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# 213 Machine Dynamics

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ME 213— MACHINE DYNAMICS

**Assignments– FALL 2010**

**TEXT:** *Design of machinery*, by R.L. Norton, Fourth Edition, McGraw Hill, 2008.

## COURSE OUTLINE

### CORE TOPICS

CHAPTER 1. Introduction.

CHAPTER 2. Kinematics Fundamentals. 2.0–2.7, 2.11, 2.14–2.17.

4–BAR LINKAGES. 2.12, 3.0–3.4, 3.6, 4.10–4.11, supplementary materials.

TEST 1

CHAPTER 4. Position Analysis. 4.0–4.2, 4.5–4.6, 4.8–4.8, 4.13.

CHAPTER 6. Velocity Analysis. 6.0–6.1, 6.3–6.4, 6.6–6.7, 6.9.

TEST 2

CHAPTER 7. Acceleration Analysis. 7.0–7.5.

CHAPTER 10. Dynamics Fundamentals. 10.0–10.8, 10.15.

CHAPTER 11. Force Analysis. 11.0–11.5, 11.11.

### APPLICATIONS

CHAPTER 12. Balancing.

TEST 3

CHAPTER 8. Cams.

CHAPTER 9. Gears.

Final Exam 10 problems, 2 hours
all tests covered, 1 cam prob, 1 gears prob.

~~211~~ 213 Machine dynamics    182 Toomey office    Notes 8-23
341-4897

LEAD Toomey 295  7:30-9:30 Wednesday  PLA Slinkard

Homework due fridays

| | |
|---|---|
| 3 Test | 60% |
| Final | 25% |
| computer assg. | 5% |
| concept quizzes | 5% |
| participation/ H/W | 5% |

participation: lead sign-up, office hours, q's in class emails

homework: any reasonable attempt = full credit

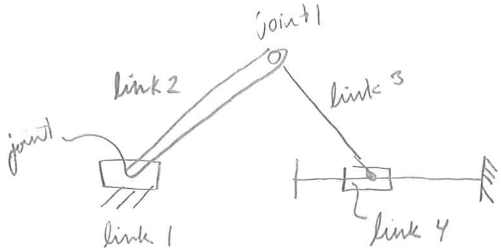
concept quiz: closed book, beginning of class

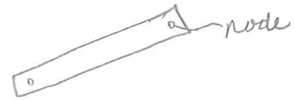
Tests  short concept questions
longer questions with calculations
open book/open notes (Final is open too)

Book: Norton, Design of Machinery

Office hours:

slider - crank mechanism



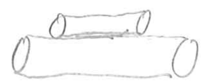




degree of freedom: # of qty's that must be specified to determine its configuration


rigid body in plane motion

lower pair - joint w/surface contact

higher pair - joint w/ point or line contact



One - freedom joint - joint w/ 1DOF relative motion (slider) R

two freedom joint - 2 DOF


(half-joint) RP

order of a joint # of bodies connected -1

Mechanism - a grounded kinematic chain

(M) MOBility = # dof

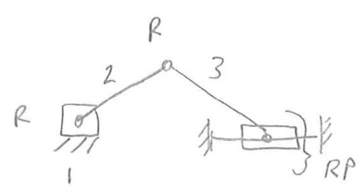
all variables can be described by angle $\theta$ $\therefore$ mobility is 1


$M = 3(L-1) - 2J_1 - J_2$

$L$ = # links

$J_1$ = # full joints

$J_2$ = # half joints




$L = 3$ $J_1 = 2$ $J_2 = 1$

Notes 8-27-10

Review

Revolute Joint [R]

Prismatic Joint [P]

Forced-closed / form closed half Joint [RP]

Toggled configuration of a mechanism: 2 adjacent links are in alignment end-to-end or overlapping

Crank: link, range of motion is full rotation

rocker: link, range of motion is partial rotation

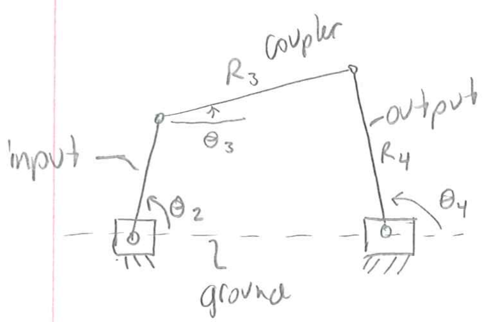
kinematic inversion: variation to mechanism by changing which link is grounded

complex motion: both rotating & translating

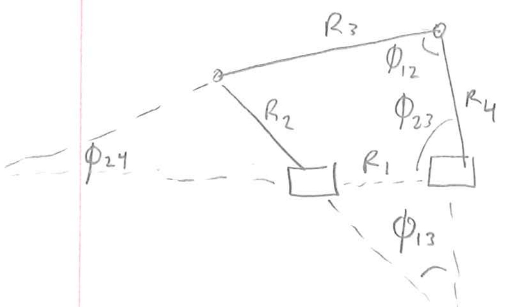
Circuit: collection of configurations, any two configs are mutually accessible without disassembly

Notes 8-30-10

4-bar mechanism




M=1




Fundamental position angles

| | Faces | Between | |
|---|---|---|---|
| $\phi_{34}$ | 3 & 4 | 1 & 2 | $= \theta_2$ |
| $\phi_{24}$ | 2 & 4 | 1 & 3 | $= \theta_3$ |
| $\phi_{23}$ | 2 & 3 | 1 & 4 | $= \pi - \theta_4$ |
| $\phi_{12}$ | 1 & 2 | 3 & 4 | $= \theta_4 - \theta_3$ |
| $\phi_{13}$ | 1 & 3 | 2 & 4 | $= \theta_2 - \theta_4$ |
| $\phi_{14}$ | 1 & 4 | 2 & 3 | $= \pi - \theta_2 + \theta_3$ |

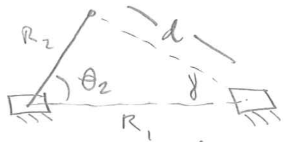
convention: $\theta_{2,3,4}$ $\quad 0 \leq \theta \leq 2\pi$

range of $\phi$'s $\quad -\pi \leq \phi \leq \pi$

Notes 8-30-10

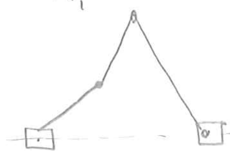
Given: lengths of R's in 4-bar linkage , $\theta_2$

1) find d

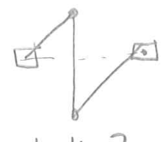


$$d^2 = R_2^2 + R_1^2 - R_1 R_2 \cos\theta_2$$

... form 1



form 2



form 1 & 2 belong to different branches

if disassembly is allowed, we can go from form 1 to 2

<u>different circuits</u> means cannot go from form 1 to 2 w/o disassembly

<u>Classify 4-bar mechanisms:</u>

s - length of shortest link

l - length of longest link $\qquad s \le s \,\&\, q \le l$

p, q. - length of others

assemblability condition: $l < s + p + q$

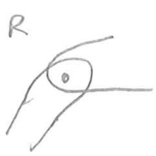
grashof condition $s + l < p + q$ ☆ in this condition, at least 1 link is a crank

(- otherwise, all links are rockers)

Notes 9-1-10

Review for Quiz today

R

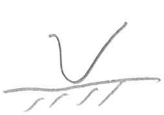


revolute

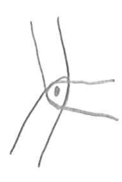
prismatic

P

helix = J



RP



double-revolute

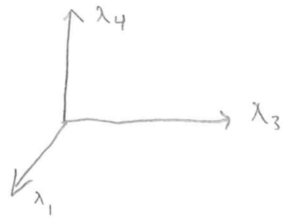
Notes 9-1-10

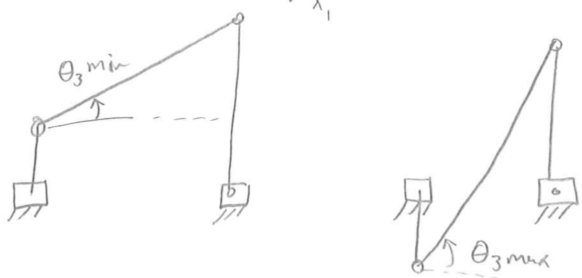
barker classification (all possible 4-bars) - Clark Barker, UMR

define: $\lambda_1 = \frac{R_1}{R_2}$    Positive octant    $\lambda_1, \lambda_2, \lambda_3 > 0$

$\lambda_3 = \frac{R_3}{R_2}$

$\lambda_4 = \frac{R_4}{R_2}$



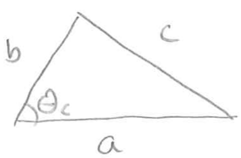

nonadjacent
if 2 links parallell,
other $\theta$ at extremum

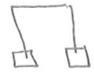
Notes 9-3-10

Thetas ($\theta$) always measured CCW from horizontal

law of cosines: memorize

$c^2 = a^2 + b^2 - 2ab\cos\theta_c$




GCCC Form 1 ⟶  Form 2 ⟶ 

notes 9-8-10

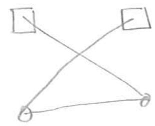
distinguish between Grashof tripple rockers

Type 1: ground link is longest

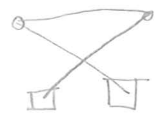
Type 2: link 2 is longest...

Notes recitation
9-8-10

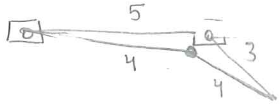
Form 1



Form 2



- most configs cannot change forms without disassembly




← this one can

Make 2 & 4 ∥, that makes $\phi_{24}$

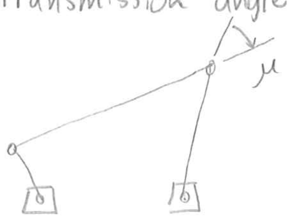
Notes 9-10-10

Open config: (Form 1) when $\theta_2$ is $(0 < \theta < 90°)$, the 2 links adjacent to the shortest link DO NOT CROSS EACHOTHER

Crossed configuration: opposite of open config

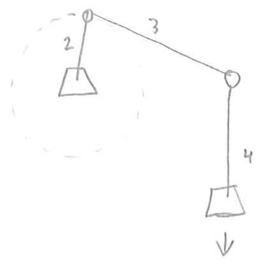
for GCCC or GCRR

Transmission angle: $\mu$, between output link & coupler

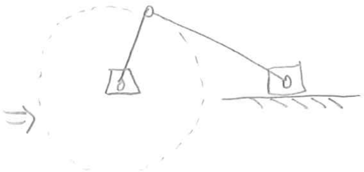



common design rule: keep $\mu \geq 40°$
"index of merit"

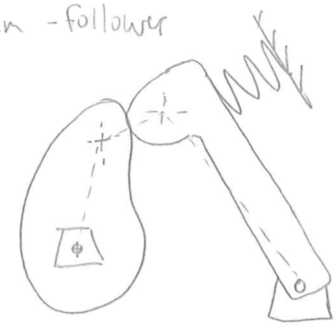
Slider-crank

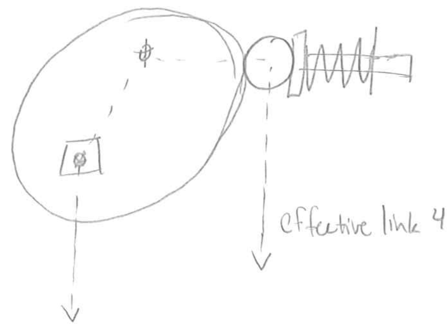



increase length of output link to $\infty$

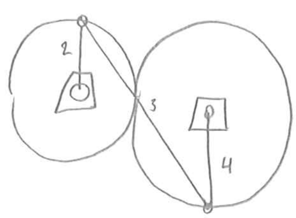


Cam-follower

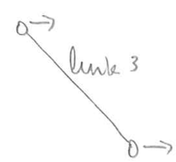





Gear train

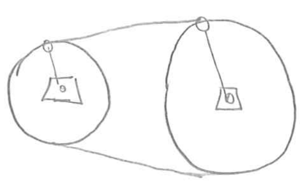



Coupler in pure translation $R_2\omega_2 = R_4\omega_4$




instantaneous~~ly~~ velocity translation only

Belt train



HW3 1) GRCR {6, 9, 3, 8} Form 1 determine limiting values of pos. angles

Notes 9-13

2) generate Barker tables for the 3 inversions of the mechanism

3) Norton prob. 3-1

- use today's lecture slides, relationship between $\phi$'s for different grounded links

analysis

mechanism → ⟶ } what does this mechanism do?

Synthesis

task ⟶ ⟶ } what mechanism does this?

design

Function generator

| Detailed math input | → * → | detailed math output |
|---|---|---|

* Func. gen. (analog computer)

Motion generation - design task requires one to control all 3 DOF of a link

Path generation - drive a particular point of a link along a specified path in plane of motion

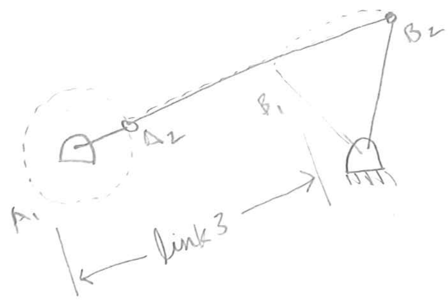
Will be a test problem over classifying "generation" of a machine

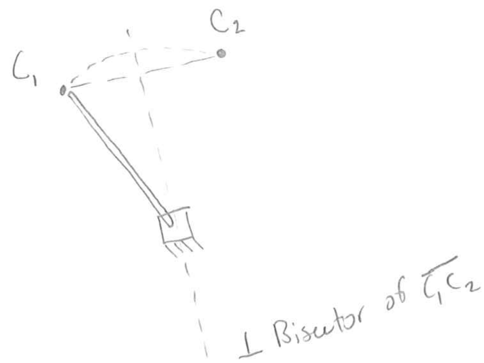
this

Graphical synthesis: (Ch. 3) pictures + trig

Analytical synthesis: (Ch. 5) equations + calculus

Ex




Notes 9-15-10

Exam 1 overview

Label joints in standard fashion & compute mobility

4-Bars: know $\theta$'s, know $\phi$'s

- classification problem, short
- long problem, give $\phi$ ranges for 3 $\phi$'s 1 O & 2 X

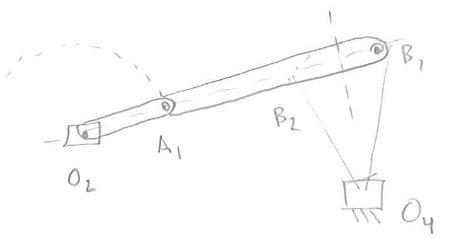
Ch. 3 synthesis

Definitions - short problem

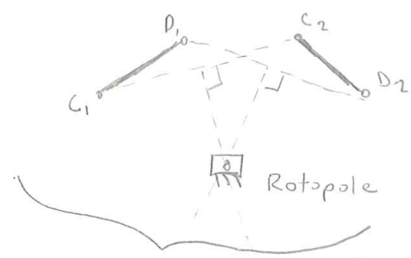
long problem, - like p #3 on homework

Notes 9-15-10

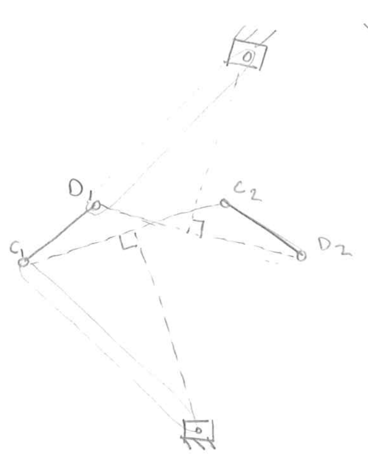
## Graphical synthesis




GCRR?

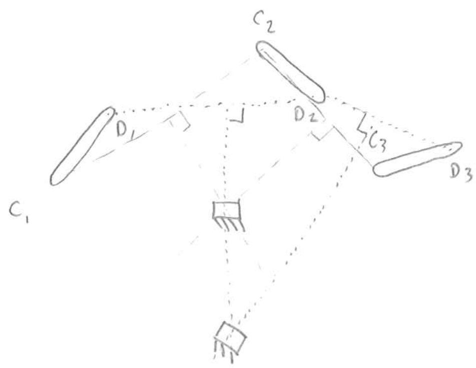



A link at the rotopole fixed to CD as shown will carry $C_1$ to $C_2$ & $D_1$ to $D_2$




Using links along the ⊥ Bisectors of lines $\overline{C_1C_2}$ & $\overline{D_1D_2}$ will give the desired motion (not including range, however)

## 3-Position problem

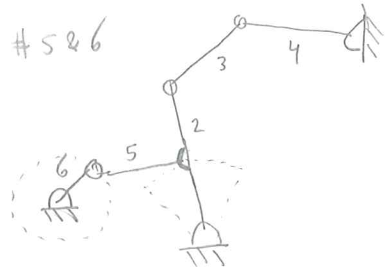


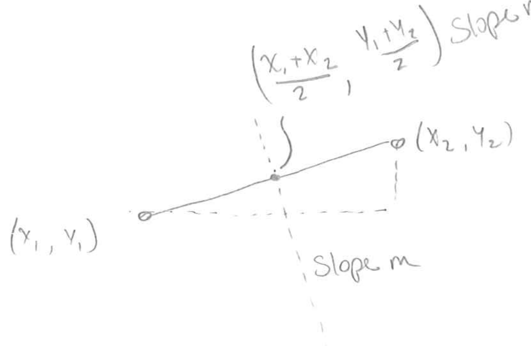

use ⊥ bisectors of both $\overline{C_1C_2}$ $\overline{C_2C_3}$ lines and intersect them.

do the same for $\overline{D_1D_2}$ $\overline{D_2D_3}$

Driving dyad: 2 extra links added to give range of motion:

driving dyad is #5&6




Perp. Slope $m = \frac{-1}{m_1}$ $\quad m_1 = \frac{y_2-y_1}{x_2-x_1}$ $\quad y - \left(\frac{y_1+y_2}{2}\right) = m\left(x - \frac{x_1+x_2}{2}\right)$

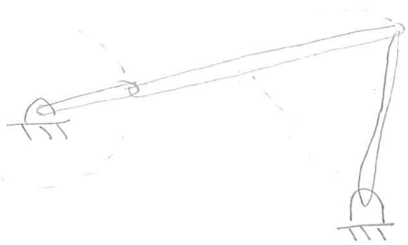
HW 1&2 ready @ 182 Toomey

Notes 9-17-10

Key to sample test posted SUNDAY

Exam: Open book/Notes Civil 125
- perp. bisector problem → required to do this algebraically
- Function vs. motion vs. path
  - start from the nature of the design tasks

- look up answers to Homework with function/path/motion
go through slides on inversion



Exam 1 tomorrow — Notes 9-20-10

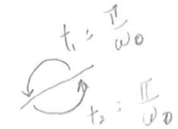
Ch. 2

Ch. 3 through example 3.4

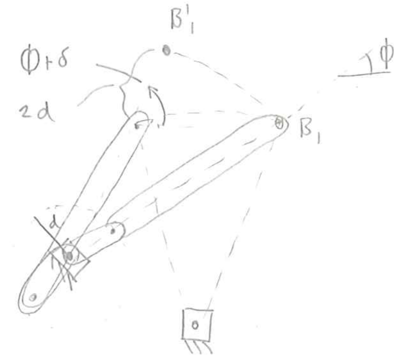
a bit of Ch. 4

## Quick-return mechanism

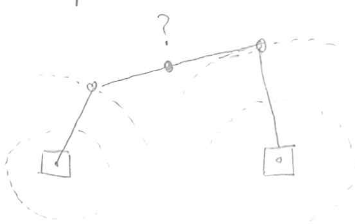
non, QR




QR


## Coupler Curves




rotating + translating

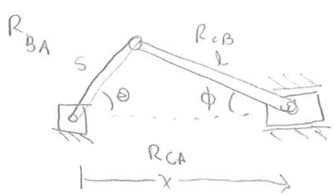
ON CD w/Book Hrones-Nelson atlas

$\boxed{\text{Position analysis}}$ ~~pe~~ complex # $Z = de^{j\theta}$ $j = \sqrt{-1}$

$Re(z) = d\cos\theta$

$Im(z) = d\sin\theta$

$R = d\sin\theta\,\hat{i} + d\sin\theta\,\hat{j}$


$\underset{\sim}{R_{BA}} + \underset{\sim}{R_{CB}} - \underset{\sim}{R_{CA}} = \underset{\sim}{0}$ $(s\cos\theta i + s\sin\theta j) + (l\cos\phi i - l\sin\phi j) - xi = 0$

$s\cos\theta + l\cos\phi - x = 0$, $s\sin\theta - l\sin\phi = 0$ $\boxed{\theta\text{ input, 1 D.O.F}}$

Notes 9-22-10

Homework 4 due Friday - prob 3-6

Vector loop equations

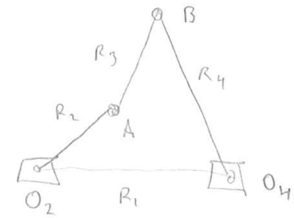
Position Analysis

- find an appropriate set of var's & identities among them to describe config of mechanism

$R_{B/A} \equiv R_{BA} = R_B - R_A$

loop:

$O_2 \to A \to B \to O_4 \to O_2$

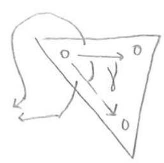



- $R_2 + R_3 - R_4 - R_1 = 0$

Variables: $\theta_2, \theta_3, \theta_4$

ternary link

two vectors part of different loops.




But, $\theta_4 - \theta_3 = \gamma =$ constant

avoid redundancy, only one variable here when link rotates

- get info on double-inversion from textbook

inverting a linkage - Figure 3-10, example 3-7 p113

loops

| | |
|---|---|
| c | v |
| v | c |
| c | c |

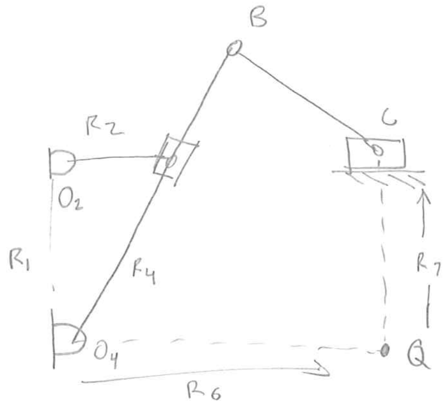
Variable count

Constraints - loop equations - completeness check - redundancy check -

- no bad loops

## Position Analysis

describe the config of a (last page)




Loop 1 $\quad O_2 \to A \to O_4 \to O_2$

$$R_2 - R_4 - R_1 = 0$$

Loop 2 $\quad O_4 \to Q \to C \to B \to A \to O_4$

$$R_6 + R_7 + R_5 - R_3 - R_4 = 0$$

| k | dR | $\theta_k$ |
|---|---|---|
| 1 | C | c |
| 2 | c | V |
| 3 | V | V |
| 4 | V | V |
| 5 | C | V |
| 6 | V | C |
| 7 | C | C |

$\theta_3 = \theta_4$

$d_3 + d_4 = \bar{d}$ (const)

$L_1$: $d_2 \cos\theta_2 - d_4 \cos\cancel{\theta_4}^{\,3} = d_1 \cos\theta_1$

$d_2 \sin\theta_2 - d_4 \sin\cancel{\theta_4}^{\,3} = d_1 \sin\theta_1$

$L_2$: $d_6 + d_5 \cos\theta_5 \underbrace{- d_3 \cos\theta_3 - d_4 \cos\theta_4}_{-\bar{d}\cos\theta_3} = 0$

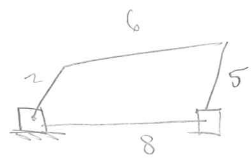
$d_7 + d_5 \sin\theta_5 \underbrace{- d_3 \sin\theta_3 - d_4 \sin\theta_4}_{-\bar{d}\sin\theta_3} = 0$

Knowns: $d_2, d_4, d_5, d_1, \bar{d}, \theta_1, \theta_2$

unknowns: $\theta_3, \theta_5, d_6, d_4$

Solve Non-linear eqn $f(x) = 0$ Numerically

Newton Algorithm $\boxed{\underbrace{x_{k+1}}_{\text{new guess}} = \underbrace{x_k}_{\text{old guess}} - \frac{f(x_k)}{f'(x_k)}}$




$$\begin{bmatrix} f(\theta_3, \theta_4) \\ f(\theta_3, \theta_4) \end{bmatrix} = \begin{bmatrix} 6\cos\theta_3 - 5\cos\theta_4 + 2\cos\overset{\text{input}}{\theta_2} - 8 \\ 6\sin\theta_3 - 5\sin\theta_4 + 2\sin\theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

unknowns: $\theta_3, \theta_4$

Matrices $[A] = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\Delta = ad - bc \neq 0$ determinant$([A])$

$$\underbrace{\frac{1}{\Delta}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}_{\text{inverse}} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Newton Raphson method → comp does all inversion for us
- Force analysis → use matrix formulation to organize things

$$[J] \text{ (using above) } = \begin{bmatrix} -6\sin\theta_3 & 5\sin\theta_4 \\ 6\cos\theta_3 & -5\cos\theta_4 \end{bmatrix}$$

$\det[J] = \Delta = \ldots -30 \sin\phi_{12}$

Notes 9-29-10

Newton Raphson

$d_4 \cos \theta_4$ ← non-linear terms

$\underbrace{\text{MATLAB}}$
"matrix"

commands

floor (a) = largest integer no larger than a

size(q) =

H = inv(Q) = gives inverse matrix

atan(Y/X) ≈ $\tan^{-1}\left(\frac{Y}{X}\right)$ → only gives $-\frac{\pi}{2}$ < angles < $\frac{\pi}{2}$

atan2(Y,X) ≈ $\tan^{-1}\left(\frac{Y}{X}\right)$ → distinguishes

Jacobian is singular when coupler & output link are in toggle

for periodic functions $(\theta_3, \theta_4) = (\hat{\theta}_3 + 2m\pi, \hat{\theta}_4 + 2n\pi)$ if $(\theta_3, \theta_4) = (\hat{\theta}_3, \hat{\theta}_4)$

Notes ~~9-3~~ 10-1-10

- gone Mon & Wed

HW 6 Posted today, LEAD Wed. night 7:30-9:30

More about MATLAB • google "matlab primer" books "Mastering MATLAB

'Rigamarole'

Notes 10-1-10

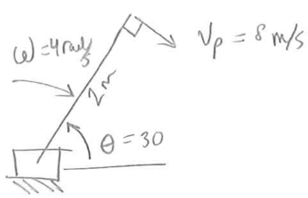
## MATLAB cont.

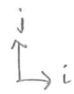
Multiple inputs

- storage for outputs
- for statement outside the existing code

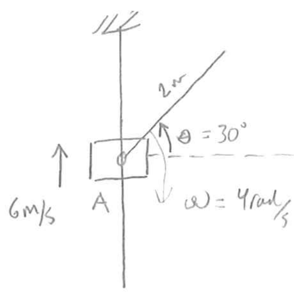
outputs become matrix entries

## Velocity anal.



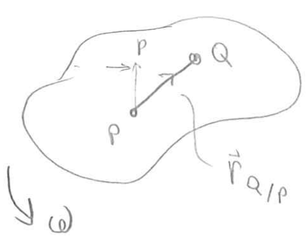




$V_p = 4\hat{i} - 4\sqrt{3}\,\hat{j}$




$V_{P/A} = 4\hat{i} - 4\sqrt{3}\,\hat{j}$

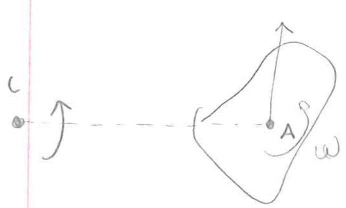
$V_p = V_{P/A} + V_A = 4\hat{i} + 6 - 4\sqrt{3}\,\hat{j}$




$V_Q = V_P + \vec{\omega} \times \vec{r}_{Q/P}$

Trans rotational

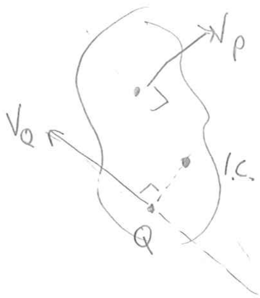
Instantaneous center of zero velocity: $V_A = \omega \times r_{A/C}$




if point B has zero velocity, B = IC

IC is only good for velocity analysis in plane motion

Notes 10-8-2010




MATLAB ①: case sensitive

② complex arguments in atan2( , )

③ legal names for M-files

lowercaseletters.m

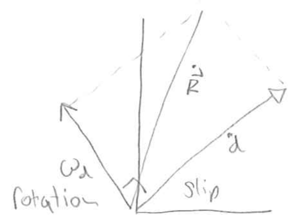

$e^{j\theta} = \cos\theta + j\sin\theta \qquad \underset{\sim}{u}(\theta) = \cos\hat{i} + \sin\theta\hat{j}$

$je^{j\theta} = -\sin\theta + j\cos\theta \quad -\sin\theta\hat{i} + \cos\theta\hat{j} = \hat{k}\times\vec{u}(\theta)$

$\vec{u}(\theta(t)) \qquad \frac{d}{dt}\{u(\theta(t))\} = \frac{d}{d\theta}\{u(\theta(t))\} = \frac{d}{d\theta}u(\theta)\dot{\theta} = \dot{\theta}(-\sin\theta i + \cos\theta j)$

$\frac{d}{dt}e^{j\theta} = \theta j e^{j\theta} \qquad \omega = \dot{\theta}\hat{k} \quad \frac{d}{dt}$

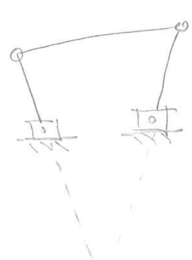
$\vec{R} \sim d(t)e^{j\theta(t)} = d\cos i + d\sin\theta\hat{j} \qquad \dot{R} \cong \dot{d}e^{j\theta} + d\dot{\theta}je^{j\theta}$




$$\omega_3 = \frac{a\sin(\theta_4 - \theta_2)\,\omega_2}{b\sin(\theta_3 - \theta_4)}\omega_2$$

$$\omega_4 = \frac{a\sin(\theta_2 - \theta_3)}{c\sin(\theta_4 - \theta_3)}\omega_2$$

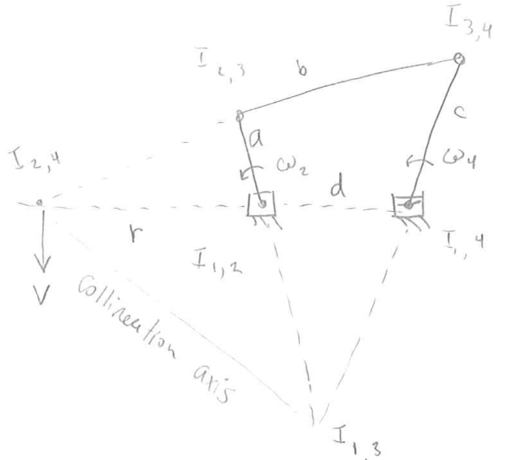
2 bodies, instant center – point where velocities of each body agrees



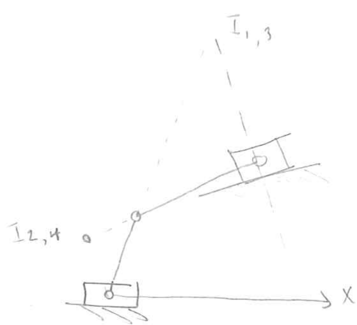
$I_{1,3}$ because $V_{link\,1} = 0$

Notes 10-11-10

instant centers continued




$V = r\omega_2 = (d+r)\omega_4$




IC of slider + link sliding on is @ ∞ along ⊥ to plane of slip

Pwr identity $\boxed{F_{in} r_{in} \omega_{in} = F_{out} r_{out} \omega_{out}}$ Mech. Adv $= \boxed{\frac{F_{out}}{F_{in}} = \frac{V_{in}}{V_{out}}}$


$F_{in} R_{in} = F_{out} R_{out}$ $\quad F_{out} = F_{in}\left(\frac{R_{in}}{R_{out}}\right)$

10-13-10

$\boxed{F_{in} V_{in} \cos\phi_{in} = F_{out} V_{out} \cos\phi_{out}}$ power identity

goal: rewrite in terms of mechanism geometry

$$\frac{F_{out}}{F_{in}} = \frac{V_{out}\cos\phi_{out}}{V_{in}\cos\phi_{in}}$$

$$M.A. = \frac{r_{in}\omega_2}{r_{out}\omega_4} = \frac{\supset R\sin\phi_{12}}{\supset R\sin\phi}$$

Notes 10-18-10

Test 2 Tues Oct 26 Venue TBA Sample test tomorrow

Topics: Position analysis (a)

Velocity analysis (b)

instant centers (c)

mechanical advantage (d)

elementary acelleration problem

(a) vector loop equations & constraints,

| | c | v |
|---|---|---|
| d | | |
| $\theta$ | | |

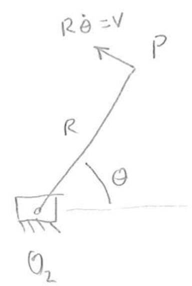
\# dof

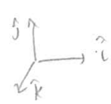
(b) given config, find $\omega$'s $V_{slip}$ "$\vec{\omega} \times \vec{R}$" "$j\omega e^{j\theta}$"

(c) "..will only ask for 2 or 3 instant centers," know Kennedy's rule

(d) $M.A. = \frac{F_{out}}{F_{in}} = \frac{\sim V_{in}}{\sim V_{out}}$ using Instant Ctrs $M.A. \sim \frac{\sim l_{in}}{\sim l_{out}}$ ✓

---




$\bar{\omega} = \dot{\theta}\hat{k} = \omega\hat{k}$ (ang. velocity)

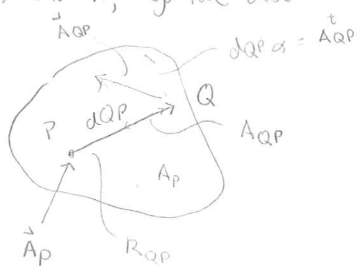
$\vec{\alpha} = \ddot{\theta}\hat{k} = \dot{\omega}\hat{k} = \alpha\hat{k}$ (ang. accel)

$\vec{R}_{PO_2} = R(\cos\theta\hat{i} + \sin\theta\hat{j}) \sim Re^{j\theta}$

$V_{PO_2} = \frac{d}{dt}\{R_{PO_2}\} = \vec{\omega} \times R_{PO_2} \sim j\omega e^{j\theta}$

$A_{PO_2} = \frac{d}{dt}\{V_{PO_2}\} = \alpha \times R_{PO_2} + \vec{\omega} \times (\vec{\omega} \times \vec{R}) \sim j\alpha Re^{j\theta} - \omega^2 Re^{j\theta}$

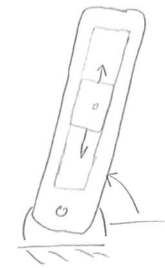
Complex motion, general case




$A^t$ = tangential accel

Notes 10-18-10

Slider on rotor



$\vec{R} \sim \underset{\llcorner V}{d}e^{j\theta}$

$\vec{V} \sim \underbrace{\dot{d}e^{j\theta}}_{V\,slip} + d\dot{\theta}je^{j\theta}$

$\vec{A} \sim \ddot{d}e^{j\theta} + 2\dot{d}\omega je^{j\theta} + d\alpha je^{j\theta} - d\omega^2 e^{j\theta}$

$A_p = \underbrace{A_{p\,slip}}_{slip} + \underbrace{\alpha_2 \times R_{PO_2} - \omega_2^2 R_{PO_2}}_{rotation} + \underbrace{2\omega_2 \times V_{p\,slip}}_{coriolis}$

General mechanism analysis

Position An. $\left(\frac{d}{dt}\right)$

$\Rightarrow$ Velocity analysis $\left(\frac{d}{dt}\right)$

$\Rightarrow$ accel analysis $\left(\frac{d}{dt}\right)$

$\Rightarrow$ jerk analysis

— Test 2 — Notes 10-20-2010

Tues 10/26

7:00 - 9:30 PM 125 Civil Eng. sample test posted

LEAD tonight

$MA = \frac{F_{out}}{F_{in}} = \sim \frac{\omega_2}{\omega_4} \qquad \frac{\omega_2}{\omega_4} = \frac{R_4 \sin(\theta_4 - \theta_5)}{R_2 \sin(\theta_2 - \theta_3)}$

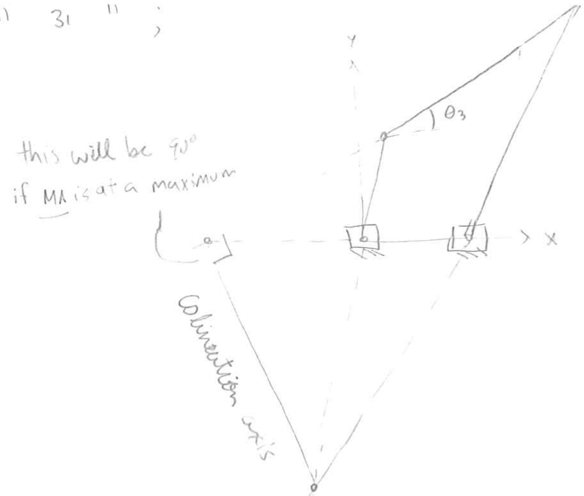
4-bar code:

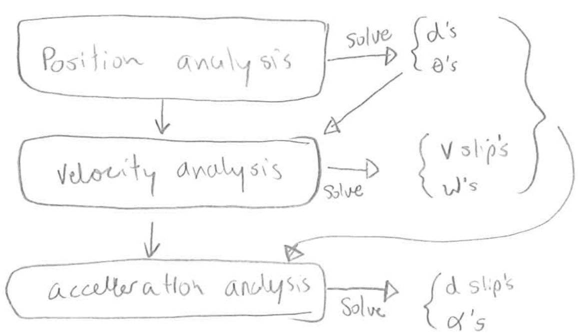
```
TH2 = 0:.01:2*pi;

Newt RAPH

TH31 =        ;
TH41 =        ;
TH32 =        ;
TH42 =        ;
```

```
fprintf (  )

n = max(size(TH2));
for k = 1:n;
        <- Direct method

end
M1 = zeros(size(TH2));
M2 = zeros(size(  ));
for k = 1:n;
```

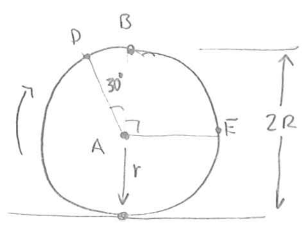
continued | #2 HW 8

```
D11 = R4 * Sin( TH41(k) - TH31(k)) ;
D12   " 2 *  "    2  "  "  31  "  ;
M1(k) = D11/D12 ;
D21 =  —
D22 =  —
M2(k) = — ;
end

TH2 = 0 : .01 : 2 * pi ;
      0 : .01 : TOGGLE 1-.1 ;
>> plot (TH2, M1) ;
>> M1        M1(19)
             TH2(19)
             TH31(19)
             TH41(19)
```




Pure rolling



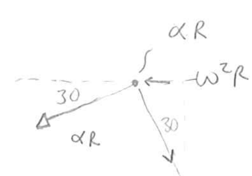

$V_c = 0$ $C$ is $I_{1,2}$ $A_c = (\omega^2 R)\hat{j}$

$V_A = -\omega R \hat{i}$

$A_A = -\alpha R \hat{i}$

$A_B = -2(\alpha R)i - (\omega^2 R)\hat{j}$

$$\boxed{\text{General: } A_Q = A_A + \alpha \times R_{QA} - \omega^2 R_{QA}}$$



10-22-10

office hours

Mon 2-4:30

Tues 1:30-4

Vector loops - will have more than 4 links, several possible loops, are yours correct?

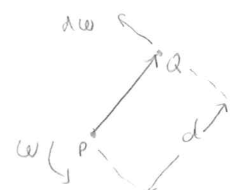
vector loop equations

constraints

velocity $\vec{V}_{QP} = V_Q - V_P$ - "relative velocity" if Q, P belong to different links

- "velocity difference" if belong to same link

$V_{QP} = V_Q - V_P = \vec{\omega} \times R_{QP}$

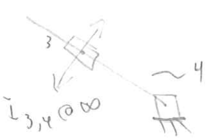



Instant centers

Vel $I_{2,3}$ considered as a pt. of Extended Body 2 = Vel of $I_{23}$ ...

- Kennedy: $I_{23}$ $I_{25}$ $I_{35}$ live on same line

- use them to get a velocity relationship ($\omega$) between nonconnected links




Mechanical advantage $P_{in} = P_{out}$ $\vec{F} \cdot \vec{V}_{in} = \vec{F}_{out} \cdot \vec{V}_{out}$

$M.A = \frac{V_{in}}{V_{out}}$

Notes 10-22

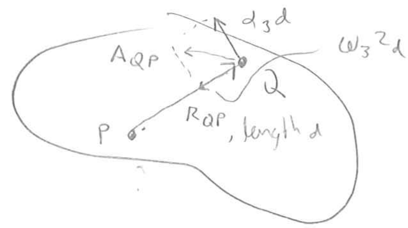
$A_{QP} = A_Q - A_P$

- Q,P on different links - relative accel
- on same link - Acceleration difference

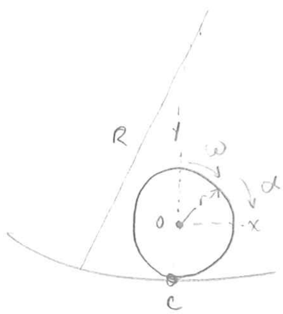
review

$$\boxed{A_{QP} = \vec{\alpha}_3 \times \vec{R}_{QP} - \vec{\omega}_3^{\,2} \vec{R}_{QP}}$$

$\vec{\alpha}_3 = \alpha_3 \hat{k}$




rolling motion continued:




$V_0 = r\omega u_t = r\omega\hat{i}$

$A_0 = (r\alpha)u_t + \left(\frac{r\omega^2}{\rho}\right)u_n = r\alpha\hat{i} + \frac{(r\omega)^2}{\ldots}$

finding accel of a link

Study notes

accel difference: find $\vec{R}_{QP}$

Formulas used on test 2:

complex forms, going from vector loops to relations of ω's & x

1) - take derivatives to get in terms of variables asked for
   - constants drop out

2) - use $\omega_3$ or $\dot{\theta}_3$ according to terms asked for
   - $(\cos\theta - \sin\theta) = je^{j\theta}$
   - Separate eq's into sin / cos eq's

example $\frac{d}{dt}\left[be^{j\theta_2} + ce^{j\theta_3} - x = 0\right]$ $\dot{\theta}_2 = -\omega_2$ asking for $\omega_3$ terms

$\Rightarrow j\dot{\theta}_2 b e^{j\theta_2} + j\omega_3 c e^{j\theta_3} - \dot{x} = 0$

complex $\Downarrow$ form

$\dot{\theta}_2 b(\cos\theta_2 - \sin\theta_2)$

Kennedy's rule: any 3 bodies in plane motion will have 3 instant centers lying on the same line

$C = \frac{n(n-1)}{2}$ where n is # links

Notes 10-27-2010

ACCell Homework due friday

LEAD tonight 7:30-9:30 Toomey 295

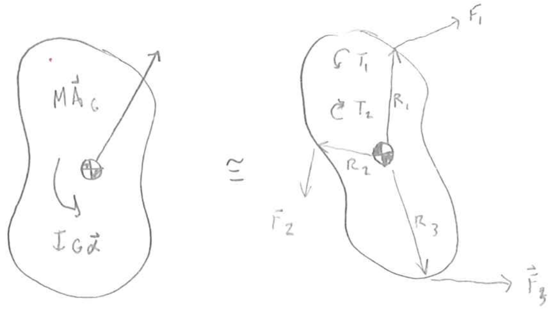
#2 $\vec{v}_{slip} = v_{slip}\, u_t$

<u>Velocity analysis</u> $\vec{V}_{slip},\ \vec{\omega}_4$

$$\vec{A}_{slip} = \dot{v}_{slip}\,\vec{u}_t + \frac{v_{slip}^2}{\rho}\,u_n$$

$$\vec{A}_{cor} = 2\omega_4 \times \vec{v}_{slip}$$

$\hookrightarrow\ \dot{v}_{slip},\ \alpha_4$

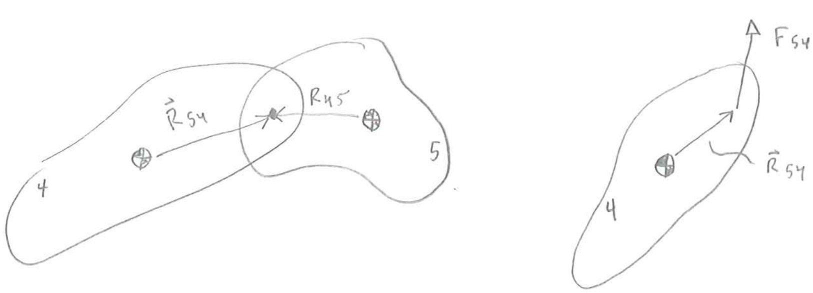



$$M\vec{A}_G = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$I_G\vec{\alpha} = T_1 - T_2 + R_1\times F_1 + R_2\times F_2 + R_3\times F_3$$

$\uparrow$ 2 component equations from these

MULTI BODY




AND

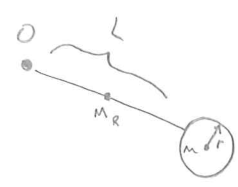
$$\vec{F}_{45} = -F_{54}$$

$$M\,\vec{a}_{G4} = \ldots\ldots + \vec{F}_{54}\ldots$$

$$I_G\,\vec{\alpha}_4 = \ldots\ldots\ \vec{R}_{54}\times\vec{F}_{54}$$

Reading assignment
Ch 10

$\boxed{I_G = \int (x^2+y^2)\,dm}$ $\boxed{I_o = I_G + md^2}$

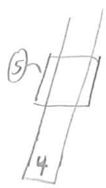



$$I_{o,rod} = \frac{1}{3}M_R l^2 \qquad I_{o,disc} = \frac{1}{2}M_d R^2 + M_d(L+R)^2$$

General irregularly shaped part ?  Notes 10-29-10

option ① locate G, find $I_G$ experimentally

② computer model of part estimate G, $\int (x^2+y^2)\,dm$ numerically




sliding contact

① smooth $F_{54_t} = 0$

② dry friction $F_{54_t} = \pm \mu F_{54_N}$

③ lubricant $F_{54t} = -\beta\, v_{slip}$

Known 4-bar lengths, inertias, CG's

Synthesize a fourbar

inputs $\theta_2, \omega_2, \alpha_2$ ↓

$\boxed{\theta_3, \theta_4, \omega_3, \omega_4 \quad \alpha_3, \alpha_4} \rightarrow \boxed{\vec{a}_{G_2}, \vec{a}_{G_3}, \vec{a}_{G_4}}$

Bottom line

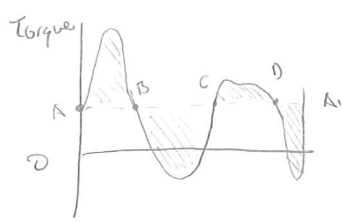
$\boxed{\text{kinetic eq's}}$

$\boxed{\vec{F}_{12}, \vec{F}_{32}, \vec{F}_{43}, \vec{F}_{14}, T_{12}}$

<u>Flywheels</u>: store kinetic energy thereby reducing  Notes 11-1-10

fluctuations in $\vec{\omega}, \vec{\tau}$.

<u>Goal</u>: - set motor to deliver $T_M = T_{avg}$ where $T_{avg}$ is average required fourbar driving torque $T_L$ = load,




Pos. lobe - flywheel gives energy to crank

Neg. lobe - flywheel takes energy from crank

Pt C = $\omega_{max}$ (flywheel)

Pt. B = $\omega_{min}$ (flywheel)

Homework 10 posted

Lead 7:30 - 9:30 problem 11-9

computer assignment to feature same 4-bar

Test 2 Avg - 90.1%

Flywheel - massive rotor used as K.E. reservoir

Print off Table 11-1, integrating the torque function

approximation will be used $\omega_{avg} = \frac{1}{2}(\omega_{max} + \omega_{min}) = \overline{\omega}$ ← known

$$\boxed{k = \frac{\omega_{max} - \omega_{min}}{\omega_{avg}}}$$ coef. of fluctuation

$$\frac{1}{2} I (\omega^2_{max} - \omega^2_{min}) = \Delta E$$ ← estimable

$$I k \omega^2_{avg} = \Delta E$$

$$\boxed{I = \frac{\Delta E}{k\, \omega^2_{avg}}}$$

prescribe

Flywheel size estimate

Note: $\overline{\omega}$ = desired average for machine - known

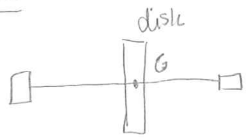
$\overline{\omega}_{avg}$ = integral average, estimated only

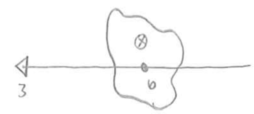
Print out slide on torque curve for engine

11-3-10

~~Machine~~ Mech. balancing – Ch 12

Example




$\vec{\alpha}_G = \vec{0}$

$\vec{\omega} \times H_G = 0$

$H_0 = -I^{0}_{xz}\omega\hat{i} - I^{0}_{yz}\omega\hat{j} + I^{0}_{zz}\omega\hat{k}$

$\omega \times H_0 = I^{0}_{yz}\omega^2\hat{i} - I^{0}_{xz}\omega^2\hat{j}$

$\vec{\omega} = \omega\hat{k}$

$\vec{v}_G = \omega \times R_{GO}$ $\qquad \vec{\alpha}_G = \vec{\alpha} \times R_{G/O} + \vec{\omega} \times (\vec{\omega} \times \vec{R}_{G/O})$

worry: big bearing forces

sol$^n$: mechanical balancing

IMBalance issues:

i) if CG is displaced from axis of rotation a distance r $\qquad a_{Gn} = r\omega^2$

→ bearing forces $O(\omega^2)$

ii) if axis of rotation is a principal axis of inertia

→ bearing forces $O(\omega^2)$

• STATIC BALANCING

- add counterweight to move CG to axis of rotation

$m_1\vec{R}_1 + m_2\vec{R}_2 + \underline{m_b\vec{R}_b} = \vec{0}$

$m_b R_{bx} = -m_1 R_{1x} - m_2 R_{2x}$ $\qquad m_b R_{by} = -m_1 R_{1y} - m_2 R_{2y}$

$m_b^2 R_b^2 = (m_1 R_{1x} + m_2 R_2)^2 + (m_1 R_{1y} + m_2 R_{2y})^2 = K^2$ $\qquad m_b R_b = K$

$\left\{ \dfrac{R_{by}}{R_{bx}} = \dfrac{m_1 R_{1y} + m_2 R_{2y}}{m_1 R_{1x} + m_2 R_{2x}} \right.$

Notes 11-8-10

good goal : have code to compute $\theta$'s $\omega$'s & $\alpha$'s completed

Exam tues 11/30 7-9:30pm (accel, kinetics, flywheels, balancing)
force analysis

>> TH41(1) -TH31(1)

–

11-15-10

Ch. 8 - Cams

Path generators – trajectory of a pt.

Motion generators – translation & rotation (back-hoe bucket)

Function generators –

kinematical input → $\boxed{\text{Math relationship}}$ → kinematical output

motion constraints

- CEP critical Extreme Position   initial config. → fixed config
  however we please

- Critical path motion CPM
  - follower motion is completely specified

6ce

Jerk

$$\vec{J} = \frac{dA}{dt} = \frac{1}{M}\frac{d}{dt}(\Sigma \vec{F})$$

discontinuous accel corresponds to $\infty$ Jerk, very rapidly varying forces

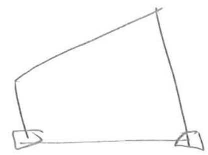
→ collision-like or percussive behavior → AVOID

require follower accel to be continuous


$\theta = w_0(t)$ $\quad V = w_0 v$ $\quad A = w_0^2 a$ $\quad J = w_0^3 j$

MATLAB ASSIGNMENT

Create q(1), q(2)



$\cancel{R}\ r_x = l \cos(\theta_3 - 31^\circ)$ $\quad l = 3.06$

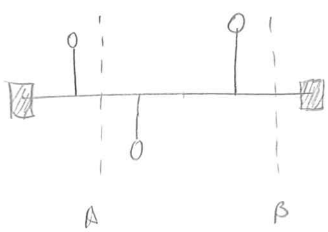
$31 \frac{\pi}{180}$

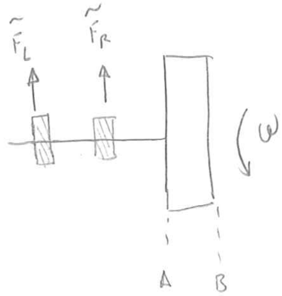
IG2 RPX AG2 X,Y ✓
IG3 RPY AG3
IG4 AG4

M2,3,4, AL2

Test 3

Tues 11/30 Civil 125



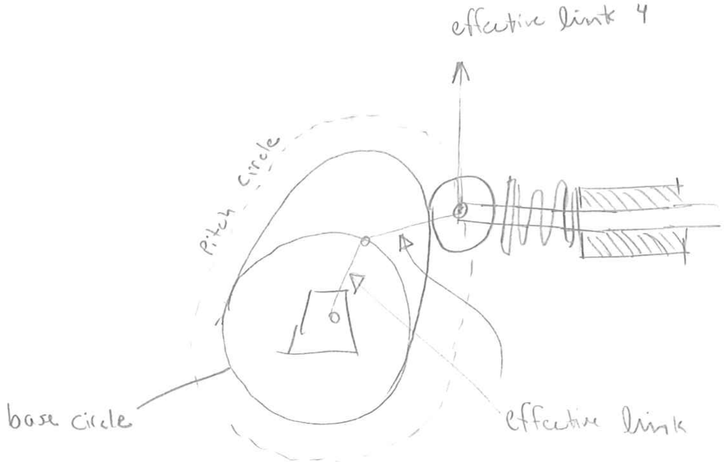

independent of $\omega$ depends on $\omega$

① accel
② force anal.

$\}$ complicated system, significant work

CAMS

11-19-10




undercutting: happens if tool that cuts cam has too great of a radius

⟶ creates spike in cam.

read section 8.7 – some part of this section will be on final exam

# Equations for exam 3

$A_{a/p} = \alpha \times R_{ap} - \omega^2 R_{ap}$     nomenclature: $R_{ap}$ = Vector from CG of link p to joint between a & p

$\tau_{CG} = R_{P/CG} \times F_P$

Flywheels: $\Delta E$ = max energy - min energy     max energy = max positive net area during cycle.

$I_{flywheel} = \frac{\Delta E}{k \omega^2_{avg}}$     k given, $\omega_{avg}$ given

$M_B l_B R_B$ should read $M_A l_B R_A$ - print out review after 221

Units

$\frac{kg \cdot m^2 \, rad}{s^2} = N \cdot m$     $\frac{kg \cdot m}{s^2}$ = Newton     $\frac{N s^2}{m} = kg$

Moment of inertia, $I = 103.89 \, kg \cdot m^2$

dynamic balancing: $F_a = m_a \omega^2 R_{a/k\,axis}$     eg,

after $\Sigma F$, check angle (- or +) by finding mass : $m_a \cos\theta_a = -2.049$

mass should come out positive, else add 180° to answer

Exams $\begin{cases} 91 \\ 100 \\ 85 \end{cases}$ carries 60%

$.920 \times .60 = .552$

computer assignment 100%
concept quizzes 100%
participation/HW 100%

carries 15% ⇒ 0.15

total .702

need .9

on Final I need .198 left

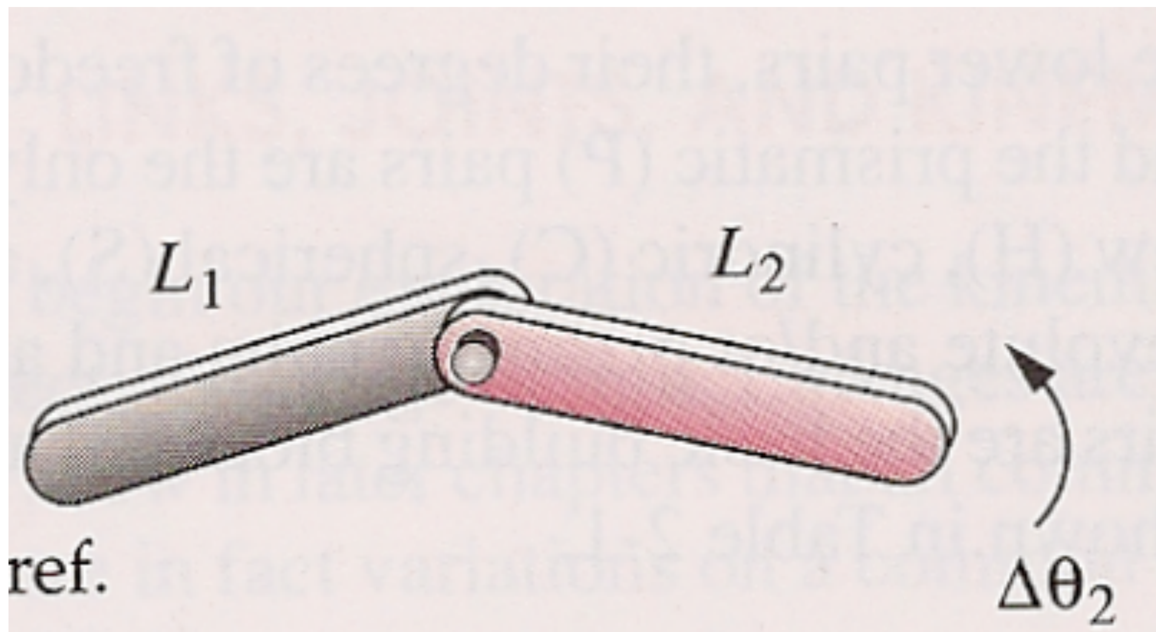
$.198 / .250 = .792$

For an A

need 79.2% on final exam

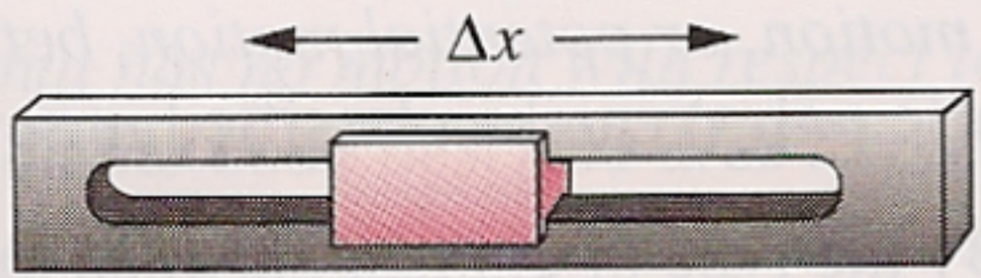
# REVIEW LECTURE 1: KINEMATICS

# REVOLUTE JOINT



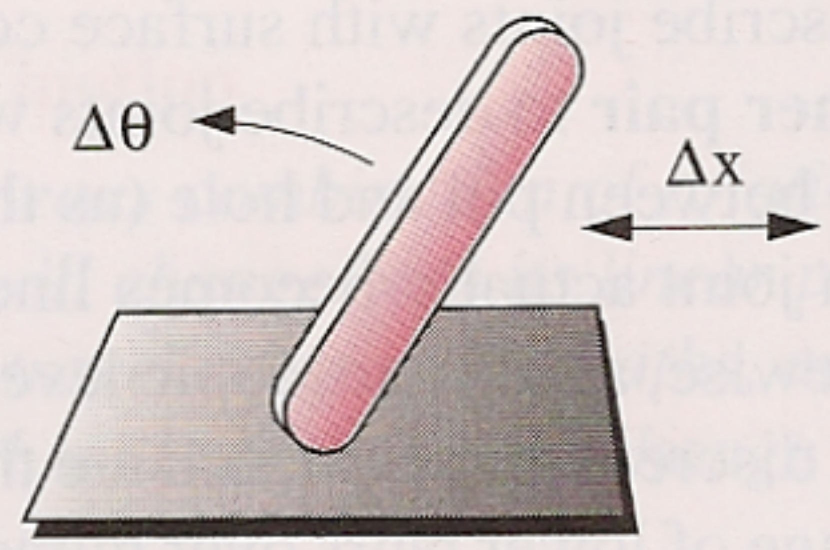

First order pin joint - one *DOF*
(two links joined)

# PRISMATIC JOINT



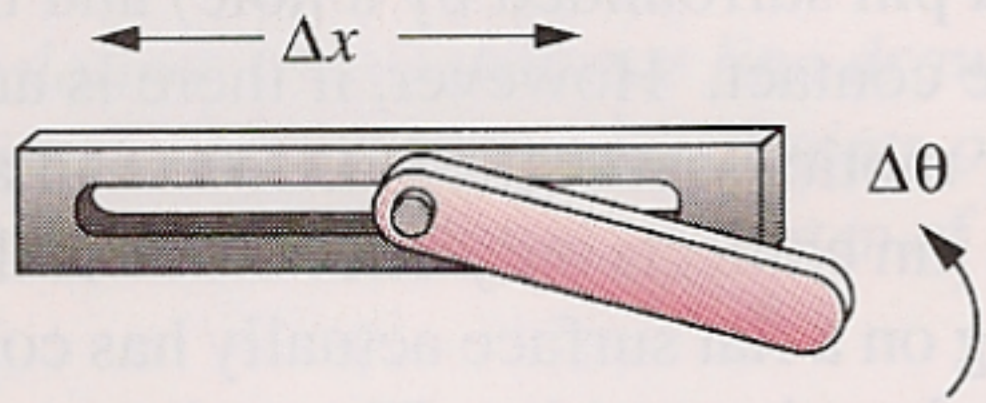

Translating full slider (P) joint (form closed)

# FORCE-CLOSED HALF JOINT




Link against plane (force closed)

# FORM-CLOSED HALF JOINT




Pin in slot (form closed)

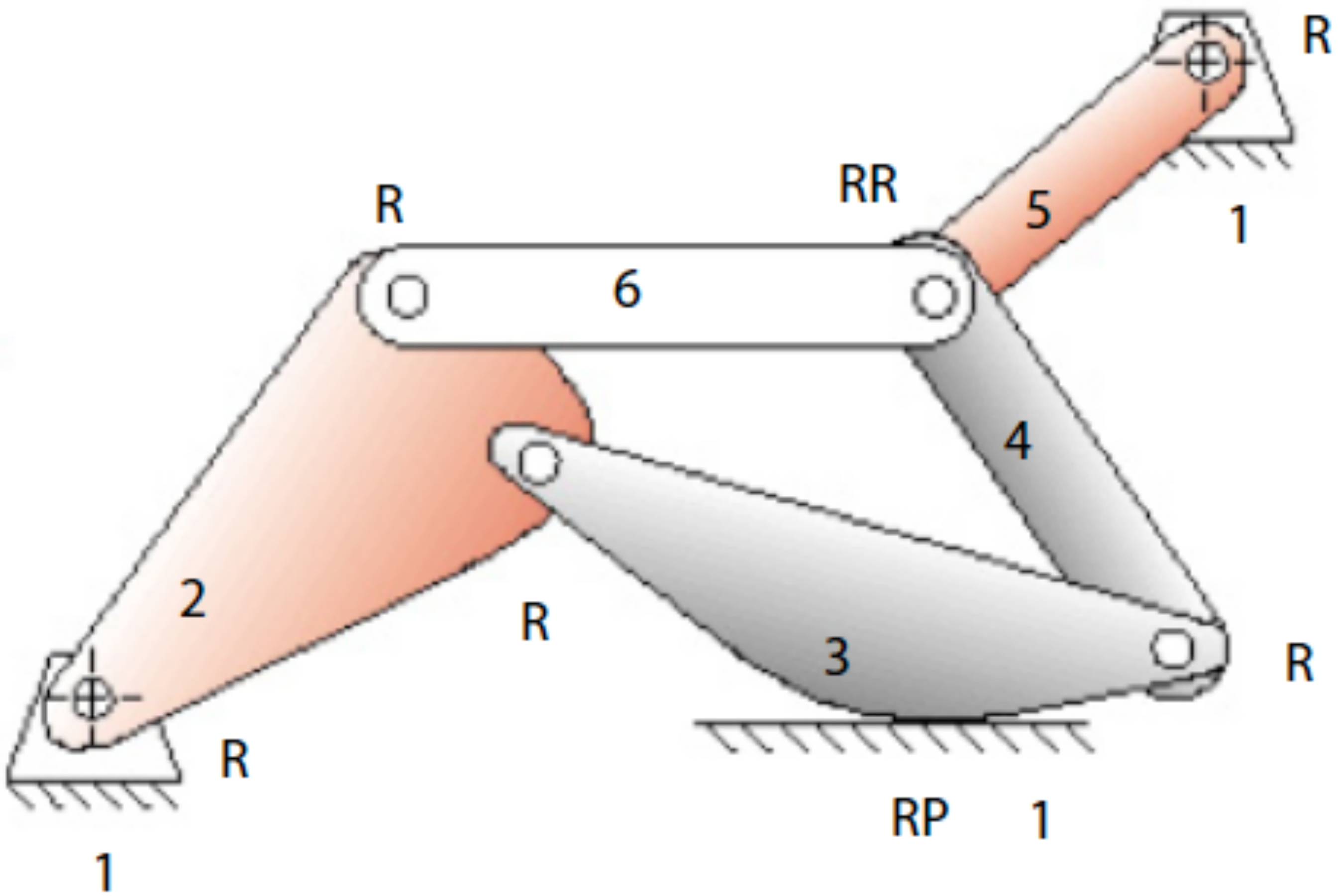
# KUTZBACH-GRUBLER MOBILITY EQUATION

$$M = 3(L - 1) - 2J_1 - J_2$$

$$L = \#\ \text{LINKS}$$

$$J_1 = \#\ \text{FULL JOINTS}$$

$$J_2 = \#\ \text{HALF JOINTS}$$

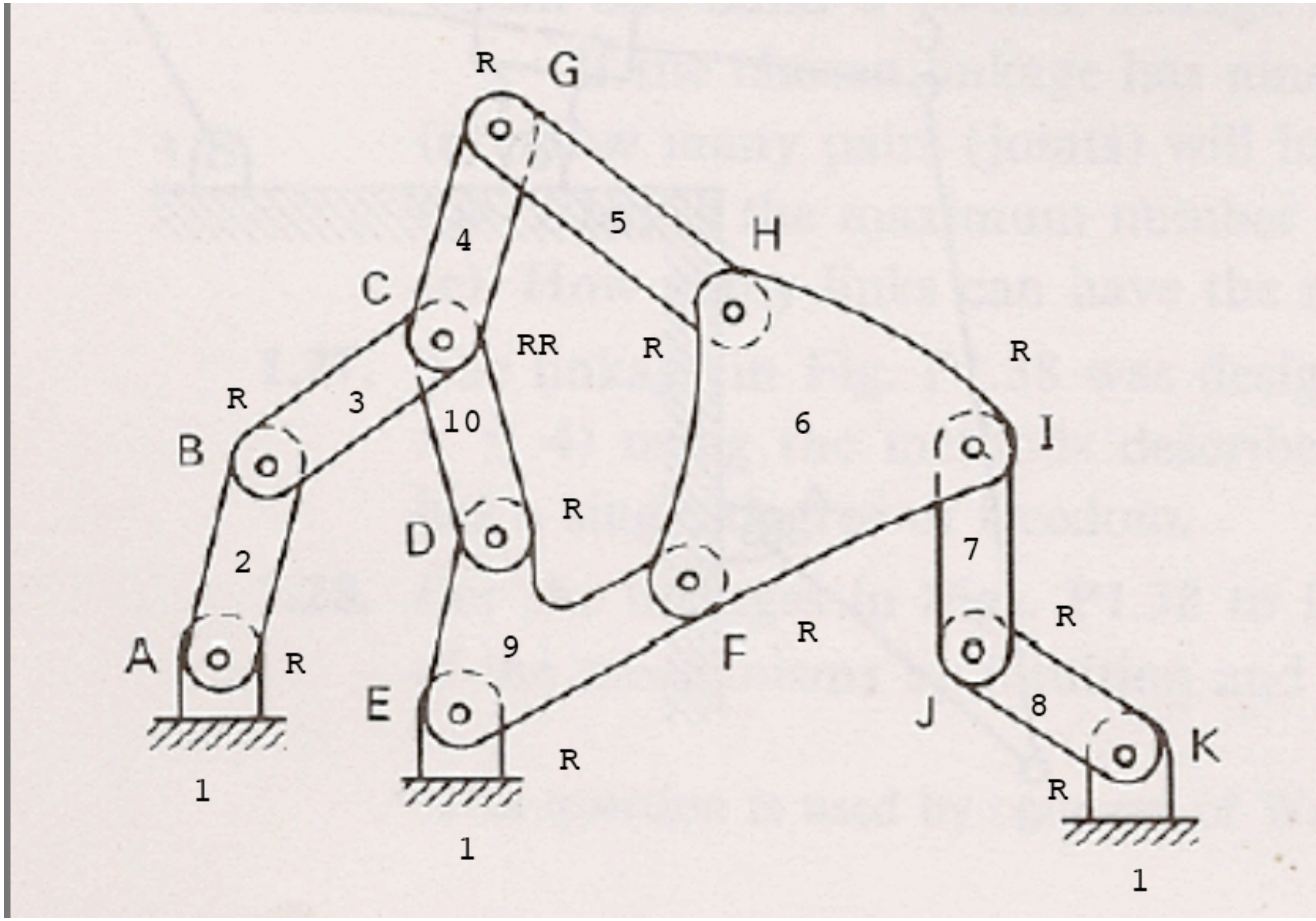



$$L = 6$$

$$J_1 = 7$$

$$J_2 = 1$$

$$M = 3(6-1) - (2)(7) - 1 = 0$$



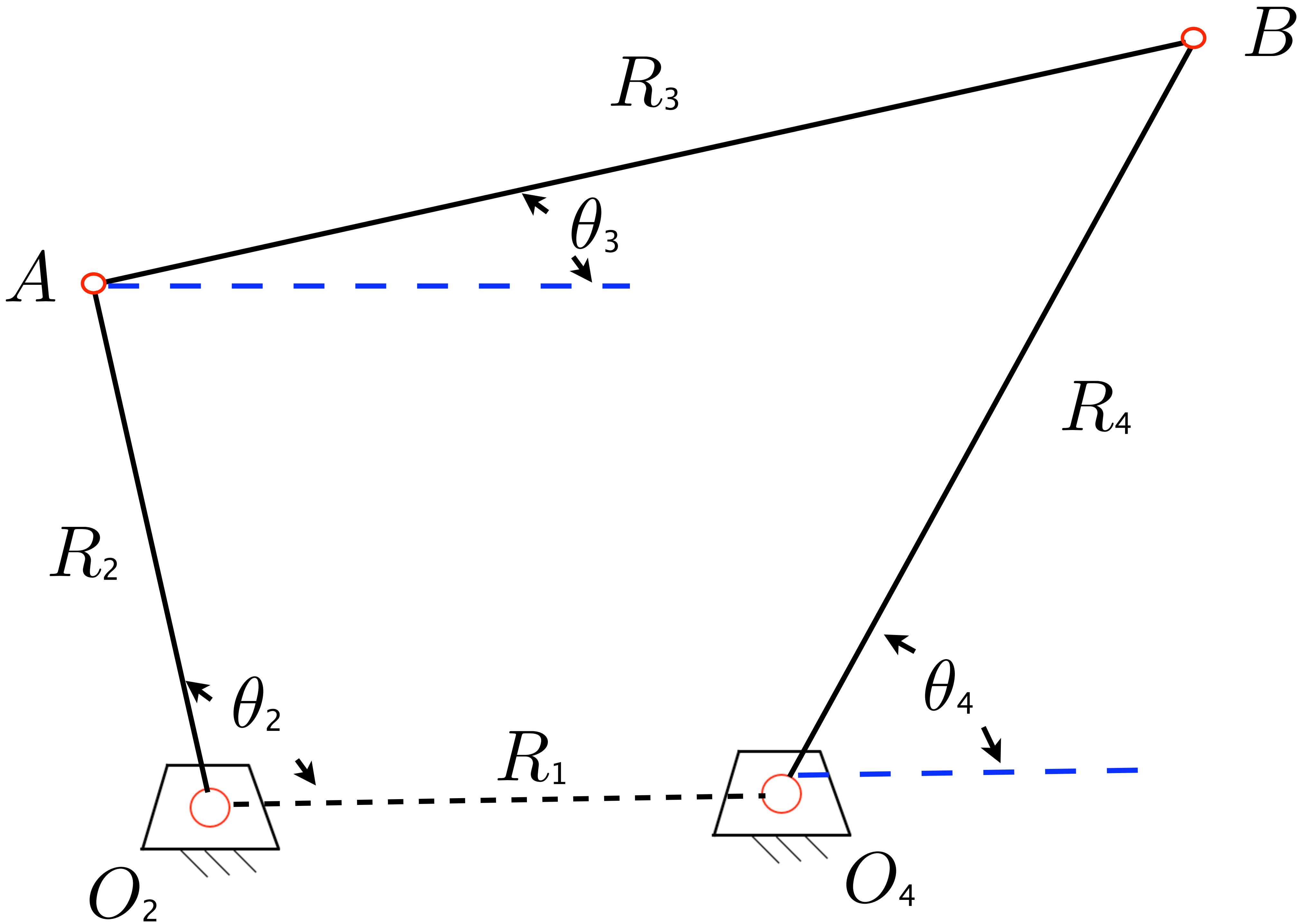

$$L = 10$$

$$J_1 = 12$$

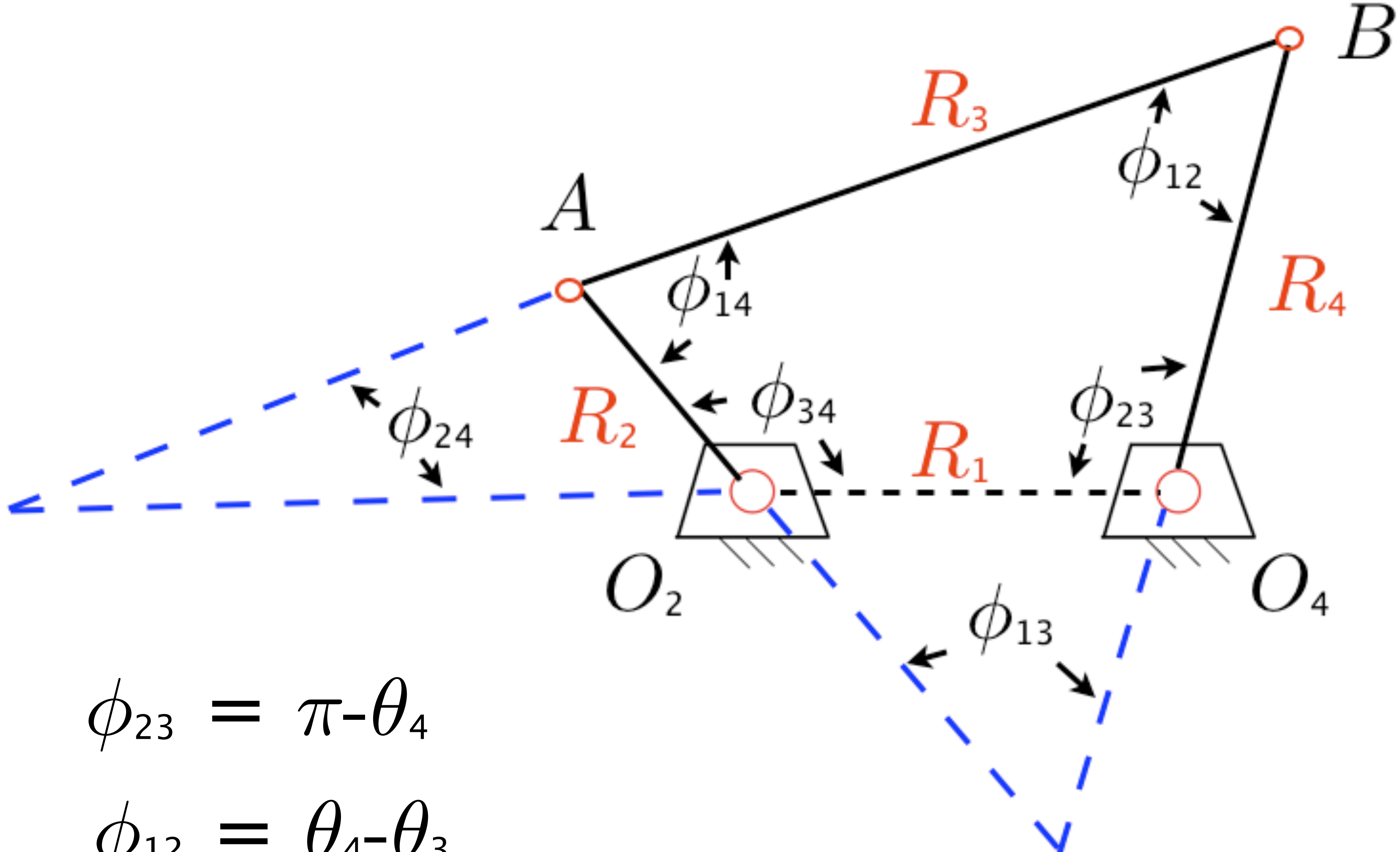
$$J_2 = 0$$

$$M = 3(L-1) - 2J_1 - J_2 = 3(9) - 2(12) - 0 = 3$$

# FOUR-BAR MECHANISM



# FUNDAMENTAL POSITION ANGLES




$\phi_{34} = \theta_2$ $\qquad$ $\phi_{23} = \pi - \theta_4$

$\phi_{24} = \theta_3$ $\qquad$ $\phi_{12} = \theta_4 - \theta_3$

$\phi_{13} = \theta_2 - \theta_4$ $\qquad$ $\phi_{14} = \pi - \theta_2 + \theta_3$

# CLASSIFICATION OF FOUR-BAR MECHANISMS

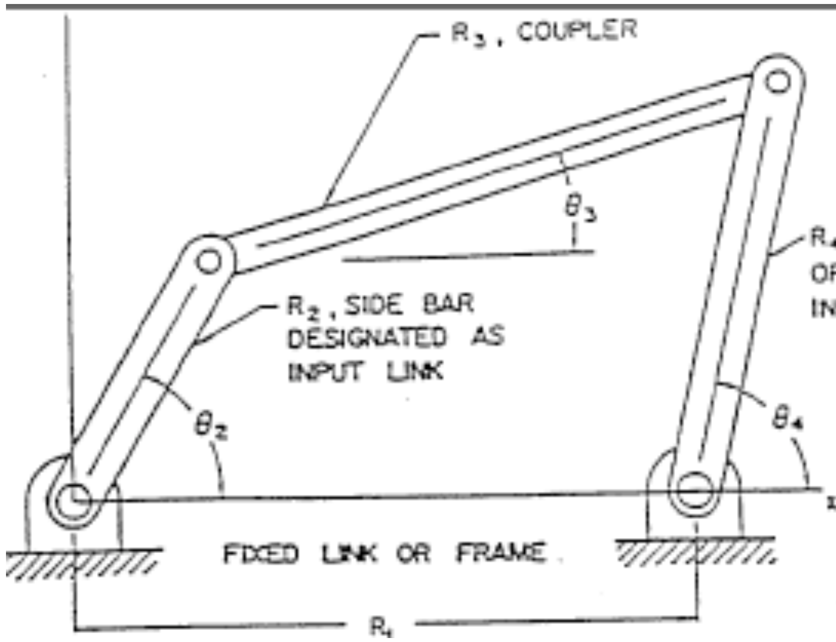



Fig. 1. Typical planar four-bar mechanism.

$s =$ **LENGTH OF SHORTEST LINK**

$l =$ **LENGTH OF LONGEST LINK**

$p, q =$ **LENGTHS OF OTHER LINKS**

$$s \leq p \leq l \qquad s \leq q \leq l$$

IN ORDER FOR THE FOUR-BAR TO BE CAPABLE OF ASSEMBLY, MUST HAVE

**ASSEMBLABILITY CONDITION:** $l < s + p + q$

GRASHOF CONDITION

**GRASHOF CONDITION:** $s + l < p + q$

IF THIS CONDITION HOLDS, AT LEAST ONE LINK IS A CRANK

# BARKER CLASSIFICATION

Table 2. Complete classification of four-bar planar mechanisms

| Number | $s + l \overset{<}{\underset{>}{=}} p + q$ | Category | Characteristic bar length | Class | Proposed name | Symbol |
|---|---|---|---|---|---|---|
| 1 | $<$ | Grashof | frame, $R_1 = s$ | 1 | Grashof crank-crank-crank | *GCCC* |
| 2 | $<$ | Grashof | input, $R_2 = s$ | 2 | Grashof crank-rocker-rocker | *GCRR* |
| 3 | $<$ | Grashof | coupler, $R_3 = s$ | 3 | Grashof rocker-crank-rocker | *GRCR* |
| 4 | $<$ | Grashof | output, $R_4 = s$ | 4 | Grashof rocker-rocker-crank | *GRRC* |
| 5 | $>$ | non-Grashof | frame, $R_1 = l$ | 1 | Class 1 rocker-rocker-rocker | *RRR*1 |
| 6 | $>$ | non-Grashof | input, $R_2 = l$ | 2 | Class 2 rocker-rocker-rocker | *RRR*2 |
| 7 | $>$ | non-Grashof | coupler, $R_3 = l$ | 3 | Class 3 rocker-rocker-rocker | *RRR*3 |
| 8 | $>$ | non-Grashof | output, $R_4 = l$ | 4 | Class 4 rocker-rocker-rocker | *RRR*4 |
| 9 | $=$ | change point | frame, $R_1 = s$ | 1 | change point crank-crank-crank | *CPCCC* |
| 10 | $=$ | change point | input, $R_2 = s$ | 2 | change point crank-rocker-rocker | *CPCRR* |
| 11 | $=$ | change point | coupler, $R_3 = s$ | 3 | change point rocker-crank-rocker | *CPRCR* |
| 12 | $=$ | change point | output, $R_4 = s$ | 4 | change point rocker-rocker-crank | *CPRRC* |
| 13 | $=$ | change point | two equal pairs | 5 | double change point | *CP2X* |
| 14 | $=$ | change point | $R_1 = R_2 = R_3 = R_4$ | 6 | triple change point | *CP3X* |

# FORM 1

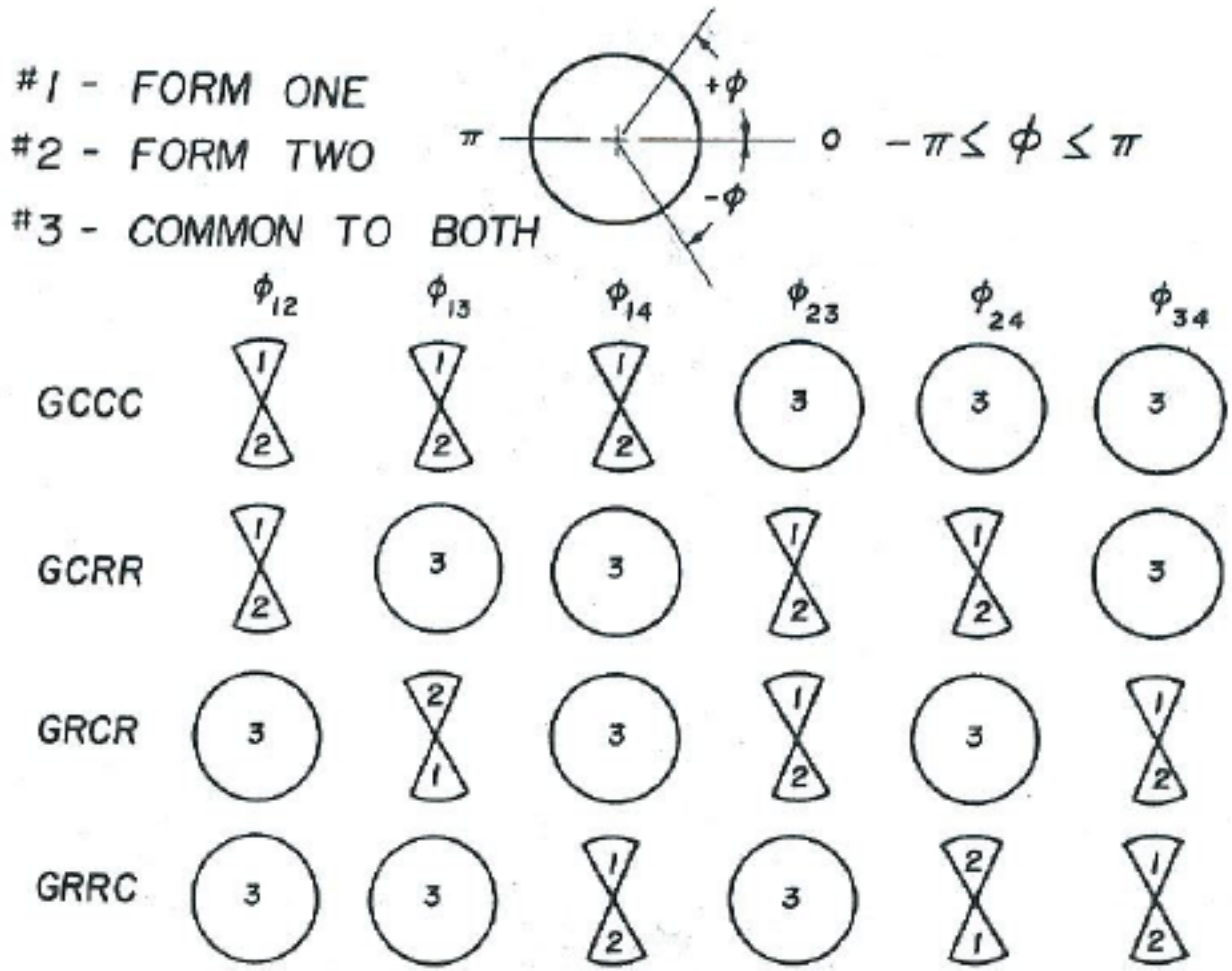
**FOR GCCC, GCRR, RRR1 ... RRR4:** $\phi_{12} > 0$

**FOR GRCR, GRRC:** $\phi_{34} > 0$

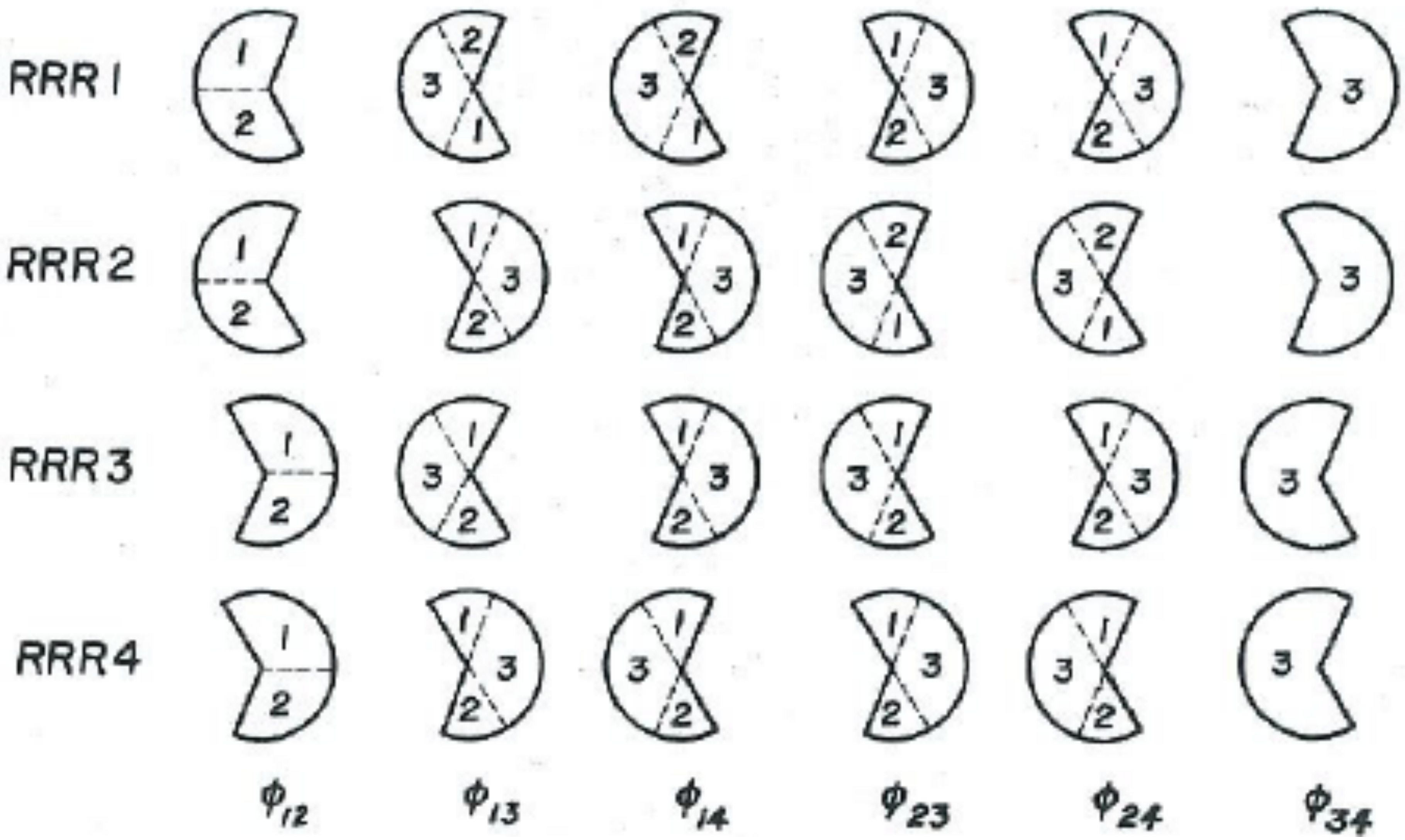
# FORM 2

**FOR GCCC, GCRR, RRR1 ... RRR4:** $\phi_{12} < 0$

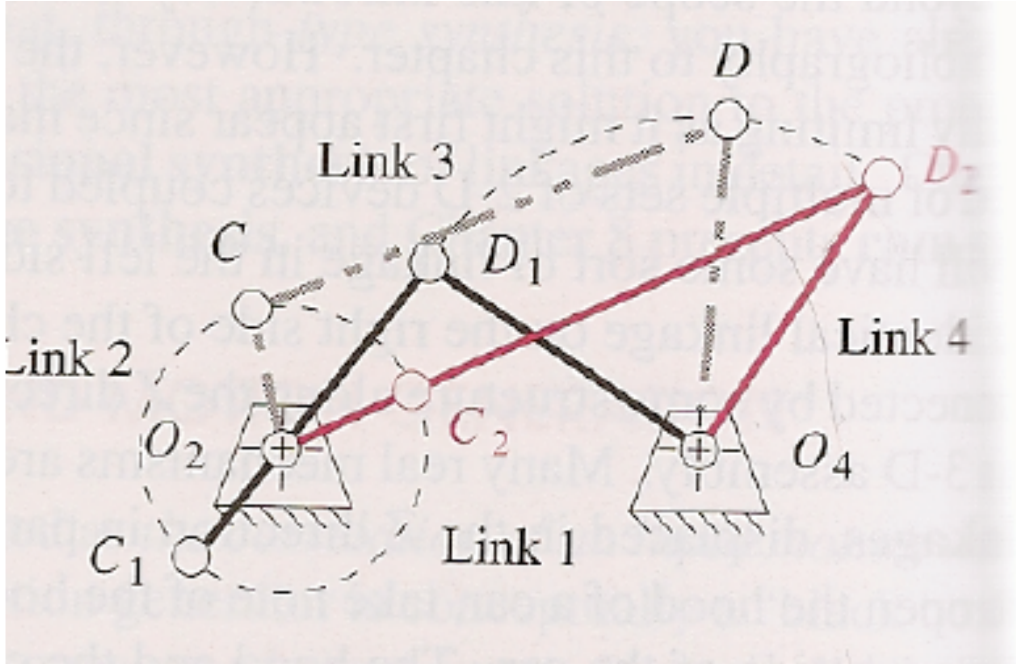
**FOR GRCR, GRRC:** $\phi_{34} < 0$



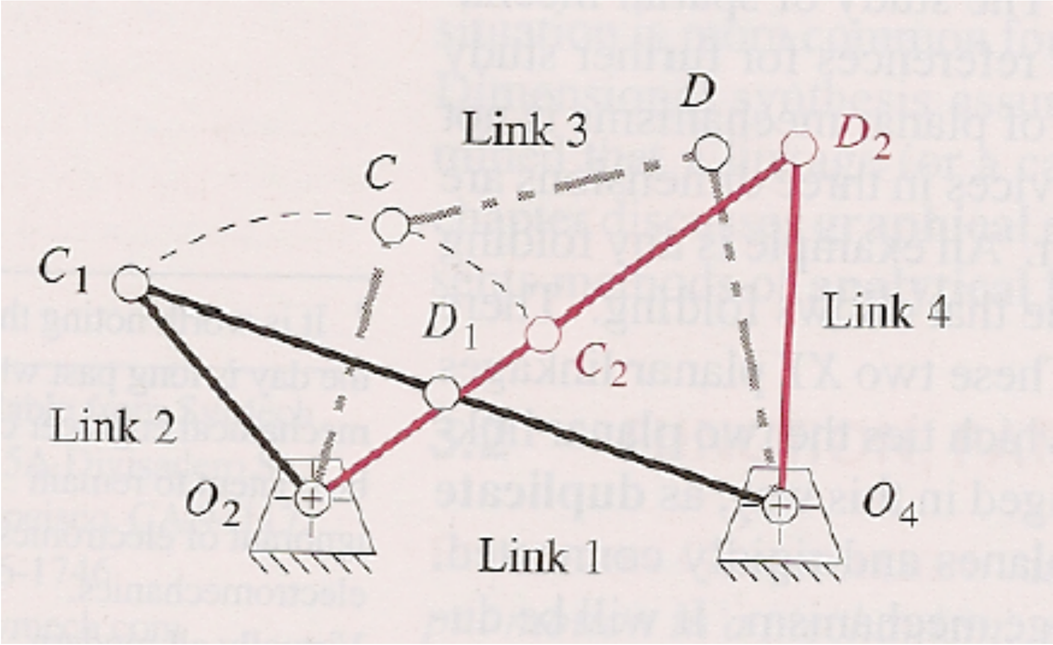
#1 - FORM ONE
#2 - FORM TWO
#3 - COMMON TO BOTH
π
+φ
0
-φ
−π ≤ φ ≤ π
φ12
φ13
φ14
φ23
φ24
φ34
GCCC
GCRR
GRCR
GRRC


RRR1
RRR2
RRR3
RRR4
φ12
φ13
φ14
φ23
φ24
φ34

# TOGGLE POINTS OF A CRANK-DOUBLE ROCKER



# TOGGLE POINTS OF A TRIPLE ROCKER



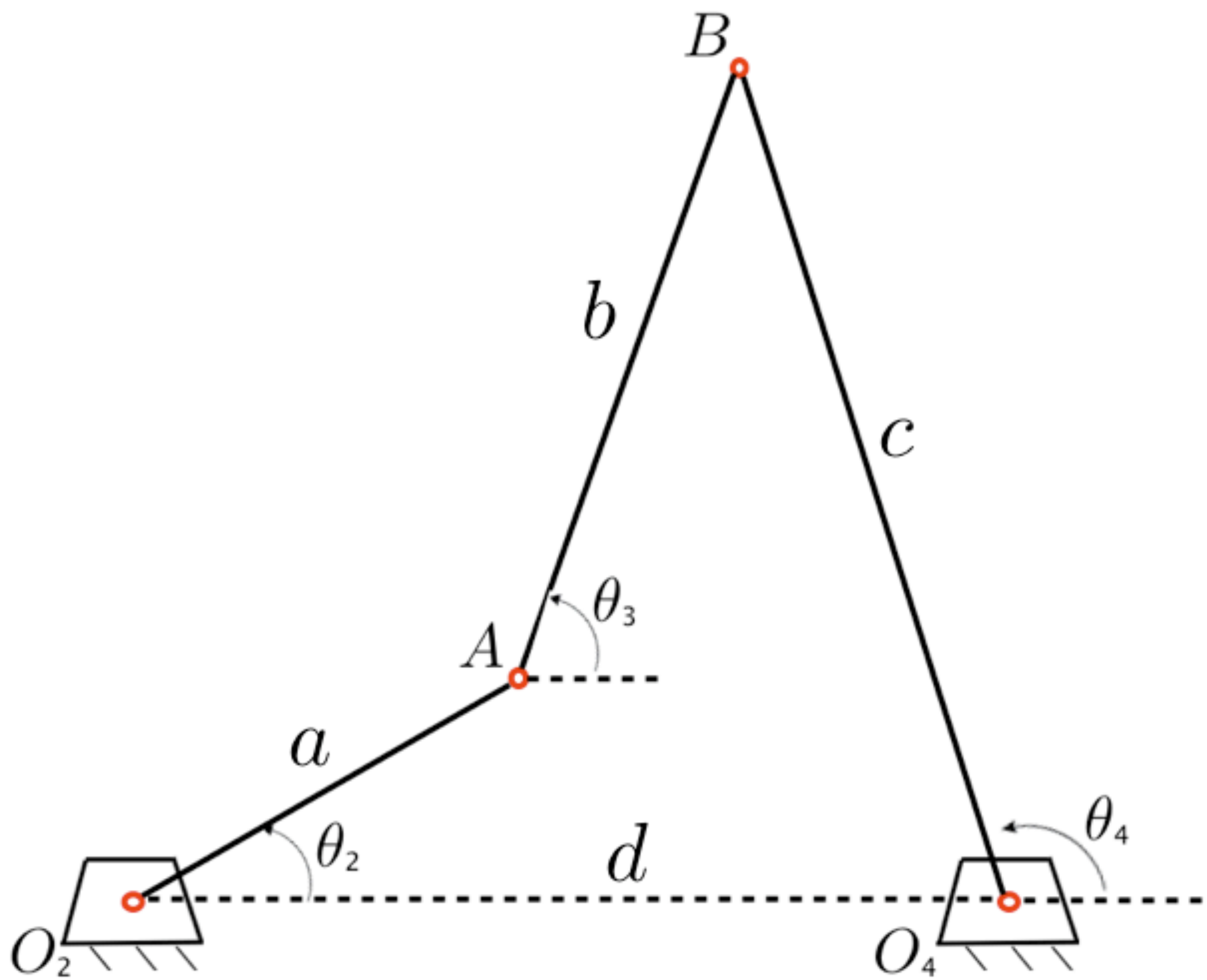
IF A FOUR-BAR MECHANISM HAS A CONFIGURATION IN WHICH TWO ADJACENT LINKS ARE TOGGLED, THE INTERNAL ANGLE BETWEEN THE OTHER TWO LINKS HAS ITS MAXIMUM IN THAT PLACEMENT.

IF TWO ADJACENT LINKS OF A FOUR-BAR MECHANISM HAVE AN EXTREME (OVERLAP) CONFIGURATION, THE INTERNAL ANGLE BETWEEN THE OTHER TWO LINKS HAS ITS MINIMUM IN THAT PLACEMENT.

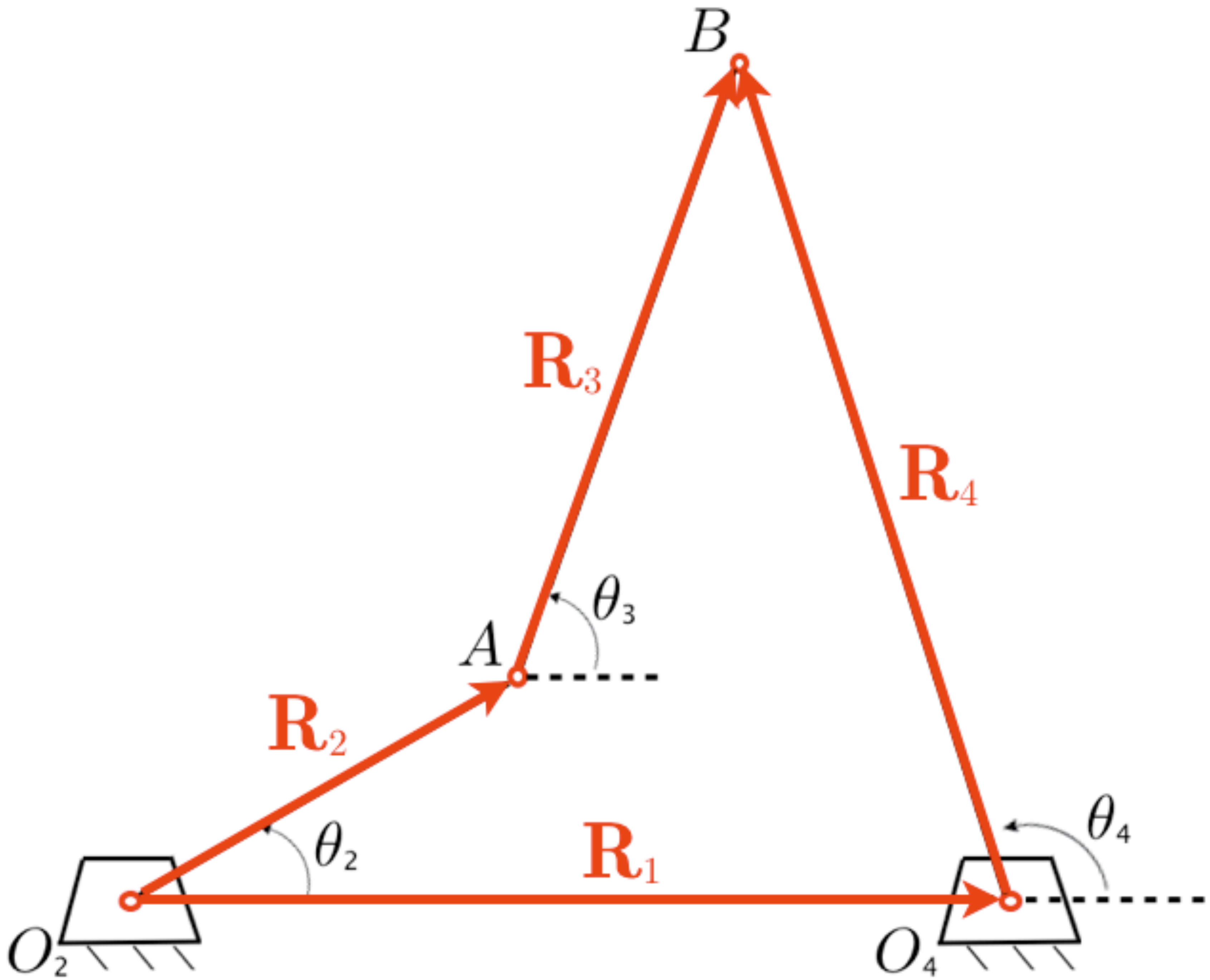
IF A FOUR-BAR MECHANISM HAS A CONFIGURATION IN WHICH TWO NON- ADJACENT LINKS ARE PARALLEL, THE INTERNAL ANGLE BETWEEN THE OTHER TWO LINKS HAS AN EXTREMUM IN THAT PLACEMENT.

# POSITION ANALYSIS

# FOUR-BAR POSITION ANALYSIS



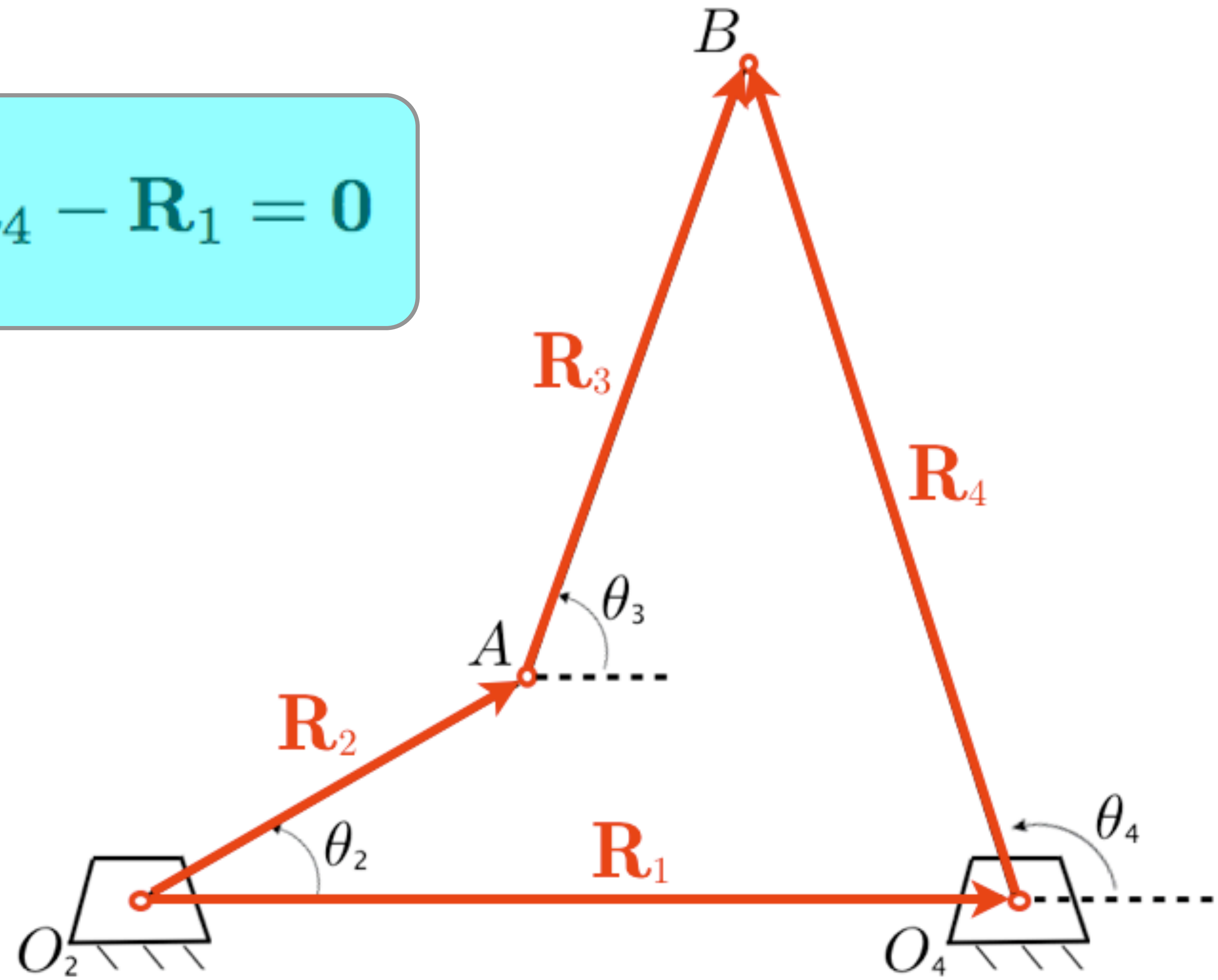
# VECTOR LOOP



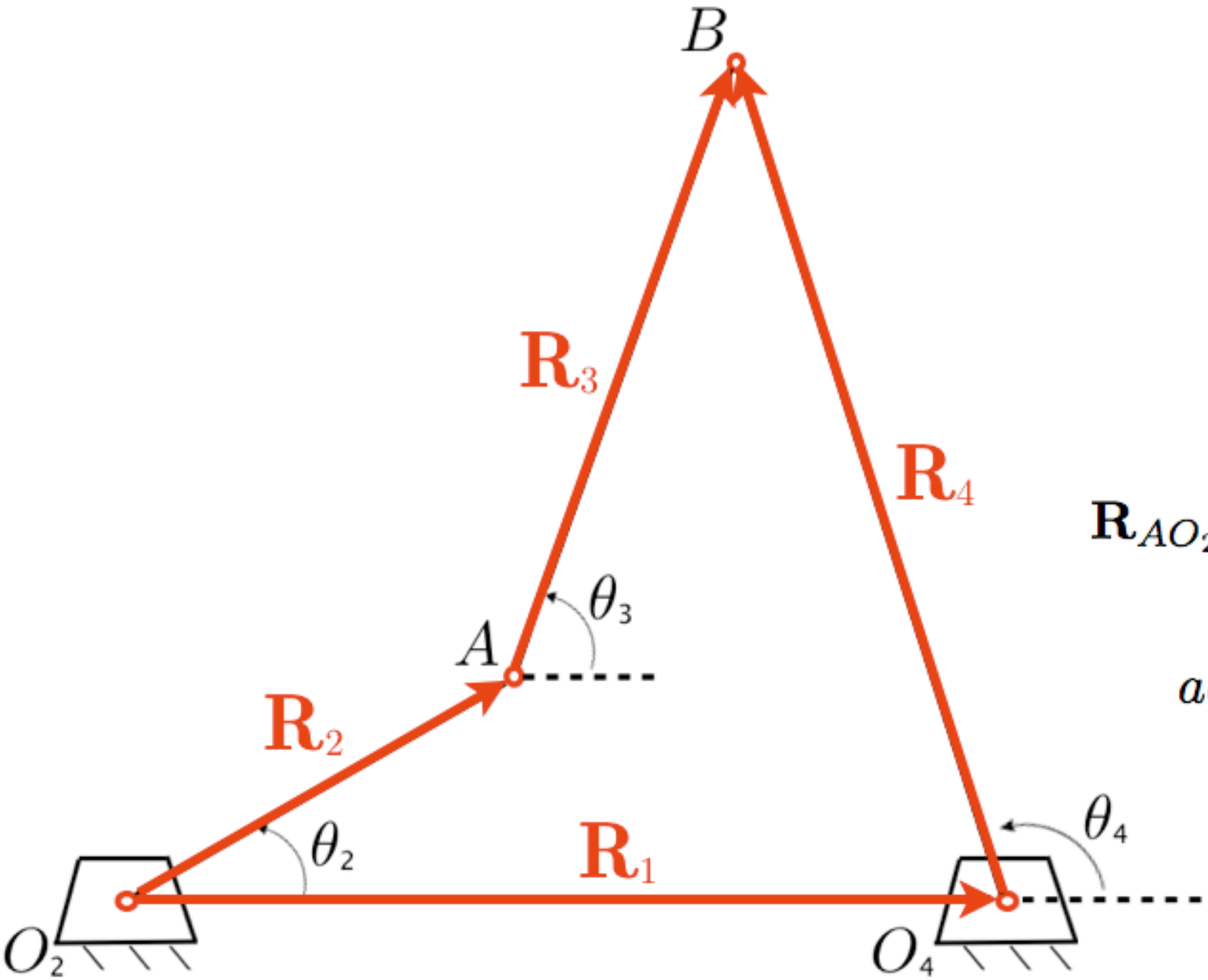

LOOP: $O_2 \rightarrow A \rightarrow B \rightarrow O_4 \rightarrow O_2$

# VECTOR LOOP EQUATION

$$\mathbf{R}_2 + \mathbf{R}_3 - \mathbf{R}_4 - \mathbf{R}_1 = \mathbf{0}$$



# VECTOR LOOP EQUATION

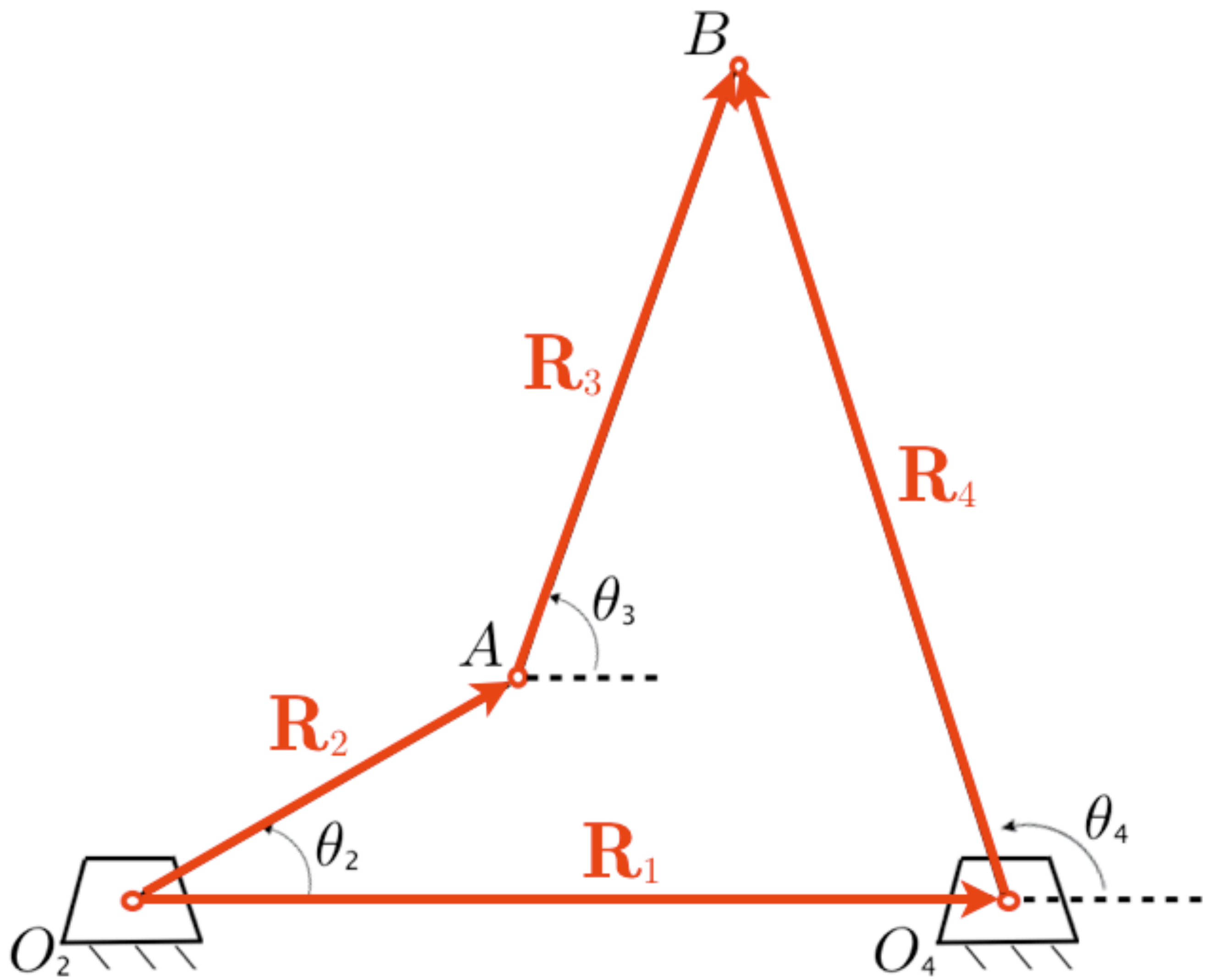



$$\mathbf{R}_2 + \mathbf{R}_3 - \mathbf{R}_4 - \mathbf{R}_1 = \mathbf{0}$$

$$\mathbf{R}_{AO_2} + \mathbf{R}_{BA} - \mathbf{R}_{BO_4} - \mathbf{R}_{O_4O_2} = \mathbf{0}$$

$$ae^{j\theta_2} + be^{j\theta_3} - ce^{j\theta_4} - d = 0$$

**VARIABLES**:– $\theta_2, \theta_3, \theta_4$.

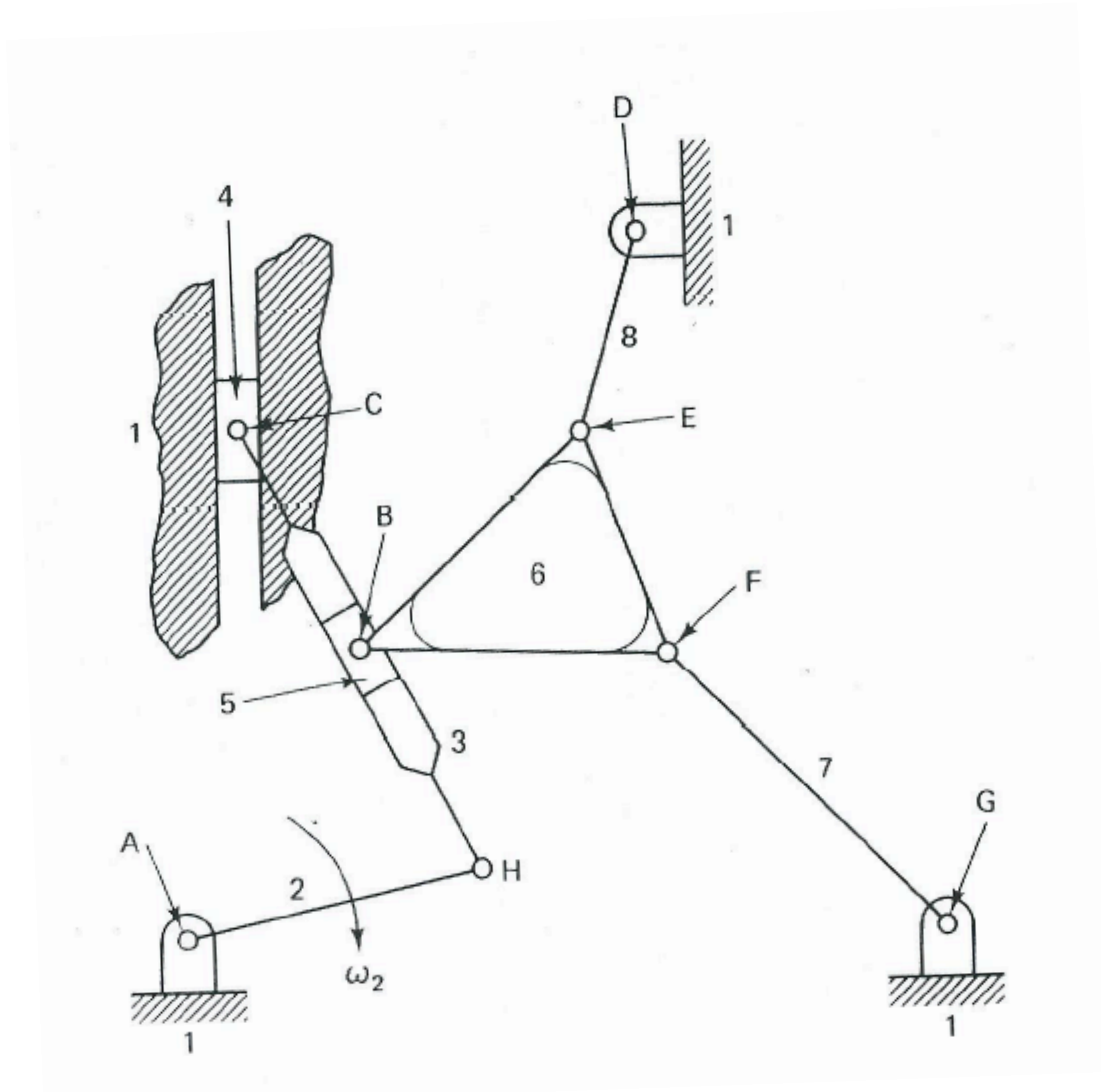


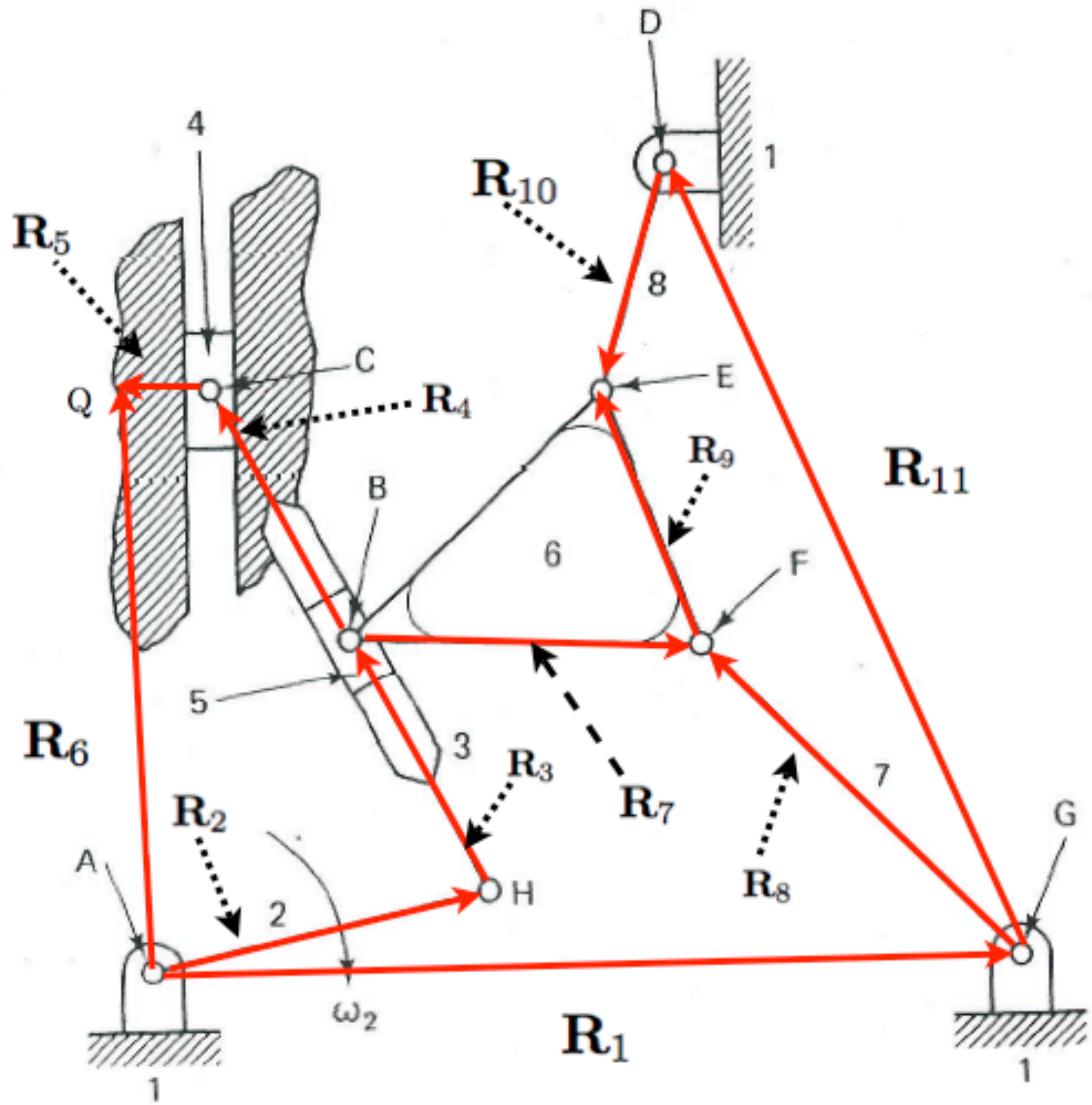

$$\Re : - \qquad a\cos\theta_2 + b\cos\theta_3 - c\cos\theta_4 - d = 0$$

$$\Im : - \qquad a\sin\theta_2 + b\sin\theta_3 - c\sin\theta_4 = 0$$

THESE MAY BE REGARDED AS TWO EQUATIONS RELATING THE TWO UNKNOWNS $\theta_3, \theta_4$ TO THE INPUT VARIABLE $\theta_2$.

# POSITION ANALYSIS EXAMPLE

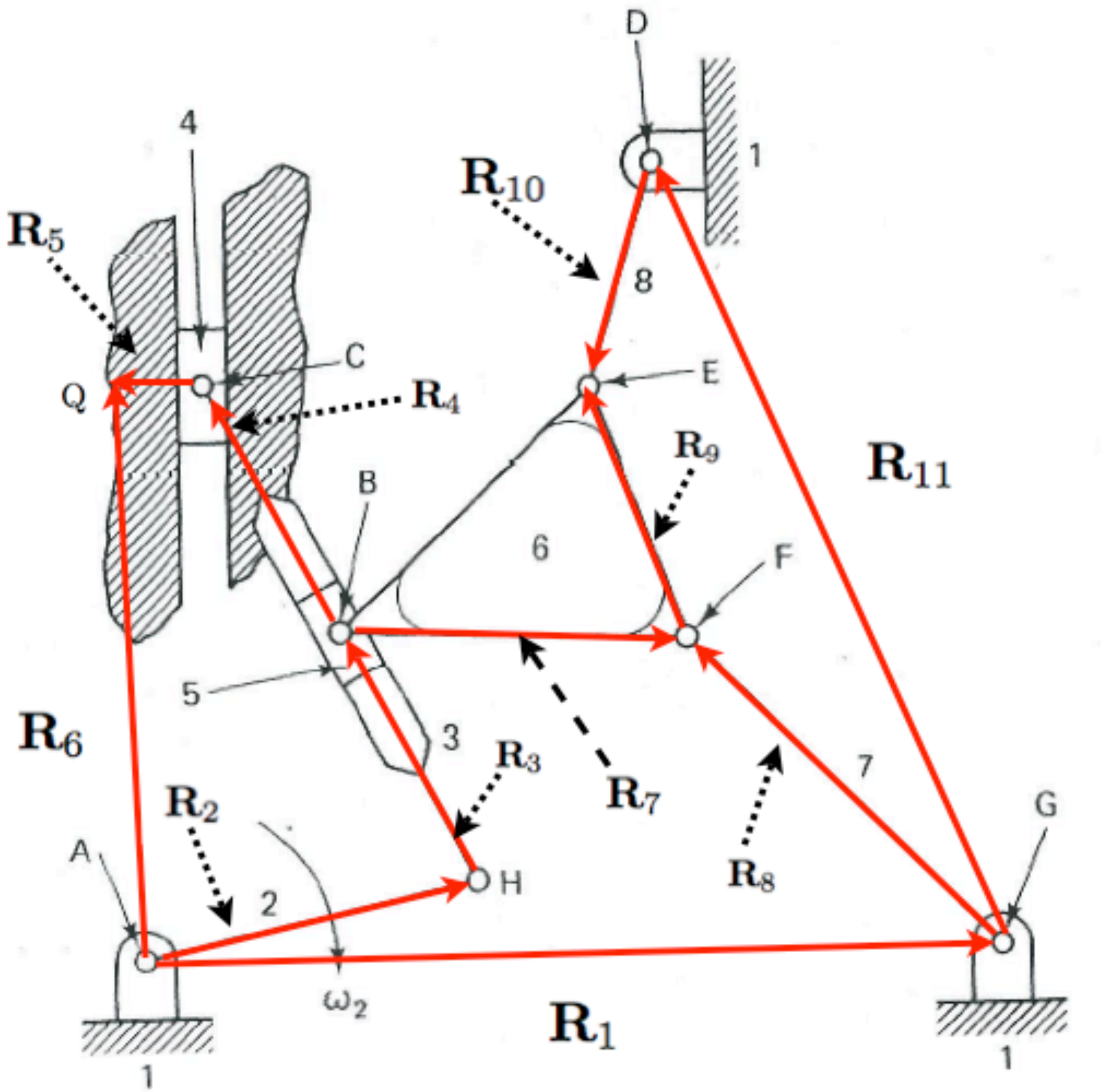





**LOOP 1:** $A \rightarrow H \rightarrow B \rightarrow C \rightarrow Q \rightarrow A$

**LOOP 2:** $A \rightarrow H \rightarrow B \rightarrow F \rightarrow G \rightarrow A$

**LOOP 3:** $G \rightarrow F \rightarrow E \rightarrow D \rightarrow G$



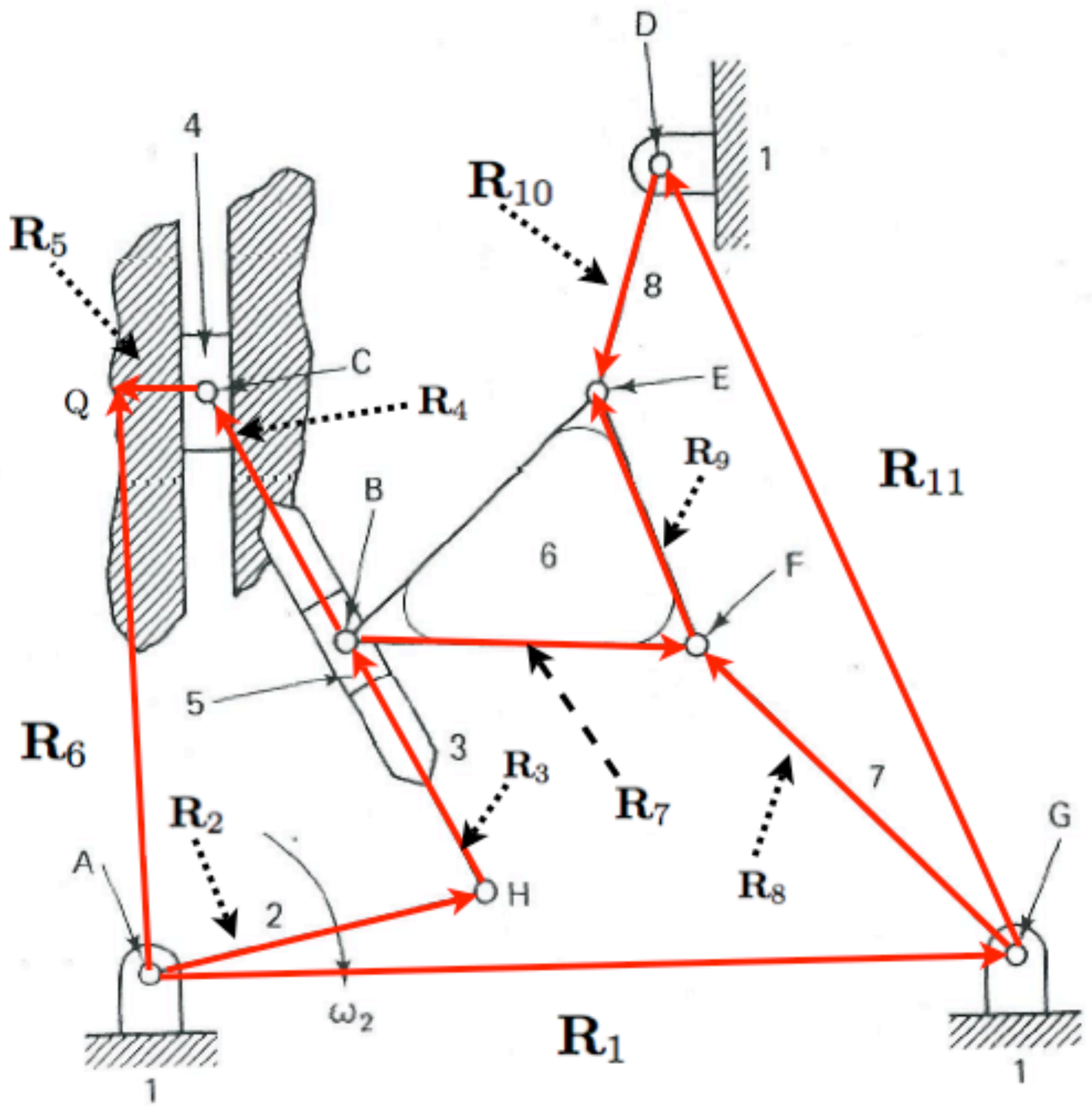

$$\mathbf{R}_2 + \mathbf{R}_3 + \mathbf{R}_4 + \mathbf{R}_5 - \mathbf{R}_6 = \mathbf{0}$$

$$\mathbf{R}_2 + \mathbf{R}_3 + \mathbf{R}_7 - \mathbf{R}_8 - \mathbf{R}_1 = \mathbf{0}$$

$$\mathbf{R}_8 + \mathbf{R}_9 - \mathbf{R}_{10} - \mathbf{R}_{11} = \mathbf{0}$$

$$\mathbf{R}_k \leftrightarrow d_k e^{j\theta_k}$$

$d_1, d_2, d_5, d_7 \ldots d_{11}$ ARE CONSTANT.
$d_3, d_4, d_6$ ARE VARIABLE.
$\theta_1, \theta_5, \theta_6, \theta_{11}$ ARE CONSTANT.
$\theta_2 \ldots \theta_4, \theta_7 \ldots \theta_{10}$ ARE VARIABLE.
$\theta_3 = \theta_4,\ \theta_9 - \theta_7 = \gamma$ (constant).
$d_3 + d_4 = \bar{d}$ (constant).

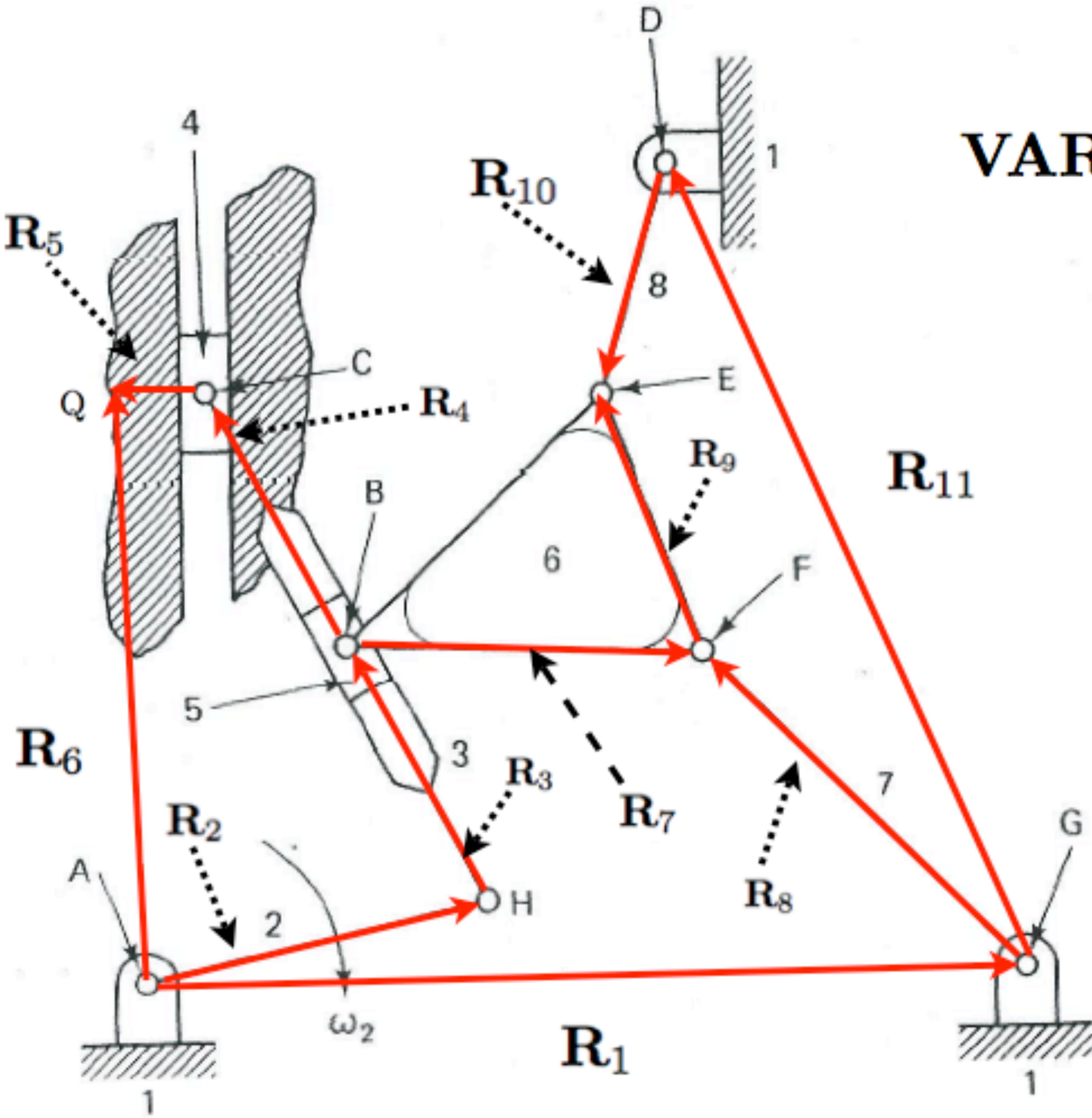



$$d_2 e^{j\theta_2} + d_3 e^{j\theta_3} + d_4 e^{j\theta_4} - d_5 - j d_6 = 0,$$

$$d_2 e^{j\theta_2} + d_3 e^{j\theta_3} + d_7 e^{j\theta_7} - d_8 e^{j\theta_8} - d_1 = 0,$$

$$d_8 e^{j\theta_8} + d_9 e^{j\theta_9} - d_{10} e^{j\theta_{10}} - d_{11} e^{j\theta_{11}} = 0,$$

$$\theta_3 = \theta_4, \;\; d_3 + d_4 = \bar{d}\,(\text{const.}), \;\; \theta_9 - \theta_7 = \gamma\,(\text{const.}).$$




**VARIABLES**: $d_3, d_6, \theta_2, \theta_3, \theta_7, \theta_8, \theta_9, \theta_{10}.$

8 VARIABLES
7 EQUATIONS
1 D.O.F.

$$d_2 e^{j\theta_2} + \bar{d} e^{j\theta_3} - d_5 - j d_6 = 0,$$

$$d_2 e^{j\theta_2} + d_3 e^{j\theta_3} + d_7 e^{j\theta_7} - d_8 e^{j\theta_8} - d_1 = 0,$$

$$d_8 e^{j\theta_8} + d_9 e^{j\theta_9} - d_{10} e^{j\theta_{10}} - d_{11} e^{j\theta_{11}} = 0,$$

$$\theta_9 - \theta_7 = \gamma\,(\text{const.}).$$

# COMPLETENESS / REDUNDANCY

EVERY JOINT OF THE MECHANISM MUST LIE ON AT LEAST ONE LOOP.

IF TWO VECTORS FROM DIFFERENT LOOPS LIE ON THE SAME LINK, THE CONSTANCY OF THE ANGLE BETWEEN THEM MUST BE STATED AS A CONSTRAINT.

IF THREE VECTORS ON THE SAME LINK ARE USED IN A VECTOR-LOOP ANALYSIS, THE FORMULATION HAS A REDUNDANCY.

# VELOCITY ANALYSIS

$$e^{j\theta} \leftrightarrow [\cos\theta\,\hat{\mathbf{\imath}} + \sin\theta\,\hat{\mathbf{\jmath}}]$$

$$je^{j\theta} \leftrightarrow [-\sin\theta\,\hat{\mathbf{\imath}} + \cos\theta\,\hat{\mathbf{\jmath}}]$$

# ANGULAR VELOCITY

$$\boldsymbol{\omega} = \dot{\theta}\,\hat{\mathbf{k}}$$

$$\frac{d}{dt}\{e^{j\theta}\} = j\dot{\theta}e^{j\theta}$$

$$\frac{d}{dt}\{[\cos\theta\,\hat{\mathbf{\imath}} + \sin\theta\,\hat{\mathbf{\jmath}}]\} = \dot{\theta}[-\sin\theta\,\hat{\mathbf{\imath}} + \cos\theta\,\hat{\mathbf{\jmath}}] = \boldsymbol{\omega} \times [\cos\theta\,\hat{\mathbf{\imath}} + \sin\theta\,\hat{\mathbf{\jmath}}]$$

## TIME DERIVATIVE OF A GENERAL VECTOR

$$e^{j\theta} \leftrightarrow [\cos\theta\,\hat{\mathbf{i}} + \sin\theta\,\hat{\mathbf{j}}]$$

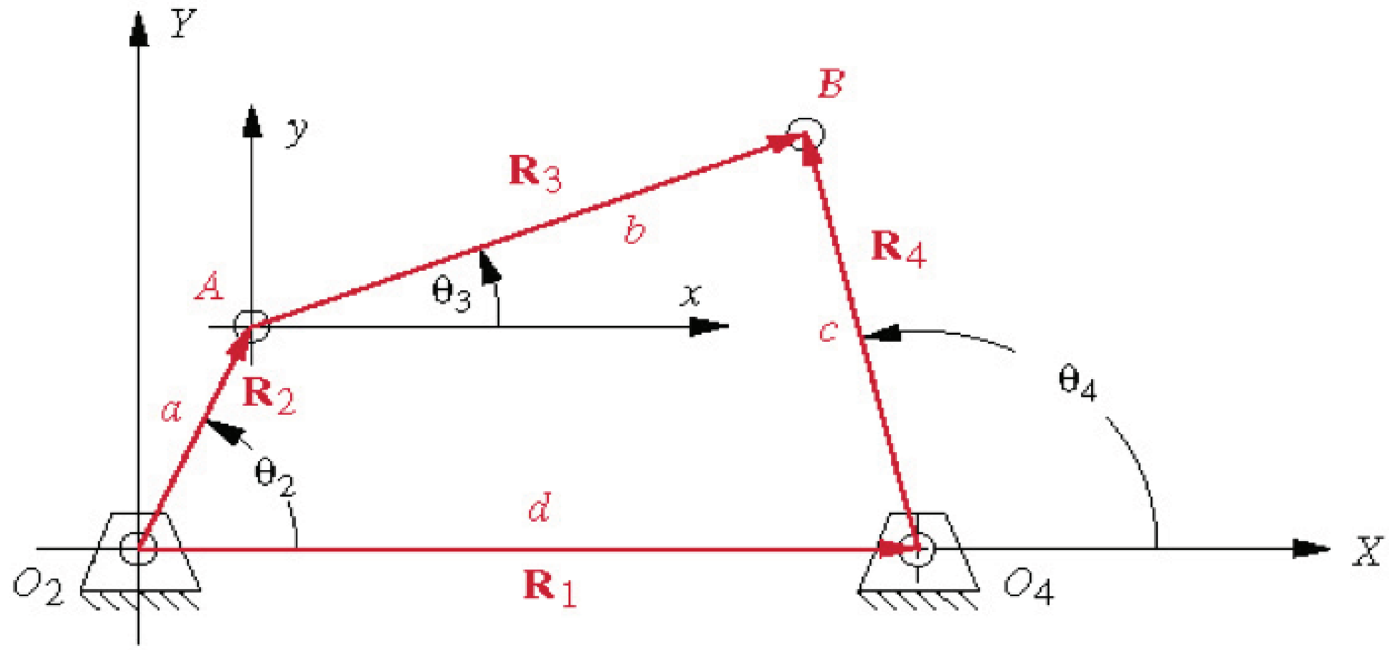
$$je^{j\theta} \leftrightarrow [-\sin\theta\,\hat{\mathbf{i}} + \cos\theta\,\hat{\mathbf{j}}]$$

$$\boldsymbol{\omega} = \dot{\theta}\,\hat{\mathbf{k}}$$

$$\frac{d}{dt}\{de^{j\theta}\} = \dot{d}e^{j\theta} + jd\dot{\theta}e^{j\theta}$$

$$\frac{d}{dt}\{d[\cos\theta\,\hat{\mathbf{i}} + \sin\theta\,\hat{\mathbf{j}}]\} = \dot{d}[\cos\theta\,\hat{\mathbf{i}} + \sin\theta\,\hat{\mathbf{j}}] + \boldsymbol{\omega} \times d[\cos\theta\,\hat{\mathbf{i}} + \sin\theta\,\hat{\mathbf{j}}]$$

# FOURBAR POSITION ANALYSIS




$$ae^{j\theta_2} + be^{j\theta_3} - ce^{j\theta_4} - d = 0$$

$$a\cos\theta_2 + b\cos\theta_3 = d + c\cos\theta_4$$

$$a\sin\theta_2 + b\sin\theta_3 = c\sin\theta_4$$

# FOURBAR VELOCITY ANALYSIS

$$j\omega_2 a e^{j\theta_2} + j\omega_3 b e^{j\theta_3} - j\omega_4 c e^{j\theta_4} = 0$$

$$a\cos\theta_2\omega_2 + b\cos\theta_3\omega_3 = c\cos\theta_4\omega_4$$

$$a\sin\theta_2\omega_2 + b\sin\theta_3\omega_3 = c\sin\theta_4\omega_4$$

$$\begin{pmatrix} b\cos\theta_3 & -c\cos\theta_4 \\ b\sin\theta_3 & -c\sin\theta_4 \end{pmatrix} \begin{pmatrix} \omega_3 \\ \omega_4 \end{pmatrix} = -a\omega_2 \begin{pmatrix} \cos\theta_2 \\ \sin\theta_2 \end{pmatrix}$$

Link 2 regarded as input link

## INSTANT CENTER

IF TWO RIGID BODIES, 1 AND 2, UNDERGO MOTION IN THE SAME PLANE, THEIR **INSTANT CENTER** - DENOTED $I_{1,2}$ - IS THE POINT OF THE PLANE WHICH HAS THE SAME VELOCITY CONSIDERED AS A POINT OF BODY 2 AS IT DOES REGARDED AS A POINT OF BODY 1.

IF $A$ IS A REFERENCE POINT ON BODY 1, $B$ A REFERENCE POINT ON BODY 2, AND IF $\boldsymbol{\omega_1}$, $\boldsymbol{\omega_2}$ ARE THE RESPECTIVE ANGULAR VELOCITIES OF THE TWO BODIES, THEN

$$\mathbf{V}_B + \boldsymbol{\omega_2} \times \mathbf{R}_{I_{1,2}B} = \mathbf{V}_A + \boldsymbol{\omega_1} \times \mathbf{R}_{I_{1,2}A}.$$

ALTERNATE DEFINITION

$I_{1,2}$ IS THE *APPARENT* INSTANTANEOUS CENTER OF BODY 2 IN THE FRAME OF REFERENCE OF BODY 1. THAT IS, IF $A$ IS A REFERENCE POINT ON BODY 1, $P$ ANY POINT ON BODY 2, AND IF $\boldsymbol{\omega_1}$, $\boldsymbol{\omega_2}$ ARE THE RESPECTIVE ANGULAR VELOCITIES OF THE TWO BODIES, THEN

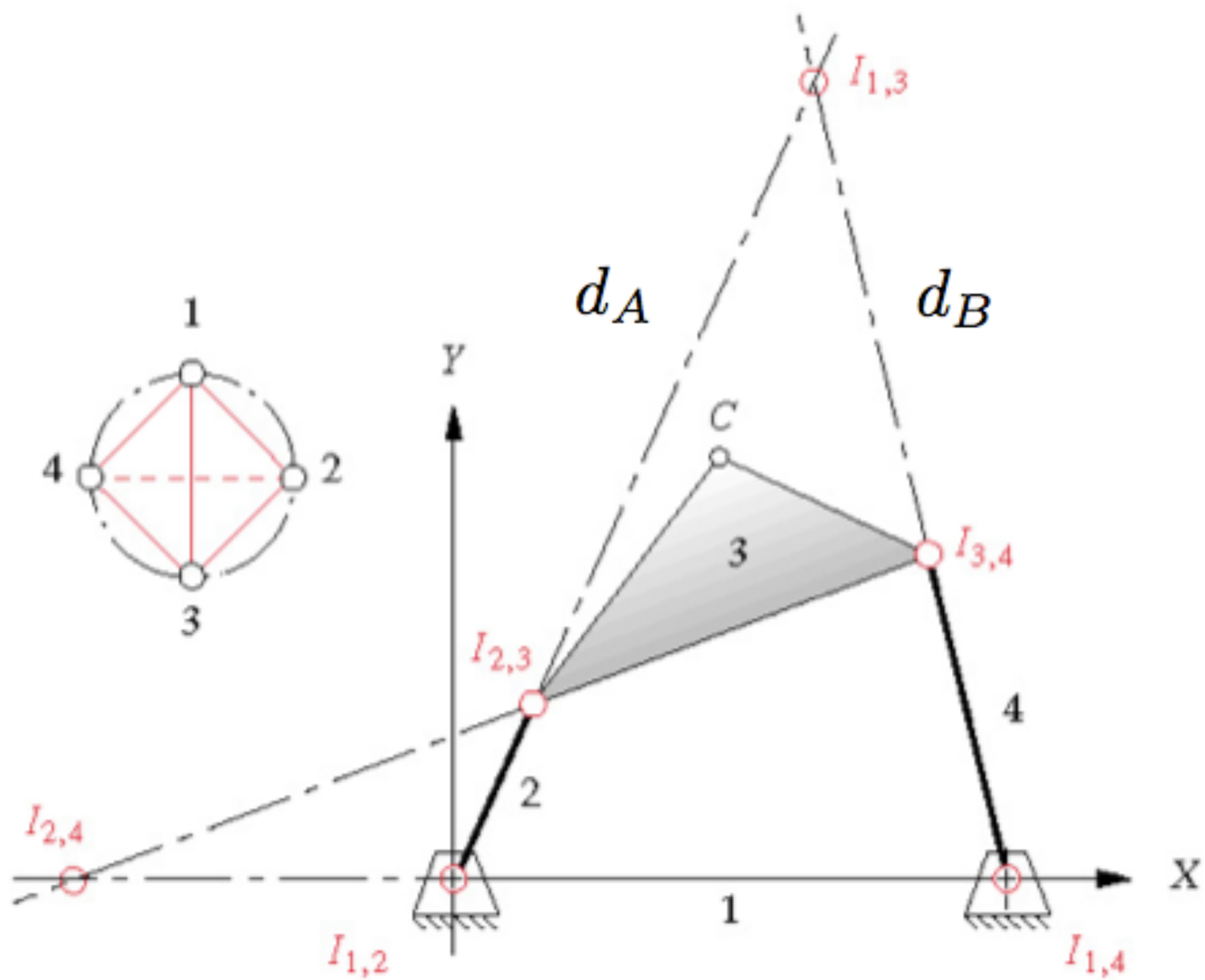
$$\left(\mathbf{V}_{PA}\right)_{Axyz} = \left(\boldsymbol{\omega_2} - \boldsymbol{\omega_1}\right) \times \mathbf{R}_{PI_{1,2}}.$$

# KENNEDY'S RULE

THE THREE INSTANT CENTERS ASSOCIATED WITH ANY TRIO OF BODIES MOVING IN THE SAME PLANE ARE COLLINEAR

$I_{1,3}$ MUST LIE ON THE LINE JOINING $I_{1,2}$ AND $I_{2,3}$.

# FOUR-BAR INSTANT CENTERS

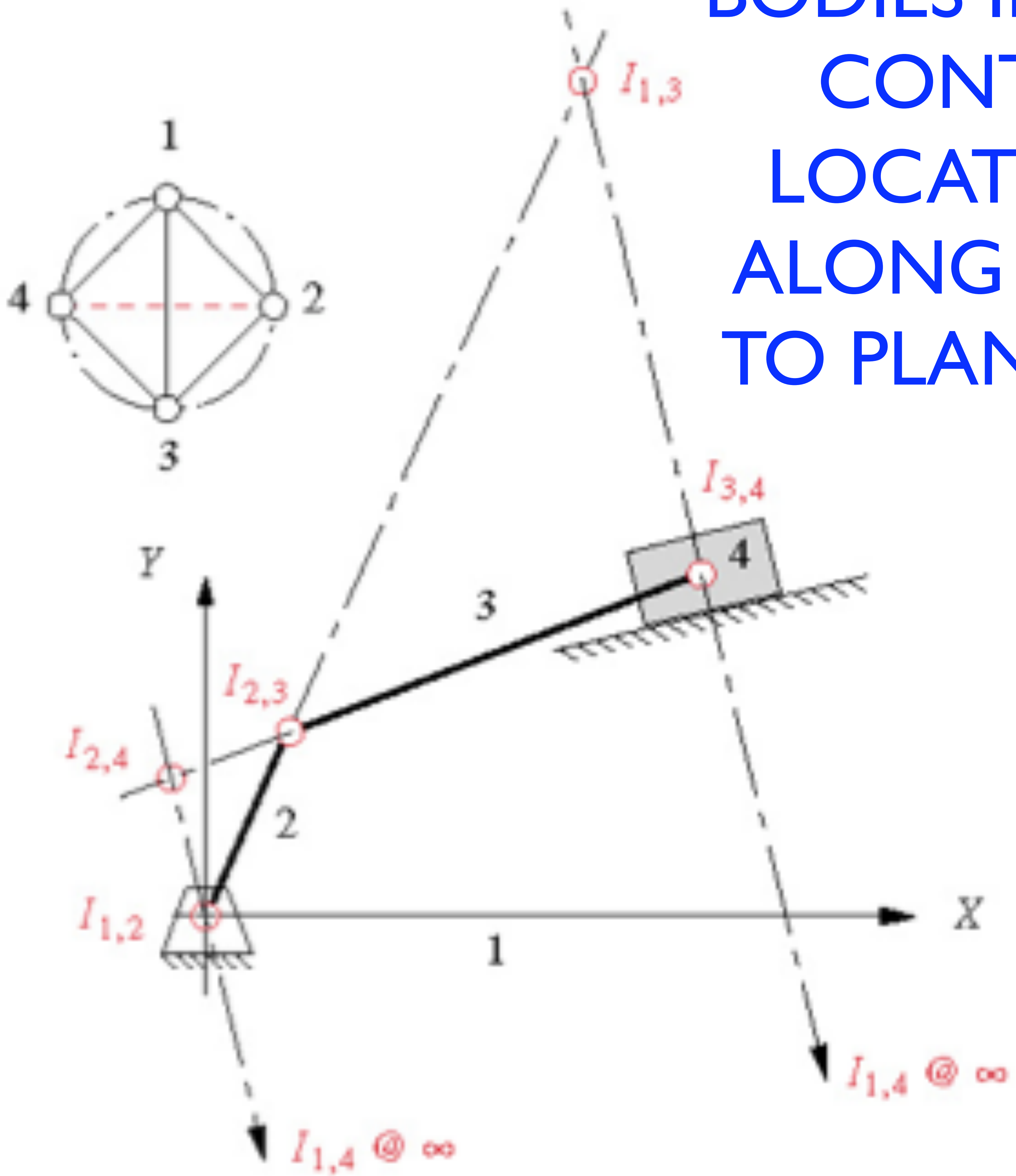



(c)

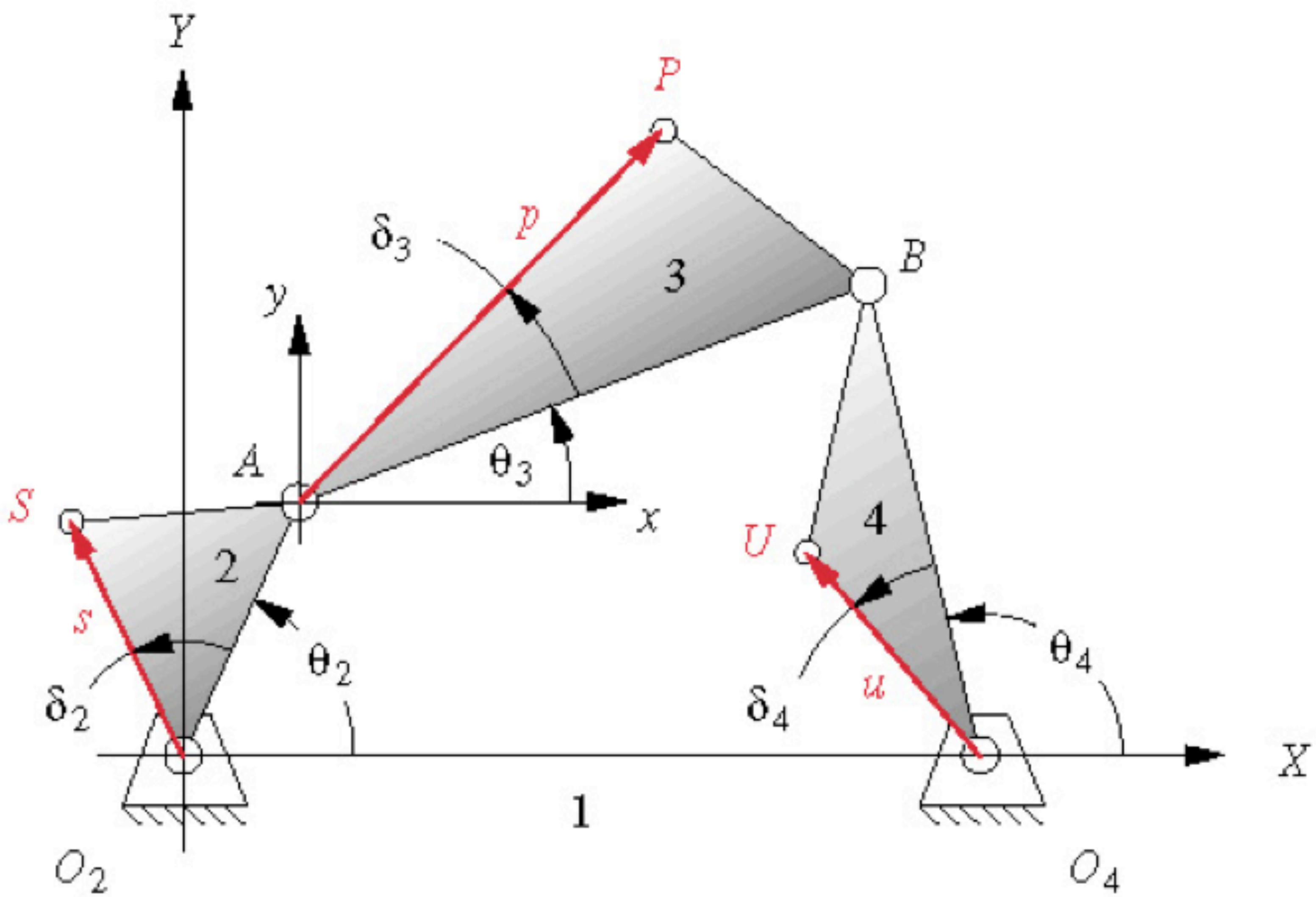
# SLIDERS

I.C. OF TWO BODIES IN SLIDING CONTACT IS LOCATED AT ∞ ALONG NORMAL TO PLANE OF SLIP.

CONNECT I.C. @ ∞ TO A FINITE POINT BY LINE THROUGH IT NORMAL TO PLANE OF SLIP.



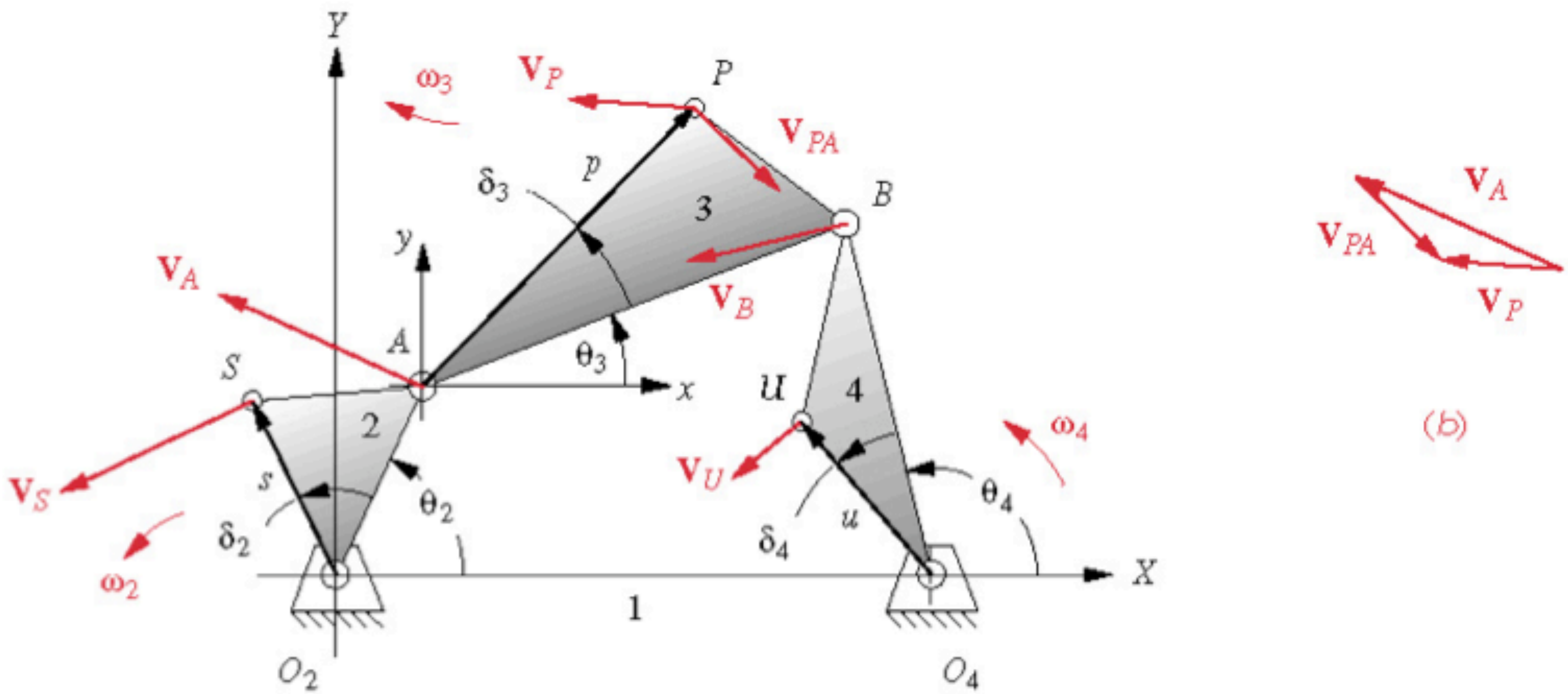
# GENERAL POINTS




$$\mathbf{R}_{SO_2} \sim se^{j(\theta_2+\delta_2)} \qquad \mathbf{R}_{UO_4} \sim ue^{j(\theta_4+\delta_4)}$$

$$\mathbf{R}_{PO_2} \sim ae^{j\theta_2} + pe^{j(\theta_3+\delta_3)}$$

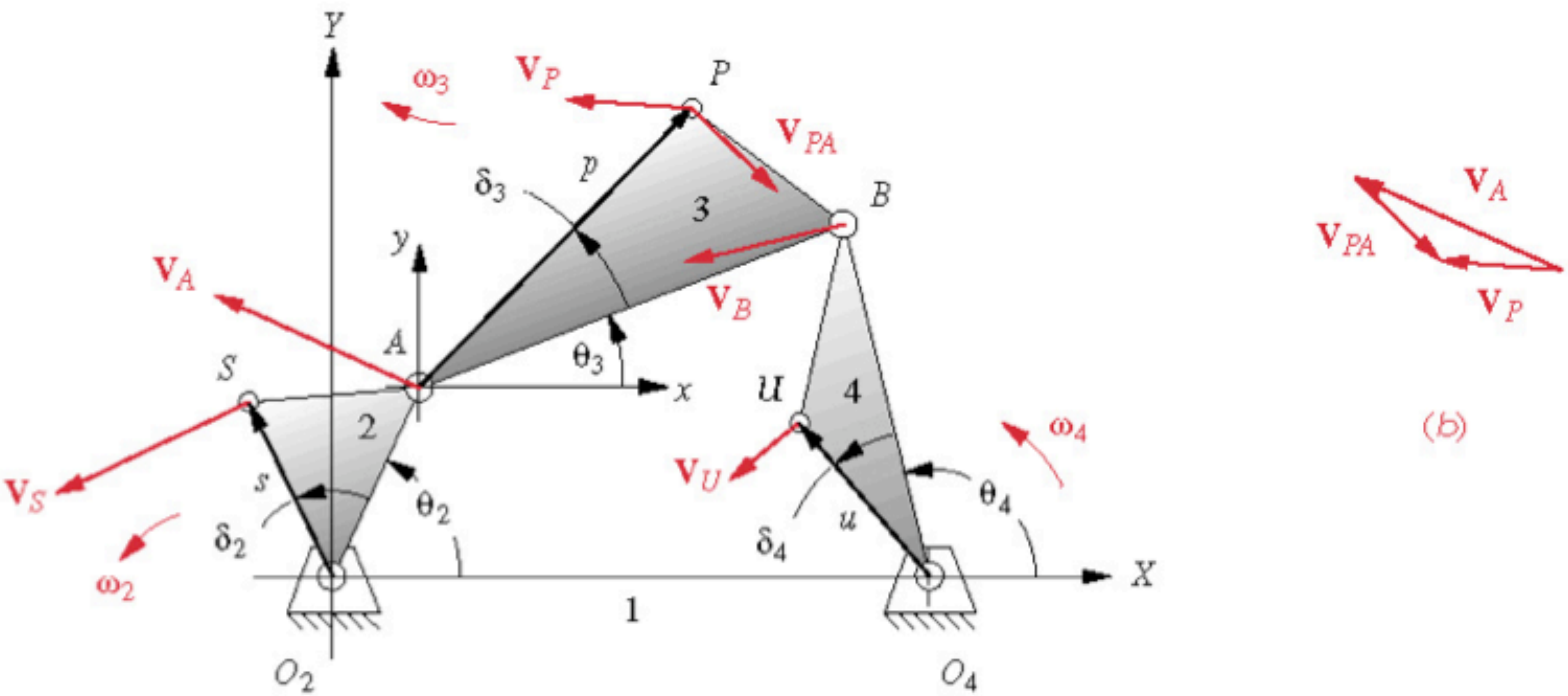
# GENERAL POINTS




$$\mathbf{V}_{SO_2} \sim j\omega_2 s e^{j(\theta_2+\delta_2)} \qquad \mathbf{V}_{UO_4} \sim j\omega_4 u e^{j(\theta_4+\delta_4)}$$

$$\mathbf{V}_{PO_2} \sim j\omega_2 a e^{j\theta_2} + j\omega_3 p e^{j(\theta_3+\delta_3)}$$

# GENERAL POINTS

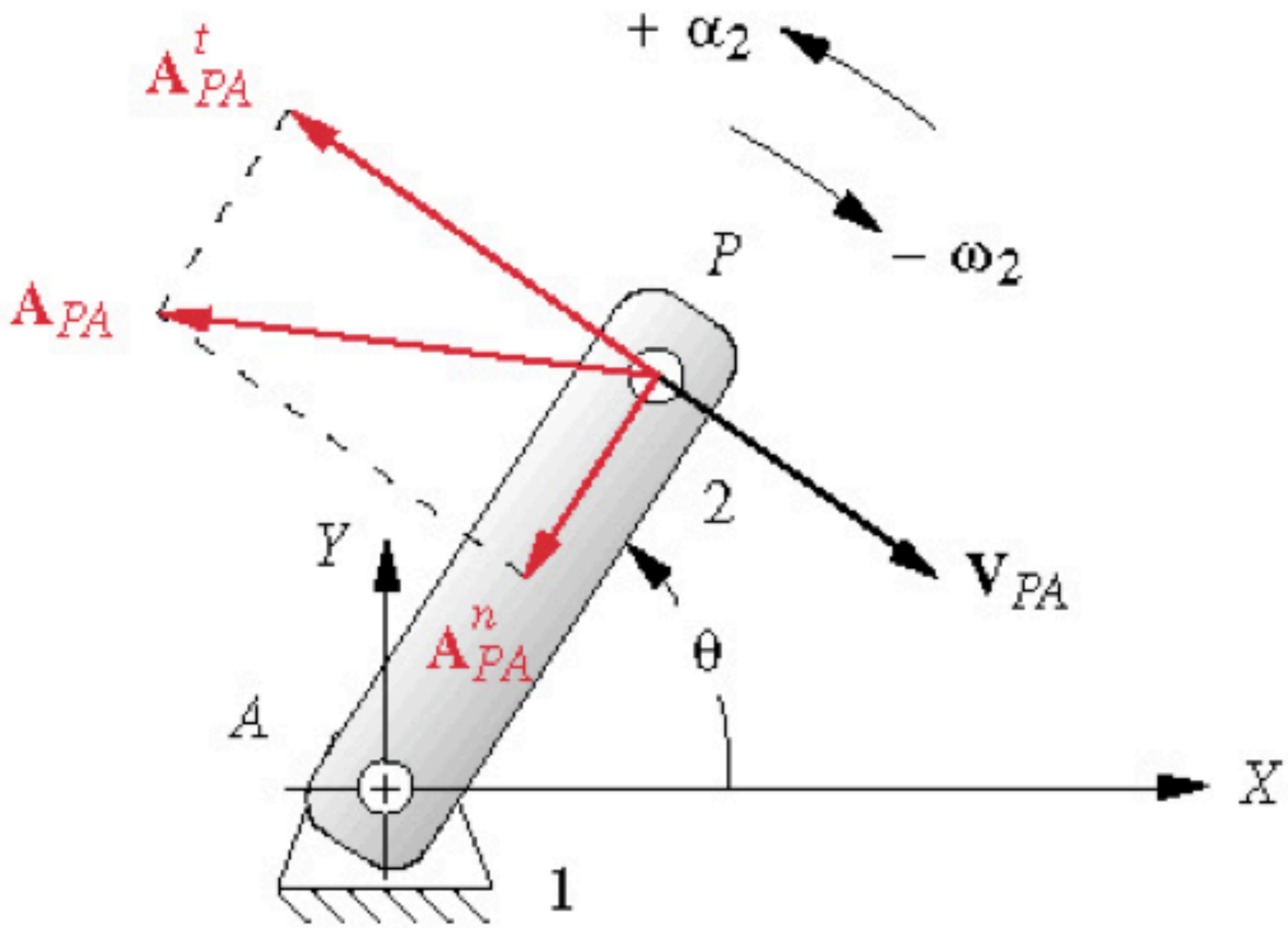



$$\mathbf{V}_S = \boldsymbol{\omega}_2 \times \mathbf{R}_{SO_2} \qquad \mathbf{V}_U = \boldsymbol{\omega}_4 \times \mathbf{R}_{UO_4}$$

$$\mathbf{V}_P = \boldsymbol{\omega}_2 \times \mathbf{R}_{AO_2} + \boldsymbol{\omega}_3 \times \mathbf{R}_{PA}$$

# ACCELERATION ANALYSIS

# PURE ROTATION

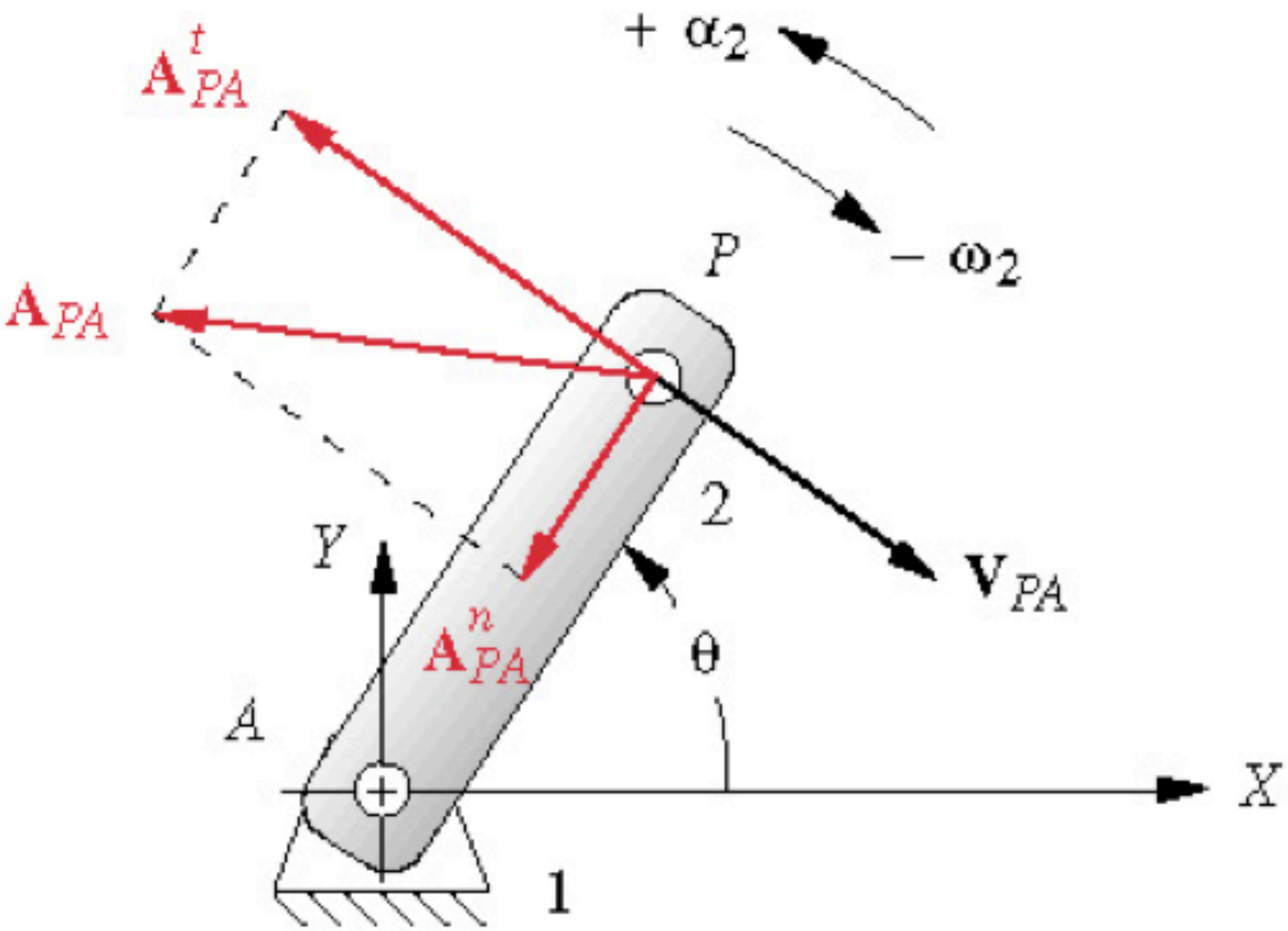



$$A_{PA}^{t} = R\alpha_2 \qquad A_{PA}^{n} = R\omega_2^2$$

$$\mathbf{V}_{PA} = \boldsymbol{\omega}_2 \times \mathbf{R}_{PA} \qquad \mathbf{A}_{PA} = \boldsymbol{\alpha}_2 \times \mathbf{R}_{PA} - \omega_2^2 \mathbf{R}_{PA}$$

$$\mathbf{R}_{PA} \sim Re^{j\theta} \qquad \mathbf{V}_{PA} \sim j\omega_2 Re^{j\theta} \qquad \mathbf{A}_{PA} \sim [j\alpha_2 - \omega_2^2]Re^{j\theta}$$

# PURE ROTATION




$$A^t_{PA} = R\alpha_2 \qquad A^n_{PA} = R\omega_2^2$$

$$\mathbf{V}_{PA} = \boldsymbol{\omega}_2 \times \mathbf{R}_{PA} \qquad \mathbf{A}_{PA} = \boldsymbol{\alpha}_2 \times \mathbf{R}_{PA} - \omega_2^2\mathbf{R}_{PA}$$

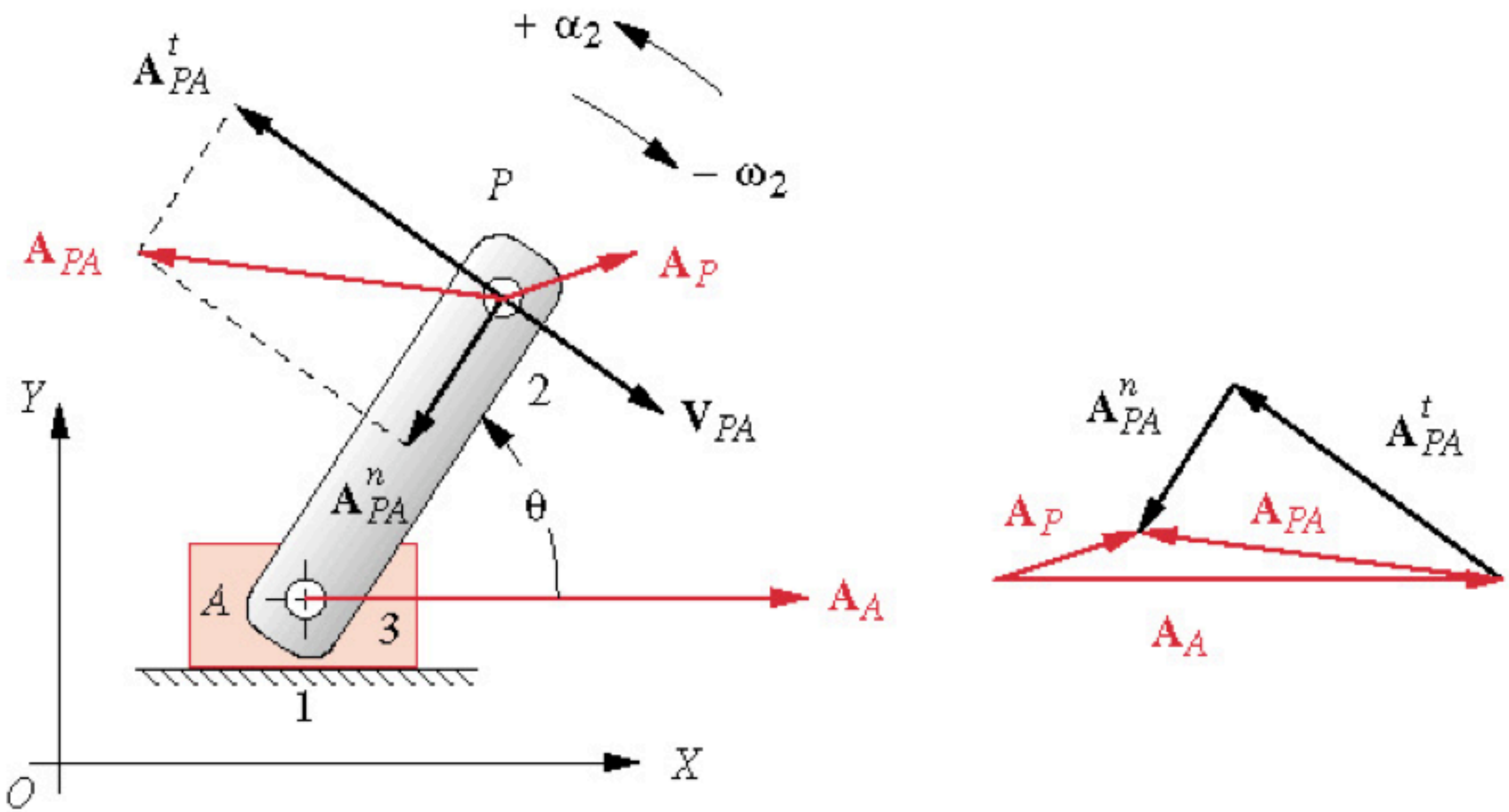
$$\mathbf{R}_{PA} \sim Re^{j\theta} \qquad \mathbf{V}_{PA} \sim j\omega_2 Re^{j\theta} \qquad \mathbf{A}_{PA} \sim [j\alpha_2 - \omega_2^2]Re^{j\theta}$$

# COMPLEX MOTION

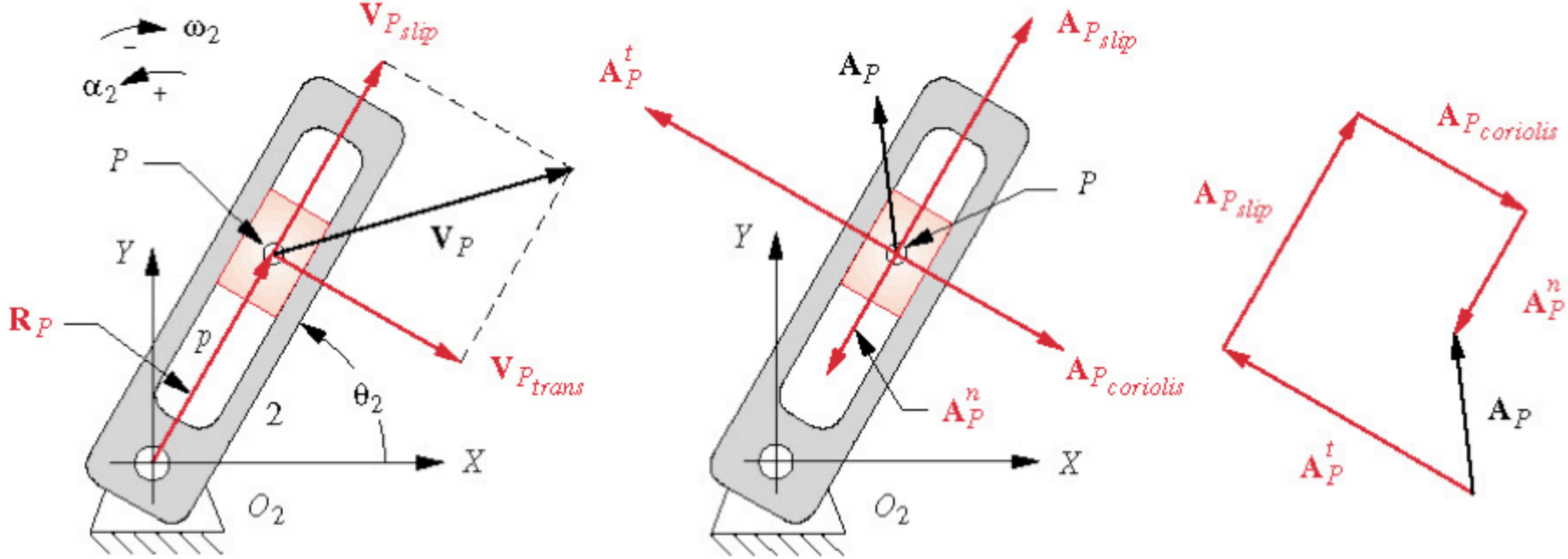
$$\mathbf{V}_{PA} = \boldsymbol{\omega}_2 \times \mathbf{R}_{PA} \qquad \mathbf{A}_{PA} = \boldsymbol{\alpha}_2 \times \mathbf{R}_{PA} - \omega_2^2 \mathbf{R}_{PA}$$

$$\mathbf{V}_P = \mathbf{V}_A + \mathbf{V}_{PA} \qquad \mathbf{A}_P = \mathbf{A}_A + \mathbf{A}_{PA}$$

$$A_{PA}^t = R\alpha_2 \qquad A_{PA}^n = R\omega_2^2$$



# SLIDERS



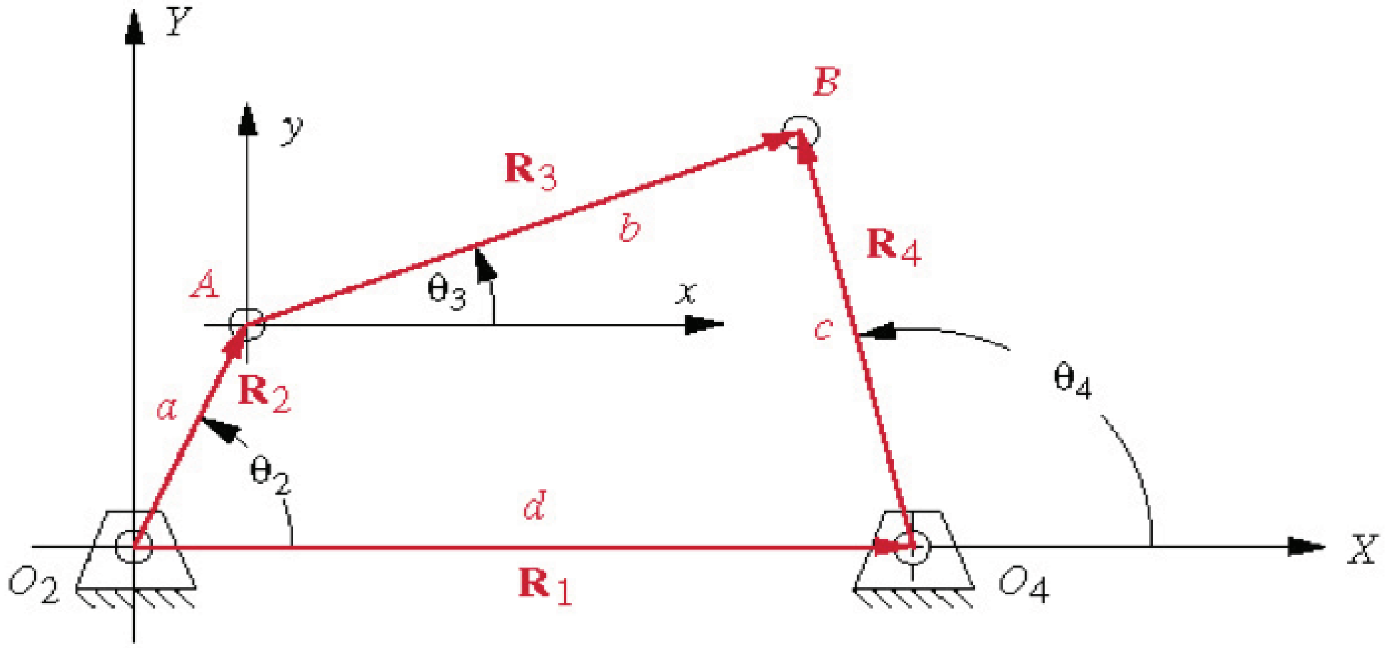

$$\mathbf{R}_{PO_2} \sim pe^{j\theta} \quad \mathbf{V}_{PO_2} \sim [\dot{p}+jp\omega_2]e^{j\theta} \quad \mathbf{A}_{PA} \sim [\ddot{p}+2j\dot{p}\omega_2+jp\alpha_2-p\omega_2^2]e^{j\theta}$$

$$\mathbf{V}_P = \mathbf{V}_{P_{slip}} + \boldsymbol{\omega}_2 \times \mathbf{R}_{PO_2}$$

$$\mathbf{A}_P = \overbrace{\mathbf{A}_{P_{slip}}}^{slip} + \underbrace{\boldsymbol{\alpha}_2 \times \mathbf{R}_{PO_2} - \omega_2^2 \mathbf{R}_{PO_2}}_{rotation} + \overbrace{2\boldsymbol{\omega}_2 \times \mathbf{V}_{P_{slip}}}^{Coriolis}$$

$$A_P^t = p\alpha_2 \qquad A_P^n = p\omega_2^2 \qquad A_{P_{slip}} = \ddot{p} \qquad A_{P_{coriolis}} = 2\dot{p}\omega_2$$

# FOURBAR POSITION ANALYSIS




$$ae^{j\theta_2} + be^{j\theta_3} - ce^{j\theta_4} - d = 0$$

$$a\cos\theta_2 + b\cos\theta_3 = d + c\cos\theta_4$$

$$a\sin\theta_2 + b\sin\theta_3 = c\sin\theta_4$$

# FOURBAR VELOCITY ANALYSIS

$$j\omega_2 a e^{j\theta_2} + j\omega_3 b e^{j\theta_3} - j\omega_4 c e^{j\theta_4} = 0$$

$$a\cos\theta_2\omega_2 + b\cos\theta_3\omega_3 = c\cos\theta_4\omega_4$$

$$a\sin\theta_2\omega_2 + b\sin\theta_3\omega_3 = c\sin\theta_4\omega_4$$

$$\begin{pmatrix} b\cos\theta_3 & -c\cos\theta_4 \\ b\sin\theta_3 & -c\sin\theta_4 \end{pmatrix} \begin{pmatrix} \omega_3 \\ \omega_4 \end{pmatrix} = -a\omega_2 \begin{pmatrix} \cos\theta_2 \\ \sin\theta_2 \end{pmatrix}$$

Link 2 regarded as input link

$$bc\sin(\theta_3-\theta_4)\begin{pmatrix}\omega_3\\ \omega_4\end{pmatrix} = -a\omega_2\begin{pmatrix}-c\sin\theta_4 & c\cos\theta_4\\ -b\sin\theta_3 & b\cos\theta_3\end{pmatrix}\begin{pmatrix}\cos\theta_2\\ \sin\theta_2\end{pmatrix}$$

$$bc\sin(\theta_3-\theta_4)\begin{pmatrix}\omega_3\\ \omega_4\end{pmatrix} = -a\omega_2\begin{pmatrix}c\sin(\theta_2-\theta_4)\\ b\sin(\theta_2-\theta_3)\end{pmatrix}$$

$$\omega_3 = \frac{a\sin(\theta_4-\theta_2)}{b\sin(\theta_3-\theta_4)}\omega_2 \quad \curvearrowleft$$

$$\omega_4 = \frac{a\sin(\theta_2-\theta_3)}{c\sin(\theta_4-\theta_3)}\omega_2 \quad \curvearrowleft$$

NOTE: Positive direction for all angular velocities is counter-clockwise.

## FOURBAR ACCELERATION ANALYSIS

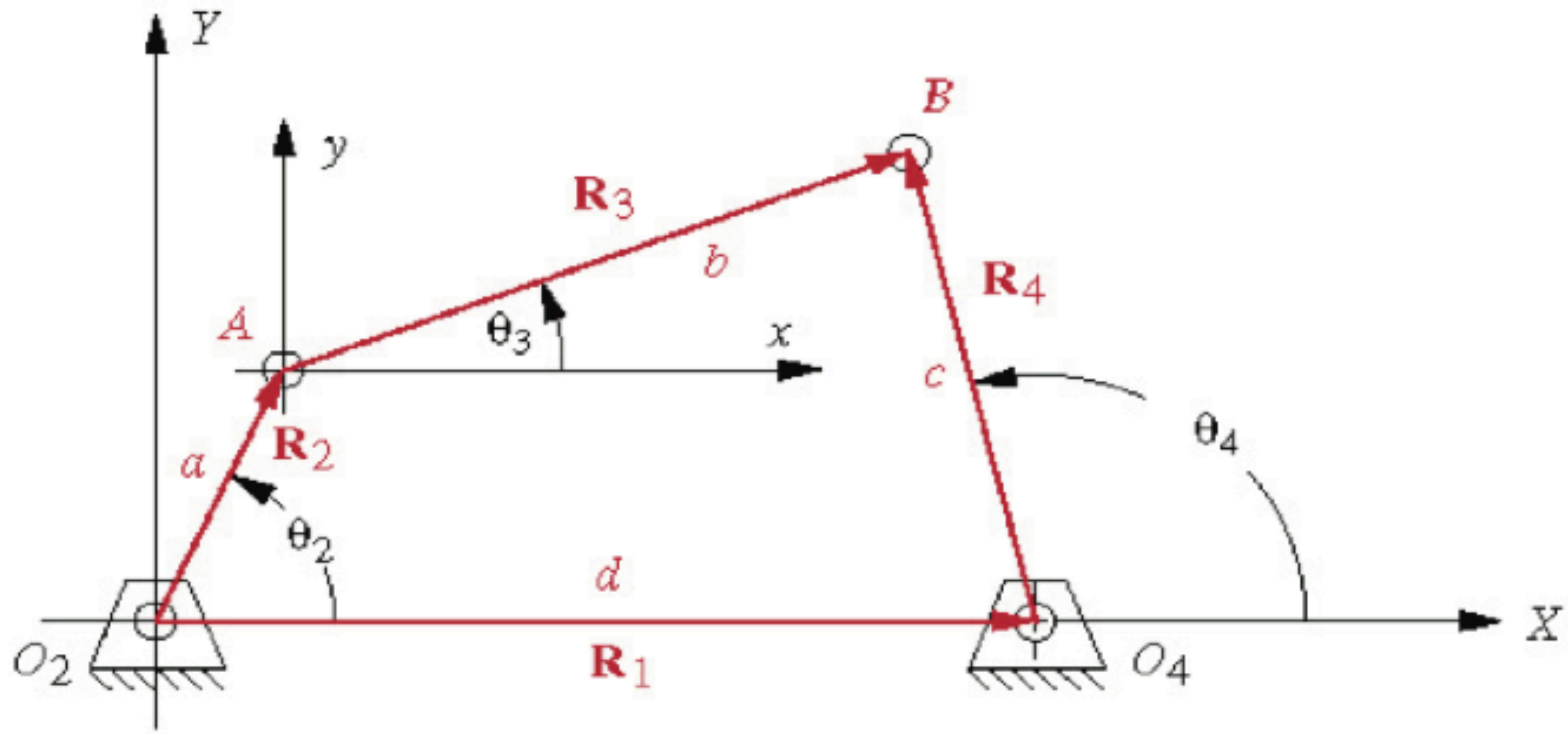
$$a[j\alpha_2 - \omega_2^2]e^{j\theta_2} + b[j\alpha_3 - \omega_3^2]e^{j\theta_3} - c[j\alpha_4 - \omega_4^2]e^{j\theta_4} = 0$$

$$-a\sin\theta_2\,\alpha_2 - b\sin\theta_3\,\alpha_3 + c\sin\theta_4\,\alpha_4 = a\cos\theta_2\,\omega_2^2 + b\cos\theta_3\,\omega_3^2 - c\cos\theta_4\,\omega_4^2$$

$$a\cos\theta_2\,\alpha_2 + b\cos\theta_3\,\alpha_3 - c\cos\theta_4\,\alpha_4 = a\sin\theta_2\,\omega_2^2 + b\sin\theta_3\,\omega_3^2 - c\sin\theta_4\,\omega_4^2$$

$$\begin{pmatrix} b\cos\theta_3 & -c\cos\theta_4 \\ b\sin\theta_3 & -c\sin\theta_4 \end{pmatrix} \begin{pmatrix} \alpha_3 \\ \alpha_4 \end{pmatrix} = \begin{pmatrix} k_1 \\ k_2 \end{pmatrix}$$

$$\begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = -a\alpha_2 \begin{pmatrix} \cos\theta_2 \\ \sin\theta_2 \end{pmatrix} + a\omega_2^2 \begin{pmatrix} \sin\theta_2 \\ -\cos\theta_2 \end{pmatrix} + b\omega_3^2 \begin{pmatrix} \sin\theta_3 \\ -\cos\theta_3 \end{pmatrix} - c\omega_4^2 \begin{pmatrix} \sin\theta_4 \\ -\cos\theta_4 \end{pmatrix}$$

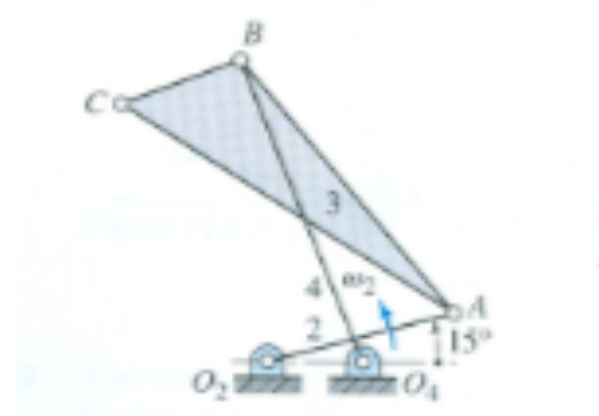


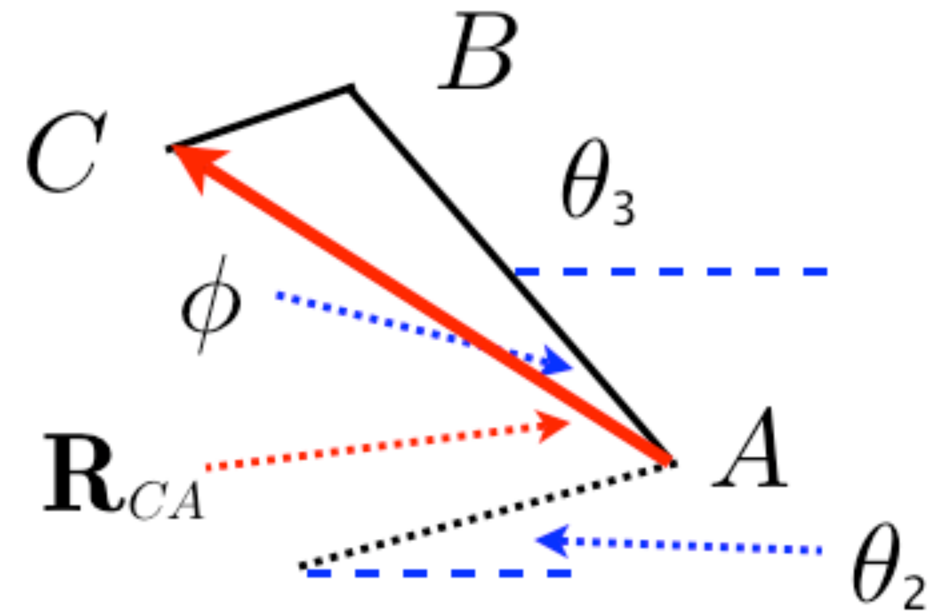

$$bc\sin(\theta_3-\theta_4)\begin{pmatrix}\alpha_3\\ \alpha_4\end{pmatrix}=\begin{pmatrix}-c\sin\theta_4 & c\cos\theta_4\\ -b\sin\theta_3 & b\cos\theta_3\end{pmatrix}\begin{pmatrix}k_1\\ k_2\end{pmatrix}$$

$$bc\sin(\theta_3-\theta_4)\begin{pmatrix}\alpha_3\\ \alpha_4\end{pmatrix}=a\alpha_2\begin{pmatrix}c\sin(\theta_4-\theta_2)\\ b\sin(\theta_3-\theta_2)\end{pmatrix}-a\omega_2^2\begin{pmatrix}c\cos(\theta_4-\theta_2)\\ b\cos(\theta_3-\theta_2)\end{pmatrix}\dots$$

$$\dots-b\omega_3^2\begin{pmatrix}c\cos(\theta_4-\theta_3)\\ b\end{pmatrix}+c\omega_4^2\begin{pmatrix}c\\ b\cos(\theta_4-\theta_3)\end{pmatrix}$$

# EXAMPLE




$$\mathbf{R}_{AO_2} = \left|\mathbf{R}_{AO_2}\right|\left[\cos(\theta_2)\hat{\mathbf{i}} + \sin(\theta_2)\hat{\mathbf{j}}\right] = \left[.1449\hat{\mathbf{i}} + .0388\hat{\mathbf{j}}\right]\text{ m}$$

$$\mathbf{R}_{CA} = \left|\mathbf{R}_{CA}\right|\left[\cos(\theta_3 + \phi)\hat{\mathbf{i}} + \sin(\theta_3 + \phi)\hat{\mathbf{j}}\right]$$

$$\mathbf{R}_{CA} = 0.3[\cos(146.44°)\hat{\mathbf{i}} + \sin(146.44°)\hat{\mathbf{j}}] = \left[-.25\hat{\mathbf{i}} + .1658\hat{\mathbf{j}}\right]\text{ m}$$

1.(30%) The link-lengths for the four-bar mechanism shown are $R_1 = 75\,\text{mm}$, $R_2 = 150\,\text{mm}$, $R_3 = R_4 = 250\,\text{mm}$ . At the instant shown, the angles, angular velocities and the angular accelerations of the links are given to be $\theta_2 = 15°$, $\theta_3 = 128.25°$, $\theta_4 = 109.85°$, $\boldsymbol{\omega}_2 = 5\,\text{rad/s} \curvearrowleft$, $\boldsymbol{\omega}_3 = 9.47\,\text{rad/s} \curvearrowleft$, $\boldsymbol{\omega}_4 = 8.73 \curvearrowleft$, $\boldsymbol{\alpha}_2 = 0\,\text{rad/s}^2 \curvearrowleft$, $\boldsymbol{\alpha}_3 = 24.00\,\text{rad/s}^2 \curvearrowright$ , $\boldsymbol{\alpha}_4 = 36.13\,\text{rad/s}^2 \curvearrowright$. $AB$ makes an angle $\phi = 18.19°$ with $AC$. $AC = 300\,\text{mm}$ Find the horizontal and vertical components of the acceleration $\mathbf{a}_C$ of the coupler-point $C$. Link angles are measured counter-clockwise relative to the ground link.

$$\mathbf{R}_{AO_2} = \left|\mathbf{R}_{AO_2}\right|\left[\cos(\theta_2)\hat{\mathbf{i}} + \sin(\theta_2)\hat{\mathbf{j}}\right] = \left[.1449\hat{\mathbf{i}} + .0388\hat{\mathbf{j}}\right] \text{ m}$$

$$\mathbf{R}_{CA} = 0.3[\cos(146.44°)\hat{\mathbf{i}} + \sin(146.44°)\hat{\mathbf{j}}] = \left[-.25\hat{\mathbf{i}} + .1658\hat{\mathbf{j}}\right] \text{ m}$$

$$\mathbf{a}_C = \mathbf{a}_A + \mathbf{a}_{CA}$$

$$\mathbf{a}_A = \boldsymbol{\alpha}_2 \times \mathbf{R}_{AO_2} - \omega_2^2\mathbf{R}_{AO_2} = \mathbf{0} - 25\left[.1449\hat{\mathbf{i}} + .0388\hat{\mathbf{j}}\right] \text{ m/s}^2$$

$$\mathbf{a}_{CA} = \boldsymbol{\alpha}_3 \times \mathbf{R}_{CA} - \omega_3^2\mathbf{R}_{CA} = -24\hat{\mathbf{k}} \times \left[-.25\hat{\mathbf{i}} + .1658\hat{\mathbf{j}}\right] - (9.47)^2\left[-.25\hat{\mathbf{i}} + .1658\hat{\mathbf{j}}\right] \text{ m/s}^2$$

$$\mathbf{a}_{C_x} = -(25)(.1449) + (24)(.1658) + (9.47)^2(.25) = 22.78\,\text{m/s}^2$$

$$\mathbf{a}_{C_y} = -(25)(.0388) + (24)(.25) - (9.47)^2(.1658) = -9.84\,\text{m/s}^2$$

# REVIEW LECTURE 2: KINETICS

# MECHANICAL ADVANTAGE

# POWER AND EFFICIENCY

## POWER OF A FORCE

$$P = \mathbf{F} \cdot \mathbf{V}$$

$\mathbf{V}$ IS THE VELOCITY OF THE POINT OF APPLICATION OF THE FORCE $\mathbf{F}$.

## POWER OF A COUPLE

$$P = \mathbf{T} \cdot \boldsymbol{\omega}$$

$\boldsymbol{\omega}$ IS THE ANGULAR VELOCITY OF THE BODY TO WHICH COUPLE $\mathbf{T}$ IS APPLIED.

## IDEAL MECHANISM

$$P_{in} = P_{out}$$

$$\boxed{\textbf{POWER IDENTITY}}$$

$$P_{in} = \mathbf{F}_{in} \cdot \mathbf{V}_{in} = F_{in} V_{in} \cos\phi_{in} = P_{out} = \mathbf{F}_{out} \cdot \mathbf{V}_{out} = F_{out} V_{out} \cos\phi_{out}$$

$$\boxed{\textbf{MECHANICAL ADVANTAGE}}$$

$$M.A. = \frac{F_{out}}{F_{in}} = \frac{V_{in} \cos\phi_{in}}{V_{out} \cos\phi_{out}}$$

# MECHANICAL ADVANTAGE ANALYSIS

GOAL: BY COMBINING THE POWER IDENTITY WITH A KINEMATICAL ANALYSIS OF THE MECHANISM, TO EXPRESS THE MECHANICAL ADVANTAGE PURELY IN TERMS OF LENGTHS ASSOCIATED WITH THE CONFIGURATION.

# EXAMPLE

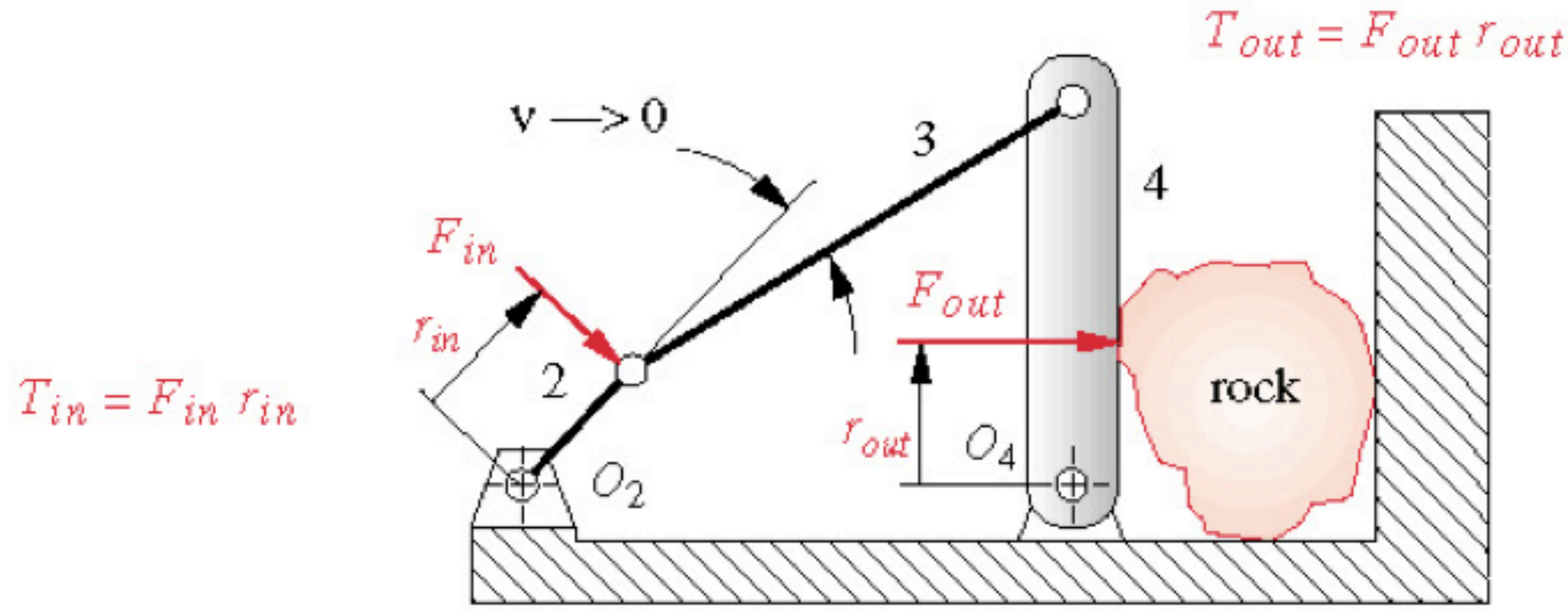



**FIGURE 6-11**

"Rock-crusher" toggle mechanism

$$F_{in} r_{in}\, \omega_2 = F_{out} r_{out}\, \omega_4$$

$$M.A. = \frac{F_{out}}{F_{in}} = \frac{r_{in}\, \omega_2}{r_{out}\, \omega_4}$$

# FOURBAR INSTANT CENTERS

$$\omega_3 = \frac{R_2}{d_A}\omega_2 \quad \curvearrowright$$

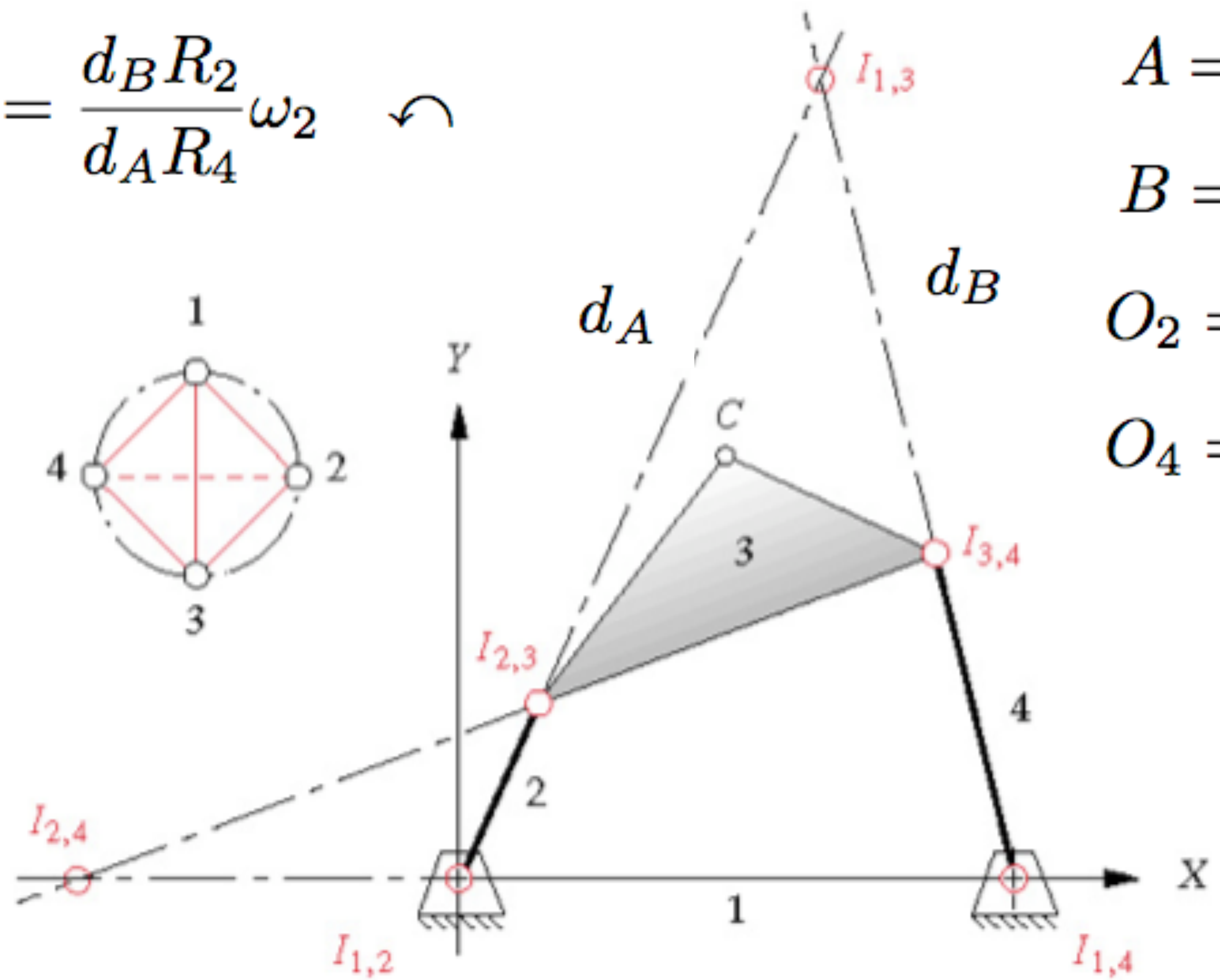
$$\omega_4 = \frac{d_B}{R_4}\omega_3 = \frac{d_B R_2}{d_A R_4}\omega_2 \quad \curvearrowleft$$

$$A = I_{2,3}$$

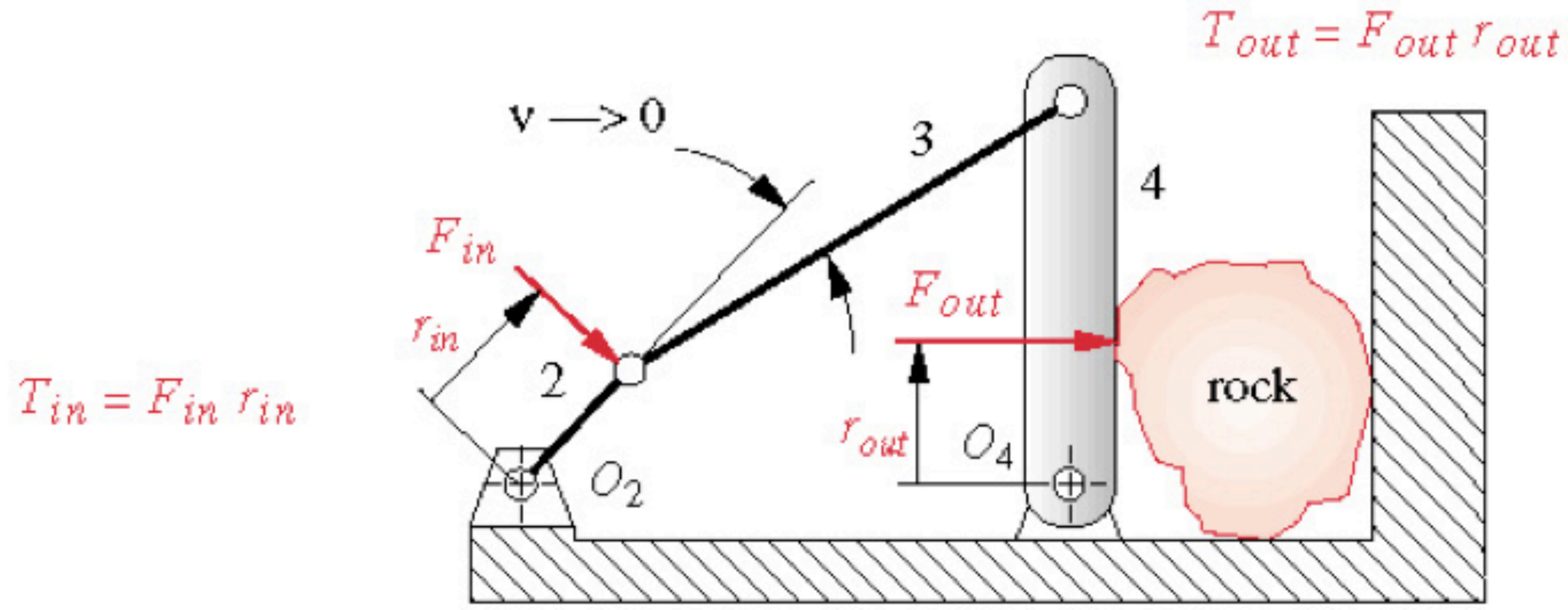
$$B = I_{3,4}$$

$$O_2 = I_{1,2}$$

$$O_4 = I_{1,4}$$



# EXAMPLE

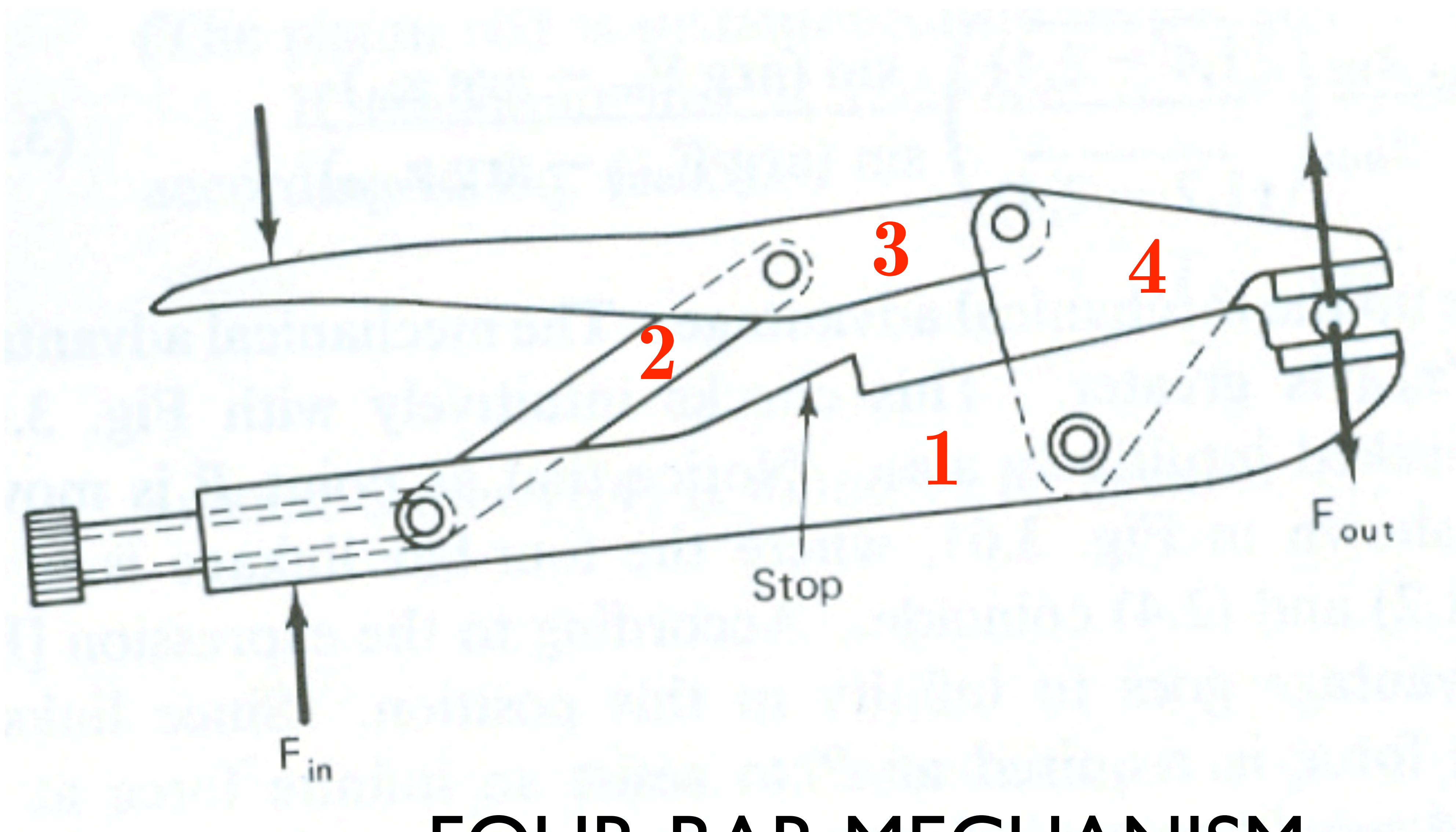



$$F_{in} r_{in}\, \omega_2 = F_{out} r_{out}\, \omega_4$$

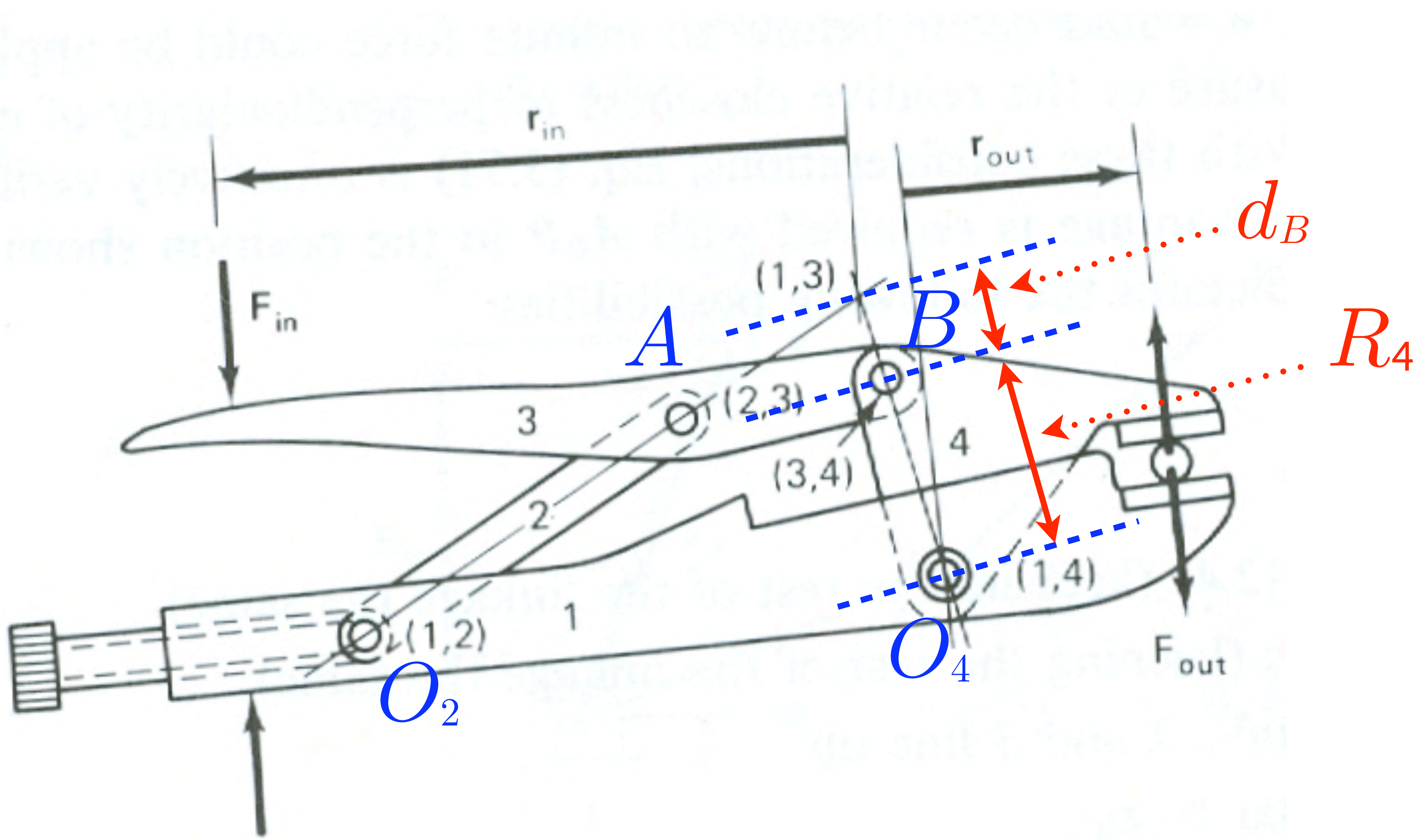
$$M.A. = \frac{F_{out}}{F_{in}} = \frac{r_{in}\, d_A R_4}{r_{out}\, d_B R_2}$$

$$\text{AS } \ \nu \to 0, I_{1,3} \to B, d_B \to 0, \text{M.A.} \to \infty.$$

# EXAMPLE: WIRE CUTTERS




FOUR-BAR MECHANISM



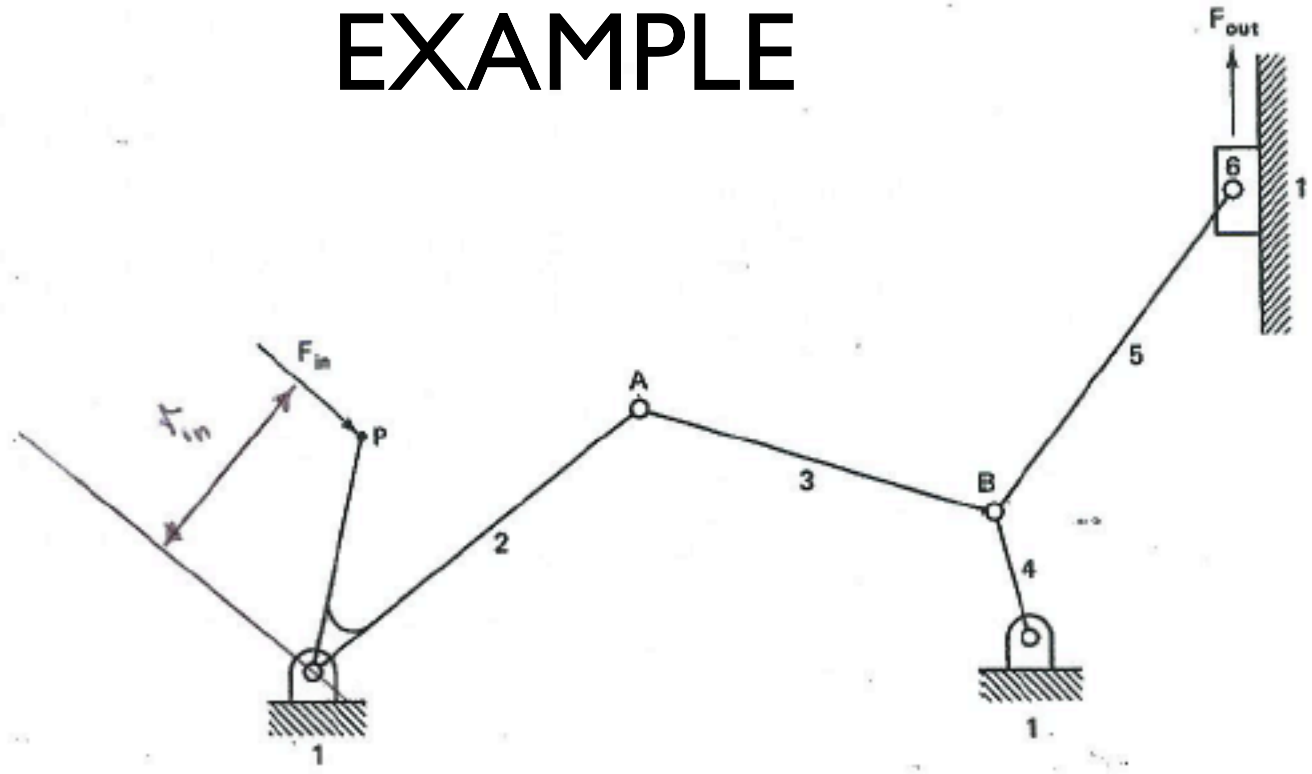

$$F_{in} r_{in}\, \omega_3 = F_{out} r_{out}\, \omega_4$$

$$M.A. = \frac{F_{out}}{F_{in}} = \frac{r_{in}\, \omega_3}{r_{out}\, \omega_4} = \frac{r_{in}\, R_4}{r_{out}\, d_B}$$

$$\text{AS } \; I_{1,3} \rightarrow B, d_B \rightarrow 0, \text{M.A.} \rightarrow \infty.$$

M.A. LARGE NEAR TOGGLE. STOP LIMITS OVERSHOOT OF TOGGLE.

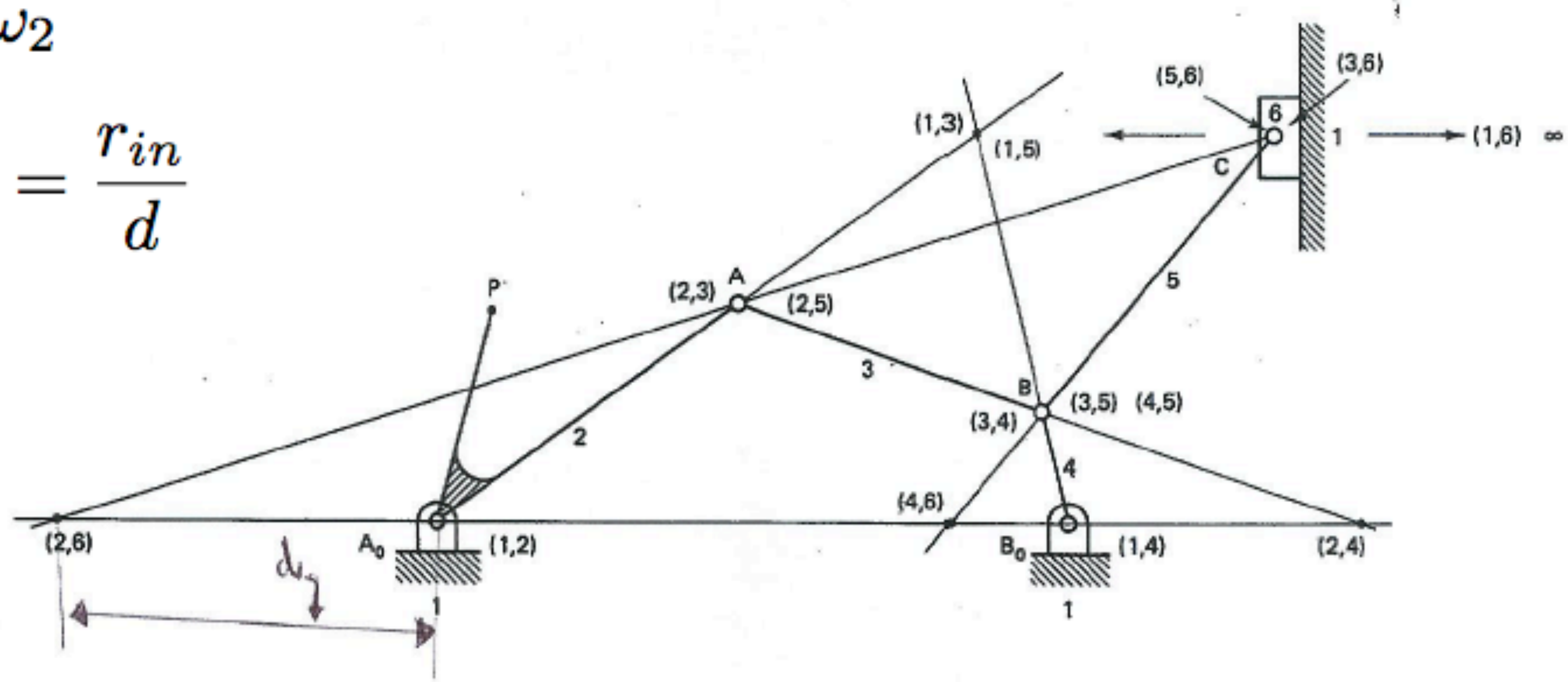
# EXAMPLE



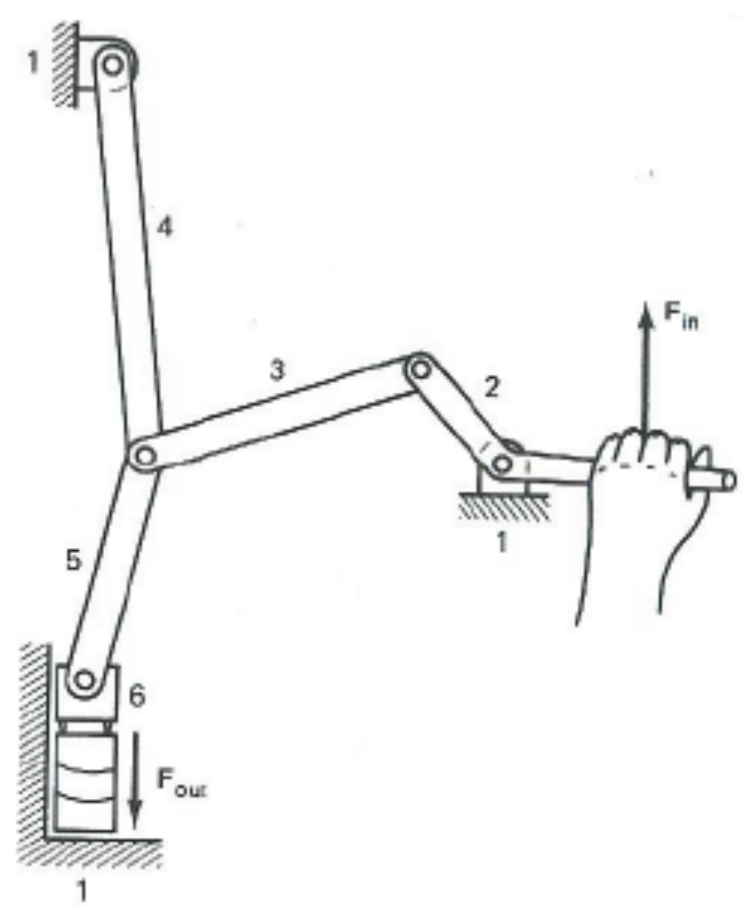

$$F_{in} r_{in} \omega_2 = F_{out} V_{out}$$

$$V_{out} = d\omega_2$$

$$M.A. = \frac{F_{out}}{F_{in}} = \frac{r_{in}}{d}$$



# EXAMPLE

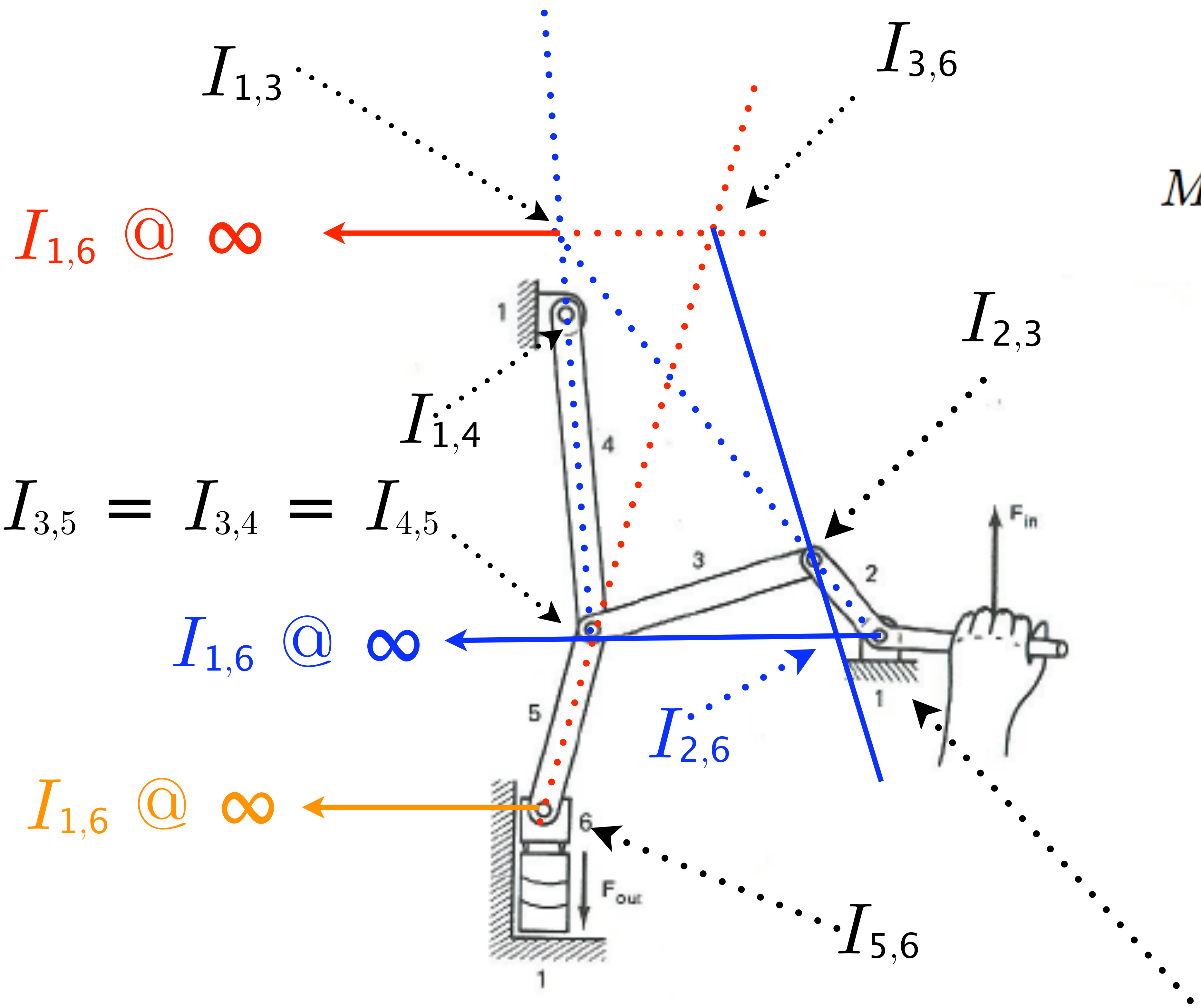


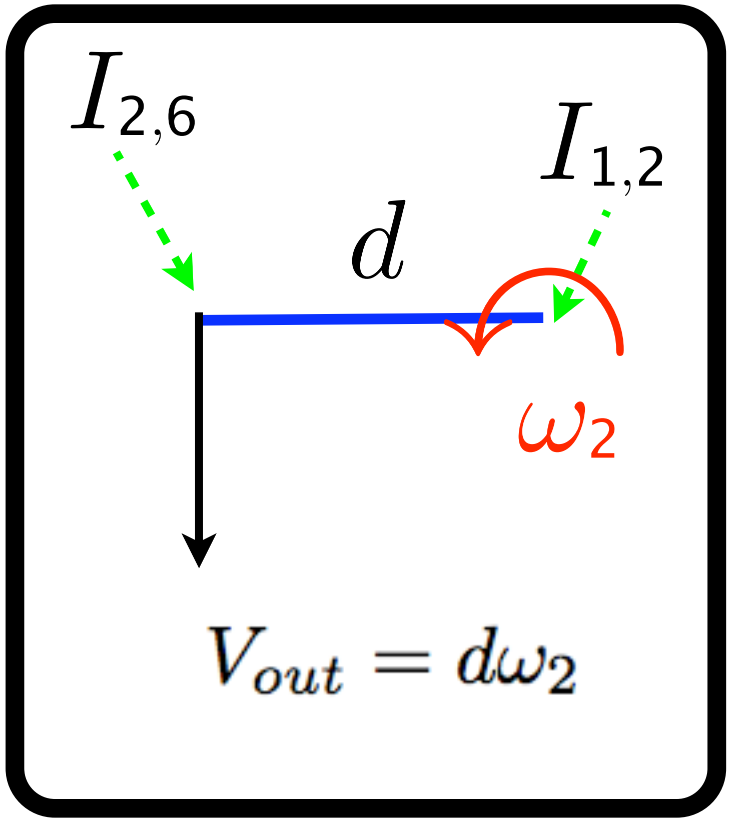
# EXAMPLE

$$F_{in} r_{in} \omega_2 = F_{out} V_{out}$$

$$V_{out} = d\omega_2$$

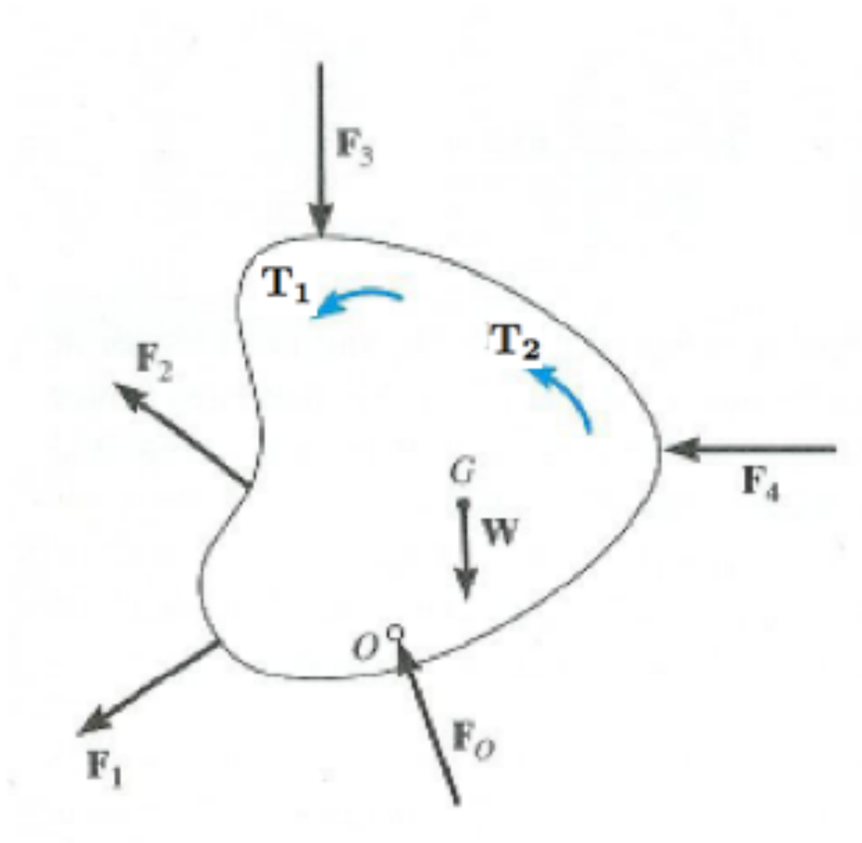
$$M.A. = \frac{F_{out}}{F_{in}} = \frac{r_{in}}{d}$$



# FORCE ANALYSIS

# EQUATIONS OF KINETICS




$$M\mathbf{A}_G = \mathbf{F}_O + \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 - W\hat{\mathbf{j}}$$

$$I_G\alpha\hat{\mathbf{k}} = \mathbf{T}_1 + \mathbf{T}_2 + \mathbf{R}_O \times \mathbf{F}_O + \sum_{k=1}^{4}\{\mathbf{R}_k \times \mathbf{F}_k\} = \sum \mathbf{T}_G$$

# WHAT'S SPECIAL ABOUT MECHANISMS ?

- MULTI-BODY PROBLEMS. THESE GET MESSY FAST. A SYSTEMATIC APPROACH IS ADVISABLE.
- KINEMATICS KNOWN UP-FRONT (DESIGNED IN).
- UNKNOWNS - DRIVING INPUTS AND JOINT FORCES.

# NOMENCLATURE

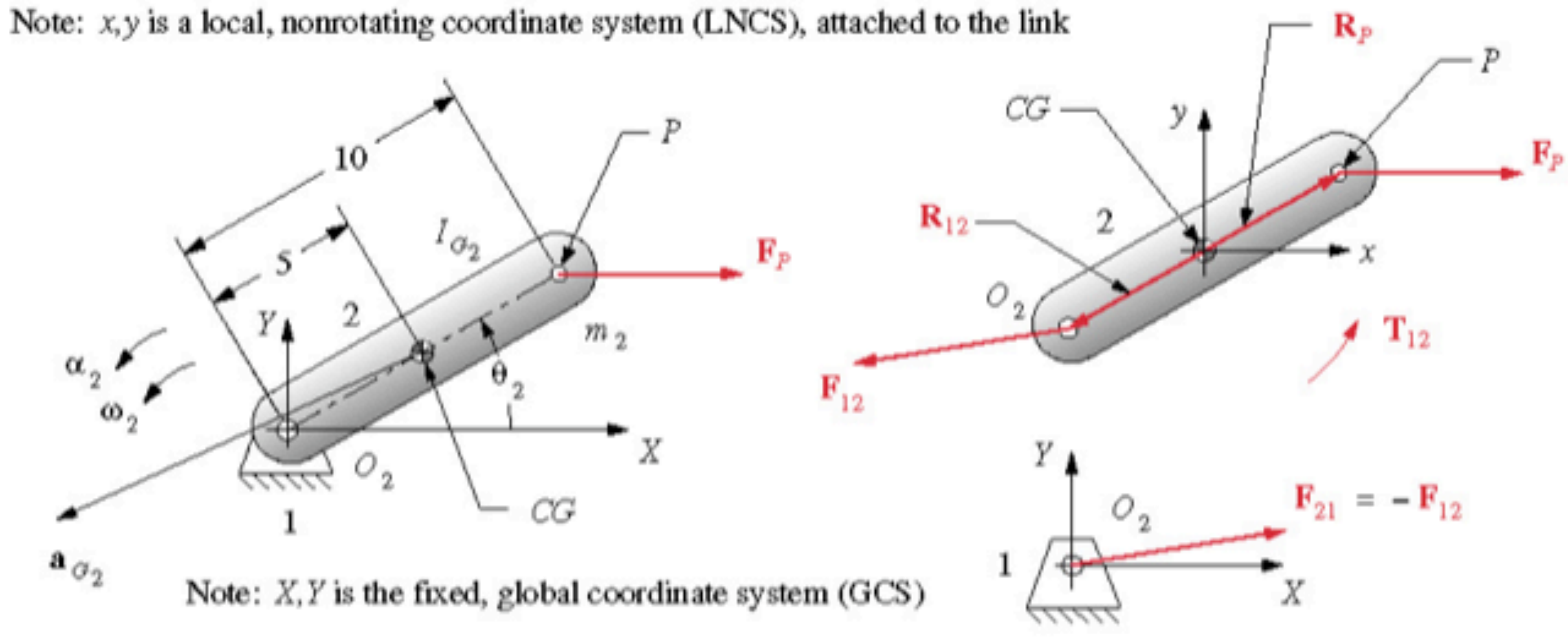
$\mathbf{R}_{ap}$ IS THE POSITION VECTOR OF THE JOINT BETWEEN LINK $a$ ("ADJACENT") AND LINK $p$ ("PARENT") RELATIVE TO THE CG of LINK $p$.

THUS, $\mathbf{R}_{32}$ IS THE POSITION VECTOR RELATIVE TO $CG_2$ OF THE JOINT BETWEEN LINKS 3 AND 2 .

$\mathbf{F}_{ap}$ IS THE FORCE EXPERIENCED BY LINK $p$ AT THE JOINT BETWEEN LINKS $a$ AND $p$.

THUS, $\mathbf{F}_{32}$ IS THE FORCE EXPERIENCED BY LINK 2 AT THE JOINT BETWEEN LINKS 3 AND 2. $\mathbf{F}_{23} = -\mathbf{F}_{32}$.

# SINGLE, GROUNDED ROTOR




(a) Kinematic diagram (b) Force (free-body) diagrams

**FIGURE 11-1**

Dynamic force analysis of a single link in pure rotation

$$m_2 \mathbf{a}_{G_2} = \mathbf{F}_{12} + \mathbf{F}_P$$

$$I_{G_2} \alpha_2 \hat{\mathbf{k}} = T_{12} \hat{\mathbf{k}} + \mathbf{R}_{12} \times \mathbf{F}_{12} + \mathbf{R}_P \times \mathbf{F}_P$$

# IN COMPONENTS

$$F_{12_x} = m_2 a_{G2_x} - F_{P_x}$$

$$F_{12_y} = m_2 a_{G2_y} - F_{P_y}$$

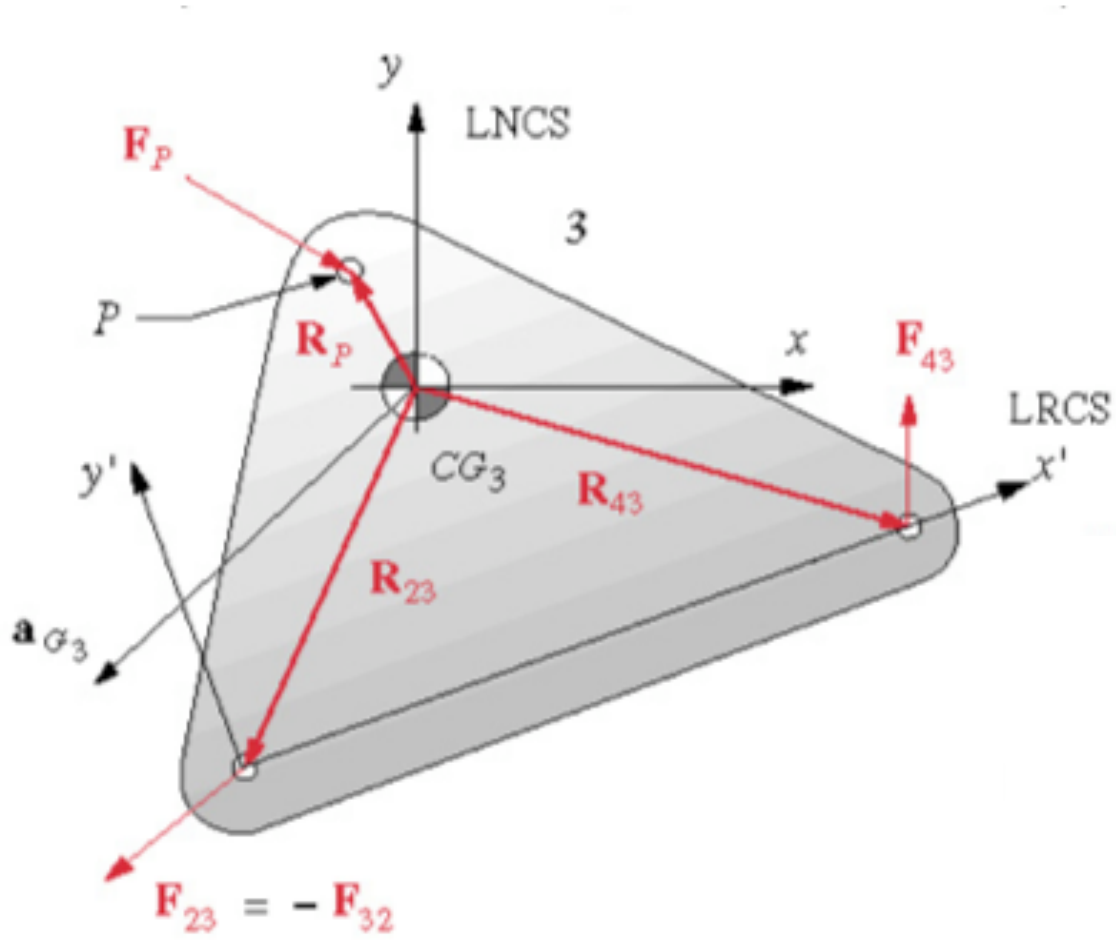
$$-R_{12_y} F_{12_x} + R_{12_x} F_{12_y} + T_{12} = I_{G_2} \alpha_2 + R_{P_y} F_{P_x} - R_{P_x} F_{P_y}$$

..... 3 LINEAR EQUATIONS IN 3 UNKNOWNS

# IN MATRIX FORM

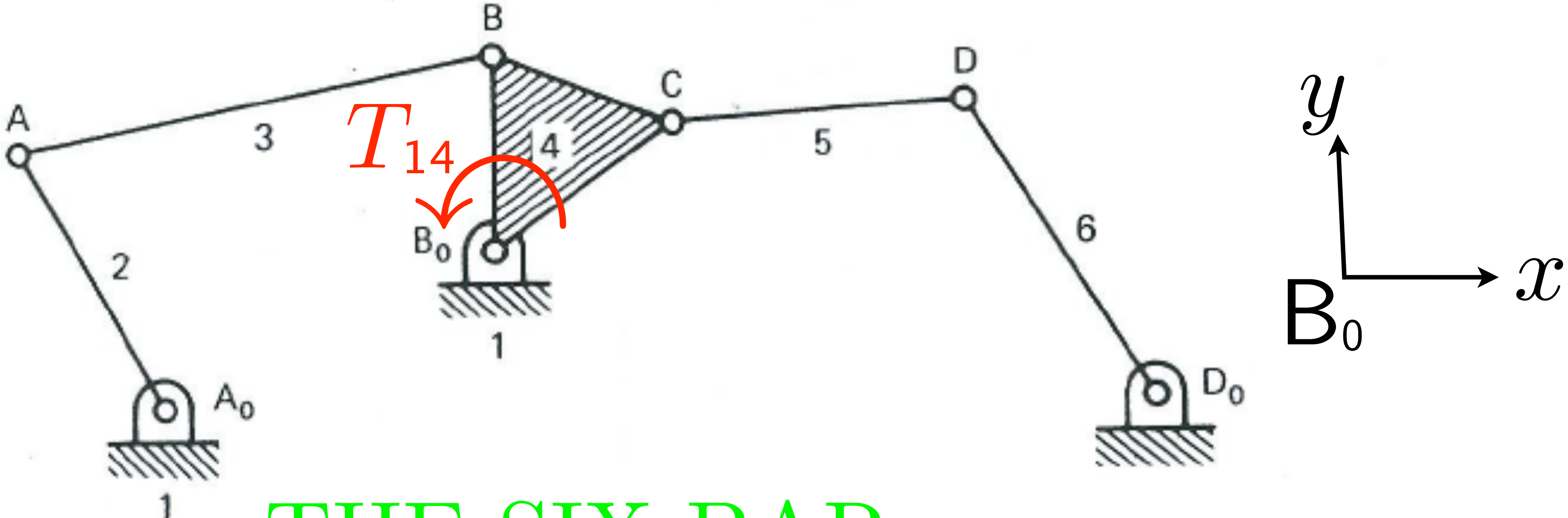
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -R_{12_y} & R_{12_x} & 1 \end{bmatrix} \begin{bmatrix} F_{12_x} \\ F_{12_y} \\ T_{12} \end{bmatrix} = \begin{bmatrix} m_2 a_{G2_x} - F_{P_x} \\ m_2 a_{G2_y} - F_{P_y} \\ I_{G_2}\alpha_2 + R_{P_y} F_{P_x} - R_{P_x} F_{P_y} \end{bmatrix}$$

# LINK 3

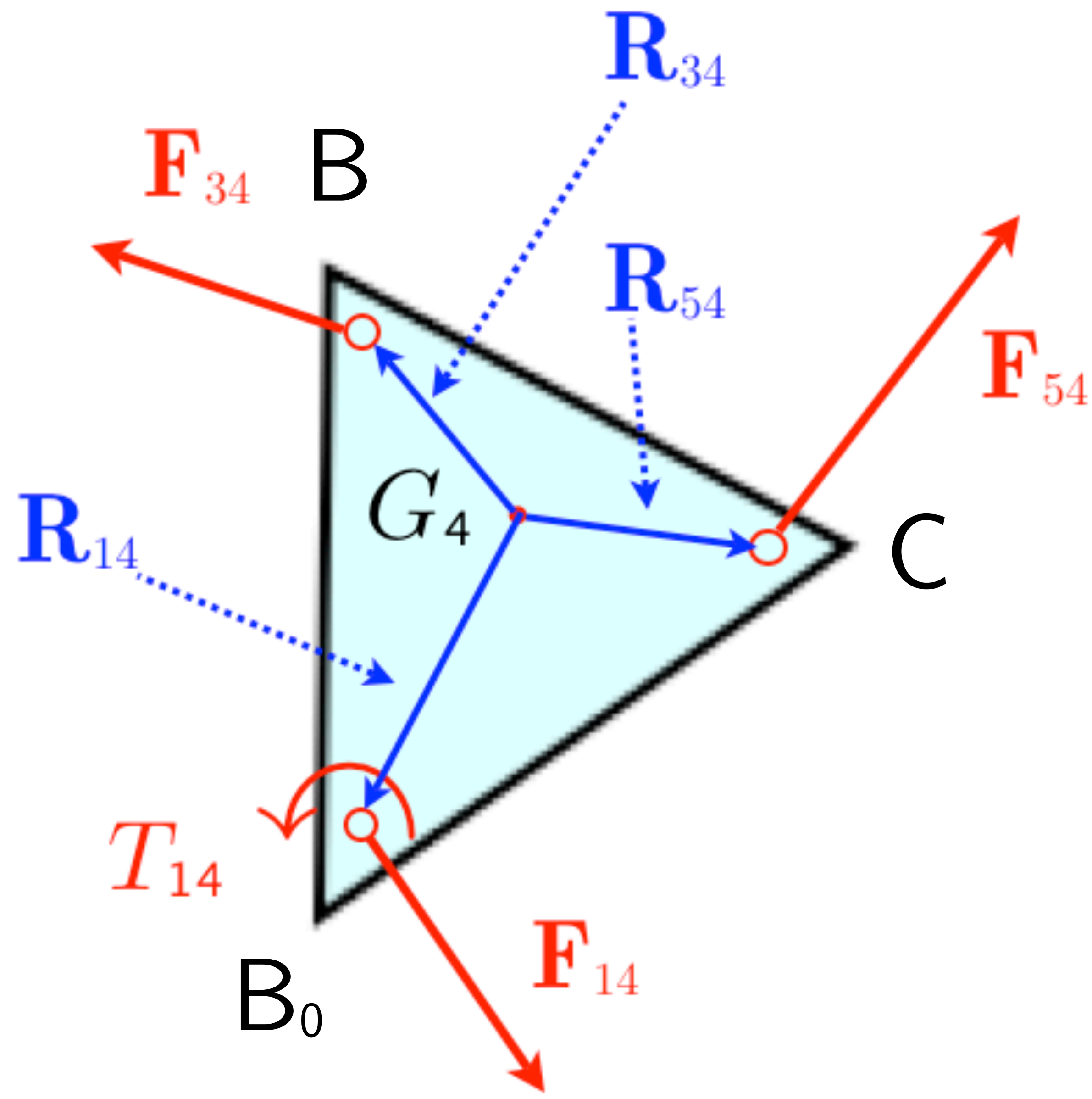


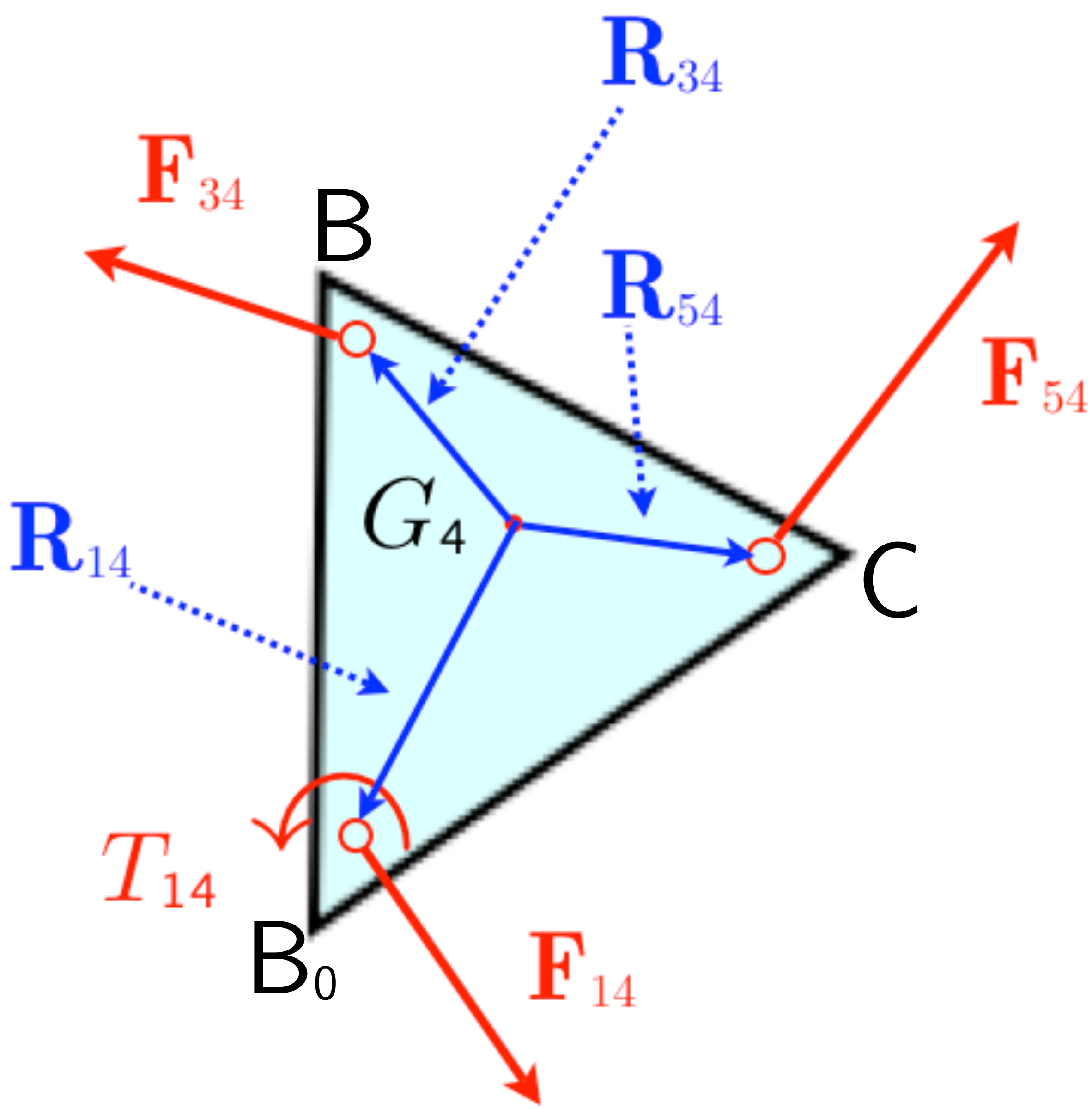

$$m_3 \mathbf{a}_{G_3} = \mathbf{F}_{43} - \mathbf{F}_{32} + \mathbf{F}_P$$

$$I_{G_3} \alpha_3 \hat{\mathbf{k}} = \mathbf{R}_{43} \times \mathbf{F}_{43} - \mathbf{R}_{23} \times \mathbf{F}_{32} + \mathbf{R}_P \times \mathbf{F}_P$$



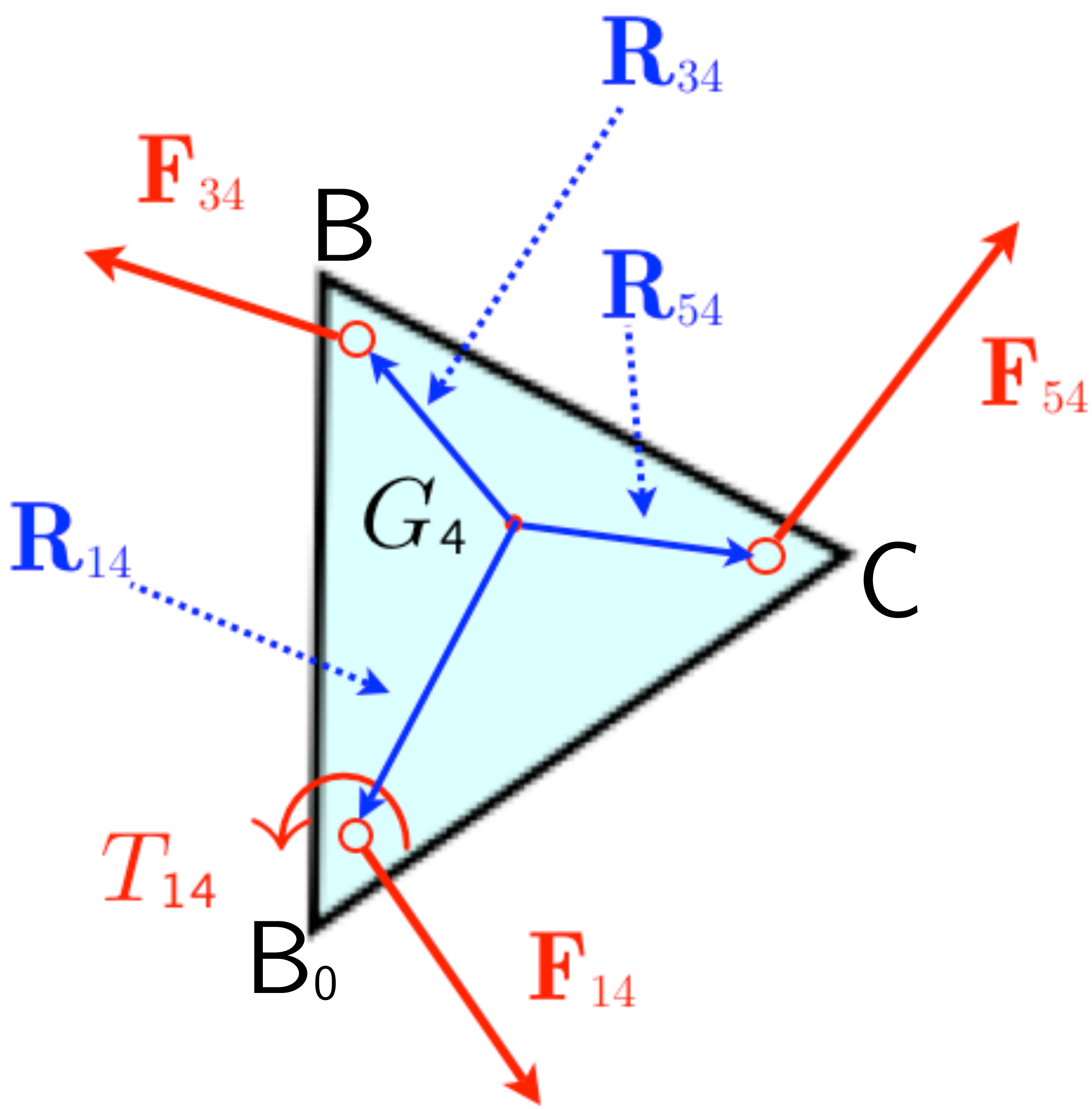

THE SIX-BAR MECHANISM IS DRIVEN BY A COUPLE APPLIED TO LINK 4. DRAW THE FBD FOR LINK 4, AND WRITE THE KINETIC EQUATIONS FOR THE LINK IN COMPONENTS WITH RESPECT TO THE LCS SHOWN.






$$\mathbf{F}_{14} + \mathbf{F}_{34} + \mathbf{F}_{54} = M_4 \mathbf{a}_{G_4}$$

$$\mathbf{R}_{14} \times \mathbf{F}_{14} + \mathbf{R}_{34} \times \mathbf{F}_{34} + \mathbf{R}_{54} \times \mathbf{F}_{54} + T_{14}\hat{\mathbf{k}} = I_{G_4}\alpha_4 \hat{\mathbf{k}}$$




$$F_{14_x} + F_{34_x} + F_{54_x} = M_4 a_{G_{4_x}}$$

$$F_{14_y} + F_{34_y} + F_{54_y} = M_4 a_{G_{4_y}}$$

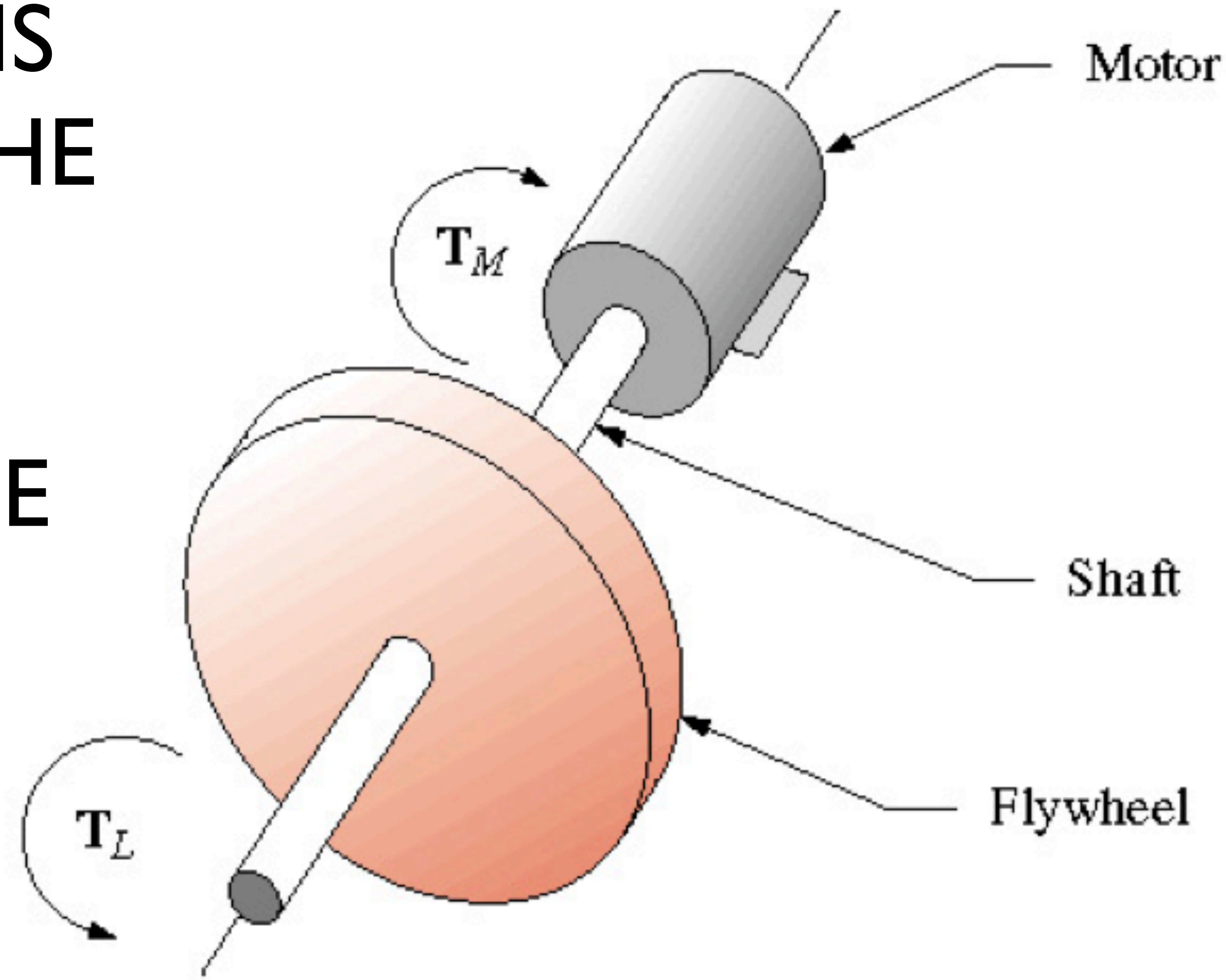
$$-R_{14_y} F_{14_x} + R_{14_x} F_{14_y} - R_{34_y} F_{34_x} + R_{34_x} F_{34_y} - R_{54_y} F_{54_x} + R_{54_x} F_{54_y} + T_{14} = I_{G_4} \alpha_4$$

# FLYWHEELS

# USE FLYWHEEL TO EASE THIS PROBLEM

THE MOTOR IMPARTS TORQUE $\mathbf{T}_M$ TO THE FLYWHEEL

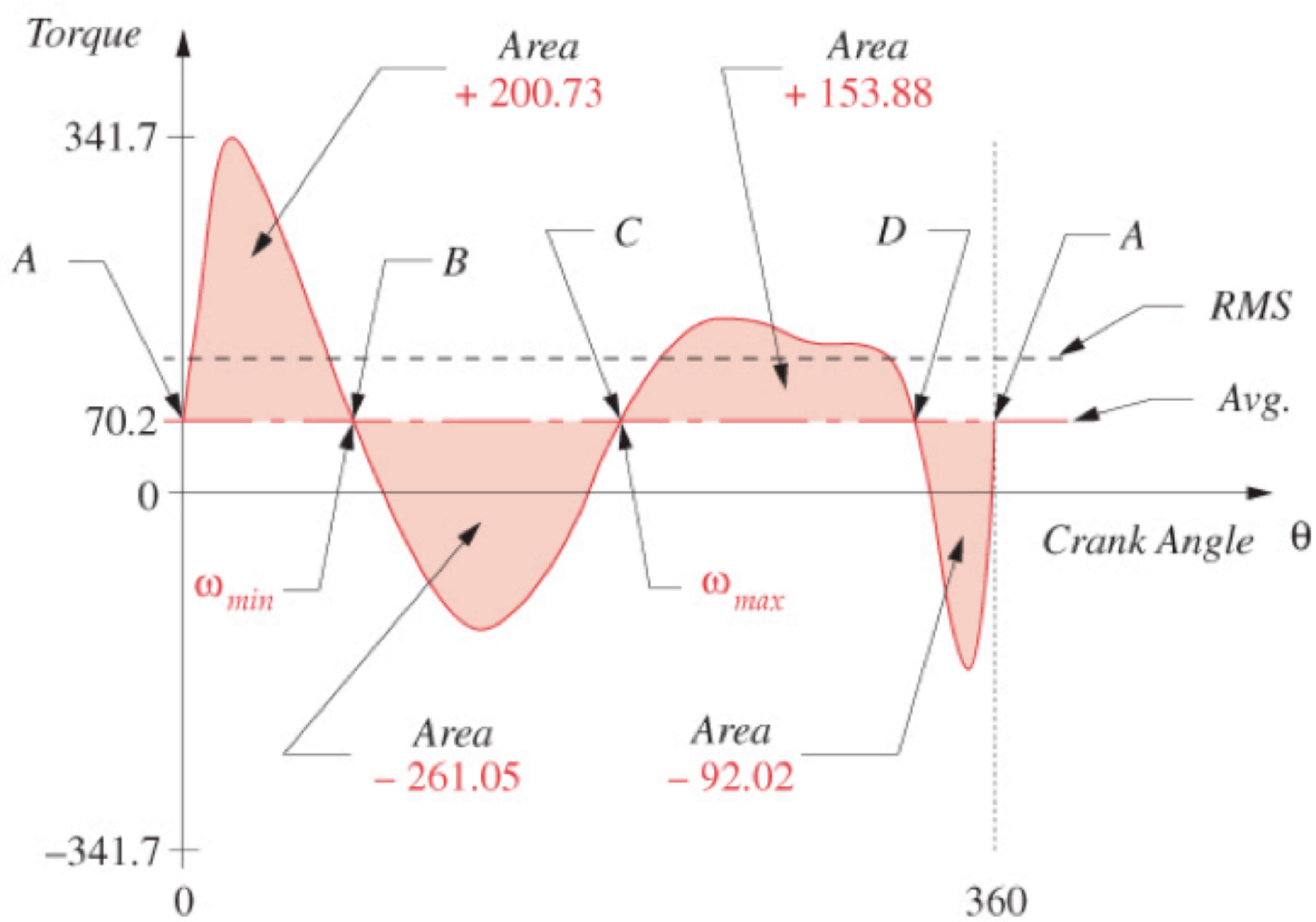
THE FLYWHEEL IS MOUNTED ON THE DRIVE-SHAFT BETWEEN THE MOTOR AND THE INPUT LINK.




THE FLYWHEEL IMPARTS TORQUE $\mathbf{T}_L$ TO THE FOUR-BAR

# $T_{12} - \theta$ PLOT

$$\text{Area} = \int_{Lobe} \{T_{12} - T_{avg}\} d\theta$$




Areas of torque pulses in order over one cycle

| Order | Neg Area | Pos Area |
|---|---|---|
| 1 | – 261.05 | 200.73 |
| 2 | – 92.02 | 153.88 |

Energy units are lb–in–rad

**FIGURE 11-11**

Integrating the pulses above and below the average value in the input torque function

**TABLE 11-1 Integrating the Torque Function**

| From | $\Delta$ Area = $\Delta E$ | Accum. Sum = $E$ | |
|---|---|---|---|
| $A$ to $B$ | +200.73 | +200.73 | $\omega_{min}@B$ |
| $B$ to $C$ | −261.05 | −60.32 | $\omega_{max}@C$ |
| $C$ to $D$ | +153.88 | +93.56 | |
| $D$ to $A$ | −92.02 | +1.54 | |

$$\text{Total } \Delta \text{ Energy} = E @ \omega_{max} - E @ \omega_{min}$$
$$= (-60.32) - (+200.73) = -261.05 \text{ in-lb}$$

POSITIVE LOBES REPRESENT ENERGY LOST BY FLYWHEEL, GAINED BY THE FOUR-BAR.

NEGATIVE LOBES REPRESENT ENERGY GAINED BY FLYWHEEL, LOST BY THE FOUR-BAR.

$$\frac{1}{2}I\left[\omega_{max}^2-\omega_{min}^2\right]\approx\int_{\theta@\omega_{min}}^{\theta@\omega_{max}}\{T_{avg}-T_{12}\}d\theta:=\left(\Delta E\right)_{Flywheel}$$

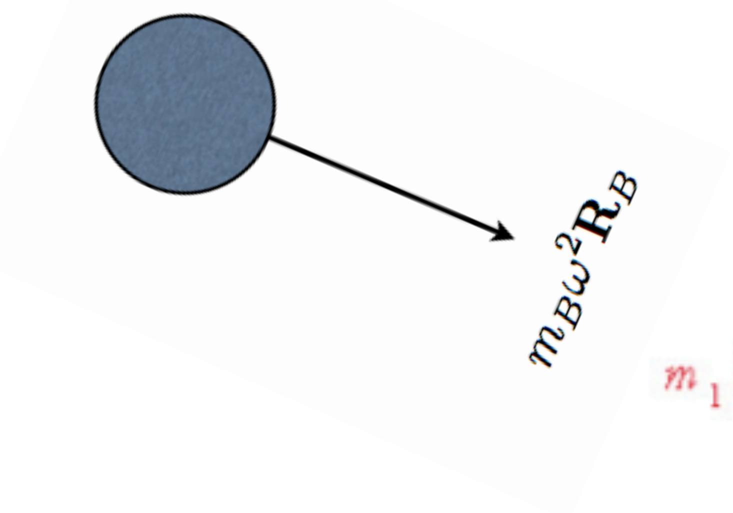
$$\left(\Delta E\right)_{Flywheel}=\frac{1}{2}I\left(\omega_{max}+\omega_{min}\right)\left(\omega_{max}-\omega_{min}\right)=I(\omega_{avg})(k\omega_{avg})$$

$$\boxed{I_s=\frac{\left(\Delta E\right)_{Flywheel}}{k\omega_{avg}^2}}$$

.... FLYWHEEL SIZE ESTIMATE

# BALANCING

# DYNAMIC BALANCING



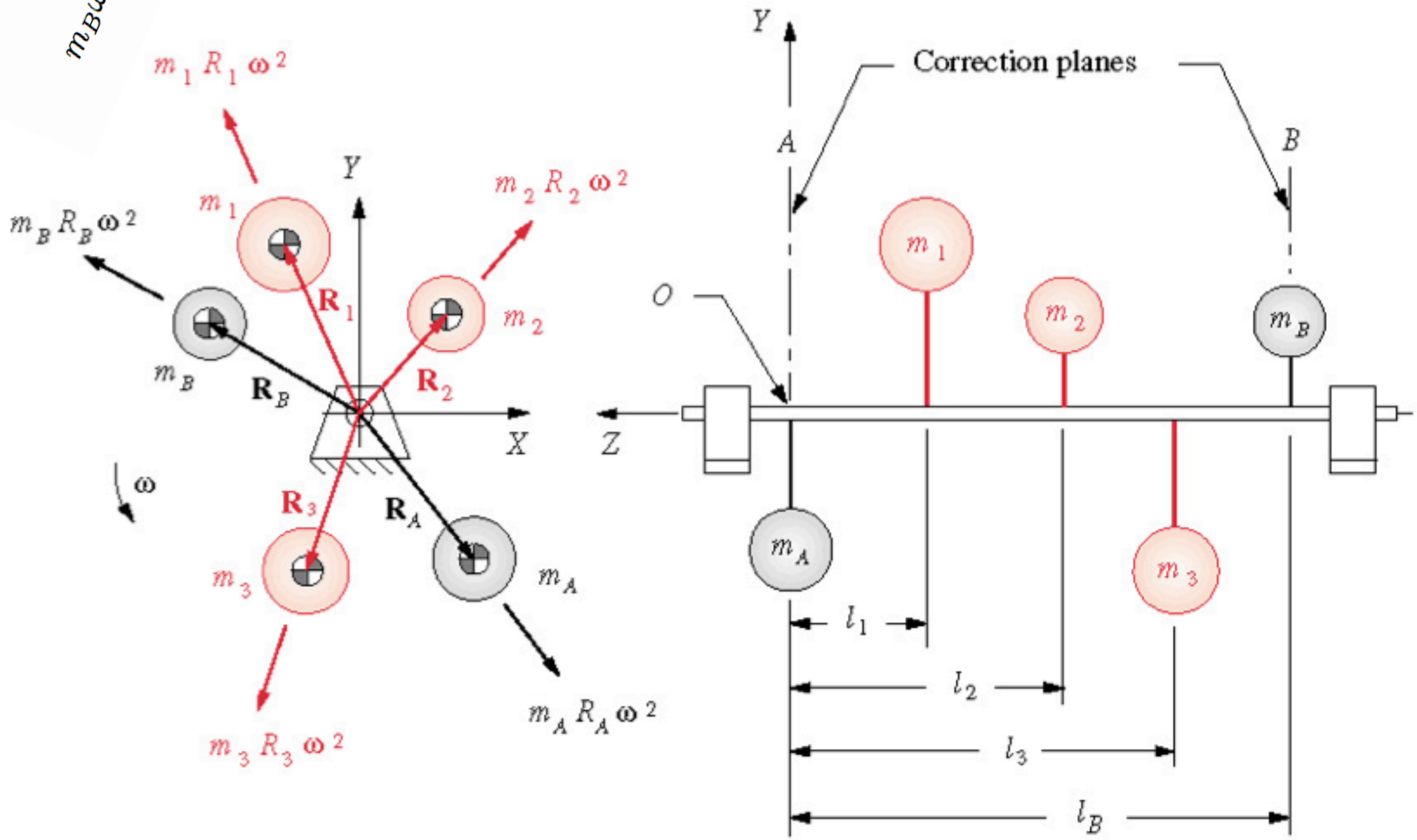
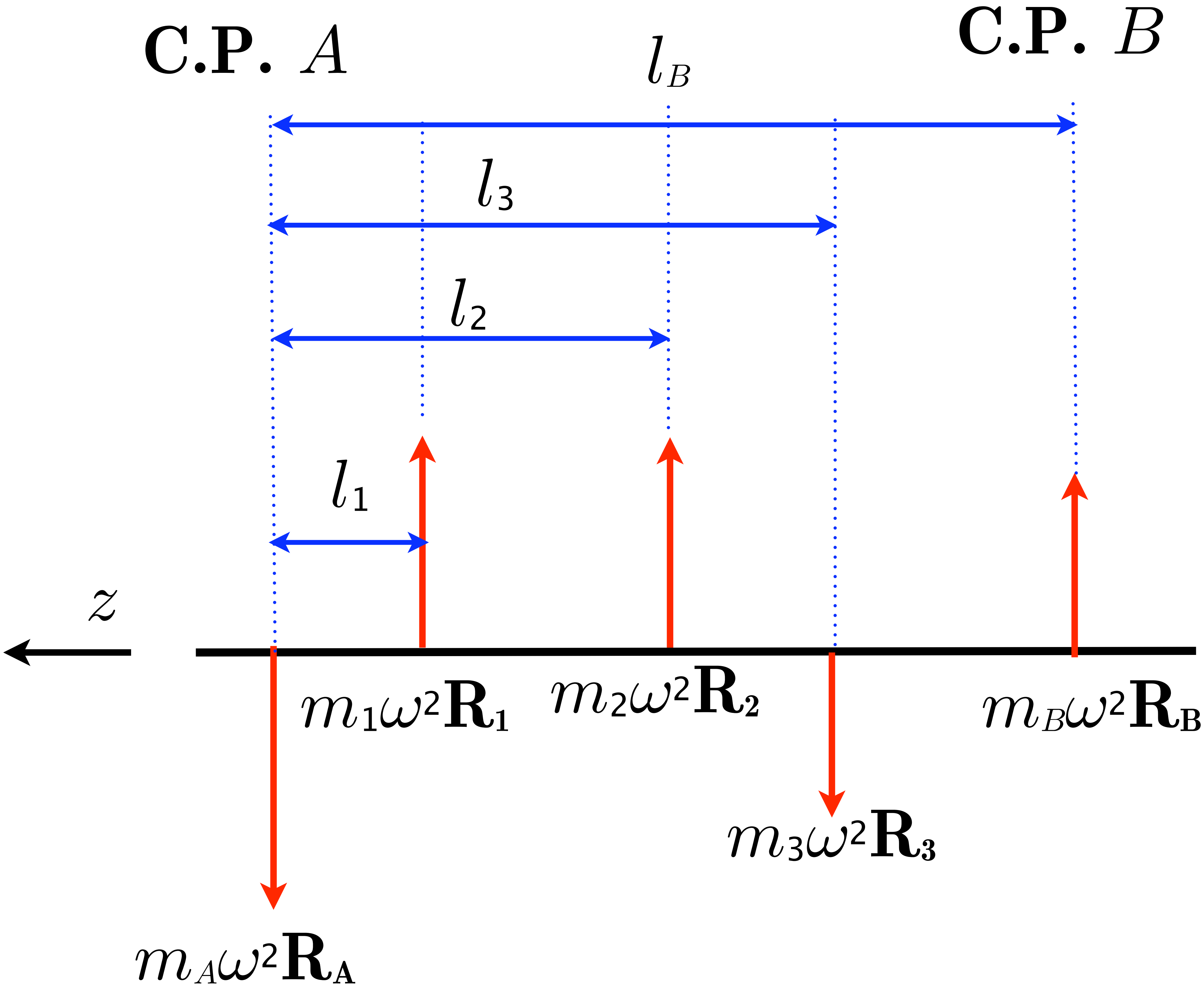




**FIGURE 12-3**

Two-plane dynamic balancing



C.P. $A$
$l_B$
C.P. $B$
$l_3$
$l_2$
$l_1$
$z$
$m_1\omega^2\mathbf{R_1}$
$m_2\omega^2\mathbf{R_2}$
$m_B\omega^2\mathbf{R_B}$
$m_3\omega^2\mathbf{R_3}$
$m_A\omega^2\mathbf{R_A}$

$$\sum \mathbf{T}_A = \mathbf{0}$$

$$-l_1\hat{\mathbf{k}} \times m_1\omega^2\mathbf{R}_1 - l_2\hat{\mathbf{k}} \times m_2\omega^2\mathbf{R}_2 - l_3\hat{\mathbf{k}} \times m_3\omega^2\mathbf{R}_3 - l_B\hat{\mathbf{k}} \times m_B\omega^2\mathbf{R}_B = \mathbf{0}$$

$$-\omega^2\hat{\mathbf{k}} \times \left\{m_1l_1\mathbf{R}_1 + m_2l_2\mathbf{R}_2 + m_3l_3\mathbf{R}_3 + m_Bl_B\mathbf{R}_B\right\} = \mathbf{0}$$

$$m_1l_1\mathbf{R}_1 + m_2l_2\mathbf{R}_2 + m_3l_3\mathbf{R}_3 + m_Bl_B\mathbf{R}_B = \mathbf{0}$$

$$\sum \mathbf{T}_B = \mathbf{0}$$

$$(l_B - l_1)\hat{\mathbf{k}} \times m_1\omega^2\mathbf{R}_1 + (l_B - l_2)\hat{\mathbf{k}} \times m_2\omega^2\mathbf{R}_2 + (l_B - l_3)\hat{\mathbf{k}} \times m_3\omega^2\mathbf{R}_3 + l_B\hat{\mathbf{k}} \times m_A\omega^2\mathbf{R}_A = \mathbf{0}$$

$$-\omega^2\hat{\mathbf{k}} \times \left\{m_1(l_B - l_1)\mathbf{R}_1 + m_2(l_B - l_2)\mathbf{R}_2 + m_3(l_B - l_3)\mathbf{R}_3 + m_Al_B\mathbf{R}_A\right\} = \mathbf{0}$$

$$m_1(l_B - l_1)\mathbf{R}_1 + m_2(l_B - l_2)\mathbf{R}_2 + m_3(l_B - l_3)\mathbf{R}_3 + m_Al_B\mathbf{R}_A = \mathbf{0}$$

**OR**

$$\sum \mathbf{F} = \mathbf{0}$$

$$m_1\omega^2\mathbf{R}_1 + m_2\omega^2\mathbf{R}_2 + m_3\omega^2\mathbf{R}_3 + m_A\omega^2\mathbf{R}_A + m_B\omega^2\mathbf{R}_B = \mathbf{0}$$

$$m_1\mathbf{R}_1 + m_2\mathbf{R}_2 + m_3\mathbf{R}_3 + m_A\mathbf{R}_A + m_B\mathbf{R}_B = \mathbf{0}$$

$$\boxed{m_1 l_1 \mathbf{R}_1 + m_2 l_2 \mathbf{R}_2 + m_3 l_3 \mathbf{R}_3 + m_B l_B \mathbf{R}_B = \mathbf{0}}$$

IN SYSTEM $Axyz$, $I_{xz} = I_{yz} = 0$.

$$\boxed{m_1(l_B - l_1)\mathbf{R}_1 + m_2(l_B - l_2)\mathbf{R}_2 + m_3(l_B - l_3)\mathbf{R}_3 + m_A l_B \mathbf{R}_A = \mathbf{0}}$$

IN SYSTEM $Bxyz$, $I_{xz} = I_{yz} = 0$.

$$\boxed{m_1 \mathbf{R}_1 + m_2 \mathbf{R}_2 + m_3 \mathbf{R}_3 + m_A \mathbf{R}_A + m_B \mathbf{R}_B = \mathbf{0}}$$

C. G. OF ASSEMBLY LIES ON $z$ - AXIS.

IF ANY PAIR OF THESE THREE CONDITIONS ARE TRUE, THE THIRD IS ALSO TRUE.

THE C.G. OF THE MODIFIED ASSEMBLY IS LOCATED ON THE AXIS OF ROTATION.
THE AXIS OF ROTATION IS A PRINCIPAL AXIS OF INERTIA FOR THE MODIFIED ASSEMBLY.

# DYNAMIC BALANCING OF A WHEEL

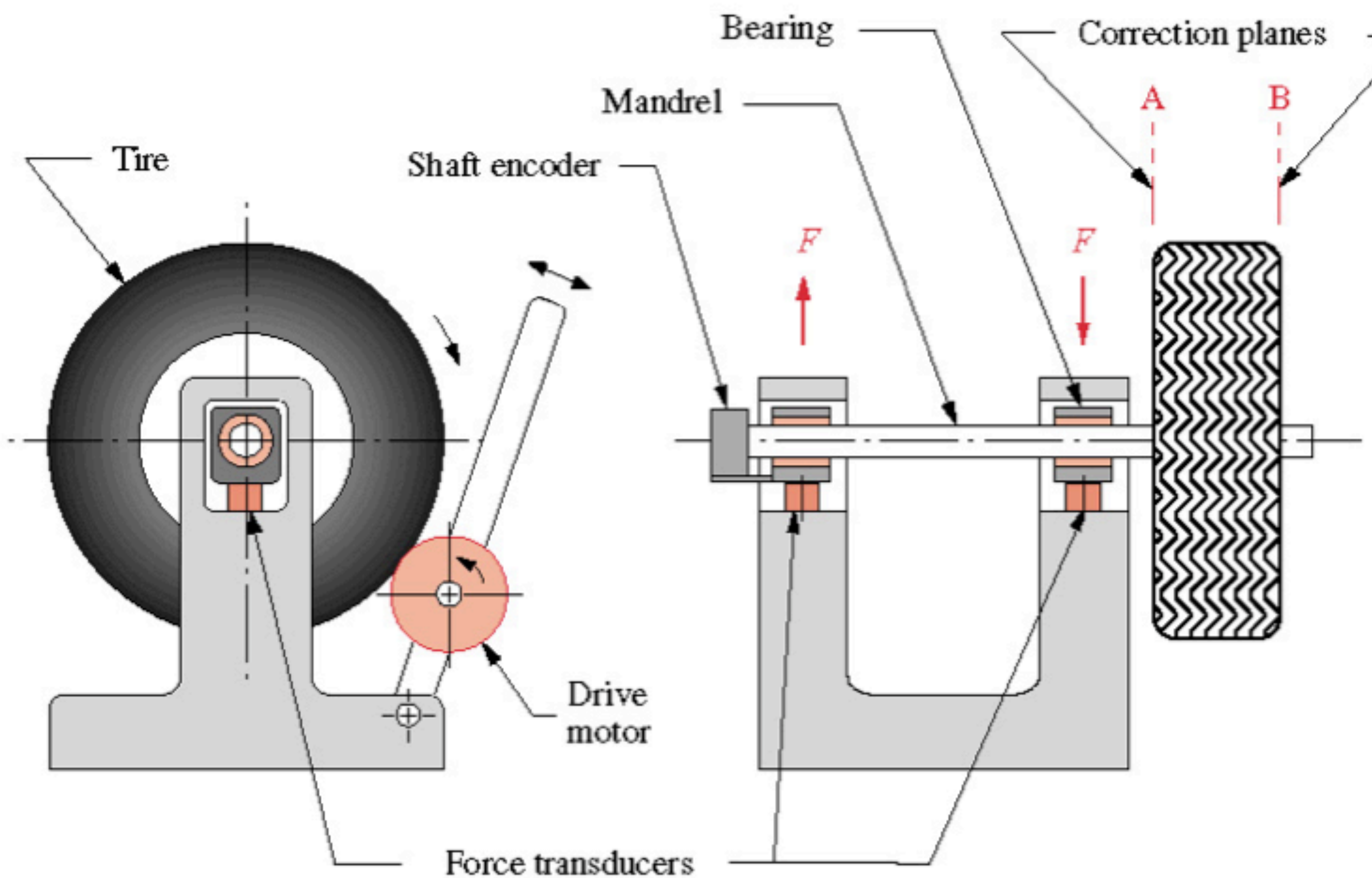


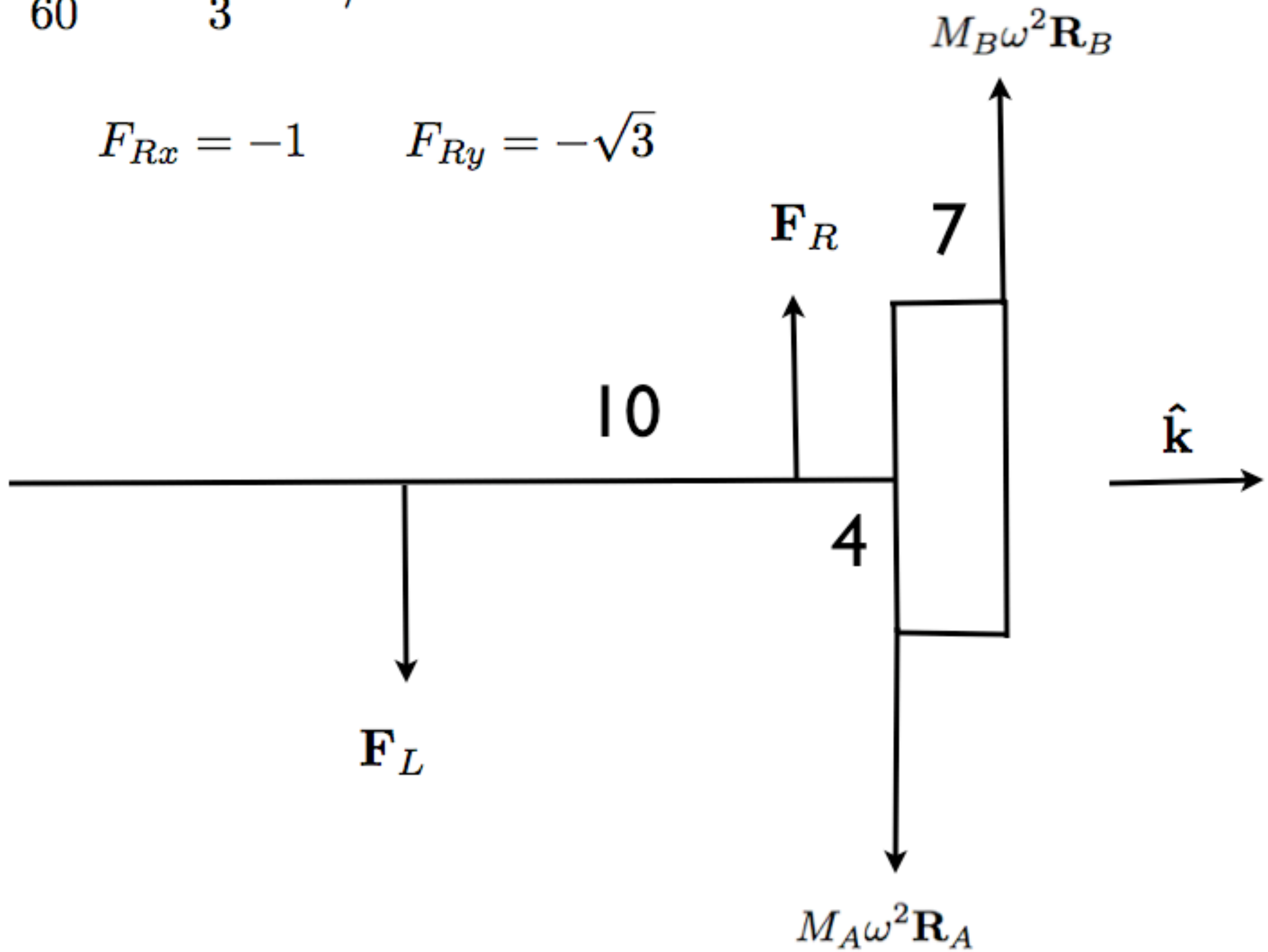

**FIGURE 12-12**

A dynamic wheel balancer

PROBLEM 12-6

$$\omega = \frac{200\pi}{60} = \frac{10\pi}{3}\,\text{rad/s}$$

$$F_{Lx} = F_{Ly} = \frac{5}{\sqrt{2}} \qquad F_{Rx} = -1 \qquad F_{Ry} = -\sqrt{3}$$



## MOMENT BALANCE (ABOUT A)

$$-14\hat{\mathbf{k}} \times \left[ \frac{5}{\sqrt{2}}\,\hat{\mathbf{i}} + \frac{5}{\sqrt{2}}\,\hat{\mathbf{j}} \right] - 4\hat{\mathbf{k}} \times \left[ -\,\hat{\mathbf{i}} - \sqrt{3}\,\hat{\mathbf{j}} \right] + 7\hat{\mathbf{k}} \times m_B \omega^2 \left[ R_{Bx}\,\hat{\mathbf{i}} + R_{By}\,\hat{\mathbf{j}} \right] = \mathbf{0}$$

$$7 m_B \omega^2 R_{Bx} = \frac{70}{\sqrt{2}} - 4 \qquad 7 m_B \omega^2 R_{By} = \frac{70}{\sqrt{2}} - 4\sqrt{3}$$

$$\mathbf{R}_B = 7.5 \left[ \cos\theta_B\,\hat{\mathbf{i}} + \sin\theta_B\,\hat{\mathbf{j}} \right] \qquad \mathbf{R}_A = 7.5 \left[ \cos\theta_A\,\hat{\mathbf{i}} + \sin\theta_A\,\hat{\mathbf{j}} \right]$$

$$5757.2692\, m_B \cos\theta_B = 45.4975 \qquad 5757.2692\, m_B \sin\theta_B = 42.5693$$

$$\tan\theta_B = .9356 \;\; \theta_B = .7521\,\text{rad} = 43.09^\circ \;\; m_B = .0108\,\text{blobs} \;\; W_B = 4.1814\,\text{lbs}$$

## FORCE BALANCE

$$m_A \omega^2 \mathbf{R}_A + m_B \omega^2 \mathbf{R}_B + \mathbf{F}_L + \mathbf{F}_R = \mathbf{0}$$

$$822.4670\, m_A \cos\theta_A = -9.0352 \qquad 822.4670\, m_A \sin\theta_A = -7.8848$$

$$\tan\theta_A = .8727 \;\; \theta_A = 3.8591\,\text{rad} = 221.11^\circ \;\; m_A = .0146\,\text{blobs} \;\; W_A = 5.6338\,\text{lbs}$$

# REVIEW LECTURE 3: CAMS, GEARS, INDICES OF MERIT

$$\boxed{\textbf{JERK}}$$

LINEAR JERK $\mathbf{J}$.

$$\mathbf{J} = \dot{\mathbf{A}} = \ddot{\mathbf{V}}.$$

ANGULAR JERK $\boldsymbol{\varphi}$.

$$\boldsymbol{\varphi} = \varphi\hat{\mathbf{k}} = \dot{\boldsymbol{\alpha}} = \dot{\alpha}\hat{\mathbf{k}} = \ddot{\boldsymbol{\omega}} = \ddot{\omega}\hat{\mathbf{k}} = \dddot{\theta}\ \hat{\mathbf{k}}.$$

## KINETIC EQUATIONS FOR JERK

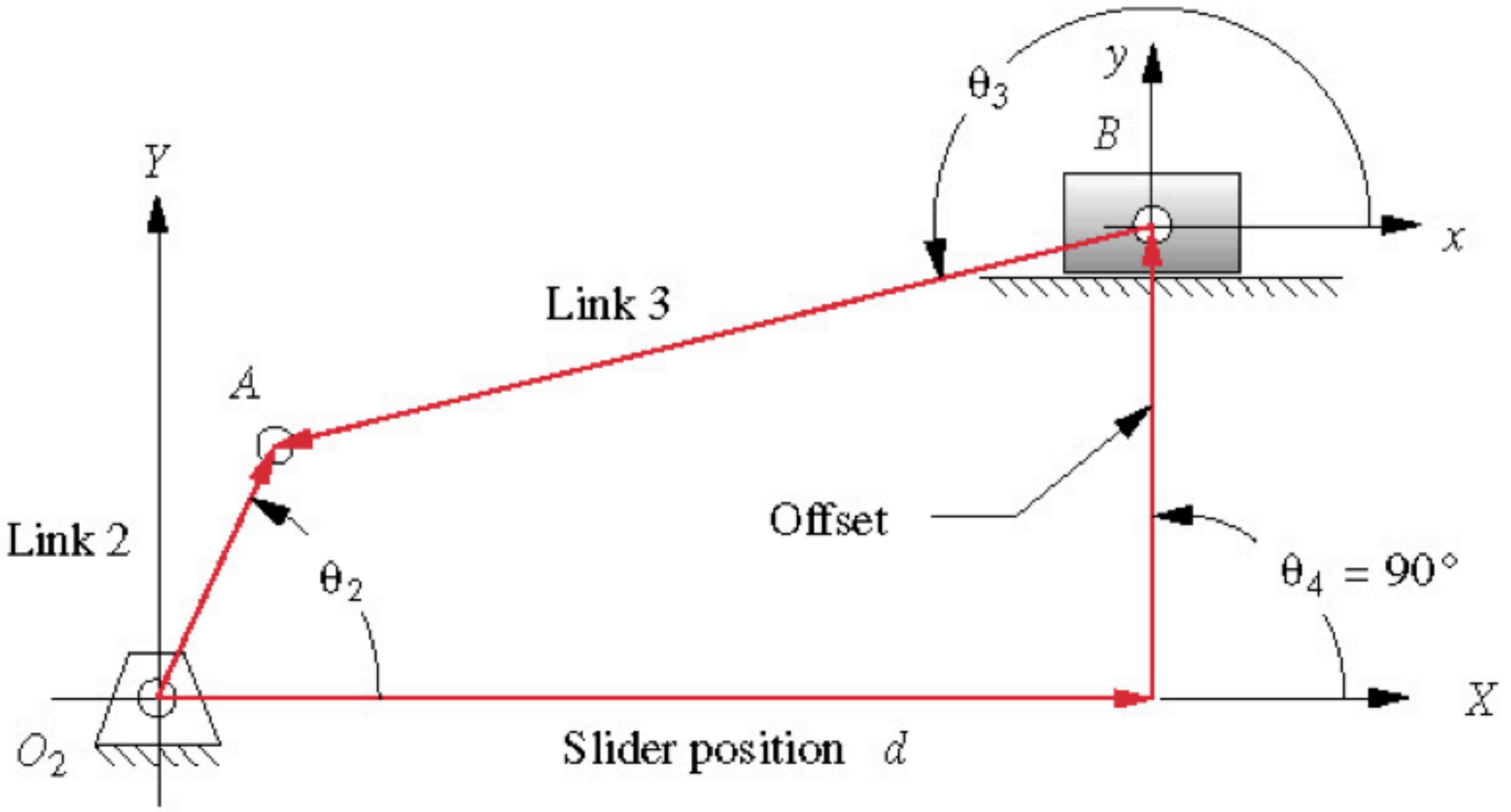
$$M\mathbf{J} = \sum \dot{\mathbf{F}}$$

$$I_G \boldsymbol{\varphi} = \sum \dot{\mathbf{T}}_G$$

LARGE JERKS $\leftrightarrow$ RAPIDLY VARYING FORCES AND TORQUES.

INFINITE JERKS $\leftrightarrow$ JUMPS IN FORCES AND TORQUES.

# SLIDER-CRANK



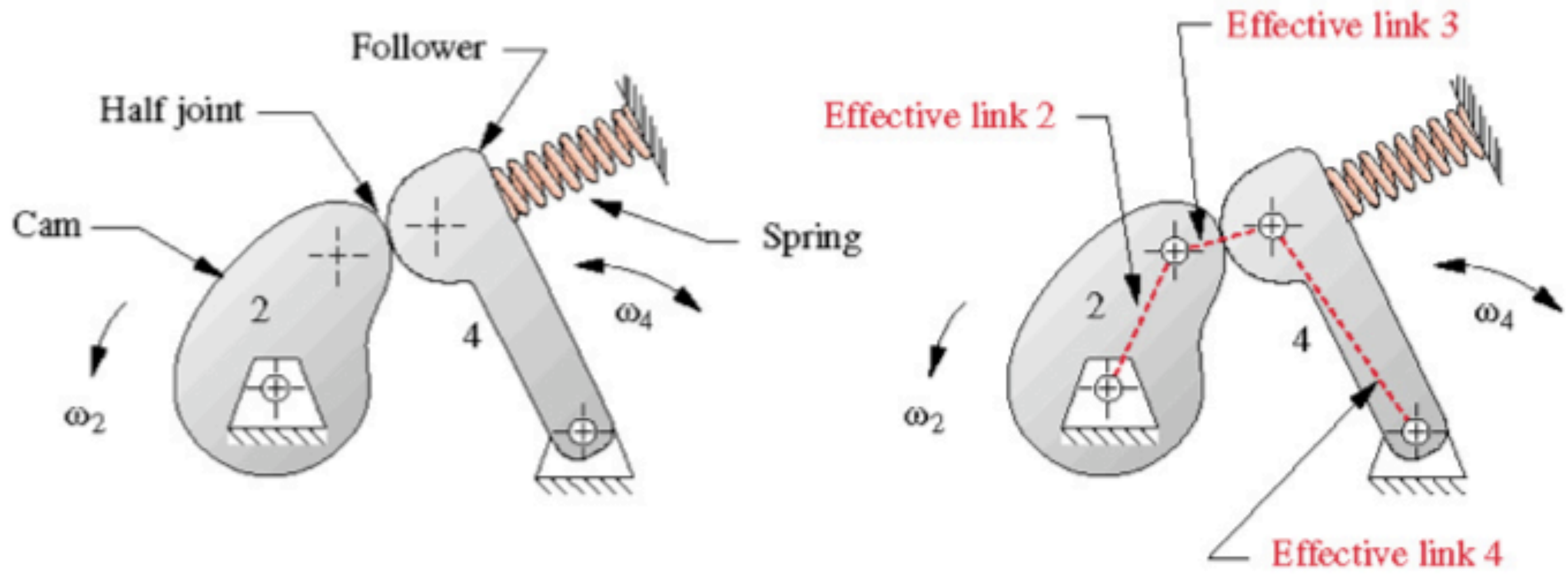

$$ae^{j\theta_2} - be^{j\theta_3} - jc - d = 0$$

$$aj\omega_2 e^{j\theta_2} - bj\omega_3 e^{j\theta_3} - \dot{d} = 0$$

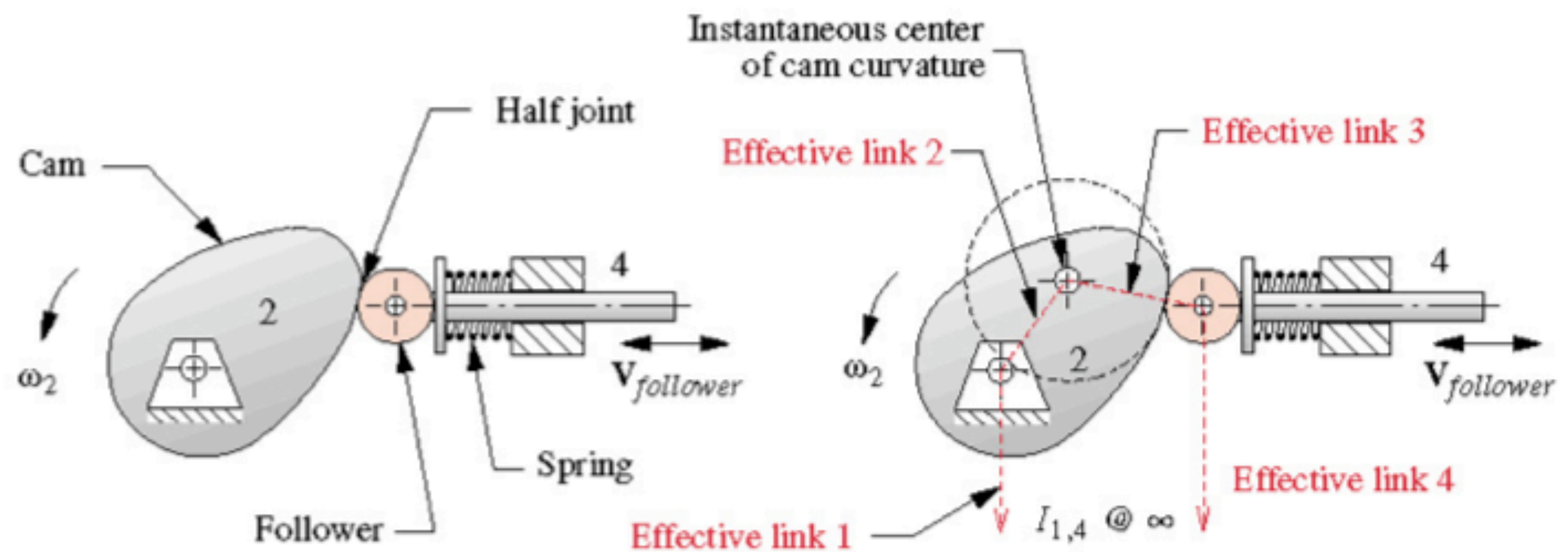
$$a\left(j\alpha_2 - \omega_2^2\right)e^{j\theta_2} - b\left(j\alpha_3 - \omega_3^2\right)e^{j\theta_3} - \ddot{d} = 0$$

$$a\left(j\varphi_2 - 3\alpha_2\omega_2 - j\omega_2^3\right)e^{j\theta_2} - b\left(j\varphi_3 - 3\alpha_3\omega_3 - j\omega_3^3\right)e^{j\theta_3} - \dddot{d} = 0$$

# CAMS




(*a*) An oscillating cam-follower has an effective pin-jointed fourbar equivalent




(*b*) A translating cam-follower has an effective fourbar slider-crank equivalent

**FIGURE 8-1**

Effective linkages in the cam-follower mechanism

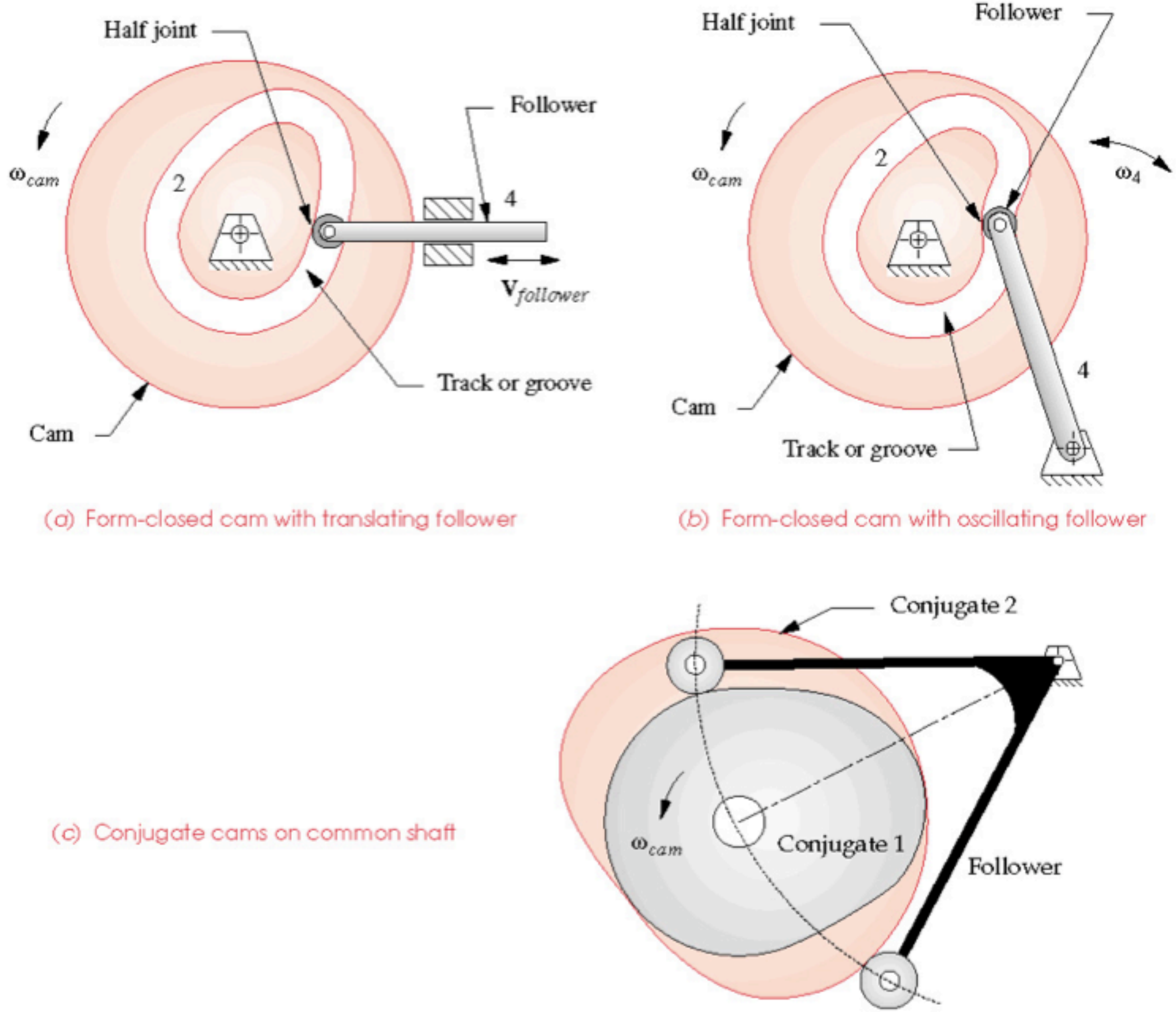
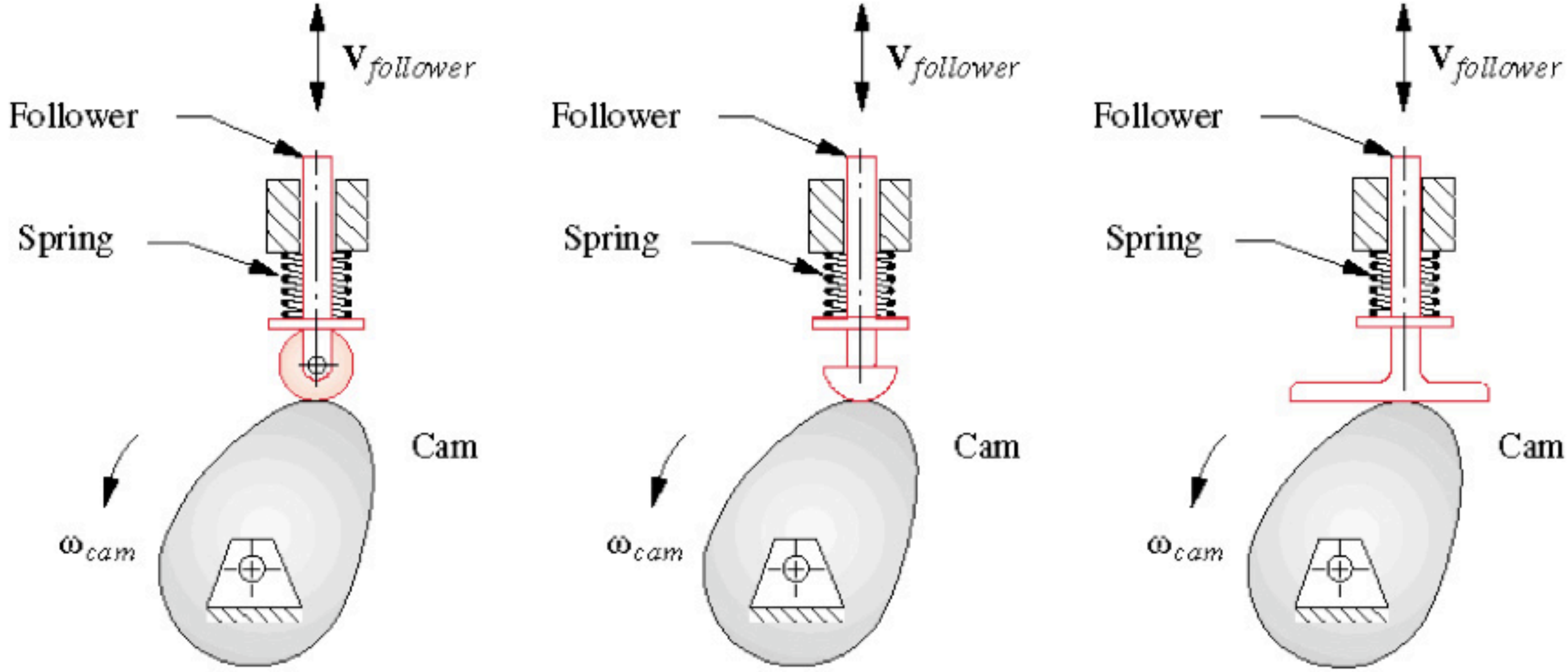



**FIGURE 8-2**

Form-closed cam-follower systems




(a) Roller follower (b) Mushroom follower (c) Flat-faced follower

**FIGURE 8-3**

Three common types of cam followers

# MOTION CONSTRAINTS

## CRITICAL EXTREME POSITION (CEP)

INITIAL AND FINAL FOLLOWER DISPLACEMENTS ARE SPECIFIED. INTERMEDIATE CONFIGURATIONS ARE UNCONSTRAINED.

# CRITICAL PATH MOTION

FOLLOWER DISPLACEMENT IS
SPECIFIED THROUGHOUT THE MOTION.
(FULL-BLOWN FUNCTION GENERATOR)

CAM SHAPE IS DESIGNED TO MEET FOLLOWER MOTION CONSTRAINTS.

FOLLOWER MOTION IS CHARACTERIZED BY svaj DIAGRAMS.

FINITE JERK REQUIREMENT: ACCELERATION MUST BE CONTINUOUS.

FOLLOWER DISPLACEMENT $s$

$s$-DIAGRAM: PLOTS $s$ AGAINST $\theta$ (IN DEGREES).

FOLLOWER VELOCITY $v$

$v$-DIAGRAM: PLOTS $v$ AGAINST $\theta$ (IN DEGREES).

FOLLOWER ACCELERATION $a$

$a$-DIAGRAM: PLOTS $a$ AGAINST $\theta$ (IN DEGREES).

FOLLOWER JERK $j$

$j$-DIAGRAM: PLOTS $j$ AGAINST $\theta$ (IN DEGREES).

## DWELLS

SUPPOSE FOLLOWER HAS TWO DWELLS

DWELL 2 ENDS @ $\theta = 0°$.

DWELL 1 BEGINS @ $\theta = \beta°$.

$$a(0) = a(\beta) = 0, \;\; v(0) = v(\beta) = 0.$$

$$\int_0^\beta a(\theta) d\theta = 0.$$

$$\int_0^\beta v(\theta) d\theta = s(\beta) - s(0).$$

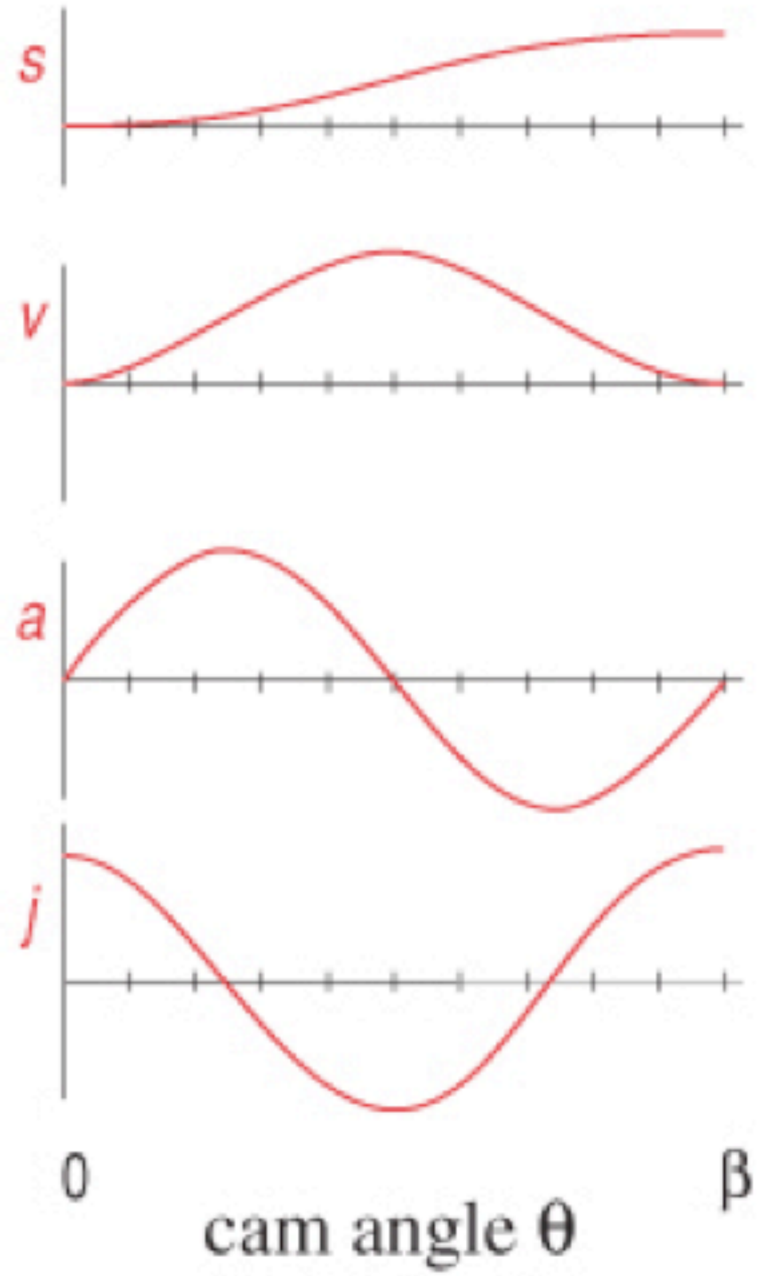



**FIGURE 8-12**

Sinusoidal acceleration gives cycloidal displacement

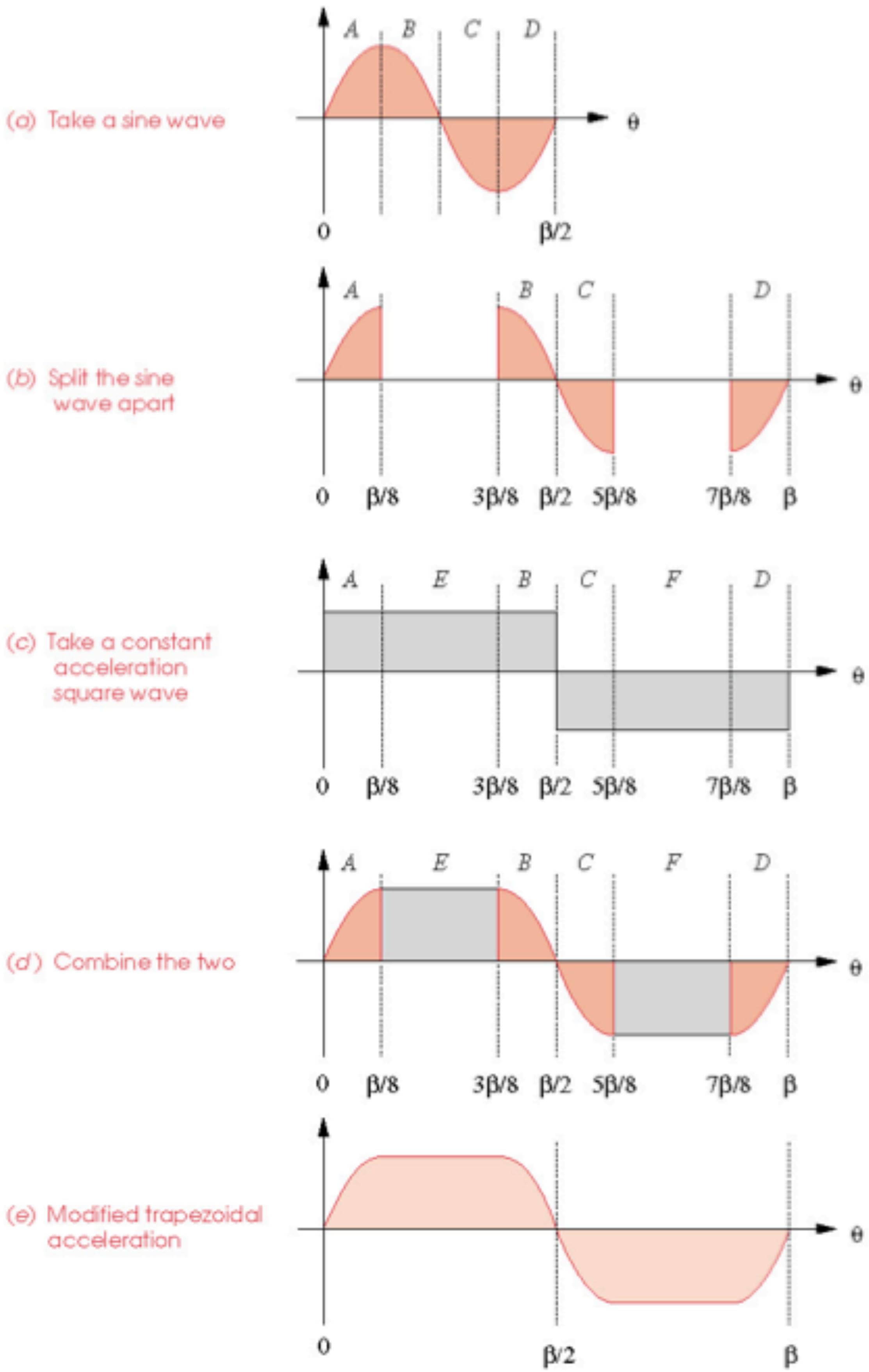



**FIGURE 8-15**

Creating the modified trapezoidal acceleration function

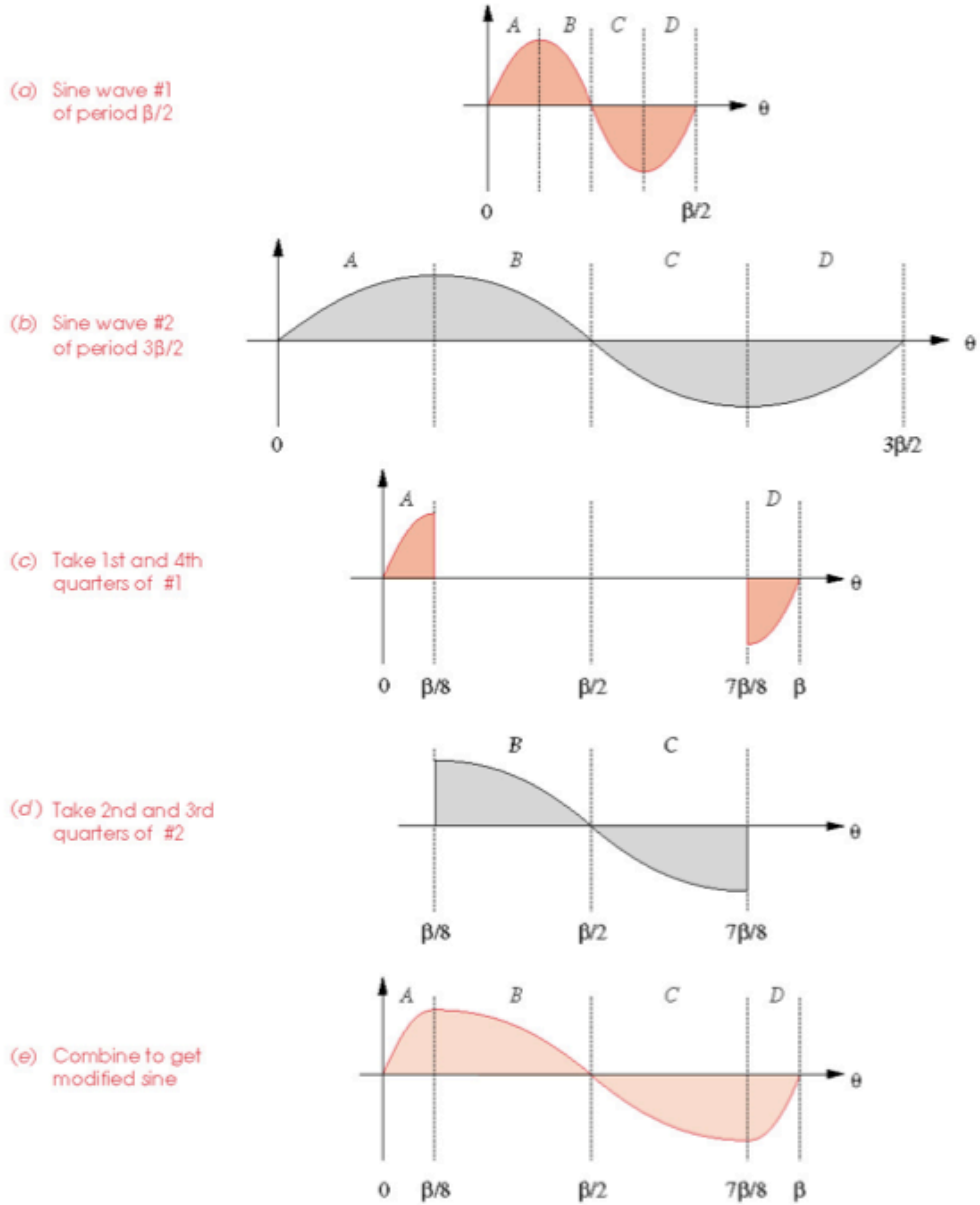



**FIGURE 8-16**

Creating the modified sine acceleration function

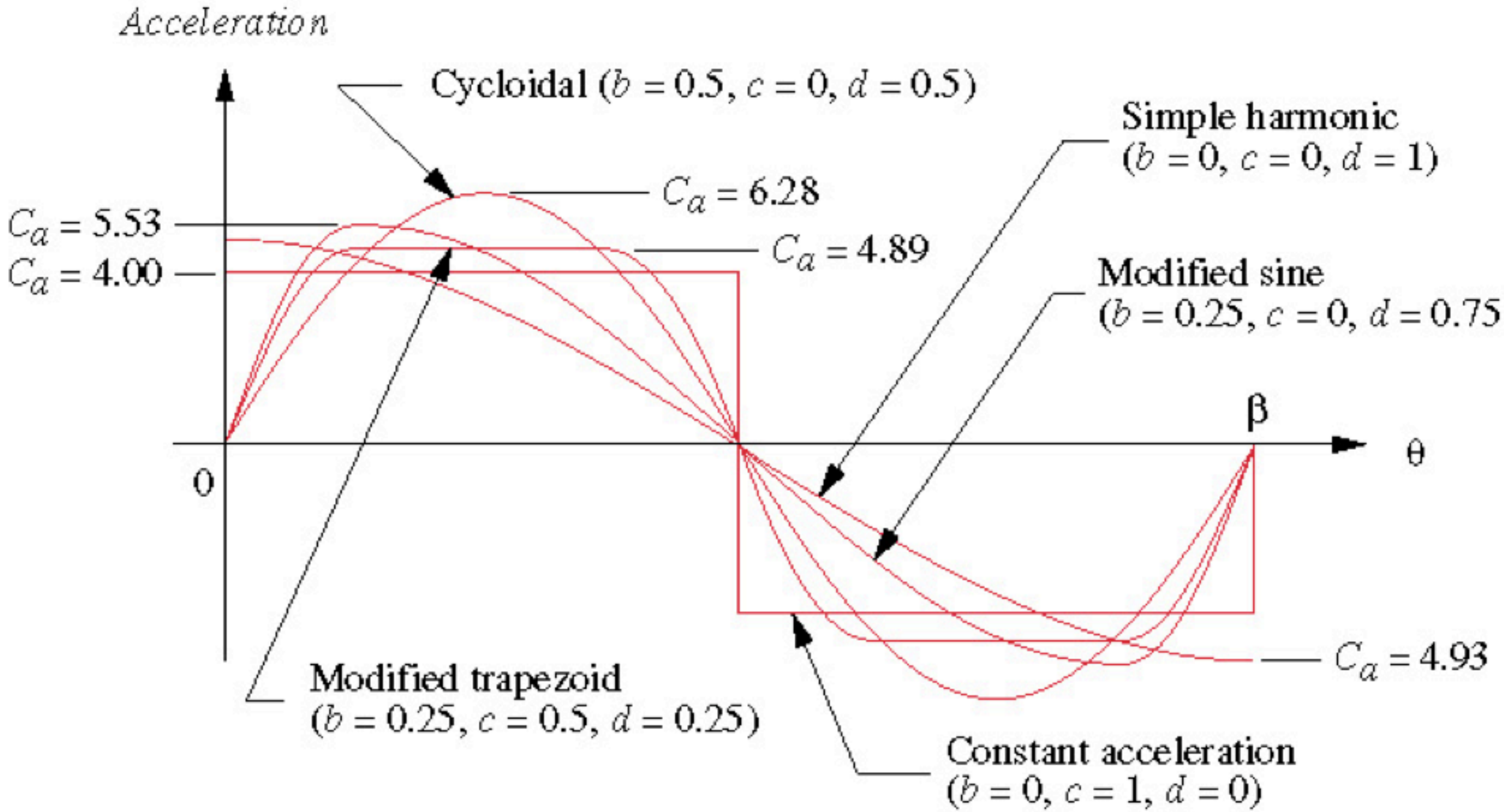



**FIGURE 8-18**

Comparison of five acceleration functions in the SCCA family

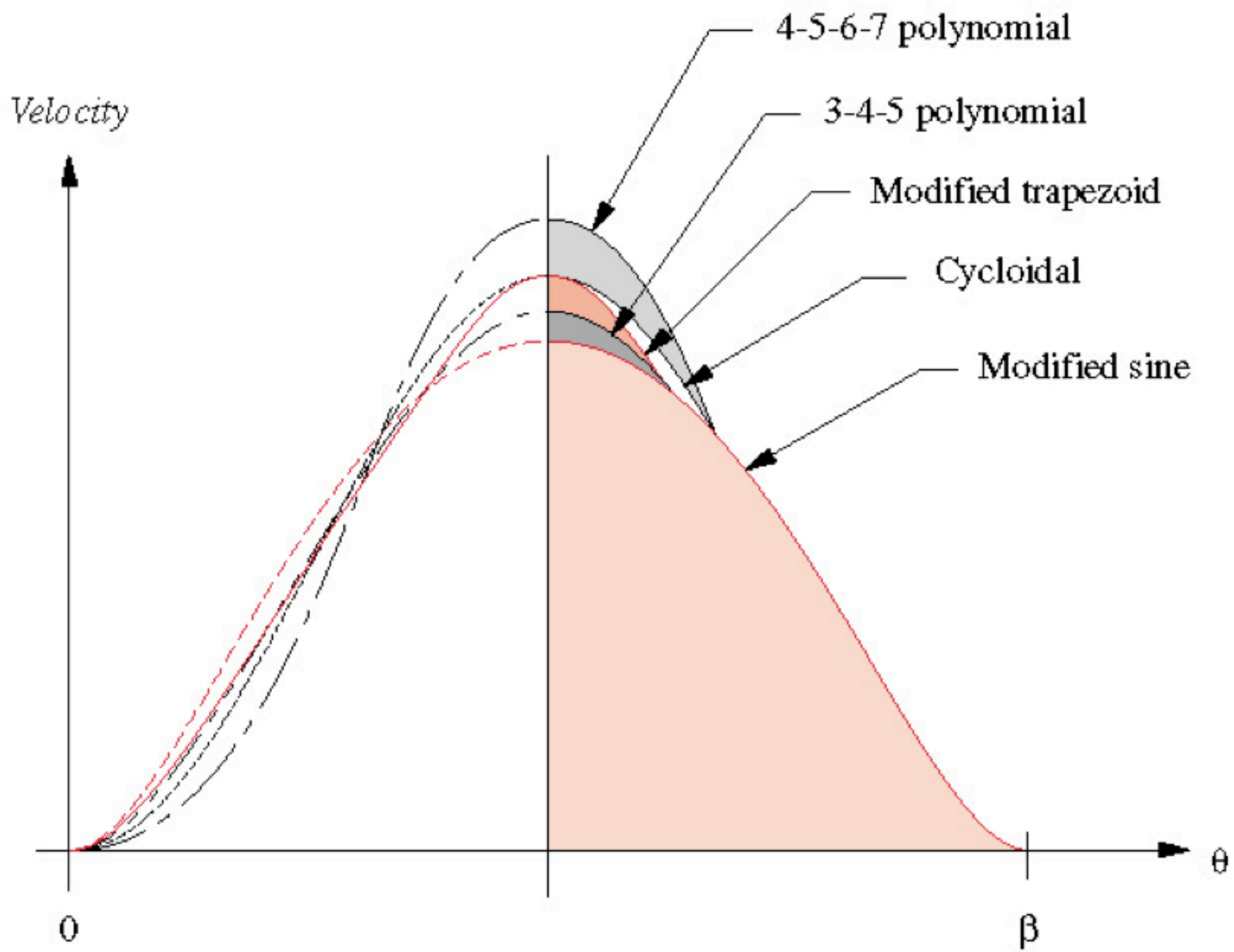



FIGURE 8-21

Comparison of five double-dwell cam velocity functions

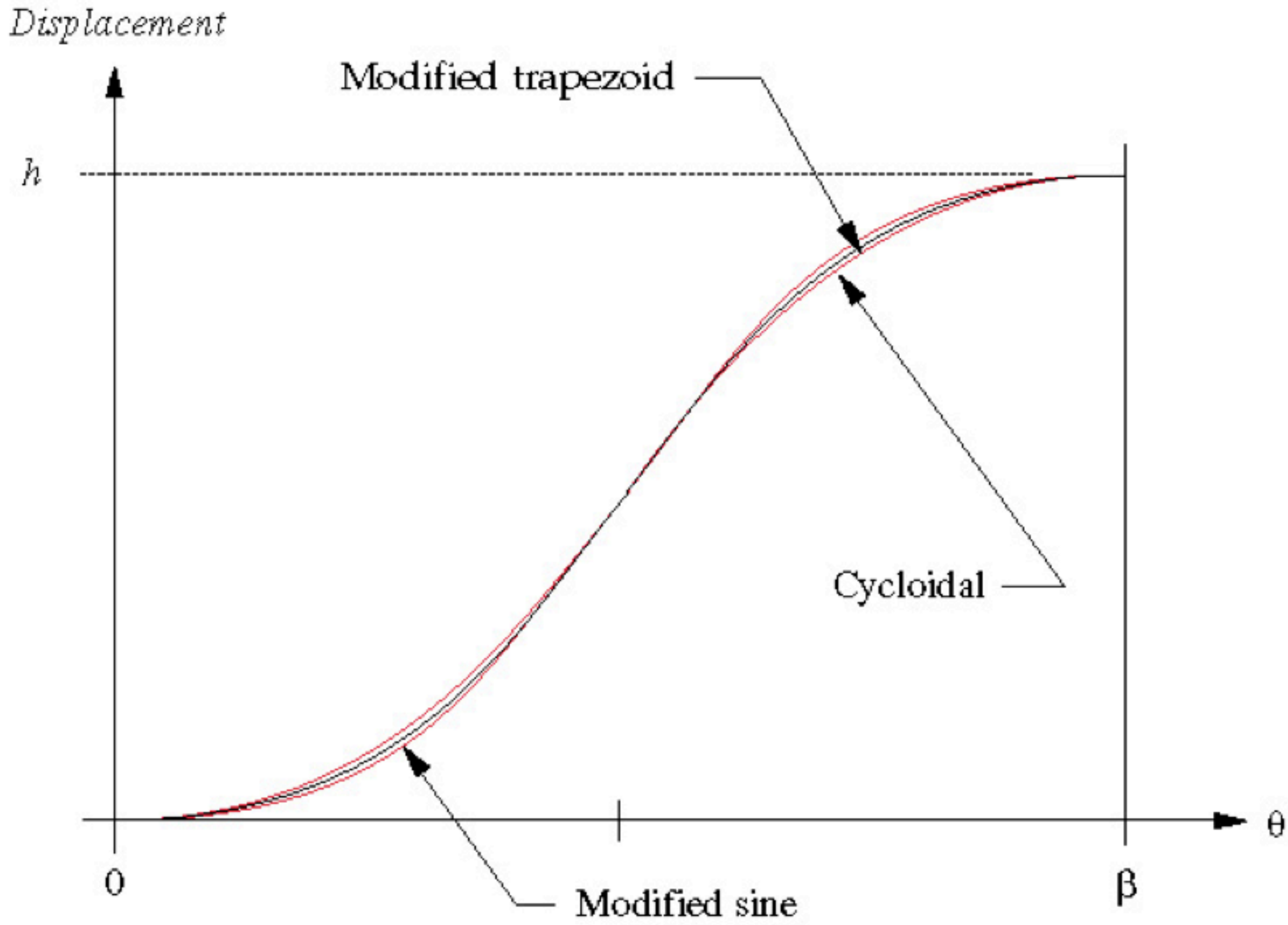



**FIGURE 8-23**

Comparison of three SCCA double-dwell cam displacement functions

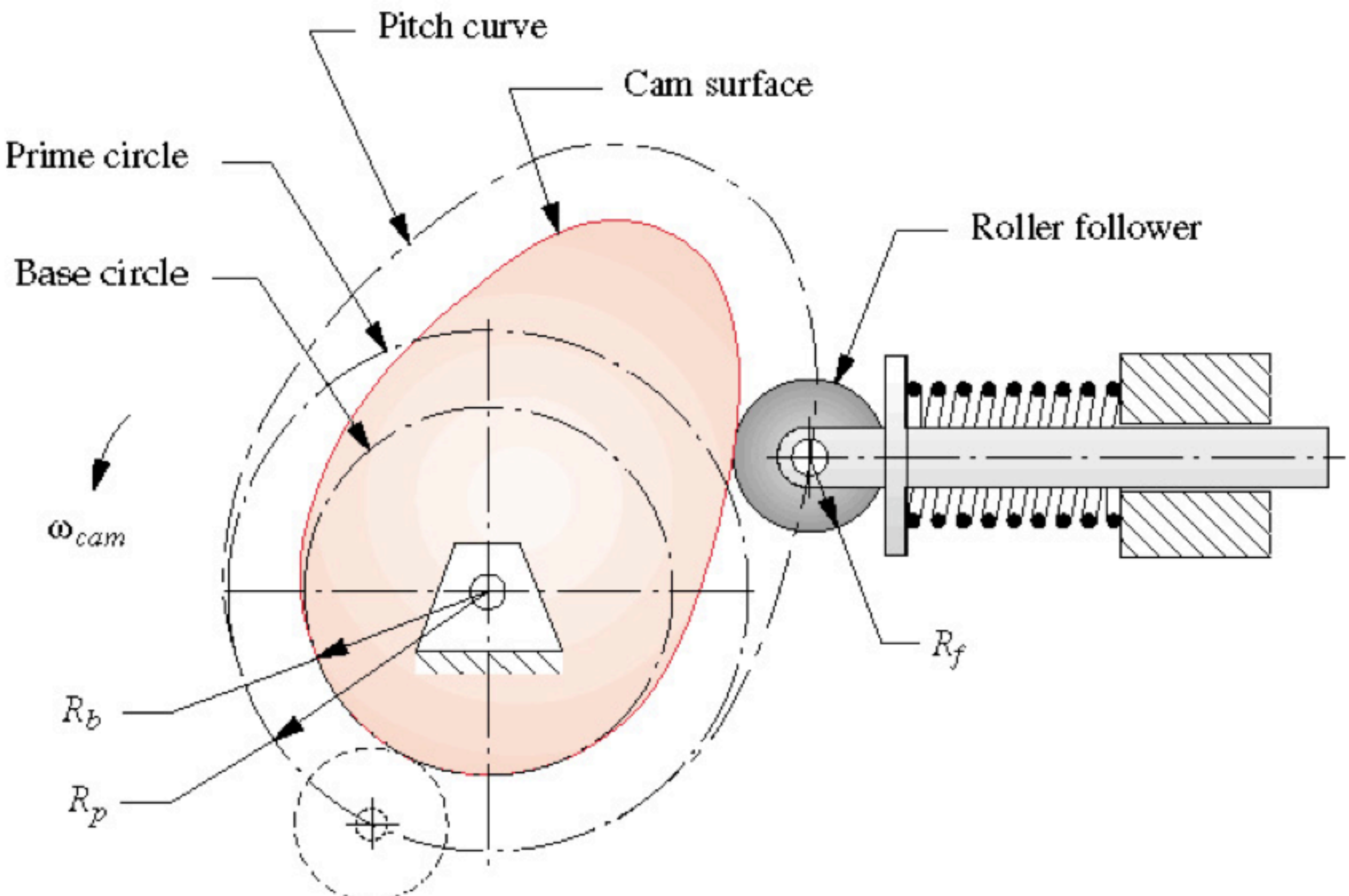



**FIGURE 8-43**

Base circle $R_b$, prime circle $R_p$, and pitch curve of a radial cam with roller follower

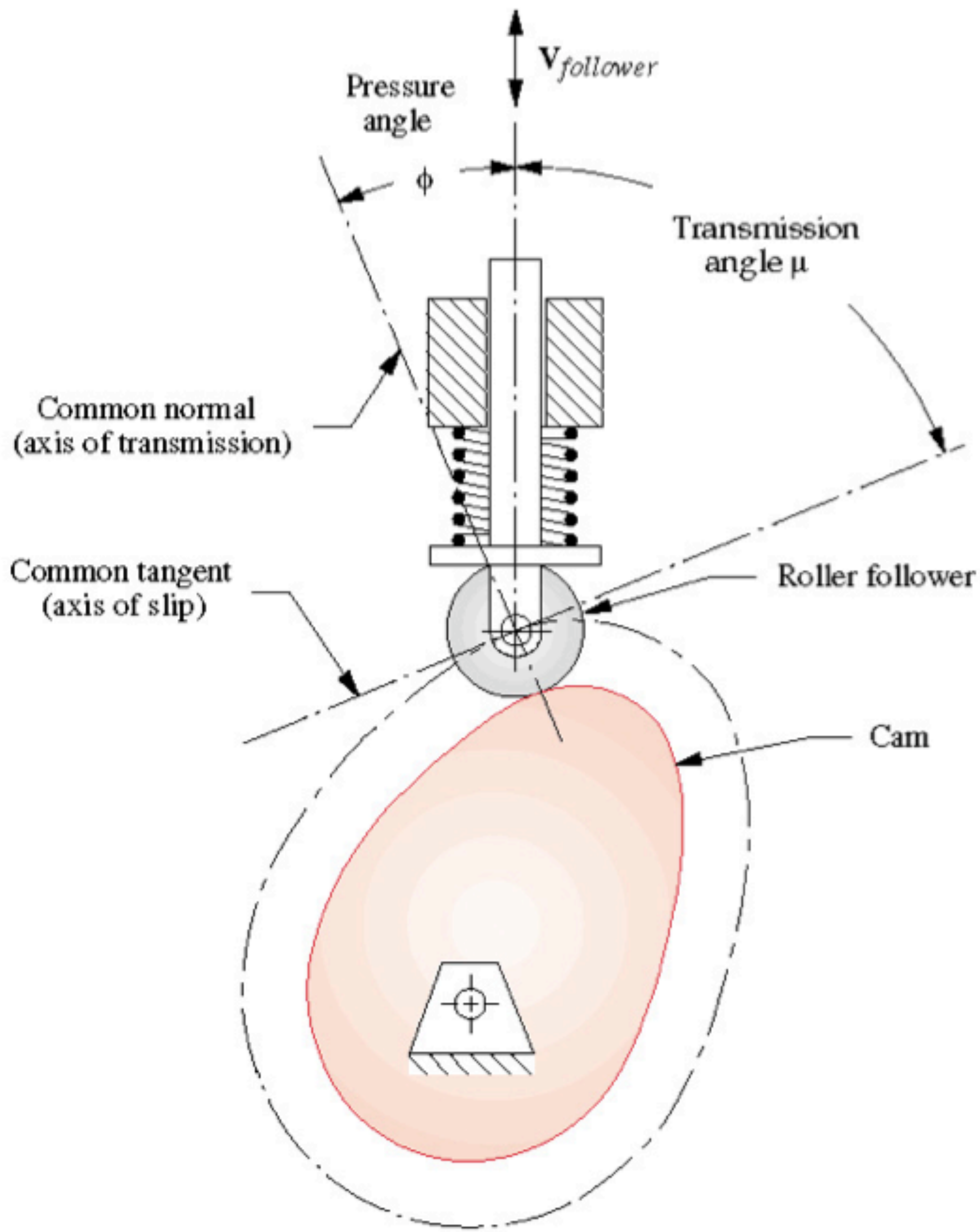
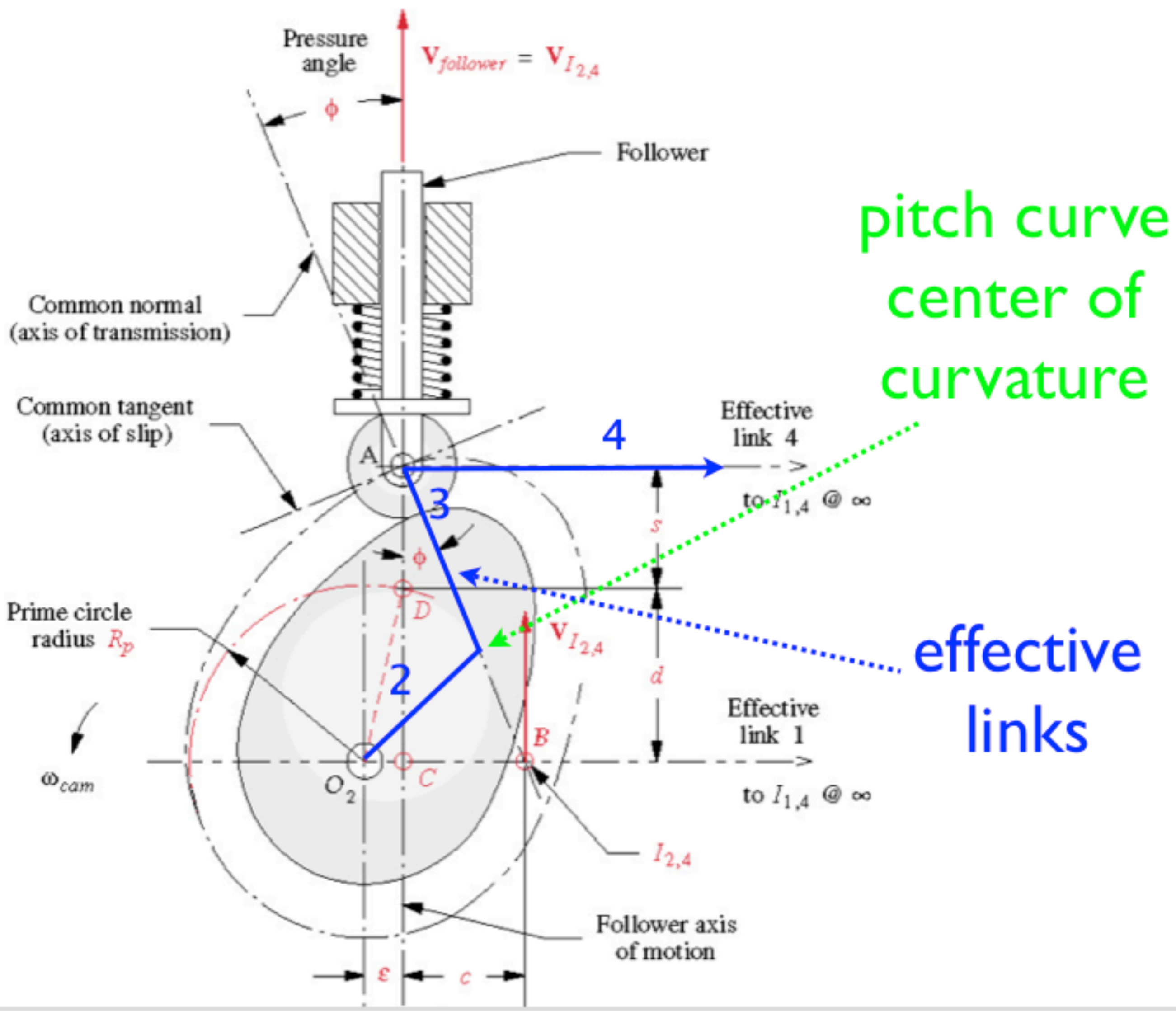



**FIGURE 8-44**

Cam pressure angle




$$\dot{s} = V_{follower} = \frac{ds}{d\theta}\omega = \nu\omega$$

$$V_{follower} = b\omega, \quad b = \nu$$

TRIANGLE $ACI_{2,4}$

$$c = b - \varepsilon = (s + d)\tan\phi$$

$$b = \varepsilon + (s + d)\tan\phi$$

TRIANGLE $CDO_2$

$$d = \sqrt{R_P^2 - \varepsilon^2}$$

$$\left(s + \sqrt{R_P^2 - \varepsilon^2}\right)\tan\phi = \nu - \varepsilon$$

$$\boxed{\phi = \arctan\frac{\nu - \varepsilon}{s + \sqrt{R_P^2 - \varepsilon^2}}}$$

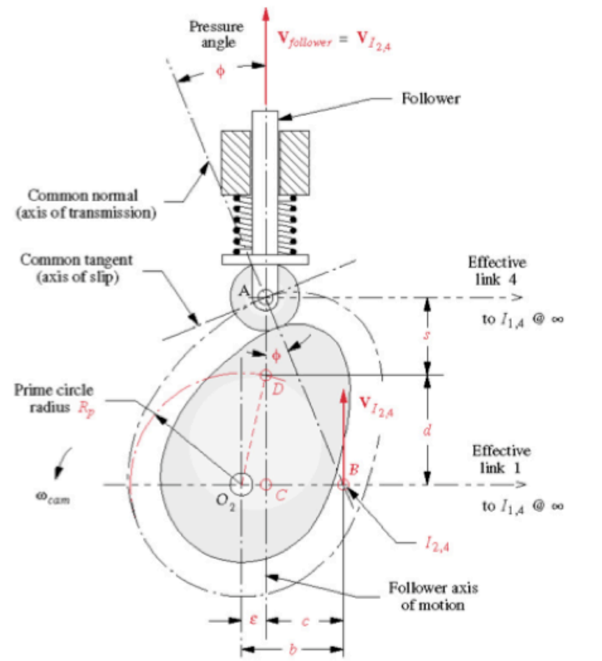


FIGURE 8-45

$$\mathbf{R}(\theta) := \varepsilon \hat{\mathbf{u}}_r + \big(d + s(\theta)\big)\hat{\mathbf{u}}_\theta$$

$$d\sigma = \sqrt{\big(d + s(\theta)\big)^2 + \big(\varepsilon + \nu(\theta)\big)^2}$$

$$\hat{\mathbf{u}}_t = \frac{1}{\sqrt{\big(d + s(\theta)\big)^2 + \big(\varepsilon + \nu(\theta)\big)^2}}\Big\{ -\big(d + s(\theta)\big)\hat{\mathbf{u}}_r + \big(\varepsilon + \nu(\theta)\big)\hat{\mathbf{u}}_\theta \Big\}$$

$$\frac{d}{d\sigma}\Big\{\hat{\mathbf{u}}_t\Big\} = \frac{1}{\rho}\hat{\mathbf{u}}_n$$

$$\rho(\theta) = \frac{\Big\{\big(d + s(\theta)\big)^2 + \big(\varepsilon + \nu(\theta)\big)^2\Big\}^{\frac{3}{2}}}{\Big[\big(d + s(\theta)\big)^2 + \big(\varepsilon + \nu(\theta)\big)\big(\varepsilon + 2\nu(\theta)\big) - \big(d + s(\theta)\big)a(\theta)\Big]}$$

GIVES RADIUS OF CURVATURE IN TERMS OF PRESCRIBED QUANTITIES.

# CONCAVE CAM: MULTIPLE CONTACT IF $R_f > \rho_{min}$

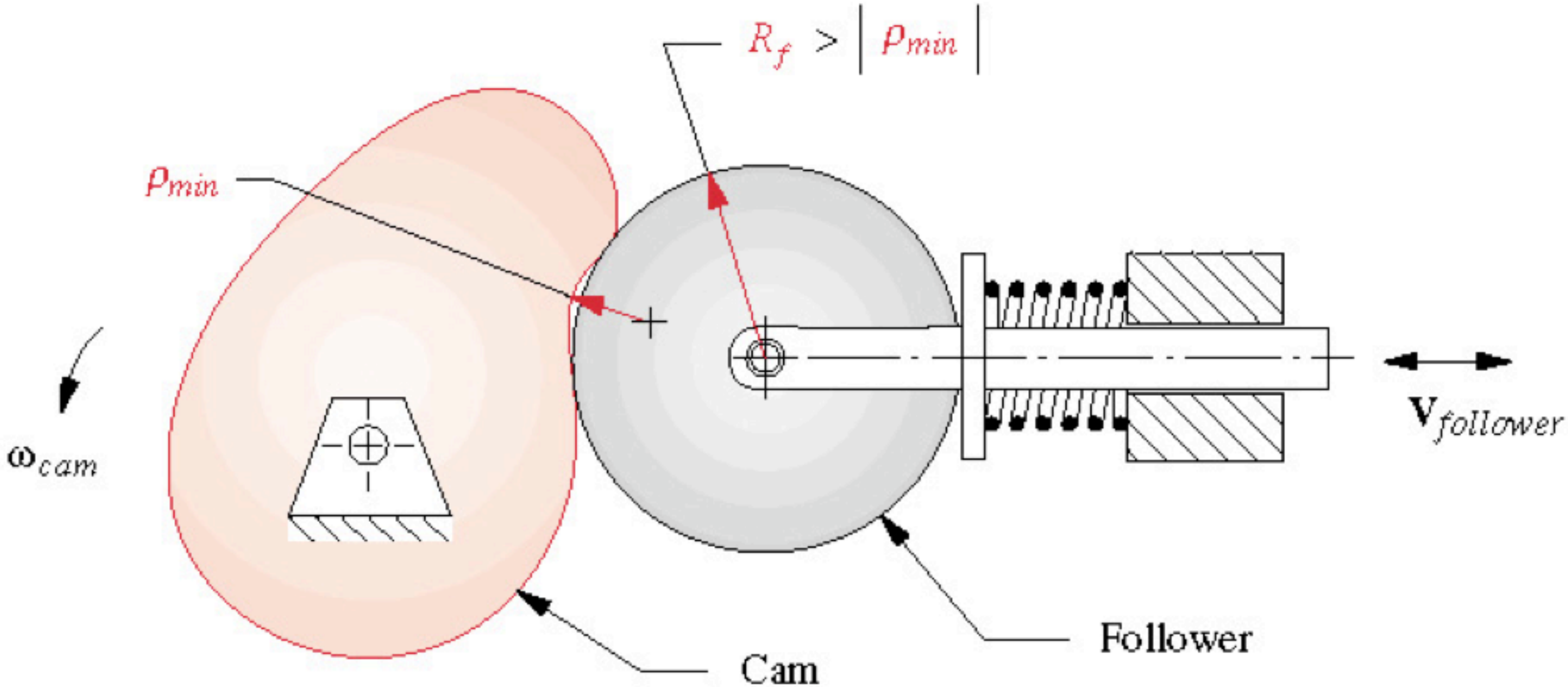
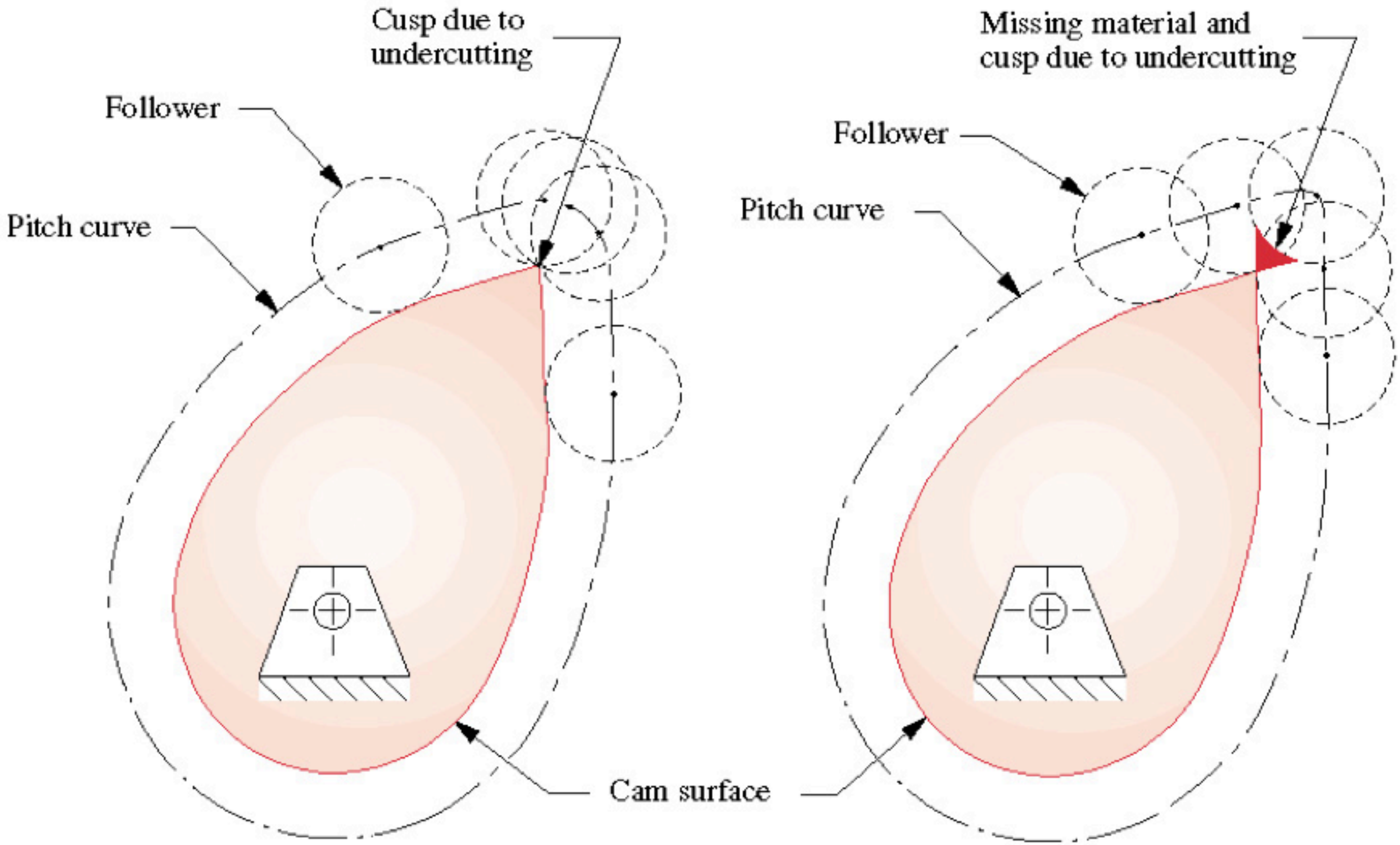



FIGURE 8-48

The result of using a roller follower larger than the one for which the cam was designed




(*a*) Radius of curvature of pitch curve equals the radius of the roller follower

(*b*) Radius of curvature of pitch curve is less than the radius of the roller follower

**FIGURE 8-49**

Small positive radius of curvature can cause undercutting

# GEARS

GEAR RELATIONSHIP: NO SLIP AT CONTACT POINT, VELOCITIES MATCH.

SIMPLEST MODEL: CYLINDERS IN PURE ROLLING CONTACT.

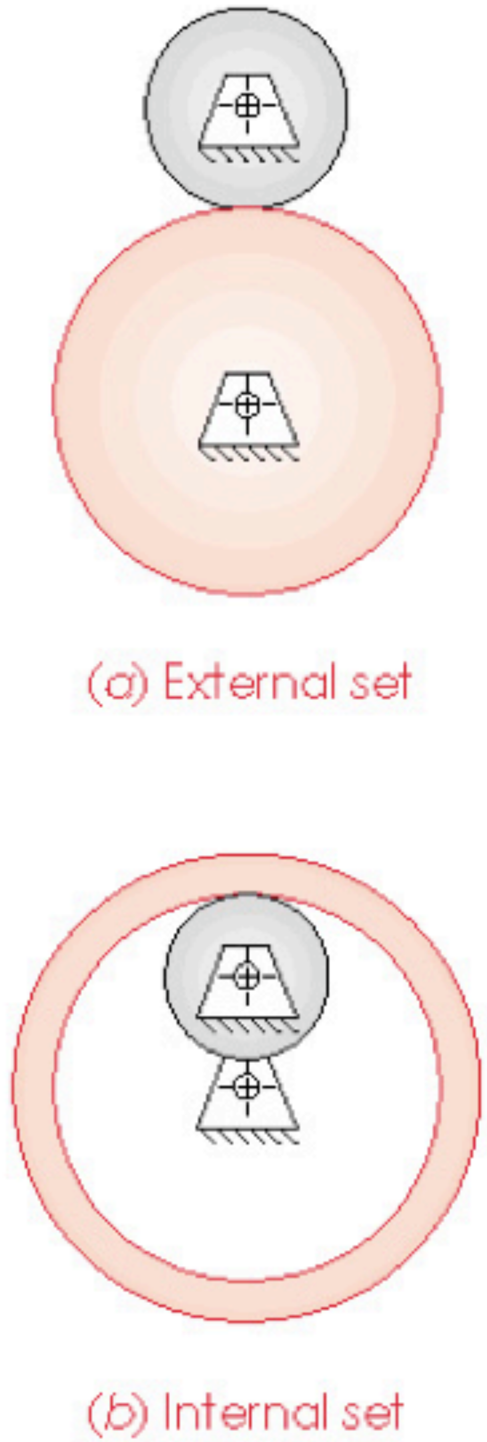



**FIGURE 9-1**

Rolling cylinders

$$\boxed{r_1\omega_1 = r_2\omega_2}$$

$$\boxed{r_1\alpha_1 = r_2\alpha_2}$$

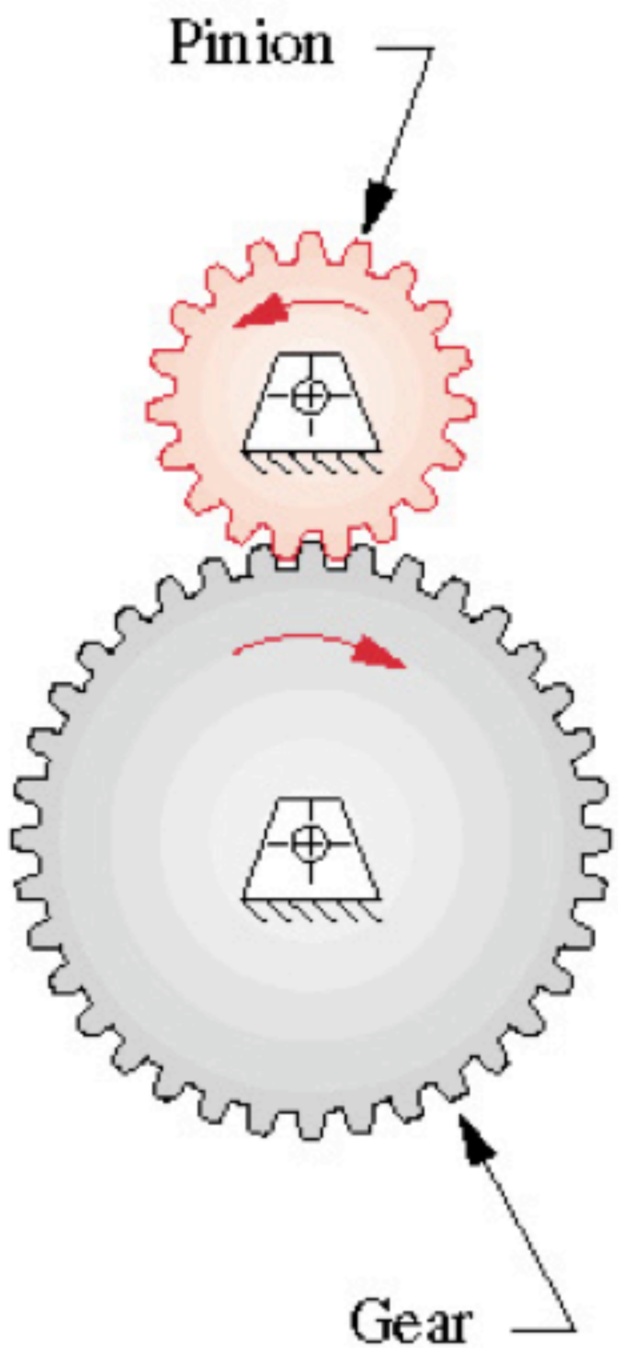
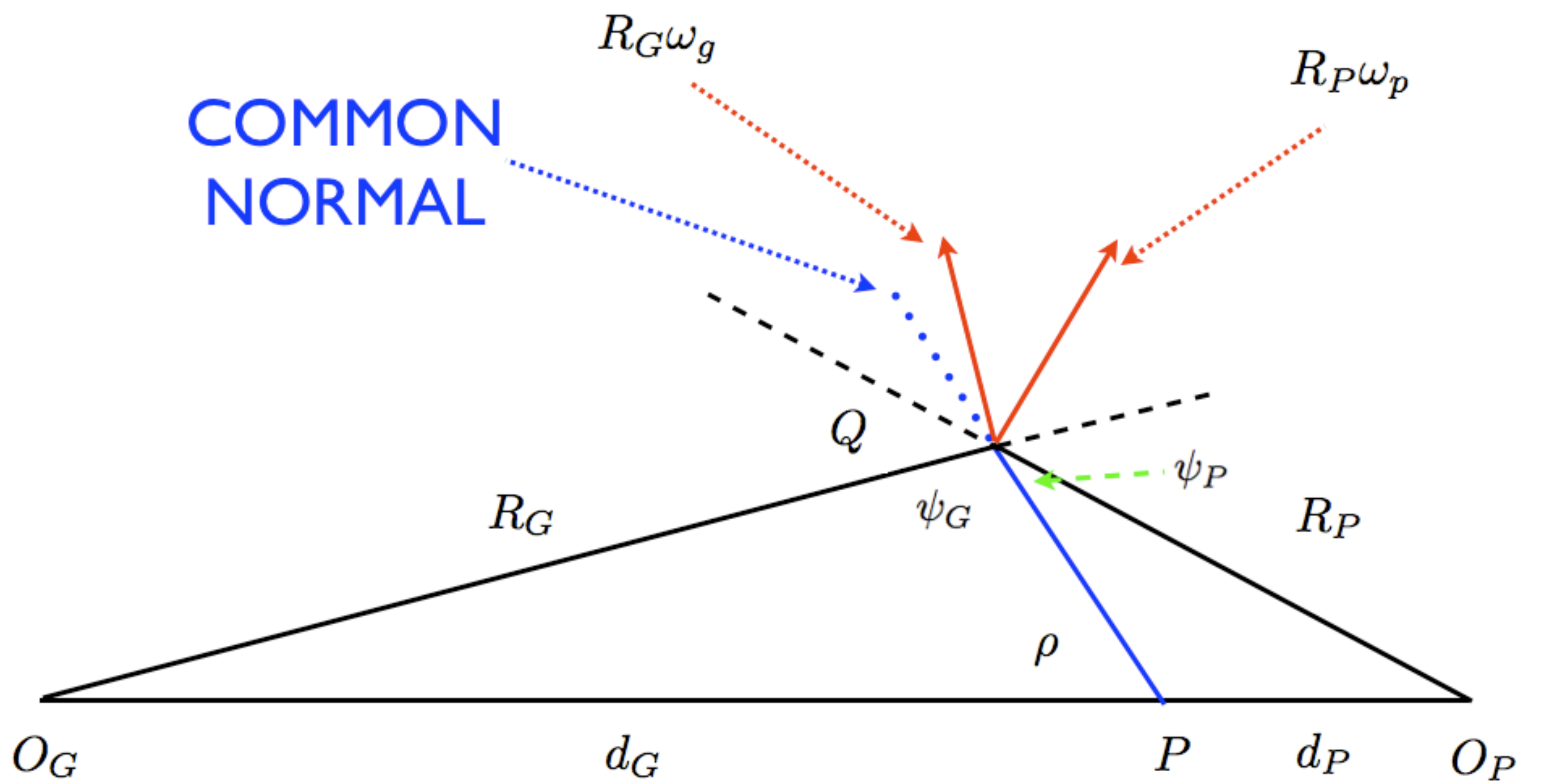



FIGURE 9-4

An external gearset




$$R_G\omega_g \sin\psi_G = R_P\omega_p \sin\psi_P$$

$$\frac{d_G}{\sin\psi_G} = \frac{R_G}{\sin\rho} \qquad \frac{d_P}{\sin\psi_P} = \frac{R_P}{\sin\rho}$$

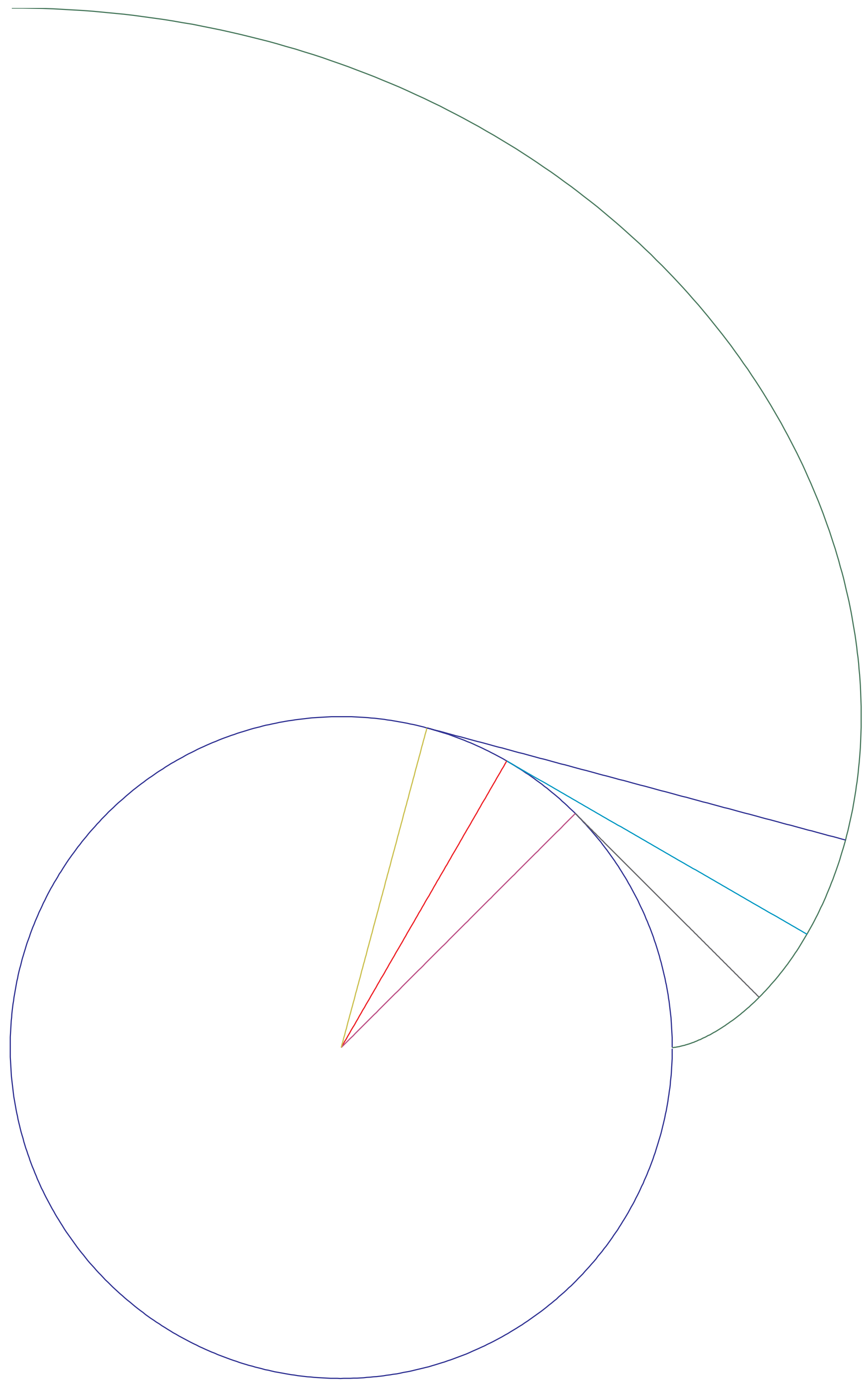
$$d_G + d_P = C \text{ (const.)}$$

$$\boxed{d_G\omega_g = d_P\omega_p}$$

IN GENERAL, THE COMMON NORMAL AND POINT $P$ VARY.

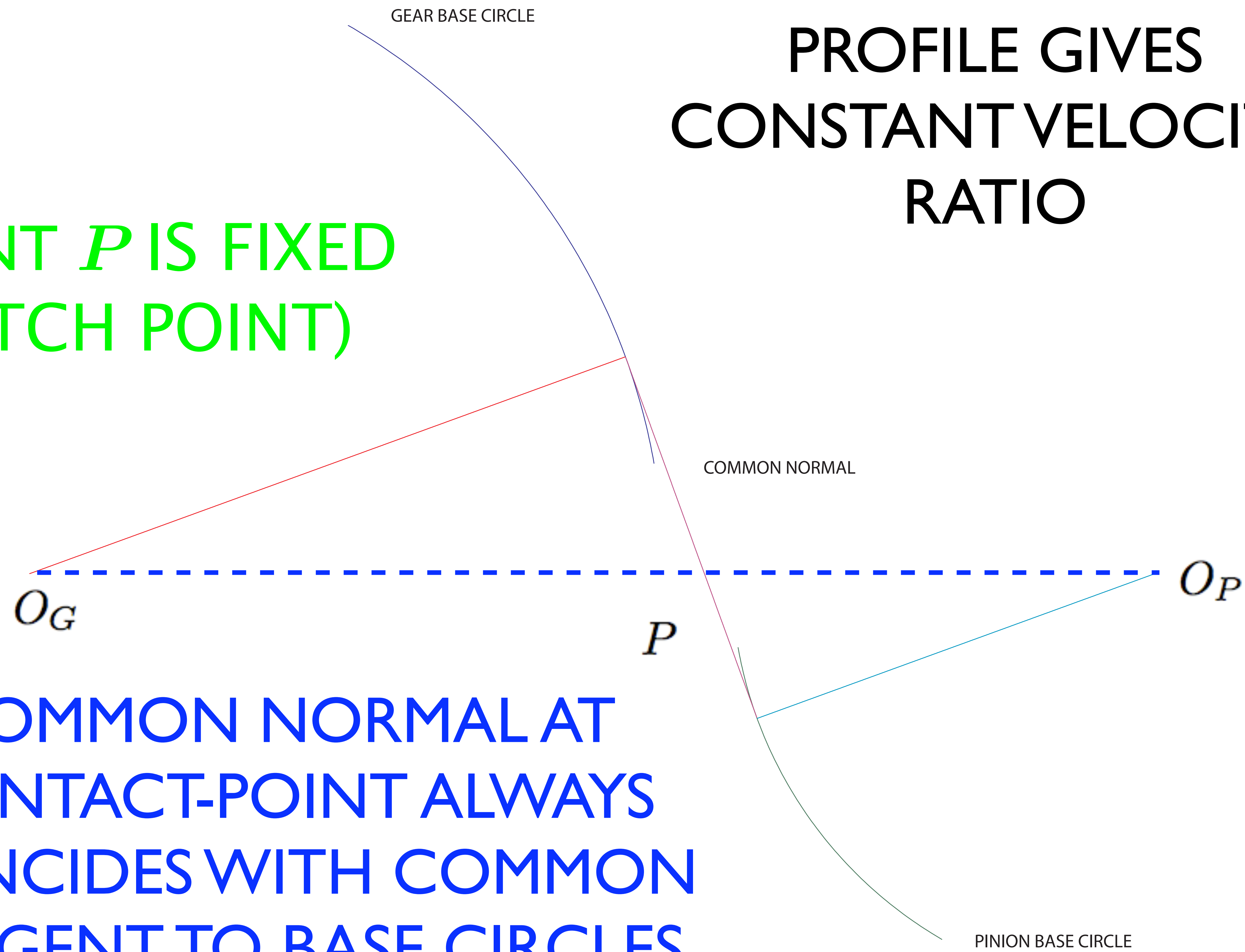
# INVOLUTE

NORMAL TO INVOLUTE IS ALWAYS TANGENT TO BASE CIRCLE



# INVOLUTE TOOTH PROFILE GIVES CONSTANT VELOCITY RATIO

POINT $P$ IS FIXED (PITCH POINT)




COMMON NORMAL AT CONTACT-POINT ALWAYS COINCIDES WITH COMMON TANGENT TO BASE CIRCLES

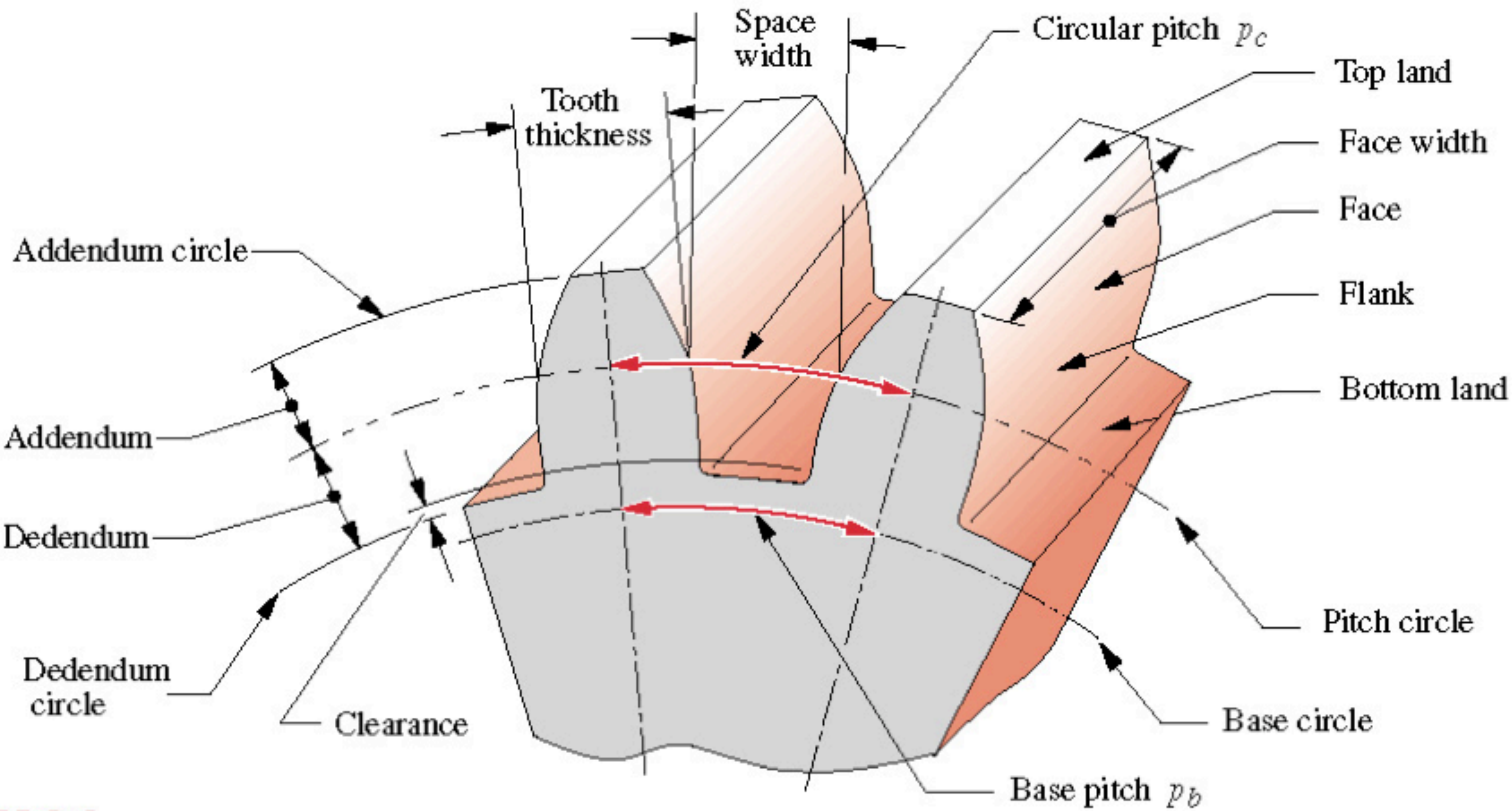
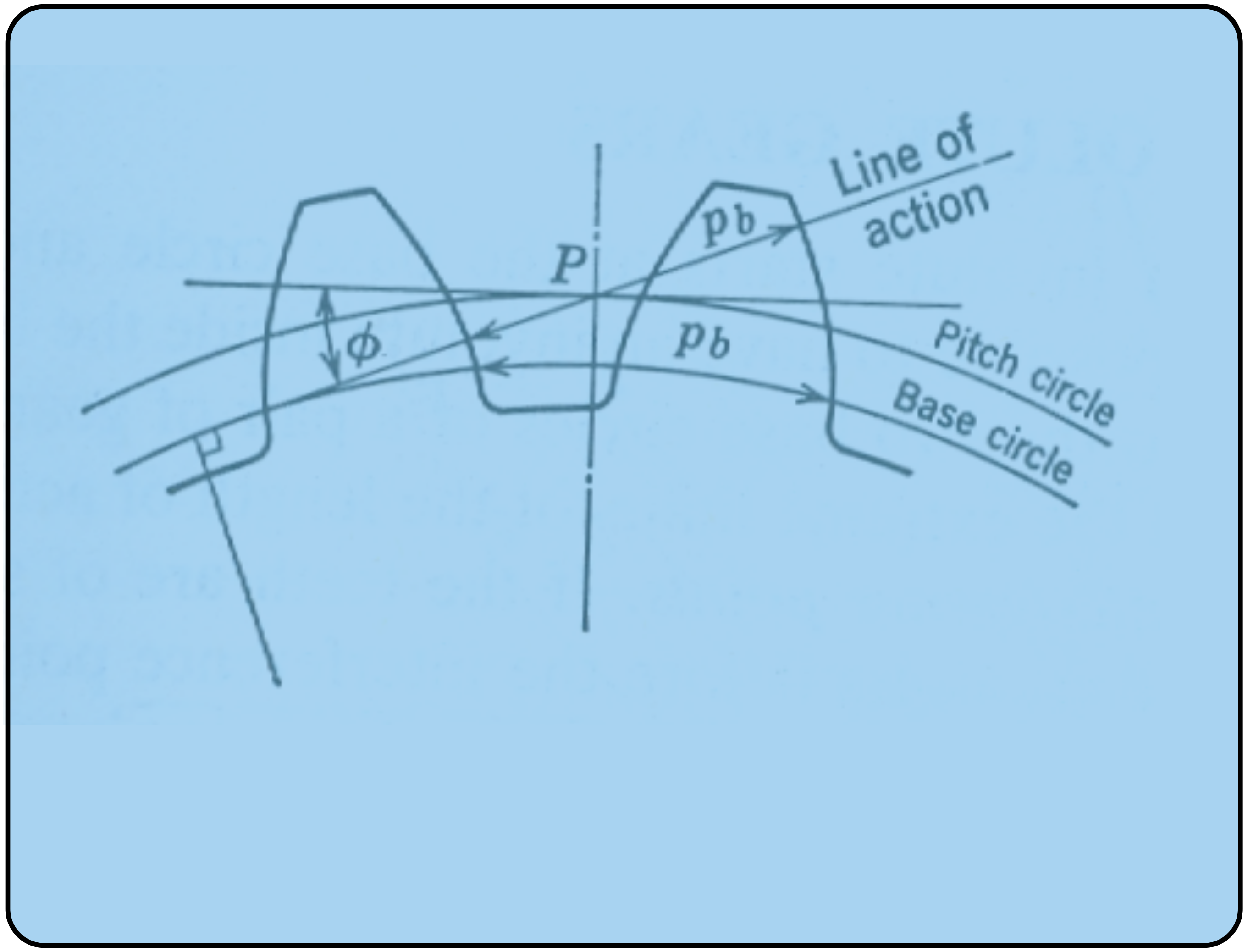



FIGURE 9-9

Gear tooth nomenclature

# BASE PITCH $p_b$




BASE CIRCLE CIRCUMFERENCE

$$p_b = \frac{2\pi r_g \cos\phi}{N_g}$$

NUMBER OF TEETH

BY DEFINITION OF INVOLUTE, $p_b$ IS ALSO THE DISTANCE BETWEEN SUCCESSIVE TEETH ALONG THE COMMON NORMAL.

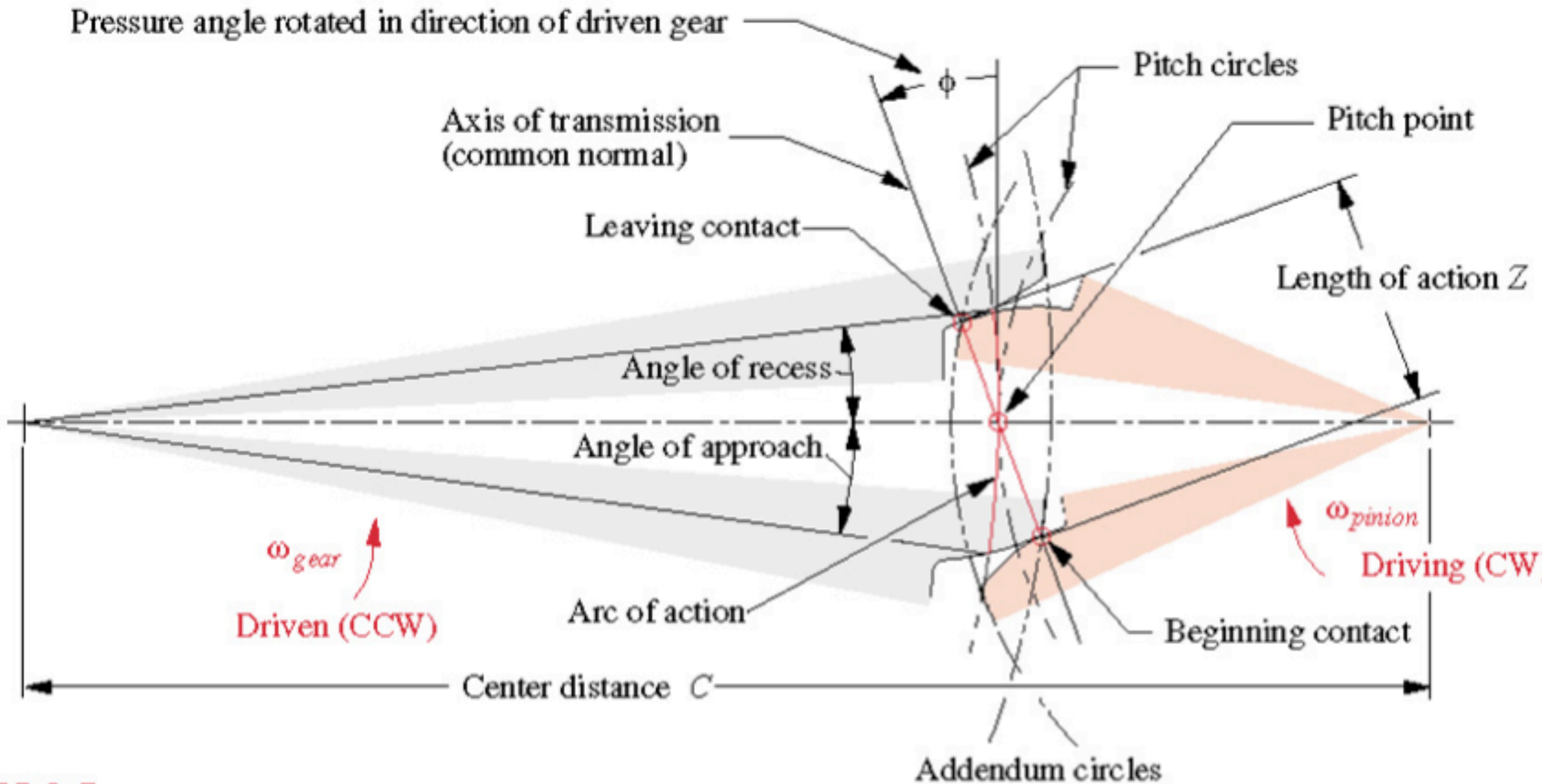
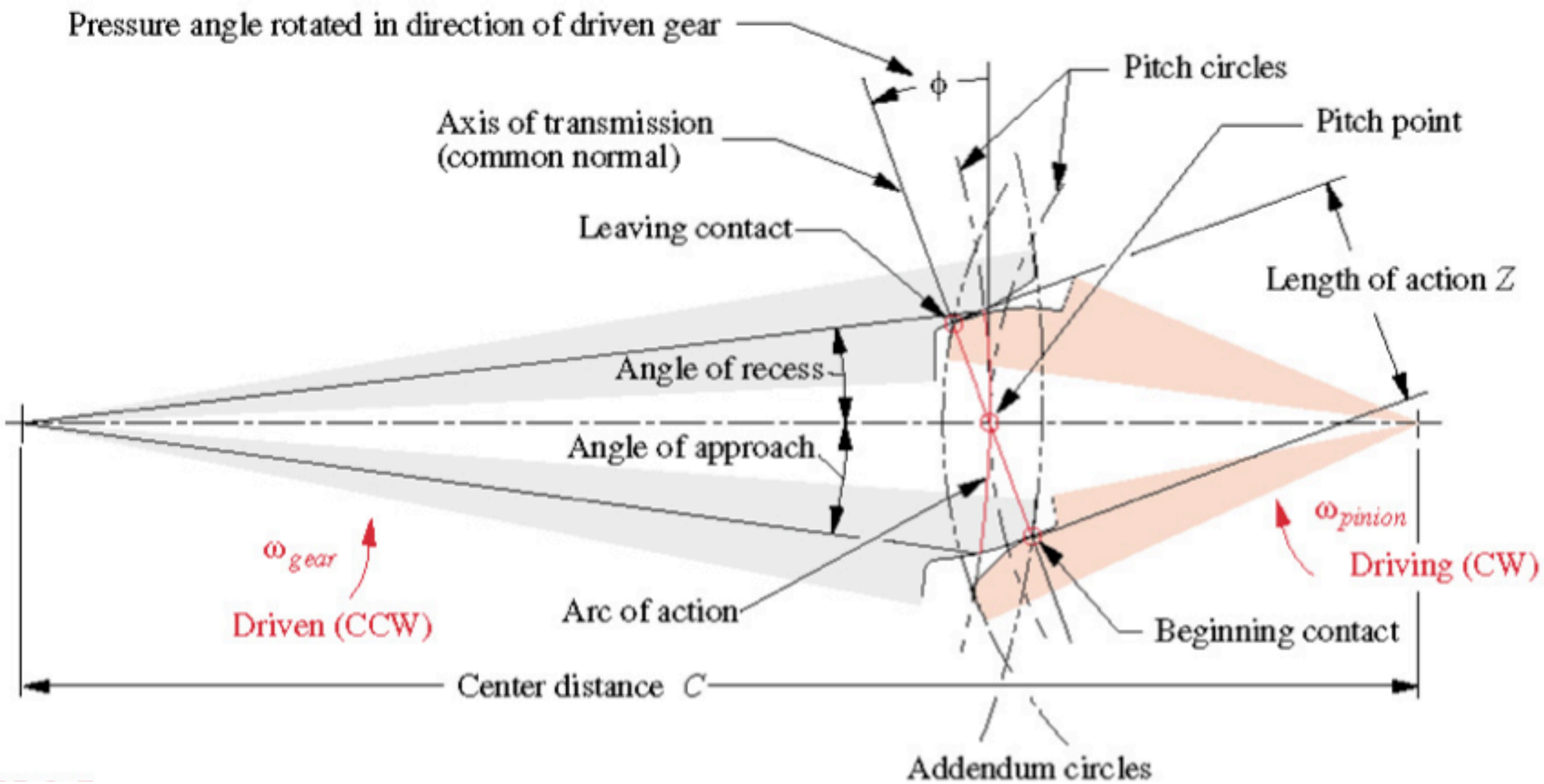



**FIGURE 9-7**

Pitch point, pitch circles, pressure angle, length of action, arc of action, and angles of approach and recess during the meshing of a gear and pinion

# LENGTH OF ACTION




$$Z = \sqrt{(r_g + a_g)^2 - (r_g \cos\phi)^2} + \sqrt{(r_p + a_p)^2 - (r_p \cos\phi)^2} - C\sin\phi$$

DISTANCE ALONG LINE OF ACTION FROM INITIAL CONTACT TO FINAL SEPARATION

# CONTACT RATIO

LENGTH OF ACTION

$$m_p = \frac{Z}{p_b}$$

BASE PITCH

AVERAGE NUMBER OF PAIRS OF TEETH IN CONTACT. THE ACTUAL NUMBER VARIES FROM INSTANT TO INSTANT. IF $m_p$ IS TOO CLOSE TO $1$, A SINGLE TOOTH MUST CARRY THE TRANSMITTED LOAD NEAR ITS TIP AND MAY BREAK.

# CONTACT RATIO

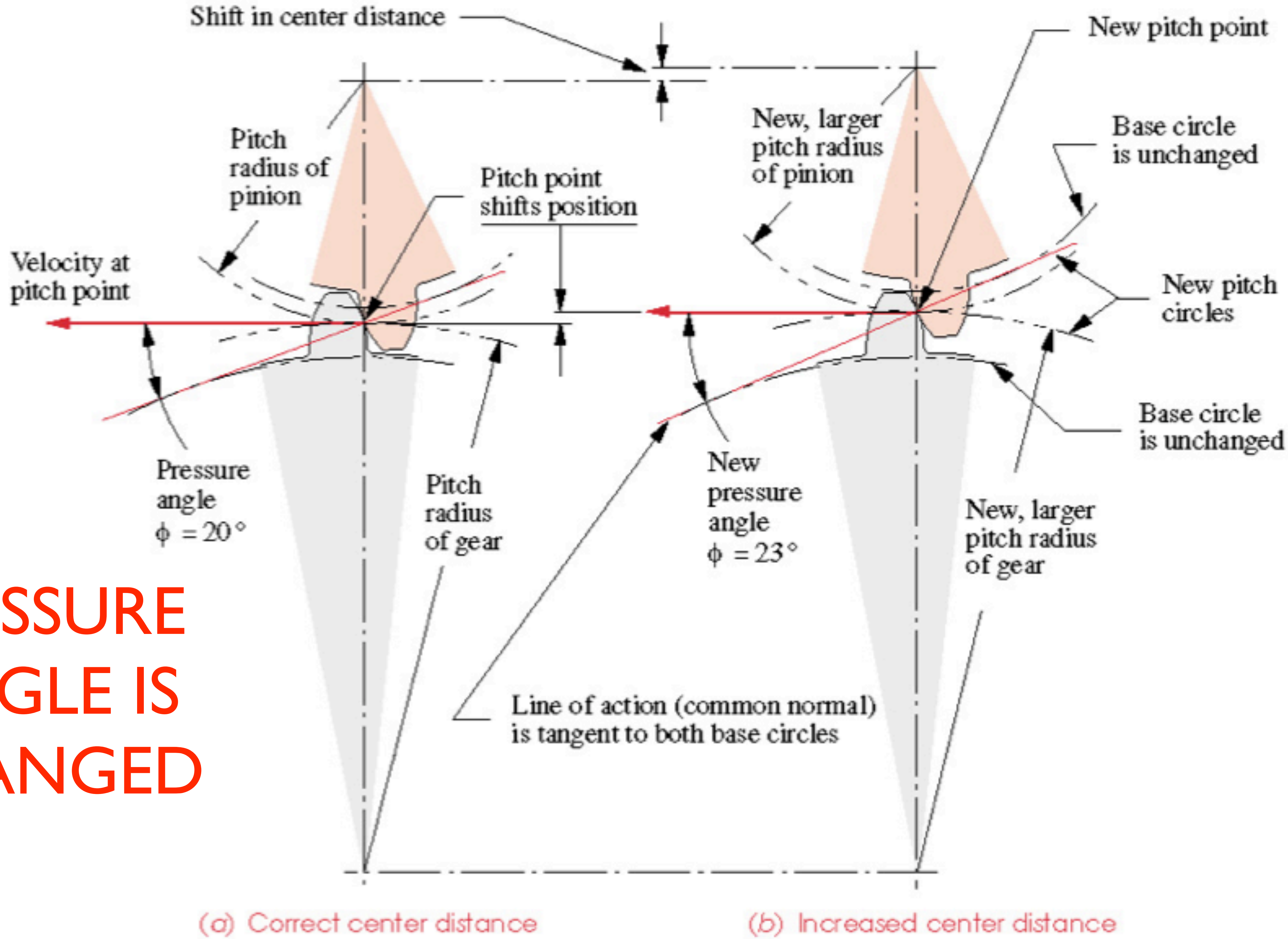
MINIMUM ACCEPTABLE VALUE:

$$\boldsymbol{m_p = 1.2.}$$

STANDARD TARGET RANGE:

$$\boldsymbol{m_p \geq 1.4.}$$

# CENTER DISTANCE ERROR




PRESSURE ANGLE IS CHANGED

**FIGURE 9-8**

Changing center distance of involute gears changes the pressure angle and pitch diameters

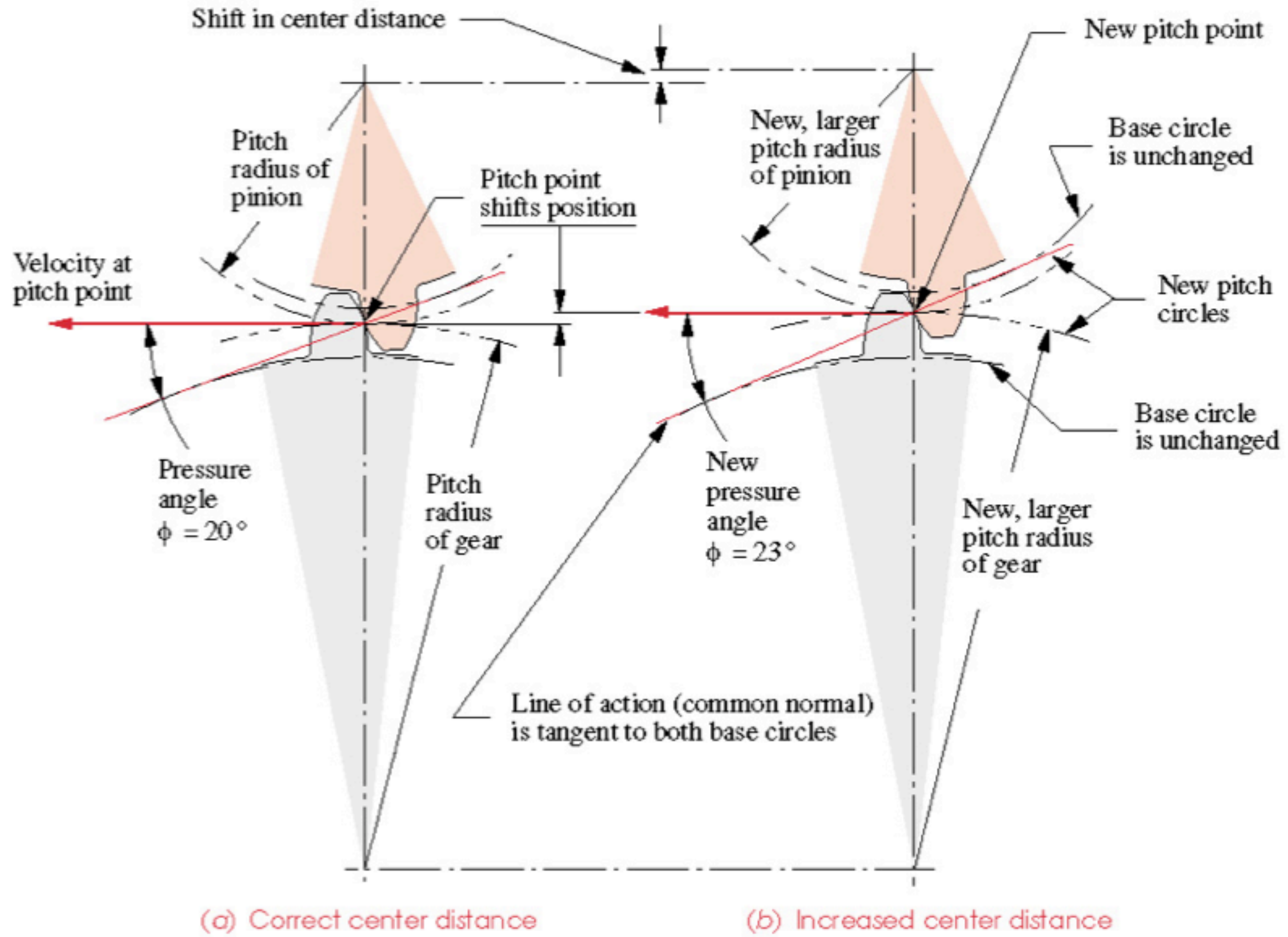



FIGURE 9-8

Changing center distance of involute gears changes the pressure angle and pitch diameters

PINION BASE CIRCLE RADIUS

$$\frac{\omega_{gear}}{\omega_{pinion}} = \frac{r_p}{r_g} = \frac{r_p \cos\phi}{r_g \cos\phi}$$

GEAR BASE CIRCLE RADIUS

VELOCITY RATIO UNCHANGED!

# INTERFERENCE

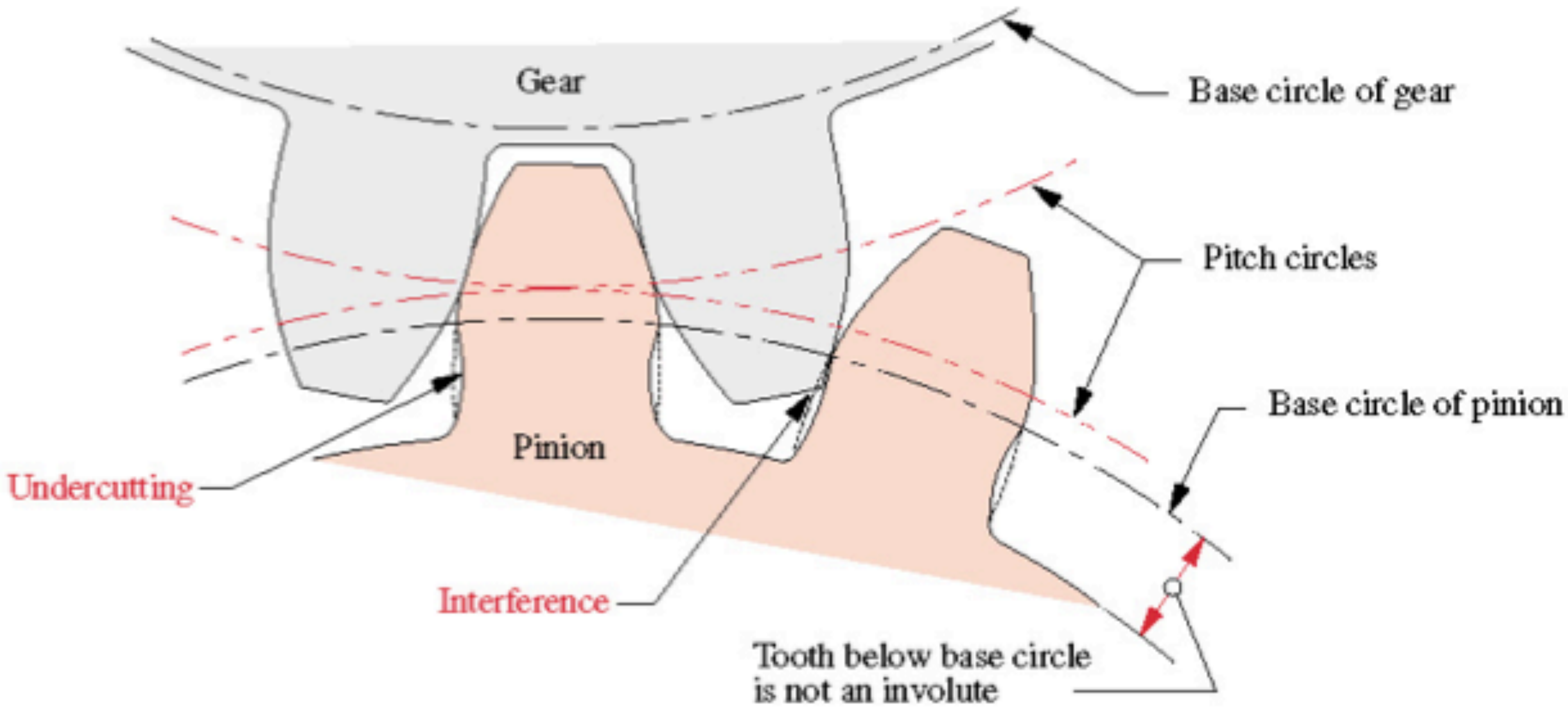



FIGURE 9-12

Interference and undercutting of teeth below the base circle

ISSUE: INVOLUTES LIE OUTSIDE THE BASE CIRCLE. IF DEDENDUM EXTENDS INSIDE THE BASE CIRCLE, THE TOOTH PROFILE IN THERE CANNOT BE INVOLUTE. CONTACT HERE IS CALLED INTERFERENCE.

# PREVENTING INTERFERENCE

## STRATEGY 1: ENOUGH TEETH ON PINION

**ADDENDUM CONSTANT:** $k = \dfrac{\pi a_g \cos\phi}{p_b} = \dfrac{\pi a_g}{p_c} = p_d a_g$

## AT ONSET OF INTERFERENCE

$$\boxed{N_g = \frac{N_p^2 \sin^2\phi - 4k^2}{4k - 2N_p \sin^2\phi}}$$

**FOR FIXED $N_g$ ,CHOOSE $N_p$ SO THAT THE EXPRESSION ON THE RIGHT EXCEEDS $N_g$ .**

# STRATEGY 2: RE-DESIGN TEETH ON PINION AND GEAR

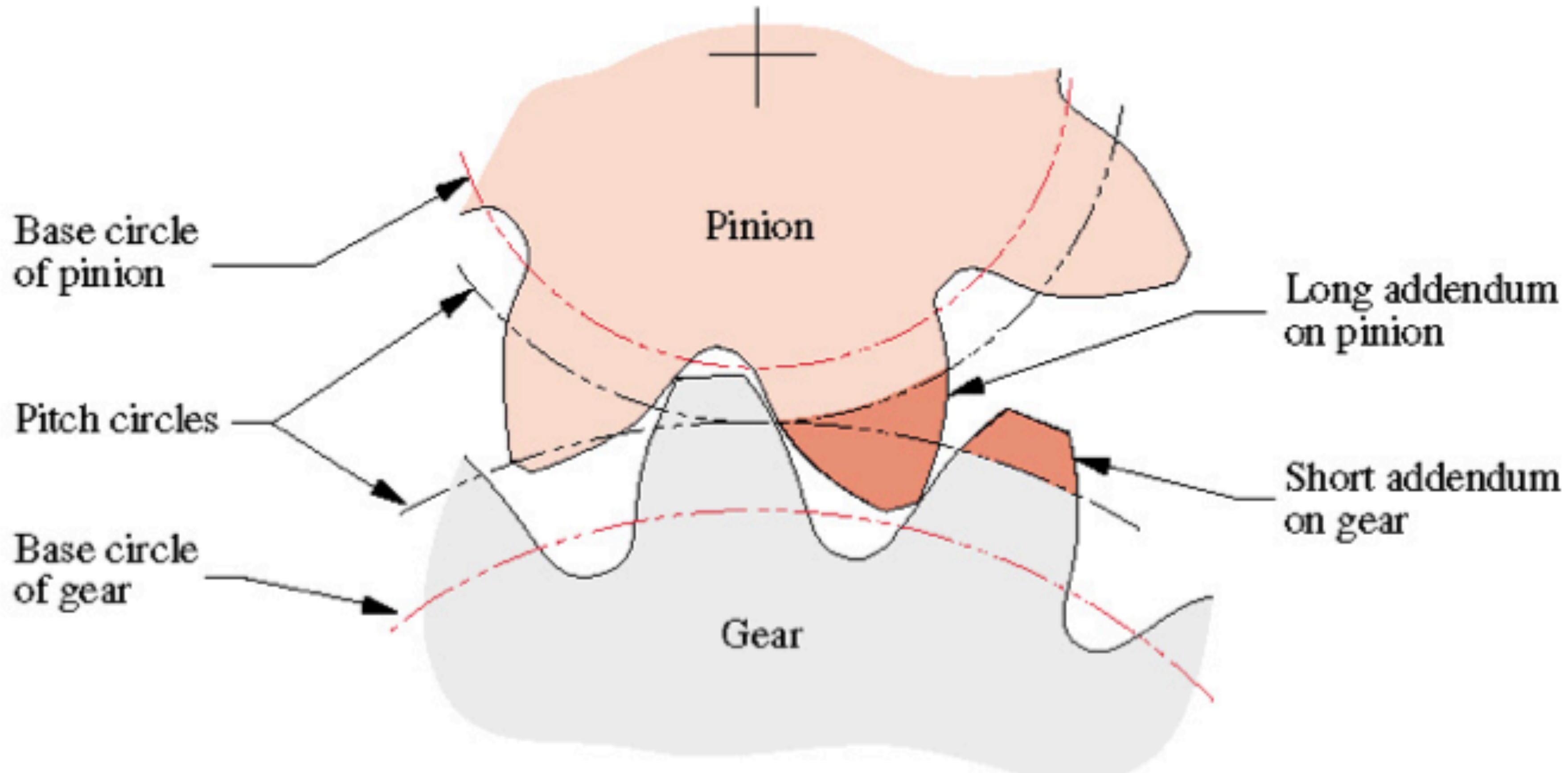



FIGURE 9-13

Profile-shifted teeth with long and short addenda to avoid interference and undercutting

...GIVES WEAKER GEAR-TOOTH, STRONGER PINION-TOOTH. THIS MAKES PINION AND GEAR TOOTH STRENGTHS CLOSER TO EQUALITY.

# BACKLASH

CLEARANCE BETWEEN MATING TEETH MEASURED AT THE PITCH CIRCLE (CAN'T BE ZERO IN REAL WORLD)

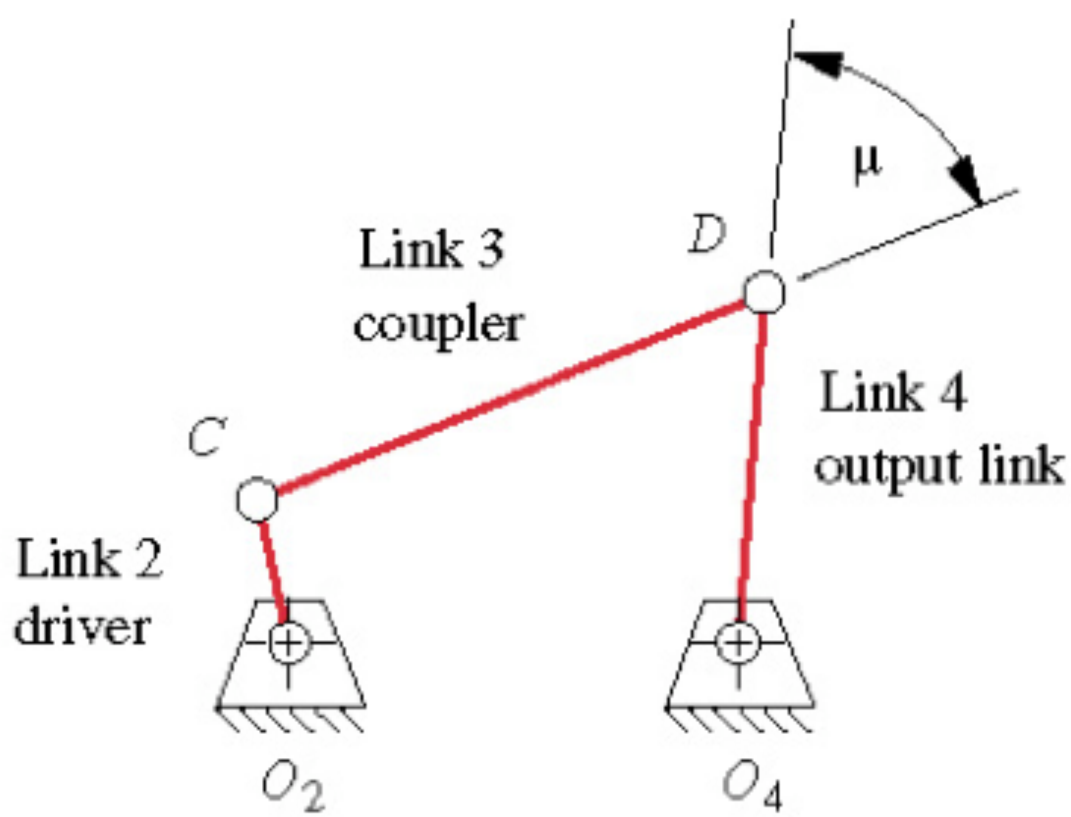
IF TORQUE IS REVERSED, CONTACTING SIDES OF TEETH SWITCH, AND TEETH SEPARATE AND REENGAGE WITH A BANG, BECAUSE OF THE GAP.

# INDICES OF MERIT

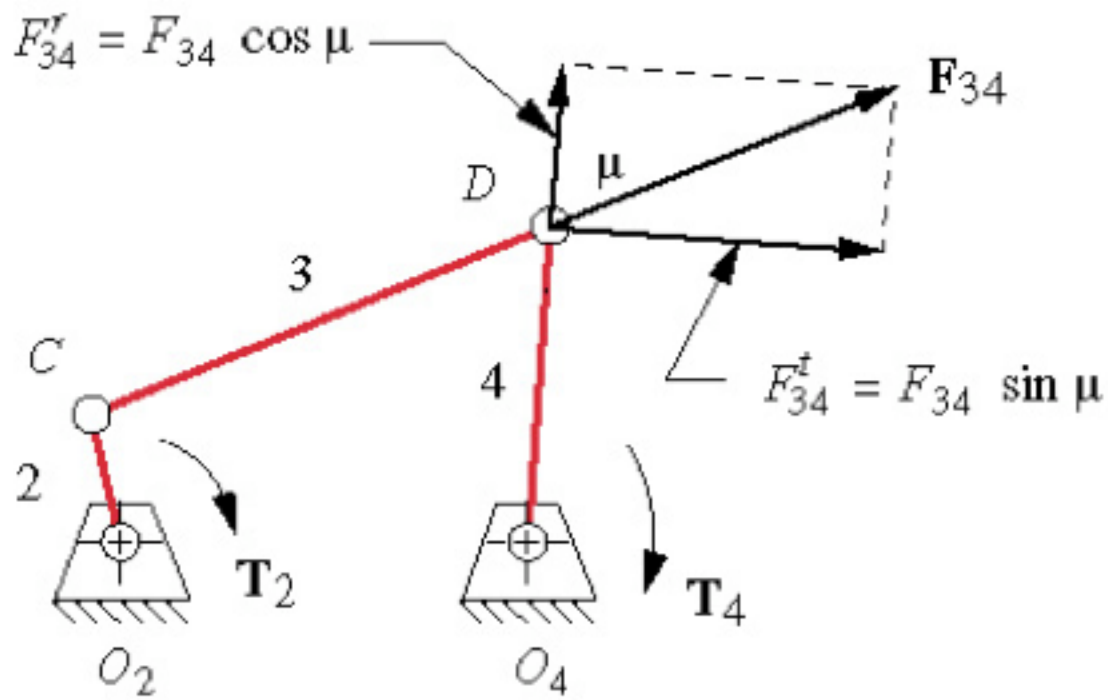
- TRANSMISSION ANGLE
- MECHANICAL ADVANTAGE
- PRESSURE ANGLE

# FOUR-BAR TRANSMISSION ANGLE

THE **TRANSMISSION ANGLE** $\mu$ IS THE ANGLE BETWEEN THE OUTPUT LINK AND THE COUPLER. IT IS TAKEN CONVENTIONALLY TO BE THE ABSOLUTE VALUE OF THE ACUTE ANGLE AT THE JOINT.



# STATIC IMPLICATIONS OF TRANSMISSION ANGLE




OPTIMUM VALUE OF $\mu$ IS 90°

COMMON DESIGNER'S RULE: KEEP $\mu$ NO LESS THAN 40°

# GCRR - MINIMUM TRANSMISSION ANGLE

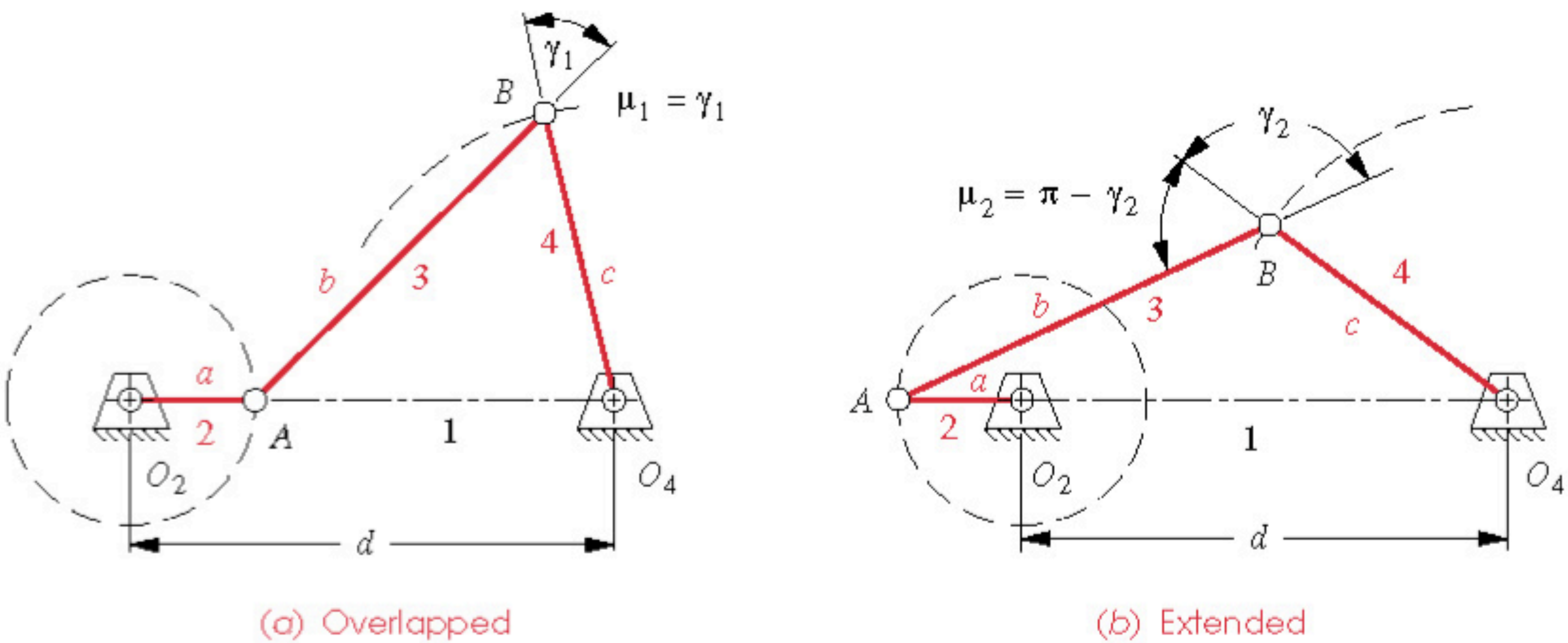



FIGURE 4-15

VALUES OF $\mu$ AT EXTREMA OF $\phi_{12}$

# CAM TRANSMISSION ANGLE

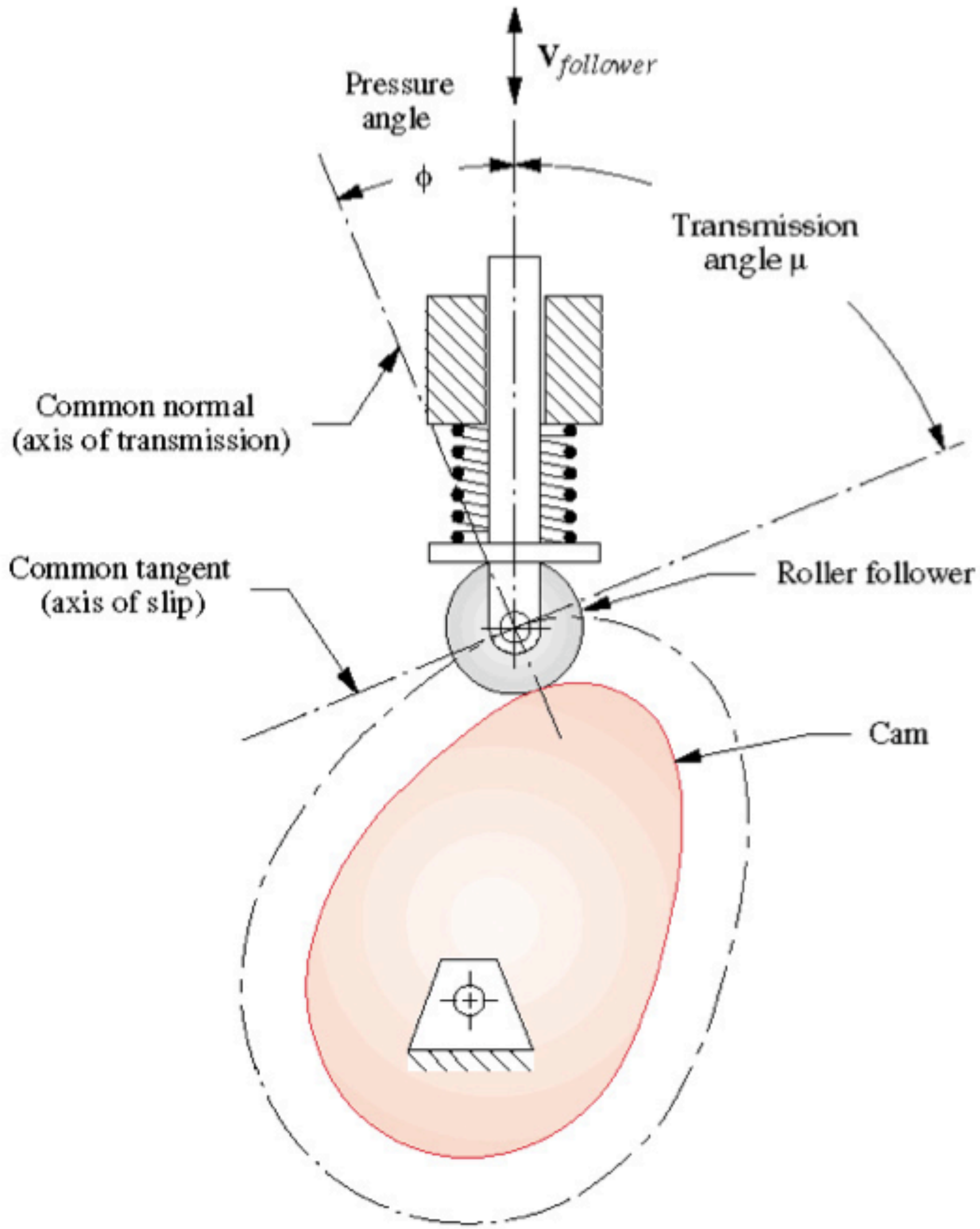
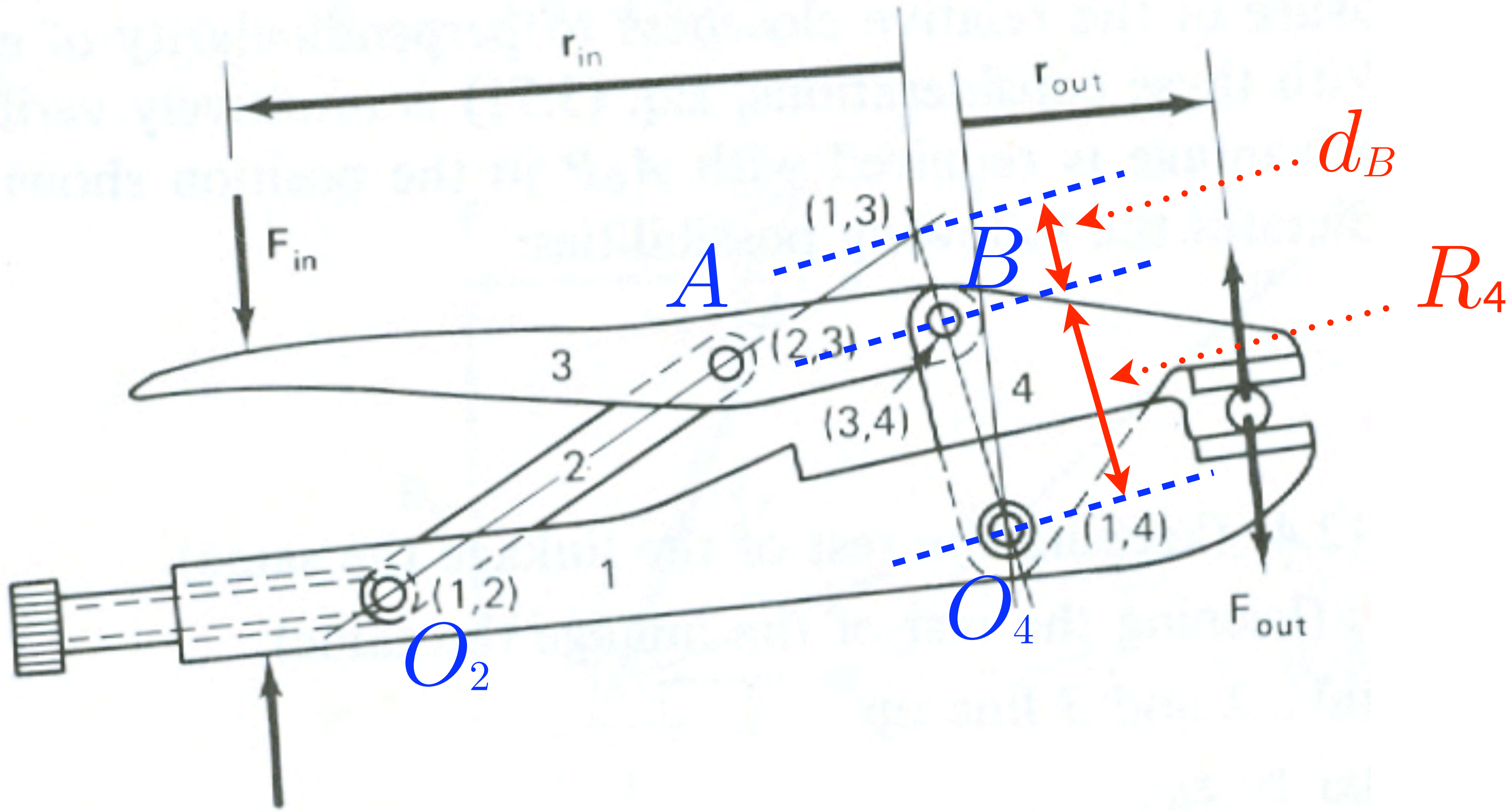



**FIGURE 8-44**

Cam pressure angle

# MECHANICAL ADVANTAGE



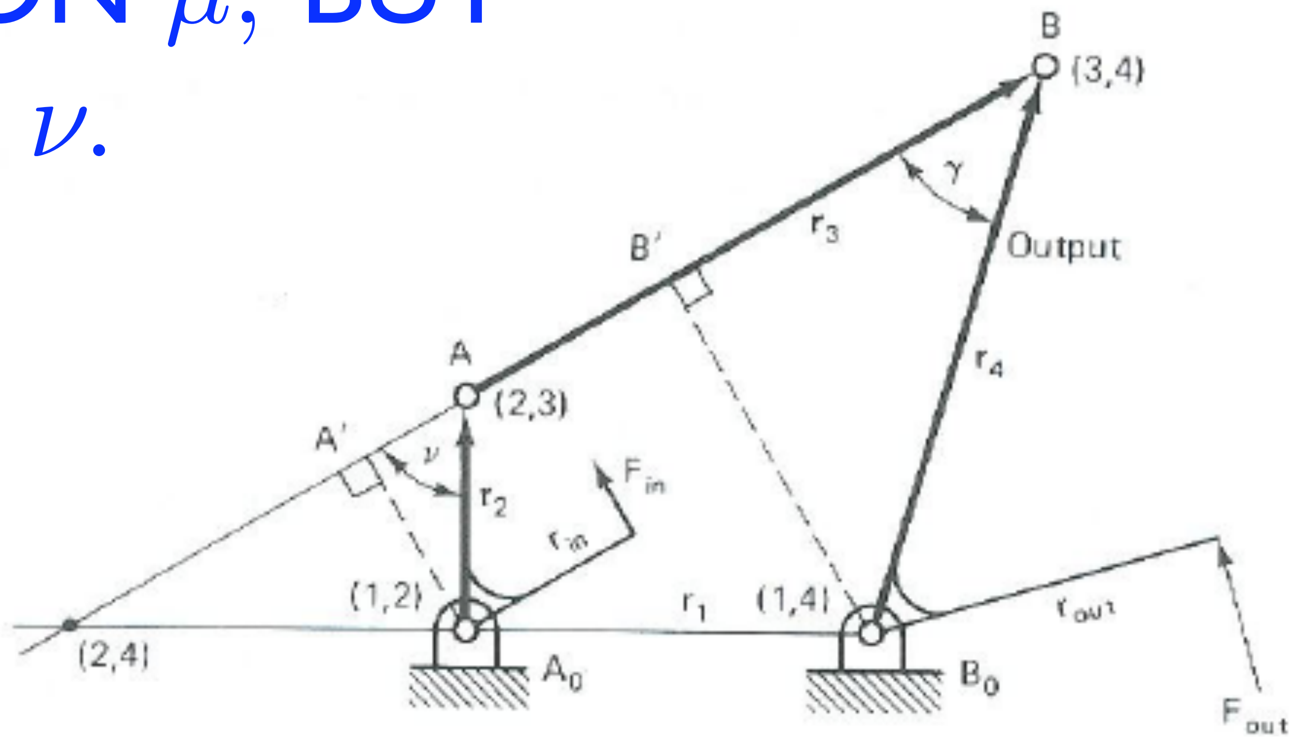

$$F_{in} r_{in}\, \omega_3 = F_{out} r_{out}\, \omega_4$$

$$M.A. = \frac{F_{out}}{F_{in}} = \frac{r_{in}\, \omega_3}{r_{out}\, \omega_4} = \frac{r_{in}\, R_4}{r_{out}\, d_B}$$

AS $I_{1,3} \to B, d_B \to 0, \text{M.A.} \to \infty.$

M.A. LARGE NEAR TOGGLE. STOP LIMITS OVERSHOOT OF TOGGLE.

# $M.\ A.$ DEPENDS ON $\mu$, BUT ALSO ON $\nu$.

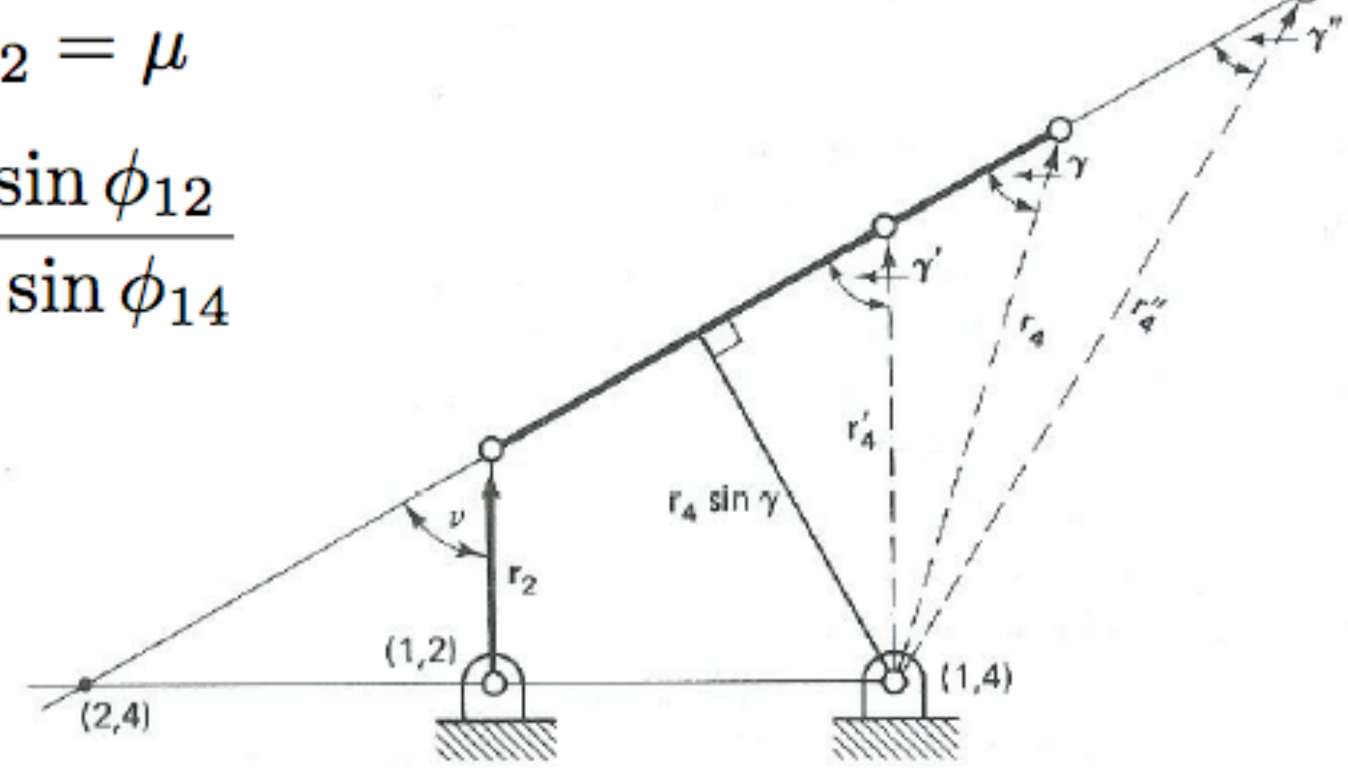



$$\nu = \pi - \phi_{14} \qquad \gamma = \phi_{12} = \mu$$

$$M.A = \frac{r_{in}\omega_2}{r_{out}\omega_4} = \frac{r_{in}R_4 \sin\phi_{12}}{r_{out}R_2 \sin\phi_{14}}$$

$$\nu = \pi - \phi_{14} \qquad \gamma = \phi_{12} = \mu$$

$$M.A = \frac{r_{in}\omega_2}{r_{out}\omega_4} = \frac{r_{in}R_4 \sin\phi_{12}}{r_{out}R_2 \sin\phi_{14}}$$




ALL THREE FOUR-BARS HAVE THE SAME M.A., BUT DIFFERENT TRANSMISSION-ANGLES.

# CAM PRESSURE ANGLE $\phi$

$$\phi + \mu = 90^\circ$$

**ACCEPTABLE RANGE OF $\phi$:** $0 \to 30^\circ$

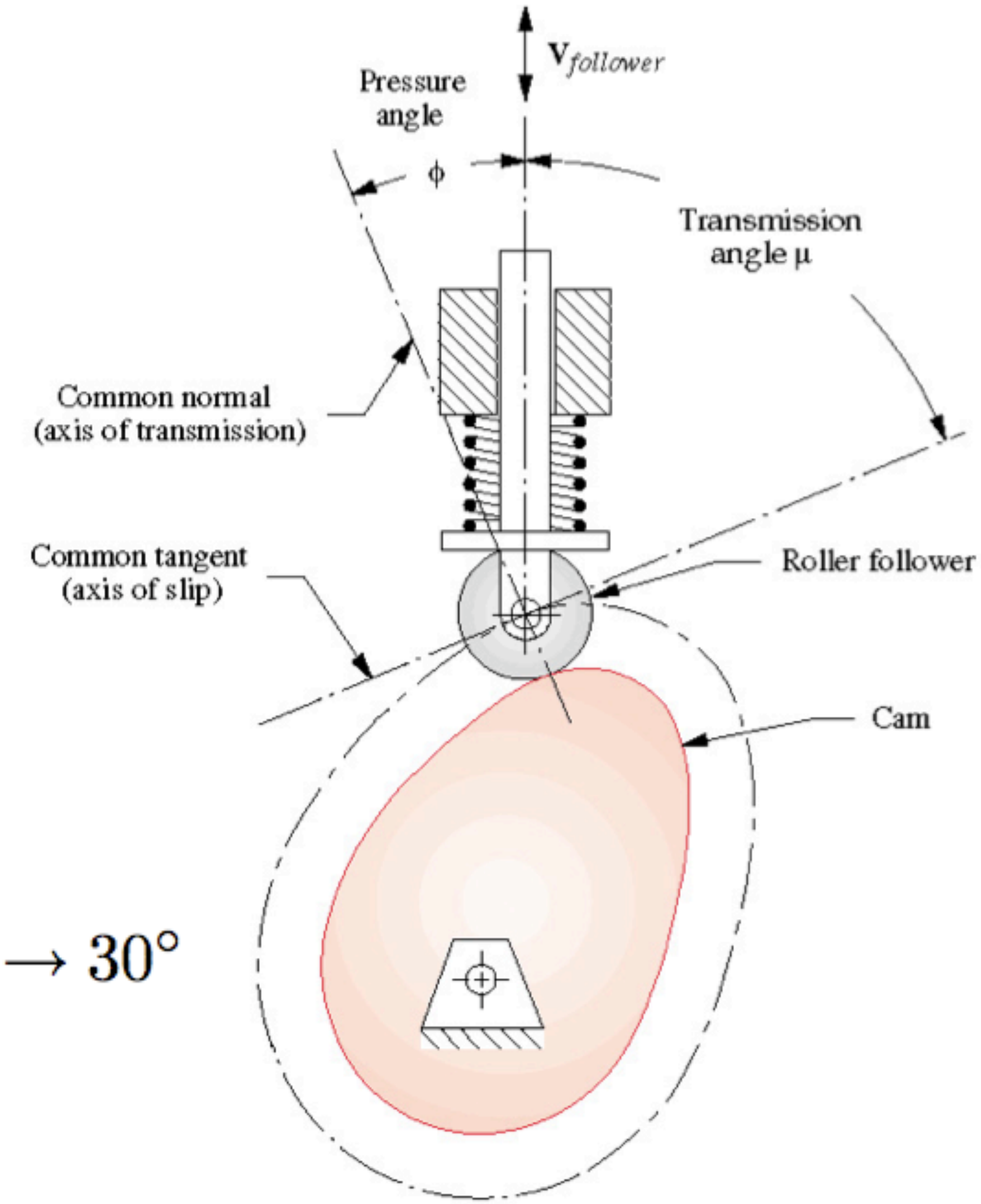



FIGURE 8-44

Cam pressure angle

# GEAR PRESSURE ANGLE $\phi$

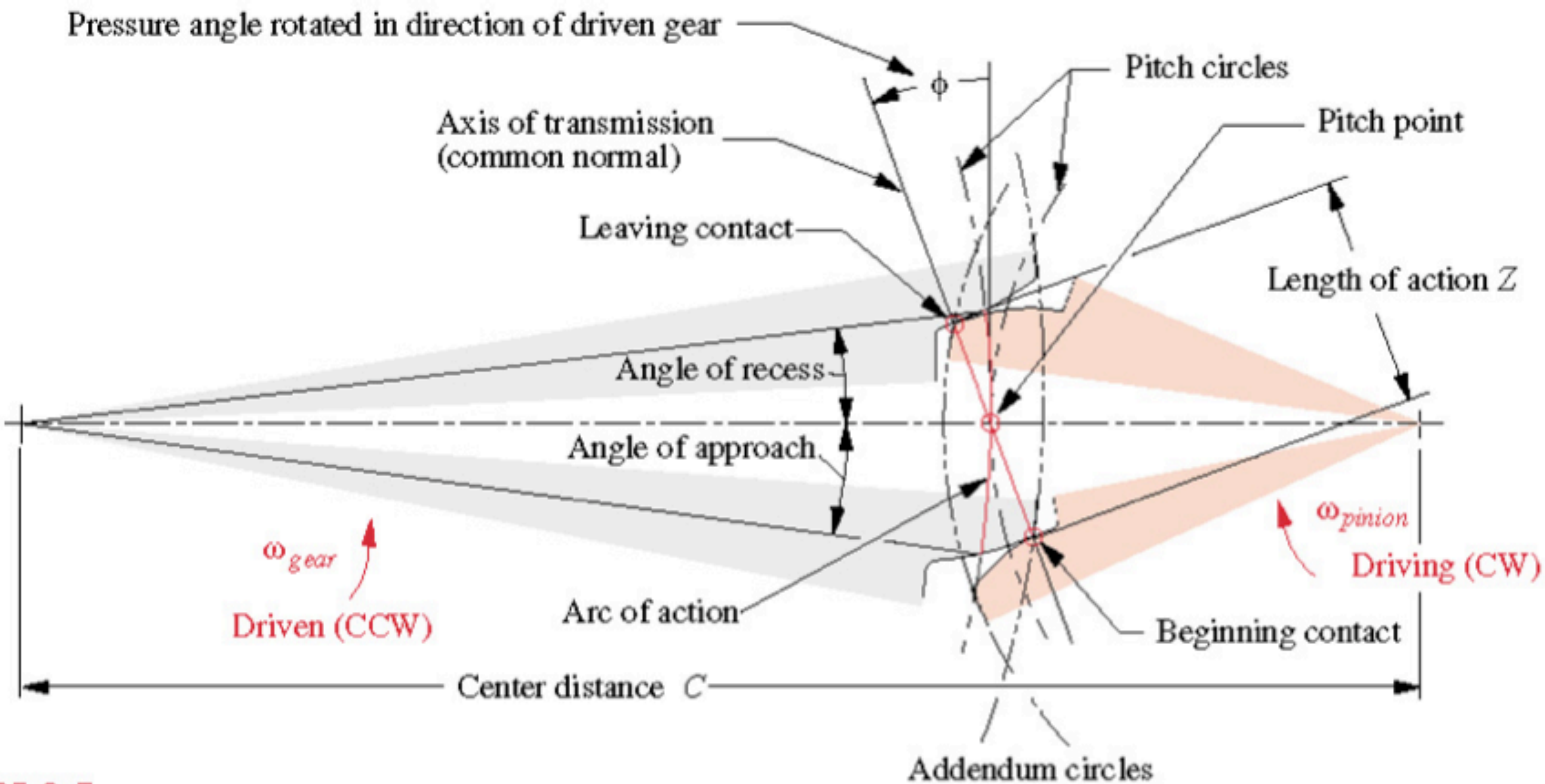



FIGURE 9-7

Pitch point, pitch circles, pressure angle, length of action, arc of action, and angles of approach and recess during the meshing of a gear and pinion

**STANDARD VALUES OF $\phi$: $14.5°,\ 20°,\ 25°$**

## ME 213 A/B HOMEWORK SET 6

(Due 10/08/10)

1. A GCRR four-bar mechanism has link-lengths $R_1 = 8$, $R_2 = 4$, $R_3 = 7$, $R_4 = 6$. Let $\theta_2$ range from $0^\circ$ to $90^\circ$ in increments of $15^\circ$. Implement the MATLAB code presented in class to find the corresponding values of $\theta_3$ and $\theta_4$, using both the direct method and Newton-Raphson.

## ME 213 A/B HOMEWORK SET 6

(Due 10/08/10)

1. A GCRR four-bar mechanism has link-lengths $R_1 = 8$, $R_2 = 4$, $R_3 = 7$, $R_4 = 6$. Let $\theta_2$ range from $0°$ to $90°$ in increments of $15°$. Implement the MATLAB code presented in class to find the corresponding values of $\theta_3$ and $\theta_4$, using both the direct method and Newton-Raphson.

## PROBLEM 6-13

***Statement:*** Find all of the instant centers of the linkages shown in Figure P6-6.

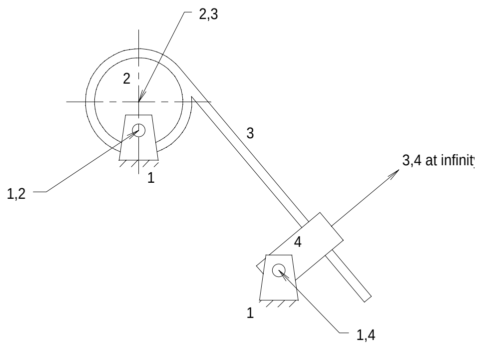
***Solution:*** See Figure P6-6 and Mathcad file P0613.

a. This is a fourbar inverted slider-crank with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

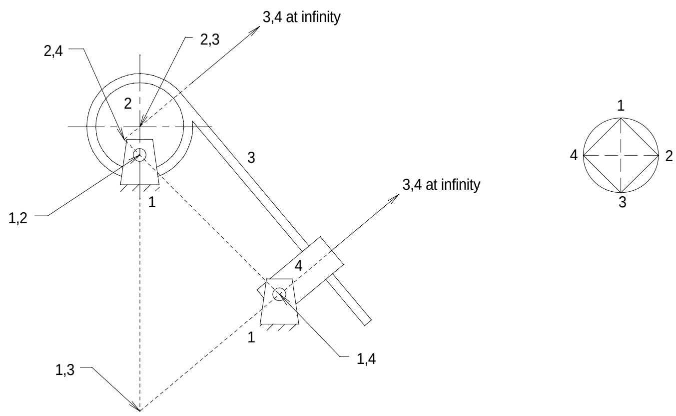
$$C := \frac{n \cdot (n-1)}{2} \qquad C = 6$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.




3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$ $\qquad$ $I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$

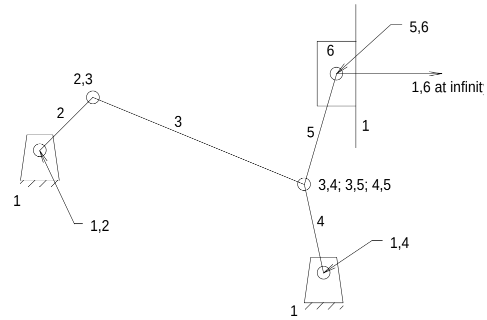



b. This is a sixbar with slider, $n := 6$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

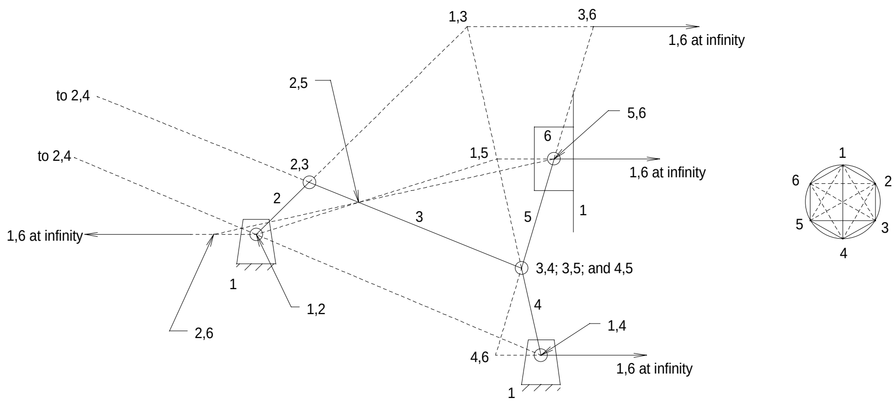
$$C := \frac{n \cdot (n-1)}{2} \qquad C = 15$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.




3. Use Kennedy's Rule and a linear graph to find the remaining 7 ICs.

| | |
|---|---|
| $I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$ | $I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$ |
| $I_{1,5}$: $I_{1,6}$-$I_{5,6}$ and $I_{1,4}$-$I_{4,5}$ | $I_{4,6}$: $I_{1,6}$-$I_{1,4}$ and $I_{4,5}$-$I_{5,6}$ |
| $I_{2,5}$: $I_{1,2}$-$I_{1,5}$ and $I_{2,4}$-$I_{4,5}$ | $I_{2,6}$: $I_{1,2}$-$I_{1,6}$ and $I_{2,5}$-$I_{5,6}$ |
| $I_{3,6}$: $I_{1,6}$-$I_{1,3}$ and $I_{3,4}$-$I_{4,6}$ | |

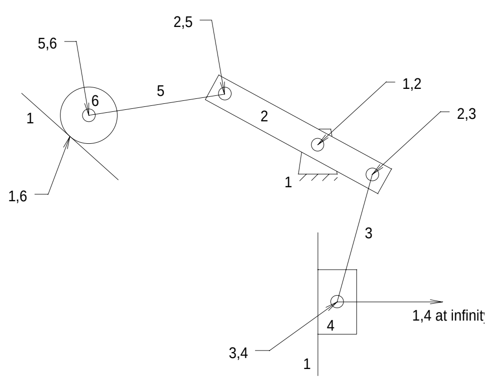



c. This is a sixbar with slider and roller with $n := 6$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

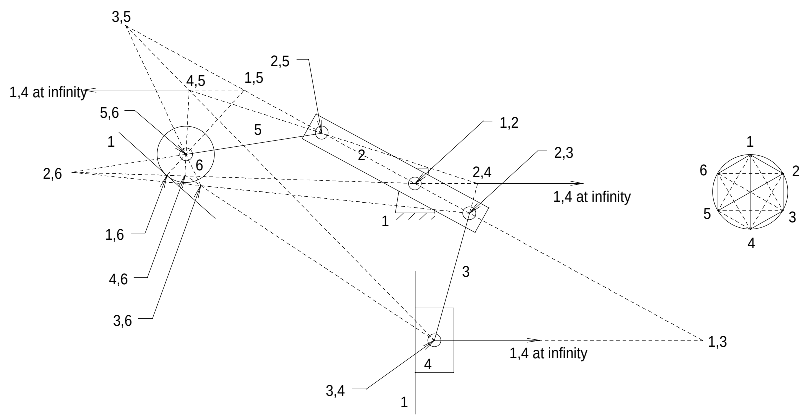
$$C := \frac{n \cdot (n-1)}{2} \qquad C = 15$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining 8 ICs.

| | |
|---|---|
| $I_{2,6}$: $I_{1,2}$-$I_{1,6}$ and $I_{2,5}$-$I_{5,6}$ | $I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$ |
| $I_{1,5}$: $I_{1,6}$-$I_{5,6}$ and $I_{1,2}$-$I_{2,5}$ | $I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,5}$-$I_{2,4}$ |
| $I_{4,5}$: $I_{1,4}$-$I_{1,5}$ and $I_{2,4}$-$I_{2,5}$ | $I_{3,5}$: $I_{3,4}$-$I_{4,5}$ and $I_{2,5}$-$I_{2,3}$ |
| $I_{3,6}$: $I_{3,6}$-$I_{5,6}$ and $I_{2,3}$-$I_{2,6}$ | $I_{4,6}$: $I_{4,5}$-$I_{5,6}$ and $I_{3,4}$-$I_{3,6}$ |

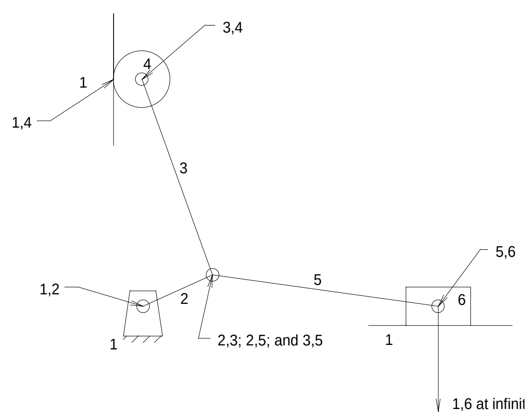



d. This is a sixbar with slider and roller with $n := 6$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n-1)}{2} \qquad C = 15$$

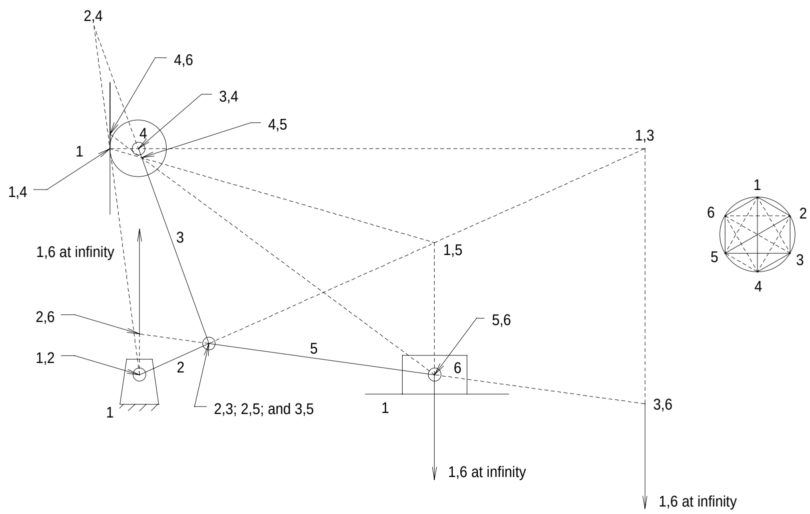
2. Draw the linkage to scale and identify those ICs that can be found by inspection.




3. Use Kennedy's Rule and a linear graph to find the remaining 7 ICs.

| | |
|---|---|
| $I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$ | $I_{3,6}$: $I_{1,6}$-$I_{1,3}$ and $I_{3,5}$-$I_{5,6}$ |
| $I_{2,6}$: $I_{1,2}$-$I_{1,6}$ and $I_{2,5}$-$I_{5,6}$ | $I_{1,5}$: $I_{1,6}$-$I_{5,6}$ and $I_{1,2}$-$I_{2,5}$ |
| $I4_{,5}$: $I_{1,4}$-$I_{1,5}$ and $I_{3,5}$-$I_{3,4}$ | $I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$ |
| $I_{4,6}$: $I_{1,6}$-$I_{1,4}$ and $I_{4,5}$-$I_{5,6}$ | |

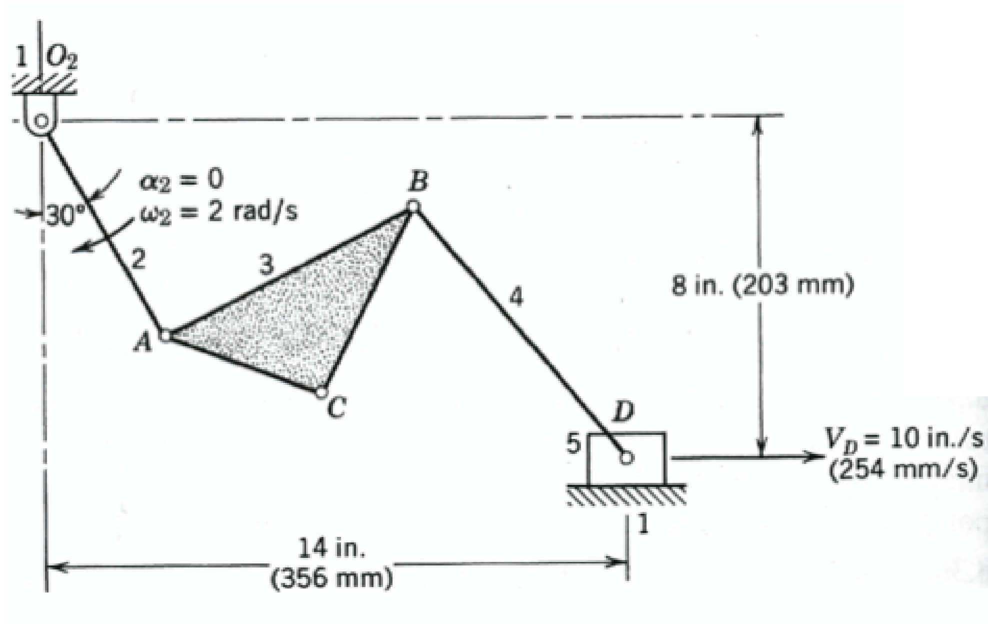
(See next page)


2,4
4,6
3,4
4,5
1,3
4
1
1,4
3
1,5
1,6 at infinity
2,6
5,6
1,2
2
5
6
1
2,3; 2,5; and 3,5
1
3,6
1,6 at infinity
1,6 at infinity
1
6
2
5
3
4

# ME 213 SPRING 2010 FINAL EXAM

**NAME:**____________________

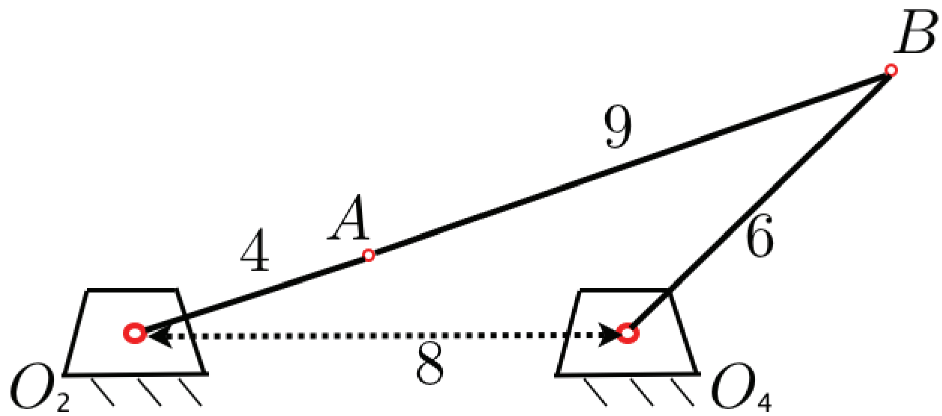
| | |
|---|---|
| 1 | __________ |
| 2 | __________ |
| 3 | __________ |
| 4 | __________ |
| 5 | __________ |
| 6 | __________ |
| 7 | __________ |
| 8 | __________ |
| 9 | __________ |
| 10 | __________ |
| Total | __________ |




1.(10%) For the 5-link mechanism shown:

(i) Label all joints,

(ii) Compute the mobility.




2.(10%) The 4-bar mechanism shown has link-lengths $\{R_1, R_2, R_3, R_4\} = \{8, 4, 9, 6\}$, and is assembled in a configuration with $\phi_{12} > 0$.

(i) Classify the linkage according to Barker's classification.

(ii) In the configuration shown, the internal angle $\phi_{23}$ attains its maximum value. Sketch the mechanism in the configuration that corresponds to the minimum value of $\phi_{12}$. Compute this value.

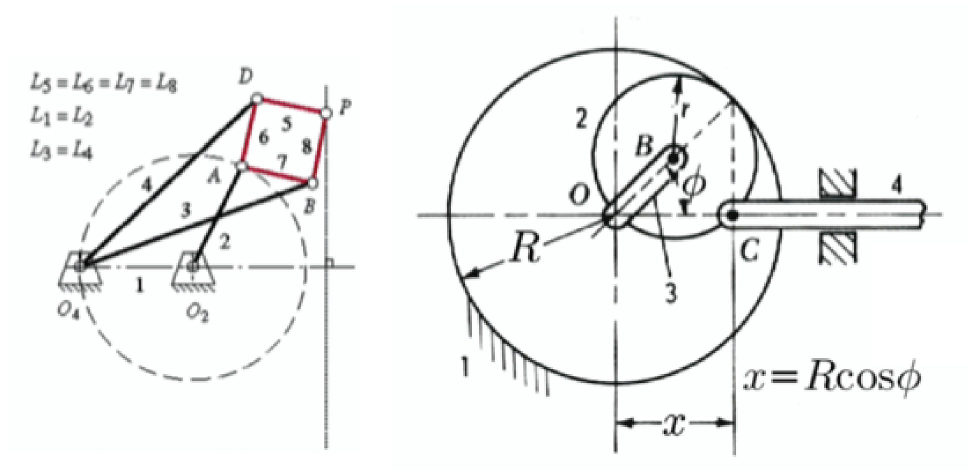
3.(10%)

Recall the following types of mechanism synthesis:

(i) Function generation:

(ii) Path generation:

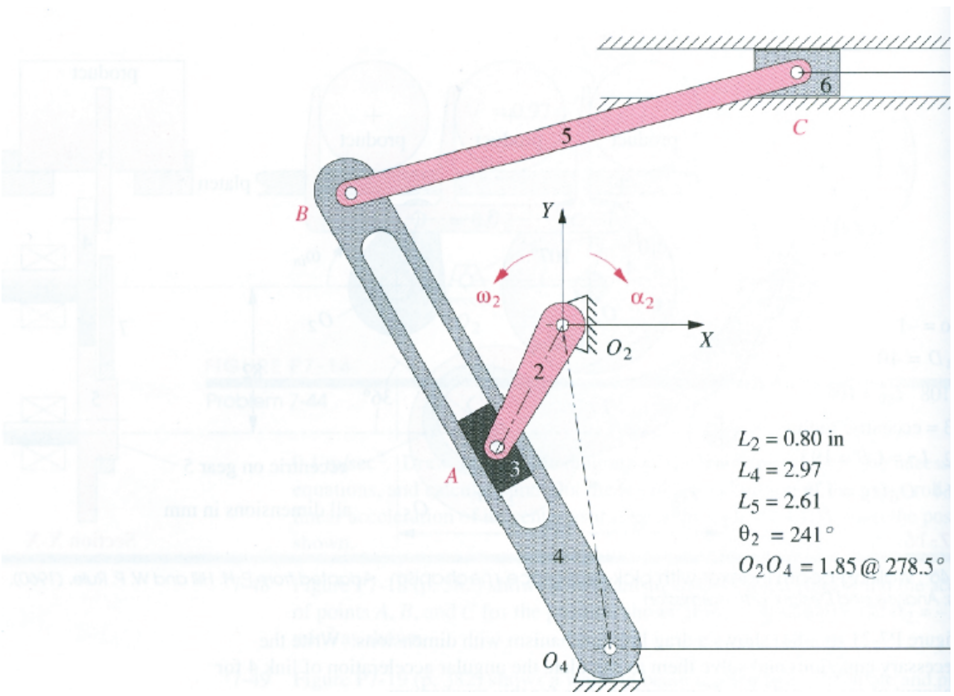
(iii) Motion generation:



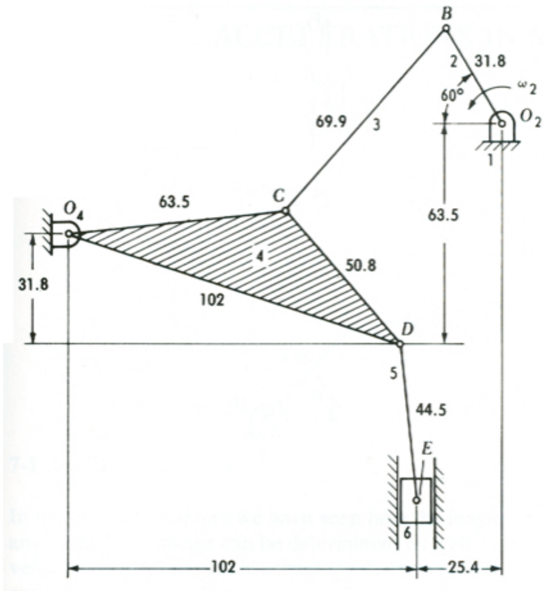

(a) (b)

(a) The device on the left is **Peaucellier's** celebrated *exact straight-line mechanism.* It is designed to move a **point** $P$ along a straight line. Which of the above types of synthesis does this mechanism represent?

(b) The device on the right is an $\mathfrak{OLDE}$–$\mathfrak{TYME}$ analog computer which, for each value of the input angle $\phi$, delivers the measured output $x = R\cos\phi$. Which of the above types of synthesis does this mechanism represent?




4.(10%) For the six-bar mechanism shown, use Kennedy's Rule to locate the instant centers $I_{1,3}$ and $I_{4,6}$.



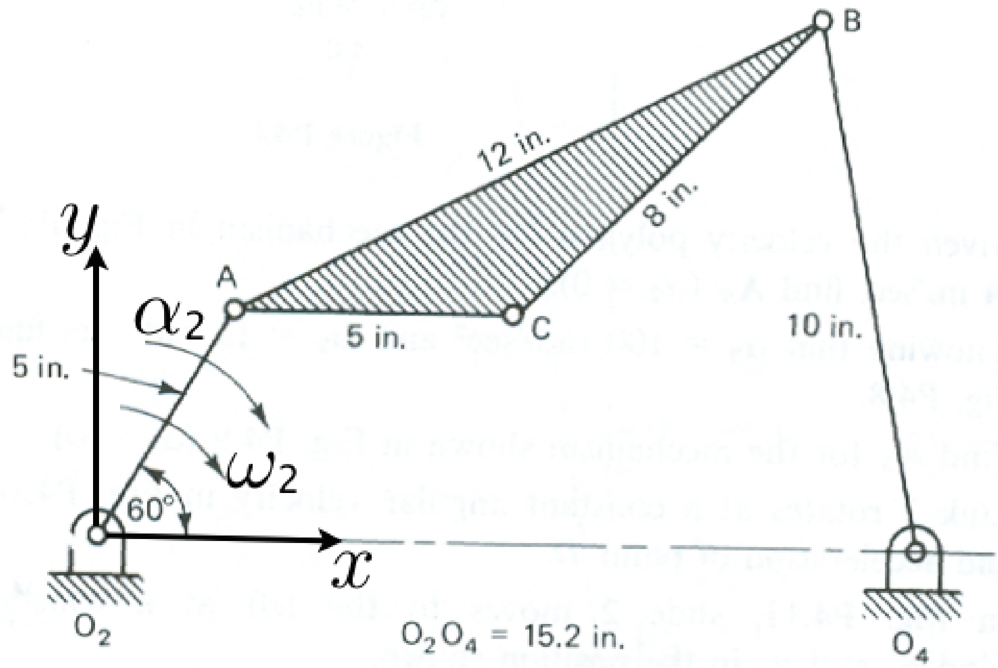

5.(10%) For the six-link mechanism shown (the lengths of all rigid links are given in mm.):

(a) Develop vector-loop equations. Explicitly specify the loop or loops being used.

(b) Consider each vector used in some vector-loop. Is its magnitude constant or variable? Is its orientation constant or variable?

(c) What, if any, geometric constraints exist among vectors used in some vector-loop?

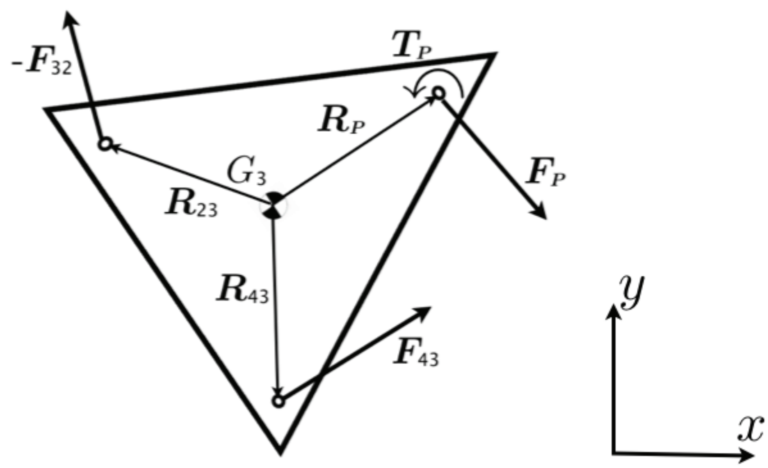
(d) Write the vector-loop equations in components and (if any exist) the constraint equations. Count equations and variables. How many of the variable quantities must be specified to make the system solvable?



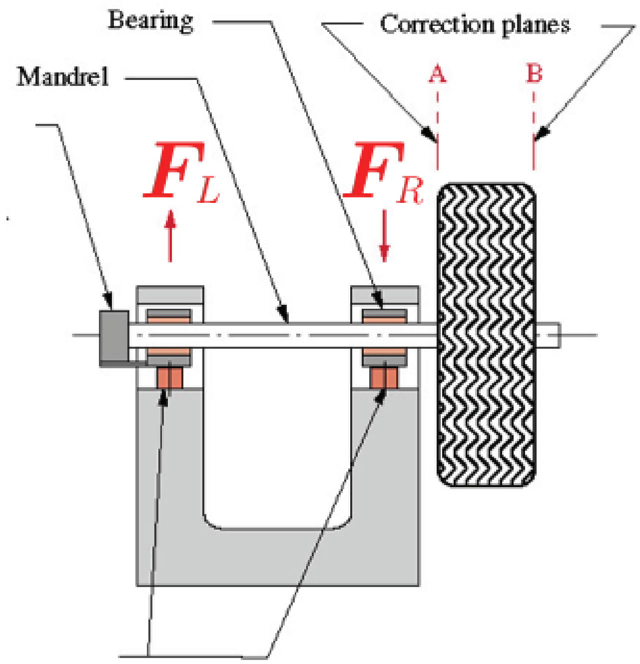

6.(10%) The link-lengths for the four-bar mechanism shown are given above. At the instant shown, the angles, angular velocities, and the angular accelerations of the links are given to be $\theta_2 = 60°$, $\theta_3 = 27.09°$, $\theta_4 = 101.63°$, $\boldsymbol{\omega}_2 = 5\,\text{rad/s} \curvearrowright$, $\boldsymbol{\omega}_3 = 7.19\,\text{rad/s} \curvearrowleft$, $\boldsymbol{\omega}_4 = 7.05\,\text{rad/s} \curvearrowleft$, $\boldsymbol{\alpha}_2 = 180\,\text{rad/s}^2 \curvearrowright$, $\boldsymbol{\alpha}_3 = 224.93\,\text{rad/s}^2 \curvearrowleft$, $\boldsymbol{\alpha}_4 = 272.08\,\text{rad/s}^2 \curvearrowleft$. AB makes an angle $\phi = 29°$ with AC. Find the horizontal and vertical components of the acceleration $\mathbf{a}_\text{C}$ of the coupler-point C. Link angles are measured counter-clockwise relative to the ground link.

$$\mathbf{a}_\text{A} = \big(-180\,\hat{\mathbf{k}}\big) \times \big[2.5\,\hat{\mathbf{\imath}} + 4.33\,\hat{\mathbf{\jmath}}\big] - 25\big[2.5\,\hat{\mathbf{\imath}} + 4.33\,\hat{\mathbf{\jmath}}\big] = \big[716.9\,\hat{\mathbf{\imath}} - 558.25\,\hat{\mathbf{\jmath}}\big]\ \text{in/s}^2$$

$$\mathbf{R}_{CA} = \big[5\cos(\theta_3 - \phi)\,\hat{\mathbf{\imath}} + 5\sin(\theta_3 - \phi)\,\hat{\mathbf{\jmath}}\big]\ \text{in}$$




7.(10%) The FBD for a link in a mechanism is shown above. The link has mass $m = 8\,\text{kg}$, and moment of inertia $I_{G_3} = 6\,\text{kg}\cdot\text{m}^2$. At the instant of concern, $\mathbf{R}_{23} = \{-4\,\hat{\mathbf{\imath}} + \hat{\mathbf{\jmath}}\}\,\text{m}$, $\mathbf{R}_{43} = \{\hat{\mathbf{\imath}} - 6\,\hat{\mathbf{\jmath}}\}\,\text{m}$, $\mathbf{R}_P = \{-\hat{\mathbf{\imath}} + 1.5\,\hat{\mathbf{\jmath}}\}\,\text{m}$, $\mathbf{a}_{G_3} = \{-4\,\hat{\mathbf{\imath}} - 5\,\hat{\mathbf{\jmath}}\}\,\text{m/s}^2$, $\alpha = 5\,\text{rad/s}^2 \curvearrowright$. The *known* force $\mathbf{F}_P$ is given to be $\mathbf{F}_P = \{3\,\hat{\mathbf{\imath}} - 5\,\hat{\mathbf{\jmath}}\}\,\text{N}$, and the *known* couple $\mathbf{T}_P$ to be $\mathbf{T}_P = 12\,\hat{\mathbf{k}}\,\text{N}\cdot\text{m}$. Using the $x-y$ horizontal-vertical coordinate system, and the quantities $\mathbf{F}_{32}$, $\mathbf{F}_{43}$ named on the diagram, write down the three scalar kinetic equations for the link in numerically explicit form.




8.(10%) The wheel and tire assembly shown has been run at 10 rad/s on a dynamic balancing machine. The force measured in the left bearing has a peak of 12 N at a phase angle of 75° with respect to the zero reference line on the tire. The force measured in the right bearing has a peak of 10 N at a phase angle of −30° with respect to the zero reference line on the tire. The center distance between the two bearings on the machine is 25 cm. The left edge of the wheel rim is 10 cm from the centerline of the closest bearing. The wheel is 17.5 cm wide at the rim. The wheel-rim **diameter** is 37.5 cm. The tire is to be balanced by attaching masses $m_A, m_B$ to the rim in correction planes $A$, $B$ at angles $\theta_A, \theta_B$ to the zero reference line on the tire. Draw the FBD for the balanced assembly, and find $m_B$ and $\theta_B$. (Hint: Take moments about Plane $A$.)

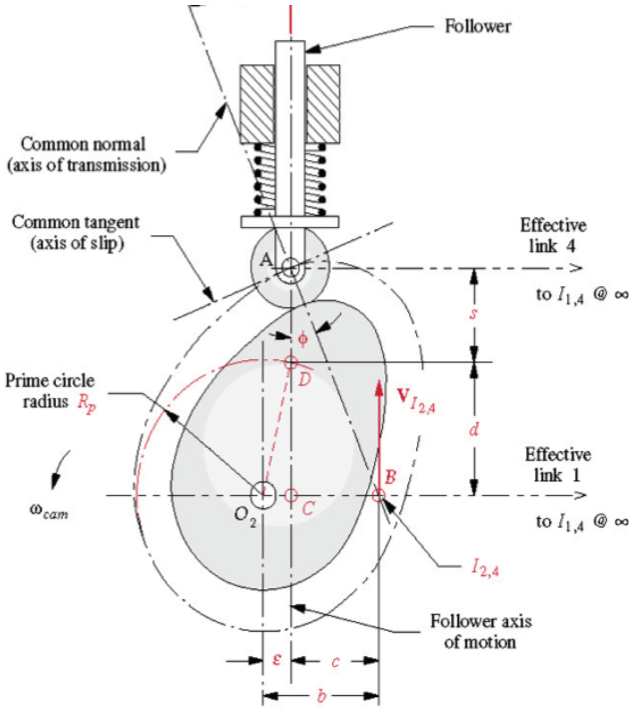



FIGURE 8-45

9.(10%) For the cam shown define in words, and, if appropriate, indicate on the picture:

(i) pitch curve,

(ii) pressure angle.

(iii) What adverse consequence arises from undercutting? How can this problem be avoided?

(iv) What rule of thumb is imposed on the pressure angle?

(v) In a consistent system of units, at a particular value of $\theta$, $s(\theta) = 0.4$, $\nu(\theta) = 1.2$. If $R_P = 2.1$, $\varepsilon = 0.1$, find the corresponding value of the pressure-angle $\phi$.

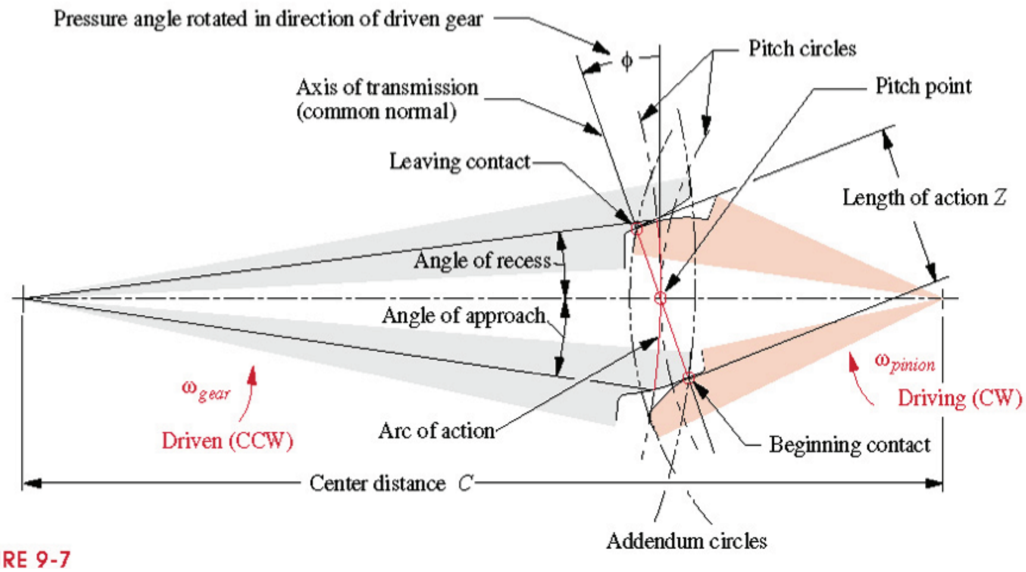


FIGURE 9-7

10.(10%) For the gear shown:

(i) What is the primary advantage of using an involute tooth profile?

(ii) Define interference.

(iii) Define base pitch,

(iv) A gearset has $\phi = 20°$, $r_g = 3.75\,\text{in}$, $a_g = .125\,\text{in}$, $r_p = 1.5\,\text{in}$, $a_p = .125\,\text{in}$. Find the length of action, $Z$, as defined in the picture above.

(v) If this gearset has base pitch $p_b = 0.3689\,\text{in}$, find the corresponding value of the contact ratio $m_p$.

David Malawey – ME213 – My Reference Sheet – 2010.12.15

213 Final Exam Reference Sheet

- Mobility:
  - Test 1 prob1
  - Labeling joints and mobility, homework 1
- 4-Bar configurations:
  - Grashof/Barker classification pg61
  - Examples, homework #2
- Mechanism synthesis:
  - Definitions, test 1 p3
  - Examples –p 158
- Instant Centers:
  - LOTS on homework 7
  - Review 2 pg2
- Vector loops:
  - Exam 2 problem1
- Acceleration:
  - With vslip: Homework 9,
  - Acceleration difference, homework 8, test 2 p5
  - Find ang. Velocity of a bar: test2 p4
- Mechanical Advantage:
  - Crimper tool: homework 8, review 3 pg 13
  - Complex problem:
- Force Analysis:
  - Kinetic eqn's, **numerically explicit form**: SPRING2010TEST3
  - Draw FBD and write kinetic eqns: Test3 prob2
- Dynamic Balancing:
  - Lollipops example: exam 3 p4
  - Tire balancing : practice exam SPRING2010TEST3
- Design a 4bar to give positions
  - Homework 3 prob2
- Cams:
  - Pitch curve p453
  - Pressure angle p445, review2 p13
  - Undercutting p450
  - Transmission angle review 3 p12,
- Gears:
  - Equation for Z: p476
  - Equation for $m_p$ p484
  - Interference & undercutting p482
  - Teeth anatomy p480
  - Involute profile p478 -center distance does not affect the velocity ratio

*homework no.3 spreadsheet - David Malawey, file type .xlsx* 9/22/2010

| | x | y |
|---|---|---|
| **c1** | 0 | 5.086 |
| **c2** | 2.019 | 3.181 |
| **c3** | 3.933 | 4.051 |
| **d1** | 0.741 | 2.703 |
| **d2** | 4.428 | 2.529 |
| **d3** | 6.304 | 4.83 |

| **Midpoint** | | | **slope** | **perpendicular slope** | **b = y - x(m)** |
|---|---|---|---|---|---|
| | **x** | **y** | | | |
| **c1 & c2** | 1.010 | 4.134 | -0.944 | 1.060 | 3.064 |
| **c2& c3** | 2.976 | 3.616 | 0.455 | -2.200 | 10.163 |
| **d1&d2** | 2.585 | 2.616 | -0.047 | 21.190 | -52.149 |
| **d2&d3** | 5.366 | 3.680 | 1.227 | -0.815 | 8.054 |

| **intersection of lines** | | | |
|---|---|---|---|
| | **mx + b = mx + b** | **x=** | **y=** |
| **C lines** | 1.06x + 3.06 = -2.20x + 10.16 | 2.178 | 5.372 |
| **D lines** | 21.19x -52.15 = -.82x + 8.05 | 2.740 | 5.911 |

| **length of links** | |
|---|---|
| **link 2** | 2.197 |
| **link 4** | 3.780 |
| **coupler** | 2.496 |
| **ground link** | 0.779 |

*homework no.4 spreadsheet - David Malawey, file type .xlsx* 9/22/2010

| | x | y |
|---|---|---|
| **O2** | 0.995 | 0 |
| **O2'** | -3.1405 | 3.9436 |
| **O2''** | 5.124 | 0.929 |
| **O4** | 5.298 | 0 |
| **O4'** | -0.927 | 0.612 |
| **O4''** | 8.4205 | 3.195 |

**mx+b=mx+b**

| **Midpoint** | | |
|---|---|---|
| | **x** | **y** |
| **o2 & o2'** | -1.073 | 1.972 |
| **o2'& o2''** | 0.992 | 2.436 |
| **o4 &o4'** | 2.186 | 0.306 |
| **o4' &o4''** | 3.747 | 1.904 |

| **slope** | **perpendicular slope** | **b = y - x(m)** |
|---|---|---|
| -0.954 | 1.049 | 3.097 |
| -0.365 | 2.741 | -0.283 |
| -0.098 | 10.172 | -21.924 |
| 0.276 | -3.619 | 15.462 |

**intersection of lines**

| **m1-m2** | **b2-b1** | **x** | **x=** | **y=** |
|---|---|---|---|---|
| -1.693 | -3.379 | 1.996258 | 1.996 | 5.190 |
| 13.790 | 37.386 | 2.711 | 2.711 | 5.652 |

| | | |
|---|---|---|
| **o2 lines** | 1.049x + 3.097 = 2.136x -.182 | 5.190 |
| **o4 lines** | 10.172x -21.924 = -1.930x + 11.440 | 5.652 |

| **length of links** | |
|---|---|
| **link 2** | 5.286 |
| **link 4** | 6.216 |
| **coupler** | 4.303 |
| **ground link** | 0.851 |