}{4}\right)}$$

8)

8.56 $H(s) = \frac{s^2+as+b}{s^2+cs+d}$ find a, b, c, d

when input is unit step, $y_{ss} = 2$

input $u(t) = A\sin 2t$, $y_{ss} = 0$

y_{tr} is critically damped, $(\zeta = 1)$

two equal roots @ $-\omega_n$

for const input $y_{ss} = H(s=0) = \frac{b}{d} = 2$

$u(t) = A(\sin 2t)$ $y_{ss} = 0 = BM\sin(\omega t + \phi + \theta)$ $B = A$, $\phi = 0$ $\omega = 2$

$$H(s=j\omega) = H(2j) = \frac{(2j)^2 + a(2j) + b}{(2j)^2 + c(2j) + d} = 0 = \frac{-4 + a2j + b}{-4 + 2jc + d}$$

$-4 + 2aj + b = 0$

$2aj + b = 4$

$\boxed{a = 0}$

$\boxed{b = 4}$

$d = \frac{b}{2}$ $\boxed{d = 2}$

$\zeta = 1$ $P(s) = s^2 + 2\omega_n \zeta s + \omega_n^2$ $s^2 + cs + d$

$\omega_n^2 = d$

$2\omega_n \zeta = c$

$2(\sqrt{2})(1) = c$ $\boxed{c = 2\sqrt{2}}$

79/80

D. Malawey

ASSIGNMENT – 10/ ME 211/ MODELING AND ANALYSIS OF DYNAMIC SYSTEMS

DUE: 11/15/2010 (Monday) IN CLASS OR BY 2PM IN MY OFFICE (TMH 129)

Problem 1: Textbook Problem 8.5; (Part b and c only); The general expression for $E_0(s)$ is given:

$$E_o(s) = \frac{sI_i(s)}{Cs^2 + \frac{1}{R}s + \frac{1}{L}} + \frac{sCe_o(0) - i_L(0)}{Cs^2 + \frac{1}{R}s + \frac{1}{L}}$$

Also, identify the transfer function.

Problem 2: Textbook Problem 8.6 (Part b only): Using the result from part a, solve part b and comment on stability.

Problem 3: Textbook Problem 8.10

Problem 4: Textbook Problem 8.19

Problem 5: Textbook Problem 8.24 (Use state variable equations #70/ Page 124)

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1) 8.5)

b) transform of output: $E_o = \dfrac{sI_i(s)}{Cs^2 + \frac{1}{R}s + \frac{1}{L}} + \dfrac{sCe_o(0) - i_L(0)}{Cs^2 + \frac{1}{R}s + \frac{1}{L}}$

$C = 1F$

$R = 1/3\ \Omega$

$L = \frac{1}{2}H$

$$E_o = \underbrace{\left(\frac{s}{s^2+3s+2}\right) I_i(s)}_{\text{zero state component}} + \underbrace{\frac{s\,e_o(0) - i_L(0)}{s^2+3s+2}}_{\text{zero-input component}}$$

c) $i_i(t) = 1 + \sin t$ for $t > 0$

$$I_i(s) = \frac{1}{s^2+1} + \frac{1}{s} = \frac{s^2+1+s}{(s^2+1)(s)} = \frac{s^2+s+1}{s(s^2+1)}$$

$$E_o(s)_{zs} = \left[\frac{\cancel{s}}{(s+1)(s+2)}\right]\left[\frac{s^2+s+1}{\cancel{s}(s^2+1)}\right] = \frac{s^2+s+1}{(s+1)(s+2)(s^2+1)} = \frac{A}{(s+1)} + \frac{B}{(s+2)} + \frac{Cs+D}{(s^2+1)}$$

$s = -1 \Rightarrow A = \frac{1}{2}$ $\quad s = -2 \Rightarrow B = \frac{3}{-5}$

$s^2+s+1 = A(s+2)(s^2+1) + B(s+1)(s^2+1) + Cs+D(s+1)(s+2)$

$s^3 = 0 = A+B+C \Rightarrow C = -A-B = \frac{1}{10}$

$s^2 = 1 = A + B + 2C + 1C + D \Rightarrow D = 0.3$

$$E_o(s)_{zs} = \frac{.5}{(s+1)} + \frac{-3/5}{s+2} + \frac{.1s}{s^2+1} + \frac{.3}{s^2+1}$$

$$\boxed{e_o(t)_{zs} = .5e^{-t} - \frac{3}{5}e^{-2t} + .1\cos(t) + .3\sin(t)}$$

$E_{ozi} = \dfrac{A}{(s+1)} + \dfrac{B}{(s+2)}$

$se_o(0) - i_L(0) = As + 2A + Bs + B$

$s: e_o(0) = A + B$

$-i_L(0) = 2A + B \Rightarrow$

$e_o(0) = A - i_L(0) - 2A \Rightarrow A = -i_L(0) - e_o(0)$

$-i_L(0) + 2i_L(0) + 2e_o(0) = B$

$B = i_L(0) + 2e_o(0)$

$$\mathcal{L}^{-1}\left[E_o(0)_{zi} = \frac{-i_L(0) - e_o(0)}{(s+1)} + \frac{i_L(0) + 2e_o(0)}{(s+2)}\right]$$

$e_o(t)_{zi} \Rightarrow (-i_L(0) - e_o(0))e^{-t} + (i_L(0) + 2e_o(0))e^{-2t}$

$$\boxed{e_o(t)_{zi} = -(i_L(0) + e_o(0))e^{-t} + [2e_o(0) + i_L(0)]e^{-2t}}$$

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2) 8.6 pt b only

find Poles $R = \frac{2}{9}\Omega$ $\frac{2}{7}\Omega$ & $\frac{2}{3}\Omega$

$$E_o(s) = \frac{-se_c(0) - 2i_L(0) + \left(s^2 + 2s\left(\frac{9}{2}\right)\right) E_i(s)}{s^2 + \left(1 + 2\left(\frac{9}{2}\right)\right)s + 16} = \frac{-se_c(0) - 2i_L(0) + (s^2 + 9s) E_i(s)}{s^2 + 10s + 16}$$

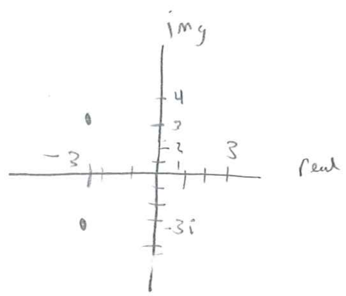
b) $D = 0 \Rightarrow s^2 + 10s + 16 = 0$ $(s+8)(s+2) = 0$ $\boxed{s = -8, -2}$ STABLE ✓

for $R = \frac{2}{7}\Omega \Rightarrow s^2 + 8s + 16 = (s+4)(s+4) \Rightarrow \boxed{s = -4, -4}$ STABLE ✓

for $R = \frac{2}{3}\Omega \Rightarrow s^2 + 3s + 16 =$ $s = \frac{-4 \pm \sqrt{4(3) - 2(16)}}{2(1)}$

$= -\frac{4}{2} \pm \sqrt{-10}$ $\boxed{s = -2 + \sqrt{10}i, \; -2 - \sqrt{10}i}$ MARGINALLY STABLE ✓

-1



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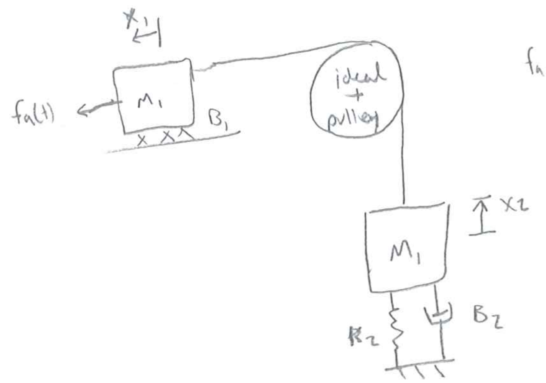
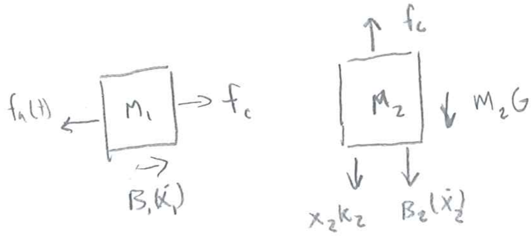
3) 8.10

$$(M_1+M_2)\ddot{X} + (B_1+B_2)\dot{X} + K_2 X = f_a(t) - M_2 g \qquad X_1 = X_2 = X$$





a)

$$M_1 \ddot{X}_1 = f_a(t) - B_1\dot{X}_1 - f_c \qquad (1)$$

$$M_2 \ddot{x}_2 = f_c - M_2 g - X_2 k_2 - B_2 \dot{X}_2$$

Plug (2) into (1)

$$f_c = -M_2\ddot{x}_2 + M_2 g + X_2 k_2 + B_2\dot{X}_2 \qquad (2)$$

$$f_a(t) = M_1\ddot{x} + B_1\dot{x} + [M_2\ddot{x} + M_2 g + X k_2 - B_2\dot{x}]$$

$$\boxed{(M_1+M_2)\ddot{x} + (B_1+B_2)\dot{x} + k_2 X = f_a(t) - M_2 g}$$

b) find damping ratio

$a_2 = (M_1+M_2) \quad a_1 = (B_1+B_2) \quad a_0 = k_2$

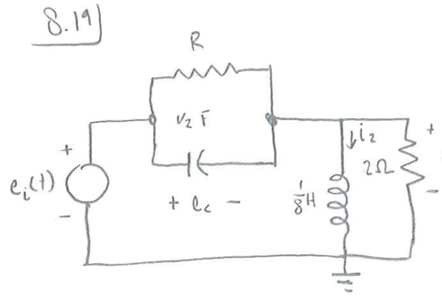
$$\xi = \frac{a_1}{2\sqrt{a_0 a_2}} = \boxed{\frac{B_1+B_2}{2\sqrt{k_2(M_1+M_2)}}} ✓ \qquad \boxed{\omega_n = \sqrt{\frac{k_2}{(B_1+B_2)}}} \quad X -1$$

c) find X_{ss}: $x = \frac{1}{k_2}(f_a(t) - M_2 g)$ But $f_a(t) = 1$

$\lim_{t\to\infty}$

$$\boxed{X_{ss} = \frac{1 - M_2 g}{k_2}} ✓$$

#4) 8.19



$H(s)$, ζ, ω_n, in terms of R

$$E_o(s) = \frac{-s e_c(0) - 2 i_L(0) + \left(s^2 + \frac{2s}{R}\right) E_i(s)}{s^2 + \left(1 + \frac{2}{R}\right)s + 16}$$

take $e_c(0) = 0$, $i_L(0) = 0$

$$E_o(s) = \frac{0 - 0 + \left(s^2 + \frac{2s}{R}\right)E_i(s)}{s^2 + \left(1 + \frac{2}{R}\right)s + 16} \Rightarrow \frac{E_o(s)}{E_i(s)} = \boxed{H(s) = \frac{\left(s^2 + \frac{2}{R}s\right)}{s^2 + \left(1 + \frac{2}{R}\right)s + 16}}$$

$$\zeta = \frac{a_1}{2\sqrt{a_0 a_2}} \qquad P(s) = s^2 + \left(1 + \frac{2}{R}\right)s + 16 \qquad a_2 = 1 \quad a_1 = \left(1 + \frac{2}{R}\right) \quad a_0 = 16$$

$$= \frac{1 + \frac{2}{R}}{2\sqrt{16}} \Rightarrow \boxed{\zeta = \frac{1}{8} + \frac{1}{4R}} \qquad \omega_n = \sqrt{\frac{16}{1}} \Rightarrow \boxed{\omega_n = 4}$$

c) $e_i(t) = 1 + e^{-2t}$ for $t > 0$ $\quad R = \frac{2}{7}\,\Omega$ $\quad$ find $e_o(t)$ $\quad$ why no y_{ss} even though i has a const.

$$E_i(s) = \frac{1}{s} + \frac{1}{s+2} = \frac{s + s + 2}{s(s+2)} = \frac{2s+2}{s(s+2)} = \frac{2(s+1)}{s(s+2)}$$

$$E_o(s) = \frac{(s^2 + 7s)\left(\frac{2(s+1)}{s(s+2)}\right)}{(s^2 + 8s + 16)} = \frac{(s^2+7s)(2)(s+1)}{s(s+4)(s+4)(s+2)} = \frac{(s+7)(2)(s+1)}{(s+4)^2(s+2)} = \frac{A}{(s+4)} + \frac{B}{(s+4)^2} + \frac{C}{(s+2)}$$

$$(s+7)(2)(s+1) = A(s+4)(s+2) + B(s+2) + C(s+4)^2$$

$$(2s + 14)(s+1) = 2s^2 + 16s + 14 = A(s^2 + 6s + 8) + B(s+2) + C(s^2 + 8s + 16)$$

$2 = A + C \Rightarrow A = 2 - C$

$16 = 6A + B + 8C \qquad 16 = 6(2 - C) + B + 8C \Rightarrow 4 = B + 2C$

$14 = 8A + 2B + 16C$

$14 = 16 - 8C + 8 - 4C + 16C \Rightarrow -10 = 4C \qquad \underline{C = -2.5} \qquad \underline{A = 4.5} \qquad \underline{B = 9}$

$$E_o(s) = \frac{4.5}{(s+4)} + \frac{+9}{(s+4)^2} + \frac{-2.5}{(s+2)}$$

$$\boxed{e_o(t) = 4.5e^{-4t} + 9te^{-4t} - 2.5e^{-2t}}$$

no y_{ss} because a zero from H(s) cancelled the pole of $E_i(s)$ @ origin

5) 8.24) use st. var-eqn from (70) p124

$$\dot{\omega} = \frac{1}{J}[-(k_1 + k_2 R^2)\phi - B\omega + k_2 R z]$$

$$\dot{v} = \frac{1}{M}[k_2 R \phi - k_2 z + f_a(t)] \Rightarrow f_a(t) = \dot{v}M + k_2 z - k_2 R\phi$$

find $H(s)$ output is z

$$f_a(t) = \ddot{z}M + k_2 z - k_2 R\phi$$

$$F_a(s) = (s^2 Z(s) - \cancel{sz(0)}^{0} - \cancel{\dot{z}(0)})M + k_2 Z(s) - k_2 R\phi(s)$$

$$\ddot{\phi}J + B\dot{\phi} + (k_1 + k_2 R^2)\phi = k_2 R z$$

$$J[s^2\phi(s) - \cancel{s\phi(0)}^{0} - \cancel{\dot{\phi}(0)}^{0}] + [s\phi(s) - \cancel{\phi(0)}^{0}]B + (k_1 + k_2 R^2)\phi(s) = k_2 R z$$

$$\phi(s)(Js^2 + sB + (k_1 + k_2 R^2)) = k_2 R Z(s)$$

$$F_a(s) = Z(s)(s^2 M + k_2) - \frac{k_2 R(k_2 R Z(s))}{Js^2 + Bs + (k_1 + k_2 R^2)}$$

$$F_a(s) = Z(s)\left[(s^2 M + k_2) - \frac{(k_2^2 R^2)}{Js^2 + Bs + k_1 + k_2 R^2}\right]$$

$$\frac{Z(s)}{F_a(s)} = \frac{Js^2 + Bs + k_1 + k_2 R^2}{-k_2^2 R^2 + (Js^2 + Bs + k_1 + k_2 R^2)(s^2 M + k_2)}$$

$$= \frac{Js^2 + Bs + k_1 + k_2 R^2}{JMs^4 + BMs^3 + k_1 M s^2 + k_2 R^2 M s^2 + Jk_2 s^2 + Bk_2 s + k_1 k_2 + \cancel{k_2^2 R^2} \cancel{- k_2^2 R^2}}$$

$$\boxed{H(s) = \frac{Js^2 + Bs + k_1 + k_2 R^2}{JMs^4 + BMs^3 + s^2(k_1 M + Mk_2 R^2 + Jk_2) + Bk_2 s + k_1 k_2}}$$

48/50

David Malawey

ASSIGNMENT – 9/ ME 211/ MODELING AND ANALYSIS OF DYNAMIC SYSTEMS

DUE: 11/10/2010 (Wednesday) IN CLASS OR BY 2PM IN MY OFFICE (TMH 129)

Problem 1: Textbook Problem 7.11 (Rough sketch using hand is acceptable)

Problem 2: Textbook Problem 7.24 (Parts a only)

Problem 3: Textbook Problem 8.21

Problem 4: Textbook Problem 8.32 (Rough sketch using hand is acceptable)

David Malawey

7.11)

input $u(t)$

output $\dot{y} + 0.5y = u(t)$ $\tau = 2$, $u(t) = A$

a) find and sketch the unit step response $y_u(t)$ $y(0) = 0$

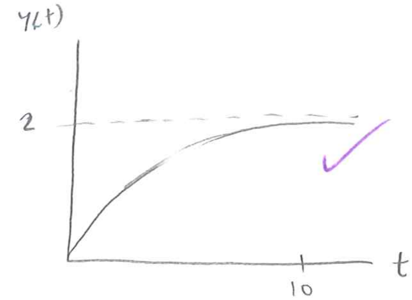
$s\,Y(s) - \cancel{y(0)}^{0} + 0.5\,Y(s) = \frac{1}{s}$

$Y(s)(s + 0.5) = \frac{1}{s} \Rightarrow Y(s) = \frac{1}{s(s+0.5)} = \frac{A}{s} + \frac{B}{(s+0.5)}$, $A = 2$, $B = -2$

$Y(s) = \frac{2}{s} + \frac{-2}{s+0.5}$

$Y(t) = 2 - 2e^{-.5t}$ $\boxed{y(t) = 2(1 - e^{-.5t})}$

$y_{ss} = 2$ $y_{tr} = -2e^{-0.5t}$



b) $u(t) = 2$ for $t > 0$ $y(0) = -1$

$s\,Y(s) - (-1) + 0.5\,Y(s) = \frac{2}{s}$

$Y(s)(s + 0.5) + 1 = \frac{2}{s}$

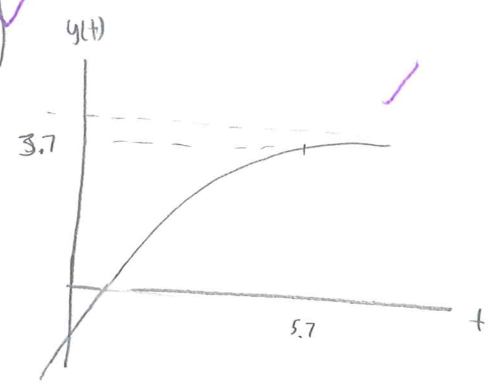
$Y(s) = \frac{-1}{s+0.5} + \frac{2}{s(s+0.5)}$ $\frac{2}{s(s+0.5)} = \frac{A}{s} + \frac{B}{(s+0.5)}$ $A = 4$, $B = -4$

$Y(s) = \frac{-1}{(s+0.5)} + \frac{4}{s} - \frac{4}{(s+0.5)}$

$Y(t) = -e^{-.5t} + 4 - 4e^{-.5t} = \boxed{4 - 5e^{-t/2}}$

$y_{ss} = 4$

$y_{tr} = -5e^{-t/2}$



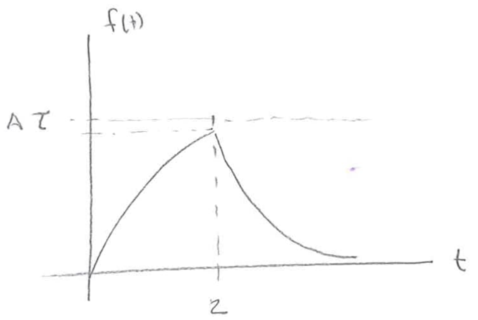
c) $u(t) = U(t) - U(t-2)$ & $y(0) = 0$

7.11, c) $u(t) = U(t) - U(t-2)$ $y(0) = 0$ by superposition,

$sY(s) - \cancel{y(0)}^{0} + 0.5\,Y(s) = \frac{1}{s^2} -$

for $t > t_1$ $y(t) = \cancel{2} - 2e^{-\frac{t}{2}} - [\cancel{2} - 2e^{-\frac{(t-2)}{2}}] = -2e^{-t/2} + 2e^{(-\frac{t}{2}+1)}$

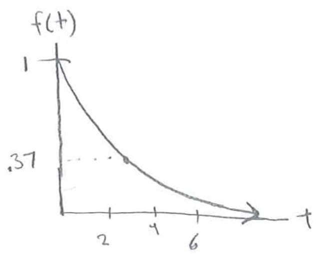
$t \le t_1$ $y(t) = 2 - 2e^{-t/2}$ ✓ for $t > t_1$ $2(-e^{-\frac{t}{2}} + e^{-\left(\frac{t-2}{2}\right)})$



d) unit impulse resp. $h(t)$ $h(t) = \frac{\mathcal{L}(\text{output})}{\mathcal{L}(\text{input})}$

$\mathcal{L}[\dot{y} + 0.5y = u(t)]$

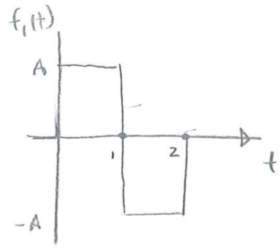
$Y(s)(s+0.5) = U(s)$ $\mathcal{L}^{-1}\frac{Y(s)}{U(s)} = \mathcal{L}^{-1}\left(\frac{1}{s+0.5}\right) = e^{-t/2}$ for $t>0$ ✓



7.24

decompose func's into step f's & ramp f's as appropriate

$f_1(t)$



a) $u(t) = \begin{cases} 0 & t \le 0 \\ 1 & t > 0 \end{cases}$ $\quad$ $f(t-a)$ $\quad u(t-1) \begin{cases} 0 & t \le 1 \\ 1 & t > 1 \end{cases}$ $\quad u(t-2) \begin{cases} 0 & t \le 2 \\ 1 & t > 2 \end{cases}$

$f(t) = A u(t) - 2A u(t-1) + A u(t-2)$ P.230

$F(s) = A U(s) - 2Ae^{-s} U(s) + Ae^{-2s} U(s)$ rewrite

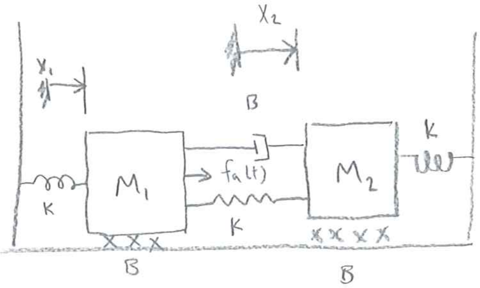
$\mathcal{L}[f(t-a) U(t-a)]$

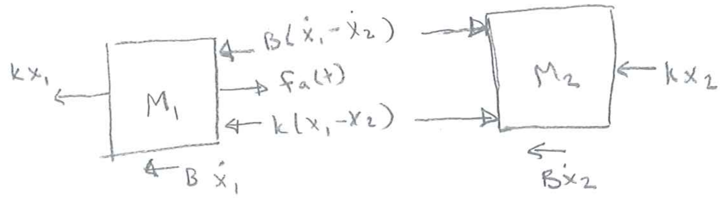
$U(s) = \frac{1}{s} \Rightarrow \frac{A}{s} - 2\frac{A}{s}e^{-s} + \frac{A}{s}e^{-2s}$

$$\boxed{F_1(s) = \frac{A}{s}(1 - 2e^{-s} + e^{-2s})}$$

8.21

M = B = k = 1 in consistent units





M_1) $M_1\ddot{x}_1 = f_a(t) - B(\dot{x}_1-\dot{x}_2) - k(x_1-x_2) - B\dot{x}_1 - kx_1$

$\cancel{M_1}\ddot{x}_1 + \cancel{B}(2\dot{x}_1-\dot{x}_2) + \cancel{k}(2x_1-x_2) = f_a(t)$

$\ddot{x}_1 + 2\dot{x}_1 - \dot{x}_2 + 2x_1 - x_2 = f_a(t)$

$$\boxed{\ddot{x}_1 + 2\dot{x}_1 + 2x_1 - \dot{x}_2 - x_2 = f_a(t)} \quad (1)$$

M_2) $\cancel{M_2}\ddot{x}_2 = \cancel{B}(\dot{x}_1-\dot{x}_2) + \cancel{k}x_1 - \cancel{k}x_2 - \cancel{k}x_2 - \cancel{B}\dot{x}_2 \longrightarrow \boxed{\ddot{x}_2 + 2\dot{x}_2 + 2x_2 - \dot{x}_1 - x_1 = 0} \quad (2)$

b) find transfer function, output is x_1

$\mathcal{L}$ (1) $s^2X_1(s) - s\cancel{x_1(0)} - \cancel{\dot{x}_1(0)} + 2[sX_1(s) - \cancel{x_1(0)}] + 2X_1(s) - [sX_2(s) - \cancel{x_2(0)}] - X_2(s) = F(s)$

$X_1(s)(s^2+2s+2) - X_2(s)(s+1) = F_a(s) \quad (3)$

$\mathcal{L}$ (2) $s^2X_2(s) - s\cancel{x_2(0)} - \cancel{\dot{x}_2(0)} + 2[sX_2(s) - \cancel{x_2(0)}] + 2X_2(s) - [sX_1(s) - \cancel{x_1(0)}] - X_1(s) = 0$

$X_2(s)(s^2+2s+2) - X_1(s)(s+1) = 0 \quad (4)$

$X_2(s) = \dfrac{X_1(s)(s+1)}{(s^2+2s+2)}$ now sub into (3): $X_1(s)(s^2+2s+2) - \dfrac{(s+1)\,X_1(s)(s+1)}{(s^2+2s+2)} = F_a(s)$

$X_1(s)\left[\dfrac{(s^2+2s+2)^2 - (s+1)^2}{(s^2+2s+2)}\right] = F_a(s)$ $\quad s^4+2s^3+2s^2+2s^3+4s^2+4s+2s^2+4s+4-s^2-2s-1$

$$\frac{X_1(s)\left(\dfrac{s^4+4s^3+7s^2+6s+3}{s^2+2s+2}\right)}{F_a(s)} = 1 \qquad \boxed{\frac{X_1(s)}{F_a(s)} = \frac{s^2+2s+2}{s^4+4s^3+7s^2+6s+3}}$$

8.21)

c) using (4) $\quad X_1(s) = X_2(s)\,\dfrac{(s^2+2s+2)}{(s+1)}$

into (3) $\quad (s^2+2s+2)\,\dfrac{(s^2+2s+2)}{(S+1)}\,(X_2(s)) - X_2(s)(s+1) = F_a(s)$

$$X_2(s)\left[\frac{s^2+2s+2)^2}{s+1} - (s+1)\right] = F_a(s)$$

$$X_2(s)\left[\frac{(s^2+2s+2)^2 - (s+1)^2}{(s+1)}\right] = F_a(s)$$

$$X_2(s)\left[\frac{2^4+2s^3+2s^2+2s^3+4s^2+2s+2s^2+4s+4-s^2-2s-1}{(s+1)}\right] = F_a(s)$$

$$X_2(s)\left[\frac{s^4+4s^3+7s^2+6s+3}{(s+1)}\right] = F_a(s)$$

$$\boxed{\frac{X_2(s)}{F_a(s)} = \left[\frac{s+1}{s^4+4s^3+7s^2+6s+3}\right]}$$

- Only the numerators differ for transfer functions $H_1(s)$ & $H_2(s)$ both functions share denominators

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8.32

$\ddot{y} + 5\dot{y} + 4y = u(t)$ find & sketch unit step response $y_u(t)$ & unit impulse response $h(t)$
use: $u(t) = F_a(t)$

$s^2Y(s) - \cancel{sy(0)} - \cancel{\dot{y}(0)} + 5[sY(s) - \cancel{y(0)}] + 4Y(s) = A$ get: $\frac{y_{ot}}{F_a(s)} =$

$Y(s)(s^2 + 5s + 4) = \frac{1}{s}$

$Y(s)(s+4)(s+1) = \frac{1}{s} \Rightarrow Y(s) = \frac{1}{(s)(s+4)(s+1)} = \frac{A}{s} + \frac{B}{s+4} + \frac{C}{s+1}$ $A = \frac{1}{4}$, $B = \frac{1}{12}$, $C = -\frac{1}{3}$

$\mathcal{L}^{-1}Y(s) = \mathcal{L}^{-1}\left[\frac{1}{4}\left(\frac{1}{s}\right) + \frac{1}{12}\left(\frac{1}{s+4}\right) - \frac{1}{3}\left(\frac{1}{s+1}\right)\right]$

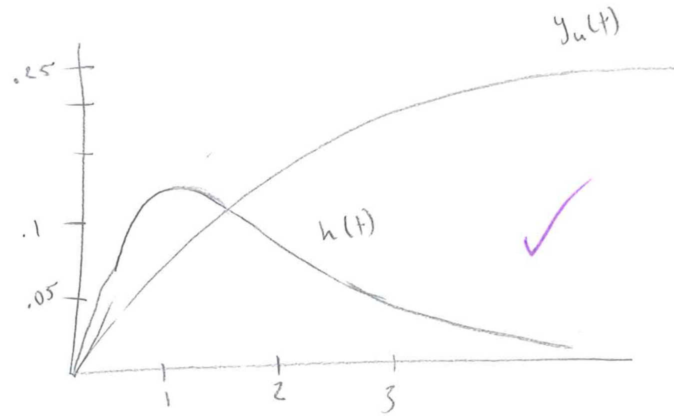
$$\boxed{y_u(t) = \frac{1}{4} + \frac{1}{12}e^{-4t} - \frac{1}{3}e^{-t}}$$ ✓

$Y(s)(s^2+5s+4) = F_a(s) \Rightarrow \frac{Y(s)}{F_a(s)} = \frac{1}{(s+1)(s+4)} = \frac{A}{(s+1)} + \frac{B}{(s+4)}$ $A = \frac{1}{3}$, $B = -\frac{1}{3}$

$\mathcal{L}^{-1}\left[\frac{Y(s)}{F_a(s)} = \frac{\frac{1}{3}}{s+1} + \frac{-1/3}{s+4} = H(s)\right]$

$h(t) = \frac{1}{3}e^{-t} - \frac{1}{3}e^{-4t}$

$$\boxed{h(t) = \frac{1}{3}\left[e^{-t} - e^{-4t}\right]}$$ ✓



✓

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ASSIGNMENT – 8/ ME 211/ MODELING AND ANALYSIS OF DYNAMIC SYSTEMS

DUE: 10/27/2010 (Wednesday) IN CLASS OR BY 2PM IN MY OFFICE (TMH 129)

Problem 1 (20 Points): Textbook Problem 7.15

Problem 2 (15 Points): Textbook Problem 7.17 (Parts b, c and d only)

Problem 3: Textbook Problem 7.19

Problem 4: Use Laplace transform to find the response $y(t)$ of the system:

$\ddot{y} + 3\dot{y} + 2y = \dot{u}(t) + 2u(t)$; Given: $u(t) = t$ for $t \geq 0$, $y(0) = 0$, and $\dot{y}(0) = 1$

Problem 5 (20 Points): Consider the first order linear system described by: $3\dot{y} + 2y = A$

a). Find the response of the system when $y(0) = 0$ and A is a constant.

b). Find the value of the time constant and the steady state response.

c). Identify the transient response.

d). Sketch the complete response when the input is unit step function.

e). Comment on the stability of the system.

Problem 6 (15 Points): Textbook Problem 7.5 (Note: The junction attached to the spring K_1 will give the input-output equation)

Problem 7: Textbook Problem 7.9 (Parts a and b only)

Note: Each problem is worth 10 points, unless otherwise noted.

#1)

7.15

find $f(t)$

a) $F(s) = \dfrac{2s^3+3s^2+s+4}{s^3}$

b) $F(s) = \dfrac{3s^2+9s+24}{(s-1)(s+2)(s+5)}$

c) $F(s) = \dfrac{4}{s^2(s+1)}$

d) $F(s)\ \dfrac{3s}{s^2+2s+26}$

a) $\dfrac{2s^3+3s^2+s+4}{s^3} = \dfrac{A}{s} + \dfrac{B}{s^2} + \dfrac{C}{s^3} + 2 \Rightarrow 2s^3+3s^2+s+4 = A(s^2)+B(s)+C$

$s=0 \Rightarrow 4 = C$ $\qquad 6s^2+6s+1 = 2As + B$ $\qquad s=0 \Rightarrow B=1$

$12s+6 = 2A \qquad s=0 \Rightarrow A=3$

$F(s) = 2+\frac{3}{s}+\frac{1}{s^2}+\frac{4}{s^3}$ $\qquad \mathcal{L}^{-1}\left(\frac{3}{s}\right) = \boxed{3}$ $\qquad \mathcal{L}^{-1}\frac{1}{s^2} : \mathcal{L}(t^n), n=1 = \frac{1!}{t} = \frac{1}{t}$

$\mathcal{L}^{-1} = \boxed{t}$

$4\mathcal{L}^{-1}\left(\frac{1}{s^3}\right) : \mathcal{L}(t^n), n=2 = \frac{2!}{t^2}$ $\qquad 4\mathcal{L}^{-1} = \boxed{2t^2}$

$\boxed{f(t) = 2\delta t + 3 + t + 2t^2}$ ✓

b) $\dfrac{3s^2+9s+24}{(s-1)(s+2)(s+5)} = \dfrac{A}{(s-1)} + \dfrac{B}{(s+2)} + \dfrac{C}{(s+5)}$ $\qquad$ set $s=1 \Rightarrow \dfrac{3+9+24}{(1+2)(1+5)} = \dfrac{36}{18} = 2 = A$

$s=-2$: $\quad B = \dfrac{3(4)+9(-2)+24}{(-3)(3)} = \dfrac{12-18+24}{} = \dfrac{+18}{-9} = -2 = B$

$s=-5$ $\quad \dfrac{75-45+24}{(-6)(-3)} \quad \dfrac{54}{18} = 3 \Rightarrow F(s) = \dfrac{2}{(s-1)} - \dfrac{2}{(s+2)} + \dfrac{3}{(s+5)}$

$\boxed{\mathcal{L}^{-1}(F(s)) = 2e^{t} - 2e^{-2t} + 3e^{-5t}}$ ✓

c) $\dfrac{4}{s^2(s+1)} = \dfrac{A}{s} + \dfrac{B}{s^2} + \dfrac{C}{(s+1)}$ $\qquad \dfrac{4}{(s+1)} = As + B + \dfrac{Cs^2}{(s+1)}$ $\qquad s=0;\ \frac{4}{1} = B$

differentiate: $4(s+1)^{-1}\frac{d}{ds} \Rightarrow \dfrac{-1(4)}{(s+1)^2} = A + \dfrac{Cs(\ )}{(\ )}$ $\qquad \overset{s=0}{\Rightarrow} A = \frac{-4}{1}$

$\dfrac{4}{s^2} = \dfrac{A(s+1)}{s} + \dfrac{B(s+1)}{s^2} + C$ $\qquad s=-1 \Rightarrow \frac{4}{1} = C$ $\qquad \Rightarrow F(s) = \frac{-4}{s} + \frac{4}{s^2} + \frac{4}{(s+1)}$

$\mathcal{L}^{-1} F(s) = -4 + 4t + 4e^{-t}$

$\boxed{f(t) = 4(-1+t+e^{-t})}$ ✓

1) continued

7.15/ d) $F(s) = \dfrac{3s}{s^2 + 2s + 26}$

$(x-(-1+5i))(x+1+5i)$

$s = -2 \pm 5i$

$s = -2 \pm \dfrac{\sqrt{4 - 4(26)}}{2} = -2 \pm \sqrt{-25}$

$F(s) = 3\left[\dfrac{(s+1)}{(s+1)^2 + 5^2}\right] - \dfrac{3}{5}\left[\dfrac{5}{(s+1)^2 + 5^2}\right]$

$\mathcal{L}^{-1}\{\} \Rightarrow 3e^{-t}\cos 5t \; - \frac{3}{5}e^{-t}\sin 5t$

$\boxed{f(t) = e^{-t}\left(3\cos 5t - \frac{3}{5}\sin 5t\right)}$ for $t > 0$

#2) 7.17 b, c, & d

b) $F(s) = \dfrac{4s^2+10s+10}{s^3+2s^2+5s} = \dfrac{A}{s} + \dfrac{B}{(s+1-2i)} + \dfrac{C}{(s+1+2i)}$

$s(s^2+2s+5)$

p 225

Find A

$= \dfrac{2}{s} + \dfrac{k_1 e^{j\phi}}{(s+1-2i)} + \dfrac{k e^{-j\phi}}{(s+1+2i)}$

$s=0$; $\dfrac{4s^2+10s+10}{\cancel{s}(s^2+2s+5)} = A = \dfrac{10}{5} = 2$

$k \angle \phi$, mag & angle of complex k_1

$k_1 = s+1-2j \; F(s)|_{s=-1+2j}$

$k_2 = s+1-2j \; F(s) \; s=-1-2j$

$\alpha = 1$

$k = \sqrt{2}$ $\quad \phi = -\frac{\pi}{4}$

$\boxed{f(t) = 2 + 2\sqrt{2}\, e^{-t} \cos(2t - \frac{\pi}{4})}$

c) $F(s) = \dfrac{3(s^3+2s^2+4s+1)}{s(s+3)^2} = \dfrac{A}{s} + \dfrac{B}{(s+3)} + \dfrac{C}{(s+3)^2}$ @ $s=0$

proper rational function, $m=n$

~~$\dfrac{3(s^3+2s^2+4s+1)}{s} = \dfrac{A(s+3)^2}{s} + B(s+3) + C \quad s=-3$~~ $\Rightarrow C = 20$

$s(s+3)^2 = s^3+6s^2+9s \Rightarrow$

$$\begin{array}{r} 3 \\ s^3+6s^2+9s \,\big)\, 3s^3+6s^2+12s+3 \\ 3s^3 - 18s^2 - 27s \\ \hline -12s^2-15s+3 \end{array} \Rightarrow$$

$F(s) = 3 + \dfrac{-12s^2-15s+3}{s(s+3)^2} = \dfrac{A}{s} + \dfrac{B}{(s+3)} + \dfrac{C}{(s+3)^2}$

$(s+3)^2 = (s^2+6s+9)$

$\div s \Rightarrow s + 6 + \frac{9}{s}$

$\dfrac{(-12s^2-15s+3)}{s} = \dfrac{A(s+3)^2}{s} + B(s+3) + C$ Now $s=-3 \Rightarrow C = 20$

$(-24s-15)s - 1(-12s^2-15s+3) = A(1+ -9s^{-2}) + B(1)$

$\dfrac{-24s^2 - \cancel{15s} + 12s^2 + \cancel{15s} + 3}{s^2}$ $\quad -12 - \dfrac{3}{s^2} = A(1-\frac{9}{s^2}) + B(1)$ $\quad s=3 \Rightarrow B = -12 - \frac{3}{9} = -\frac{37}{3}$

$-3(-2)(\cancel{s^{-3}}) = A \; -9(-2)\cancel{s^{-3}}$ $\quad \Rightarrow A = \frac{1}{3}$

$F(s) = 3 + \frac{1}{3}(\frac{1}{s}) + -\frac{37}{3}(\frac{1}{s+3}) + 20(\frac{1}{(s+3)^2})$

$\mathcal{L}^{-1} = 3\delta_s + \frac{1}{3} - \frac{37}{3}e^{-3t} + 20te^{-3t}$

$\boxed{\mathcal{L}^{-1} = 3\delta_s + \frac{1}{3} + e^{-3t}\left[-\frac{37}{3} + 20t\right]}$

#2

7.17|a

$$F(s) = \frac{s^3-4s}{(s+1)(s^2+4s+4)} = \frac{s^3-4s}{s^2+4s+4+s^3+4s^2+4s} = s^3+5s^2+8s+4 \overline{\big) \; s^3-4s} \quad \begin{matrix} 1 \\ s^3+5s^2+8s+4 \end{matrix}$$

$$= 1 + \left\{ \frac{-5s^2-4s+8s+4}{s^3+5s^2+8s+4} \right\} = 1 + \left[-5s^2-4s+4 = \frac{A}{s+1} + \frac{B}{(s+2)} + \frac{C}{(s+2)^2} \right]$$

$$-5s^2-4s+4 = A(s^2+4s+4) + B(s+2)(s+1) + C(s+1)$$

$$B(s^2+3s+2)$$

$\cancel{5s^2}$ $-5 = A+B$

$$\left.\begin{matrix} -4 = 4A + 3B + C \\ 4 = 4A + 2B + C \end{matrix}\right\} -8 = B \Rightarrow A = 3 \Rightarrow C = 0$$

$$F(s) = 1 + \frac{3}{s+1} + \frac{-8}{(s+2)} + 0$$

$$\boxed{\mathcal{L}^{-1} = f(s) = \delta(t) + 3e^{-t} - 8(e^{-2t})}$$

3, 7.19

$$F(s) \quad \frac{4(s^2+8s+72)}{s^3+8s^2+32s} = \frac{A}{s} + \frac{Bs+C}{s^2+8s+32}$$

$$A + \frac{(Bs+C)s}{''} = \frac{4(s^2+8s\ 72)}{s+8s+32} \qquad s=0, \Rightarrow A = \frac{4(72)}{32} = \frac{72}{8} = \boxed{9 = A}$$

$$\times \text{ denominator} \Rightarrow 4(s^2+8s+72) = \frac{Bs+C}{s^2+8s^2+32}(s^3+8s^2+32s)$$

$$4s^2+32s+288 = (Bs+C)(s) + 9(s^2+8s+32)$$

$$4s^2+32s+\cancel{288} - 9s^2 - 72s - \cancel{288} = Bs^2+Cs$$

$$-5s^2 = Bs^2 \Rightarrow \boxed{B = -5}$$

$$-40s = Cs \Rightarrow \boxed{C = -40}$$

$$F(s) = \frac{9}{s} + \frac{-5s-40}{s^2+8s^2+32} = \frac{9}{s} + -5\left(\frac{s+8}{s^2+8s+32}\right)$$

$$\uparrow$$

$$\frac{(s+4)+4}{(s+4)^2+4^2}$$

$$(F(s)) = \frac{9}{s} + (-5)\left[\frac{(s+4)}{(s+4)^2+4^2} + \frac{4}{(s+4)^2+4^2}\right]$$

$$\mathcal{L}^{-1}(F(s)) = 9 + (-5)\left[e^{-4t}\cos(4t) + e^{-4t}\sin 4t\right]$$

$$\boxed{f(t) = 9 + (-5e^{-4t})\left[\cos(4t) + \sin(4t)\right]}$$

#4) find $y(t)$ of system

$\mathcal{L}[\ddot{y} + 3\dot{y} + 2y = \dot{u}(t) + 2u(t)]$ $\quad u(t) = t$ for $t \geq 0$, $\quad \dot{y}(0) = 1 \quad y(0) = 0$

$s^2 Y(s) - s\cancel{y(0)}^{0} - \cancel{\dot{y}(0)}^{1} + 3[-\cancel{y(0)}^{0} + sY(s)] + 2Y(s) = sU(s) + \cancel{u(0)}^{0} + 2U(s)$

$-1 + s^2 Y(s) + 3sY(s) + 2Y(s) = sU(s) + 2U(s)$ $\quad$ where $U(s) = \mathcal{L}(u(t)) = \mathcal{L}\, t = \frac{1}{s^2}$

$\cancel{-1} + (s^2 + 3s + 2)(Y(s)) = \frac{1}{s} + \frac{2}{s^2} + 1$ $\quad Y(s) = \dfrac{s + 2 + s^2}{(s^2)(s^2 + 3s + 2)}$

$Y(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{(s+1)} + \frac{D}{(s+2)} = As + B + \frac{Cs^2}{s+1} + \frac{Ds^2}{s+2} = \dfrac{s + 2 + s^3}{\cancel{s^2}(s^2 + 3s + 2)}$

$\Rightarrow B = \frac{2}{2} = 1$ $\quad A + (C + D)\dfrac{(2s(s+1) - 1(s^2))}{(s+1)^2} = \dfrac{(2s + 1)(s^2 + 3s + 2) - (2s + 3)(s + 2 + s^2))}{(s+2)^2(s+1)^2}$

$s = 0 \Rightarrow A = \dfrac{1(2) - 3(2)}{2^2(1)^2} = \dfrac{2 - 6}{4} \Rightarrow A = -1$

$\dfrac{s^2 + s + 2}{s^2(s^2 + 3s + 2)} = \frac{-1}{s} + \frac{1}{s^2} + \frac{C}{s+2} + \frac{D}{s+1}$ $\quad s = -2 \Rightarrow C = \dfrac{4 - 2 + 2}{4(-2+1)} = \dfrac{4}{-4}$

$s^2(s+2)(s+1)$

$C = -1$ $\quad s = -1 \Rightarrow D = \dfrac{1 + (-1) + 2}{1(-1+2)} = \frac{2}{1}$ $\quad D = 2$

$(Y(s)) = -\frac{1}{s} + \frac{1}{s^2} + \frac{-1}{(s+1)} + \frac{2}{(s+2)}$

$\mathcal{L}^{-1}(Y(s)) = -1 + t + (e^{-t}) + 2e^{-2t}$ ✓

$\boxed{y(t) = t - 1 + e^{-2t} - 2e^{-2t}}$ ✗ - 1

5) $3\dot{y} + 2y = A$ a) response when $y(0) = 0$ & A = const.

$$\mathcal{L}\left[\dot{y} + \tfrac{2}{3}y = \tfrac{A}{3}\right] \Rightarrow \mathcal{L}\left[\dot{y} + \frac{y}{\tau} = \frac{A}{3}\right] \Rightarrow sY(s) - \cancel{y(0)} + \frac{1}{\tau}Y(s) = \frac{A}{3s} \qquad \tau = \frac{3}{2}$$

$$Y(s) = \frac{A}{3s(s+\frac{2}{3})} + \cancel{\frac{Y_0}{(s+\frac{2}{3})}} = \frac{B}{3s} + \frac{C}{(s+\frac{2}{3})} \qquad s=0 \Rightarrow B = \frac{A}{(\frac{2}{3})} \qquad B = \frac{3A}{2}$$

$$s = -\tfrac{2}{3} \Rightarrow C = \frac{A}{-2} \qquad C = \qquad Y(s) = \frac{\frac{3}{2}A}{3s} + \frac{\frac{A}{-2}}{(s+\frac{2}{3})} = \frac{A}{2s} + \frac{A}{-2}\left(\frac{1}{s+2/3}\right)$$

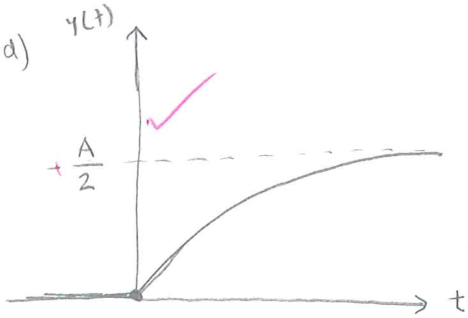
$$\boxed{\mathcal{L}^{-1}(Y(s)) = \text{OR } y(t) = \frac{A}{2} + \frac{A}{-2}e^{-\frac{2}{3}t}}$$

b) Value of time const & steady state Y_t & Y_{ss}

$\boxed{\tau = \text{time const} = \frac{3}{2}}$ $\boxed{Y_{ss} = Y_t \lim_{t\to\infty} \Rightarrow \frac{A}{2} \text{ OR } A\frac{\tau}{3}}$

c) transient response = $y_{tr} = y(t) - Y_{ss} = \boxed{-\frac{A}{2}e^{-\frac{2}{3}} \text{ OR } -\frac{A\tau}{3}e^{-t/\tau}}$

d)

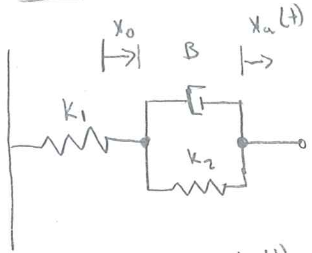


$y = 0$

Check pt. d

e) $\tau > 0$, system is stable

\# 6) 7.5)



relate x_o to $x_a(t)$

$$\cancel{m\ddot{x}_o}^{\,0} = B\dot{x}_a(t) - B\dot{x}_o + k_2 x_a(t) - k_2 x_o - k_1 x_o$$

$$B\dot{x}_a(t) - B\dot{x}_o - (k_1+k_2)x_o + k_2 x_a(t) = 0$$

$k_1 x_o \leftarrow \bullet \rightarrow B(\dot{x}_a(t) - \dot{x}_o)$

$\rightarrow k_2(x_a(t) - x_o)$

$$\frac{B\dot{x}_o + (k_1+k_2)x_o = +B\dot{x}_a(t) + k_2 x_a(t)}{B}$$

a)

$$\boxed{\dot{x}_o + \frac{(k_1+k_2)}{B}x_o = \dot{x}_a(t) + \frac{k_2}{B}x_a(t)}$$

b) time const. $\boxed{\tau = \frac{B}{(k_1+k_2)}}$

c) $K_1 = 1\,N/m$ $\quad K_2 = 2\,N/m$ $\quad B = 1\,N\text{-}s/m$

$x_o = f(t)$ when $t > 0$ no energy in the springs

$$\mathcal{L}\left[\dot{x}_o + x_o(3) = \dot{x}_a(t) + \frac{2\,N/m}{1\,N\text{-}s/m}x_a(t)\right] \Rightarrow [sX_o(s) - \cancel{x(0)}^{\,0}] + X_o(3s) = sX_a(s) + \cancel{x_a(0)}^{\,0} + 2X_a(s)$$

$$X_o(s)(s+3) = X_a(s)(s+2) \qquad X_o(s) = \frac{X_a(s)(s+2)}{(s+3)} = \frac{\cancel{s+2}}{s+3}\left(\frac{2}{s(\cancel{s+2})}\right) = \frac{2}{s(s+3)}$$

$$X_o(s) = \frac{2}{s(s+3)} = \frac{A}{s} + \frac{B}{s+3} \qquad s=0 \rightarrow A = \frac{2}{3} \qquad s=-3 \Rightarrow B = -\frac{2}{3}$$

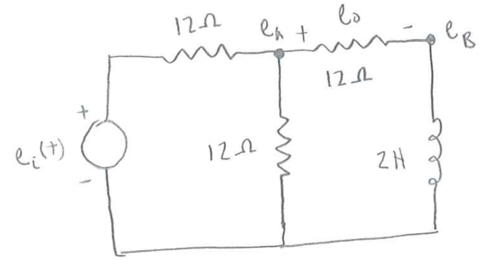
$$x_o(s) = \frac{2}{3}\left(\frac{1}{s}\right) + \left(-\frac{2}{3}\right)\left(\frac{1}{s+3}\right)$$

$$\mathcal{L}^{-1}\{x_o(s)\} = \frac{2}{3}(1) - \frac{2}{3}e^{-3t}$$

$$\boxed{x_o(t) = \frac{2}{3}(1 - e^{-3t})}$$ for $t > 0$

#7)

7.9) a, b only



a) $\dot{e}_o + 9e_o = 3e_i(t)$

$e_A = e_B + e_o$

$\dot{e}_B - \dot{e}_A = -\dot{e}_o$

KCL Node e_B

$\frac{e_B - e_A}{12}$ ←

$\frac{1}{2}\int e_B \, d\lambda + \cancel{i(0)}^{0}$ ↓

$\left[\frac{1}{2}\int e_B \, d\lambda + \frac{e_B - e_A}{12} = 0\right] \frac{12\,d}{dt}$

$6e_B + \dot{e}_B - \dot{e}_A = 0$

$6e_B - e_o = 0$

$e_B = \frac{e_o}{6}$

Node e_A KCL

← $\frac{e_A - e_i(t)}{12}$ $\frac{e_A - e_B}{12}$ →

↓ $\frac{e_A}{12}$

$e_A - e_i(t) + e_A - e_B + e_A = 0$

$3e_A - e_i(t) - e_B = 0$

$3e_o + 3e_B - e_B = e_i(t)$

$3e_o + 2e_B = e_i(t)$

$\left[3e_o + 2\left(\frac{\dot{e}}{3}\right) = e_i(t)\right]3$

$9e_o + \dot{e}_o = 3e_i(t)$

time const. $\tau = \frac{1}{9}$

B) $\dot{e}_o + 9e_o = 3e_i(t)$

$sE(s) - \cancel{E(0)}^{0} + 9E(s) = 3\cancel{E_i(s)}^{1}$

$(s+9)E(s) = \frac{3}{s}$

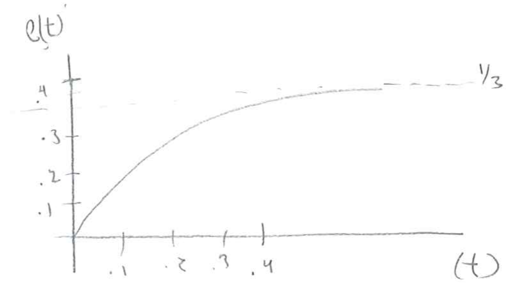
$E(s) = \frac{3}{s(s+9)} = \frac{A}{s} + \frac{B}{(s+9)}$ $s = 0 \rightarrow A = \frac{1}{3}$ $s = -9 \Rightarrow B = -\frac{1}{3}$

$\mathcal{L}^{-1}\left[\frac{1}{3}\frac{1}{s} + -\frac{1}{3}\left(\frac{1}{s+9}\right)\right] = \frac{1}{3} - \frac{1}{3}e^{-9t} = e_o(t)$

$= \mathcal{L}^{-1}[E(s)]$

$$\boxed{e_o(t) = \frac{1}{3}\left(1 - e^{-9t}\right)}$$ $t > 0$

$E_o(t)$



99/100

David Malawey

ASSIGNMENT – 7/ ME 211/ MODELING AND ANALYSIS OF DYNAMIC SYSTEMS

DUE: 10/15/2010 (Friday) IN CLASS OR BY 2PM IN MY OFFICE (TMH 129)

Problem 1 (20 Points): Textbook Problem 7.1 (Scientific calculators not allowed)

Problem 2 (10 Points): Textbook Problem 7.4

Problem 3 (20 Points): Textbook Problem 7.23

David Malaway

1) 7.1 Sc: calc not allowed

2) 7.4 √

3) 7.23 √

7.1) a) √ $f_1(t) = t^2$ b) √ $f_2(t) = e^{-at}\cos\omega t$ c) √ $f_3(t) = te^{-at}$ d) ? X $f_4(t) = \begin{cases} \sin 2t & 0<t<\pi \\ 0 & \text{otherwise} \end{cases}$

$\mathcal{L}(t) = \int_0^\infty e^{-st} f(t)\,dt$

$\int_0^\infty e^{-st} t^2\,dt$

$= \int u \frac{dv}{-s}$

$\int u\,dv = uv - \int v\,du$

$u = t^2 \quad \therefore du = 2t\,dt$

$v = e^{-st} \quad dv = -se^{-st}dt \quad \frac{dv}{-s} = e^{-st}dt$

$= -\frac{1}{s}\, uv\Big|_0^\infty - \int_a^b v\,du$

$= -\frac{1}{s}\underbrace{\left[t^2 e^{-st}\right]_0^\infty}_{0} + \frac{1}{s}\int_0^\infty e^{-st}\,2t\,dt = \frac{2}{s}\int_0^\infty te^{-st}\,dt$

$u = t \quad \therefore du = 1\,dt$

$v = e^{-st} \quad \therefore dv = -se^{-st}$

$= \frac{2}{-s^2}\underbrace{\left[te^{-st}\right]_0^\infty}_{0} + \frac{2}{s^2}\int_0^\infty e^{-st}\,dt = -\frac{2}{s^2}\left[\frac{e^{-st}}{-s}\right]_0^\infty = -\frac{2}{s^3}\left[\cancel{e^{-\infty}}^{0} - e^0\right] = -\frac{2}{s^3}(-1)$

$\boxed{\mathcal{L} = \frac{2}{s^3}}$ ✓

b) $\mathcal{L}(e^{-at}\cos\omega t) = \int (e^{-at}\cos\omega t)e^{-st}\,dt = \int \cos\omega t\, e^{-(st+at)} = \int \cos\omega t e^{-t(s+a)}$

$= F(s+a)$

$F(s) =$ continued next page

David Malaney

7.1 b) continued

$\mathcal{L}(\cos\omega t)$: $u = \cos\omega t$ $du = -\omega\sin(\omega t)$ $dv = e^{-st}$ $v = -\frac{1}{s}e^{-st}$

$$= \int \cos\omega t\, e^{-st}\, dt = -\frac{1}{s}e^{-st}\cos\omega t - \frac{\omega}{s}\underbrace{\int e^{-st}\sin\omega t\, dt}$$

$\hookrightarrow$ $u = \sin\omega t$ $du = \omega\cos\omega t$
$dv = e^{-st}$ $v = \frac{-1}{s}e^{-st}$

$$= \frac{e^{-st}}{-s}\cos\omega t - \frac{\omega}{s}\left[-\frac{e^{-st}}{s}\sin\omega t + \frac{\omega}{s}\int e^{-st}\cos\omega t\, dt\right]$$

$$= \frac{e^{-st}}{-s}\cos\omega t + \frac{\omega}{s^2}\left(e^{-st}\sin\omega t\right) - \frac{\omega^2}{s^2}\underbrace{\int e^{-st}\cos\omega t\, dt}$$ Solving *

$$* \quad \left(1 + \frac{\omega^2}{s^2}\right)\int \cos\omega t\, e^{-st}\, dt = -\frac{e^{-st}}{s}\cos\omega t + \frac{\omega}{s^2}e^{-st}\sin\omega t$$

$$= \frac{s^2+\omega^2}{s^2}\int \cos\omega t\, e^{-st}\, dt = -\frac{e^{-st}}{s}\cos\omega t + \frac{\omega}{s^2}e^{-st}\sin\omega t$$

$$\int \cos\omega t \cdot e^{-st}\, dt = \left[\frac{s^2}{s^2+\omega^2}\right]\left(\frac{e^{-st}}{-s}\cos\omega t + \frac{\omega}{s^2}e^{-st}\sin\omega t\right) = -s\cos(\omega t) + \omega\sin\omega t\left(\frac{e^{-st}}{s^2+\omega^2}\right)$$

$$\mathcal{L}(\cos(\omega t)) = \int_0^\infty \cos\omega t\, e^{-st}\, dt = \left[-s\cos\omega t + \omega\sin\omega t\left(\frac{e^{-st}}{s^2+\omega^2}\right)\right]_0^\infty$$

$$= 0 - \left(-s(1) + \omega(0)\right)\left(\frac{1}{s^2+\omega^2}\right) =$$

$$F(s) = \frac{s}{s^2+\omega^2} \Rightarrow \boxed{F(s+a) = \frac{(s+a)}{(s+a)^2+\omega^2}}$$

Dawid Malawey

7.1 c)

$\mathcal{L}(te^{-at})$

$u = t \quad du = dt$

$v = e^{-t(a+s)} \quad dv = -(a+s)e^{-t(a+s)}$

$\frac{dv}{-(a+s)} = e^{-t(a+s)}$

$= \int te^{-at}e^{-st}\,dt$

$= \int te^{-t(a+s)}\,dt$

$= \frac{-1}{(a+s)}\left[\cancel{te^{-t(a+s)}\Big|_0^\infty} - \int_0^\infty e^{-t(a+s)}\,dt\right] = \frac{-1}{(a+s)}\int_0^\infty e^{-t(a+s)}\,dt$

$= \left(\frac{-1}{(a+s)}\right)\frac{-1}{(a+s)}e^{-t(a+s)}\Big|_0^\infty = \frac{e^{-(a+s)t}}{(a+s)^2}\Big|_0^\infty = \boxed{\frac{1}{(a+s)^2}}$ ✓

d) $f(t) = \begin{cases} 0 & \text{for } t<0 \\ \sin 2t & \text{for } 0<t<\pi \\ 0 & t>\pi \end{cases}$

$\mathcal{L}(\sin 2t) = \int_0^\pi \sin(2t)e^{-st}\,dt \qquad \sin 2t = \frac{e^{i2t}-e^{-i2t}}{2i}$

$\int_0^\pi \frac{e^{i2t}-e^{-i2t}}{2i}e^{-st}\,dt = \frac{1}{2i}\left[\int_0^\pi e^{(2i-s)t}\,dt - \int_0^\pi e^{(-2i-s)t}\,dt\right]$

$\frac{1}{2i}\left[\frac{e^{(2i-s)t}}{2i-s}\Big|_0^\pi - \frac{e^{(-2i-s)t}}{-(2i+s)}\Big|_0^\pi\right] = \frac{1}{2i}\left[\frac{e^{(2i-s)\pi}-1}{2i-s} + \frac{e^{(-2i-s)\pi}-1}{2i+s}\right]$

$= \frac{e^{(2i-s)\pi}-1}{-4-2is} + \frac{e^{(-2i-s)\pi}-1}{-4+2is} = \frac{-4e^{(2i-s)\pi}+2ise^{(2i-s)\pi}+4-2is-4e^{(-2i-s)\pi}-2ise^{(-2i-s)\pi}+4+2is}{16\ \cancel{-4i^2s^2} = +4s^2}$

$= \frac{(-4+2is)e^{(2i-s)\pi}+(-4-2is)e^{(-2i-s)\pi}}{16+4s^2} = \frac{(-2+is)e^{(2i-s)\pi}+(-2-is)e^{(-2i-s)\pi}}{8+2s^2}$

$\left\{\begin{array}{l} e^{2i\pi}e^{-s\pi} = (\cancel{\cos 2\pi}^{1} + \cancel{i\sin 2\pi}^{0})(e^{-s\pi}) = e^{-s\pi} \\ e^{-2i\pi}e^{-s\pi} = (\cancel{\cos 2\pi}^{1} + \cancel{i\sin(-2\pi)}^{0})(e^{-s\pi}) = e^{-s\pi} \end{array}\right\} \Rightarrow \frac{-2e^{-s\pi}\cancel{+ise^{-s\pi}}-2e^{-s\pi}\cancel{-ise^{-s\pi}}}{8+2s^2}$

$= \frac{-4e^{-s\pi}}{8+2s} = \boxed{-\frac{2e^{-s\pi}}{4+s}} = -\frac{e^{-s\pi}}{2} - \frac{e^{-s\pi}}{s}$ ✗ −2

David Malawey

7.4

a) prove $\mathcal{L}\left(f\left(\frac{t}{a}\right)\right) = aF(as)$

b) apply with $f(t) = \cos\omega t$ to get $\mathcal{L}[\cos 2\omega t]$

$\mathcal{L} f(t) = \int f(t) e^{-st} dt = F(s)$

let $f(t) = t \Rightarrow F(s) = \int te^{-st} dt = \frac{1}{s^2} \therefore F(as) = \frac{1}{s^2a^2} \therefore aF(as) = \boxed{\frac{1}{s^2 a}}$

let $f\left(\frac{t}{a}\right) = \frac{t}{a} \Rightarrow \mathcal{L}\left(f\left(\frac{t}{a}\right)\right) = \int \frac{t}{a} e^{-st} dt$ $\left\{ \begin{array}{ll} u = t & du = dt \\ v = \frac{1}{a}e^{-st} & dv = -\frac{s}{a}e^{-st} \end{array} \right\}$

$\int u dv = \int t\left(\frac{-s}{a}\right)e^{-st}$

$\frac{1}{-s}\int u dv = \frac{1}{-s}\int \frac{t}{a}e^{-st} = -\frac{1}{s}\left[uv\Big|_0^\infty - \int_0^\infty v du\right] = -\frac{1}{s}\left[t\left(-\frac{s}{a}\right)e^{-st}\right]_0^\infty - \left(-\frac{1}{s}\right)\int \frac{1}{a}e^{-st} dt$

$= -s[0] + \frac{1}{as}\int e^{-st} dt = \frac{1}{as}\left[\frac{e^{-st}}{-s}\right]_0^\infty = \frac{1}{as}\left[0 - \frac{e^0}{-s}\right]$

$= \boxed{\frac{1}{as^2}}$

$\mathcal{L}(\cos\omega t) = \frac{s^2}{s^2+\omega^2}$ let $a = 2^{-1}$ $\mathcal{L} f\left(\frac{t}{a}\right) = \mathcal{L}\left(\cos\left(\omega \frac{t}{a}\right)\right) = \mathcal{L}(\cos 2\omega t)$

$F(s) = \frac{s^2}{s^2+\omega^2}$

$aF(as) = a\left(\frac{a^2s^2}{a^2s^2+\omega^2}\right) = \frac{a^3s^2}{a^2s^2+\omega^2}$

X -5

$\mathcal{L} f\left(\frac{t}{a}\right) = \int f\left(\frac{t}{a}\right)e^{-st} dt = a\int f\left(\frac{t}{a}\right) e^{-as\left(\frac{t}{a}\right)} d\left(\frac{t}{a}\right)$

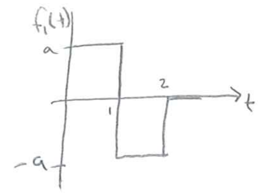
$\mathcal{L} f\left(\frac{t}{a}\right) = aF(as)$

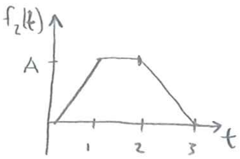
+5

$f(t) = \cos\omega t$ $\mathcal{L}(\cos 2\omega t) = \int_0^\infty \cos\omega t\, e^{-st} dt = \frac{s}{s^2+\omega^2}$ $a = \frac{1}{2}$

$\frac{1}{2}\left(\frac{\frac{1}{2}s}{\left(\frac{1}{2}s\right)^2+\omega^2}\right) = .5\left(\frac{.5s}{.25s^2+\omega^2}\right) = \frac{s}{4\left(\frac{1}{4}s^2+\omega^2\right)} = \boxed{\frac{s}{s^2+4\omega^2}}$ ✓

David Mahaney

7.23 | evaluate $\mathcal{L}$ for a) 

b) 

a) $u(t) = \begin{cases} a & 0 < t < 1 \\ -a & 1 < t < 2 \\ 0 & \text{otherwise} \end{cases}$

$$\mathcal{L}(a) = \int_0^1 a e^{-st}\,dt = a\int_0^1 e^{-st}\,dt = a\left[\frac{e^{-st}}{-s}\right]_0^1 = a\left[\frac{e^{-s}}{-s} - \frac{1}{-s}\right]$$

$$= \boxed{\frac{-a}{s}\left(e^{-s}-1\right)}$$

$$\mathcal{L}(-a) = -a\left[\frac{e^{-st}}{s}\right]_1^2 = -a\left[\frac{e^{-2s}}{s} - \frac{e^{-s}}{s}\right] = \boxed{-\frac{a}{s}\left(e^{-2s}-e^{-s}\right)}$$

$$-\frac{a}{s}\left(e^{-s}-1+e^{-2s}-e^{-s}\right) = \boxed{\frac{a}{s}\left(1-2e^{-s}+e^{-2s}\right)}$$

b) $y = mx+b$ $\quad f(x) = \begin{cases} at & 0 < t < 1 \\ a & 1 < t < 2 \\ -at+3a & 2 < t < 3 \end{cases}$

$$\mathcal{L}(at) = \boxed{\frac{a}{s^2}}$$

$$\mathcal{L}(a) = \boxed{\frac{a}{s}}$$

$$\mathcal{L}(3a - at) = \mathcal{L}(3a) - \mathcal{L}(at) = \boxed{\frac{3a}{s} - \frac{a}{s^2}}$$

continued next page

David Malawey

7.23 b)

$$\int u\,dv = uv - \int v\,du$$

$u = t \quad du = dt$

$v = e^{-st} \quad dv = -se^{-st}\,dt$

$$\mathcal{L}(at) = \int_0^1 ate^{-st}\,dt = a\int_0^1 te^{-st}$$

$$-\frac{a}{s}\int u\,dv = \frac{a}{-s}\int te^{-st}\,dt = \frac{a}{-s}\left[te^{-st}\Big|_0^1 - \int_0^1 e^{-st}\,dt\right]$$

$$= -\frac{a}{s}(e^{-s}) + \frac{a}{s}\left[\frac{e^{-st}}{-s}\right]_0^1 = \frac{a}{s}\left(-e^{-s} + \frac{e^{-s}}{-s} - \frac{1}{-s}\right)$$

$$\mathcal{L}(a) = \int_1^2 ae^{-st}\,dt = a\int_1^2 e^{-st}\,dt = a\left[\frac{e^{-st}}{-s}\right]_1^2 =$$

$$= \frac{ae^{-2s}}{-s} - \frac{ae^{-s}}{-s}$$

$$\mathcal{L}(3a - at) = 3a\left[\frac{e^{-st}}{-s}\right]_2^3 - \frac{a}{-s}\left[te^{-st}\Big|_2^3 - \int_2^3 e^{-st}\,dt\right]$$

$$= \frac{3ae^{-3s}}{-s} - \frac{3ae^{-2s}}{-s} - \frac{a}{-s}\left[\left(3e^{-3t} - 2e^{-2s}\right) - \left[\frac{e^{-st}}{-s}\right]_2^3\right]$$

$$\frac{3ae^{-3s}}{-s} + \frac{3ae^{-2s}}{s} - \frac{3ae^{-3s}}{-s} + \frac{2ae^{-2s}}{-s} + \frac{a}{-s}\left(\frac{e^{-3s}}{-s} - \frac{e^{-2s}}{-s}\right)$$

$$= -\cancel{\frac{3ae^{-3s}}{s}} + \cancel{\frac{3ae^{-2s}}{s}} + \cancel{\frac{3ae^{-3s}}{s}} - \cancel{\frac{2ae^{-2s}}{s}} + \frac{ae^{-3s}}{s} - \cancel{\frac{ae^{-2s}}{s}}$$

$$+\frac{ae^{-3s}}{s} + \cancel{\frac{6ae^{-2s}}{s}}$$

$$\mathcal{L}\,u(t) = -\cancel{\frac{ae^{-s}}{s}} - \frac{ae^{-s}}{s^2} + \frac{a}{s^2} - \frac{ae^{-2s}}{s} + \cancel{\frac{ae^{-s}}{s}} + \frac{ae^{-3s}}{s}$$

$$\boxed{\frac{a}{s^2}\left[-e^{-s} + 1 - e^{-2s} + e^{-3s}\right]}$$

48/50

43/50

David Malawey

ASSIGNMENT – 6/ ME 211/ MODELING AND ANALYSIS OF DYNAMIC SYSTEMS

DUE: 10/11/2010 (Monday) IN CLASS OR BY 2PM IN MY OFFICE (TMH 129)

Problem 1: Textbook Problem 6.15; (For this problem; take $e_0 = 0$ and find R_{eq})

Problem 2: Textbook Problem 6.19

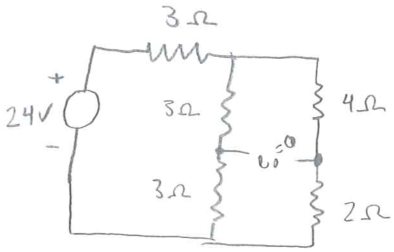
Problem 3: Textbook Problem 6.22 (Think: How many input(s) do you have??)

Problem 4: Textbook Problem 6.29 (Note: There are two capacitor elements and no inductor element)

David Malawey 1)

6.151 $e_0 = 0$, find Req

parallel $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$ Series $R_{eq} = r_1 + r_2$



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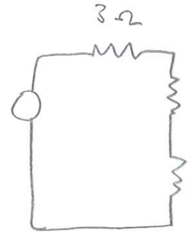
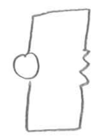


$\left(\frac{1}{3} + \frac{1}{4}\right)^{-1} = \frac{12}{7}$

$\left(\frac{1}{3} + \frac{1}{2}\right)^{-1} = \frac{6}{5}$

=



$3 + \frac{12}{7} + \frac{6}{5}$

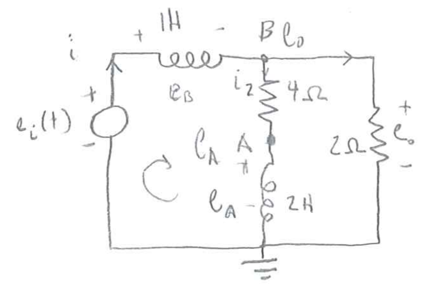
$\boxed{R_{eq} = 5.914\ \Omega}$ X

-2

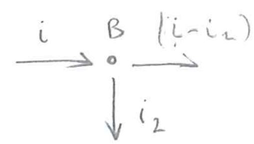
David Malawey 2)

State vars: i_x i_2

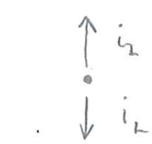
6.19 find state var eq'ns, & eqn for e_0



Node B KCL



A



A

1) $-e_0 + e_i(t) = 1\frac{di}{dt}$

2) $e_0 - e_A = 4i_2$

3) $e_A = 2\frac{di_2}{dt}$

4) $e_0 = 2(i - i_2)$

$-i + i_2 + \frac{e_0}{2} = 0$

$\frac{e_0}{2} + \frac{e_0 - e_A}{4} = i$

$\frac{3e_0}{4} - \frac{1}{4}\left(\frac{2di_1}{dt}\right) = i$

$i = \frac{3}{4}2(i - i_2) - \frac{1}{2}\frac{di}{dt}$

$\frac{(e_A - e_0)}{4} = \cancel{i_2(0)}^{0} + \frac{1}{2}\int_0^t e_A \lambda\, d\lambda$

$\frac{\dot{e}_A}{4} - \frac{\dot{e}_0}{4} = \frac{e_A}{2}$

state variables: i, i_2, $e_i(t)$

using 1) & 4)

$\frac{di}{dt} = -2(i - i_2) - e_i(t)$

$\boxed{\frac{di}{dt} = 2i_2 - 2i - e_i(t)}$

$\frac{di_2}{dt} = \frac{1}{2}(e_0 - 4i_2)$

$\frac{di_2}{dt} = i - i_2 - 2i_2$

$\boxed{\frac{di_2}{dt} = i - 3i_2}$

KCL Node

$-i + i_2 + \frac{1}{2}e_0 = 0$

output

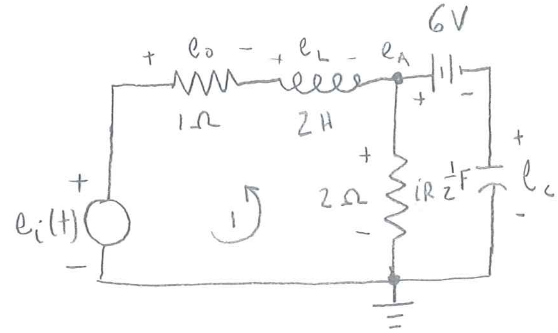
Voltages across inductors

$\boxed{\begin{array}{l} e_B = e_i(t) - e_0 = e_i(t) - 2i + 2i_2 \\ e_B = e_0 - 4i_2 = 2i - 6i_2 \end{array}}$

substitute

expressions

David Malawey 3)



6.22/ Find a set of state-variable eqn's & an algebraic output for e_o

- How many inputs do we have?

KVL loop 1

$$e_A + e_L + e_o - e_i(t) = 0$$

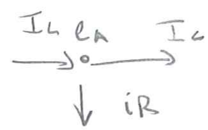
$$\dot{I}_L = \frac{e_L}{L}$$

$$q = \begin{bmatrix} e_c \\ I_L \end{bmatrix}$$

$$(e_c + 6V) + e_L + i_L - e_i(t) = 0$$

$$\boxed{\dot{I}_L = \frac{1}{2}\left(-6 - e_c - i_L + e_i(t)\right)}$$

$$\dot{e}_c = \frac{I_c}{C}$$



$$i_L - i_c - i_R = 0$$

$$i_c = i_L - i_R$$

$$i_c = i_L - \frac{1}{2}(e_c + 6)$$

$$\boxed{e_o = 1 I_L}$$

$$\dot{e}_c = \frac{i_c}{\frac{1}{2}F} = 2\left(i_L - .5\,(e_c + 6)\right)$$

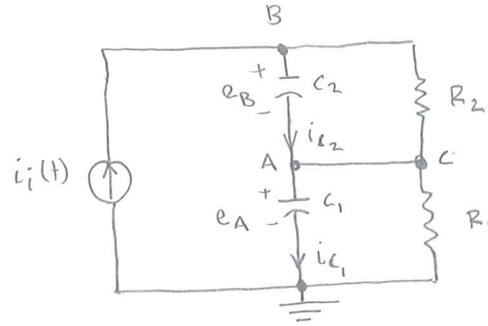
$$\boxed{\dot{e}_c = 2i_c - e_c - 6}$$

David Malawey 4)

6.29 ~~Voltage source~~ current src $i_i(t)$ ↑

write output eq'n for i_{c_1}

find st. var. eq'n's



$i_i(t)$ B $i_i(t) - i_{c_2}$

i_{c_2}

$$q = \begin{bmatrix} e_A \\ e_B \end{bmatrix}$$

$$i = \frac{e}{R}$$

$$i_i(t) = \frac{e_B}{R_2} + i_{c_2}$$

$$\boxed{i_{c_2} = -\frac{e_B}{R_2} + i_i(t)}$$

$$\dot{e}_c = \frac{I_c}{C}$$

$$\boxed{\dot{e}_B = \frac{1}{C_2}\left[i_i(t) - \frac{1}{R_2} e_B \right]}$$ ✓

$i_i(t)$ i_{c_1} $(i_i(t) - i_{c_1})$

$$i_i(t) - i_{c_1} - (i_i(t) - i_{c_1}) = 0$$

$$i_i(t) - i_{c_1} - \frac{e_A}{R_1} = 0$$

$$\dot{e}_c = \frac{I_c}{C}$$

$$\boxed{\dot{e}_A = \frac{1}{C_1}\left[i_i(t) - \frac{e_A}{R_1} \right]}$$ ✓

$$\boxed{i_{c_1} = i_i(t) - \frac{e_A}{R_1}}$$ ✓

38/40

David Malawey

ASSIGNMENT – 5/ ME 211/ MODELING AND ANALYSIS OF DYNAMIC SYSTEMS

DUE: 10/6/2010 (Wednesday) IN CLASS OR BY 2PM IN MY OFFICE (TMH 129)

Problem 1: Textbook Problem 5.18 (Parts a and b only); Given: N (Gear ratio) = $R_2/R_1 = \theta_1/\theta_2$

Problem 2: Textbook Problem 5.21 Take $\tau_L = 0$; Also, N (Gear ratio) = $R_b/R_a = \omega_a/\omega_b$

Problem 3: Textbook Problem 6.3 (Using loop-equation method).

Problem 4: Textbook Problem 6.6 (Using loop-equation method).

Problem 5: Verify your answer to problem 3 using node-equation method.

Problem 6: Verify your answer to problem 4 using node-equation method.

Problem 7: Textbook Problem 6.4 (Using node-equation method). Note: The input is $i_i(t)$ and output is e_0.

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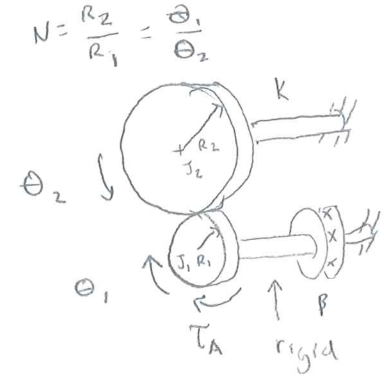
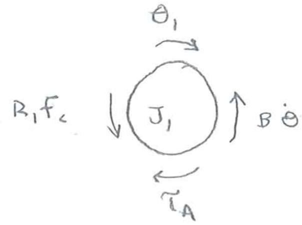
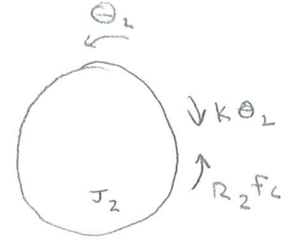
1) $\boxed{5.18}$ $N = \frac{R_2}{R_1} = \frac{\theta_1}{\theta_2}$







$\theta_2 = \frac{R_1}{R_2}\theta_1$

$k\theta_2\left(\frac{R_1}{R_2}\right)$

$\uparrow \Sigma \tau_1 = J_1 \ddot{\theta}_1$

$J_1 \dot{\omega}_1 = \tau_A(t) - B\dot{\theta} - R_1 f_c$

$\boxed{\dot{\omega}_1 = \frac{1}{J_1}\left[\tau_A(t) - B\omega - R_1 f_c\right]}$ ✓

$\Sigma \tau_2 = J_2 \ddot{\theta}_2$

$\boxed{\dot{\omega}_2 = \frac{1}{J_2}\left(R_2 f_c - k\theta_2\right)}$ ✓

$R_2 f_c = J_2 \dot{\omega}_2 + k\theta_2$

$f_c = \frac{1}{R_2}\left(J_2 \dot{\omega}_2 + k\theta_2\right)$

$\dot{\omega}_1 = \frac{1}{J_1}\left[\tau_a(t) - B\omega_1 - R_1\left(\frac{1}{R_2}\right)\left(J_2\dot{\omega}_2 + k\theta_2\right)\right]$

$\dot{\theta}_1 = N\dot{\theta}_2$

$\dot{\omega}_1 = N\dot{\omega}_2$

$J_1 N \dot{\omega}_2 = \left[\tau_a(t) - BN\omega_2 - \left(\frac{1}{N}\right)\left(J_2\dot{\omega}_2 + k\theta_2\right)\right]$

$\tau_a(t) = J_1 N \dot{\omega}_2 + BN\omega_2 + \frac{J_2\dot{\omega}_2 + k\theta_2}{N}$

$J_1 N^2 \dot{\omega}_2 + BN^2\omega_2 + J_2\dot{\omega}_2 + k\theta_2 = \tau_a(t)$

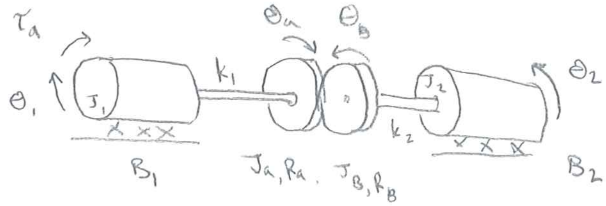
$\boxed{(J_1 N^2 + J_2)\dot{\omega}_2 + BN^2\omega_2 + k\theta_2 = \tau_a(t) N}$ X −2

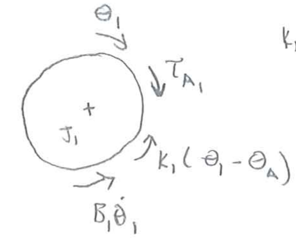
can only have ω_1 & θ_1 in this equation

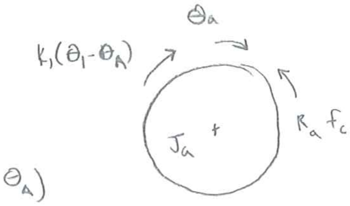
-ask kumar if this is complete

David Malaney

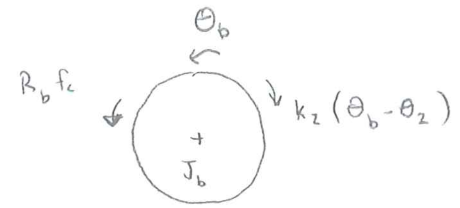
2) $\boxed{5.21}$ $\tau_L = 0$ $N = \frac{R_b}{R_a} = \frac{\omega_a}{\omega_b}$

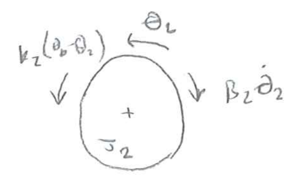






(1) $-k_1\theta_1 + k_1\theta_A - B_1\dot{\theta}_1 + \tau_A = J_1\ddot{\theta}_1$





(2) $k_1(\theta_1 - \theta_A) - R_a f_c = J_a\ddot{\theta}_a$

State Vars: ω_1 ω_2 ω_a

$\phi_a = \theta_1 - \theta_a$ $\phi_b = \theta_2 - \theta_b$ -2

(3) $R_b f_c - k_2(\theta_b - \theta_2) = \ddot{\theta}_b J_b$

(4) $K_2(\theta_b - \theta_2) - B_2\dot{\theta}_2 = J_2\ddot{\theta}_2$

$-\phi_b$

$N = \frac{R_b}{R_a}$ $N\omega_b = \omega_a$ $\dot{\omega}_a = ?$

(2 & 3) $f_c = \frac{1}{R_b}\left[\ddot{\theta}_b J_b + k_2(\theta_b - \theta_2)\right]$

(4) $\boxed{J_2\dot{\omega}_2 + B_2\dot{\theta}_2 - k_2\phi_b = 0}$ ✓

$k_1\phi_a - \frac{R_a}{R_b}\left[\ddot{\theta}_b J_b - k_2\phi_b\right] = J_a\ddot{\theta}_a$

$k_1\phi_a - \frac{1}{N}\left[\ddot{\theta}_b J_b - k_2\phi_b\right] = J_a N\ddot{\theta}_b$

$k_1\phi_a - \ddot{\theta}_b\left(\frac{J_b}{N} + J_a N\right) - k_2\phi_b = 0$

$f_c = \dfrac{\left(\frac{J_b k_1\phi_a}{J_a N} - k_2\phi_b\right)}{\left(R_b + \frac{J_b R_a}{J_a N}\right)}$

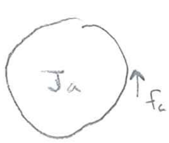
$= \dfrac{J_b k_1\phi_a - k_2\phi_b N J_a}{R_b J_a N + J_b R_a}$

(1) $\tau_A - k_1\phi_A - B_1\dot{\theta}_1 = J_1\dot{\omega}_1$

$\boxed{J_1\dot{\omega}_1 + k_1\phi_A + B_1\dot{\theta}_1 = \tau_a(t)}$ ✓

$= \boxed{\dfrac{J_b k_1\phi_a - J_a N k_2\phi_b}{(J_a N^2 + J_b)R_a}}$ ✓

b)



$f_c = \frac{1}{R_b}\left[\dot{\omega}_b J_b - k_2\phi_b\right]$

$f_c = \frac{1}{R_a}\left[k_1\phi_a - J_a\dot{\omega}_a\right]$ eliminate $\dot{\omega}$'s

$f_c = \frac{1}{R_a}\left\{k_1\phi_a - J_a\dot{\omega}_b N\right\}$

$f_c = \frac{1}{R_b}\left[\frac{J_b\left[k_1\phi_a - f_c R_a\right]}{J_a N} - k_2\phi_b\right]$

$\frac{-f_c R_a + k_1\phi_a}{J_a N} = \dot{\omega}_b$

$f_c R_b = \frac{J_b k_1\phi_a}{J_a N} - \frac{J_b f_c R_a}{J_a N} - k_2\phi_b$

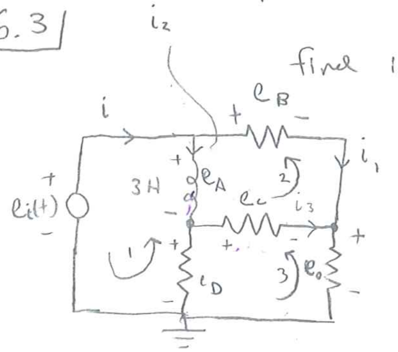
$f_c\left(R_b + \frac{J_b R_a}{J_a N}\right) = \frac{J_b k_1\phi_a}{J_a N} - k_2\phi_b$ ✓

need to solve equations for state variables

David Malawey 3)

6.3

find input-output d.e. relating e_o & $e_i(t)$ use loop eqn method



② KVL eqn's

loop 1 $\quad e_D + e_A - e_i(t) = 0 \quad (1)$

loop 2 $\quad e_B - e_A - e_C = 0 \quad (2)$

loop 3 $\quad e_o + e_C - e_D = 0 \quad (3)$

$V = \frac{i}{r}$

$i = Vr$

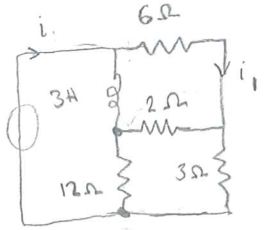
$i = e\,\Omega$

$\boxed{i = \frac{1}{R} e}$

$\boxed{i = C \frac{de}{dt}}$

$\boxed{e = L \frac{di}{dt}}$

1) const. relations



③ constitutive relations

$e_A = (3)\left(\frac{di}{dt}\right)$

$e_D = (12)(i_1 - i_2)$

$e_B = (6)(i - i_1)$

$e_C = (2)(i_2$

$e_O = (3)(i - i_1 + i_2)$

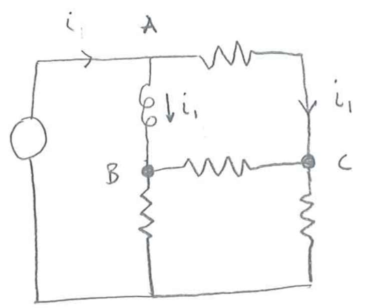
(1) $e_i(t) = 12i - 12i_2 + 3\frac{di}{dt}$

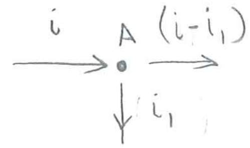
(3) $e_o + 2i_2 - 12i + 12i_2 = 0$

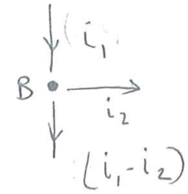
$e_o + 14i_2 - 12i_1 = 0$

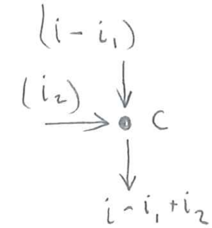
(2) $6i_1 - 6i_2 - 3\frac{di}{dt} - 2i_2 = 0$

① apply KCL to express current through each element









(1 & 2) $\quad 6i_1 - 6i_2 - 2i_2 + 12i_1 - 12i_2 = e_i(t)$

$e_i(t) = 6i + 6i_1 - 14i_2$

$-e_o = 14i_2 - 12i_1$

$e_o = 3i - 3i_1 + 3i_2$

$$\begin{bmatrix} 6 & 6 & -14 \\ 0 & -12 & 14 \\ 3 & -3 & 3 \end{bmatrix} \begin{bmatrix} i \\ i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} e_i \\ -e_o \\ e_o \end{bmatrix} \xrightarrow{\text{Row ops}} \left[\begin{array}{ccc|c} 1 & 1 & -7/3 & e_i/6 \\ 0 & 12 & 14 & -e_o \\ 0 & 0 & -6 & -3e_o + e_i \end{array}\right]$$

$-6i_2 = -3e_o + e_i$

$i_2 = +\frac{e_o}{2} - e_i/6$

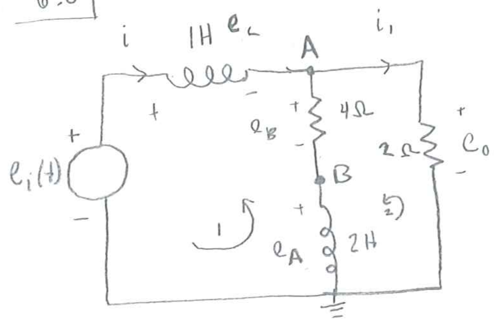
$i_1 = \frac{2}{3}e_o - \frac{7}{30}e_i$

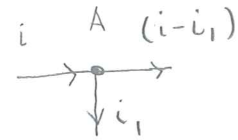
$\frac{di_1}{dt} = \frac{2}{3}\dot{e}_o - \frac{7}{36}\dot{e}_i$

KVL loop 1 $\quad 12\left(\frac{2}{3}e_o - \frac{7}{36}e_i - \left(\frac{1}{2}e_o - \frac{1}{6}e_i\right)\right) + 3\left(\frac{2}{3}\dot{e}_o - \frac{7}{36}\dot{e}_i\right) = e_i$

.... $\boxed{\dot{e}_o + e_o = \frac{2}{3}e_i + \frac{7}{24}\dot{e}_i}$

4) 6.6





$i = \frac{1}{R} e$

$i = C \frac{de}{dt}$

$e = L \frac{di}{dt}$

② Write KVL

Loop 1 $\quad e_c + e_B - e_i(t) + e_A = 0 \quad \Rightarrow \quad 2\frac{di}{dt} + 4i_1 + 1\frac{di}{dt} - e_i = 0$ ✓

loop 2 $\quad e_o - e_B - e_A = 0$ ✓ $\Rightarrow e_o - 4i - 2\frac{di}{dt} = 0$ ✓

③ relations, constitutive

$e_A = 2\left(\frac{di}{dt}\right)$

$e_B = 4(i_1)$

$e_c = 1\left(\frac{di}{dt}\right)$

$e_o = 2(i - i_1)$

$\frac{di}{dt} = -e_o + e_i \quad$ integrate

$i = \int e_i - \int e_o \quad \leftarrow e_o = 2i - 2i_1$

$e_o = 2\int e_i - 2\int e_o - 2i_1$

$\frac{e_o}{2} = \int e_i - \int e_o - i_1$

$i_1 = -\frac{e_o}{2} + \int e_i - \int e_o$

$\frac{di}{dt} = \frac{-\dot{e}_o}{2} + e_i - e_o \quad$ plug into KVL loop 2

$e_o - 4\left(-\frac{e_o}{2} + \int e_i - \int e_o\right) = 2\left(-\frac{\dot{e}_o}{2} + e_i - e_o\right)$

$e_o + 2e_o - 4\int e_i + 4\int e_o = -\dot{e}_o + 2e_i - 2e_o$

$+2e_o \qquad\qquad + 2e_o$

$\frac{d}{dt}\left[\ 5e_o - 4\int e_i + 4\int e_o = -\dot{e}_o + 2e_i\right]$

$5\dot{e}_o - 4e_i + 4e_o = -\ddot{e}_o + 2\dot{e}_i$

$\boxed{\ddot{e}_o + 5\dot{e}_o + 4e_o = 2\dot{e}_i + 4e_i}$ ✓

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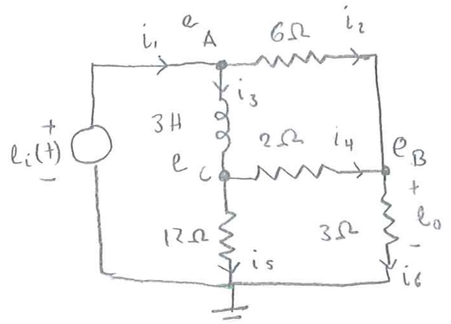
#5) Verify #3 with node-equation method

6.31

1) label e's wrt ground



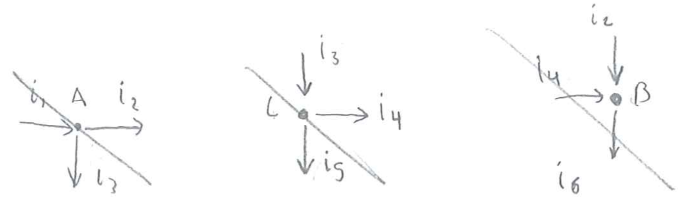
$\cancel{e_A = e_i(t)}$ (1)

$e_B = e_o - 0$ (2)

$e_A - e_B = 6i_2$ (3) $\quad e_a = 6i_2 + e_0$ (2 & 3)

$e_A - e_c = \frac{i_3}{3}$ (4)

$e_c = 12 i_5$ (5)



(2) KCL

For Node A

i_1, $\frac{(e_A - e_B)}{6}$, $\frac{1}{3}\int_0^t (e_A - e_K)\,d\lambda$

KCL For node B

$\frac{(e_B - e_c)}{2}$, $\frac{(e_B - e_a)}{6}$, $\frac{e_0}{3}$

KCL For node C

$\frac{1}{3}\int (e_c - e_a)\,d\lambda$, $\frac{(e_c - e_B)}{2}$, $\frac{e_c}{12}$

$$\frac{3e_B}{\cancel{2}} - \frac{3e_c}{\cancel{2}} + \frac{e_B}{\cancel{6}} - \frac{e_A}{\cancel{6}} \frac{2e_0}{\cancel{3}} = 0$$

$$4e_B - 3e_c - e_A - 2e_0 = 0$$

$$e_c = e_0, \; e_A = e_i$$

$$4e_B - 5e_0 - e_A = 0$$

$$e_B = 2e_0 - \frac{e_i}{3}$$

$$\dot{e}_B = 2\dot{e}_0 + -\frac{\dot{e}_i}{3} \quad (2)$$

$$\frac{1}{3}\int (e_c - e_a)\,d\lambda - \frac{e_c}{12} + \frac{e_B}{12} - \frac{e_a}{12} = 0$$

$$\frac{e_c}{3} - \frac{e_a}{3} - \frac{\dot{e}_c}{2} + \frac{\dot{e}_B}{2} - \frac{\dot{e}_c}{12} = 0$$

$$4e_0 - 4e_i - 7\dot{e}_0 + 6\dot{e}_B = 0 \quad (1)$$

(1 & 2) $\quad 4e_0 - 4e_i - 7\dot{e}_0 + 12\dot{e}_0 - 2\dot{e}_i = 0$

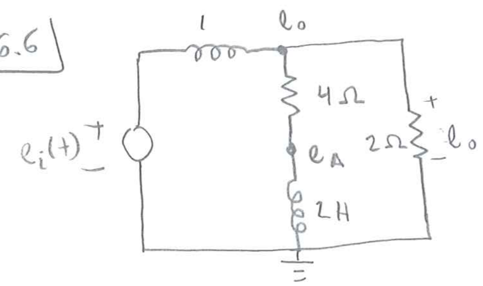
$$4e_0 - 4e_i + 5\dot{e}_0 - 2\dot{e}_i = 0$$

$$\boxed{\dot{e}_0 + e_0 = \frac{2}{3}e_i(t) + \frac{7}{24}\dot{e}_i}$$

David Malawey

#6 Verify problem 4, using node voltage

6.6



KCL @ e_A

$\downarrow \frac{e_A - e_o}{4}$

$\downarrow \frac{1}{2}\int e_A \, d\lambda$

$$\frac{e_A - e_o}{4} + \frac{1}{2}\int e_A \, d\lambda = 0$$

$$\frac{e_A}{2} + \frac{\dot{e}_A}{4} - \frac{\dot{e}_o}{4} = 0$$

$$2e_A - \dot{e}_o + \dot{e}_A = 0 \quad (1)$$

KCL Node e_o

$\frac{1}{1}\int (e_o - e_i(t)) \, d\lambda \rightarrow \cdot \xrightarrow{e_o/2}$, $\downarrow \frac{(e_o - e_A)}{4}$

$$\int e_o - e_i(t) \, d\lambda + \frac{e_o}{2} + \frac{e_o - e_A}{4} = 0$$

$$e_o - e_i(t) + \frac{\dot{e}_o}{2} + \frac{\dot{e}_o}{4} - \frac{\dot{e}_a}{4} = 0$$

$$e_o - e_i(t) + \frac{3}{4}\dot{e}_o - \frac{\dot{e}_a}{4} = 0$$

$$(2) \quad 4e_o - 4e_i(t) + 3\dot{e}_o - \dot{e}_a = 0$$

$$\dot{e}_a = 4\dot{e}_o - 4\dot{e}_i t + 3\ddot{e}_o$$

$$2(4e_o - 4e_i(t) + 3\dot{e}_o) - \dot{e}_o + \cancel{\dot{e}_A} = 0 + 4\dot{e}_o - 4\dot{e}_i(t) + 3\ddot{e}_o = 0$$

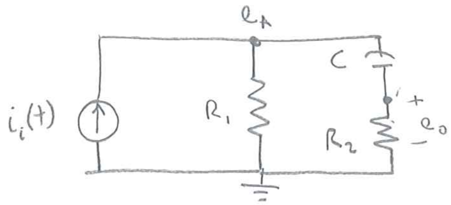
$$8e_o - 8e_i(t) + 6\dot{e}_o - \dot{e}_o + 4\dot{e}_o - 4\dot{e}_i(t) + 3\ddot{e}_o = 0$$

$$\ddot{e}_o + 5\dot{e}_o + \frac{8e_o}{2} = \frac{8e_i(t)}{2} + \frac{4\dot{e}_i(t)}{2}$$

$$\boxed{\ddot{e}_o + 5\dot{e}_o + 4e_o = 2\dot{e}_i(t) + 4e_i(t)}$$ ✓

#7)

6.4



KCL @ Node e_A

$i_i(t) \longrightarrow \bullet \longrightarrow C(e_A - e_o)\frac{d}{dt}$

$\downarrow \frac{e_A}{R_1}$

$$-i_1(t) + \frac{e_A}{R_1} + C\left(\frac{de_A}{dt} - \frac{de_o}{dt}\right) = 0$$

$$-i_1(t) + \frac{e_A}{R_1} + C\dot{e}_A - C\dot{e}_o = 0 \quad (1)$$

KCL @ Node e_o

$\downarrow C(e_o - e_A)\frac{d}{dt}$

$\bullet$

$\downarrow \frac{e_o}{R_2}$

$$C\dot{e}_o - C\dot{e}_A + \frac{e_o}{R_2} = 0$$

$$\dot{e}_A = \dot{e}_o + \frac{e_o}{CR_2} \quad (2)$$

$$-i_1(t) + \frac{e_A}{R_1} + C\left[\dot{e}_o - \frac{e_o}{CR_2}\right] - C\dot{e}_o = 0$$ Sub (2) into (1)

$$\left[\frac{e_A}{R_1} + C\dot{e}_o - \frac{e_o}{R_2} - C\dot{e}_o = i_1(t)\right]\frac{d}{dt}$$

$$\frac{1}{R_1}e_o - \frac{1}{R_1}\frac{e_o}{CR_2} + \cancel{C\ddot{e}_o} - \frac{\dot{e}_o}{R_2} - \cancel{C\ddot{e}_o} = \frac{d}{dt}i_1(t)$$

$$\frac{\dot{e}_o}{R_1} + \frac{e_o}{CR_1R_2} - \frac{\dot{e}_o}{R_2} = \frac{d}{dt}i_1(t)$$

$$CR_2\dot{e}_o - CR_1\dot{e}_o + e_o = CR_1R_2\frac{d}{dt}i_1(t)$$

$$\boxed{\dot{e}_o(CR_2 + CR_1) + e_o = CR_1R_2\frac{d}{dt}i_1(t)}$$

−1

65/70

ASSIGNMENT – 4/ ME 211/ MODELING AND ANALYSIS OF DYNAMIC SYSTEMS

DUE: 09/20/2010 (MONDAY) IN CLASS OR BY 2PM IN MY OFFICE (TMH 129)

Problem 1: Textbook Problem 5.1

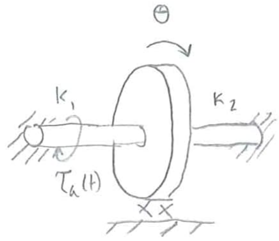
Problem 2: Textbook Problem 5.6 only part a)

Problem 3: Textbook Problem 5.12 (Instead of y, take θ as the coordinate defining the angular displacement of the lever)

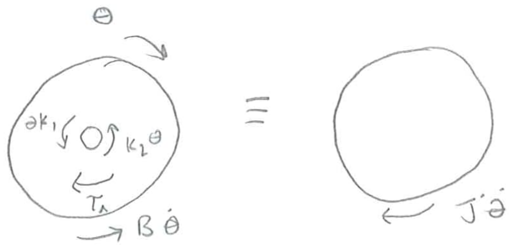
Problem 4: Textbook Problem 5.14

David Malawey

5.1



Write D.E, determine equivalent stiffness const.



$$J\ddot{\theta} = \tau_a(t) - (k_1 + k_2)\theta - B\dot{\theta}$$

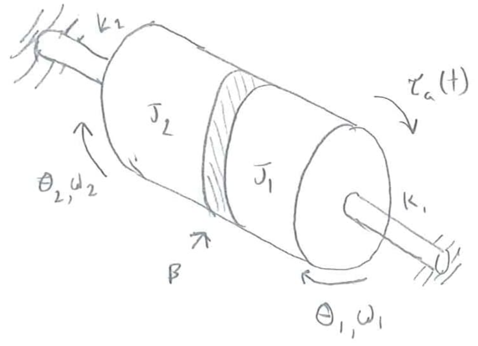
$$\boxed{J\ddot{\theta} + B\dot{\theta} + (k_1 + k_2)\theta = \tau_a(t)}$$ ✓

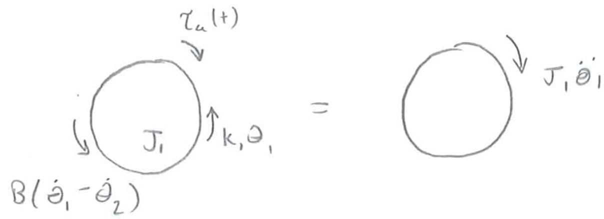
$$\boxed{k_{eq} = (k_1 + k_2}$$ ✓

5.6) a)

input $\tau_a(t)$

Output = Viscous T on J_2

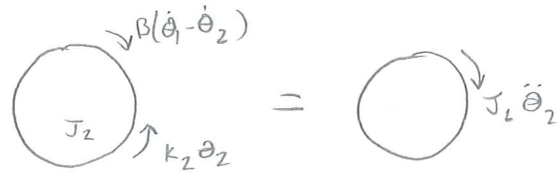




$$\tau_a(t) - k_1\theta_1 - B(\dot\theta_1 - \dot\theta_2) = J\ddot\theta$$

$$\boxed{\dot\omega_1 = \frac{1}{J}\left[\tau_a(t) - B\omega_1 - k_1\theta_1 + B\omega_2\right]}$$

$$\boxed{\begin{array}{l}\theta_1 \\ \omega_1 = \dot\theta_1\end{array}}$$



$$B(\dot\theta_1 - \dot\theta_2) - k_2\theta_2 = J_2\ddot\theta_2$$

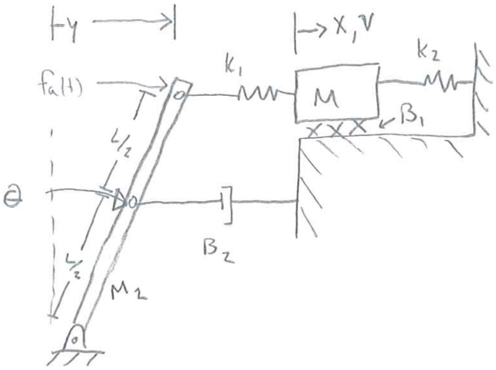
$$\boxed{\dot\omega_2 = \frac{1}{J_2}\left[-B\omega_2 - k_2\theta_2 + B\omega_1\right]}$$

$$\boxed{\begin{array}{l}\theta_2 \\ \omega_2 = \dot\theta_2\end{array}}$$

$$\boxed{\text{output equation} \quad \circlearrowright + \quad \tau = -B(\omega_1 - \omega_2)}$$

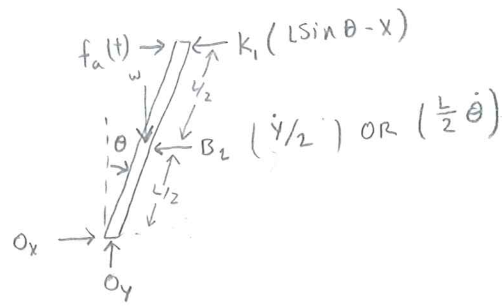
David Malawey

5.12) take θ to define angular displacement of lever
output: displacement x



$k_1(L\sin\theta - x) \rightarrow \boxed{M} \leftarrow k_2 x \quad \equiv \quad \boxed{M}\ M\ddot{x} \rightarrow$

$\leftarrow B_1 \dot{x}$



Mass of block

$k_1(L\sin\theta - x) - k_2 x - B_1\dot{x} = M\ddot{x}$

$\dot{v} = \frac{1}{M}\left[k_1(L\sin\theta - x) - B_1 v - k_2 x\right]$

$$\boxed{\dot{v} = \frac{1}{M}\left[k_1 L\sin\theta - x(k_1+k_2) - B_1 v\right]}$$

simplify $\sin\theta \approx \theta$. -.5

assumptions for lever:

θ is small $\Rightarrow \sin\theta \approx \theta$
$\cos\theta \approx 1$
W causes no torque

evenly distributed M

$J_0 = \frac{1}{3}ML^2$

Lever

$\circlearrowleft + \sum \tau_0 = J\ddot{\theta}$

$f_a(t)\, L\cos\theta - k_1(L\sin\theta - x)(L\cos\theta) - B_2(\frac{L}{2}\dot{\theta})(\frac{L}{2}\cos\theta)$
$= \frac{M_2 L^2}{3}\ddot{\theta}$

$f_a(t)L - k_1(L\theta - x)(L) - B_2(\frac{L}{2}\dot{\theta})(\frac{L}{2}) = M_2\frac{L^2}{3}\ddot{\theta}$

$\frac{f_a(t)\cancel{L}}{L} - k_1 L^{\cancel{2}}\theta + \frac{k_1 x \cancel{L}}{L} - \frac{B_2 L^{\cancel{2}}\dot{\theta}}{4} = M_2\frac{\cancel{L}^{\cancel{2}}}{3}\ddot{\theta}$

$$\boxed{\dot{\omega} = \frac{3}{LM_2}\left[f_a(t) - Lk_1\theta + k_1 x - \frac{B_2\omega L}{4}\right]}$$

X -1

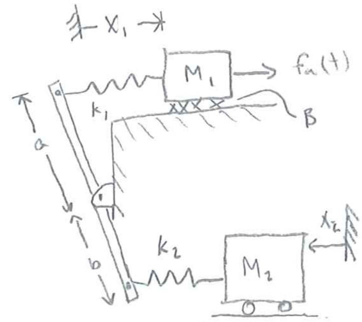
$\boxed{\omega = \dot{\theta}}$

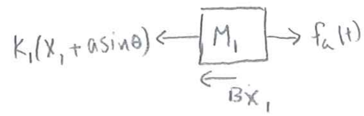
Output

X is a state variable
no equation needed

David Malawey

5.14 use prob 5.11

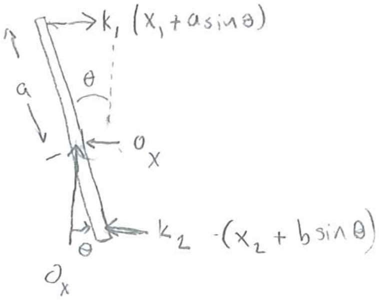




$\Sigma F_x = M_1\ddot{x}_1$

$f_a(t) - B\dot{x}_1 - k_1(x_1 + a\sin\theta) = M_1\ddot{x}_1$

$$\boxed{\begin{aligned} &x_1 \\ &\dot{x}_1 = v_1 \checkmark \\ &\dot{v}_1 = \frac{1}{M_1}\left[-Bv_1 - k_1x_1 - k_1a\theta + f_a(t)\right] \end{aligned}} \checkmark$$



$\curvearrowleft + \Sigma M_o = J_o\ddot{\theta}$

$k_1(x_1+a\sin\theta)(a\cos\theta) + k_2(x_2+b\sin\theta)(b\cos\theta) = \frac{M_3(a^3+b^3)}{3(a+b)}\ddot{\theta}$

$\dot{\omega}_1 = \frac{3(a+b)}{M_3(a^3+b^3)}\left[k_1x_1a + k_1a^2\theta + k_2x_2b + k_2b^2\theta\right]$

$J_o = J_a + J_B$

$J_o = \frac{1}{3}M\left(\frac{a}{a+b}\right)a^2 + \frac{1}{3}M\left(\frac{b}{a+b}\right)b^2$

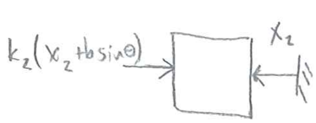
$J_o = \frac{M}{3}\left[\frac{a^3}{a+b} + \frac{b^3}{a+b}\right]$

$J_o = \frac{M(a^3+b^3)}{3(a+b)}$

$$\boxed{\begin{aligned} &\dot{\omega} = \frac{3(a+b)}{M_3(a^3+b^3)}\left[ak_1x_1 + bk_2x_2 + (k_1a^2+k_2b^2)\theta\right] \\ &\theta \\ &\dot{\theta} = \omega \checkmark \end{aligned}} \times -1$$

$\overset{+}{\leftarrow} \Sigma F_x = M_2\ddot{x}_2$

$-k_2(x_2 + b\sin\theta) = M_2\ddot{x}_2$



$$\boxed{\begin{aligned} &x_2 \\ &\dot{x}_2 = v_2 \checkmark \\ &\dot{v}_2 = \frac{1}{M_2}(-k_2x_2 - k_2b\theta) \end{aligned}} \checkmark$$

$\boxed{\text{Output: } x_1 \text{ is a state variable, no eqn}}$

37.5/40

ASSIGNMENT – 3/ ME 211/ MODELING AND ANALYSIS OF DYNAMIC SYSTEMS/ FALL 2010

DUE: 09/15/2010 (WEDNESDAY) IN CLASS OR BY 2PM IN MY OFFICE (TMH 129)

Problem 1: : Textbook Problem 3.5 (Show only output equations and identify the matrices C and D)

Problem 2: Textbook Problem 3.19 (Show only output equations and identify the matrices C and D)

Problem 3: Textbook Problem 3.22 (Show only output equations and identify the matrices C and D)

Problem 4: Textbook Problem 3.28 (Use equation 22/ page 59 as the state variable form).

David Maloney

37/40

3.5

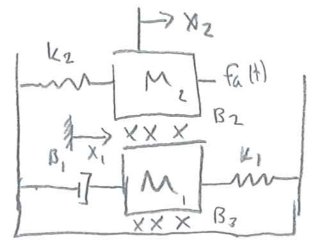


fig. 2.15

Y_1 = viscous force M_2 on M_1 $\xrightarrow{+}$

state var

Y_2 = tensile force in k_2

$\left\{ \begin{array}{l} x_1 \\ x_2 \\ v_1 \\ v_2 \end{array} \right.$

$Y_1 = -(\dot{x}_2 - \dot{x}_1) B_2$ $\boxed{Y_1 = B_2(v_1 - v_2)}$

$\boxed{Y_2 = k_2 x_2}$

10

$$\boxed{\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & B_2 & 0 & -B_2 \\ 0 & 0 & k_2 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ v_1 \\ x_2 \\ v_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{D} f_a(t)}$$

ans.

David Malawey

3.19

state vars

x_1

x_2

$$\dot{x}_1 + 2\dot{x}_2 = 3x_1 + 4x_2 - 5u(t) \qquad Y = \dot{x}_1 + 2x_2$$

$$\dot{x}_1 - \dot{x}_2 = 2x_1 + x_2 + u(t)$$

$$\dot{x}_1 + 2\dot{x}_1 + \cancel{2\dot{x}_2} - \cancel{2\dot{x}_2} = 3x_1 + 4x_2 - 5u(t) + 4x_1 + 2x_2 + 2u(t)$$

$$3\dot{x}_1 = 7x_1 + 6x_2 - 3u(t)$$

$$\dot{x}_1 = \frac{7}{3}x_1 + 2x_2 - u(t)$$

$$\boxed{Y = \frac{7}{3}x_1 + 4x_2 - u(t)}$$

$$\boxed{[Y] = \underbrace{\begin{bmatrix} \frac{7}{3} & 4 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} -1 \end{bmatrix}}_{D} u(t)}$$

10

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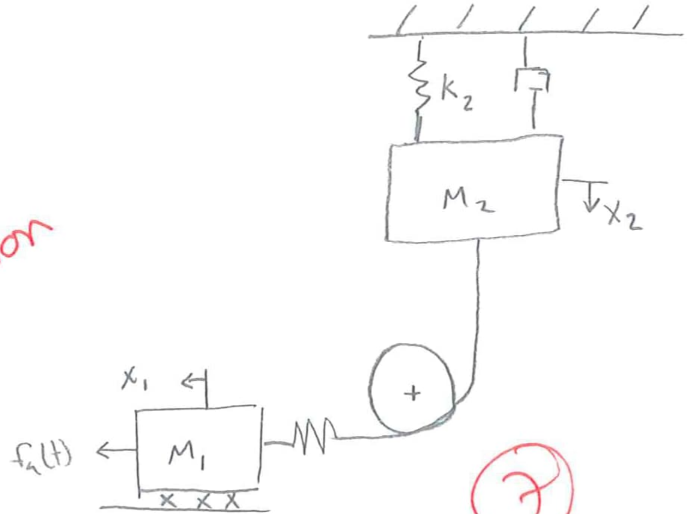
3.22) fig. 2-24

outputs = spring elongation



$Y_1 = k_1(x_1 - x_2)$

$Y_2 = k_2 x_2$

This is not elongation

?

$$\underbrace{\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}}_{2\times 1} = \underbrace{\begin{bmatrix} k_1 & 0 & -k_1 & 0 \\ 0 & 0 & k_2 & 0 \end{bmatrix}}_{C \; 2\times 4} \underbrace{\begin{bmatrix} x_1 \\ v_1 \\ x_2 \\ v_2 \end{bmatrix}}_{4\times 1} + \underbrace{\begin{bmatrix} 0 & 0 \end{bmatrix}}_{D \; 1\times 2} \underbrace{f_a(t)}_{1\times 1}$$

cannot multiply

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3.28/ eq. 22: $\dot{x} = q + \frac{B}{M_1} x_2(t)$

state var. form $\dot{q} = \frac{1}{M_1}\left[-(k_1+k_2)x_1 - Bq + \left(k_2 - \frac{B_2}{M_1}\right) x_2(t)\right]$

input $x_2(t)$

output x_1, V_1

state vars

identify:
- Matrices A B C D
- State vector q
- input vector u
- output vector y

Give values of n, m, p

$$\underset{2\times1}{\begin{bmatrix} \dot{x}_1 \\ \dot{q}_1 \end{bmatrix}} = \underset{A \quad 2\times2}{\begin{bmatrix} 0 & 1 \\ \frac{-(k_1+k_2)}{M_1} & -\frac{B}{M_1} \end{bmatrix}} \underset{2\times1}{\begin{bmatrix} x \\ q \end{bmatrix}} + \underset{B \quad (2\times1)}{\begin{bmatrix} \frac{B_2}{M_1} \\ \left(k_2 - \frac{B_2}{M_1}\right) \end{bmatrix}} \underset{(1\times1)}{x_2(t)}$$

eq 21 V_1 = terms of q

$$\boxed{\begin{bmatrix} x_1 \\ V_1 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{C}(2\times2) \begin{bmatrix} x \\ q \end{bmatrix} q(2\times1) + \underbrace{\begin{bmatrix} 0 \\ B/M_1 \end{bmatrix}}_{D} (2\times1) \; x_2(t)}$$

$$\boxed{\begin{aligned} p &= 2 \\ n &= 2 \\ m &= 1 \end{aligned}}$$

$y = Cq + Du$

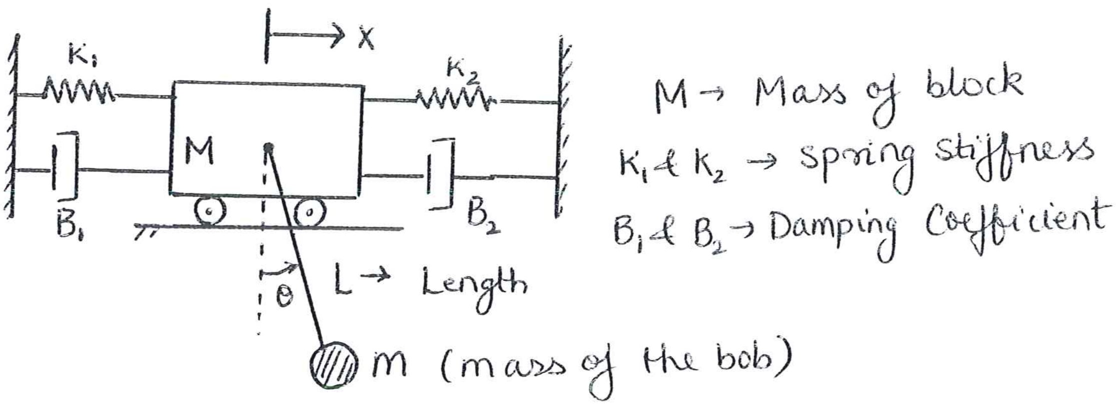
$V_1 = q + \frac{B}{M_1} x_2(t)$

10

ASSIGNMENT – 2/ ME 211/ MODELING AND ANALYSIS OF DYNAMIC SYSTEMS/ FALL – 2010

~~DUE: 09/8/2010 (WEDNESDAY) IN CLASS~~

Problem 1 (20 Points): Derive the differential equation of motion for the system shown in the figure below. Assume the angle θ to be small. Find the state variable model for the system. Write in matrix form the state-variable model.



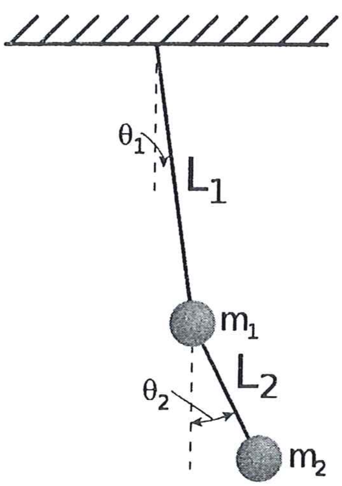
Problem 2 (10 Points): Textbook Problem 3.2/ Page 70

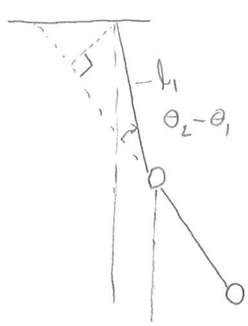
Problem 3 (10 Points): Textbook Problem 3.4/ Page 70

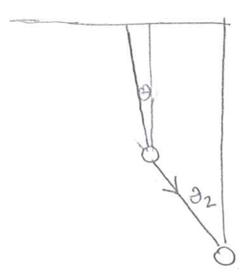
Problem 4 (10 Points): Textbook Problem 3.10/ Page 70 (Note: No need to include/ solve algebraic output equations)

Problem 5 (10 Points): Textbook Problem 3.19/ Page 72

Extra Credit (10 Points): Write in matrix form the state-variable model for the double pendulum shown below. The angles θ_1 & θ_2 can be arbitrarily large. Show all the steps for deriving eq'n



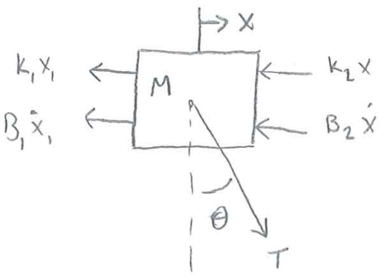




David Malawey ME211 assignment 2

Pendulum: solving for $\ddot{r}$

#1)



$\overset{x}{\rightarrow} \sum F_x = M\ddot{x}$

$-T\sin\theta = m\ddot{x}$ (1)

$\vec{r} = xi + L\sin\theta\, i - L\cos\theta\, j$

$= (x + l\sin\theta)i - (l\cos\theta)j$

$\dot{r} = (\dot{x} + L\dot{\theta}\cos\theta)i + (\dot{\theta}l\sin\theta)j$

$\ddot{r} = (\ddot{x} + L\ddot{\theta}\cos\theta + L\dot{\theta}^2(-\sin\theta))i + (\ddot{\theta}l\sin\theta + \dot{\theta}^2 l\cos\theta)j$

Block

$\overset{+}{\rightarrow} \sum F_x = M\ddot{x}$

$M\ddot{x} = -x(k_2+k_1) - \dot{x}(B_2+B_1) + T\sin\theta$

$M\ddot{x} = -x(k_2+k_1) - \dot{x}(B_2+B_1) - m(\ddot{x} + l\ddot{\theta} - L\dot{\theta}^2\theta)$ *

Ball

$\sum F_x : m\ddot{r}_i$

$-T\sin\theta = m(\ddot{x} + l\ddot{\theta}\cos\theta - L\dot{\theta}^2\sin\theta)$ θ is small

$-T\theta = m(\ddot{x} + l\ddot{\theta} - L\dot{\theta}^2\theta)$

$T = \frac{m}{-\theta}(\quad)$

$+\uparrow \sum F_y = m\ddot{y}$

$-mg + T\cos\theta = m\ddot{y}$ or $m\ddot{r}_j$

$T\cos\theta - mg = m(\ddot{\theta}l\sin\theta + \dot{\theta}^2 l\cos\theta)$

$T\cos\theta = m(\ddot{\theta}l\sin\theta + \dot{\theta}^2 l\cos\theta + g)$

Block

* $M\ddot{x} + m\ddot{x} = -x(k_2+k_1) - \dot{x}(B_2+B_1) - mL\ddot{\theta} + mL\dot{\theta}^2\theta$

$$\boxed{\ddot{x} = \frac{1}{(M+m)}\left[-\dot{x}(B_2+B_1) - x(k_2+k_1) - mL\ddot{\theta} + mL\dot{\theta}^2\theta\right]}$$ ✓

Ball

$\frac{-T\sin\theta}{T\cos\theta} = \frac{m(\ddot{x} + l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta)}{m(\ddot{\theta}l\sin\theta + \dot{\theta}^2 l\cos\theta + g)}$

$\cancel{m}(\ddot{x}\cos\theta + l\ddot{\theta}\cos^2\theta \cancel{- l\dot{\theta}^2\sin\theta\cos\theta}) = \overset{(-)}{\cancel{m}}(\ddot{\theta}l\sin^2\theta \cancel{- \dot{\theta}^2 l\cos\theta\sin\theta} - g\sin\theta)$

$\ddot{x}\cos\theta + l\ddot{\theta}\cancel{(\cos^2\theta + \sin^2\theta)} + g\sin\theta = 0$

$$\boxed{\ddot{x} + l\ddot{\theta} + g\sin\theta = 0}$$

θ

-.5 simplify

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ME 211 HW 2

input output

#2/ 3.2) $\dddot{x} + \alpha\ddot{x} + \beta\dot{x} + \gamma x = f_a(t)$ $y = \ddot{x} + \dot{x}$

put in state var. form x, $v = \dot{x}$ $a = \ddot{x}$

$\dddot{x} = f_a(t) - \alpha\ddot{x} - \beta\dot{x} - \gamma x$

$\boxed{\dot{a} = f_a(t) - \alpha a - \beta v - \gamma x}$ (1) $y = a + v$ (2) ask if this is ok.

$\boxed{a = y - v}$

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state vars:

#3 3.4 page 70 input: $f_a(t)$ output $y = \dot{x}_1 - \dot{x}_2$ x_1, $v_1 = \dot{x}_1$ x_2, and $v_2 = \dot{x}_2$

$$M_1 \ddot{x}_1 + B(\dot{x}_1 - \dot{x}_2) + K_1 x_1 = 0$$

$$M_2 \ddot{x}_2 + B(\dot{x}_2 - \dot{x}_1) + K_2 x_2 = f_a(t) \longrightarrow \ddot{x}_2 = \frac{1}{M_2}\left[f_a(t) - B(\dot{x}_2 - \dot{x}_1) - K_2 x_2\right]$$

$$\boxed{\dot{v}_2 = \frac{1}{M_2}\left[f_a(t) - B v_2 - K_2 x_2 + B v_1\right] \quad (2)}$$

$$\ddot{x}_1 = \frac{1}{M_1}\left[-B(\dot{x}_1 - \dot{x}_2) - k_1 x_1\right]$$

$$\dot{v}_1 = \frac{1}{M_1}(-B\dot{x}_1 - k_1 x_1 + B\dot{x}_2)$$

$$\boxed{\dot{v}_1 = \frac{1}{M_1}(-B v_1 - k_1 x_1 + B v_2) \quad (1)}$$

$$\boxed{y = v_1 - v_2 \quad (3)}$$

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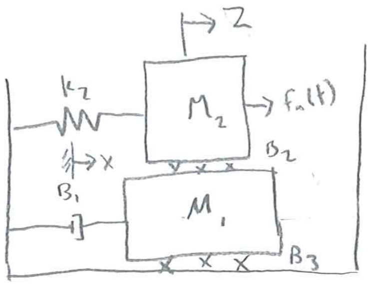
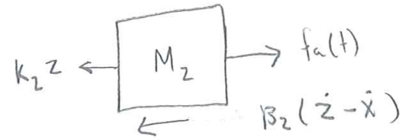
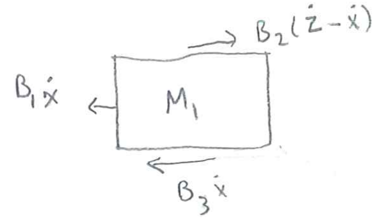
#4 3.10)

use $\cancel{x_2}\ Z$, V_1, V_2 as state vars

$\boxed{\text{no forces depend on } X_1}$







M_2 | $\overset{+}{\rightarrow} \Sigma F_x = m\ddot{z}$

$$m_2\ddot{z} = f_a(t) - B_2(\dot{z}-\dot{x}) - k_2 Z$$

$$\ddot{z} = \frac{1}{M_2}\left(f_a(t) - B_2\dot{z} - k_2 Z + B_2\dot{x}\right)$$

$$\boxed{\begin{aligned} \dot{V}_2 &= \frac{1}{M_2}\left(f_a(t) - B_2V_2 - k_2 Z + B_2V_1\right) \\ \dot{z} &= V_2 \\ \dot{x} &= V_1 \end{aligned}}$$

M_1 | $\overset{+}{\rightarrow} \Sigma F_x = m\ddot{x}$

$$\ddot{x} = \frac{1}{M_1}\left[+B_2(\dot{z}-\dot{x}) - B_3\dot{x} - B_1\dot{x}\right]$$

$$\dot{V}_1 = \frac{1}{M_1}\left[-\dot{x}(B_1+B_2+B_3) + B_2\dot{z}\right]$$

$$\boxed{\dot{V}_1 = \frac{1}{M_1}\left[-V_1(B_1+B_2+B_3) + B_2V_2\right]}$$

X_1 is not needed as a state variable because no forces (no motion) depends on X_1.

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#5 3.19) input $u(t)$ output y state vars x_1, x_2

$\dot{x}_1 + 2\dot{x}_2 = 3x_1 + 4x_2 - 5u(t)$

$\dot{x}_1 - \dot{x}_2 = 2x_1 + x_2 + u(t)$

$y = \dot{x}_1 + 2x_2$

$3(\dot{x}_1) + \cancel{2\dot{x}_2} - \cancel{2\dot{x}_2} = 3x_1 + 4x_2 - 5u(t) + 4x_1 + 2x_2 + 2u(t)$

$3\dot{x}_1 = 7x_1 + 6x_2 - 3u(t)$

$\boxed{\dot{x}_1 = \frac{7}{3}x_1 + 2x_2 - u(t)}$ ✓

$\cancel{\dot{x}_1} - \cancel{\dot{x}_1} + 2\dot{x}_2 + \dot{x}_2 = 3x_1 + 4x_2 - 5u(t) - 2x_1 - x_2 - u(t)$

$3\dot{x}_2 = 3x_2 + x_1 - 6u(t)$

$\boxed{\dot{x}_2 = x_2 + \frac{1}{3}x_1 - 2u(t)}$ ✓

output equation

$\boxed{y = \frac{7}{3}x_1 + 4x_2 - u(t)}$ ✓

$\frac{59.5}{60}$

ASSIGNMENT – 1/ ME 211/ MODELING AND ANALYSIS OF DYNAMIC SYSTEMS/ FALL – 2010
DUE: 09/1/2010 (WEDNESDAY) IN CLASS

Problem 1: For each of the following ODE systems, determine the order of the system, whether the system is linear or nonlinear (if nonlinear, identify the nonlinearity) and comment if the ODE is homogeneous/ non-homogeneous. (10 Points)

1) $\ddot{x} + \dot{x}(x^2-1) + x = P\cos 2t$ (P & 2 are constant)

2) $\dot{x} + t^3 x = \sin t$

3) $\dot{y} = \alpha - \beta(y^{1/2})$ α and β are constant

4) $\dfrac{d^4x}{dt^4} + 6xte^{-t} = (t^2+2t+1)$

Problem 2: Consider the ordinary differential equation; $a_1\, d^2x/dt^2 + a_2\, dx/dt + a_3\, x = f(t)$ where a_1, a_2 and a_3 are constants and input f(t) is a function of time. Suppose that when $f(t) = f_1(t)$, the solution is $x_1(t)$. Suppose that when $f(t) = f_2(t)$, the solution is $x_2(t)$. Show that when $f(t) = \alpha f_1(t) + \beta f_2(t)$, the solution is $x(t) = \alpha x_1(t) + \beta x_2(t)$ (i.e., the principle of superposition holds). 10 Points

Prob 3

2.3 For the system shown in Figure P2.3, the springs are undeflected when $x_1 = x_2 = 0$. The input is $f_a(t)$. Draw free-body diagrams and write the modeling equations.

10 points

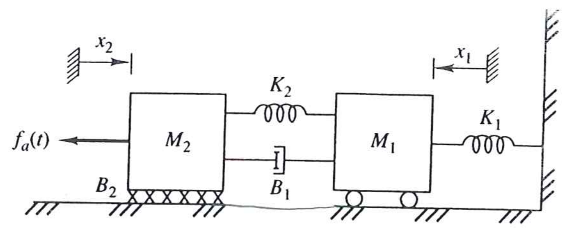


FIGURE P2.3

Prob (4): Draw the FBD's and write the ODE for the two masses, shown in Fig. 2.24 and 2.16 (20 Points)

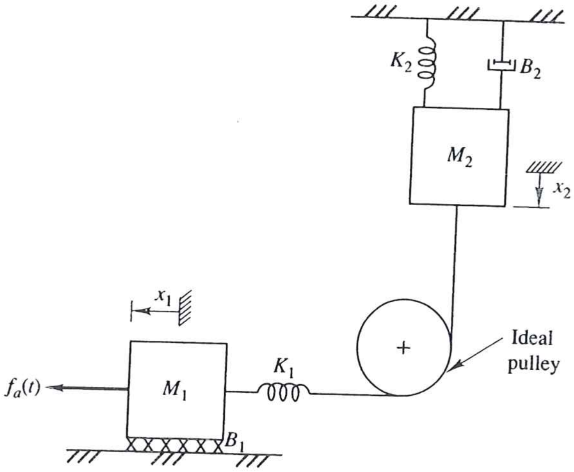


FIGURE P2.24

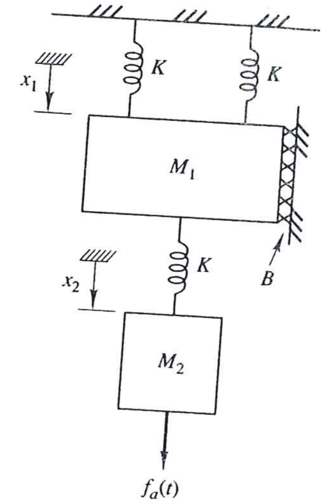


FIGURE P2.16

David Malawey
ME211
Homework due 9-1-10

#1) 1) $\ddot{x} + \dot{x}(x^2-1) + x = P\cos \Omega t$

2nd order eq'n
nonlinear (term "x^2-1")
ODE is non-homogeneous

2) $\dot{x} + t^3 x = \sin t$

1st order, linear ~~non~~ inhomogeneous

3) $\dot{y} = \alpha - \beta\sqrt{y} \Rightarrow$

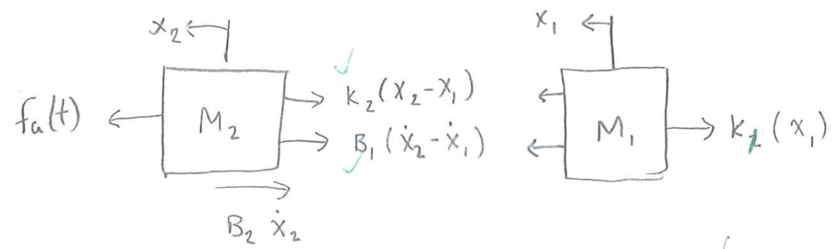
$\dot{y} + \beta\sqrt{y} = \alpha$

1st order, non-linear ($\sqrt{y}$), ~~non~~ inhomogeneous (can it be non-linear & homogeneous)

4) $\dfrac{d^4x}{dx^4} + 6xte^{-t} = \underbrace{(t^2 + 2t + 1)}_{g(t)}$

4th order, linear, ~~non~~ inhomogeneous

#3)



$f_a(t) - k_2(x_2-x_1) - B_1(\dot{x}_2-\dot{x}_1) - B_2\dot{x}_2 = M_2\ddot{x}_2$

$f_a(t) = M_2\ddot{x}_2 + B_1(\dot{x}_2-\dot{x}_1) + B_2\dot{x}_2 + k_2(x_2-x_1)$

$= M_2\ddot{x}_2 + B_1\dot{x}_2 + B_2\dot{x}_2 + k_2x_2 - B_1\dot{x}_1 - k_2x_1$

$$\boxed{f_a(t) = M_2\ddot{x}_2 + (B_1+B_2)\dot{x}_2 + k_2x_2 - B_1\dot{x}_1 - k_2x_1}$$

(−) due to reversed axes from original

$M_1\ddot{x}_1 = k_2(x_2-x_1) + B_1(\dot{x}_2-\dot{x}_1) - k_1(x_1)$

$$\boxed{M_1\ddot{x}_1 + B_1\dot{x}_1 + (k_2+k_1)x_1 - k_2x_2 - B_1\dot{x}_2 = 0}$$

(−)

David Malawey hw due 8-30-10

#2) $a_1 \frac{d^2x}{dt^2} + a_2 \frac{dx}{dt} + a_3 x = f(t)$

$f_1(t) \Rightarrow x_1(t)$

$f_2(t) \Rightarrow x_2(t)$

$\alpha f_1(t) + \beta f_2(t) \Rightarrow \alpha x_1(t) + \beta x_2(t)$

$a_1 \ddot{x} + a_2 \dot{x} + a_3 x = f(t)$

two inputs applied

$a_1(\alpha x_1(t) + \beta x_2(t))'' + a_2(\alpha x_1(t) + \beta x_2(t))' + a_3(\alpha x_1(t) + \beta x_2(t)) = f(t)$

(distribute)

$f(t) = \alpha f_1(t) + \beta f_2(t) = \alpha(a_1\ddot{x}_1 + a_2\dot{x}_1 + a_3 x_1) + \beta(a_1 x_2'' + a_2 x_2' + a_3 x_2) =$

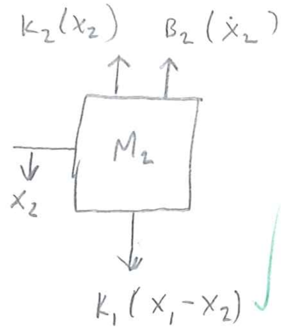
sum of individual responses

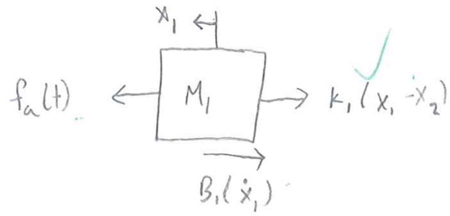
$a_1 \alpha \ddot{x}_1 t + a_1 \beta \ddot{x}_2(t) + a_2 \alpha \dot{x}_1(t) + a_2 \beta \dot{x}_2(t) + a_3 \alpha x_1(t) + a_3 \beta x_2 +$

.5

" response to several inputs applied simultaneously must be equal to the sum of responses to inputs applied individually " ← proves superposition

#4)





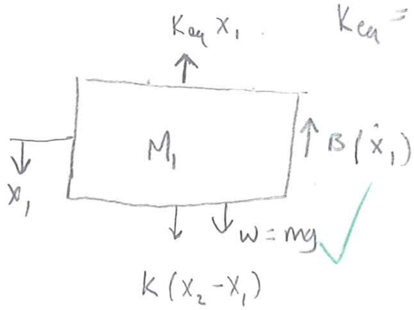
$$M_1\ddot{x}_1 = f_a(t) - B_1(\dot{x}_1) - k_1(x_1 - x_2)$$

$$\boxed{f_a(t) = M_1\ddot{x}_1 + B_1\dot{x}_1 + k_1x_1 - k_1x_2}$$

$$M_2\ddot{x}_2 = k_1(x_1 - x_2) - k_2(x_2) - B_2(\dot{x}_2)$$

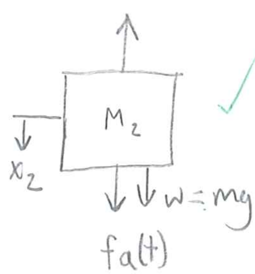
$$\boxed{M_2\ddot{x}_2 + B_2\dot{x}_2 + (k_1 + k_2)x_2 - k_1x_1 = 0}$$

$K_{eq} = 2K$



$$M_1\ddot{x}_1 = k_1(x_2 - x_1) - B_1\dot{x}_1 - 2Kx_1$$

$$\boxed{M_1\ddot{x}_1 + B\dot{x}_1 + 3Kx_1 - Kx_2 - mg = 0}$$



$$M_2\ddot{x}_2 = f_a(t) - K(x_2 - x_1)$$

$$\boxed{f_a(t) = M_2\ddot{x}_2 + Kx_2 - Kx_1 - mg}$$

$\frac{45.5}{50} + 1.5 = \frac{47}{50}$

use engineering paper

CRIB SHEETS

David Malawey

General 1st order system

$\dot{y} + \frac{y}{\tau} = f(t) \Rightarrow y(t) = \tau A + (y_0 - \tau A)e^{-t/\tau}$

- if $f(t) =$ const, $f(t)$ is your A
- find response means find $\mathcal{L}(f(t))$

Stable $\tau > 0$ Marginally stable $\tau \to \infty$

unstable $\tau < 0$

τ = time const., measures how quickly sys. reaches steady state

Definition of Laplace:

$\mathcal{L}[f(t)] = \int_0^\infty f(t)e^{-st}dt$

Transform derivatives:

$\mathcal{L}\left[\frac{d^n f(t)}{dt^n}\right] = s^n F(s) - s^{n-1}f(0) + \dots f^{n-1}(0)$

Int. by Parts

$\int_a^b u\,dv = uv|_a^b - \int_a^b v\,du$

- make e^{xt} the V term then $dv = xe^{xt}dt$

Matrix, st. var form

$Y = Cq + Du$ outputs

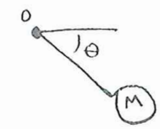
$\dot{q} = Aq + Bu$

st. vars input

$$\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\\ \dot{q}_3\end{bmatrix} = \begin{bmatrix}a_1 & a_2 & a_3\\ & & \\ & & \end{bmatrix}\begin{bmatrix}q_1\\ q_2\\ q_3\end{bmatrix} + \begin{bmatrix}b\\ b\\ b\end{bmatrix}[f_a(t)]$$

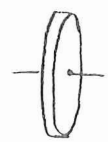
H_0 = angular momentum of particle about O

H_0 for mass at a distance



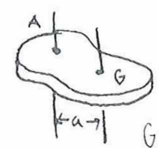
$H_0 = \vec{r} \times \vec{p} = \vec{r} \times mv = ml^2\dot{\theta}$

$M_0 = \dot{H}_0 = ml^2\ddot{\theta} = F d_{perp.} = I\alpha$



$J = \frac{MR^2}{2}$

Parallel Axis thm



$J_{AA'} = J_0 + Ma^2$

G = centroid

Pendulum, moving support

- create r vector, get $\dot{r}$ in $\hat{i}$ & $\hat{j}$

static equilibrium deflection

— $\dot{y} \to 0$ $\ddot{y} \to 0$ $y \to y_0$

Gears

$N = \frac{R_2}{R_1} = \frac{\theta_1}{\theta_2}$ $V = r\omega$ $\Sigma T = J\ddot{\theta}$

K_{eq} or B_{eq}

parallel

$= k_1 + k_2$

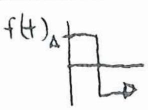
series

$= \frac{k_1 k_2}{k_1 + k_2}$

State Var form: terms w/x = terms w/out

Y_{tr}, transient response $t \to \infty$, $Y_{tr} \to 0$

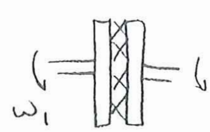
Y_{ss}, steady state, $= \lim_{t\to\infty} f(t)$



$u(t) = \begin{cases} 0 & t \le 0 \\ 1 & t > 1 \end{cases}$ $u(t-t_1) = \begin{cases} 0 & t \le t_1 \\ 1 & t > t_1 \end{cases}$

$f(t) = Au(t) - 2Au(t-t_1)$

torque



$\tau = B\Delta\omega$

levers: $\circlearrowleft + \Sigma M_0 = J_0\alpha$ small θ: $\sin\theta = \theta$, $\cos\theta = 1$

$J = \frac{ML^2}{12}$

$J = \frac{ML^2}{3}$

EXAM III 211

David Malawey

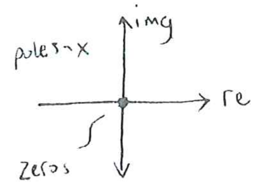
n^{th} order differential eqn $a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \dots a_1 \frac{dy}{dt} + a_0 y = b_m \frac{d^n u}{dt^n} + \dots b_1 \frac{du}{dt} + b_0 u$

then $H(s) = \dfrac{b_m s^n + \dots b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \dots a_1 s + a_0} = \dfrac{Y(s)}{U(s)} = \dfrac{Y_{zs}}{Fa(s)} = \dfrac{\text{output}}{\text{input}}$

$P(s)$ = characteristic eqn where $\mathcal{L}[u(t)] = U(s)$, $\mathcal{L}[y(t)] = Y(s)$

Dont need initial condition to find H(s). It is always taken at zero st.

stability: P(s), denominator of H(s)



1) all poles have neg. real points ⇒ stable
2) all " " " " " & 1 or more distinct poles lies on img. axis → → Marginally (critically) stable
3) at least one pole lies in R.H. plane OR has double roots on imaginary axis

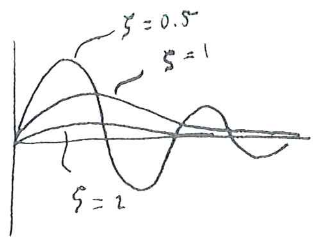
Given: $P(s) = as^2 + bs + c$ zeros of p(s) ⇒ $s = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Damping & natural frequency

standard form, 2nd order:

$\ddot{y} + 2\zeta\omega_n \dot{y} + \omega_n^2 y = f(t)$

• stable if $0 < \zeta < 1$



CLASSIFICATION

$0 < \zeta < 1$ underdamped

$\zeta > 1$ overdamped

$\zeta = 1$ critically damped

Cannot take $y(0)$ & $\dot{y}(0)$ = 0 for Y_{zi}, only for transfer function

Zero input response, Y_{zi} general form

$a_2\ddot{y} + a_1\dot{y} + a_0 y = \cancel{u(t)}\ 0$ $\mathcal{L}[\quad] = 0$

$Y_{zi} = k_1 e^{s_1 t} + k_2 e^{s_2 t}$ s_1 & s_2 are real & distinct

$Y_{zi} = k_1 e^{s_1 t} + k_2 t e^{s_2 t}$ $s_1 = s_2$

$Y_{zi} = k e^{\alpha t} \cos(\beta t + \phi)$ $s_{1,2} = \alpha \pm i\beta$

initial conditions must be given to find k, φ. need $y(0)$ & $\dot{y}(0)$. Set t=0 then $y(0) = y_{zi}$

Unit step response $u(t) = 1$, for $\dfrac{1}{s(as^2 + bs + c)}$ use $\dfrac{A}{s} + \dfrac{Bs + C}{(as^2 + bs + c)}$ Also $\dfrac{1}{(s+c)^2} = \dfrac{A}{(s+c)} + \dfrac{B}{(s+c)^2}$

$y_u(t) = \mathcal{L}^{-1}\left[\dfrac{H(s)}{s}\right]$

impulse input response $h(t) = \dfrac{d\, y_u(t)}{dt}$ OR $H(s)$ $h(t) = \mathcal{L}^{-1}(H(s))$

Given $U(s)$ find $Y(t)$

$Y(s) = H(s)\, U(s)$

steady state response $u(t) = $ const. A. $y_{ss} = A H(0)$

$u(t) = B\sin(\omega t + \phi) \Rightarrow y_{ss}(t) = y_p = BM\sin(\omega t + \phi + \theta)$

FRF, Frequency Response function: $= H(s = j\omega)$

M = magnitude of FRF groups $\sqrt{(\text{real part})^2 + (\text{img part})^2}$ ← ignore j's within this pl

$\arg = \tan^{-1}\left(\dfrac{\text{img}}{\text{real}}\right)$ $\theta(\omega) = \arg(\text{num}) - \arg(\text{denom})$

David Malawey

2nd order D.E

$$a_2\ddot{y} + a_1\dot{y} + a_0 y = b_2\ddot{u} + b_1\dot{u} + b_0 u$$

$$H(s) = \frac{b_2 s^2 + b_1 s + b_0}{a_2 s^2 + a_1 s + a_0}$$

Operating Point

$x \to \bar{x}$ $\quad u \to \bar{u}$ $\quad$ where $\dot{\bar{x}} = \ddot{\bar{x}} = 0$ always

$\bar{x}$ = x-coord of O.P.

$\bar{f}$ = y-coord of O.P.

a) replace output terms $y(t)$ or $x(t)$ by $\bar{y}$ or $\bar{x}$

b) same for input

c) for input terms, $u(t)$, all time-dependent terms are ignored

General form of 3rd order DE

$$a_3\dddot{y} + a_2\ddot{y} + a_1\dot{y} + a_0 y = b_3\dddot{u} + b_2\ddot{u} + b_1\dot{u} + b_0 u$$

$$H(s) = \frac{b_3 s^3 + b_2 s^2 + b_1 s + b_0}{a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

Procedure for linearization:

1) determine O.P.'s

2) identify nonlinear terms, linearize using Taylor S.

Taylor Series: about pt. $\bar{x}$

$$f(x) = f(\bar{x}) + \frac{df}{dx}\Big|_{\bar{x}}(x-\bar{x}) + \frac{1}{2!}\frac{d^2f}{dx^2}\Big|_{\bar{x}}(x-\bar{x})^2 + \dots$$

since $x - \bar{x} = \hat{x}$ (increment. var) $\quad \hat{x}^2 \approx 0$

$$f(x) = f(\bar{x}) + \frac{df}{dx}\Big|_{\bar{x}}\hat{x} + \cancel{\frac{1}{2!}\frac{d^2f}{dx^2}\Big|_{\bar{x}}(\hat{x})^2}^{\,0} + \text{H.O.T}$$

3) rewrite all linear terms as $x = \hat{x} + \bar{x}$

4) Solve the linear eqn

- if you get a $|x|x$, choose $x > 0$

ignore $\bar{x}$ values that are (−)

EXAM III 211

n^{th} order differential eqn $\quad a_n \frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1} y}{dt^{n-1}} + \dots a_1 \frac{dy}{dt} + a_0 = b_m \frac{d^m u}{dt^m} + \dots b_1 \frac{du}{dt} + b_0 u$

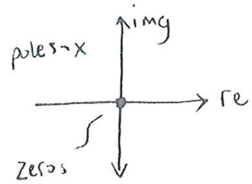
then $H(s) = \dfrac{b_m s^m + \dots b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \dots a_1 s + a_0} = \dfrac{Y(s)}{U(s)} = \dfrac{Y_{zs}}{F_a(s)} = \dfrac{\text{output}}{\text{input}}$

Dont need initial condition to find H(s). It is always taken at zero st.

$P(s)$ = characteristic eqn where $\mathcal{L}[u(t)] = U(s)$, $\quad \mathcal{L}[y(t)] = Y(s)$

Given: H(s) Find general form: Take coefficients and plug in backwards

stability: P(s), denominator of H(s)



1) all poles have neg. real points → stable
2) all " " " " & 1 or more distinct poles lies on img. axis → → Marginally (critically) stable
3) at least one pole lies in R.H. plane OR has double roots on imaginary axis

Given: $P(s) = as^2 + bs + c \quad$ zeros of $p(s) \Rightarrow s = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Damping & natural frequency

standard form, 2nd order: $\ddot{y} + 2\zeta\omega_n \dot{y} + \omega_n^2 y = f(t)$ (input)

Given $\ddot{y} + a\dot{y} + by = f(t)$ find ω_n & $\zeta \Rightarrow$ compare to std. fm.

• stable if $0 < \zeta < 1$

CLASSIFICATION

$0 < \zeta < 1$ underdamped

$\zeta > 1$ overdamped

$\zeta = 1$ critically damped

general form of zero input $\quad a\ddot{y} + b\dot{y} + cy = \cancel{u(t)}\ 0$

$\mathcal{L}[\qquad\qquad] = 0$

Cannot take $y(0)$ & $\dot{y}(0) = 0$ for Y_{zi}, only for transfer function

Y_{zi}, zero input

2) $Y_{zi} = k_1 s e^{s_1 t} + k_2 e^{s_2 t} \quad s_1$ & s_2 are real & distinct

$Y_{zi} = k_1 e^{s_1 t} + k_2 t e^{s_2 t} \quad s_1 = s_2$

$Y_{zi} = k e^{\alpha t}\cos(\beta t + \phi) \quad s_{1,2} = \alpha \pm i\beta$

initial conditions must be given to find k, ϕ. need $y(0)$ & $\dot{y}(0)$. Set $t=0$ then $y(0) = y_{zi}$

Unit step response $\quad u(t) = 1 \qquad$ for $\dfrac{1}{s(as^2+bs+c)}$ use $\dfrac{A}{s} + \dfrac{Bs+C}{(as^2+bs+c)}$

Also $\dfrac{\sim}{(s+c)^2} = \dfrac{A}{(s+c)} + \dfrac{B}{(s+c)}$

$y_u(t) = \mathcal{L}^{-1}\left[\dfrac{H(s)}{s}\right]$

~~Y_{tr} = goes to zero as $t \to \infty$~~

~~$X_{ss} = \lim_{t\to\infty} x(t)$~~

impulse input response $\quad h(t) = \dfrac{d\, y_u(t)}{dt} \quad$ OR $\quad H(s) \quad h(t) = \mathcal{L}^{-1}(H(s))$

Given $U(s)$ find $Y(t)$

$Y(s) = H(s)\,U(s)$

steady state response $\quad u(t) = $ const. A. $\quad y_{ss} = A\,H(0)$

$u(t) = B\sin(\omega t + \phi) \Rightarrow y_{ss}(t) = y_p = BM\sin(\omega t + \phi + \theta)$

FRF, Frequency Response function: $= H(s = j\omega)$

M = magnitude of FRF $\quad$ groups $\sqrt{(\text{real part})^2 + (\text{img part})^2}$ ← ignore j's within this p

$\arg = \tan^{-1}\left(\dfrac{\text{img}}{\text{real}}\right) \qquad \theta(\omega) = \arg(\text{num}) - \arg(\text{denom})$

EXAM III 211

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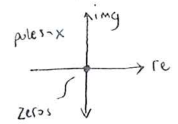

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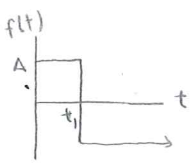


$u(t) = \begin{cases} 0 & t \le 0 \\ 1 & t > 1 \end{cases}$ $\quad u(t - t_1) = \begin{cases} 0 & t \le t_1 \\ 1 & t > t_1 \end{cases}$

$f(t) = Au(t) - 2A\,u(t - t_1)$

State Var form

$Y = Cq + Du$ outputs

$\dot{q} = Aq + Bu$ → inputs

↓

state vars

$$\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\\ \dot{q}_3\end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ & & \\ & & \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \\ q_3\end{bmatrix} + \begin{bmatrix} b \\ b \\ b\end{bmatrix}(f_a(t))$$

$\mathcal{L}(f(t)) = \int_0^\infty e^{-st} f(t)\,dt$

$y = y_{ss} + y_{tr}$ $\quad$ y_{tr} = part that → 0 as $t \to \infty$

System stabil if $y(t) = y_{ss}$ as $t \to \infty$

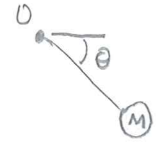
David Malawey Exam I

H_o = angular momentum of a particle about o

H_o for mass at a distance

$H_o = \vec{r} \times \vec{p} = (\vec{r} \times m\vec{v}) = ml^2\theta$

$M_o = \dot{H}_o = ml^2\dot{\theta} = F d_{perp.} = I\alpha$



Pendulum w/moving support

- create r vector, get $\ddot{r}$ in $\hat{i}$ & $\hat{j}$

Static equilibrium deflection

$\dot{y} \to 0 \quad \ddot{y} \to 0 \quad y \to y_o$

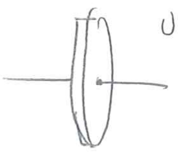
Keq or Beq

parallel

$= k_1 + k_2$

series

$= \dfrac{k_1 k_2}{k_1 + k_2}$



Uniform disk

$J = \frac{1}{2} MR^2$

State Var form

$y = Cq + Du$ outputs

$\dot{q} = Aq + Bu$ → input

↓ state vars

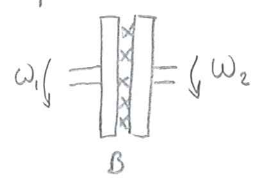
$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} a_1\ a_2\ a_3 \\ \\ \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ \end{bmatrix} + \begin{bmatrix} b \\ b \\ b \end{bmatrix} [f_a(t)]$$

state vars   inputs

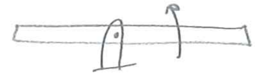
state var form

terms with x = terms w/o x or, F(t)

torque



$q = B\Delta\omega$

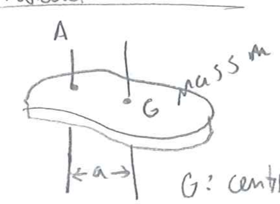


$J = \dfrac{ML^2}{12}$

ok

$J = \dfrac{ML^2}{3}$

Parallel axis thm.



$J_{AA'} = J_o + Ma^2$

G: centroid

$\perp \Rightarrow V = 0$

KCL equ: Current in each node sums to zero

KVL equ: Voltage around loop sums to zero - try to get only 1 inductor in loop

$e = L\frac{di}{dt} = Ri \quad \dot{e}_c = \frac{I_c}{c}$

$i = C\frac{di}{dt} \quad i_L = i_L(+0) + \frac{1}{L}\int_0^t e_0(\lambda)\, d\lambda$

R resistance, Ω

c capacitance,

e voltage

L inductance, Henries

State var eqns

$\dot{I}_L = \frac{e_L}{L} \quad \dot{e}_c = \frac{I_c}{c}$

- one state var eqn for each inductor & capacitor
- nodes for capacitors

Gears $\frac{R_2}{R_1} = n = \frac{\theta_1}{\theta_2} \quad V = r\omega \quad \Sigma T = J\ddot{\theta}$

$y_{tr} \to$ transient response : immediately after turned on

$y_{ss} \to$ steady state response = after y_{tr} has disappeared $t \to \infty$

$\tau \to$ time constant (seconds) measures how quickly system reaches steady state

1st order system : Stable $\tau > 0$
unstable $\tau < 0$
marginally stable $\tau \to \infty$

definition of laplace

$\mathcal{L}(f(t)) = F(s) = \int_0^\infty f(t) e^{-st}\, dt$

common transforms

$\mathcal{L}(A) = \frac{A}{s} \quad A \to$ const

$\mathcal{L}\, e^{at} = \frac{1}{s-a} \quad a \to$ real #

$\mathcal{L}(t) = \frac{1}{s^2}$

$\mathcal{L}[\sin(\omega t)] = \frac{\omega}{s^2+\omega^2}$

$\mathcal{L}[\cos(\omega t)] = \frac{s}{s^2+\omega^2}$

$\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$

transform of derivatives

$\mathcal{L}(f'(t)) = -f(0) + sF(s)$

$\mathcal{L}[\ddot{f}(t)] = s^2 F(s) - s f(0) - \dot{f}(0)$

$\mathcal{L}\left(\frac{d^n f(t)}{dt^n}\right) = s^n F(s) - s^{n-1} f(0) + \dots f^{n-1}(\ )$

Integration by parts

$\int_a^b u\,dv = uv\big|_a^b - \int_a^b v\,du \quad$ sub $\frac{-dv}{s} = e^{-st} dt$

General 1st order sys. where $\tau = \frac{a_2}{a_1}$

$a_2\dot{y} + a_1 y = a_0 g(t) \qquad \frac{a_0}{a_2} g(t) = f(t)$

$\dot{y} + \frac{a_1}{a_2} y = \frac{a_0}{a_2} g(t) \Rightarrow \dot{y} + \frac{y}{\tau} = f(t)$

$\Rightarrow y(t) = \tau A + (y_0 - \tau A) e^{-t/\tau}$

if $f(t)$ is constant, $f(t)$ is your A

final response means find $\mathcal{L}[f(t)]$

Levers

Small θ :
$\sin\theta = \theta$
$\cos\theta = 1$

$\circlearrowleft^{+} \Sigma M_0 = J_0 \alpha \qquad J_{\text{slender rod}} = \frac{1}{12}ML^2$ center axis
$\frac{1}{3}ML^2$ end