

# **ME 225**

# **HEAT**

# **TRANSFER**

## **DR. KOYLU**

Compiled Mechanical Engineering Coursework by David Malawey  
Class of 2011 December at Missouri University of Science & Technology  
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Course Binder Part 1 of 2  
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## ME 225 HEAT TRANSFER

Spring 2011

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*Office Hours:* TW: 13:00-14:00; Other times: open door or by appointment

*Schedule:* TTh 9:30-10:45, 254 Toomey

*Background:* Differential Equations, Thermodynamics (Fluid Mechanics +)

*Textbook:* "Introduction to Heat Transfer" by Incropera et al., 5<sup>th</sup> Edition, Wiley

*Suggested Reading:* Any heat transfer book, e.g., "Heat Transfer" by Cengel, McGraw-Hill

***Objective:*** To introduce students to the fundamentals of conduction, convection and radiation heat transfer, and to provide the necessary theoretical/practical tools to tackle real engineering applications involving energy systems

| <i>Work:</i>   | <u>Chapters Covered</u> | <u>Tentative Date<sup>†</sup></u> | <u>Weight</u> |
|----------------|-------------------------|-----------------------------------|---------------|
| Test 1         | 1, 2, 3, 5              | 02/15/11                          | 25%           |
| Test 2         | 6, 7, 8                 | 03/24/11                          | 25%           |
| Test 3         | All                     | 05/06/11                          | 25%           |
| 8 Problem Sets | All                     | Weekly                            | 10%           |
| 2 Projects     |                         | To be announced                   | 15%           |

*General Outline:*

| <u>Book Chapter</u> | <u>Subject</u>                          |
|---------------------|-----------------------------------------|
| 1                   | General Introduction                    |
| 2                   | Introduction to Conduction              |
| 3                   | One-Dimensional Steady-State Conduction |
| 5                   | Transient Conduction                    |
| 6                   | Introduction to Convection              |
| 7                   | Forced Convection - External Flows      |
| 8                   | Forced Convection - Internal Flows      |
| 11                  | Heat Exchangers                         |
| 12                  | Radiation: Processes and Properties     |
| 13                  | Radiation Exchange between Surfaces     |

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<sup>†</sup> It is the student's responsibility to follow the most up-to-date announcements for all tests.

## Course Policies

Grades: The grades will be treated on a relative basis, that is, the average score of the entire class,  $X$ , and the standard deviation,  $\sigma$ , will determine the final outcome. For a typical class, this approximately corresponds to a “B” for  $X < \text{Grade} \leq X + \sigma$ , and a “C” for  $X - \sigma < \text{Grade} \leq X$  at the end of the semester. Any further positive or negative deviations from the mean will result in an “A” or a “D”, respectively. A 60% cut-off may be enforced for a passing grade when applicable. The instructor reserves minor discretion over grades for some critical cases.

Exams: There will be three tests, which will be related to the material covered in the lectures/textbook, problems solved in the class, and homework/project assignments. Students are allowed to bring the textbook, their own class notes, handouts, and assignments to each test. Because all exams are required, an unexcused absence will result in a zero for that exam. Therefore, a student, who cannot attend an exam because of illness or emergency, must contact the instructor and receive permission before the exam. After the test, only a last minute real emergency will be considered as a possible excused absence for which the student will be expected to notify the instructor about the situation as soon as possible. In any case, necessary official paperwork must be supplied to prove the emergency. The instructor has the final authority to determine whether a student will be given a make-up exam. Finally, academic dishonesty is a serious offense and will be treated accordingly.

Assignments: A total of eight homework assignments will be given on a regular basis. A problem set will be usually due in one week at the end of a class period after it is assigned. Solutions to homework will be e-mailed to students after due dates. It is extremely important that you first attempt to solve the problems on your own, and, if unsuccessful, contact the instructor with your specific questions. Group work is also encouraged. Late assignments are not accepted.

Projects: There will be two projects related to the practical and design aspects of heat transfer (details to be given during the semester). For each project, two students will team up to prepare a professional report. Projects are not to be discussed with other students/groups. Similar to assignments, late submissions of projects are strictly unacceptable.

Attendance: A regular attendance is expected from students to be successful in this course although it is not directly counted toward students' final grades.

Expected Performance: Attend class regularly, participate in discussion and ask questions in the class, solve homework problems on a regular basis, submit assignments and projects on time, and not miss an exam.

Feedback: The instructor can provide feedback about your performance in this course and specifically what you need to do to improve it only per your request. He also would appreciate your feedback about how the course material and presentation could be further improved. Please see him confidentially for any suggestions/concerns you may have related to the course.

## — Heat Exchanger —

$$G = \dot{m} C_p$$

$$h_i = \frac{Nu k_{fluid}}{D}$$

$$Re_D = \frac{V_m \rho D}{\mu}$$

$$NTU = \frac{UA}{G_{min}}$$

$$\epsilon = \frac{q}{q_{max}}$$

$$q_{max} = G_{min} (T_{hi} - T_{ci})$$

Thin-wall tube:  $U = \left[ \frac{1}{h_i} + \frac{1}{h_o} \right]^{-1}$

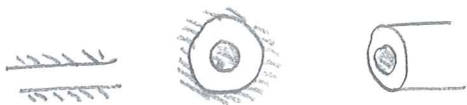
With fouling & wall:  $\frac{1}{U_i A_i} = \left[ \frac{1}{h_i A_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi K L} + \frac{R_{f,o}}{A_o} + \frac{1}{h_o A_o} \right]$

## — Ch. 12 Radiation —

- gray = •  $\alpha$  is independent of  $\lambda$
- $\alpha = \epsilon$

- F factors, Pg 702

- 2-surface enclosures p 793



$$\epsilon(T_s) = \frac{\int_0^\infty \epsilon_\lambda E_{\lambda b} d\lambda}{E_b}$$

$$E_b = \sigma T^4; \quad E = \epsilon \sigma T^4$$

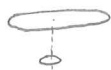
$$J, \text{ radiosity} = E + G_{\text{reflected}}$$

$$G, \text{ irradiation} = \text{all incident rad.}$$

## — Ch. 13 View factors & Rad. —

Basic View factors: p 775

$$A_i \ll A_j \quad \text{p 781}$$



$$F_{12} = \frac{A_2}{A_1} F_{21}$$

$$F_{11} + F_{12} + F_{13} + F_{14} (\text{enclosed}) = 1$$

$$\text{net } q_{12}, \text{ general} = \frac{\sigma(T_1^4 - T_2^4)}{\left[ \frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\epsilon_2}{\epsilon_2 A_2} \right]}$$



## Example Problems:

### Flat Plates

|                                                      |                   |
|------------------------------------------------------|-------------------|
| Constant ( $T_{inf}-T_s$ ) Re increase, find q ratio | Summer 2010 #3 p3 |
| Boundary layer thickness, local                      | HW 7.2            |
| Velocity(inf) and $T(inf)$ given                     | HW 7.11           |
| Convection & solar absorptivity, irradiation         | HW 7.20           |
| Cooling time with changed fluid                      | Spring 2010 #3 p7 |

### Tubes

#### Circular

|                                |                   |
|--------------------------------|-------------------|
| Constant $T_s$ , find $T_{so}$ | Spring 2010 #4 p8 |
| Annular, find L to cool        | HW 8.93           |
| Cross flow, tube conduction    | HW 8.50           |
| Pump Power                     | HW 8.31           |
| Pump power                     | Fall 2008 #1 p12  |
| Air outside, fluid inside      | example p11       |

#### Noncircular

|                                                      |                   |
|------------------------------------------------------|-------------------|
| Constant $T_s$ , thin wall, find $T_{si}$ , $T_{so}$ | Summer 2011 #1 p4 |
| Constant $q''$ , find $T_s$ , $T_{outlet}$           | HW 8.74           |

### Arbitrary surfaces

|                                         |                   |
|-----------------------------------------|-------------------|
| Velocity profile given, find average Nu | Summer 2010 #1 p1 |
| Temp profile given, find average h      | Spring2011 #1 p9  |

### Pin fin

|                                        |         |
|----------------------------------------|---------|
| Diameter doubles, find new q           |         |
| Diameter doubles, inf long, find new q | HW 7.45 |

### Couette flow

|                                               |                    |
|-----------------------------------------------|--------------------|
| Surface 1: Constant $T_s$ , Surface 2: no HT  | Summer 2010 #2 p6  |
| Both Surfaces constant $T_s$ . Find $T_{max}$ | Spring 2010 #2 p10 |

### Drag coefficient

|                 |         |
|-----------------|---------|
| Cooling spheres | HW 7.73 |
|-----------------|---------|

## Exam 3 Help Index

### Chapter 11:

①<sup>22</sup> Overall U with fouling: 11.2

2008, #1 fine fouling factor given  $q$

④ Shell and Tube HX: 11.22

oc pg 3

, also 2008 #3 ps ⑦

② Concentric Tube: 11.18

④ Cross-flow HX 11.34

⑤ Condensing steam 11.35

### Chapter 12

②②  $G_{abs}$  as a function of  $F$  and  $\alpha$

12.46

②② step function  $\epsilon$ , 12.32

①⑦  $G$  with angle of approach,  $Q$  received by lens

12.47 Redo:

①① Absorptivity,  $f(\lambda)$  and Emissive power

12.51

①③ Transmitted through car window

2008, #2

①③ Earth's radiation from sun

Example 12.20

①⑤ Find  $\alpha$  from  $G_{abs}/G$  (step function)  
find detector incident radiation

2008 #2

### Chapter 13:

①⑧, ①⑨ View factors, all sorts

13.1

⑧ Net radiation to  $A_1$  in enclosed surface  
or ⑨ -dot in cylinder

13.19



⑧ Net radiation of close parallel plates

13.41

notes Radiation shield, flat plates Ex 13.56

②③ Radiation shield,  $q$  balance w/convection

13.62

①④ Long oven view factors

2005 #3

②① Net HT between two enclosed surfaces  
-side in semicircle

A #3,ii

# Review for Exam 2

Air properties P.852

## CH.7. EXTERNAL FLOW

$\bar{C}_f$ , average friction coefficient

$\delta$ , boundary layer thickness (where  $\frac{u}{u_\infty} = 0.99$ )

$\alpha$  = absorptivity

$\epsilon$  = emissivity

$$R_{\text{cylinder}} = \frac{\ln(r_2/r_1)}{2\pi kL}$$

Reminder:

$q$  heat trans. rate, W

$q'$  heat trans rate per length, W/m

$q''$  heat flux, W/m<sup>2</sup>

Memorize  $q'' = A \frac{(T_s - T_\infty)}{R_{\text{tot}}}$   $R_{\text{tot}} = \left( \frac{1}{h_1} + \frac{1}{h_2} + \frac{L}{k} \right)$   $q = \frac{\delta T}{R_{\text{tot}}}$

heat transfer correlations Table 7.9 p 431

$T_f$ , flow temperature

$C_D$ , drag coefficient p 401

$Re_D = \text{cross flow} = \frac{V_\infty D}{\nu}$   $Re_{cr} = 5 \times 10^5$

$Re_x = \frac{V_\infty x}{\nu} = \frac{\rho V_\infty x}{\mu}$

$Nu = \frac{h_x x}{k}$

## CH 8 INTERNAL FLOW

friction factor, MOODY CHART, p 461

Circular pipe,  $Re_D = \frac{U_m D}{\nu}$   $Re_{D,c} \approx 2300$

$\dot{m}$ , mass flow =  $\rho U_m A_c \Rightarrow U_m = \frac{\dot{m}}{\rho A_c}$

$\bar{h}_i = \frac{Nu_D k_{\text{fluid}}}{D_h}$  (for local  $\equiv$  avg  $Nu$ ) p 489

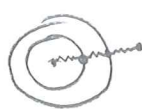
$D_h = \frac{4A_c}{\text{Perimeter}}$

Pumping Pwr =  $\Delta P \frac{\dot{m}}{\rho} = \Delta P \dot{V}$   $\Delta P = f \left( \frac{L}{D} \right) \left( \frac{\rho U_m^2}{2} \right)$

$Re_D = \frac{\rho V_\infty D}{\mu} = \frac{V_\infty D}{\nu} = \frac{4\dot{m}}{\pi D_i \mu}$  circular only

Energy balance

$T_m(x) = T_{m,i} + \frac{q''_s P}{\dot{m} c_p} x$

  $\frac{q}{L} = \frac{T_\infty - T_m}{\frac{1}{h_o A_o} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{1}{h_i A_i}}$

$q'' = A \frac{(T_s - T_\infty)}{R_{\text{tot}}}$

$q = \frac{\Delta T}{R_{\text{tot}}}$

Re<sub>x</sub>, Reynolds by distance p 355 (6.11)

Re<sub>d</sub>, Reynolds by diameter p 457 (8.1)

hydraulic diameter p 488. (8.66)

Relate  $T_s$  &  $T_m$   $T_m \sim T_s$  p. 463 (8.27)

Annular tubes •  $T_s$  const p 490

•  $q$  const p 491

$h$  from  $T(x)$  function (6.5) p 350

$$h = \frac{-k_f \frac{\partial T}{\partial y}|_{y=0}}{T_s - T_\infty}$$

cooling time  $t$  (5.5) (lumped capacitance)

$$t = \frac{\rho V c}{h A_s} \ln \left( \frac{T_i - T_\infty}{T - T_\infty} \right) \quad p 258$$

Pipe flow  $T_m$  &  $T_{\infty, \text{outside}}$  NOT  $q$  const  
NOT  $T_s$  const

$U$  (8.45a) p 472

$$U = \left( \frac{1}{h_i} + \frac{1}{h_o} \right)^{-1}$$

cross flow relationships p 431 chapter 7

|          |   | hrs |
|----------|---|-----|
| ME 240   | A | 2   |
| ME 279   | A | 3   |
| EMGT 124 | A | 1   |
| EE 281   |   | 3   |
| ME 225   |   | 3   |

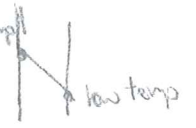


Notes Jan 11, 2011

$$q_x = -kA \frac{dT}{dx} \leftarrow \text{Fourier's Law}$$

Temperature gradient: slope of temperature.

(Within solid or nonmoving fluid)



Conduction:

• Thermal diffusivity  $\alpha = \frac{\text{heat conducted}}{\text{heat stored}} = \frac{k}{\rho c_p}$

$\alpha \uparrow \Rightarrow$  fast response to thermal environment

• In this class:

Scott K.

Zach Jennings

Stephen Arnaud

~~Catzone~~~~Kayla Bittadeau~~

Ch 2 Intro to Conduction

cylindrical coordinates, \* radius is changing \*

Analysis: determine T field in a solid/stationary fluid resulting from conditions imposed on system boundaries

- define a C.V.
  - identify relevant energy transfer processes
  - write down governing eqs
- } diff eq's & boundary init. conditions  
↓  
temp. field

governing eq for pure conduction problems = heat equation

Energy conservation:  $\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$

$\dot{E}_g = \dot{q} dV = \dot{q} dx dy dz$   
 $\uparrow$  rate of energy gen. per unit volume ( $W/m^3$ )

$\dot{E}_{st} = m C_p \frac{\partial T}{\partial t} = \rho dV C_p \frac{\partial T}{\partial t} = \rho dx dy dz C_p \frac{\partial T}{\partial t}$

$q_x = -k dy dz \frac{\partial T}{\partial x}$      $q_y = -k dz dx \frac{\partial T}{\partial y}$      $q_z = -k dx dy \frac{\partial T}{\partial z}$

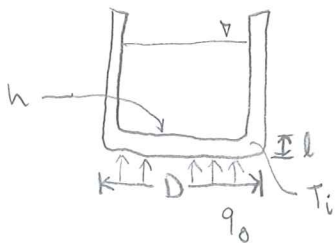
$(q_x + q_y + q_z) - \left( q_x + \frac{dq_x}{dx} dx + q_y + \frac{dq_y}{dy} dy + q_z + \frac{dq_z}{dz} dz \right) + \dot{q} dx dy dz = \rho C_p \frac{\partial T}{\partial t} dx dy dz$

Heat  
Equation

$\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho C_p \frac{\partial T}{\partial t}$  ← Cartesian coords

• also forms for cylindrical & spherical

Example (2.21)



stage 1: (assuming h.t. is in 1 direction) (no heat gen)

$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$  boundary cond:  $q_0 = -k \left( \frac{\pi D^2}{4} \right) \frac{\partial T}{\partial x} \Big|_{x=0}$  ①

b.c ②  $-k \frac{\partial T}{\partial x} \Big|_{x=L} = h [T(L, t) - T_{\infty}(t)]$

initial cond:  $T(x, 0) = T_i$

Stage 2:  $\frac{d^2 T}{dx^2} = 0$

$\frac{dT}{dx} = C_1$      $T = C_1 x + C_2$     B.C.'s:  $q_0 = -k \left( \frac{\pi D^2}{4} \right) \frac{dT}{dx} \Big|_{x=0}$      $T(L) = T_h$

\* get Boundary conditions from Table 2.2.

Jan 20 notes

Ch. 3

- steady state  $\frac{dT}{dt} = 0$
- 1-D  $\Rightarrow \frac{\partial T}{\partial y} \approx \frac{\partial T}{\partial z} \ll \frac{\partial T}{\partial x}$
- no energy generation  $\Rightarrow \dot{q} = 0$

H.E.:  $\frac{d}{dx} \left( k \frac{dT}{dx} \right) = 0$

$\frac{dq_x''}{dx} = 0 \Rightarrow q_x'' = \text{const}$

$k = \text{const} \Rightarrow \frac{d^2 T}{dx^2} = 0 \Rightarrow \frac{dT}{dx} = \text{const.}$

$\Rightarrow T = C_1 x + C_2 \text{ (linear)}$

• Conduction thru wall

$q = \frac{kA}{L} (T_{s1} - T_{s2}) = \frac{kA}{L} \Delta T = \frac{\Delta T}{(L/kA)} = \frac{\Delta T}{R_{\text{cond}}} = \frac{\text{driving potential for conduction}}{\text{thermal resistance}}$

heat transfer  $\rightarrow q = \frac{-kA}{L} (-\Delta T) = kA \frac{\Delta T}{L} = \frac{\Delta T}{L/kA}$

$R_{\text{cond}} = \frac{L}{kA}$

outside the wall

similarly,  $q_{\text{conv}} = hA (T_s - T_{\infty}) = \frac{T_s - T_{\infty}}{(1/hA)} = \frac{\Delta T}{R_{\text{conv}}}$

$R_{\text{convection}} = \frac{1}{hA}$

Fig 3.1  $R_{\text{tot}} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A}$  total thermal resistance  $\Rightarrow q_x'' = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{tot}}}$

Multilayer Plane Walls

i) walls in a series:  $R_{\text{tot}} = \sum R_i$

ii) walls of parallel layers  $\frac{1}{R_{\text{tot}}} = \sum \frac{1}{R_i}$

Ch 3 slide 6

case a)  $q = \frac{\Delta T}{R_{\text{tot}}} = \frac{T_{\infty i} - T_{\infty o}}{\frac{1}{h_i A} + \frac{L_p}{k_p A} + \frac{L_f}{k_f A} + \frac{L_w}{k_w A} + \frac{1}{h_o A}} = \frac{20 - (-15)}{(0.2 + 0.059 + 1.92 + 0.083 + 0.067)/1}$

$q = \frac{35}{2.33} = 15.0 \text{ W}$

assume  $A = 1 \text{ m}^2$

b)  $q = \frac{T_{\infty i} - T_{\infty o}}{\frac{1}{h_i A} + \frac{L_g}{k_g A} + \frac{1}{h_o A}} = \frac{35}{0.269} = 130 \text{ W}$

c)  $q = \frac{T_{\infty i} - T_{\infty o}}{\frac{1}{h_i A} + \frac{L_g}{k_g A} + \frac{L_o}{k_g A} + \frac{L_g}{k_g A} + \frac{1}{h_o A}} = 76 \text{ W}$

## Notes Jan 25)

• for cylindrical wall, temperature gradient is logarithmic (not linear)

Overall H.T. coefficient

Emissivity is  $0 \leq \epsilon < 1$ , refers to how much radiation passes thru

Ex.) with insulated tube

Energy balance on outer surface  $q'_{\text{cond}} = q'_{\text{conv}} + q'_{\text{rad}}$

$$\frac{T_{s,i} - T_{s,o}}{\frac{1}{h_o A} + \frac{\ln(r_2/r_1)}{2\pi k_{\text{st}} \cdot l} + \frac{\ln(r_3/r_2)}{2\pi k_{\text{ins}} \cdot l}} = \frac{T_{s,o} - T_{\infty,o}}{h_o 2\pi r_3 \cdot l} + (2\pi r_3 \cdot l)(\epsilon \sigma)(T_{s,o}^4 - T_{\text{sun}}^4)$$

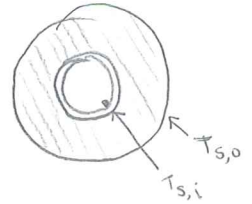
↑ would need this term had they not said "inside wall same temp as fluid"

- will need calculators to solve this

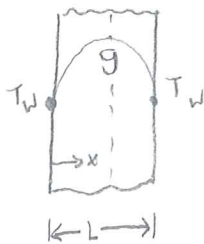
$$\Rightarrow r_3 = .394 \text{ m} \quad t_{\text{ins}} = r_3 - r_2 = 394 - 180 = 214 \text{ mm}$$

$$q' = 420 \text{ W/m}$$

\* using  $q'$  denotes "for 1 m length of the pipe"



ex)



$$\text{B.C.'s} \quad T(x=0) = T_w \Rightarrow T_w = \left(-\frac{\dot{q}}{2k}\right)0^2 + C_1 \cdot 0 + C_2 \Rightarrow C_2 = T_w$$

$$T(x=L) = T_w \Rightarrow T_w = \left(-\frac{\dot{q}}{2k}\right)L^2 + C_1 L + T_w \Rightarrow C_1 = \frac{\dot{q}L}{2k}$$

$$T(x) = -\frac{\dot{q}}{2k}x^2 + \frac{\dot{q}}{2k}Lx + T_w$$

Practice

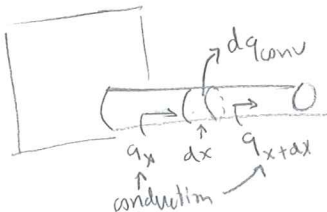
$$\frac{1}{r} \frac{d}{dr} \left( kr \frac{dT}{dr} \right) + \dot{q} = 0$$

Slide #18

## Notes Jan 27)

see cylindrical cooling fin slide 22

(pin fin) • assumed 1 dimensional temperature gradient - only depends on x



$$\text{Energy balance for } dx \quad q_{\text{cond},x} = q_{\text{cond},x+dx} + dq_{\text{conv}}$$

$$q'_x = -kA_c \frac{dT}{dx} \quad q'_x + \frac{dq}{dx} dx = h(T - T_\infty) dA_s$$

$$= -kA_c \frac{dT}{dx} - kA_c \frac{d^2T}{dx^2} dx$$

$$0 = -kA_c \frac{d^2T}{dx^2} dx + h(T - T_\infty) P dx \Rightarrow \frac{d^2T}{dx^2} - \frac{hP}{kA_c} (T - T_\infty) = 0$$

⇒

Notes Jan 27

$$\theta = T - T_{\infty}, \quad m^2 \frac{hP}{kA_c} \Rightarrow \frac{d^2\theta}{dx^2} - m^2\theta = 0$$

$$\theta = C_1 e^{mx} + C_2 e^{-mx} \quad (1. b.c.): x=0 \Rightarrow T = T_b \quad T_b - T_{\infty} = C_1 e^{m \cdot 0} + C_2 e^{-m \cdot 0}$$

$$\theta_b = T_b - T_{\infty} = C_1 + C_2$$

2<sup>nd</sup> b.c.: 4 possibilities slide 24

Note  $\cosh x = \frac{e^x + e^{-x}}{2}$   $\sinh x = \frac{e^x - e^{-x}}{2}$

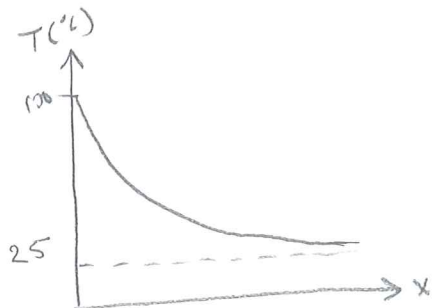
ex)  $T_b = 100^\circ\text{C}$

$AL (k = 180 \text{ W/m}\cdot\text{K})$   
 $D = 5 \text{ mm}$   
 $h = 100 \text{ W/m}^2\cdot\text{K}$   
 $T_{\infty} = 25^\circ\text{C}$

Very long fin ( $L \rightarrow \infty$ )  $\frac{\theta}{\theta_b} = e^{-mx}$

$$\Rightarrow T = T_{\infty} + (T_b - T_{\infty}) e^{-mx}$$

$$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{h}{k} \frac{\pi D}{\pi D^2/4}} = \sqrt{\frac{4h}{kD}} = 21 \Rightarrow T = 25 + 75 e^{-21x}$$



$$q_f = m = \sqrt{hP k A_c} \theta_b$$

$$q_f = (0.075)(75) = 5.6 \text{ W}$$

\* Note; in general,  $q_f = q_b$  heat lost by fin = heat entering fin @ base  
 $= -kA_c \left. \frac{d\theta}{dx} \right|_{x=0}$  OR  $q_f = \int_{A_f} h(T - T_{\infty}) dA_s$

• What should be the length of a fin with negligible heat loss at the tip that can transfer 80% of  $q_{f, L \rightarrow \infty}$ ?

$$\frac{q_{f, \text{adiabatic}}}{q_{f, L \rightarrow \infty}} = \frac{M \tanh(mL)}{M} = 0.8 \Rightarrow \tanh(mL) = 0.8 \Rightarrow mL = 1.1$$

$$\Rightarrow L = \frac{1.1}{21} = 0.05 \text{ m} = \boxed{5 \text{ cm}}$$

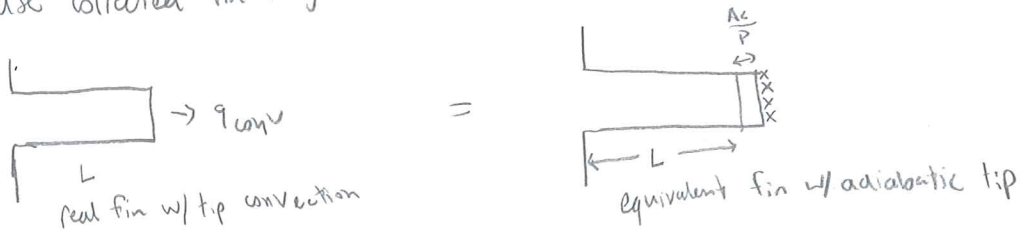
$$q_f = M \tanh(mL) = 4.5 \text{ W}$$

plug this into

Jan 27 #3

Engineering Solution:

use corrected fin length  $L_c = L + \frac{A_c}{P}$  instead of  $L$  in case B (slide 22)



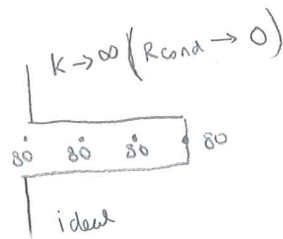
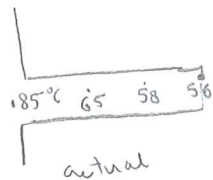
$$w + \frac{A_c}{P} \approx \frac{t}{2}$$

$$\frac{A_c}{P} = \frac{D}{4}$$

— Fin Efficiency —

SLIDE 25 IMPORTANT

ex)



$$q_{f, \max} = h A_f \theta_b = h P L (T_b - T_\infty)$$

$$q_f = \eta_f q_{f, \max} \quad \text{AND} \quad \eta_f = \frac{q_f}{q_{f, \max}} = \frac{\sqrt{h P k A_c} \theta_b \tanh(m L_c)}{h P L \theta_b}$$

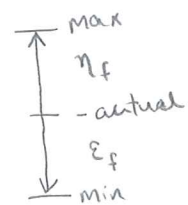
$$\eta_f = \frac{\tanh(m L_c)}{m L_c}$$

★ See table 3.5 for  $\eta_f$   
or figs. 3.18, 3.19

— Fin Effectiveness —

$$\varepsilon_f = \frac{q_{f, \text{actual}}}{q_{\text{no fin}}}$$

$\varepsilon_f \geq 2$  to justify use of fins



$$L \rightarrow \infty \text{ (upper limit for } \varepsilon_f \text{): } \varepsilon_f = \dots \left( \frac{k P}{h A_c} \right)^{1/2}$$

to max.  $\varepsilon_f$ : —  $k \uparrow$  use metals

—  $P/A_c \uparrow$  use thin fins

—  $h \downarrow$  use fins on surfaces w/ low  $h$

Feb 3, 2011 Notes

## Arrays of fins

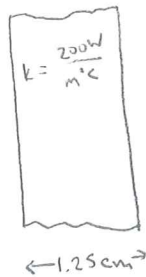
tot. h.t. rate from a surface of  $N$  fins

$$\begin{aligned}
 q_t &= q_{\text{finned}} + q_{\text{unfinned}} = Nq_f + h_{\text{unfinned}} A_{\text{unfinned}} \Theta_b \\
 &= \underset{\substack{\uparrow \\ \text{calculate}}}{N} \underset{\substack{\uparrow \\ \text{calculate}}}{\eta_f} \underset{\substack{\uparrow \\ \text{calculate}}}{h_{\text{finned}}} \underset{\substack{\uparrow \\ \text{calculate}}}{A_f} \Theta_b + h_{\text{unfinned}} \underset{\substack{\uparrow \\ \text{calculate}}}{A_{\text{unfinned}}} \Theta_b
 \end{aligned}$$

to study for test 1, consider all configurations of phys/ fins on a surface

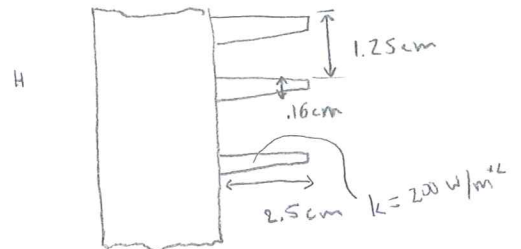
Ex)

$$\begin{aligned}
 T_{\infty 1} &= 120^\circ \text{C} \\
 h_1 &= 450 \frac{\text{W}}{\text{m}^2 \text{K}}
 \end{aligned}$$



$$\begin{aligned}
 T_{\infty 2} &= 20^\circ \text{C} \\
 h_2 &= 25 \frac{\text{W}}{\text{m}^2 \text{K}}
 \end{aligned}$$

H.T. enhancement due to fin addition?



$$\frac{q_{\text{w/fin}}}{q_{\text{w/o fin}}} = \frac{N \eta_f h A_f \Theta_b + h A_{\text{total unfinned}} \Theta_b}{h A_{\text{total base}} \Theta_b} = \frac{N \eta_f A_f + A_{\text{total unfinned}}}{A_{\text{total base}}}$$

$$mL_c = \sqrt{\frac{hP}{kA_c}} L_c = \sqrt{\frac{h \cdot 2w}{k \cdot 2t}} \left( L + \frac{t}{2} \right) \quad mL_c = \sqrt{\frac{(25 \times 2)}{(200)(0.16 \times 10^{-2})}} \left( 2.5 + \frac{0.16}{2} \right) \times 10^{-2} = 0.3225$$

$$\eta_f = \frac{\tanh(mL_c)}{mL_c} = 0.967 \quad A_f = 2wL_c, \quad A_{\text{total unfinned}} = w(H - Nt)$$

$$A_{\text{total base}} = HW$$

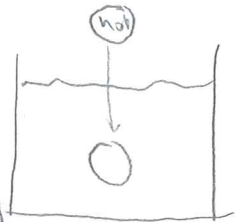
$$\frac{q_{\text{w/fin}}}{q_{\text{w/o fin}}} = \frac{2\eta_f L_c + \frac{H}{N} - t}{\frac{H}{N}} = \frac{2\eta_f L_c + S - t}{S} = \frac{2 \cdot (0.967) \cdot (2.58 \times 10^{-2}) + 1.25 \times 10^{-2} - 0.16 \times 10^{-2}}{1.25 \times 10^{-2}} = 4.76$$



First, neglect spatial variations of temp within the body,  $T = T(t)$

⇒ lumped analysis

$$\Rightarrow R_{\text{cond}} \ll R_{\text{conv}} \Rightarrow \frac{q_{\text{conv}}}{q_{\text{cond}}} = \frac{R_{\text{cond}}}{R_{\text{conv}}} = \frac{\frac{h_c}{k A_s}}{\frac{1}{h A_s}} = \frac{h h_c}{k} \equiv \text{Biot number} = Bi$$



\* in this chapter,  $h_c$  means "characteristic length", not corrected length

⇒  $Bi < 1$   $Bi \leq 0.1 \Rightarrow$  use the following lump analysis

— Energy balance:  $\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}} + \dot{Q}_g = \dot{E}_{\text{st}}$

$$-h A_s (T - T_{\infty}) = \rho V c_p \frac{dT}{dt} \quad \Theta = T - T_{\infty} \Rightarrow \frac{dT}{dt} = \frac{d\Theta}{dt} \quad -h A_s \Theta = \rho V c_p \frac{d\Theta}{dt}$$

$$\int_0^t -dt = \int_{\Theta_i}^{\Theta} \frac{\rho V c_p}{h A_s} \frac{d\Theta}{\Theta} \Rightarrow t = \frac{\rho V c_p}{h A_s} \ln \frac{T_i - T_{\infty}}{T - T_{\infty}} \quad \text{or} \quad \frac{T - T_{\infty}}{T_i - T_{\infty}} = e^{\left( \frac{-h A_s}{\rho V c_p} t \right)}$$

$$h_c = \frac{V}{A_s} = \begin{cases} L & \text{for plane wall of thickness } 2L \\ r_o/2 & \text{for very long cylinder of radius } r_o \\ r_o/3 & \text{for sphere of radius } r_o \end{cases} \quad p.261$$

$$\text{Now } \frac{h A_s t}{\rho V c_p} = \frac{h t}{\rho c_p h_c} = \frac{h h_c}{k} \frac{\rho t}{h_c} = Bi \quad \text{NON DIMENSIONAL}$$

$$\Theta^* = \frac{\Theta}{\Theta_i} = e^{-Bi \cdot F_0} \quad F_0 = \text{Fourier number}$$

$$T(t) \Rightarrow q(t) = h A_s [T(t) - T_{\infty}] = h A_s \Theta(t)$$

$$\text{Total heat transfer up to time } t \quad Q = \rho V c_p [T_i - T(t)]$$

$$Q_{\text{max}} = Q_0 = \rho V c_p (T_i - T_{\infty})$$

Notes

2-8-11

$$Bi > 1 \Rightarrow T(x, t)$$

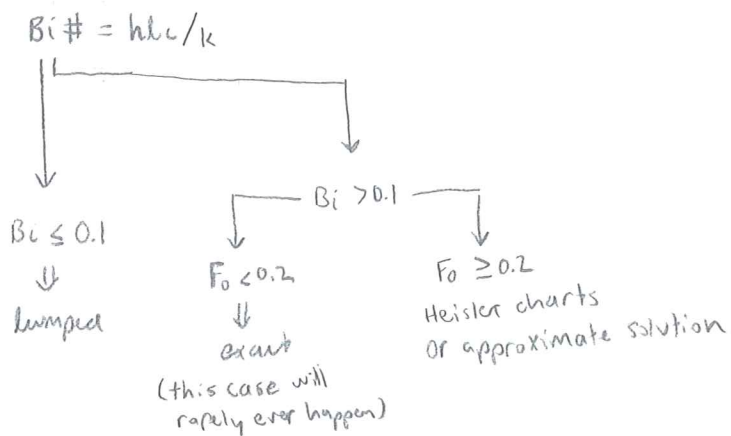
$$\text{B.C.'s } \left. \frac{\partial T}{\partial x} \right|_{x=0} = 0$$

(symmetry)

$$\text{Heat equation: } \frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

$$-k \left. \frac{\partial T}{\partial x} \right|_{x=L} = h [T(L, t) - T_\infty]$$

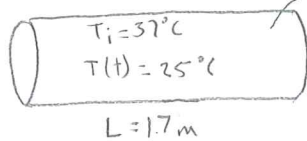
$$\text{Initial condition: } T(x, 0) = T_i$$



Example CSI Roller

$T_\infty = 20^\circ\text{C}$   $h = 8 \text{ W/m}^2\text{K}$

$D = 0.3 \text{ m}$



Water, table A-6 ( $\bar{T} \approx 305 \text{ K}$ )

$\rho = 995 \text{ kg/m}^3$

$C_p = 4178 \text{ J/kg}\cdot\text{K}$

$k = 0.62 \text{ W/m}\cdot\text{K}$

$\alpha = \frac{k}{\rho C_p} = 1.5 \times 10^{-7} \frac{\text{m}^2}{\text{s}}$

$l_c = \frac{V}{A_s} = \frac{\frac{\pi D^2}{4} L}{\pi D L + 2 \left( \frac{\pi D^2}{4} \right)} = \frac{1}{\frac{4}{D} + \frac{2}{L}}$

Note: long cyl.  $L \gg D \Rightarrow l_c = \frac{D}{4} = \frac{r_o}{2}$

$Bi = \frac{h l_c}{k} = 0.89 > 0.1 \Rightarrow \text{Heister charts}$

$\theta_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = \frac{25 - 20}{37 - 20} = 0.29$

$Bi^{-1} = \frac{k l_c}{h r_o} = 1.04 \Rightarrow Fo = \frac{\alpha t}{r_o^2} \approx 0.6$   
from Fig D.4

$t = \frac{(0.6)(0.15)^2}{1.5 \times 10^{-7}} = 90,000 \text{ s} = 25 \text{ hr.}$

? Lumped analysis?

$t = \frac{\rho C_p l_c}{h} \ln \left( \frac{T_i - T_\infty}{T - T_\infty} \right)$

$t = 43815 \text{ s} = 12.2 \text{ hr}$  Error  $\approx 100\%$  underestimate

Feb 10 2011

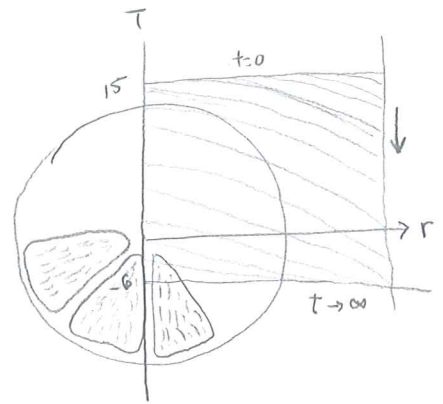
Ex. Oranges in cold weather

$$Bi = \frac{h(r_o/3)}{k} = \frac{30(4 \times 10^{-3})/3}{613 \times 10^{-3}}$$

$$= 0.65 > 0.1 \Rightarrow \text{Heisler}$$

$$Bi^{-1} = \frac{k}{hr_o} = 0.51 \quad Fo = \frac{\alpha t}{r_o^2} = \frac{(1.5 \times 10^{-7})(4 \times 3600)}{(4 \times 10^{-3})^2} = 1.35$$

$$\theta_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = 0.007 \quad \text{from Fig. D.7} = \frac{T_o - (-6)}{15 - (-6)} \Rightarrow T_o = -5.8^\circ\text{C}$$



Feb 17 2011

$$h = \frac{q_{conv}}{A_s(T_s - T_\infty)} = f(\rho, \nu, k, c_p, V, \text{type of flow, surface geometry})$$

use empirical correlations

solve boundary layer eq's (for simple flows)

fluid viscosity momentum transport  $\Rightarrow$  relates gradients with b.l.  $\Rightarrow$  drag force

$$\text{shear stress } \tau_{s,x} = \frac{\text{drag force}}{\text{surface area}} = \frac{F_D}{A_s} = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0} \quad \text{N/m}^2$$

↑  
viscosity

$$\text{If you know } u(y) \text{ w/in b.l.} \Rightarrow \left. \frac{\partial u}{\partial y} \right|_{y=0} \Rightarrow \tau_{s,x} \Rightarrow F_D$$

$$\text{Practical approach: } \tau_{s,x} = C_{f,x} \frac{\rho V_\infty^2}{2} \quad ;$$

↑  
local friction coef.

$$F_D = \int_{A_s} \tau_{s,x} \cdot dA_s = \int_{A_s} C_{f,x} \frac{\rho V_\infty^2}{2} dA_s$$

$$= \bar{C}_{f,L} A_s \frac{\rho V_\infty^2}{2} \quad (\text{N})$$

$$\bar{C}_{f,L} = \frac{1}{A_s} \int_{A_s} C_{f,x} \cdot dA_s = \frac{1}{L} \int_0^L C_{f,x} \cdot dx$$

Flow regime depends on Reynolds #

$$Re = \frac{V_\infty L}{\nu} = \frac{\text{inertia forces}}{\text{viscous forces}}$$

$$\text{local } Re_x = \frac{V_\infty x}{\nu}$$

$$\text{Nusselt # ; } = \frac{q_{conv}}{q_{cond}} = \frac{h \Delta T}{k_f \frac{\Delta T}{L}} = \frac{hL}{k_f}$$

$Nu = 1$ , pure conduction

$Nu \uparrow$ , more enhanced h.t.

Notes 2-22-11)

Non dimensional #'s

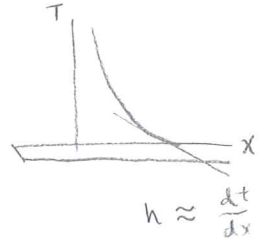
$$Nu = \frac{q_{conv}}{q_{cond}} = \frac{hL}{kf}$$

non-dimensional convection h.t. coeff.

$$Pr = \frac{\text{momentum transport}}{\text{heat transport}} = \frac{\nu}{\alpha} = \frac{\frac{\mu}{\rho}}{\frac{k}{\rho c_p}} = \frac{\mu c_p}{k}$$

For laminar b.l.'s:  $\frac{\delta V}{\delta t} \approx Pr^{-1/2}$

Convection h.t. coefficient is  $\approx$  temperature gradient right at surface



Boundary layer approximation For incompressible ( $\rho = \text{const}$ ), 2-D, steady flow over surfaces,

- B.L. thickness is typically very small so that

$$u \gg v, \quad \frac{\partial u}{\partial y} \gg \frac{\partial u}{\partial x}, \quad \frac{\partial v}{\partial y}, \quad \frac{\partial v}{\partial x} \Rightarrow \tau = \mu \frac{\partial u}{\partial y}$$

$$\frac{\partial T}{\partial y} \gg \frac{\partial T}{\partial x} \Rightarrow q_y'' \gg q_x''$$

Then, mass:  $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

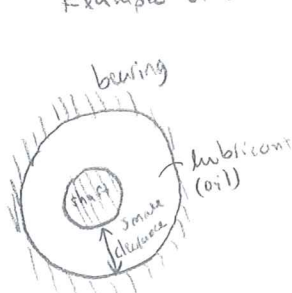
x-momentum:  $\underbrace{u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}}_{\text{acceleration}} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \underbrace{\nu \frac{\partial^2 u}{\partial y^2}}_{\text{viscous forces}}$

y-momentum:  $\frac{\partial P}{\partial y} = 0$

Energy:  $\underbrace{u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}}_{\text{advection}} = \underbrace{\alpha \frac{\partial^2 T}{\partial y^2}}_{\text{conduction}} + \underbrace{\frac{\nu}{c_p} \left( \frac{\partial u}{\partial y} \right)^2}_{\text{viscous dissipation}} + \dot{q}$   
(Only important at high speeds)

advection: bulk motion of the fluid

Example of solving bl eqns for simple flows: Couette flow



mass:  $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$  so  $\frac{\partial u}{\partial x} = 0$  so  $u = f(y)$

momentum:  $\frac{\partial u}{\partial x} = 0, v = 0, \frac{\partial P}{\partial x} = 0 \Rightarrow \frac{\partial^2 u}{\partial y^2} = 0, \frac{\partial u}{\partial y} = C_1, u(y) = C_1 y + C_2$

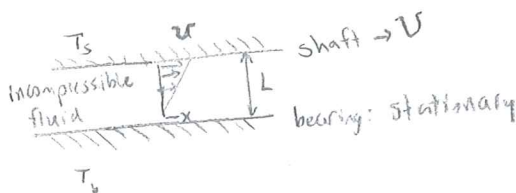
B.c.'s  $u(y=0) = 0 \Rightarrow C_2 = 0, u(y=L) = U \Rightarrow C_1 = \frac{U}{L}$

$$u = \frac{U}{L} y$$

Energy  $\frac{\partial T}{\partial x} = 0, v = 0, \dot{q} = 0 \Rightarrow 0 = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p} \left( \frac{\partial u}{\partial y} \right)^2 \Rightarrow \frac{\partial^2 T}{\partial y^2} = -\frac{\mu}{k} \left( \frac{\partial u}{\partial y} \right)^2$

$$= -\frac{\mu}{k} \left( \frac{U}{L} \right)^2 \Rightarrow \frac{dT}{dy} = -\frac{\mu}{k} \left( \frac{U}{L} \right)^2 y + C_3$$

$$T = -\frac{\mu}{2k} \left( \frac{U}{L} \right)^2 y^2 + C_3 y + C_4$$



2-22]

\* Prandtl number is a property of the fluid

B.C's  $T(y=0) = T_b$   $T_b = 0 + 0 + C_4$   $T_b = C_4$

$T(y=L) = T_s$  ....  $C_3 = \frac{T_s - T_b}{L} + \frac{\mu}{2k} \left(\frac{U}{L}\right)^2$

Thus  $T = T_b + \frac{\mu}{2k} U^2 \left(\frac{y}{L} - \frac{y^2}{L^2}\right) + (T_s - T_b) \frac{y}{L}$

2-24-11]  $\bar{C}_{fL} = f(Re_L)$

- don't worry about  $St$  # in book

$\frac{\bar{C}_f}{2} = \frac{\bar{Nu}_L}{Re_L \cdot Pr^{1/3}}$

$\bar{Nu}_L = \frac{0.025}{2} (1.23 \times 10^7)^{1/3} = 13,780 = \frac{\bar{h}_L L}{k_f}$

Local Reynolds number

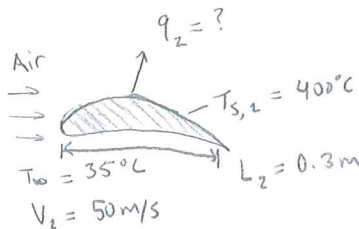
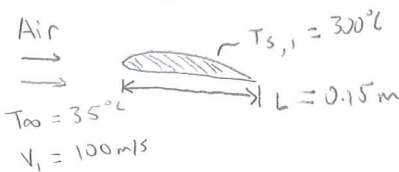
$Re_L = \frac{u L}{\nu}$   $Re_x = \frac{u \cdot x}{\nu}$

$\frac{V_{\infty} L}{\nu}$

Prob. 6.43]  $Re_L = \frac{V \cdot L}{\nu} = \frac{100(2)}{16.3 \times 10^{-6}} = 1.23 \times 10^7$  ;  $\bar{h}_L = \frac{(13780)(0.022)}{2} = 152 \frac{W}{m^2 \cdot K}$

$\bar{q}_{conv}'' = \bar{h}_L (T_s - T_{\infty}) = (152)(5 - (-12.3)) = 4260 \text{ W/m}^2$

Prob 6.19]  $q_1 = 1500 \text{ W}$



$Re_{L,1} = \frac{V_1 L_1}{\nu} = \frac{15}{\nu}$   $Re_{L,2} = \frac{V_2 L_2}{\nu} = \frac{50(0.3)}{\nu} = \frac{15}{\nu}$  }  $Nu_{L,1} = Nu_{L,2}$

$\frac{\bar{h}_{L,1} L_1}{k} = \frac{\bar{h}_{L,2} L_2}{k}$   $\bar{h}_{L,2} = \frac{L_1}{L_2} \bar{h}_{L,1} = \frac{L_1}{L_2} \frac{q_1}{A_1 (T_{s,1} - T_{\infty})}$

$q_2 = \bar{h}_{L,2} A_2 (T_{s,2} - T_{\infty})$

$q_1 = \dots = \frac{T_{s,2} - T_{\infty}}{T_{s,1} - T_{\infty}} q_1 = 2066 \text{ W}$

March 1 2011

Reynolds analogy connects fluid dynamics to heat transfer

$$\frac{C_f}{2} = \frac{Nu}{Re Pr^{1/3}}$$

$$C_{f,x} = f(x^*, Re) \Rightarrow \bar{C}_{f,L} = f(Re_L)$$

$$Nu_x = f(x^*, Re_x, Pr) \Rightarrow \bar{Nu}_L = f(Re_L, Pr)$$

- be able to solve couette flow problem (using heat flux instead of temp gradient)

## — FORCED CONVECTION: EXTERNAL FLOWS (Ch. 7)

The Empirical Method

measure  $h$  for a variety of conditions  $\Rightarrow \bar{Nu}_L = C Re_L^m Pr^n$

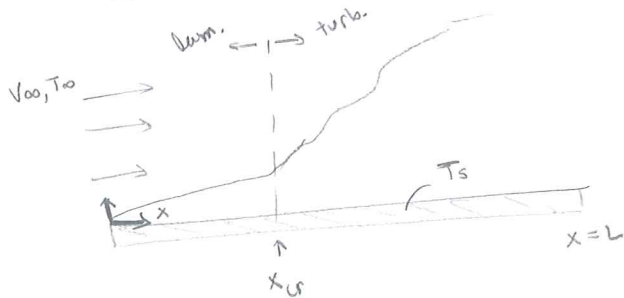
$m, n = f(\text{surface geometry, type of flow})$

assume const. fluid properties at  $T_f = \frac{T_s + T_{\infty}}{2}$

Table 7.9 ← Print this on full page for exam

last 3 under geom WILL NOT USE

Flow over flat plates

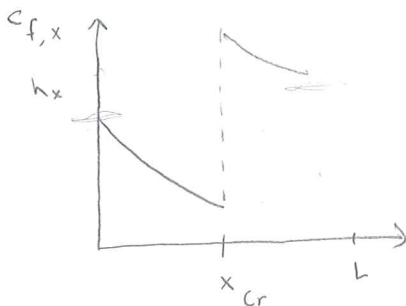


$$Re_{cr} = \frac{V_{\infty} x_{cr}}{\nu} = 5 \times 10^5$$

$$F_D = F_{\text{pressure}} + F_{\text{friction}}$$

$$F_D = \bar{C}_{f,L} \cdot A_s \cdot \frac{\rho V_{\infty}^2}{2}$$

- See table 7.9  
for specific  
correlations  
p431



For laminar flow,  $\delta_{v,t} \sim x^{1/2}$ ,  $C_{f,x} \sim x^{-1/2}$ ,  $Nu_x \sim x^{1/2}$ ,  $h_x \sim x^{-1/2}$

For turbulent flow  $\delta_{v,t} \sim x^{4/5}$ ,  $C_{f,x} \sim x^{-1/5}$ ,  $Nu_x \sim x^{4/5}$ ,  $h_x \sim x^{-1/5}$

$$\bar{C}_{f,L} = \frac{1}{L} \int_0^L C_{f,x} dx$$

$$\bar{h}_L = \frac{1}{L} \int_0^L h_x dx = \frac{1}{L} \left[ \int_0^{x_{cr}} h_{x,lam} dx + \int_{x_{cr}}^L h_{x,turb} dx \right]$$



March 1 2011

# Flow over flat plates continued

Methodology:

1. calculate  $Re_L$  using fluid properties at  $T_f = \frac{T_s + T_\infty}{2}$

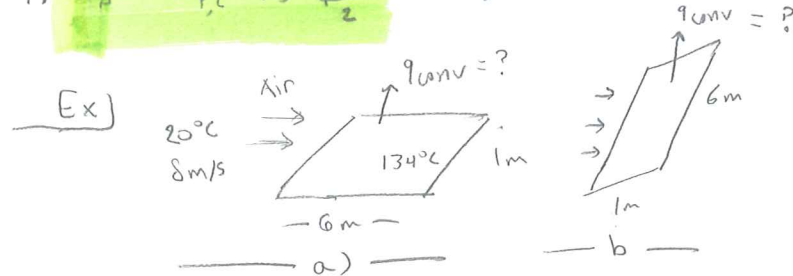
2. If  $Re_L < 5 \times 10^5 \Rightarrow$  entirely laminar flow, use laminar conditions

3. If  $Re_L > 5 \times 10^5 \Rightarrow$  mixed laminar & turbulent

- for local  $\sigma_x, t, f_{f,x}, Nu_x$ , use either lam or turb local correlations depending on  $Re_x$

- for average  $\bar{C}_{f,L}$  &  $\bar{Nu}_L$  use mixed avg. correlations

4.  $\dot{Q}_A = \bar{C}_{f,L} \cdot A_s \cdot \frac{\rho V_\infty^2}{2}$   $\dot{Q}_{conv} = \bar{h}_L \cdot A_s \cdot (T_s - T_\infty)$  where  $\bar{h}_L = \frac{\bar{Nu}_L \cdot k_f}{L}$



$$T_f = \frac{T_s + T_\infty}{2} = \frac{134 + 20}{2} = 77^\circ\text{C} = 350^\circ\text{K}$$

$$\Rightarrow \nu = 20.92 \times 10^{-6} \text{ m}^2/\text{s} \quad k = 30 \times 10^{-3} \text{ W/m}\cdot\text{K}$$

$$Pr = 0.7$$

a)  $L = 6\text{m} \Rightarrow Re_L = \frac{8(6)}{20.92 \times 10^{-6}} = 2.29 \times 10^6 \quad Re_{cr} = 5 \times 10^5 \Rightarrow$  mixed flow

P431 find Table 7.9 relationship flat plate  $\checkmark$ , mixed flow  $\checkmark$  average  $Nu \checkmark$

$$\bar{Nu}_L = (0.037 Re_L^{4/5} - 874) Pr^{1/3} = 3249 \Rightarrow \bar{h}_L = \frac{(3249)(30 \times 10^{-3})}{6} = 16.25 \text{ W/m}^2\cdot\text{K}$$

$$\dot{Q}_{conv} = (16.25)(6 \times 1)(134 - 20) = \boxed{11,15 \text{ W}}$$

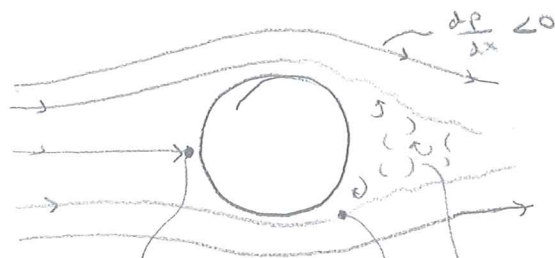
b)  $Re_L = \frac{8(1)}{20.92 \times 10^{-6}} = 3.82 \times 10^5 < Re_{cr} \Rightarrow$  entirely laminar

$$\bar{Nu}_L = 0.664 Re_L^{1/2} Pr^{1/3} = 364$$

$$\bar{h}_L = \frac{(30 \times 10^{-3})(364)}{(1)} = 10.9 \text{ W/m}^2\cdot\text{K}$$

$$\dot{Q}_{conv} = 7469 \text{ W}$$

cross flow

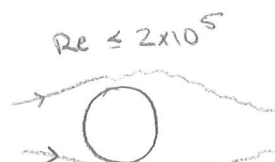


Stagnation point  $\frac{dp}{dx} = 0$   
wake region, has backflow  
Separation point,  $\frac{dp}{dx} = 0$

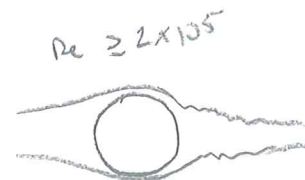
$$Re = \frac{\rho V D}{\mu} = \frac{\text{inertia}}{\text{viscosity}}$$

location of separation

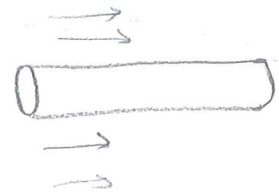
Fig 7.8, important P401



$$Re_D = \frac{4\dot{m}}{\pi D \mu} = \frac{VD}{\nu}$$

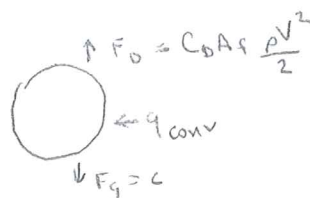


counter flow



ex) 7.73

$T_\infty = 280^\circ\text{C}$   
Water



Cu sphere,  $D = 20\text{mm}$   
 $T_i(r,0) = 360\text{K}$   
 $T_c(0,t_f) = 320\text{K}$

Find: terminal velocity  
tank height at point  
when  $T = T_f$

Balance forces

$$C_D A_s \frac{\rho V^2}{2} = C_D \frac{\pi r^2}{4} \rho \frac{V^2}{2} = g(\rho_{Cu} - \rho_{H_2O}) \dots \Rightarrow C_D V^2 = \frac{4}{3} D \frac{\rho_{Cu} - \rho}{\rho} g$$

$$C_D V^2 = 2.07 \text{ m}^2/\text{s}$$

Table 7.8  $Re_D = \frac{VD}{\nu}$

make a guess and do iterations P401

$$V = 2.1 \text{ m/s}$$

$$Re_D = 27850$$

$$Nu_D = 2 + (.4 Re_D^{1/2} + .06 Re_D^{2/3}) \left( \frac{\mu}{\mu_s} \right)^{1/4} \left( \frac{\mu_s}{\mu} \right)^{1/4} = 439$$

$$\bar{h} = 12775 \text{ W/m}^2 \cdot \text{K}$$

$$Bi = h \left( \frac{r_o}{3} \right) k_{Cu} = 12775 \times \left( \frac{.021}{3} \right) 398 = 0.11$$

$$\Theta^* = C_1 e^{(-3^2 F_0)} \rightarrow 5.1$$

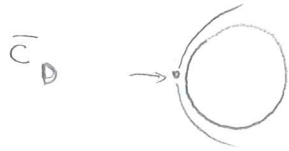
$$F_0 = 0.87 = \frac{\alpha t_f}{r_o^2}$$

$$H = 2.1 \times 0.76 \cdot 9890 =$$

$$= 1.6 \text{ m}$$



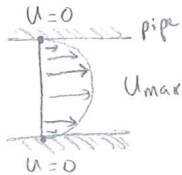
only friction drag



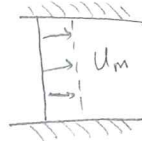
friction and pressure drag

summary for last class

Momentum Considerations:



actual



-Idealized

$$\dot{m} = \int_{A_c} \rho u(r) dA_c = \boxed{\rho u_m A_c}$$

$$Re = \frac{\rho u_m D_h}{\mu} \quad \text{where } D_h = \frac{4A_c}{P} \text{ (hydraulic diameter)}$$

Circular pipe with Diameter  $D \Rightarrow D_h = D$

$$\boxed{Re = \frac{4\dot{m}}{P\mu} = \frac{4\dot{m}}{\pi D\mu}}$$

$Re_{cr} \approx 2300$  (onset of turbulence)

Fully turbulent flow:  $Re \approx 10,000$

hydrodynamic entry length:  $\left(\frac{x_{fd,h}}{D}\right)_{lam} = .05 Re_D$

$$10 \lesssim \left(\frac{x_{fd,h}}{D}\right)_{turb} \lesssim 60$$

→ Fully developed conditions

pumping power =  $\Delta P \cdot \dot{V} = \boxed{\Delta P \cdot \frac{\dot{m}}{\rho}}$

pressure drop =  $\Delta P = P_1 - P_2 = \boxed{f \left(\frac{L}{D}\right) \left(\frac{\rho u_m^2}{2}\right) N/m}$

Friction factor =  $f = \frac{-(dp/dx)D}{\left(\frac{\rho u_m^2}{2}\right)} = \frac{\Delta P/L \cdot D}{\left(\frac{\rho u_m^2}{2}\right)} = f(Re) \text{ for smooth surfaces}$

Smooth Surfaces

- laminar flow in circular tube:  $f = \frac{64}{Re_D}$
- turbulent flow circular tube:  $\rightarrow f = (0.79 \ln Re_D - 1.64)^{-2}$

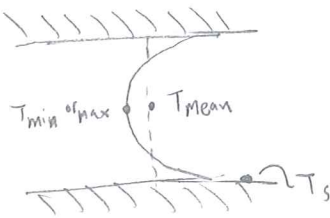
• in general;  $f = f(Re, e/D) \leftarrow \text{Moody chart (Fig 8.3) p461}$

↑  
relative pipe roughness

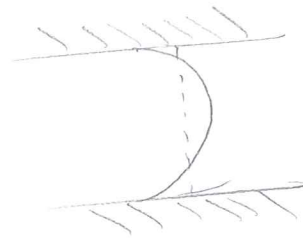
$\frac{\partial u}{\partial r} \Big|_{r=\frac{D}{2}} \neq f(x) \Rightarrow \tau_w \text{ constant} \Rightarrow f \text{ constant for } x > x_{fd,h} \leftarrow \text{fully developed description}$

$\frac{\partial u}{\partial x} = 0 \Rightarrow u_m = \text{const.}$

Thermal considerations:



OR



$$\dot{E}_f = \int_{A_c} \rho u c_p T dA_c = \dot{m} c_p T_m \quad \text{Local heat flux: } q_s'' = h(T_s - T_m)$$

$$\text{Thermal entry length: } \left(\frac{x_{fd,t}}{D}\right) \approx 0.05 Re_D Pr \quad \left(\frac{x_{fd,t}}{D}\right)_{turb} \approx 10$$

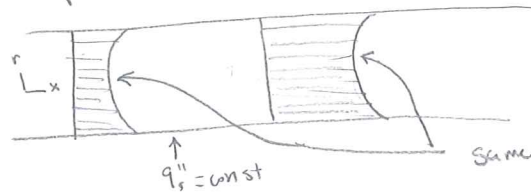
Note:  $Pr \gg 1$ ,  
then  $x_{fd,t} \approx x_{fd,h}$

Fully developed conditions:

$$\frac{\partial u}{\partial x} = 0 \quad \text{but} \quad \frac{dT_m}{dx} \neq 0 \quad \text{and} \quad \frac{\partial T}{\partial x} \neq 0 \quad \text{as} \quad T_m = T_m(x)$$

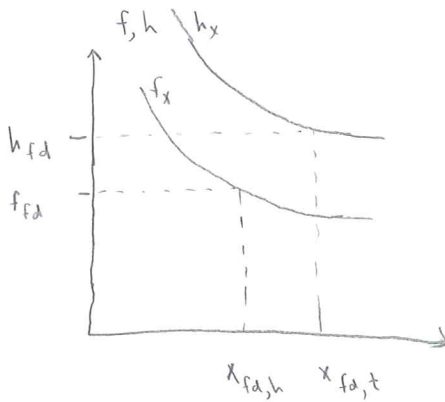
Although  $T(r) = f(x)$ , relative shape of the temperature profile does not change w/x,

$$\text{i.e.) } \frac{\partial}{\partial x} \left( \frac{T_s - T}{T_s - T_m} \right)_{fd,t} = 0$$



Same profile, greater T's

similar to f,  $h = \text{const}$  for  $x > x_{fd,t}$

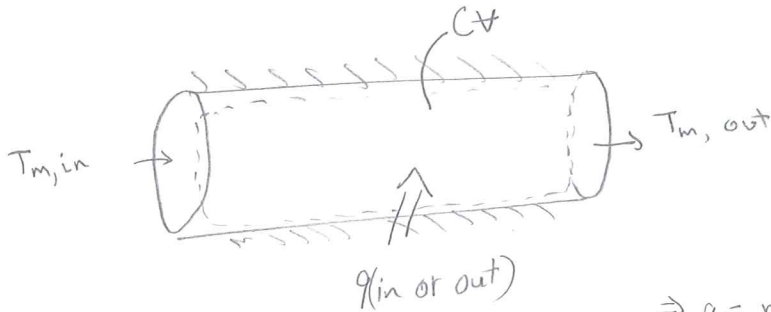


Fully developed internal flows

effect of friction felt all over x-section

$$\frac{\partial u}{\partial x} = 0 \Rightarrow u_m \text{ const, but; } \frac{\partial T}{\partial x} \neq 0 \Rightarrow T_m = T_m(x) = ?$$

$$f = \text{const (at fully developed)} \quad h = \text{const.}$$



$$\dot{m} = \text{const}$$

$$E_{in} = E_{out}$$

$$\dot{q} + \underbrace{\dot{m} c_p T_{m,i}}_h = \dot{m} c_p T_{m,o}$$

$$\Rightarrow \dot{q} = \dot{m} c_p (T_{m,o} - T_{m,i})$$

$$d\dot{q} = \dot{m} c_p dT_m$$

$$\leftarrow \dot{q}'' p dx = \dot{m} c_p dT_m \Rightarrow \frac{dT_m}{dx} = \frac{\dot{q}_s'' P}{\dot{m} c_p}$$

Solve this D.E. for 2

Surface conditions:-

$$1) \dot{q}_s'' = \text{const}$$

$$2) T_s = \text{const}$$

heat tape

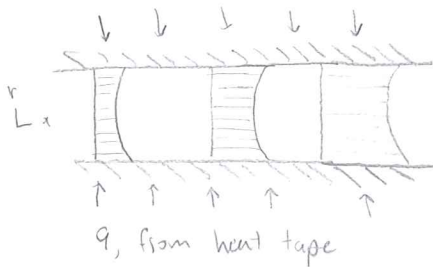
$$\dot{q}_s'' = h(T_s - T_m) = \text{const. } h = \text{const, for } x > x_{fd,t} \Rightarrow T_s - T_m = \frac{\dot{q}_s''}{h} = \text{const.}$$

i)  $\dot{q}_{\text{surface}} = \text{constant}$ 

$$\frac{dT_m}{dx} = \frac{\dot{q}_s'' \cdot P}{\dot{m} c_p} = \text{const} \Rightarrow T_m(x) = T_{m,i} + \frac{\dot{q}_s'' \cdot P}{\dot{m} c_p} x \quad (\text{linear})$$

$$T_{m,o} = T_{m,i} + \frac{\dot{q}_s'' \cdot P \cdot L}{\dot{m} c_p}$$

$$T_{m,o} = T_{m,i} + \frac{\dot{q}_s'' A_s}{\dot{m} c_p}$$



ii)  $T_s = \text{const}$  (phase change)

$$\frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m} c_p} = \frac{h(T_s - T_m) P}{\dot{m} c_p}$$

$$\Delta T \equiv T_s - T_m \Rightarrow \frac{d(\Delta T)}{dx} = -\frac{dT_m}{dx}$$

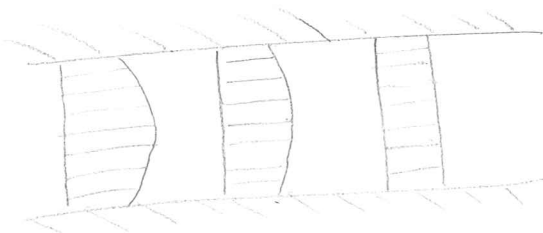
$$-\frac{d\Delta T}{dx} = \frac{h \Delta T P}{\dot{m} c_p} \Rightarrow \frac{d\Delta T}{\Delta T} = -\frac{Ph}{\dot{m} c_p} dx \quad \text{integration} \Rightarrow \ln \frac{\Delta T}{\Delta T_i} = \frac{-Phx}{\dot{m} c_p}$$

$$\frac{\Delta T}{\Delta T_i} = \frac{T_s - T_m(x)}{T_s - T_{m,i}} = e^{\left(\frac{-Phx}{\dot{m} c_p}\right)}$$

$$\text{or} \quad \frac{\Delta T_o}{\Delta T_i} = \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = e^{\left(\frac{-PLh}{\dot{m} c_p}\right)}$$

$$\text{or} \quad \ln \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = -\frac{\bar{h} A_s}{\dot{m} c_p}$$

$$\text{or} \quad T_{m,o} = T_s - (T_s - T_{m,i}) e^{\left(\frac{-\bar{h} A_s}{\dot{m} c_p}\right)}$$



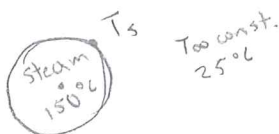
constant surface temp cooling

$$\frac{h A_s}{\dot{m} c_p} = \text{NTU (number of transfer units)}$$

$$L \rightarrow \infty, \text{NTU} \rightarrow \infty \Rightarrow T_{m,o} \rightarrow T_s$$

$$q = \dot{m} c_p (T_{m,o} - T_{m,i}) = -\frac{\bar{h} A_s}{\ln \left( \frac{T_s - T_{m,o}}{T_s - T_{m,i}} \right)} (T_{m,o} - T_{m,i}) = -\Delta T_{\text{lm}}, \text{ log mean Temp. difference}$$

- If the external fluid temp  $T_{\infty}$  is constant (instead of pipe surface temp  $T_s$  const) then replace  $T_s$  w/  $T_{\infty}$  &  $\bar{h}$  with  $\bar{U}$



$$\frac{T_{\infty} - T_{m,o}}{T_{\infty} - T_{m,i}} = e^{\left(\frac{-A_s \bar{U}}{\dot{m} c_p}\right)}$$

$$q = \bar{U} A_s \Delta T_{\text{lm}}$$

↑  
overall heat transfer coefficient

$$U A_s = \frac{1}{R_{\text{total}}}$$

Review, ex  $R_{\text{tot}} = R_{\text{conv},i} + R_{\text{cond}} + R_{\text{conv},o}$

$$= \frac{1}{h_i A} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{1}{h_o A}$$

$h = ?$  Table 8.4 summarizes Nu & correlations

For example i) fully developed laminar flow inside circular pipes:

$$\text{Nu}_D = \frac{hD}{k} = \begin{cases} 4.36 & \text{for } q''_s = \text{const} \\ 3.66 & \text{for } T_s = \text{const} \end{cases}$$

for noncircular tubes use hydraulic diameter,

$$D_h = \frac{4A_c}{P} \text{ w/ Table 8.1}$$

3-15-11

- i) fully-developed turbulent flow inside circular pipes  
 $Nu_D = 0.023 Re_D^{0.8} Pr^n$  where  $n = \begin{cases} 0.4 & \text{for heating } (T_s > T_m) \\ 0.3 & \text{for cooling} \end{cases}$

Turbulent correlations are valid for both surface conditions of

$q''_s = \text{const} \ \& \ T_s = \text{const.}$

• for noncircular pipes, use the correlations with

$D_h = \frac{4A_c}{P}$

3-17-11

ex)

$T_{m,i} = 85^\circ\text{C} \rightarrow$

$\rightarrow T_{m,o} = 40^\circ\text{C}$

smooth, thin-walled tube  
 $T_s = 20^\circ$

$\rho = 1000 \text{ kg/m}^3$

$c_p = 3000 \text{ J/kg}\cdot\text{K}$

$\mu = 0.04 \text{ N/s}\cdot\text{m}^2$

$k = 0.26 \text{ W/m}\cdot\text{K}$

$L = ? \quad i) D = 10 \text{ cm}, T_s = 20^\circ\text{C const} \quad \dot{m} = 1 \text{ kg/s}$

$(\text{power}) \quad P = ?$   
 $L = \frac{\dot{m} c_p}{\pi D \bar{h}} \ln \left( \frac{T_s - T_{m,i}}{T_s - T_{m,o}} \right)$

$q = \dot{m} c_p (T_{m,o} - T_{m,i}) = \bar{h} A_s \Delta T_{\ln}$   
 $= 1(3000)(40 - 85) = -135 \text{ kW}$

$P = \Delta P \dot{V} = f \frac{L}{D} \frac{\rho u_m^2}{2} \cdot \frac{\dot{m}}{\rho}$

$Re = \frac{\rho u_m D}{\mu}; u_m = \frac{\dot{m}}{\rho \frac{\pi D^2}{4}} = \frac{1}{1000 \left( \frac{\pi}{4} \right) (0.1)^2} = 0.127 \text{ m/s}$

$Re = \frac{1000(0.127)(0.1)}{0.04}$

relationship 8.53 used for Nu

Table 8.4 p 449

$Nu = 3.66 = \frac{\bar{h} D}{k_f} \Rightarrow \bar{h} = \frac{(3.66)(0.26)}{(0.1)} = 9.5 \text{ W/m}^2\cdot\text{K}$

$L = \frac{1(3000)}{\pi(0.1)(9.5)} \ln \left( \frac{20 - 85}{20 - 40} \right) \Rightarrow L = 1182 \text{ m}$

$P = (0.2) \frac{(1182)}{(0.1)} \frac{(1000)(0.127)^2}{2} \frac{(1)}{(1000)} \Rightarrow P = 19.2 \text{ NM/s}$

8.62  $Nu = \frac{f(Re - 1000)Pr}{1 + 12.7 \left( \frac{f}{8} \right)^{1/2} (Pr^{1/3} - 1)} = \bar{h} = 250 \text{ W/m}^2\cdot\text{K}$

$L = 450 \text{ m}$

$q = -1350$

$P = 1533 \text{ NM/s}$

Calculate this with  $Re$  doubled from original - still laminar



3-17-2011

ex continued

(ii) a square duct of equal perimeter  $P = \pi D = \pi(1) = 3.14 \text{ m} = 4a \quad a = .0785$

$$D_h = \frac{4Ac}{P} = \frac{4(a^2)}{4a} = a = .0785 \quad \text{See table 8.1}$$

$$u_m = .162 \text{ m/s} \Rightarrow Re_{D_h} = \frac{\rho u_m D_h}{\mu} = 318; \text{ laminar}$$

$$f = \frac{57}{Re} = .179; \quad Nu = 2.98 \Rightarrow \bar{h} = 9.9 \text{ W/m}^2\text{K}$$

$$L = 1137 \text{ m} \quad q = -135 \text{ kW} \quad P = 34 \text{ Wm/s}$$

iv)  $D = 10 \text{ cm} \quad T_\infty = 20^\circ \text{C}, \text{ const } \dot{m} = 1 \text{ kg/s} \quad \text{air, } V_\infty = 10 \text{ m/s}$

$$L = \frac{\dot{m} c_p}{\pi D \bar{h}} \ln \left( \frac{T_\infty - T_{mi}}{T_\infty - T_{mo}} \right)$$

$$UA = \frac{1}{R_{tot}} \Rightarrow$$

$$U = \left( \frac{1}{h_i} + \frac{1}{h_o} \right)^{-1}$$

9.5 ch 46.2

$$\bar{h}_o = 46.2 \text{ W/m}^2\text{K}$$

$$L = 1425 \text{ m}$$

Calculate this on your own, this is most common problem type

Project help - no need to draw air

3-22-11 Notes

Example test prob

$Pr = 4 \quad Re_1 = 2000$ , circular pipe,  $T_s = \text{const} > T_{mi}$   $\dot{m} = \text{const}$   $D_2 = \frac{D_1}{5}$

$T_{mi} = \text{const} \quad T_{mo} = \text{const} \quad \frac{L_2}{L_1} = ? \quad \frac{f_2}{f_1} = ? \quad \frac{P_2}{P_1} = ? \quad Re_2 = 5Re_1 = 10,000 \rightarrow \text{turbulent}$

$$L = \frac{\dot{m} c_p}{\pi D \bar{h}} \ln \left( \frac{T_s - T_{mi}}{T_s - T_{mo}} \right)$$

$$\frac{L_2}{L_1} = \frac{\frac{1}{D_1 \bar{h}_1}}{\frac{1}{D_2 \bar{h}_2}} = \frac{D_1 \bar{h}_1}{D_2 \bar{h}_2} = 5 \frac{\bar{h}_1}{\bar{h}_2}$$

$$\bar{Nu}_2 = .023 (10,000)^{.8} (4)^{.4} = 63.5$$

$$Nu = \frac{hD}{k} \Rightarrow h = \frac{Nu k}{D}$$

$$\frac{L_2}{L_1} = 5 \frac{\frac{Nu_1 k}{D_1}}{\frac{Nu_2 k}{D_2}} = 5 \frac{D_2}{D_1} \frac{\bar{Nu}_1}{\bar{Nu}_2}$$

$$\frac{\bar{Nu}_1}{\bar{Nu}_2} = \frac{3.66}{63.5} = \boxed{0.06}$$

$$\frac{f_2}{f_1} = \frac{.316 Re_2^{-.25}}{.316 Re_1^{-.25}} = \frac{.316 (10,000)^{-.25}}{.316 (2000)^{-.25}} = .9875$$

$$\frac{P_2}{P_1} = \frac{\dot{V}_2 \Delta P_2}{\dot{V}_1 \Delta P_1} = \frac{\frac{\dot{m}_2}{\rho} f_2 \frac{L_2}{D_2} \frac{\bar{u}_{m2}^2}{2}}{\frac{\dot{m}_1}{\rho} f_1 \frac{L_1}{D_1} \frac{\bar{u}_{m1}^2}{2}} = \frac{f_2}{f_1} \frac{L_2}{L_1} \frac{D_1}{D_2} \frac{D_1^4}{D_2^4} \frac{\bar{u}_{m2}^2}{\bar{u}_{m1}^2}$$

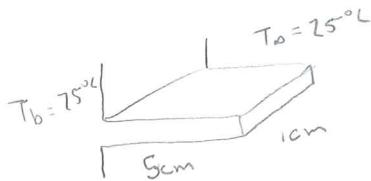
$$\frac{P_2}{P_1} = (.9875) (.04) (5)^5$$

$$\frac{P_2}{P_1} = \boxed{185}$$

Notes 3-22-11

We Never use local Nu for cylinder or sphere, Sometimes with flat plate

2)



dissipate  
 $\eta_f = 0.9 \quad q = 2W = \eta_f \bar{h} A_f (T_b - T_o) = (0.9) \bar{h} (2 \times 5 \times 10^{-4}) (75 - 25)$   
 $\downarrow$   
 $\bar{Nu} \rightarrow Re \rightarrow V_\infty$

$\Rightarrow \bar{h} = 44.5 \frac{W}{m^2 K}$

what to use for fluid temp

$T_f = \frac{T_s - T_o}{2} = 50^\circ C$

$\Rightarrow$  use air @ 300K

$\nu = 15.89 \times 10^{-6}$

$k = 26.3 \times 10^{-3}$

$Pr = .707$

$\bar{Nu} = \frac{\bar{h} L}{k_{fluid}} = \frac{44.5 (5 \times 10^{-2})}{26.3 \times 10^{-3}} = .664 Re_L^{1/2} Pr^{1/3} = .664 Re_L^{1/2} (.707)^{1/3}$

$\Rightarrow Re_L = 20,455 = \frac{V_\infty L}{\nu} = \frac{V_\infty \times 5 \times 10^{-2}}{15.89 \times 10^{-6}} \quad \boxed{V_\infty = 6.5 \text{ m/s}}$

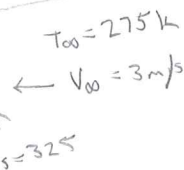
ii) water with same velocity above (or use 20 m/s)

$\nu = \frac{\mu}{\rho} = 1.011 \times 10^{-3} \text{ m}^2/\text{kg} \quad \mu = 577 \times 10^{-6} \quad k = 640 \times 10^{-3} \text{ W/mK} \quad Pr = 3.71$

$Re_L = \frac{(6.5)(5 \times 10^{-2})}{1.011 \times 10^{-3}} = 5.5 \times 10^5 > Re_{cr} \Rightarrow \text{mixed flow}$

$\bar{Nu} = (\text{expression 7.31}) = 897 \quad \bar{h}_L = \frac{640 \times 10^{-3}}{(5 \times 10^{-2})} (897) = 11,476 \text{ W/m}^2 K$

$q_f = (0.9)(11,476)(2 \times 5 \times 10^{-4})(50) = 516 \text{ W}$



2008  
#3

arbitrary surface,  $A_s = 4 \text{ m}^2 \quad T_s = 325 \text{ K}$

$L = 2 \text{ m}$

$Re_{cr} = 10^6$

Temp profile w/in bl:

$T = 325 + 50e^{-500y/x^{1/2}}$  when flow is locally laminar

$T = 325 - 20,000 \frac{y}{x^{1/5}}$  " " " " turbulent

Air at  $\frac{325+275}{2} = 300 \text{ K}$

$\nu = 15.89 \times 10^{-6} \text{ m}^2/\text{s}$

$k = 26.3 \times 10^{-3}$

$Pr = .707$

$\rho = 1.1616 \text{ kg/m}^3$

i)  $Re_L = \frac{V_\infty L}{\nu} = \frac{(3)(2)}{15.89 \times 10^{-6}} = 3.78 \times 10^5 < Re_{cr} = 10^6$  laminar everywhere

ii)  $h(x) = \frac{-kf \frac{\partial T}{\partial y}|_{y=0}}{T_s - T_o}$

Know definition of Nusselt #

$Nu_x = \frac{h_x x}{k}$

iii)  $\bar{h}_L = \frac{1}{L} \int_0^L h_x dx$

iv)  $\bar{C}_{f,L} = \frac{2 Nu_L}{Re_L Pr^{1/3}} = \dots$  (Reynolds analogy)

v)  $q = \bar{h}_L A_s (T_s - T_o)$

$F_D = \bar{C}_{f,L} \frac{\rho V_\infty^2}{2} A_s$

Fouling Factor: fluid impurities & rust formation  $\Rightarrow$  deposition of a film layer on the surface  
 $\Rightarrow$  decrease heat transfer due to additional conduction resistance

$\downarrow$   
 $R_f'' = f(T, v, t, \text{fluid properties})$

• for an unfouled H.X.:  $\frac{1}{UA} = \left( \frac{1}{U_i A_i} \right) = \frac{1}{U_o A_o} = \boxed{\frac{1}{h_i A_i} + \frac{R_{f,i}''}{A_i} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{R_{f,o}''}{A_o} + \frac{1}{h_o A_o}}$

a) without fouling  $\boxed{\frac{1}{U_o} = \frac{1}{h_i} \frac{D_o}{D_i} + \frac{D_o \ln(D_o/D_i)}{2k} + \frac{1}{h_o}}$   $Re_{D_i} = \frac{4\dot{m}_i}{\pi D_i \mu} = 21,220$

\* See Ch. 11 slide 8  $h_i = \frac{k_f}{D_i} (0.023 Re_{D_i}^{0.8} Pr^{0.4}) = 3400 \text{ W/m}^2\text{K}$

$U_o = \frac{1}{4.43 \times 10^{-4}} = 2255 \text{ W/m}^2\text{K}$

b) with fouling  $\left( 4.43 \times 10^{-4} + \frac{D_o}{D_i} R_f'' \right)^{-1} = (5.55 \times 10^{-4})^{-1} = 1800 \text{ W/m}^2\text{K}$

c)  $\dot{m}_h h_{fg} = \dot{m}_c c_p (T_{mp} - T_{m,i}) \Rightarrow T_{m,o} = T_{m,i} + \frac{\dot{m}_h h_{fg}}{\dot{m}_c c_p} = 29.4^\circ\text{C}$

$\uparrow$   
 Condensation rate

• note that example gives mass flow of water "per tube"

$= 15 + \frac{10(2414 \times 10^3)}{400(4180)}$

$q = \dot{m}_{\text{cond/evap}} \cdot h_{fg}$

— Analysis: —

assume - steady state

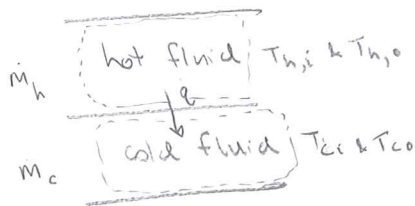
-  $\dot{m} = \text{const.}$

- negligible  $\Delta KE$  &  $\Delta PE$

-  $C_p = \text{const.}$

- negligible axial heat conduction

- perfectly insulated H.X.



$q = \dot{m}_h C_{p,h} (T_{h,i} - T_{h,o}) = C_h (T_{h,i} - T_{h,o})$

$q = \dot{m}_c C_{p,c} (T_{c,o} - T_{c,i}) = C_c (T_{c,o} - T_{c,i})$

$C = \dot{m} C_p = \text{heat capacity rate}$

$q = C \Delta T$

$q = UA \Delta T_{\text{lm}}$  where  $\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$

Parallel flow (PF):  $\Delta T_1 = T_{h,i} - T_{c,i}$   $\Delta T_2 = T_{h,o} - T_{c,o}$

Counter flow (CF):  $\Delta T_1 = T_{h,i} - T_{c,o}$   $\Delta T_2 = T_{h,o} - T_{c,i}$

• For the same in/outlet temps,

$\Delta T_{\text{lm, CF}} > \Delta T_{\text{lm, PF}}$  Thus

• for the same  $U$ ,  $A_{\text{CF}} < A_{\text{PF}}$  for a prescribed  $q$

• note that possibility of  $T_{c,o} > T_{h,o}$  is only for a C.F.

Notes 4-12-11) continued

$$q = C_h (T_{h,i} - T_{h,o}) = C_c (T_{c,o} - T_{c,i})$$

Notes 4-14-11)

HX analysis:  $\dot{q} = \dot{m} c_{p,c} (T_{c,o} - T_{c,i}) = \dot{m} c_{p,h} (T_{h,i} - T_{h,o}) = \underbrace{UA \Delta T_{lm}}_{\text{only for PF \& CF HX's}}$

$\epsilon$ -NTU method:

$$\epsilon, \text{ effectiveness} = \frac{q}{q_{\max}} \quad (0 \leq \epsilon \leq 1) \quad q_{\max} = C_{\min} (T_{h,i} - T_{c,i})$$

The fluid with  $C_{\min}$  experiences the largest temp. difference

$$\epsilon = f(\text{geometry, flow arrangement}) = f(NTU, C_r)$$

where  $NTU = \frac{UA}{C_{\min}}$ ;  $C_r = \frac{C_{\min}}{C_{\max}}$

Tables 11.3 or Figs 11.10-11.15 give  $\epsilon$ -NTU relationships

$C \rightarrow \infty$  for a condensing fluid or phase change

$$q = \epsilon q_{\max} = \epsilon C_{\min} (T_{h,i} - T_{c,i})$$

During phase change (Boiler or condenser):  $C_r \rightarrow 0 \Rightarrow \epsilon = 1 - e^{-NTU}$   
or  $NTU = -\ln(1 - \epsilon)$

— Discuss prob 11.63 (slides)



surface area  $\pi D_i L$

$$\epsilon = \frac{q}{q_{\max}}$$

$U$ , overall HT. coefficient

$$q = C_c (T_{c,o} - T_{c,i}) = C_h (T_{h,i} - T_{h,o}) = \underbrace{\epsilon C_{\min} (T_{h,i} - T_{c,i})}_{q_{\max}}$$

$$= UA \Delta T_{lm}$$

$$\epsilon = 0.65$$

$$C_r = \frac{C_{\min}}{C_{\max}} = \frac{0.125 (2100)}{125 (4200)} = 0.5$$

$$NTU = \frac{UA}{C_{\min}}$$

$$\Rightarrow \frac{A_{CF}}{A_{PF}} = \frac{NTU_{CF}}{NTU_{PF}}$$

$$\approx \frac{1.3}{2.6} \approx 0.5$$

Fig 11.10

on slides they solve exactly using  $\Delta T_{lm}$ , slide 15

Problem 11.38

Notes 4-14-2011

11.38

$$A_s = 2\pi D L N$$

q. for condenser: use Eq. 11.35 instead of figures

Notes 4-19-2011

## Ch 12, — RADIATION —

Blackbody:

Stefan Boltzmann law: the radiation energy emitted by a blackbody over all  $\lambda$ 's per unit time and surface area is

total blackbody emissive power  $\rightarrow E_b = \sigma T^4$  (W/m<sup>2</sup>)  
(must be in K!)

$$\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$$

The radiation emitted by a blackbody per unit wavelength is given by Planck's Law

$$E_{b\lambda} = \frac{C_1}{\lambda^5 (e^{C_2/\lambda T} - 1)} \quad (\text{W/m}^2 \cdot \mu\text{m})$$

spectral blackbody emissive power

$$C_1 = 3.742 \times 10^8 \text{ W}\mu\text{m}^4/\text{m}^2$$

$$C_2 = 1.439 \times 10^4 \mu\text{m}\cdot\text{K}$$

As  $T \uparrow$ ,  $E_{b\text{max}}$  shifts towards shorter  $\lambda$ :  $\lambda_{\text{max}} = f(T)$

$$\frac{\partial E_{b\lambda}}{\partial \lambda} = 0 \Rightarrow \lambda_{\text{max}} = T = C_3 = 2897.8 \mu\text{m}\cdot\text{K} \quad (\text{Wien's law})$$

$$\text{Sun} \Rightarrow T = 5762 \text{ K} \quad \lambda_{\text{max}} = \frac{2897.8}{5762} \approx 0.50 \mu\text{m} \text{ (Vis)}$$

$$T = 298 \text{ K} \Rightarrow \lambda_{\text{max}} = 9.72 \mu\text{m} \text{ (IR)}$$

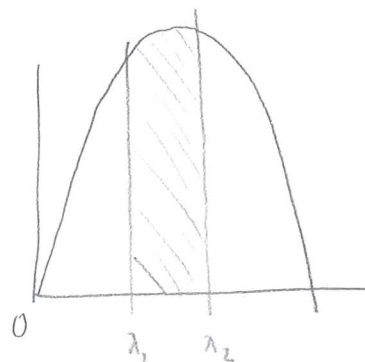
$$\text{Flame} \Rightarrow T \approx 2000 \text{ K} \Rightarrow \lambda_{\text{max}} = 1.5 \mu\text{m}$$

Fraction of blackbody energy from 0 to  $\lambda$

$$F_{(0 \rightarrow \lambda)} = \frac{\int_0^\lambda E_{b\lambda} d\lambda}{\int_0^\infty E_{b\lambda} d\lambda} = \frac{\int_0^\lambda E_{b\lambda} d\lambda}{\sigma T^4} \quad 0 \leq F \leq 1$$

Fraction of blackbody energy between  $\lambda_1$  &  $\lambda_2$

$$F_{(\lambda_1 \rightarrow \lambda_2)} = \frac{\int_{\lambda_1}^{\lambda_2} E_{b\lambda} d\lambda}{\sigma T^4} = F_{(0 \rightarrow \lambda_2)} - F_{(0 \rightarrow \lambda_1)}$$



Notes 4-19-2011

(5800K)

P. 12.23: fraction of sun's radiation is emitted in visible range (VIS 0.4-0.7  $\mu\text{m}$ )

$$F_{(0.4-0.7)} = F_{(0 \rightarrow 0.7)} - F_{(0 \rightarrow 0.4)}$$

$$\lambda_1 T = 0.4(5800) = 2320 \mu\text{m}\cdot\text{K}$$

$$\lambda_2 T = 0.7(5800) = 4060 \mu\text{m}\cdot\text{K}$$

$$= 0.48 - 0.14$$

$$= 0.34 = \boxed{34\%}$$

Radiation properties of Real Surfaces:

(i) Emissivity:  $\epsilon = \frac{\text{energy emitted by a surface @ } T}{\text{" " " blackbody @ } T}$

$$0 \leq \epsilon \leq 1$$

$$\epsilon = 1 \Rightarrow \text{blackbody}$$

$\epsilon_\lambda$  = spectral emissivity

$\epsilon$  = total emissivity = average  $\epsilon_\lambda$  over all  $\lambda$ 's

$$\epsilon_\lambda(T) = \frac{E_\lambda(T)}{E_{b\lambda}(T)} ; \epsilon(T) = \frac{E(T)}{E_b(T)} = \frac{E(T)}{\sigma T^4} = \frac{1}{\sigma T^4} \int_0^\infty \epsilon_\lambda E_{b\lambda} d\lambda$$

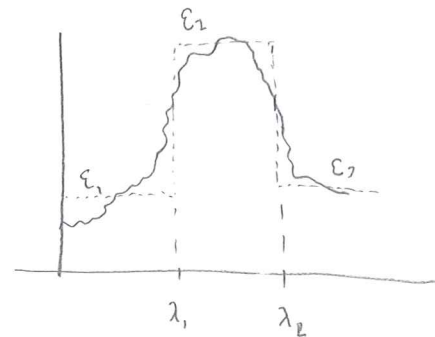
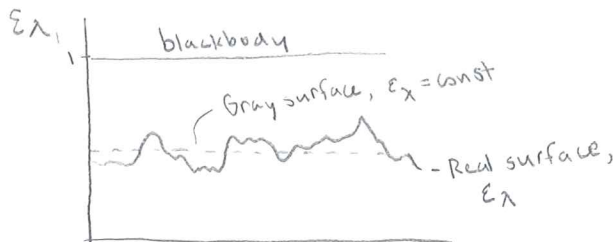
Real surface:  $\epsilon_\lambda \neq \text{const}$   $\epsilon_\theta \neq \text{const}$

Diffuse surface:  $\epsilon_\theta = \text{const}$

Gray surface:  $\epsilon_\lambda = \text{const}$

} diffuse & gray surface:  $\epsilon = \epsilon_\theta = \epsilon_\lambda = \text{const}$

Notes 4-21-11



$$\epsilon = \frac{1}{\sigma T^4} \left\{ \epsilon_{\lambda_1} \int_0^{\lambda_1} E_{b\lambda_1} d\lambda + \epsilon_{\lambda_2} \int_{\lambda_1}^{\lambda_2} \dots \right\}$$

$$\epsilon = \epsilon_{\lambda_1} F_{(0 \rightarrow \lambda_1)} + \epsilon_{\lambda_2} F_{(\lambda_1 \rightarrow \lambda_2)} + \epsilon_{\lambda_3} F_{(\lambda_2 \rightarrow \infty)}$$

$$= (F_{0 \rightarrow \lambda_2} - F_{0 \rightarrow \lambda_1})$$

Approximation:

$$\epsilon_\lambda: \begin{cases} \epsilon_1 = \text{const. } 1, & 0 \leq \lambda \leq \lambda_1 \\ \epsilon_2 & \lambda_1 \leq \lambda \leq \lambda_2 \\ \epsilon_3 & \lambda_2 \leq \lambda < \infty \end{cases}$$



Notes 4-21-11 continued

ii) Absorptivity, Reflectivity, and transmissivity:

$$\alpha = \frac{\text{absorbed radiation}}{\text{Irradiation}} = \frac{G_{\text{abs}}}{G} \quad \rho = \frac{G_{\text{ref}}}{G} \quad \tau = \frac{G_{\text{tra}}}{G}$$

$$G = G_{\text{abs}} + G_{\text{ref}} + G_{\text{tra}} \quad 1 = \alpha + \rho + \tau$$

• opaque surface:  $\tau = 0 \Rightarrow \alpha + \rho = 1$  ( $G_{\text{tr}} = 0$ )

• black surface:  $\rho = \tau = 0 \Rightarrow \alpha = 1$

Spectral properties:

$$\alpha_\lambda = \frac{G_{\lambda, \text{abs}}}{G_\lambda} \quad \rho_\lambda = \frac{G_{\lambda, \text{ref}}}{G_\lambda} \quad \tau_\lambda = \frac{G_{\lambda, \text{tra}}}{G_\lambda} \quad \alpha_\lambda + \rho_\lambda + \tau_\lambda = 1$$

$$\alpha = \frac{\int_0^\infty \alpha_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda}; \quad \rho = \frac{\int_0^\infty \rho_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda} \quad \tau = \frac{\int_0^\infty \tau_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda}$$

$$\sigma = 5.67 \times 10^{-8}$$

$\rho$  reflectivity

$\epsilon$  emissivity

$\alpha$  absorptivity

$\tau$  transmissivity

$G$  irradiation

$J$  radiosity

Kirchoff's law:

$$\alpha_\lambda = \epsilon_\lambda$$

ex, 12.69) opaque  $\tau_\lambda = 0$   $\alpha_\lambda + \rho_\lambda + \tau_\lambda = 1$   $\alpha_\lambda = 1 - \rho_\lambda \Rightarrow \rho_\lambda = \begin{cases} 1 - 0.4 = 0.4 & 0 \leq \lambda \leq 3 \mu\text{m} \\ 1 - 0.2 = 0.8 & 3 \leq \lambda \leq \infty \mu\text{m} \end{cases}$

$$\alpha = \frac{\int_0^\infty \alpha_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda} = \frac{0.4 \int_0^3 G_\lambda d\lambda + 0.8 \int_3^6 G_\lambda d\lambda + 0.8 \int_6^\infty G_\lambda d\lambda}{\int_0^3 G_\lambda d\lambda + \int_3^6 G_\lambda d\lambda + \int_6^\infty G_\lambda d\lambda} = \frac{0.4 \frac{12(500)}{2} + 0.8(3)(500) + 0.8(2)(500)}{2500}$$

$$= \frac{1800}{2500} = 0.72$$

$$\epsilon_\lambda = \alpha_\lambda$$

$$\epsilon = \epsilon_{\lambda_1} F_{(0 \rightarrow 3)} + \epsilon_{\lambda_2} F_{(3 \rightarrow \infty)} = \underbrace{\alpha_{\lambda_1}}_{0.4} \cdot F_{(0 \rightarrow 3)} + \underbrace{\alpha_{\lambda_2}}_{0.8} (1 - F_{(0 \rightarrow 3)})$$

$$\lambda T = 3(750) = 2250 \mu\text{m} \cdot \text{K} \Rightarrow F_{(0 \rightarrow 3)} = 0.1$$

$$\epsilon = 0.756$$

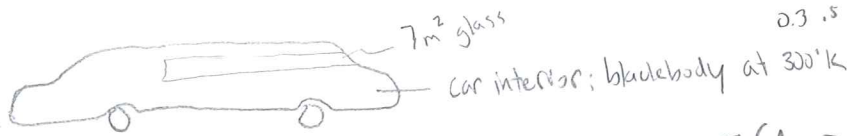
$$q''_{\text{radiation}} = G_{\text{absorber}} - E = \alpha \cdot G - \epsilon \sigma T_s^4 = (0.72)(2500) - 0.756 (5.67 \times 10^{-8})$$

$$= -11,763 \text{ W/m}^2$$

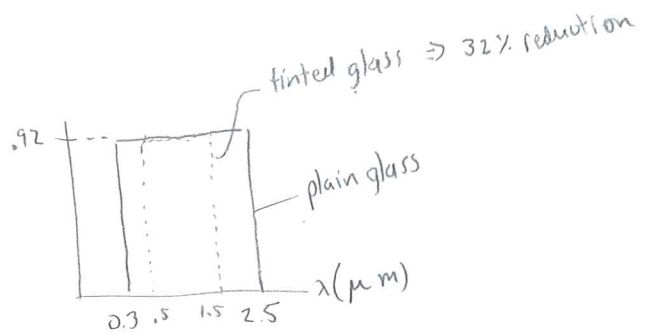


continued

Greenhouse effect:

7 m<sup>2</sup> glass

car interior; blackbody at 300 K



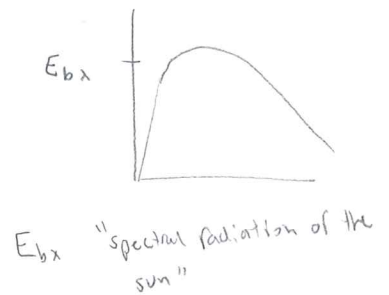
a) energy of sun transmitted through windows =  $\tau G A_s = ?$

$$\tau = \frac{\int_0^\infty \tau_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda} = \frac{\int_0^\infty \tau_\lambda E_{b\lambda} d\lambda}{\int_0^\infty E_{b\lambda} d\lambda} = \frac{.92 \int_{.3}^{2.5} E_{b\lambda} d\lambda}{\int_0^\infty E_{b\lambda} d\lambda}$$

$$= (0.92) (F_{(0 \rightarrow 2.5)} - F_{(0 \rightarrow 0.3)}) = 0.85$$

$$\lambda_2 T = 2.5 (5800) = 14,500 \mu\text{m} \Rightarrow F_{(0 \rightarrow 2.5)} \cong 0.96$$

$$\lambda_1 T = 0.3 (5800) = 1740 \mu\text{m} \Rightarrow F_{(0 \rightarrow 0.3)} \cong 0.04$$



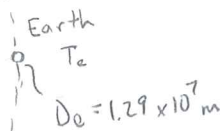
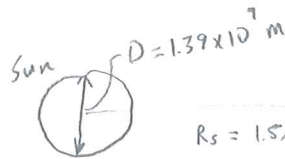
b) radiant energy from car's interior transmitted back to the surroundings thru windows =  $\tau E_b A_s$  (300 K) =  $1.35 \times 10^{-5} (5.67 \times 10^{-8}) (300^4) (1) = .004 \text{ W}$

$$(2.5)(300) = 750 \mu\text{m} \cdot \text{K} \Rightarrow F_{(0 \rightarrow 2.5)} \cong .000015 \quad \left. \vphantom{\begin{matrix} (2.5)(300) = 750 \mu\text{m} \cdot \text{K} \\ (0.3)(300) = 90 \mu\text{m} \cdot \text{K} \end{matrix}} \right\} \tau = 1.35 \times 10^{-5}$$

$$(0.3)(300) = 90 \mu\text{m} \cdot \text{K} \Rightarrow F_{(0 \rightarrow 0.3)} \cong 0$$

Notes 4-26-2011

Ex) Problem 12.20

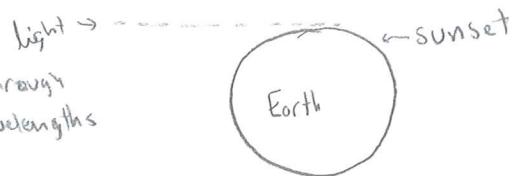


Solar flux at outer edge of Earth's atmosphere  $q_s'' = 1353 \text{ W/m}^2$

$$\underbrace{E_s (\pi D_s^2)}_{4\pi R_s^2} = q_s'' 4\pi (R_{se}^2) \Rightarrow E_s = 6.3 \times 10^7 \text{ W/m}^2 = \sigma T_s^4 \Rightarrow T_s = 5774 \text{ K}$$

gas molecules & small particles  $C_{sca} \sim \frac{1}{\lambda^4}$ ;  $C_{abs} \sim \frac{1}{\lambda}$

• more absorption can occur through a greater length & longer wavelengths will be preserved



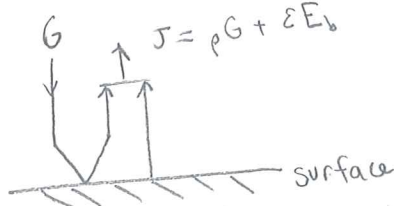
## Radiation Exchange Between Surfaces (Ch.13)

Consider two surfaces separated by a non-participating medium (most gases)

Governing factors

- Temperature
- Radiation Properties
- Geometries & orientations

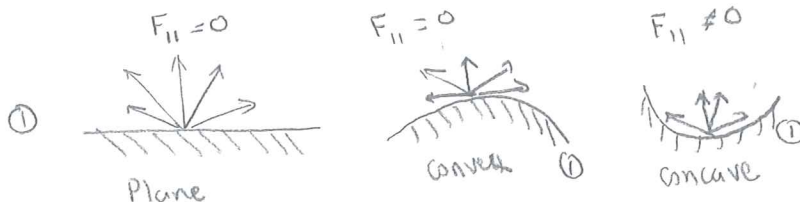
• Radiosity:  $(J)$



• View factor: fraction of radiation leaving a surface that is intercepted by the other surface

•  $J$ : emitted radiation + reflected radiation

$F_{12}$  = how much does surface 1 "see" surface 2.



i) inspection

ii) Reciprocity rule:  $A_1 F_{12} = A_2 F_{21}$

iii) Summation rule  $\sum_{j=1}^N F_{ij} = 1$  for an enclosure w/  $N$  surfaces

iv) Symmetry rule: if surfaces 2 & 3 are symmetric about surface 1  $F_{12} = F_{13}$

$N$  surfaces  $\Rightarrow N^2$  view factors but only  $\frac{N(N-1)}{2}$  of them are needed directly

# Notes 4-26 continued:

## 1. Integration:

See tables 13.1-13.2 Figures 13.4-13.6 for selected geometries

## 2. View factor Algebra

(i)   $\bar{F}_{11} = 0$   $\bar{F}_{22} = 0$   
 $\bar{F}_{12} = 1 = \bar{F}_{21}$

(ii)



$$\bar{F}_{11} = 0 \quad A_1 \bar{F}_{12} = A_2 \bar{F}_{21}$$

$$\bar{F}_{12} = 1 \Rightarrow \bar{F}_{21} = \frac{A_1}{A_2} = \frac{D_1}{D_2}$$

$$\bar{F}_{21} + \bar{F}_{22} = 1 \Rightarrow \bar{F}_{22} = 1 - \frac{A_1}{A_2}$$

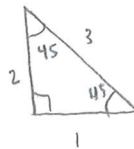
(iii)



$$\bar{F}_{12} = \bar{F}_{13} = \frac{1}{2}$$

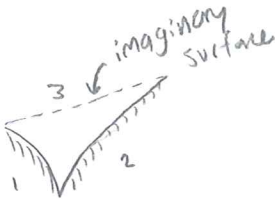
$$\bar{F}_{21} = \bar{F}_{23} = \frac{1}{2} \dots$$

(iv)



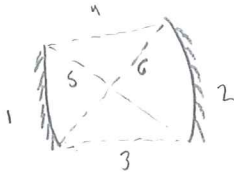
9 view factors?  
 • find these

v)



$$A_1 \bar{F}_{12} = A_2 \bar{F}_{21} = \frac{A_1 + A_2 - A_3}{2}$$

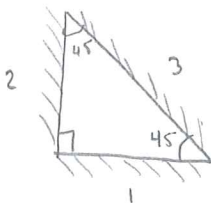
vi)



$$A_1 \bar{F}_{12} = A_2 \bar{F}_{21} = \frac{\sum (\text{Crossed strings}) - \sum (\text{uncrossed strings})}{2}$$

$$= \frac{(A_5 + A_6) - (A_3 + A_4)}{2}$$

4-28-2011



$$\bar{F}_{11} + \bar{F}_{12} + \bar{F}_{13} = 1$$

$$\bar{F}_{31} + \bar{F}_{32} + \bar{F}_{33} = 1$$

$$\bar{F}_{31} = \bar{F}_{32} = 1/2$$

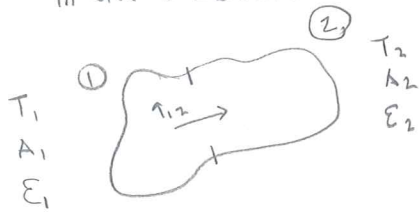
$$A_1 \bar{F}_{13} = A_3 \bar{F}_{31} = A_3 (1/2)$$

$$\bar{F}_{13} = \frac{A_3}{2A_1}$$

~~$A_1 \bar{F}_{12} = A_2 \bar{F}_{21}$~~   $\Rightarrow \dots$  stopped taking notes

$$\bar{F}_{21} = \frac{2A_1 - A_3}{A_2} \quad F$$

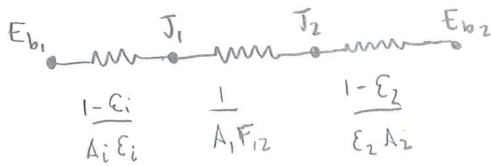
Radiation Exchange Between Two Diffuse, Gray, opaque surfaces in an enclosure



$$q_{12} = \frac{\sigma (T_1^4 - T_2^4)}{\frac{1-\epsilon_1}{A_1 \epsilon_1} + \frac{1}{A_1 F_{12}} + \frac{1-\epsilon_2}{A_2 \epsilon_2}}$$

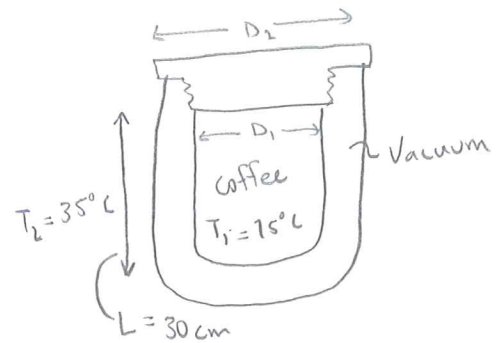
$R_i = \frac{1-\epsilon_i}{A_i \epsilon_i}$ ; surface resistance to radiation, a measure of departure from black-body ( $\epsilon=1 \Rightarrow R=0$ )

$R_{12} = \frac{1}{A_1 F_{12}}$ : a measure of how two surfaces "see" each other



ex) Thermos bottle 13.06, but with vacuum treat as long concentric cylinder

$$q_{12} = \frac{\sigma \pi D_1 L (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1-\epsilon_2}{\epsilon_2} \left( \frac{r_1}{r_2} \right)} = 3.2 \text{ W}$$



Thickness of an equivalent conduction insulation (say,  $k_{\text{ork}} = 0.04 \frac{\text{W}}{\text{mK}}$ )

$$q_{12} = 3.2 = \frac{T_1 - T_2}{\left( \frac{\ln r_2/r_1}{2\pi k L} \right)} \Rightarrow \ln \frac{D_2}{7} = \dots \quad D_2 = 18 \text{ cm}$$

— Radiation shields —

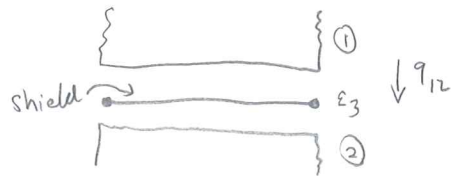
$$\epsilon \downarrow \Rightarrow \frac{1-\epsilon}{A_i \epsilon_i} \uparrow \Rightarrow q_{\text{rad}} \downarrow$$

special case: multiple shields w/ same emissivity:  $(q_{12})_{N \text{ shields}} = \frac{1}{N+1} (q_{12})_{0 \text{ shields}}$



Ex. 13.56)

$$\epsilon_1 = \epsilon_2 = 0.8 \quad F_{13} = F_{12} = 1$$



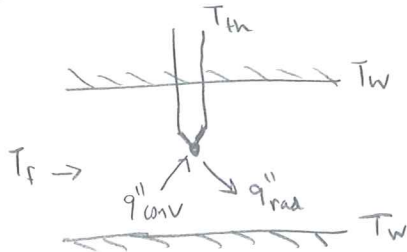
$$\epsilon_3 = ?$$

$$(q_{12})_{w/o \text{ shield}} = \frac{A_1 \sigma (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

$$(q_{12})_{w/shield} = \frac{A_1 \sigma (T_1^4 - T_2^4)}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1\right) + \left(\frac{1}{\epsilon_3} + \frac{1}{\epsilon_3} - 1\right)} \quad (\text{pg 793})$$

$$\frac{q_{w/shield}}{q_{w/o \text{ shield}}} = \frac{1}{10} = \frac{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1\right)}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} + \frac{2}{\epsilon_3} - 2\right)} \Rightarrow \epsilon_3 = 0.138$$

→ Radiation Correction → on temperature measurements



$$q_{conv} = q_{rad}$$

$$h(T_f - T_{th}) = \epsilon_{th} \sigma (T_{th}^4 - T_w^4)$$

$$T_f = T_{th} + \underbrace{\frac{\epsilon_{th} \sigma (T_{th}^4 - T_w^4)}{h}}_{\text{radiation correction}}$$

• try to minimize  $\epsilon$



11.2 (a,b), 18, 22, 34(a,b,c), 35(a)

11.2

$$D_i = 22 \text{ mm} \quad R_{f,i}'' = .0004 \text{ m}^2 \text{ K/W}$$

$$D_o = 27 \text{ mm} \quad R_{f,o}'' = .0002 \text{ m}^2 \text{ K/W}$$

a) determine overall h.t. coefficient,  $U_o$ 

$$U_o = A_o \left[ \frac{1}{h_i A_i} + \frac{R_{f,i}''}{A_i} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{R_{f,o}''}{A_o} + \frac{1}{h_o A_o} \right]^{-1}$$

$$\frac{\ln(D_o/D_i)}{2\pi k L} = .00216$$

 $h_o$ :

$$Re_{D,o} = \frac{V_o D}{\nu} = \frac{20 \text{ m/s} (0.027)}{1.589 \times 10^{-6}} = 33984$$

$$Nu_{D,o} = C = .26 \quad m = 0.6$$

pg. 431  
(7.46)  
why?

$$Nu_D = 0.3 + \frac{0.62 Re_D^{1/2} Pr_o^{1/4}}{\left[1 + (0.4/Pr_o)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5}$$

$$Nu_{D,o} = 103 \quad \bar{h}_o = \frac{Nu_{D,o} k}{D} = 104 \text{ W/m}^2 \text{ K}$$

$$\frac{1}{U_o} = A_o \left[ \begin{array}{ccccc} R_{cv,i} & R_{f,i} & R_w & R_{f,o} & R_{cv,o} \\ .00429 & .00579 & .00216 & .00236 & .1134 \end{array} \right]^{-1}$$

$\swarrow$  convection       $\swarrow$  fouling       $\swarrow$  conduction

$$R_{tot} = 0.126$$

$$U_o = 92.1$$

$$57.8$$

$$R_{cv,o,2} = .002689$$

$$A_o = .0348$$

fouling contributes resistance as large as the wall and convection resistances. Convection resistance for air is the greatest.

Part b) on back

K =

$$\text{Water } T_{m,i} = 75^\circ \text{C} \approx 350 \text{ K}$$

$$U_{m,i} = 0.5 \text{ m/s}$$

$$Re = \frac{U_m D}{\nu} = \frac{0.5 (0.022)}{(1.027 \times 10^{-7}) (365 \times 10^6)} = 29,345$$

$$Pr = 2.29$$

$$Nu_d = .023 Re^{4/5} Pr^{0.3} = 111$$

$$\bar{h}_i = \frac{Nu k}{D} = \frac{111 (.668)}{0.022} = 3370$$

302 stainless steel

 $C_p =$ 

$$k = 15.1 \text{ W/m} \cdot \text{K}$$

Air  $15^\circ \text{C} \sim 300 \text{ K}$ 

$$\nu = 1589 \times 10^{-6}$$



David Maloney

11.2) b) cross flow water,  $15^\circ\text{C}$ ,  $V_o = 1\text{m/s}$   $Pr = 7.56$   $k = .598$   $D_o = .027\text{m}$

$$Nu_{D,o} : Re = \frac{V_o D}{\mu} = \frac{1(.027)}{(1.001) \times 10^{-3} (1080) \times 10^{-6}} \Rightarrow Re = 24,975 \quad 31, \times \times \times$$

$$Nu_D = CR_{Re}^m Pr^{1/4} = 198 \quad \text{Table 7.2}$$

$$\bar{h}_o = \frac{Nu_{D,o} k}{D_o} = 4385 \Rightarrow Re_{v,o} = \frac{1}{h_o A_o} = .002689 \Rightarrow \boxed{U = 681} \quad \checkmark$$

11.18/

concentric tube,

$A_s = 50\text{m}^2$   
Hot fluid

Cold

a) find  $T_{h,o}$

heat capacity rate  $\text{kJ/k}$

$T_i, ^\circ\text{C}$

60

30

$T_o, ^\circ\text{C}$

48

54

$$q = C_h (T_{h,i} - T_{h,o}) = C_c (T_{c,o} - T_{c,i}) \Rightarrow \boxed{T_{h,o} = 48^\circ} \quad \checkmark$$

b) system is in counterflow because  $T_{h,o} < T_{c,o}$

$$c) \text{ find } U, \quad U = \frac{q}{A \Delta T_{lm}} \quad \Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = 19.4 \Rightarrow \boxed{U = .0742} \quad \text{check}$$

$$d) \text{ effectiveness, } \varepsilon = \frac{q}{q_{\max}} = \frac{72}{90} \quad C_{\min} (T_{h,i} - T_{c,i}) = 3(60 - 30) = 90$$

$$\boxed{\varepsilon = 0.80} \quad \checkmark$$

e) effectiveness if length were very long?

$$q = C_c (T_{c,o} - T_{c,i}) \quad \& \quad T_{c,o} \rightarrow T_{h,i}$$

$$\Rightarrow q \rightarrow C_c (T_{h,i} - T_{c,i})$$

$$\Rightarrow \boxed{\varepsilon \rightarrow 1} \quad \checkmark$$

# David Maloney HW 9

11.22, 34(a,b,c), 35(a)

11.22 — Shell & Tube H.X. — water oil  
 2.5 kg/s of water  $T_{ci} = 15^\circ\text{C}$   $T_{hi} = 160^\circ\text{C}$   
 $N = 10$   $T_{co} = 85^\circ\text{C}$   $T_{ho} = 100^\circ\text{C}$   
 thin walled  
 $D = 25\text{ mm}$  — what is flow rate of oil?  
 tubes make 8 passes ea.

$$\dot{Q}_c (T_{co} - T_{ci}) = \dot{Q}_h (T_{hi} - T_{ho})$$

$$2.5(4.182)(85 - 15) = \dot{m}_{oil} (2.337)(160 - 100)$$

$$\dot{m}_{oil} = 5.22 \text{ kg/sec}$$

— How long must tubes be?  $NTU = \frac{UA}{\dot{Q}_{min}}$

$$\frac{1}{UA} = A$$

Oil  
 $h_o = 400 \text{ W/m}^2$  outside tubes  
 $c_p @ 130^\circ\text{C} = 2.337$  (p 857)

Water  
 $c_p @ 325\text{K} = 4.182 \times 10^3$   
 $\mu = 528 \times 10^{-6}$   
 $Pr = 3.42$   
 $k = .645$

$$\epsilon = \frac{q}{\dot{Q}_{max}} = \frac{C_c (T_{co} - T_{ci})}{C_{min} (T_{hi} - T_{ci})} = \frac{732}{1515}$$

$$\epsilon = .483$$

$$\frac{\dot{Q}_{min}}{\dot{Q}_{max}} = 0.857$$

fig 11.12  
 $NTU = 1$

$$\frac{1}{UA} = \left[ \frac{1}{h_i A_i} + \frac{1}{h_o A_o} \right]$$

$$hi \quad Re = \frac{V_m D}{\nu} = \frac{4\dot{m}}{\pi D \mu} = 24114 \quad \overline{Nu} = .023 Re_p^{4/5} Pr^{0.4} = 121$$

$$h_i = \frac{\overline{Nu} k}{D} = 3122 \quad \Rightarrow \frac{1}{U} = \left[ \frac{1}{h_i} + \frac{1}{h_o} \right] \Rightarrow U = 355$$

$$h_o = 400$$

$$\frac{NTU}{U} (\dot{Q}_{min}) = 10 \pi (.025) L \quad \Rightarrow \quad \frac{1}{355} (2.5)(4.182) \frac{1}{(10 \pi (.025))} = L = 37.5 \text{ m}$$

11.34 | a, b, c      cross-flow  
hx      5 liter/min

$$U = 750 \text{ W/m}^2\text{K}$$

a) exchanger heat transfer rate?

$$q = \dot{m}_b C_{p_b} (T_i - T_o)$$

blood

$$T_{hi} = 37^\circ\text{C}$$

$$T_{ho} = 25^\circ\text{C}$$

$$\rho = 1050 \text{ kg/m}^3$$

$$C_p = 3740 \text{ J/kg}\cdot\text{K}$$

ice bath

$$T_{ci} = 0^\circ\text{C}$$

$$T_{co} = 15^\circ\text{C}$$

$$C_p = 4.198 \text{ kJ/kg}\cdot\text{K}$$

$$= .005 \text{ m}^3/\text{min} \left( \frac{1 \text{ min}}{60 \text{ s}} \right) (1050 \text{ kg/m}^3) (3740) (37 - 25)$$

$$q = 3927 \text{ J/s} = \boxed{3.927 \text{ kW}} \quad \checkmark$$

b) water flow rate       $\dot{m}_w C_{p_w} (15) = 3.927 \text{ kW}$

$$\dot{m}_w = \boxed{.0624 \text{ kg/s}}$$

c) find  $A_s$ :  $q = UA \Delta T_{\text{em}}$

$$\frac{C_{\min}}{C_{\max}} = \frac{\dot{m}_w C_{p_w} 262}{\dot{m}_b C_{p_b} 327.3} = .800$$

$$\varepsilon = \frac{q}{q_{\max}} = \frac{3.927 \text{ kW}}{262(37-0)} = 0.405$$

Fig 11.14

NTU = 0.7

$$NTU = \frac{UA}{C_{\min}} \Rightarrow A_s = \frac{0.7(262)}{750}$$

$$\boxed{A_s = 0.245 \text{ m}^2} \quad \checkmark$$

11.35

Steam (sat) 0.14 bar, condensing  $C_s \rightarrow \infty$

— Shell & Tube —

- 1 shell pass, 2 tube passes
- 130 brass tubes, 2m per pass

find  $U, T_{c,o}, \dot{m}_{\text{condense}}$

$$hi: Re = \frac{\rho V_{\infty} D}{\mu} = \frac{1}{1005 \times 10^{-3}} \frac{1.25 (0.0134)}{769 \times 10^{-6}} = 21,673$$

$$Nu_D = 0.023 Re^{0.8} Pr^{0.4} = 130.9$$

$$hi = \frac{k_f Nu_D}{D} = 6057$$

$$\frac{1}{U_{A_0}} = \left[ \frac{1}{hi A_i} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{1}{h_o A_o} \right]$$

$$\frac{1}{U_o} = \left[ \frac{1}{hi A_i} + \frac{\ln(D_o/D_i) \pi D_o L}{2\pi k L} + \frac{1}{h_o} \right] = \left[ \frac{1}{6057 \left( \frac{159}{134} \right)} + \frac{\ln \left( \frac{159}{134} \right) \cdot 0.0159}{2 (114)} + \frac{1}{13.5 \times 10^3} \right]$$

$$U_o = 3549 \text{ W/m}^2 \text{ K}$$

$$T_{c,o}: q = \epsilon q_{\max} \quad q_{\max} = C_{\min} (T_{hi} - T_{ho})$$

$$\dot{m}_c = 1.25 \text{ m/s} (\pi \cdot 0.0134) (130 \text{ tubes}) \left( \frac{1}{1005 \times 10^{-3}} \frac{\text{kg}}{\text{m}^3} \right) = 22.8 \text{ kg/sec}$$

$$C_c = 22.8 \text{ kg/sec} (4178 \text{ J/kg}) = 95,258 = C_{\min}$$

$$q = C_c (T_{c,o} - T_{c,i}) =$$

$$NTU = \frac{U_o A_o}{C_{\min}}, \quad A_o = \pi (0.0159) (2 \text{ m}) (130) (2 \text{ pass}) = 26.0 \text{ m}^2$$

$$NTU = .969 \Rightarrow \text{Fig 11.12} \Rightarrow \epsilon = 0.61$$

$$\epsilon = 1 - e^{-NTU} \Rightarrow \epsilon = 0.621$$

$$T_{c,o} = \frac{q}{C_c} + T_{c,i} =$$

$$T_{c,o} = 39.8^\circ \text{C}$$

$$q = \epsilon C_c (T_{hi} - T_{c,i}) \Rightarrow q = 1.89 \times 10^6$$

$$\dot{m}_{\text{condense}} = \frac{\dot{m}_c C_p (T_{hi} - T_{hi})}{h_{fg}} = 0.793 \text{ kg/s}$$

Cooling  $H_2O$

$$T_{c,i} = 20^\circ \text{C}$$

$$T_{c,o} =$$

$$\dot{m} =$$

$$V = 1.25 \text{ m/s}$$

$$h_o = 13.5 \text{ kW/m}^2 \text{ K}$$

$$\text{Water guess } \sim 305 \text{ K}$$

$$\rho = \frac{1}{1.005}$$

$$Pr = 5.2$$

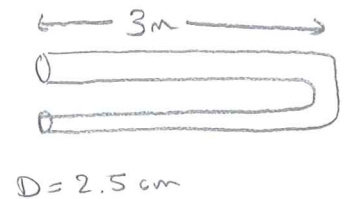
$$k = .620$$

$$C_p = 4178$$

# EXAM 3 2008

#1) Shell & tube. 2 passes, 1 shell  
 condensing steam,  $C_s = \infty$   
 $P = 1 \text{ bar}$   
 $T = 373 \text{ K}$   
 $h = 2000 \text{ W/m}^2\text{K}$

tube side:  
 Water  
 $T_i = 270 \text{ K}$   
 $T_o = 320 \text{ K}$   
 $\dot{m} = .05 \text{ kg/s}$   
 $h = 500 \text{ W/m}^2\text{K}$



find: fouling factor

$$\frac{1}{UA} = \left[ \frac{1}{h_i A} + \frac{R''_f}{A} + \frac{1}{h_o A} \right] = \frac{1}{U} = \left[ \frac{1}{h_i} + R''_f + \frac{1}{h_o} \right]$$

$$q_{\text{per tube}} = \frac{\dot{m} C_p (T_o - T_i)}{1}$$

$$= .05 (4181) (320 - 270)$$

$$= 5774 \text{ W}$$

$$\epsilon = 1 - e^{(-NTU)}$$

$$q_{\text{max}} = .05 (4181) (373 - 270) = 21110 \text{ W}$$

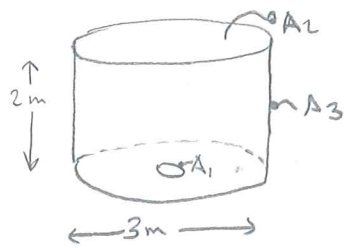
$$\epsilon = .485 \quad NTU = .664$$

$$NTU = \frac{UA}{\dot{q}_{\text{min}}} \Rightarrow .664 = \frac{U \cdot 6 \times \pi (0.025)}{.05 (4181)} \quad U = 294.6$$

$$.00339 = \left[ \frac{1}{500} + R''_f + \frac{1}{2000} \right] \quad R = .00089$$

# Homework

13.19] - 3 black surfaces



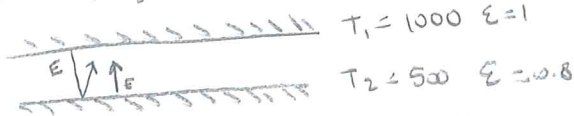
find  $F_{13}$ , net  $q''$  from  $A_1 \rightarrow A_3$   $A_1 = 0.05 \text{ m}^2$   
 $T_1 = 1000 \text{ K}$   
 $T_3 = 500 \text{ K}$

$$F_{13} + F_{12} = 1 \quad F_{12} = \frac{D^2}{D^2 + 4L^2} \quad (P781) \quad F_{12} = 0.36 \quad \boxed{F_{13} = 0.64}$$

$$\text{net } q'' : q_{13} = \frac{\sigma(T_1^4 - T_3^4)}{\frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1 F_{13}} + \frac{1-\epsilon_3}{\epsilon_3 A_3}} = \frac{\sigma(1000^4 - 500^4)}{\frac{1}{0.05(0.55)}}$$

$$\boxed{q_{13} = 1701 \text{ W}}$$

13.41] 2 large diffuse, gray surfaces



Find:

- $G_1$  &  $J_1$
- net  $q''_{\text{rad}}$

$$G_1 = E_2 + G_{1p2} = \epsilon_2 \sigma T_2^4 + (\epsilon_1 \sigma T_1^4) p_2$$

$$= \boxed{14175 \text{ W/m}^2}$$

$$J_1 = E_1 + E_2 p_1 = \epsilon_1 \sigma T_1^4 =$$

$$= \boxed{56700 \text{ W/m}^2}$$

$$q_{1 \rightarrow 2} = G_1 - J_1 = \boxed{42525 \text{ to } A_1}$$

# HW 9 — Exam Practice —

11.22)

Shell-and-tube HX

10 tubes

shell side

engine oil  $\dot{m} =$

Water:  $\dot{m} = 2.5 \text{ kg/s}$

$T = 320 \text{ K}$

$T_i = 15^\circ \text{C}$

$T_o = 85^\circ \text{C}$

$C_p = 4180$

$\nu = 1.011 \text{ m}^2/\text{s} \times 10^{-3}$

$\mu = 577 \times 10^{-6}$

$k = .640$

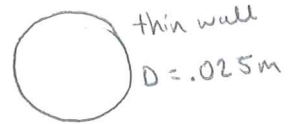
$Pr = 3.77$

$T_i = 160^\circ \text{C}$

$T_o = 100^\circ \text{C}$

$h_o = 400 \text{ W/m}^2\text{K}$

$C_p = 2340$



8 passes

$$\dot{m}_{\text{oil}}? \quad 2.5 \text{ kg/s} (4180)(85-15) = \dot{m}_{\text{oil}} (2340)(160-100) \Rightarrow \boxed{\dot{m}_{\text{oil}} = 5.21 \text{ kg/s}}$$

$$h_i = \bar{Nu}_i k$$

$$V = \frac{2.5 \text{ kg/s} (1.011 \text{ m}^2/\text{s})}{10 \times \pi \times \frac{(0.025)^2}{4}} = 0.514 \text{ m/s}$$

$$Re_D = \frac{0.514 \rho D}{\mu} = 22,028$$

$$\bar{Nu}_D = 0.023 Re_D^{0.8} Pr^{0.4} = 117$$

$$h = \frac{\bar{Nu}_D k}{D} = 3000$$

$$NTU = \frac{UA}{C_{\min}}$$

$$\varepsilon = \frac{q}{q_{\max}}$$

$$\varepsilon = .481$$

$$q_{\max} = C_{\min} (T_{hi} - T_{lc}) = 2.5(4180)(160-15)$$

$$q_{\max} = 1.52 \times 10^6 \text{ W}$$

$$q = 731,500$$

$$\frac{C_{\min}}{C_{\max}} = .857$$

$$NTU \approx 1 \text{ (Table 11.12)}$$

$$U = \frac{1}{\frac{1}{h_c} + \frac{1}{h_h}} = 353$$

$$1 = \frac{353 \times 10 \times \pi \times 0.025 \times L}{(2.5)(4180)}$$

$$\boxed{L = 37.7 \text{ m}}$$



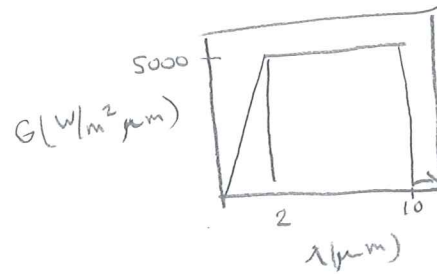
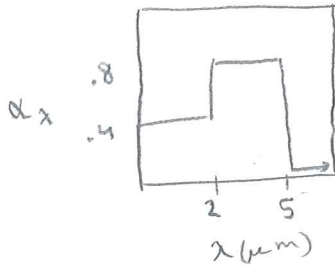
HW 10

12.51)

Find

- total absorptivity,
- Emissive Power
- Surface temp going  $\uparrow$  or  $\downarrow$ ?

$$T_{\text{surface}} = 1250 \text{ K}$$



$$\alpha = \frac{G_{\text{abs}}}{G} = \frac{0.4 \left( \frac{2 \times 5000}{2} \right) + 0.8 (5 - 2)(5000)}{\frac{2 \times 5000}{2} + 8 \times 5000} = \frac{14,000}{45,000} = \boxed{0.311}$$

$$\epsilon = 0.4 (F_{0 \rightarrow 2}) + 0.8 (F_{2 \rightarrow 5}) = 0.4 (.161) + 0.8 (.754 - .161) = \boxed{0.5388}$$

$$E = \epsilon E_b = 0.54 \sigma T^4 = 74,750 \text{ W/m}^2$$

$$q_{\text{net}} = G_{\text{abs}} - E = 14,000 - 74,750 = -60,750$$

Temperature is dropping

HW 10

$$G_{\text{solar}} = 900 \text{ W/m}^2$$

$$\alpha = 0.9$$

$$h = 20 \text{ W/m}^2 \cdot \text{K}$$

$$\epsilon = 0.1$$

$$T_{\text{sur}} = 290 \text{ K}$$

Find:  $T_{\text{ss}}$



$$\text{q-balance: } G_{\text{abs}} - E - q_{\text{conv}} = 0$$

$$G_{\text{abs}} = \alpha (G_{\text{solar}} + G_{\text{surroundings}})$$

$$E = 0.1 \sigma T^4$$

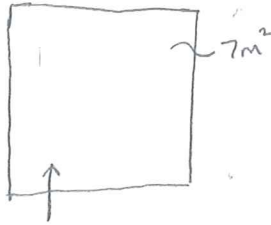
$$q_{\text{conv}} = h(T_s - T_{\text{air}})$$

$$0.9(900 \text{ W/m}^2) + (0.1)(\sigma)(290)^4 = 0.1 \sigma T^4 + 20(T - 290 \text{ K})$$

$$\Rightarrow T = 329 \text{ K}$$

# Exam 3 2008

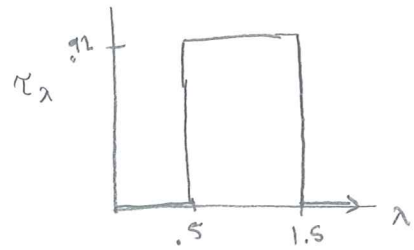
#2, car window



$$G_{\text{solar}} = 400 \text{ W/m}^2$$

find  $G_{\text{trans}}$

- $E_{\text{interior}}$  that is transmitted
- net rad trapped in car in 1 hour



interior, blackbody, 300K

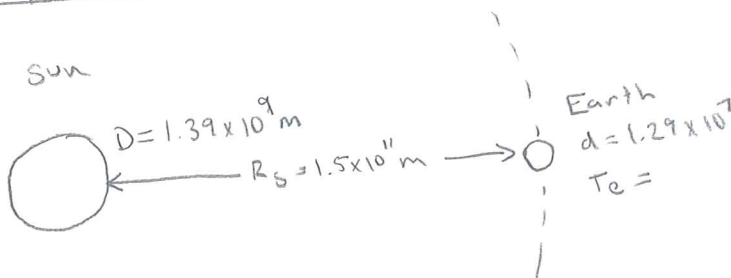
$$G_{\text{trans}}: \tau = 0.92 (F_{.5 \rightarrow 1.5}) = 0.92 (.880 - .250) = 0.580$$

$$G_{\text{trans}} = \tau (400) (7 \text{ m}^2) = 162 \text{ W}$$

$$E_{\text{trans}} = 0.92 (F_{(1.5 \times 300)} - F_{(0.5 \times 300)}) = (.92) \times 0$$

$$\text{net heat trapped in car} = 162 \text{ W} = 162 \text{ J/s} \times (3600 \text{ s}) = 583 \text{ kJ}$$

Example 12.20



Sphere Surface Area:

$$4\pi r^2 = \pi D^2$$

$$\text{solar flux @ edge of earth atmosphere} = 1353 \text{ W/m}^2$$

$$E_s (\pi D_s^2) = q_{\text{earth}}'' 4\pi (R_{\text{se}})^2$$

$$T_s = 5774$$

# Exam 3 2005

#3)

long oven

$$F_{12}, F_{21}, F_{13}, F_{23,34}$$

$$F_{12}: F_{41} = F_{42} = \frac{1}{2} \text{ Symmetry}$$

$$F_{14} = \frac{A_4}{A_1} F_{41} = 1.73 \left(\frac{1}{2}\right) = .865$$

$$\cos(30) = \frac{L_4}{2L_1} = 1.73$$

$$F_{12} + F_{14} = 1 \Rightarrow$$

$$F_{21} = F_{12} = 0.135 \checkmark$$

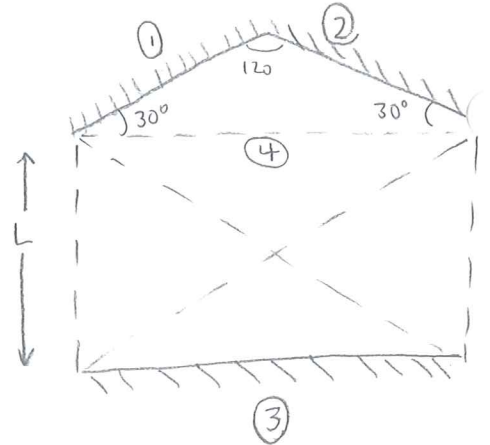
$$F_{43} = \frac{W_i = 1 \quad W_j = 1}{F_{ij} = \frac{(2^2 + 4)^{1/2} - (0^2 + 4)^{1/2}}{2(1)}} \quad F_{43} = 0.414$$

$$F_{34} = .414 \quad F_{31} = F_{32} = 0.207$$

$$F_{13} = \frac{A_3}{A_1} F_{31} = 1.73 (.207) =$$

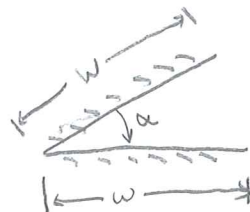
$$0.358 = F_{13}$$

$$F_{23} = .358$$



— Finding  $F_{14}$  —

Table 13.1 pg 775

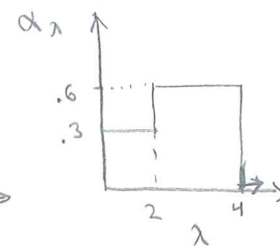
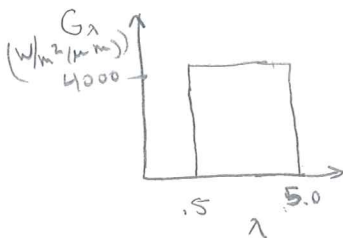
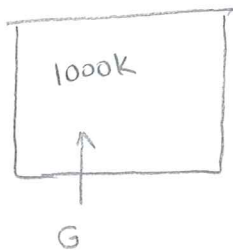


$$F_{ij} = 1 - \sin\left(\frac{\alpha}{2}\right)$$

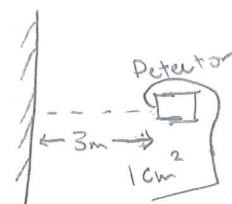
MAY 2008

#2

$2 \text{ m}^2 \quad T = 727^\circ\text{C}$



surf



Find:  $\alpha, \rho, \epsilon$

$$G_{\text{abs}} = 0.3(4000)(2 - 0.5) + 0.6(4000)(4 - 2) = 6600 \text{ W/m}^2$$

$(\text{W/m}^2 \cdot \mu\text{m})(\mu\text{m})$

$$\alpha = \frac{G_{\text{abs}}}{G} \Rightarrow \alpha = \frac{6600}{18000} = \boxed{0.367}$$

$$\rho_\lambda = 1 - \alpha_\lambda \Rightarrow G_{\text{ref}} = 0.7(4000)(2 - 0.5) + 0.4(4000)(4 - 2) + 1(4000)(5 - 4)$$

$$= 1140 \Rightarrow \rho = \frac{11400}{18000} = \boxed{0.633}$$

$$\rho = \frac{G_{\text{ref}}}{G} \Rightarrow$$

Find: Temp change over time:

$$\epsilon = \alpha = 0.367$$

$$E = \epsilon \sigma T^4: \epsilon = 0.3(F_{0 \rightarrow 2}) + 0.6(F_{2 \rightarrow 4})$$

$$= 0.3(.067) + 0.6(.481 - .067) = .269$$

$$E = .269(5.67 \times 10^{-8})(1000)^4 = 15,252$$

$$q_{\text{entering plate}} = G_{\text{abs}} - E = 6600 - 15,252 = \boxed{-8,650 \text{ plate is cooling}}$$

$$J = ? \quad J = G_{\text{ref}} + E = 11,400 + 15,252 = 26,650$$

Find: radiant power reaching detector

$$q_d = J A_s \frac{A_d}{\pi L^2} = 26,650 \frac{2}{\pi} \frac{.0001}{9} = \boxed{0.188 \text{ W}}$$

heat flux to the detector  $\uparrow$   
 $\text{W/m}^2(\text{m}^2) \left( \frac{\text{detector area}}{\text{alt area}} \right)$

surface of sphere  
 $= \pi D^2$

225]

Suggested problems for last material:

12.47, 13.1, 19, 41, 62

12.47] diffuse, opaque surface, 700 K  
 radiant flux of  $1000 \text{ W/m}^2$   
 uniform  $1 < \lambda < 6 \mu\text{m}$  is incident  
 at angle  $30^\circ$  to normal

$$\epsilon_\lambda = \begin{cases} 0.5 & 3 \mu\text{m} < \lambda \leq 10 \mu\text{m} \\ 0.9 & 10 \mu\text{m} < \lambda \leq \infty \end{cases}$$

total radiant Power = ? (received by detector)  
 for  $10^{-4} \text{ m}^2$  area

$$\text{aperture} = 10^{-5} \text{ m}^2, d = 1 \text{ m}$$

$$q_d = I_e + r A_s \cos \theta_d - s \Delta W_d - s$$

$$q_d = \frac{\epsilon \sigma T_s^4 + \rho G}{\pi} A_s \frac{A_d}{L^2}$$

$$\epsilon = \frac{\int_0^\infty \epsilon_\lambda E_{b\lambda}(700\text{K}) d\lambda}{E_b(700\text{K})} = 0.5(0.8081 - 0.0838) + 0.9(1 - 0.8081) = 0.535$$

$$\rho = \frac{\int_0^\infty \rho_\lambda G_\lambda d\lambda}{G} = \frac{\int_0^\infty (1 - \epsilon_\lambda) q''_\lambda d\lambda}{q''} = 1 \times F_{(0 \rightarrow 3 \mu\text{m})} + 0.5 \times F_{(3-6 \mu\text{m})}$$

fraction of blackbody radiation in wavelength band

$$= 1 \times (0.4) + 0.5(0.6) = 0.7 = \rho$$

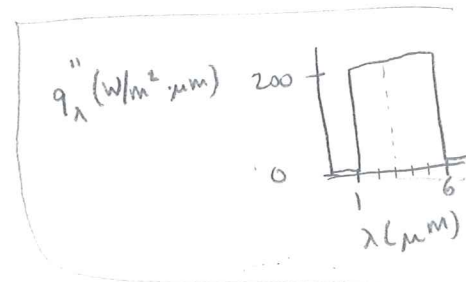
$$G = q'' \cos \theta_i = 1000 \cos(30^\circ) = 866 \text{ W/m}^2$$

$$q_d = \frac{\epsilon \sigma T_s^4 + \rho G}{\pi} A_s \frac{A_d}{L^2}$$

$\swarrow$  aperture ( $10^{-5} \text{ m}^2$ )  
 $\swarrow$  (1 m)

$$q_d = \frac{7889}{\pi} (1 \times 10^{-9}) = 2.51 \times 10^{-6} \text{ W}$$

Given:



$$\sigma = 5.67 \times 10^{-8}$$

13.1

a) long duct



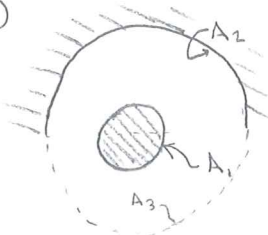
$$A_1 F_{12} = A_2 F_{21}$$

$$F_{12} = \frac{A_2}{A_1} F_{21} \quad \frac{A_2}{A_1} = \frac{\frac{3}{4}\pi D}{D} = \frac{3\pi}{4}$$

$$F_{11} + F_{12} = 1$$

$$F_{21} = \frac{4}{3\pi}(1) \Rightarrow F_{21} = \frac{4}{3\pi} = 0.424$$

b)



$$A_2 = 2A_1$$

$$\text{Symmetry} \quad F_{12} = F_{13} = \frac{1}{2}$$

$$\text{Summation: } F_{11} + F_{12} + F_{13} = 1$$

$$A_1 F_{12} = A_2 F_{21} \quad F_{21} = \frac{A_1}{A_2} F_{12} = \frac{1}{2} \left( \frac{1}{2} \right) = 0.25$$

$$F_{12} = \frac{1}{2}$$

$$F_{21} = 0.25$$

c)



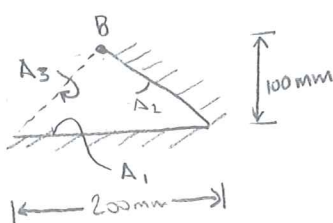
$$F_{22} = ?$$

$$F_{12} = 1 \quad F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\pi D^2/4}{\pi D^2/2} (1) \Rightarrow F_{21} = \frac{2}{\pi}$$

$$F_{22} = 1 - \frac{2}{\pi}$$

$$F_{22} = 0.363$$

d)



$$F_{12} + F_{13} = 1 \Rightarrow F_{12} = F_{13} = \frac{1}{2}$$

$$A_1 F_{12} = A_2 F_{21} \quad F_{21} = \frac{A_1}{A_2} F_{12} = \frac{200}{\sqrt{100^2 + 100^2}} \left( \frac{1}{2} \right) = 0.707$$

$$F_{21} = 0.707$$

$$F_{12} = \frac{1}{2}$$

e)

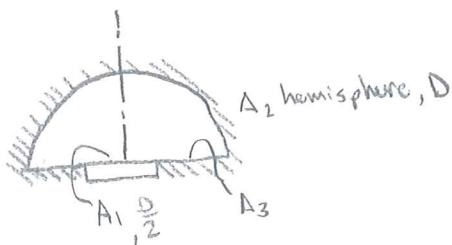


$$F_{13} + F_{12} = 1 \Rightarrow F_{12} = F_{13} = \frac{1}{2}$$

$$F_{21} = \frac{A_1}{A_2} F_{12} \rightarrow 0$$

$$F_{21} = 0$$

f)



$$F_{12} = 1 \text{ by obs.}$$

$$F_{32} = 1$$

$$F_{21} = \frac{A_1}{A_2} F_{12}$$

$$\frac{\pi(D/2)^2}{\frac{\pi D^2}{2}} =$$

$$= 0.125$$

$$F_{21} = 0.125$$

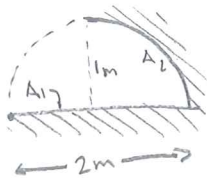
$$F_{23} = \frac{A_3}{A_2} F_{32} = 0.375$$

$$F_{23} = 0.375$$



13.1)

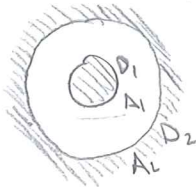
g) long, open channel  $F_{11} + F_{12} + F_{13} = 1$   
 $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{2 \times L}{\frac{(2\pi L)}{4} \times L} = \frac{4}{\pi} \times .50 = 0.637$



$$\boxed{F_{12} = 0.5}$$

$$\boxed{F_{21} = 0.637}$$

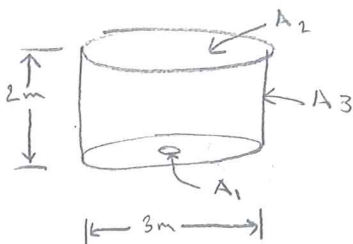
h) concentric cylinders (long)



$$F_{12} = 1$$

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\pi D_1}{\pi D_2} (1) \Rightarrow \boxed{F_{21} = \frac{D_1}{D_2}}$$

13.19



$$A_1 < A_2 \text{ \& } A_3 \quad A_1 = .05 \text{ m}^2$$

$$F_{12} + F_{13} = 1$$

$$F_{12} = \frac{D^2}{D^2 + 4L^2} \text{ (Eq 13.8) pg 781} \quad F_{12} = 0.36$$

$$F_{13} = 1 - F_{12} = \boxed{0.64}$$

$$r_1 = \left( \frac{A_1}{\pi} \right)^{1/2} = \left( \frac{.05 \text{ m}^2}{\pi} \right)^{1/2} = .126$$

Net radiation H.T. from  $A_1$  to  $A_3$   $T_1 = 1000 \text{ K}$   $T_3 = 500 \text{ K}$  Black surfaces,  $\epsilon = 1$

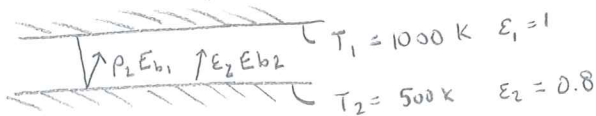
$$q_i = \frac{E_{bi} - J_i}{(1 - \epsilon_i) / \epsilon_i A_i} \quad \text{Black surfaces}$$

$$q_i = A_1 F_{13} \sigma (T_1^4 - T_3^4)$$

$$= .05 \text{ m}^2 (0.64) (5.67 \times 10^{-8}) (1000^4 - 500^4) = \boxed{1700 \text{ W}}$$

13.41)

diffuse, gray surfaces



Find:

- irradiation & radiosity for upper plate?
- radiosity for lower plate? (J)
- net radiation exchange?

$$G_1 = \epsilon_2 E_{b2} + \rho_2 E_{b1} = \epsilon_2 \sigma T_2^4 + (1 - \epsilon_2) \sigma T_1^4$$

$$G_1 = .8(5.67 \times 10^{-8})(500^4) + (0.2)(5.67 \times 10^{-8})(1000^4) = \boxed{14175 \text{ W/m}^2}$$

$G_1$ , irradiation =  
flux emitted by  
surface 2 + reflected  
flux emitted by  
surface 1

Radiosity = radiant flux leaving the surface by emission & reflection

$$J_1 = E_{b1} = \sigma T_1^4 = \boxed{56,700 \text{ W/m}^2}$$

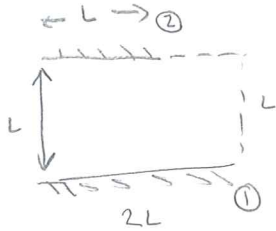
$$J_2 = \epsilon_2 E_{b2} + \rho_2 G_2 = 0.8(\sigma T_2^4) + (0.2)(\sigma T_1^4) = \boxed{14,175 \text{ W/m}^2}$$

$$\begin{matrix} \checkmark & \checkmark & \checkmark \\ (\rho = 1 - \epsilon) & G_2 = E_{b1} \text{ (blackbody)} \end{matrix}$$

$$\text{net radiation exchange} = \frac{56,700 - 14,175}{J_1 - J_2} = \boxed{42,525 \text{ towards } A_2} \text{ (W/m}^2\text{)}$$

Exam A

3)

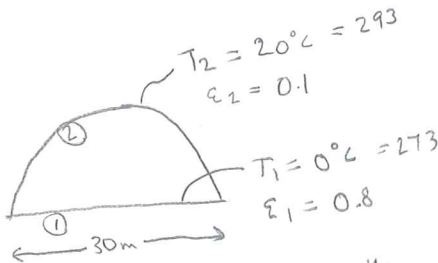


$$F_{11} = \frac{((2+2)^2 + 4)^{1/2} - (4)^{1/2}}{2(2)} = 0.618$$

$$F_{12} = \frac{.618}{2} = .309$$

$$F_{21} = \frac{A_2}{A_1} F_{12} = .618$$

ii)

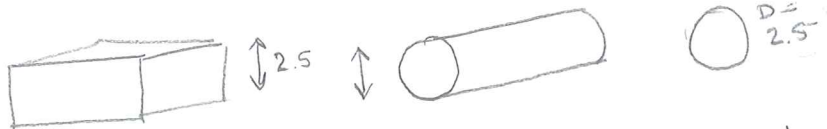


$$F_{12} = 1$$

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{1}{\frac{\pi(1)}{2}} (1) = .636$$

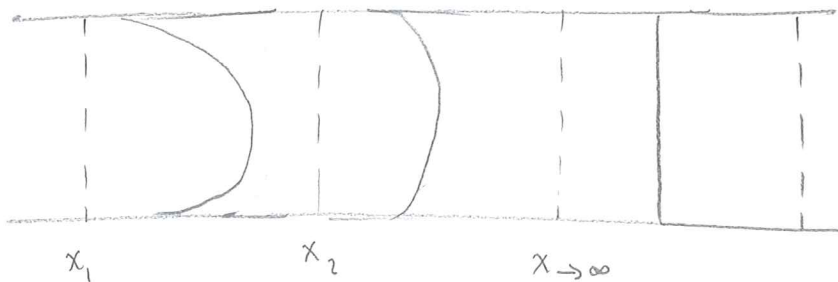
$$q_{12} = \frac{\sigma (273^4 - 293^4)}{\frac{0.2}{30(.8)} + \frac{1}{30} + \frac{0.9}{\pi(1)(.1)}} = -442 \text{ W}$$

4) i)



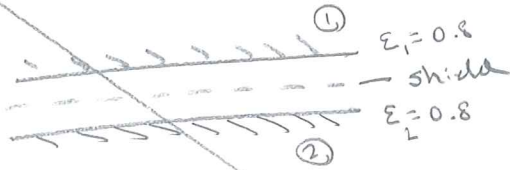
Sphere will cool fastest because it has the shortest distance for heat to escape in all directions. smallest volume too.

ii



13.56

2 large, diffuse, gray surfaces

Find:  $\epsilon_{\text{shield}}$  to  
reduce  $q'' \times 10$  $q$  w/o shield = 10  $q$  w/ shield

(p792)

$$\frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\epsilon}{\epsilon} + \frac{1-\epsilon}{\epsilon}} = 10 \frac{\sigma(T_1^4 - T_2^4)}{\frac{2}{\epsilon} + \frac{2-2\epsilon_s}{\epsilon_s}}$$

$$\frac{1}{\frac{2-1.6}{.8}} = \frac{10}{\frac{2}{.8} + \frac{2-2\epsilon_s}{\epsilon_s}}$$

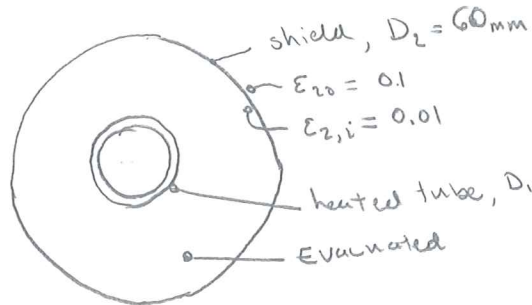
$$2 = \frac{10}{2.5} + \frac{10\epsilon_s}{2-2\epsilon_s}$$

$$-2 = \frac{10\epsilon_s}{2-2\epsilon_s}$$

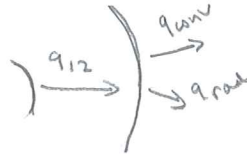
$$\Rightarrow \epsilon = 0.444$$

13.62

diffuse, gray radiation shield 60mm diam



Walls 17°C

convection, air  $h = 10 \text{ W/m}^2\text{K}$ shield  $T = 42^\circ\text{C}$  constant

$$q_{\text{radiation}} + q_{\text{convection}} = q_{12}$$

$$q_{12} = \frac{\sigma A_1 (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1 - \epsilon_2}{\epsilon_2} \left(\frac{r_1}{r_2}\right)^2} \quad (13.21)$$

$$q_{\text{convection}} = hA$$

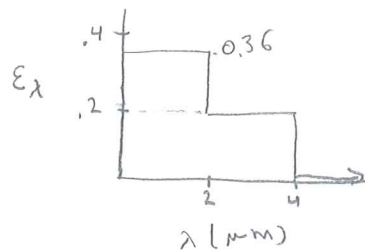
$$q_{\text{rad}} = \epsilon_{2,o} A_2 \sigma (T_2^4 - T_{\text{wall}}^4) \quad (13.22), \text{ pg 793}$$

$$\frac{\sigma D_1 (T_1^4 - T_2^4)}{1 + \frac{1 - \epsilon_2}{\epsilon_2} \left(\frac{D_1}{D_2}\right)^2} = \epsilon_{2,o} \sigma T_2^4 D_2 + h_o (T_2 - T_{\text{amb}}) D - \alpha_{2,o} \sigma T_{\text{wall}}^4 D$$

$$3.34 \times 10^{-11} (T_1^4 - T_2^4) = 3.35 + 9 - 2.41$$

$$\boxed{T_1 = 745 \text{ K}} \Rightarrow T_1 = 472^\circ\text{C}$$

12.32



a) Find: total hemi emissivity  $\epsilon$  & total emissive power  $E$  @ 2000 K

$\epsilon$  = average  $\epsilon_\lambda$  over all  $\lambda$ 's

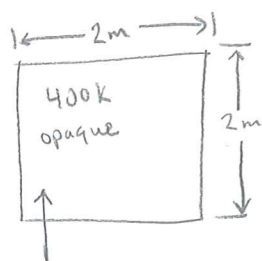
$$\epsilon(T) = F_{(0 \rightarrow 2)} + F_{(2 \rightarrow 4)} = 0.36(0.481) + 0.2(0.856 - 0.481)$$

$$\epsilon(T) = 0.248$$

$$\text{Total emissive power @ 2000 K} = \epsilon \sigma T^4 = 0.248 (5.67 \times 10^{-8}) (2000 \text{ K})^4$$

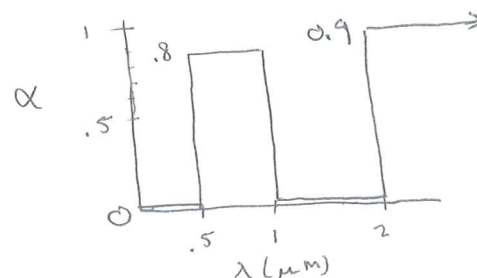
$$E = 225,000 \text{ W}$$

12.46



diffuse

- Find:
- absorbed irradiation,  $G_{abs}$
  - emissive power,  $E$
  - radiosity
  - net h.t.



$$G_{solar} = 1200 \text{ W/m}^2 \quad T_{sun} = 5800 \text{ K}$$

$$G_{abs} = [F_{(0 \rightarrow 1)} \alpha + F_{(1 \rightarrow \infty)} \alpha] G_{solar} \Rightarrow \alpha = 0.43$$

$$= [0.8(0.720 - 0.250) + 0.9(1 - 0.940)] (1200) * \text{Area}$$

$$= 516 \text{ W/m}^2 \times 4 \text{ m}^2 = 2064 \text{ W}$$

$$\text{Emissivity } \epsilon = \frac{\int_0^\infty \epsilon_\lambda E_{\lambda_b}(400) d\lambda}{E_b(400 \text{ K})} = (0.8)(F_{0 \rightarrow 1}) + (0.9)(F_{1 \rightarrow \infty}) = 0.9(1 - 0) = 0.9$$

$$\text{Emissive Power, } E = \epsilon \sigma T^4 = 1306 \text{ W/m}^2 \times 4 = 5224 \text{ W}$$

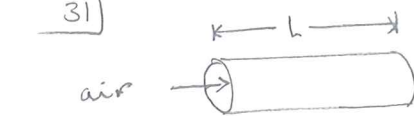
$$J, \text{ radiosity} = E + \rho G = 5224 + (1 - 0.43)(1200 \times 4 \text{ m}^2) = 7960 \text{ W}$$

$$q_{net, \text{ re-radiation heat trans}} = 2064 - 5224 = 3164 \text{ W} \quad (\text{heat leaving surface})$$

( $G_{abs} - J$ )

8.31, 50, 74, 93, 94

31)



$$T_{m,i} = 29^\circ\text{C} \quad T_{m,o} = 21^\circ\text{C}$$

$$V_i = 0.025 \text{ m}^3/\text{s}$$

Smooth surface

find L

$$k = 15 \text{ W/m}\cdot\text{K}$$

$$D_i = 0.15 \text{ m}$$

$$D_o = 0.17 \text{ m}$$

$$T_{\infty, \text{water}} = 17^\circ\text{C}$$

$$h_o \approx 15,000 \text{ W/m}^2\cdot\text{K} \text{ @ outer pipe surface}$$

$$V_{\text{air}, 25^\circ\text{C}} = 15.89 \times 10^{-6} \text{ m}^2/\text{s} \quad Pr = 0.707$$

$$\text{discharge temp } T_{m,o} = 21^\circ\text{C}$$

$$k_{\text{air}} = 26.3 \times 10^{-3} \text{ W/m}\cdot\text{K}$$

$$Re_D = \frac{U_m D}{\nu} = \frac{1.41 (0.15)}{15.89} = 13,310 > 2300, \text{ Turbulent}$$

$$\frac{V_i}{A} = \frac{0.025 \text{ m}^3/\text{s}}{\pi (0.15)^2/4} = U_m = 1.41 \text{ m/s}$$

$$Nu_{D,i} = 3.66 \Rightarrow h = \frac{Nu_D k_a}{D_h}$$

$$R_{\text{tot}} = \frac{1}{h_i A} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{1}{h_o A}$$

$$A = \pi D_o L$$

$$(.267 + .133 + 2.12 \times 10^{-5}) \frac{1}{L} = \frac{0.40}{L}$$

$$Nu_D = 0.023 Re_D^{4/5} Pr^{0.3} = 45.35 \quad \bar{h}_i = 7.95 \text{ W/m}^2\cdot\text{K}$$

$$(\bar{U} A_s)^{-1} = \frac{1}{L} (R_{\text{total}})$$

$$\frac{T_{\infty} - T_{m,o}}{T_{\infty} - T_{m,i}} = e^{\left(\frac{-A_s \bar{U}}{\dot{m} c_p}\right)} \quad \text{where } \bar{U} A_s = \frac{1}{R_{\text{total}}}$$

$$\ln\left(\frac{17-21}{17-29}\right) = -\frac{1}{R_{\text{tot}}} \left(\frac{1}{\dot{m} c_p}\right) =$$

$$-1.099 = -(2.5 \text{ L}) \left(\frac{1}{0.029 (1007)}\right)$$

$$L = 12.84 \text{ m}$$

Pump Power

$$\Delta P \cdot \dot{V} = \frac{f \rho_i U_{m,i}^2}{2 D_i} L \cdot \dot{V}_i$$

$$f = .316 (13,310)^{-1/4} = .0294$$

$$\rho \dot{V} = .0290$$

$$U_m = 1.41$$

$$L = 12.84 \text{ m}$$

$$D_i = 0.15 \text{ m}$$

$$\text{Power} = .073 \text{ W}$$

$$\dot{V} = \dot{m} RT$$

$$\dot{m} = \rho \dot{V} = .0290 \text{ kg/s}$$

8.50

134a

$$\dot{m} = 1 \text{ kg/s}$$

$$T = 240 \text{ K}$$

$$q' = \frac{(T_{\infty} - T_m)}{R_{\text{tot}}}$$

$$Re_{D,i} = \frac{\rho V D}{\mu} \quad (\text{Iterations}) = \frac{4 \dot{m}}{\pi D_i \mu} = 12126 \quad \text{Turbulent}$$

$$Nu = 0.023 Re_D^{4/5} Pr^{0.4} = 80.96$$

$$\bar{h}_i = \frac{Nu_D k}{D_i} = 347$$

$$\bar{h}_o \quad ② \quad Nu_{D,o} = C Re_{D,o}^m Pr^{1/3} \quad \text{Pg 431 lookup}$$

$$= 0.027 (44,050)^{0.805} (.707)^{1/3} = 131.7$$

$$① \quad Re_D = \frac{VD}{\nu} = \frac{25 \text{ m/s} (.028)}{15.89 \times 10^{-6}} = 44,050$$

$$③ \quad \bar{h}_o = \frac{k Nu_o}{D} = \frac{.0263 (132)}{.028} = 124 \text{ W/m}^2 \text{K}$$

$$q' = \frac{T_{\infty} - T_m}{\frac{1}{347 \pi (.025)} + \frac{\ln \frac{.28}{.25}}{2 \pi (.35)} + \frac{1}{124 \pi (.028)}}$$

$$q' = 333 \text{ W/m}$$

find q per unit length

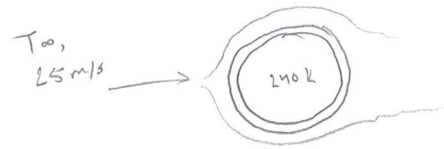
Air

$$V = 25 \text{ m/s}$$

$$T = 300 \text{ K}$$

$$Pr = .707$$

$$k = .0263$$



$$T_m \quad \frac{1}{h_i A} + \frac{\ln(D_o/D_i)}{2 \pi k} + \frac{1}{h_o A} \quad T_{\infty}$$

134a

$$\rho = 1398$$

$$C_p = 1.267$$

$$\mu = .042 \times 10^{-2}$$

$$\nu = .3006 \times 10^{-6}$$

$$k = .107$$

$$Pr = 5.0$$

Teflon

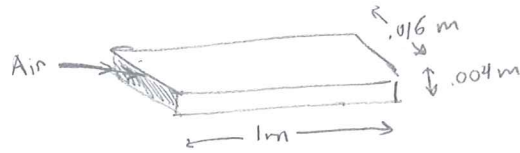
$$k = 0.35$$

$$= \frac{T_{\infty} - T_m}{.0367 + .0515 + .0917} = \frac{T_{\infty} - T_m}{.180} = \frac{300 - 240}{.180}$$



8.74

Air  
 $\dot{m} = 3 \times 10^{-4} \text{ kg/s}$   
 $T = 27^\circ\text{C} = 300\text{K}$



$q'' = 600 \text{ W/m}^2$   
 Temp of air & surface @ outlet?

$C_p = 1007$   
 $\mu = 184.6 \times 10^{-7}$   
 $k = 0.0263 \text{ W/mK}$   
 $Pr = 0.707$

$q = q'' \times A_s = 600 \times (1.004 \times 1 + 0.016 \times 1) \times 2 = 24 \text{ W}$

$T_{m,o} = T_{m,i} + \frac{q}{\dot{m} C_p} = \boxed{379 \text{ K}}$  ✓

$D_h = \frac{4A_c}{P} = \frac{4(1.004 \times 0.016)}{0.04 \text{ m}} = 0.064$

$T_{s,o} = T_{m,o} + q''/h \quad (8.27) \text{ p463}$

$Re_D = \frac{\rho u_m D_h}{\mu} = \frac{\dot{m} D_h}{A_c \mu} = 1625$

$h_{\text{local}} = \frac{k Nu}{D_h} = \frac{0.0263}{0.064} 5.33 = 21.9$

Laminar

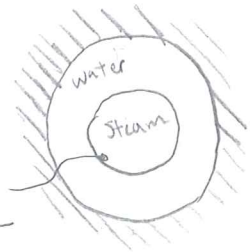
$T_{s,o} = 379 \text{ K} + \frac{600}{21.9} = \boxed{406 \text{ K}}$  ✓

8.93

Water,  $\dot{m} = 0.02 \text{ kg/s}$   
 $T_{m,i} = 20^\circ\text{C}$   
 $T_{m,o} = 75^\circ\text{C}$

Tube  
 $D_i = 0.025 \text{ m}$   
 $D_o = 0.100 \text{ m}$

Steam  
 $T_{s,i} = 100^\circ\text{C}$   
 find length of tube L

Constant  $T_{s,i}$ 

$L = -\frac{\dot{m} C_p}{P \bar{h}_L} \ln \left( \frac{T_s - T_{s,m}}{T_s - T_{m,i}} \right)$

(8.41) pg 471

$H_2O$   
 $320^\circ\text{C}$   
 $C_p = 4180$   
 $\mu = 577 \times 10^{-6}$   
 $Pr = 3.77$   
 $k = 0.640$

$\bar{h}_L = \frac{Nu_i k}{D_h}$

$Nu_i = 7.37$   
 $D_h = D_o - D_i = 0.075 \text{ m}$

Table 8.2 annular tubes p490

$\bar{h}_L = 62.9$

$Re_D = \frac{\dot{m} D_h}{A_c \mu}$

$A_c = \pi \frac{D_o^2}{4} - \pi \frac{D_i^2}{4} = \frac{\pi}{4} (D_o^2 - D_i^2) = 0.00834 \text{ m}^2$

$Re_D = \boxed{35.3}$

$L = \boxed{19.7 \text{ m}}$  ✓

8.94

how long must annulus be if  $\dot{m} = 0.3 \text{ kg/s}$ ?

$$Re_D = 5296 > 2300 \text{ turbulent}$$

Water  
320°

$$Pr = 3.77$$

$$k = .640$$

$$Nu_D = .023 Re_D^{4/5} Pr^{0.4} = 33.8$$

$$\bar{h} = \frac{k}{D_h} Nu = 318$$

$$L = -\frac{\dot{m} C_p}{Ph_L} \ln \left( \frac{T_s - T_{m,o}}{T_s - T_{m,i}} \right)$$

$$\frac{L_2}{L_1} = \frac{.3}{.02} \frac{62.9}{318}$$

$$\Rightarrow L_2 = 58.5 \text{ m}$$



7.2 a, b, c, 11, 20, 45, 73,

$$\nu_{\text{oil}, 330\text{K}} = 96.6 \times 10^{-6} \text{ m}^2/\text{s} \quad \text{Table A.5}$$

7.2) oil 100°C  
0.1 m/s



$$Re_L = \frac{uL}{\nu} = \frac{0.1 \text{ m/s} (1 \text{ m})}{96.6 \times 10^{-6} \text{ m}^2/\text{s}} = \frac{1035}{4546} < 5 \times 10^5$$

laminar flow velocity & thermal boundary layer thickness?

a)  $\delta = \frac{5x}{\sqrt{Re_x}} = \frac{5(1)}{\sqrt{4546}} = 0.1554 \text{ m}$   $\delta_t = \delta Pr^{-1/3} = 0.1554 (1205)^{-1/3}$   
 $\delta_t = 0.0146 \text{ m}$

b) local heat flux & surface shear stress @ the trailing edge

$$C_{f,x} = \frac{\tau_{s,x}}{\rho u_{\infty}^2} = 0.664 Re_x^{-1/2}$$

$$\tau_{s,x} = 0.664 \sqrt{\frac{1}{1035}} \left( 865.8 \frac{\text{kg}}{\text{m}^3} \right) \left( \frac{0.1 \text{ m}}{\text{s}} \right)^2 \frac{1}{2}$$

$$\tau_s = 0.0894 \text{ N/m}^2$$

$$Pr = 1205 \quad (\text{Table A.5})$$

$$k = 141 \times 10^{-3} \text{ W/m}$$

heat flux  $Nu_x \equiv \frac{h_x x}{k}$

$$Nu_x = 0.332 Re_x^{1/2} Pr^{1/3} = 114$$

$$h_x = \frac{(141 \times 10^{-3} \frac{\text{W}}{\text{m}})(114)}{(1 \text{ m})} = 16.06 \text{ W/m}^2 \cdot \text{K}$$

$$q'' = h(T_s - T_{\infty}) = 16.06 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} (20 - 100) =$$

$$-1.286 \frac{\text{kW}}{\text{m}^2}$$

c) Total avg h.t. of plate

$$\bar{Nu}_x = 0.664 Re_x^{1/2} Pr^{1/3} = \frac{\bar{h}_x x}{k} \Rightarrow \bar{h} = 32.05 \text{ W/m}^2 \cdot \text{K}$$

$$\bar{C}_f = 1.328 Re_x^{-1/2}$$

$$q'' = -80/(32.05) = -2564 \times 2 \text{ plates}$$

$$q'' = -5128 \text{ W/m}^2$$

$$\Delta D' = 2L \left( \frac{\rho u_{\infty}^2}{2} \right) 1.328 Re_L^{-1/2} = 0.337 \text{ N/m}$$

$$D' = 2L \bar{\tau}_{s,L}$$

a) find velocity & BL thickness @ trailing edge

7.111



Given  
 $u_{\infty} = 5 \text{ m/s}$   
 $T_{\infty} = 20^{\circ}\text{C}$

a) find  $\bar{h}$ ,  $q$ ,  $D'$ 

$L = 2 \text{ m}$  x BOTH SIDES  
 $W = 2 \text{ m}$

Air flow,  $T_s = 50^{\circ}\text{C}$ ,  $80^{\circ}\text{C}$ 

$$\frac{20+50}{2} = 35^{\circ}$$

$$\frac{20+80}{2} = 40^{\circ}$$

$$313 \text{ K}$$

$$308 \text{ K}$$

$$\bar{C}_f = 0.074 Re_L^{-1/2} - 1742 Re_L^{-1}$$

$$= 2.35 \times 10^{-3}$$

$$F_{D,1} = \bar{C}_f \frac{\rho u_{\infty}^2}{2} A_s \Rightarrow \boxed{F_D = 0.273 \text{ N}}$$

For BOTH TEMPERATURES

$$\bar{h}_L = \frac{\bar{Nu}_L \cdot k_f}{L}$$

$$\bar{Nu}_L = (0.037 Re_L^{1/2} - 871) Pr^{1/3} = 660.2$$

$$= \frac{660(0.0263)}{2}$$

$$\boxed{\bar{h}_L = 8.679}$$

$$q_{conv} = \bar{h}_L \cdot A_s (T_s - T_{\infty})$$

$$\boxed{q_{50^{\circ}\text{C}} = 2083 \text{ W} \quad q_{80^{\circ}\text{C}} = 3471 \text{ W}}$$

b)  $L = 0.1 \text{ m}$   $W = 0.1 \text{ m}$   $A_s = 0.2 \text{ m}^2$ 

$$Re_1 = \frac{u_{\infty} L}{\nu} = 6.86 \times 10^5$$

$$Re_2 = 9.02 \times 10^5$$

$$\bar{C}_{f,1} = 0.074 Re_1^{-1/2} - 1742 Re_1^{-1} = 2.49 \times 10^{-3}$$

$$C_{f,2} = 2.84 \times 10^{-3}$$

$$F_{D,1} = \bar{C}_f \frac{\rho u_{\infty}^2}{2} A_s = \boxed{0.620 \text{ N}}$$

$$\boxed{F_{D,2} = 0.700 \text{ N}}$$

$$\bar{Nu}_1 = (0.037 Re_1^{1/2} - 871) Pr^{1/3}$$

$$\bar{Nu}_1 = 1450$$

$$\bar{Nu}_2 = 1952$$

$$\bar{h}_1 = \frac{\bar{Nu}_1 \cdot k_f}{L}$$

$$\boxed{\bar{h}_1 = 9050 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} \quad q_1 = 5430 \text{ W}}$$

$$\boxed{\bar{h}_2 = 12,600 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} \quad q_2 = 15100 \text{ W}}$$

$$q = \bar{h}_L A_s (T_s - T_{\infty})$$

Air, 300 K 1 atm

$$\rho = 1.161 \text{ kg/m}^3$$

$$\nu = 15.89 \times 10^{-6} \text{ m}^2/\text{s}$$

$$k = 0.0263 \text{ W/m}\cdot\text{K}$$

$$Pr = 0.706$$

H<sub>2</sub>O, liquid 315 K

$$\rho = 991 \text{ kg/m}^3$$

$$\nu = 1584 \text{ m}^2/\text{s}$$

$$k = 0.634$$

$$Pr = 5.83$$

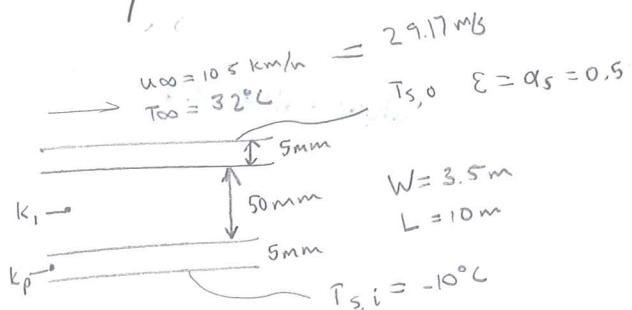
310

.993

1439

David Mataway

7.20]



$k_p = 180 \text{ W/m}\cdot\text{K}$   
 $k_i = 0.026 \text{ W/m}\cdot\text{K}$   
 $\text{irradiation} = 750 \text{ W/m}^2$

$\nu = 15.89 \times 10^{-6} \frac{\text{m}^2}{\text{s}}$

a) find  $T_{s,o}$

$$\alpha_s G_s + q''_{\text{conv}} - E = q''_{\text{cond}} = \frac{T_{s,o} - T_{s,i}}{R''_{\text{tot}}}$$

$$\alpha_s G_s + \bar{h} (T_{\infty} - T_{s,o}) - \epsilon \sigma T_{s,o}^4 = \frac{T_{s,o} - T_{s,i}}{2R''_p + R''_i}$$

$$\bar{h} = \frac{k_p}{L} \cdot 0.37 Re_L^{1/2} Pr^{1/3} = 56.2 \text{ W/m}^2\cdot\text{K}$$

calculator  $\Rightarrow T_{s,o} = 307 \text{ K} = 34^{\circ}\text{C}$

$$q = A \frac{(T_s - T_{\infty})}{R_{\text{tot}}} \Rightarrow q = 780 \text{ W}$$

b)  $\alpha = 0.15$   $\epsilon = 0.8$

$$\alpha_s G_s + \bar{h} (T_{\infty} - T_{s,o}) - \epsilon \sigma T_{s,o}^4 = \frac{T_{s,o} - T_{s,i}}{2R''_p + R''_i} \Rightarrow T_{s,o} = 300 \text{ K} = 27^{\circ}\text{C}$$

$$q = 675 \text{ W}$$

c) no insulation

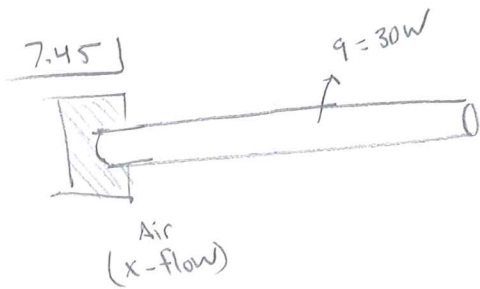
$$R''_{\text{tot}} = 2 \times (2.78 \times 10^{-5}) = 5.56 \times 10^{-5}$$

$$\bar{h} = 56.2 \text{ W/m}^2\cdot\text{K}$$

$$\alpha_s = \epsilon = 0.5$$

$$T_{s,o} = 263.1 \text{ K}$$

$$q = 90.63 \text{ kW}$$



$D = 10 \text{ mm}$   
 $k = 240 \text{ W/mK}$   
 $\rho = 2700 \text{ kg/m}^3$   
 $c_p = 900 \text{ J/kgK}$   
 $Re_c = 4000$

infinitely long pin  
Ch 3 slide 24

Cylinder,  $Re_D$   
 $4 < Re_D < 4 \times 10^5$   
 $\bar{Nu}_D = C Re_D^m Pr^{1/3}$   
 $C = .683$   $m = 0.466$

$\bar{h} \sim D^{-1} D^{-.466} = D^{-1.466}$   
 $q_{conv} \sim (D^{-1.466} \cdot D \cdot D^2)^{1/2}$   
 $q_{conv} \sim D^{-.534}$

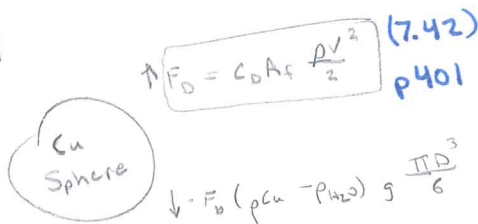
$q_{conv} = (\bar{h} L P \Delta T_c)^{1/2} \Rightarrow q_{conv} \sim D^{(-.534 \times D \times D^2)^{1/2}}$

$q_{conv} \sim D^{1.23}$

$q_2 = q_1 \left( \frac{D_2}{D_1} \right)^{1.23} = 30 \text{ W} (2)^{1.23}$

$q_2 = 70.4 \text{ W}$  ✓

7.73



$C_u, 35014$   
 $\rho = 8933$   
 $k = 398$   
 $C_p = 387$

$H_2O, 280K$   
 $\rho = 1000$   
 $\mu = 1.422 \times 10^{-4}$   
 $k = .582$   
 $Pr = 10.26$

$T_s \approx 340$   
 $\mu_s = 420 \times 10^{-5} \text{ N.s/m}^2$

$C_D V^2 = \frac{4D}{3} \frac{\rho_{Cu} - \rho_{H2O} g}{\rho} = 2.07 \text{ m}^2/\text{s}^2$

Fig 7.8  $\rightarrow \sqrt{2.1} \text{ m/s}$

$Re_D = 29580$   $C_D \approx 0.46$

$\bar{Nu}_D = 2 + (.4 (29580)^{1/4} + .06 (29580)^{2/3}) (10.26)^{1/4} \left( \frac{1422}{420} \right)^{1/4} = 439$

$\bar{h} = \bar{Nu}_D \frac{k}{D} = 12,775 \text{ W/m}^2\text{K}$

$Bi = \bar{h} \left( \frac{r_o}{3} \right) / k_{Cu} = 0.11$  No lumped method, single term approximation  
 $\theta_o^* = C_1 e^{(-\zeta_1^2 Fo)}$   $C_1 = 1.0937$   $\zeta_1 = .9472$   $0.5 = 1.094 e^{-.9472 Fo} \Rightarrow Fo = 0.87 = \frac{\alpha t_f}{r_o^2}$

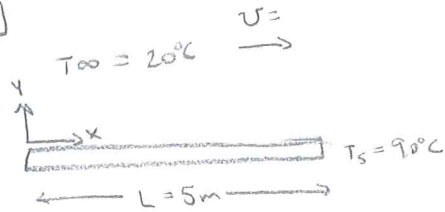
$q_{Cu} = \frac{k}{\rho c_p} = 1.15 \times 10^{-4} \text{ m}^2/\text{s}$   $t_f = 0.76 \text{ s}$

$H = t_f \cdot V = 1.6 \text{ m}$  ✓

6.8, 15, 18, 32, 35

100

6.8)



$$T(^{\circ}\text{C})(x) = 20 + 70 e^{-600x}$$

find  $h(x)$ ,  $\bar{h}$ 

$$h = \frac{-k_f \left( \frac{dT}{dy} \right)_{y=0}}{T_s - T_{\infty}}$$

$$\frac{dT}{dx} = -70(600) e^{-600x} \bigg|_{y=0} \Rightarrow -42000x$$

$$\bar{T} = \frac{90 - 20}{2} + 273 = 328\text{K} \Rightarrow k_f \approx .03 \text{ W/mK} \quad (300\text{K})$$

$$h = \frac{-k_f (-42000x)}{90 - 20} = \boxed{18x \text{ W/m}^2\text{K} = h}$$

$$\text{average coefficient, } \bar{h} = \frac{1}{L} \int_0^L h dx = \frac{18}{5} \int_0^5 x dx = \frac{18}{5} \left[ \frac{x^2}{2} \right]_0^5 = \boxed{45 \text{ W/m}^2\text{K} = \bar{h}}$$

6.15)  $V_{\max} = 50 \text{ m/s}$  a) find  $L_{\text{minimum}}$ 

$$P = 1 \text{ ATM}$$

$$T = 25^{\circ}\text{C}$$

$$Re_x = 10^8$$

$$Re_{x,c} = \frac{\rho u_{\infty} x_c}{\mu} = \frac{u_{\infty} x_c}{\nu}$$

$$x_c = \frac{10^8 (15.89 \times 10^{-6})}{50 \text{ m/s}}$$

$$\boxed{L_{\min} = 31.8 \text{ m}}$$

$$Pv = RT$$

$$\rho = \frac{P}{RT} = \frac{101 \text{ kPa}}{8.314 (298)} = .0408 \text{ kg/m}^3$$

$$b) Re_{x,c} = 5 \times 10^5 \Rightarrow 31.8 \left( \frac{5 \times 10^5}{10^8} \right) = \boxed{0.159 \text{ m}} x_c$$

$$Re_L \equiv \frac{VL}{\nu}$$

$$V_1 = V_2 \Rightarrow$$

$$5(20) = 1(100) \Rightarrow Re_{L1} = Re_{L2}$$

6.18)

$$L = 1 \text{ m}$$

$$T_s = 400 \text{ K}$$

$$T_{\infty} = 300 \text{ K}$$

$$V = 100 \text{ m/s}$$

$$\bar{q}'' = 20,000 \text{ W/m}^2$$

$$L_2 = 5 \text{ m}$$

$$T_s = 400 \text{ K}$$

$$T_{\infty} = 300$$

$$V = 20 \text{ m/s}$$

$$h_1 = \frac{q_{\text{conv}}}{A_s (T_s - T_{\infty})} = \frac{20,000}{100} = 200 \text{ W/m}^2\text{K}$$

$$\bar{h}_2 = \frac{1}{\frac{A_2}{A_1} (1)} (200 \text{ W/m}^2\text{K}) \cdot \frac{q_{\text{conv}}}{(T_s - T_{\infty}) (5 \text{ A}_s)} \Rightarrow \boxed{40 \text{ W/m}^2\text{K}} \bar{h}_2$$

6.32

$L = 800 \text{ mm}$   
 $t = 6 \text{ mm}$   
 $V = 70 \text{ mph} = 31.29$   
 $T_{\infty} = -15^{\circ}\text{C}$

Table A.3:  $k_g = 1.4 \text{ W/m}\cdot\text{K}$

Model:  $\overline{Nu}_L = .030 Re_L^{0.8} Pr^{1/3}$

$Re_L = \frac{VL}{\nu}$   
 $k_{air} = .023 \text{ W/m}\cdot\text{K}$   
 $\nu = 12.5 \times 10^{-6} \text{ m}^2/\text{s}$   
 $Pr = 0.71$

$T_{DP} = 10^{\circ}\text{C}$   
 $T_{\infty,i} = 50^{\circ}\text{C}$   
 $\bar{h}_i \text{ minimum} = ?$

$R_{tot} = \frac{1}{h_i} + \frac{t}{k_g} + \frac{1}{h_o}$

$\frac{T_{\infty,i} - T_{s,i}}{1/h_i} = \frac{T_i - T_{\infty,o}}{\frac{t}{k_g} + \frac{1}{h_o}}$

$\frac{50 - 10}{1/\bar{h}_i} = \frac{10 - (-15)}{.006/1.4 + 1/h_o}$

$Re_L = \frac{VL}{\nu} = \frac{31.29 \text{ m/s} (0.8)}{12.5 \times 10^{-6} \text{ m}^2/\text{s}}$   
 $= 2.00 \times 10^6$

$Pr = 0.71$

$\bar{h}_o = ? \quad Nu = \frac{hL}{k_f} = .030 (Re_L)^{0.8} Pr^{1/3}$   
 $= 2940$

$\Rightarrow \bar{h}_o = 84.5$

$40(\bar{h}_i) = \frac{25}{.00429 + .0118}$

$\bar{h}_{i \text{ min}} = 38.8 \text{ W/m}^2\cdot\text{K}$

6.35

$u_{\infty} = 15 \text{ m/s}$   
 $T_{\infty} = 15^{\circ}\text{C}$   
 $T_s = 140^{\circ}\text{C}$   
 $A_s = .25 \text{ m}^2$   
 drag force  $0.25 \text{ N}$   
 power needed = ?  
 $Pr = .707$

$q_{in} = q_{out} = h A_s (T_s - T_{\infty})$

$\bar{h}_o = \frac{C_F}{2} Pr^{2/3} \rho u_{\infty} c_p$

$\bar{h}_o = \frac{\tau_s}{\rho(u_{\infty})} \frac{\rho u_{\infty} c_p Pr^{2/3}}{Pr^{2/3}} = \frac{\tau_s}{u_{\infty}} c_p Pr^{-2/3}$

$\bar{h}_o = \frac{F_D c_p}{A u_{\infty} Pr^{2/3}} = \frac{0.25}{0.25} \frac{1009}{15} (0.707)^{-2/3}$

$\bar{h}_o = 84.8 \frac{\text{W}}{\text{m}^2\cdot\text{K}}$

$q = \frac{T_s - T_{\infty}}{\frac{1}{h}} (A) = 85(140 - 15)(0.25) = 2.66 \text{ kW}$



3.116, 129, 134b, 145 adiabatic tip

$$\begin{aligned} 3.116) \quad k &= 20 \text{ W/m}\cdot\text{K} & T_{\infty} &= 1200^{\circ}\text{C} \\ L &= 50 \text{ mm} & h &= 250 \text{ W/m}^2\cdot\text{K} \end{aligned}$$

$$A_c = 6 \times 10^{-4} \text{ m}^2 \quad \text{Perimeter } P = 110 \text{ mm} \quad T_{\text{base}} = 300^{\circ}\text{C} \quad \text{find } T_{\text{tip}}$$



$$\frac{\theta}{\theta_b} = \frac{T - T_{\infty}}{T_b - T_{\infty}} = \frac{\cosh m(L_c - x)}{\cosh mL_c} = \frac{1}{5.52} = .1811 \Rightarrow T = 1037^{\circ}\text{C}$$

$$a) \quad M = \sqrt{hPkA_c} = 47.87$$

1037° ≤ 1050. Yes, cooking is satisfactory

$$b) \quad q_f = M \tanh(mL_c) =$$

$$M = \sqrt{hPkA_c} \theta_b = -517.0$$

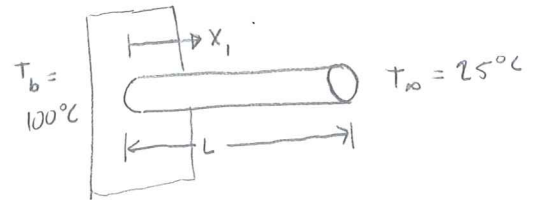
$$q_f = -508 \text{ W}$$

3.129

$$k_A = 200 \text{ W/m}\cdot\text{K}$$

find  $k_B$ 

$$\left. \begin{aligned} T_A &= 75^{\circ}\text{C} \\ T_B &= 60^{\circ}\text{C} \end{aligned} \right\} \text{ at } x_1$$



$$\text{Temp. dist.} \quad \frac{T - T_{\infty}}{T_b - T_{\infty}} = e^{-mx, A} = 0.667 \quad -mx, A = -.4055$$

$$e^{-mx, B} = 0.533 \quad -mx, B = -.7621$$

$$\frac{-.4055}{-.7621} = \frac{-m_A L}{-m_B} = \sqrt{\frac{k_B}{k_A}} \Rightarrow k_B = .2831 k_A$$

$$k_B = 56.62 \text{ W/m}\cdot\text{K}$$

134) b)

$$T_{max} = 75^\circ\text{C}$$

4x4 array, Cu pins

$$q_c = ?$$

$$T_c = 75^\circ\text{C}$$

$$D_p = .0015\text{ m}$$

$$L_p = 15\text{ mm}$$

Conditions 3.27

$$h_o = 1000\text{ W/m}^2\cdot\text{K}$$

$$T_{\infty, o} = 20^\circ\text{C}$$

$$r_{chip/board} = 10^{-4}\text{ m}^2\cdot\text{K/W}$$

$$L_{board} = 5\text{ mm}$$

$$k_b = 1\text{ W/m}\cdot\text{K}$$

$$\text{air } h_i = 40\text{ W/m}^2\cdot\text{K} \quad T_{\infty, i} = 20^\circ\text{C}$$

$$k_{cu} = 400\text{ W/m}\cdot\text{K}$$

$$q = N \eta_f h_{finned} A_f \Theta_b + h_{unfinned} A_{unfinned} \Theta_b$$

$$(16)(.6768)(1000)\left(\frac{1.77 \times 10^{-6}}{7.25 \times 10^{-5}}\right)(75 - 20)$$

$$= 43.2\text{ W}$$

$$+ (1000)(1.33 \times 10^{-4})(75 - 20) = 7.3\text{ W}$$

$$q_{total} = 50.5\text{ W}$$

$$145) (L_c)^{3/4} h / k A_p^{1/4} = 0.11$$

$$q_f = \eta_f h A_f \Theta_b =$$

$$= 2\pi(.87)(40\text{ W/m}^2\cdot\text{K})[.042^2 - .025^2](180)$$

$$a) q_f = 50\text{ W}$$

$$\epsilon_f = \frac{q_f}{h A_{c,b} \Theta_b} = 11.05$$

$$b) \bar{q} = N' q_f + h(1 - N') (2\pi r_c) \Theta_b$$

$$\bar{q} = 6820\text{ W/m}$$

$$\eta_f = \frac{\tanh mL_c}{mL_c} = .6768$$

$$m = \sqrt{hP/kA_c} = 81.65\text{ m}^{-1}$$

$$L_c = L + \frac{D}{4} = .01538$$

$$A_f = \frac{\pi}{4} D^2 (N)$$

$$A_{unfinned} = [(12.7 \times 10^{-3})^2 - 16 \left(\frac{\pi}{4}\right) (.0015^2)]$$

$$= 1.33 \times 10^{-4}\text{ m}^2$$

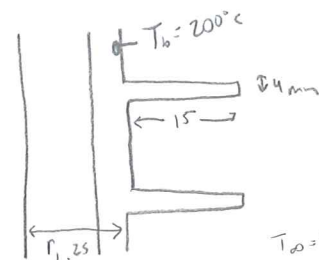
$$r_{2c} = r_2 + \frac{t}{2} = .042\text{ m}$$

$$r_{2c}/r_1 = 1.68$$

$$L_c = L + t/2 = .017\text{ m}$$

$$A_p = L_c t = 6.8 \times 10^{-5}\text{ m}^2$$

$$k_{Al} = 240\text{ W/m}\cdot\text{K}$$

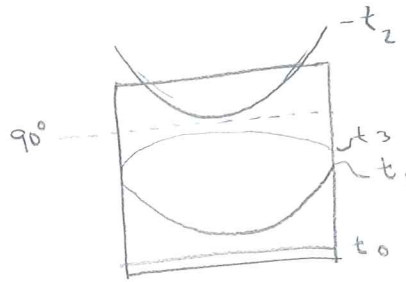
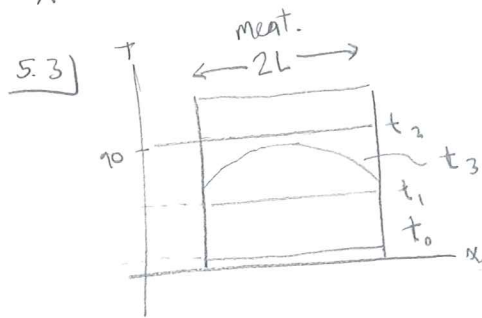


$$T_{\infty} = 20^\circ\text{C}$$

$$h = 40\text{ W/m}^2\cdot\text{K}$$

David Malawey 225 HW 5

5.3, 11, 43(a,b) 57,



microwave

oven

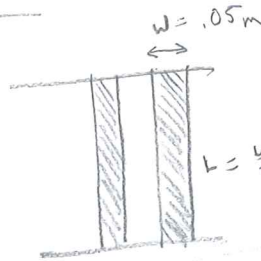
5.11)  $T_{\infty} = 600^{\circ}\text{C}$

$h = 100 \text{ W/m}^2 \cdot \text{K}$

- Time to achieve 75% of capacity

- Temp of aluminum @ 75%?

$$R_{\text{conduction}} = \frac{L}{kA} = \frac{0.025}{237 \text{ A}}$$



$k = 237 \text{ W/m} \cdot \text{K}$

$T_i = 25^{\circ}\text{C}$

$L = \frac{w}{2} = 0.025 \text{ m}$

assuming constant specific heat

$$T_{75\%} = 456^{\circ}\text{C}$$

$\rho = 2702 \text{ kg/m}^3$

$c_p = 1033 \text{ J/kg} \cdot \text{K}$

$$R_{\text{conv}} = \frac{1}{hA} = \frac{1}{100 \text{ A}} \quad B_i = \frac{0.025}{\frac{237 \text{ A}}{100 \text{ A}}} = 0.11$$

$$\frac{T - T_{\infty}}{T_i - T_{\infty}} = e^{\left( \frac{-hA_s t}{\rho V c_p} \right)}$$

$$t = \frac{\rho V c_p}{hA_s} \ln(0.75) =$$

$$\frac{2702(1033) \cdot \frac{w}{2}}{100 \text{ W/m}^2 \cdot \text{K} \cdot A} \ln(0.75)$$

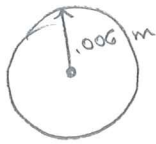
$$\frac{w}{A} = \frac{L k w}{h w} = 0.025 \text{ m}$$

$$t = 968 \text{ s}$$

$$\frac{456 - 600}{25 - 600} = 0.25$$

5.5)

Sphere



$$T_i = 1150 \text{ K}$$

$$T_f = 400 \text{ K}$$

$$T_\infty = 325 \text{ K}$$

$$h = 20 \text{ W/m}^2\text{K}$$

$$\rho = 7800 \text{ kg/m}^3$$

$$C = 600 \text{ J/kg}\cdot\text{K}$$

$$k = 40 \text{ W/m}\cdot\text{K}$$

$$\text{find } Bi \# = \frac{h L_c}{k} = \frac{20 \frac{\text{W}}{\text{m}^2\text{K}} \left( \frac{0.006 \text{ m}}{3} \right)}{40 \text{ W/m}\cdot\text{K}} = .001$$

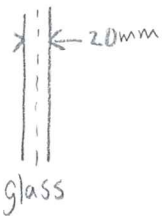
Time required for cooling?

$$t = \frac{\rho V C_p}{h A_s} \ln \frac{T_i - T_\infty}{T_f - T_\infty}$$

$$= \frac{7800 \left( \pi \left( \frac{0.012}{2} \right)^3 \right) 600}{20 \pi \left( \frac{0.012}{2} \right)^2} \ln \left( \frac{1150 - 325}{400 - 325} \right)$$

$$t = 1122 \text{ seconds}$$

5.43)



$$\text{diffusivity } \alpha = 6 \times 10^{-7} \text{ m}^2/\text{s}$$

$T_i \rightarrow T_s$  suddenly

$$L = .01 \text{ m}$$

$$Bi = \frac{h L}{k} \Big|_{h \rightarrow \infty} \rightarrow Bi^{-1} = 0$$

$$-k \frac{\partial T}{\partial x} \Big|_{x=L} = h [T(L, t) - T_\infty]$$

$$\text{Heisler charts, } \theta^* = \frac{T - T_\infty}{T_i - T_\infty} = 0.5 \text{ (to reach 50\% of temp. reduction)}$$

$$Fo = 0.375 = \frac{\alpha t}{L^2} \Rightarrow t = \frac{0.375 (.01)^2}{6 \times 10^{-7}} \Rightarrow t = 62.5 \text{ s}$$



b)  $T_i - T_s = 300^\circ\text{C}$ , find max temp gradient

$$\theta^* = C_1 e^{(-\zeta_1^2 Fo)} \cos(\zeta_1 x^*) \quad \frac{\partial \theta^*}{\partial x^*} = C_1 e^{(-\zeta_1^2 Fo)} - \zeta_1 \sin(\zeta_1 x^*)$$

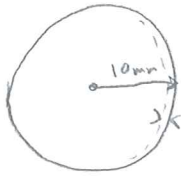
$$\zeta_1 = (\text{append. B3}) 1.5708 \quad C_1 = 1.2733 \text{ (table S.1)} \quad x^* = 1 \quad \frac{\partial \theta^*}{\partial x^*} \Big|_{x^*=1} = -0.793$$

$$\frac{\partial \theta^*}{\partial x^*} = \frac{\partial \left( \frac{T - T_\infty}{T_i - T_\infty} \right)}{\partial \left( \frac{x}{L} \right)} = \frac{L}{T_i - T_\infty} \frac{\partial T}{\partial x} \Rightarrow \frac{\partial T}{\partial x} \Big|_{x=L} = \frac{T_i - T_\infty}{L} \frac{\partial \theta^*}{\partial x^*} \Big|_{x^*=1} = \frac{300}{0.01} (-0.793)$$

$$\frac{\partial T}{\partial x} = -23560 \frac{^\circ\text{C}}{\text{m}}$$

David Maloney

5.57/



$$c = 500 \text{ J/kg} \cdot \text{K}$$

$$\rho = 7800 \text{ kg/m}^3$$

$$k = 50 \text{ W/m} \cdot \text{K}$$

$$T_{\infty} = 1300 \text{ K}$$

$$h = 5000 \text{ W/m}^2 \cdot \text{K}$$

$$T_i = 300 \text{ K}$$

$$\alpha = \frac{k}{\rho c} = 1.28 \times 10^{-5}$$

find  $t$  for outer 1mm to reach 1000 K

$$Bi = \frac{k}{hr} = \frac{50}{5000(0.01)} = 1.0$$

$$\frac{r}{r_o} = \frac{9}{10} = 0.90 \Rightarrow \text{chart D.8} \quad \frac{\theta}{\theta_o} = 0.475$$

Surface  $\frac{T - T_{\infty}}{T_o - T_{\infty}} = 0.6 = \frac{1000 - 1300}{T_o - 1300} \Rightarrow T_o = 800$

$$\theta_o^* = \frac{T_o - T_{\infty}}{T_i - T_{\infty}} = \frac{800 - 1300}{300 - 1300} = 0.5$$

Chart:  $Bi^{-1} = 1.0 \quad \theta_o^* = 0.5 \quad F_o = 0.4 \Rightarrow t = \frac{F_o r_o^2}{\alpha} = \frac{.4(.01)^2}{1.28 \times 10^{-5}} = \boxed{3.12 \text{ sec}}$

① find  $Bi, Bi^{-1}$

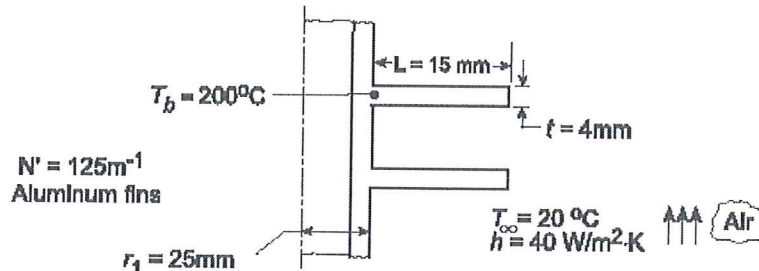
② use chart for surface to find  $\frac{\theta}{\theta_o} = 0.6$

### PROBLEM 3.145

**KNOWN:** Dimensions and base temperature of aluminum fins of rectangular profile. Ambient air conditions.

**FIND:** (a) Fin efficiency and effectiveness, (b) Rate of heat transfer per unit length of tube.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Steady-state conditions, (2) One-dimensional radial conduction in fins, (3) Constant properties, (4) Negligible radiation, (5) Negligible base contact resistance, (6) Uniform convection coefficient.

**PROPERTIES:** Table A-1, Aluminum, pure ( $T \approx 400$  K):  $k = 240$  W/m·K.

**ANALYSIS:** (a) The fin parameters for use with Figure 3.19 are

$$\begin{aligned} r_{2c} &= r_2 + t/2 = 40 \text{ mm} + 2 \text{ mm} = 0.042 \text{ m} & L_c &= L + t/2 = 15 \text{ mm} + 2 \text{ mm} = 0.017 \text{ m} \\ r_{2c}/r_1 &= 0.042 \text{ m}/0.025 \text{ m} = 1.68 & A_p &= L_c t = 0.017 \text{ m} \times 0.004 \text{ m} = 6.8 \times 10^{-5} \text{ m}^2 \\ L_c^{3/2} (h/kA_p)^{1/2} &= (0.017 \text{ m})^{3/2} \left[ 40 \text{ W/m}^2 \cdot \text{K} / 240 \text{ W/m} \cdot \text{K} \times 6.8 \times 10^{-5} \text{ m}^2 \right]^{1/2} = 0.11 \end{aligned}$$

The fin efficiency is  $\eta_f \approx 0.97$ . From Eq. 3.86,

$$\begin{aligned} q_f &= \eta_f q_{f,\max} = \eta_f h A_{f(\text{ann})} \theta_b = 2\pi \eta_f h \left[ r_{2c}^2 - r_1^2 \right] \theta_b \\ q_f &= 2\pi \times 0.97 \times 40 \text{ W/m}^2 \cdot \text{K} \left[ (0.042)^2 - (0.025)^2 \right] \text{m}^2 \times 180^\circ \text{C} = 50 \text{ W} \end{aligned} \quad <$$

From Eq. 3.81, the fin effectiveness is

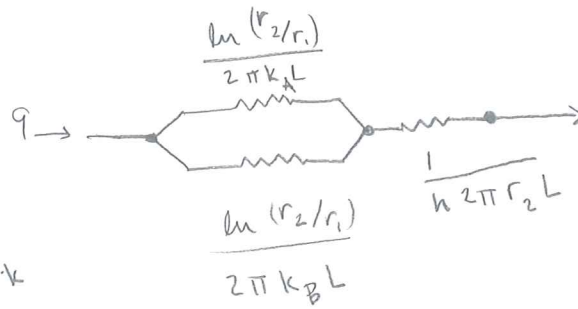
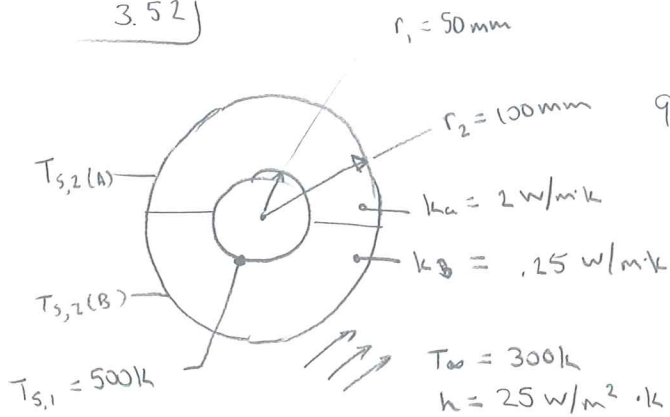
$$\varepsilon_f = \frac{q_f}{h A_{c,b} \theta_b} = \frac{50 \text{ W}}{40 \text{ W/m}^2 \cdot \text{K} \cdot 2\pi (0.025 \text{ m})(0.004 \text{ m}) 180^\circ \text{C}} = 11.05 \quad <$$

(b) The rate of heat transfer per unit length is

$$\begin{aligned} q' &= N' q_f + h(1 - N') (2\pi r_1) \theta_b \\ q' &= 125 \times 50 \text{ W/m} + 40 \text{ W/m}^2 \cdot \text{K} (1 - 125 \times 0.004) (2\pi \times 0.025 \text{ m}) \times 180^\circ \text{C} \\ q' &= (6250 + 565) \text{ W/m} = 6.82 \text{ kW/m} \end{aligned} \quad <$$

**COMMENTS:** Note the dominant contribution made by the fins to the total heat transfer.

3.52)



$$\ln \frac{100}{50} = .6931$$

$$\dot{Q}_r = \frac{\Delta T}{R_{\text{tot}}}$$

$$\frac{1}{R_{AB}} = \frac{\cancel{\pi} k_a L}{\ln(r_2/r_1)} + \frac{\cancel{\pi} k_b L}{\ln(r_2/r_1)}$$

$$\frac{1}{R_{AB}} = \frac{\cancel{\pi} L (k_a + k_b)}{\ln(r_2/r_1)} \Rightarrow R_{AB} = \frac{\ln(r_2/r_1)}{\cancel{\pi} L (k_a + k_b)}$$

$$R_{\text{total}} = \frac{\ln(r_2/r_1)}{\cancel{\pi} L (k_a + k_b)} + \frac{1}{h 2\pi r_2 L} = .1617$$

b)

$$\dot{Q} = \frac{500 - 300}{.1617} = 1236 \text{ Watts/meter}$$

find  $T_{s,2,A}$   $T_{s,2,B}$  \*

$$\frac{1}{r} \frac{d}{dr} \left( r \frac{dT}{dr} \right) + \frac{\dot{q}}{k} = 0$$

above calculations assume  
uniform temperature outer surface.  
- not realistic.

$$T(r=0) = T_s$$

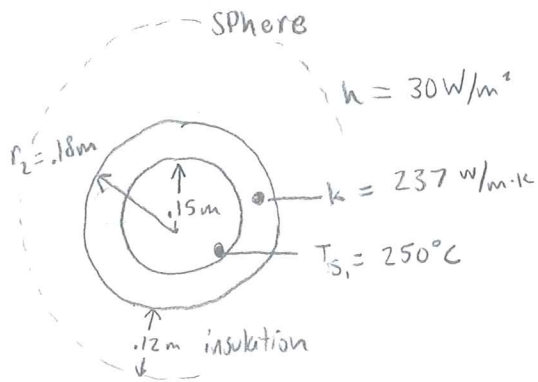
$$T(r=\infty) = 300 \text{ K}$$

$$q_{\text{outside surface}} = \dot{q}_{\text{inside}}$$

$$q = \frac{T_s - T_{\infty}}{R} = h 2\pi r L (T_s - T_{\infty}) = 2\pi (100) (25 \text{ W/m}^2 \cdot \text{K}) (T_s - 300) = 1236 \text{ W/m}$$

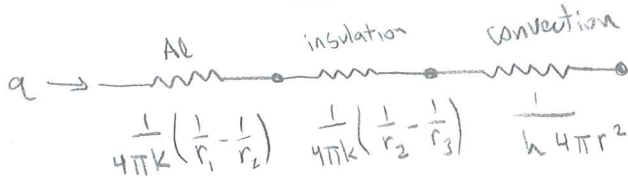
$$T_s = 379 \text{ K}$$

3.57)



80 W dissipated  
find  $k_{\text{insulation}}$

$$q = \frac{\Delta T}{R_{\text{eq}}}$$



$$R_{\text{Al}} = 3.731 \times 10^{-4}$$

$$R_{\text{ins.}} = \frac{1}{k} (0.1768)$$

$$R_{\text{air}} = 0.0295$$

$$80 \text{ W} = \frac{250 - 20}{0.0295 + 3.731 \times 10^{-4} + \frac{0.1768}{k}}$$

$$\frac{14.14}{k} = 227.61 \Rightarrow k = 0.0621 \text{ W/mK}$$

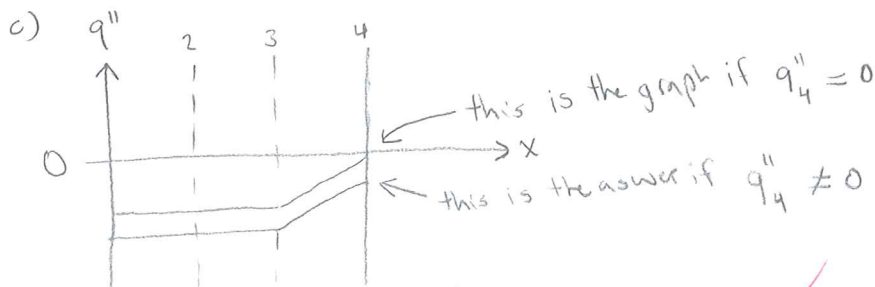
3.71)

a)  $q''_2 = q''_3$  because temp gradients are linear

$q''_3 > q''_4$  because the curved temp indicates heat generation in layer 'c'

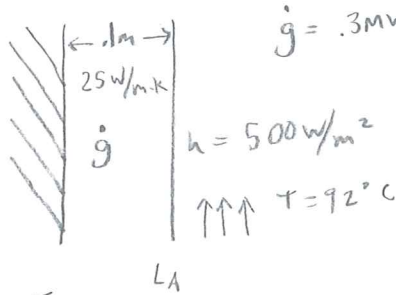
b)  $k_A < k_B$  because slope of temp grad. for A is larger

$k_B > k_C$  because slope of temp gradient is less but  $q''$  is equal





3.72



determine  $T_{max}$  in wall

$$\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) = -\dot{g}$$

$$k \frac{d^2 T}{dx^2} = -\dot{g} \quad \frac{d^2 T}{dx^2} = -\frac{\dot{g}}{k}$$

$$T = -\frac{\dot{g}}{2k} x^2 + C_1 x + C_2$$

B.C.'s

$$\left. x(0) \right| \frac{dT}{dx} = 0 = -\frac{\dot{g}}{k} x + C_1 \Rightarrow C_1 = 0$$

at surface

$$-\dot{q} = \frac{T_{\infty} - T_L}{R} \quad \dot{q} R - T_{\infty} = -T_L \quad R = \frac{L}{h} \quad T_L = 92 + \frac{3 \times 10^6 (0.1)}{500} =$$

$$T_L = 152^\circ\text{C} = -\frac{\dot{g}}{2k} x^2 + C_1 = -\frac{0.3 \times 10^6}{2(25)} (0.1)^2 + C_1$$

$$152^\circ\text{C} + 60 = C_1 = 212$$

$$T_0 = 212^\circ\text{C}$$

3.94

$$\dot{g} = \dot{g}_0 [1 - (r/r_0)^2]$$

↑ local gen. rate    ↑ const    ↑ container radius



Find  $T(r)$

use eqn on slide 18 ~~X~~ No.

3.94)  $\dot{g} = \dot{g}_0 \left[ 1 - \left( \frac{r}{r_0} \right)^2 \right]$  AND  $\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{dT}{dr} \right) + \frac{\dot{g}}{k} = 0$

$$\frac{d}{dr} \left( r^2 \frac{dT}{dr} \right) = -\frac{\dot{g}_0}{k} \left[ 1 - \left( \frac{r}{r_0} \right)^2 \right] r^2 \quad \text{INTEGRATE}$$

$$r^2 \frac{dT}{dr} = \int \left[ -\frac{\dot{g}_0 r^2}{k} + \frac{\dot{g}_0 r^4}{k r_0^2} \right] = -\frac{\dot{g}_0 r^3}{3k} + \frac{\dot{g}_0 r^5}{5k r_0^2} + C_1 \quad \begin{array}{l} \text{DIVIDE BY } r^2, \\ \text{INTEGRATE} \end{array}$$

$$T = \int \left[ -\frac{\dot{g}_0 r}{3k} + \frac{\dot{g}_0 r^3}{5k r_0^2} + \frac{C_1}{r^2} \right] = -\frac{\dot{g}_0 r^2}{6k} + \frac{\dot{g}_0 r^4}{20k r_0^2} - \frac{C_1}{r} + C_2$$

BOUNDARY CONDITIONS

①  $\frac{dT}{dr} \Big|_{r=0} = 0$

$$0 = -\frac{\dot{g}_0 r}{3k} + \frac{\dot{g}_0 r^3}{5k r_0^2} + C_1 \Rightarrow C_1 = 0$$

②  $-k \frac{dT}{dr} \Big|_{r=r_0} = h [T(r_0) - T_\infty]$

$$h [T(r_0) - T_\infty] = \frac{k \dot{g}_0 r_0}{3k} - \frac{\dot{g}_0 r_0^3}{5k r_0^2} + C_0 = \frac{\dot{g}_0 r_0}{3} - \frac{\dot{g}_0 r_0^2}{5}$$

$$\dot{g}_0 \left[ \frac{r_0}{3} - \frac{r_0^2}{5} \right] = h \left[ -\frac{\dot{g}_0}{k} \left( \frac{r_0^2}{6} - \frac{r_0^2}{20} \right) + C_2 - T_\infty \right]$$

PLUG IN  
 $T(r)$  &  $r_0$  for  $r$ 's

$$\Rightarrow C_2 = \frac{2r_0 \dot{g}_0}{15h} + \frac{7\dot{g}_0 r_0^2}{60k} + T_\infty$$

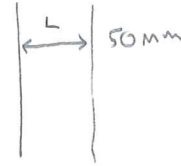
$$T(r) = T_\infty + \frac{2r_0 \dot{g}_0}{15h} + \frac{\dot{g}_0 r^2}{k} \left[ \frac{7}{60} - \frac{1}{6} \left( \frac{r}{r_0} \right)^2 + \frac{1}{20} \left( \frac{r}{r_0} \right)^4 \right]$$

2.23, 25, 46 3.2, 3.13

2.23) therm. conductivity  $k = \frac{50 \text{ W}}{\text{m} \cdot \text{K}}$

$$T = a + bx^2 \quad a = 200^\circ\text{C} \quad b = -2000^\circ\text{C}/\text{m}^2$$

$x$  (meters)



a)  $\dot{q} = ?$  heat eq.  $\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho C_p \frac{\partial T}{\partial t}$

$$\dot{q} = - \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right)$$

$$\dot{q} = - \frac{\partial}{\partial x} (2bkx) = -2bk$$

$$\dot{q}(L) - \dot{q}(0) = -2bk(L) + 2bk(0) = -2 \left( -\frac{2000^\circ\text{C}}{\text{m}^2} \right) \left( \frac{50 \text{ W}}{\text{m} \cdot \text{K}} \right) (0.05 \text{ m})$$

$$\dot{q} = 10 \frac{\text{KW}}{\text{m}^2}$$

2.25)  $2L = 40 \text{ mm}$  (thickness)  
 $k = 5 \frac{\text{W}}{\text{m} \cdot \text{K}}$   
 $\dot{q}$  volumetric heat gen rate

What is  $\dot{q}$  in wall?

$$\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \dot{q} = 0$$

$$\frac{\partial}{\partial x} (kb + 2kCx) + \dot{q} = 0$$

b)  $\dot{q} = -2kC = -2 \frac{5 \text{ W}}{\text{m} \cdot \text{K}} \left( -2 \times 10^4 \frac{^\circ\text{C}}{\text{m}^2} \right) = 2 \times 10^5 \frac{\text{W}}{\text{m}^3}$

c)  $\ddot{q}_x(x) = -k \left( \frac{\partial^2 T}{\partial x^2} \right) = -kb - 2kCx = -[5(-210) + 2(5)(-2 \times 10^4)(0.02 \text{ m})]$

$$\ddot{q}_x(L) = +5050 \text{ W/m}^2$$

$$\ddot{q}_x(-L) = -2950 \text{ W/m}^2$$

$$T(-L) = 82 - 210(-0.02) - 20000(-0.02)^2 = 78.2$$

$$T(L) = 82 - 210(0.02) - 20000(0.02)^2 = 69.8$$

a) convection coefficients for surfaces?

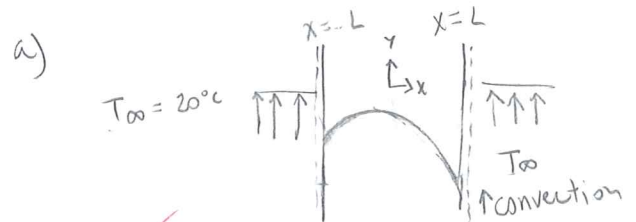
$$q''_{\text{conv},L} = \dot{q}_x(-L) =$$

$$h_L [T_\infty - T(-L)] = h_L (20 - 78.2) \text{ K} = -2950 \text{ W/m}^2$$

$$h_R [T(L) - T_\infty] = h_R (69.8 - 20) \text{ K} = 5050 \frac{\text{W}}{\text{m}^2}$$

$$h_L = 51 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}$$

$$h_R = 101 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}$$



$$T_x = a + bx + cx^2 \quad a = 82.0^\circ\text{C}$$

$x$  - meters  
 $T$  -  $^\circ\text{C}$

$$b = -210^\circ\text{C}/\text{m}$$

$$c = -2 \times 10^4 \frac{^\circ\text{C}}{\text{m}^2}$$

2.25] continued

e) heat flux distribution  $q''(x)$

$$q''_x(x) = -k \left( \frac{\partial T}{\partial x} \right) = -k(b + 2cx) = -5(-210 + 2(-2 \times 10^4)x) \\ = 1050 + 2 \times 10^5 x$$

$$-1050 = -2(10^5)x \quad x = \frac{1050}{2(10^5)}$$

$$\text{flux} = 0 @ \quad x = -5.25 \text{ mm} \quad t_{\max} \text{ is here}$$

f)  $\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \dot{q}''' = \rho C_p \frac{\partial T}{\partial t}$

$$k(2c) = 5 \frac{\text{W}}{\text{m} \cdot \text{K}} (-2(10^4) \frac{\text{K}}{\text{m}}) = -2 \times 10^5 \frac{\text{W}}{\text{m}^2}$$

g)

$$\rho = 2600 \text{ kg/m}^3 \quad \text{wall will eventually reach } 20^\circ\text{C}$$

$$C_p = 800 \frac{\text{J}}{\text{kg} \cdot \text{K}}$$

$$E_{\text{in}}'' - E_{\text{out}}'' = \Delta E_{\text{st}}'' = E_f'' - E_i''$$

$$E_{\text{out}}'' = \rho C_p \int_{-L}^L (T_i - T_\infty) dx$$

$$= \rho C_p \int_{-L}^L [a + bx + cx^2 - T_\infty] dx = \rho C_p \left[ ax + \frac{bx^2}{2} + \frac{cx^3}{3} - T_\infty x \right]_{-L}^L$$

$$= \rho C_p \left[ aL(2) + \frac{2cL^3}{3} + -2T_\infty L \right]$$

$$= 2(2600) \frac{\text{kg}}{\text{m}^3} 800 \frac{\text{J}}{\text{kg} \cdot \text{K}} \left[ 82^\circ\text{C}(.02\text{m}) + \frac{-2 \times 10^4 \frac{\text{K}}{\text{m}^2} (.02)^3}{3} - 20^\circ\text{C}(.02\text{m}) \right]$$

$$= 4.94 \times 10^6 \frac{\text{J}}{\text{m}^2}$$

Danar Malaney

2.46

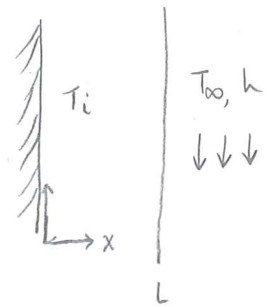
$$a) \quad \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) = \rho c_p \frac{\partial T}{\partial t}$$

$$k \frac{\partial^2 T}{\partial x^2} = \rho c_p \frac{\partial T}{\partial t}$$

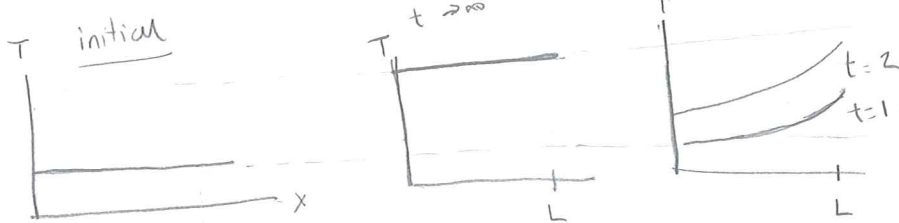
initial conditions  $t \leq 0, \quad T_x = T_i$

boundary cond. at  $x=0, \quad q''_x(0) = 0 \quad \frac{\partial T}{\partial x} = 0$

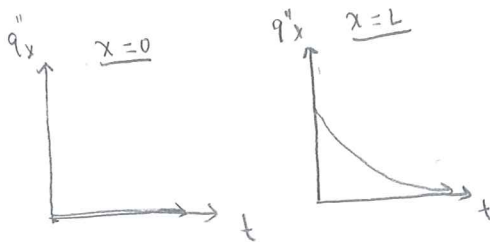
at  $x=L \quad -k \frac{\partial T}{\partial x} = h [T(L,t) - T_\infty]$



b)



c)



d) Total energy transferred to the wall ( $J/m^2$ )

$$E_{in} = \int_0^\infty q''_{conv} A_s dt = h A_s \int_0^\infty (T_\infty - T(L,t)) dt$$

$$\boxed{\frac{E_{in}}{V} = \frac{h}{L} \int_0^\infty (T_\infty - T(L,t)) dt}$$



3.2)

$$T_{\infty, i} = 40^{\circ}\text{C}$$

$$h_i = 30 \text{ W/m}^2 \cdot \text{K}$$

find T of glass surfaces

$$k_{\text{glass}} = 1.4 \text{ W/m} \cdot \text{K}$$

$$q'' = \frac{T_{\infty, i} - T_{\infty, o}}{\frac{1}{h_o} + \frac{L}{k} + \frac{1}{h_i}} = \frac{50^{\circ}\text{C}}{\frac{1}{65} + \frac{0.004}{30} + \frac{1}{30}} = 969 \frac{\text{W}}{\text{m}^2}$$

$$T_{s, i} = T_{\infty, i} - \frac{q''}{h_i} = 40 - \frac{969}{30} = 7.7^{\circ}\text{C} \text{ inside}$$

$$T_{s, o} = T_{\infty, o} + \frac{q''}{h_o} = -10 + \frac{969}{65} = 4.9^{\circ}\text{C} \text{ outside}$$

b)? -5

3.13)

a)

$$R_{\text{tot}} = \sum R_i = \frac{L_p}{k_p A} + \frac{L_b}{k_b A} + \frac{L_s}{k_s A} + \frac{1}{h_i A} + \frac{1}{h_o A}$$

b)

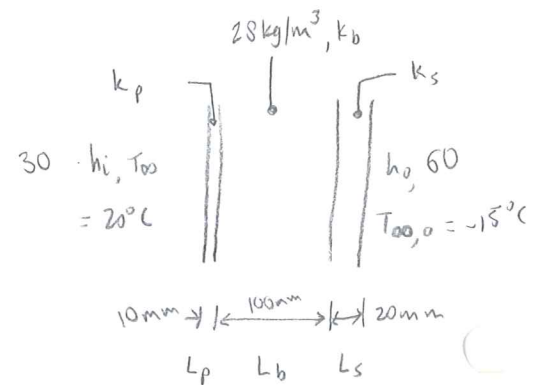
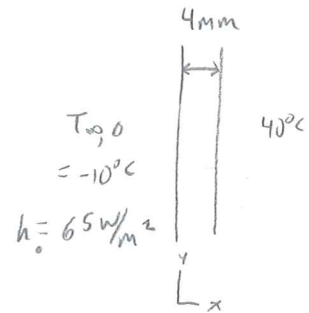
$$q = \frac{20 - (-15)}{R_{\text{tot}}} = \frac{35}{831 \times 10^{-5}} = 4.21 \text{ kW}$$

c)

$$R_{\text{tot}} \Rightarrow 826 \times 10^{-5} \Rightarrow q = \uparrow 0.6\% \text{ with wind blowing}$$

d)

glass fiber blanket is the controlling resistance,  $\frac{L_b}{k_b A} = \frac{1}{0.38} = 2.631$



$$\frac{1}{r} \frac{d}{dr} \left( r \frac{dT}{dr} \right) + \frac{\dot{q}}{k} = 0$$

$$\frac{1}{r} \frac{d}{dr} \left( r \frac{dT}{dr} \right) = - \frac{\dot{q}_0 (1 - (r/r_0)^2)}{k}$$

$$\frac{d}{dr} \left( r \frac{dT}{dr} \right) = - \frac{r \dot{q}_0}{k} + \frac{r \dot{q}_0}{k} \left( \frac{r^2}{r_0^2} \right) \quad \text{integrate}$$

$$r \frac{dT}{dr} = - \frac{r^2}{2} \frac{\dot{q}_0}{k} + \frac{\dot{q}_0}{k r_0^2} \left( \frac{r^4}{4} \right) + C_1 \quad \text{divide by } r \text{ \& integrate}$$

$$T(r) = - \frac{r^2}{4} \frac{\dot{q}_0}{k} + \frac{\dot{q}_0}{k r_0^2} \frac{r^4}{16} + C_1 \ln r + C_2$$

B.C.'s: ①  $\left. \frac{dT}{dr} \right|_{r=0} = 0$

$$- \frac{r}{2} \frac{\dot{q}_0}{k} + \frac{\dot{q}_0}{k r_0^2} \left( \frac{r^3}{4} \right) + \frac{C_1}{r} = 0 \Rightarrow C_1 = 0$$

②  $T(r=r_0) = T_s$

$$- \frac{r^2}{4} \frac{\dot{q}_0}{k} + \frac{\dot{q}_0 r^4}{k r_0^2 16} + C_2 = T_s$$

$$- \frac{r_0^2 \dot{q}_0}{4k} + \frac{\dot{q}_0 r_0^2}{k 16} + C_2 = T_s \Rightarrow C_2 = T_s + \left( \frac{3}{16} \right) \frac{\dot{q}_0 r_0^2}{k}$$

$$T(r) = \frac{r^4 \dot{q}_0}{16 k r_0^2} - \frac{r^2 \dot{q}_0}{4k} + \left( \frac{3}{16} \right) \frac{\dot{q}_0 r_0^2}{k} + T_s$$

mesh 1  $150 + 20i_1 + 10(i_1 - i_2) = 0$

mesh 2  $16i_2 - 100 + 10(i_2 - i_1) = 0$

$25i_2 - 10i_1 = 100$

$2.5i_2 = 10 + i_1$

heat flux  $q_x''$  is the heat transfer rate in x-direction  $W/m^2$   
per unit area perpendicular to direction of transfer

$q_x'$  = heat transfer rate per unit length  $W/m$

$\dot{q}$  = rate of energy generation per unit volume  $W/m^3$

Pg 133

OK

Repeated

integration

correctly

$$\int \left[ \frac{1}{r} \frac{d}{dr} \left( r \frac{dT}{dr} \right) \right] dr \Rightarrow r \left( \frac{dT}{dr} \right)$$

$$\frac{1}{r} \frac{d}{dr} \left( r \frac{dT}{dr} \right) + \frac{\dot{q}}{k} = 0$$

$$\int \frac{d}{dr} \left( r \frac{dT}{dr} \right) = \int - \frac{\dot{q}r}{k}$$

$$r \frac{dT}{dr} = - \frac{\dot{q}r^2}{2} + C_1$$

$$\int \left( \frac{dT}{dr} \right) = \int \left( - \frac{\dot{q}r}{2} + \frac{C_1}{r} \right)$$

$$T(r) = - \frac{\dot{q}r^2}{4} + C_1 \ln r + C_2$$

$$\int \int -\dot{q}r = \int -\frac{\dot{q}r^2}{2} + C = -\frac{\dot{q}r^3}{6} + C_1r + C_2$$

$$\frac{1}{r} \left( -\frac{\dot{q}r^3}{6} + C_1r + C_2 \right)$$

$$= -\frac{\dot{q}r^2}{6} + C_1 + \frac{C_2}{r}$$



225 math prob.

$$\frac{1}{r} \frac{d}{dr} \left( kr \frac{dT}{dr} \right) + \dot{q} = 0$$

$$u = \frac{1}{r} \quad du = \frac{d}{dr} \left( kr \frac{dT}{dr} \right)$$

$$du = -r^{-2} \quad v = kr \left( \frac{dT}{dr} \right)$$

$$\int = uv - \int v du = \frac{1}{r} \left( kr \frac{dT}{dr} \right) - \int \underbrace{kr \left( \frac{dT}{dr} \right)}_{\textcircled{1}} \frac{dr}{r^2}$$

$$\textcircled{1} \quad \int \frac{kr}{r^2} \left( \frac{dT}{dr} \right) dr = k \int \frac{1}{r} \left( \frac{dT}{dr} \right) dr$$

$$u = \frac{1}{r} \quad dw = \left( \frac{dT}{dr} \right) dr$$

$$du = -r^{-2} \quad v = T(r) = \underbrace{\left( \frac{1}{r} \right) T(r) - \int \frac{T(r)}{-r^2} dr}_{\textcircled{2}}$$

$$\textcircled{2} \quad \int \frac{T(r)}{-r^2} dr = \frac{T}{r} + C$$

$$\Rightarrow \frac{1}{r} \left( kr \frac{dT}{dr} \right) - \left[ \left( \frac{1}{r} \right) T - \left( \frac{T}{r} + C \right) \right]$$

$$= k \frac{dT}{dr} + \cancel{\frac{T}{r}} - \cancel{\frac{T}{r}} + C$$

$$= k \frac{dT}{dr} + q' r + C$$

David Maloney

100

225 HW #1

P. 1.6, 22, 24, 55, 57

1.6]  $W = 1\text{ m}$   
 $H = 2\text{ m}$

5 mm thick  $k_g = 1.4\text{ W/m}\cdot\text{K}$

$$\frac{dT}{dx} = \frac{T_2 - T_1}{L} = \frac{(-20 - 15)}{.005\text{ m}}$$

$$T_{in} = 15^\circ\text{C}$$
$$T_{out} = -20^\circ\text{C}$$

$$\dot{Q}_{out} = ?$$

$$q''_x = -k \frac{dT}{dx} = -1.4 \frac{\text{W}}{\text{m}\cdot\text{K}} \left( \frac{-35\text{ K}}{.005\text{ m}} \right)$$

$$q''_x = 9800\text{ W/m}^2 \quad q_x = (9800)(2\text{ m}^2) = 19600\text{ W} \quad \boxed{19.6\text{ kW}}$$

10 mm air  $10^\circ\text{C}, -15^\circ\text{C}$   $1 \times 2\text{ m window}$

$$k_a = .024 \frac{\text{W}}{\text{m}\cdot\text{K}}$$

$$q_x = (-.024) \left( \frac{(-15 - 10)}{(.01)} \right) (2\text{ m}^2) = \boxed{120\text{ W}} \quad \checkmark$$

1.22]

Plate  $T = 225^\circ\text{C}$

$$\frac{dT}{dt} = -.022\text{ K/s}$$

$$T_{air} = 25^\circ\text{C}$$

plate  $.3 \times .3\text{ m}$

$$W = 3.75\text{ kg}$$

find convection coefficient  $h$  ( $\text{W/m}^2\cdot\text{K}$ )

$$c_p = 2770 \frac{\text{J}}{\text{kg}\cdot\text{K}}$$

$$\dot{Q} = -228.5\text{ W}$$

$$\dot{Q} = c_p \frac{\Delta T}{\Delta t} = \frac{-.022\text{ K}}{\text{s}} (2770) = -60.9 \frac{\text{J}}{\text{kg}\cdot\text{s}} (3.75\text{ kg})$$

$$q'' = h(T_s - T_{\infty}) \quad T_s = 225^\circ\text{C} \quad T_{\infty} = 25^\circ\text{C}$$

$$h = \frac{-228.5\text{ W}}{2 \times (.3)^2\text{ m}^2 (200\text{ K})} = \boxed{6.35 \frac{\text{W}}{\text{m}^2\cdot\text{K}}} = h \quad \checkmark$$

1.24

$$T_{\text{air}} = 20^\circ\text{C}$$

$$T_{\text{walls}} \begin{matrix} 27^\circ\text{C summer} \\ 14^\circ\text{C winter} \end{matrix} \quad T_{\text{person}} = 32^\circ\text{C}$$

$$h = 2 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}$$

$$q''_{\text{rad}} = \epsilon \sigma (T_s^4 - T_{\text{sur}}^4) \quad q''_{\text{summer}} = (.9)(5.67 \times 10^{-8}) \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4} (305^4 - 300^4) = 28.3 \text{ W/m}^2$$

$$q''_{\text{winter}} = (.9)(5.67 \times 10^{-8}) \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4} (305^4 - 287^4) = 95.4 \text{ W/m}^2 \quad \checkmark$$

heat is lost from the body at 3.4 times faster in the winter

1.55

$$\text{absorb from } 800 \text{ W/m}^2$$

$$h = 12 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$a) \quad T_{\text{air}} = 20^\circ\text{C} \quad T_{\text{roof}} = ? \quad (\text{steady state})$$

$$q''_{\text{rad}} - q''_{\text{cond}} - q''_{\text{conv}} = 0$$

$$800 \frac{\text{W}}{\text{m}^2} - h(T_s - 20^\circ\text{C}) = 0$$

$$800 = 12(T_{\text{roof}} - 20) \quad \boxed{T_{\text{roof}} = 86^\circ\text{C}}$$

$$b) \quad \text{surface emissivity} = 0.8$$

$$q''_{s, \text{abs}} A_s - q''_{\text{conv}} - E \cdot A_s = 0$$

$$q''_{s, \text{abs}} A_s - h A_s (T_s - T_{\infty}) - \epsilon A_s \sigma T_s^4 = 0 \quad 800 \frac{\text{W}}{\text{m}^2} - 12 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} (T_s - 293\text{K}) - 0.8(5.67 \times 10^{-8}) = 0$$

$$12T_s + 4.536 \times 10^{-8} T_s^4 = 4316$$

$$\boxed{T_s = 320\text{K} = 47^\circ\text{C}} \quad \checkmark$$

c) ?

1.57

$$A = 1.8 \text{ m}^2 \quad 2100 \text{ kcal} \quad 2000 \text{ kcal thermal energy} \quad T_{\text{air}} = 20^\circ\text{C} \quad h = 3 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}$$

a)

$$T_{\text{skin}} = ? \quad \epsilon = 0.95$$

$$\frac{2000 \times 10^3 \text{ cal}}{0.239 \frac{\text{cal}}{\text{J}} (86400 \text{ s/day})} =$$

$$96.9 \text{ W}$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$96.9 \text{ W} = h A (T_s - T_{\infty}) + \epsilon \sigma A (T_s^4 - T_{\text{sur}}^4) = 3(1.8)(T_s - 293) + .95(5.67 \times 10^{-8})(1.8)(T_s^4 - 293^4)$$

$$\boxed{T_s = 299\text{K} = 26^\circ\text{C}}$$

b)

$$\dot{E}_{\text{out}} = \dot{m} h_{fg}$$

$$\dot{m} =$$

$$\frac{\dot{E}_{\text{out}}}{h_{fg}} =$$

$$\frac{96.9 \text{ W}}{2421 \text{ kJ/kg}}$$

$$=$$

$$\boxed{4.0 \times 10^{-5} \text{ kg/s}}$$

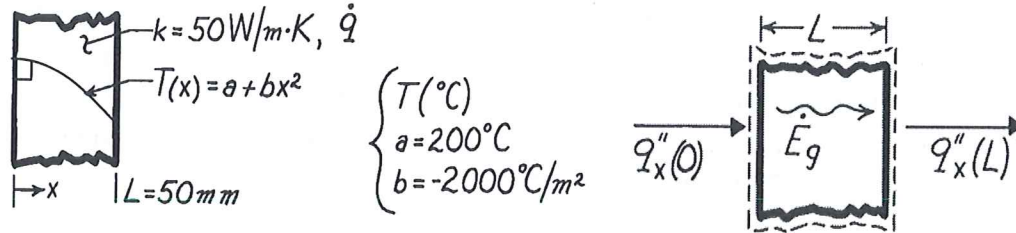
$$= \boxed{4 \times 10^{-5} \text{ l/s}} \quad \checkmark$$

**PROBLEM 2.23**

**KNOWN:** Temperature distribution in a one-dimensional wall with prescribed thickness and thermal conductivity.

**FIND:** (a) The heat generation rate,  $\dot{q}$ , in the wall, (b) Heat fluxes at the wall faces and relation to  $\dot{q}$ .

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Steady-state conditions, (2) One-dimensional heat flow, (3) Constant properties.

**ANALYSIS:** (a) The appropriate form of the heat equation for steady-state, one-dimensional conditions with constant properties is Eq. 2.19 re-written as

$$\dot{q} = -k \frac{d}{dx} \left[ \frac{dT}{dx} \right]$$

Substituting the prescribed temperature distribution,

$$\dot{q} = -k \frac{d}{dx} \left[ \frac{d}{dx} (a + bx^2) \right] = -k \frac{d}{dx} [2bx] = -2bk$$

$$\dot{q} = -2(-2000^\circ\text{C}/\text{m}^2) \times 50 \text{ W}/\text{m} \cdot \text{K} = 2.0 \times 10^5 \text{ W}/\text{m}^3.$$

(b) The heat fluxes at the wall faces can be evaluated from Fourier's law,

$$q''_x(x) = -k \left. \frac{dT}{dx} \right|_x.$$

Using the temperature distribution  $T(x)$  to evaluate the gradient, find

$$q''_x(x) = -k \frac{d}{dx} [a + bx^2] = -2kbx.$$

The fluxes at  $x = 0$  and  $x = L$  are then

$$q''_x(0) = 0$$

$$q''_x(L) = -2kLb = -2 \times 50 \text{ W}/\text{m} \cdot \text{K} (-2000^\circ\text{C}/\text{m}^2) \times 0.050 \text{ m}$$

$$q''_x(L) = 10,000 \text{ W}/\text{m}^2.$$

**COMMENTS:** From an overall energy balance on the wall, it follows that, for a unit area,

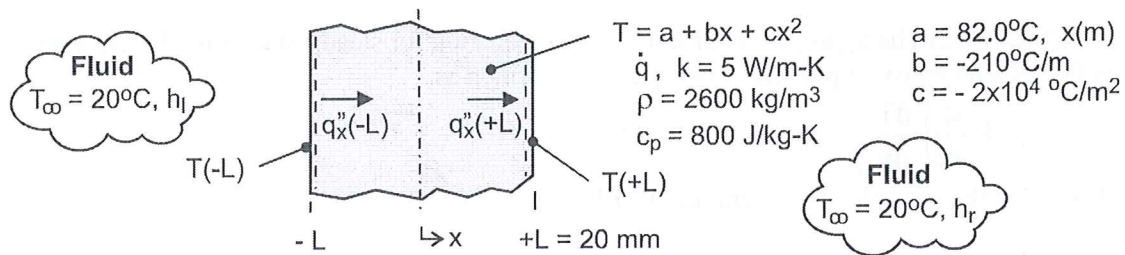
$$\begin{aligned} \dot{E}_{\text{in}} - \dot{E}_{\text{out}} + \dot{E}_g &= 0 & q''_x(0) - q''_x(L) + \dot{q}L &= 0 \\ \dot{q} &= \frac{q''_x(L) - q''_x(0)}{L} = \frac{10,000 \text{ W}/\text{m}^2 - 0}{0.050 \text{ m}} = 2.0 \times 10^5 \text{ W}/\text{m}^3. \end{aligned}$$

## PROBLEM 2.25

**KNOWN:** Analytical expression for the steady-state temperature distribution of a plane wall experiencing uniform volumetric heat generation  $\dot{q}$  while convection occurs at both of its surfaces.

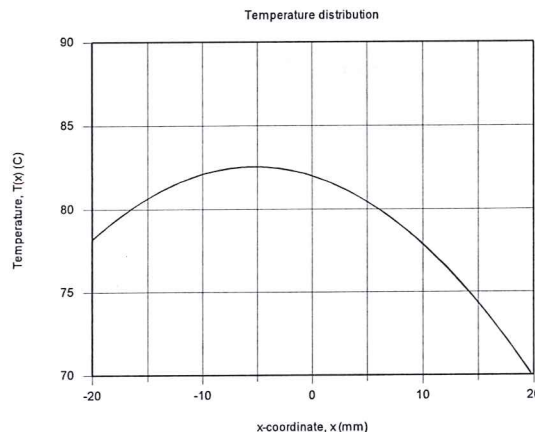
**FIND:** (a) Sketch the temperature distribution,  $T(x)$ , and identify significant physical features, (b) Determine  $\dot{q}$ , (c) Determine the surface heat fluxes,  $q''_x(-L)$  and  $q''_x(+L)$ ; how are these fluxes related to the generation rate; (d) Calculate the convection coefficients at the surfaces  $x = L$  and  $x = -L$ , (e) Obtain an expression for the heat flux distribution,  $q''_x(x)$ ; explain significant features of the distribution; (f) If the source of heat generation is suddenly deactivated ( $\dot{q} = 0$ ), what is the rate of change of energy stored at this instant; (g) Determine the temperature that the wall will reach eventually with  $\dot{q} = 0$ ; determine the energy that must be removed by the fluid per unit area of the wall to reach this state.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Steady-state conditions, (2) Uniform volumetric heat generation, (3) Constant properties.

**ANALYSIS:** (a) Using the analytical expression in the Workspace of IHT, the temperature distribution appears as shown below. The significant features include (1) parabolic shape, (2) maximum does not occur at the mid-plane,  $T(-5.25 \text{ mm}) = 83.3^\circ\text{C}$ , (3) the gradient at the  $x = +L$  surface is greater than at  $x = -L$ . Find also that  $T(-L) = 78.2^\circ\text{C}$  and  $T(+L) = 69.8^\circ\text{C}$  for use in part (d).



(b) Substituting the temperature distribution expression into the appropriate form of the heat diffusion equation, Eq. 2.19, the rate of volumetric heat generation can be determined.

$$\frac{d}{dx} \left( \frac{dT}{dx} \right) + \frac{\dot{q}}{k} = 0 \quad \text{where} \quad T(x) = a + bx + cx^2$$

$$\frac{d}{dx} (0 + b + 2cx) + \frac{\dot{q}}{k} = (0 + 2c) + \frac{\dot{q}}{k} = 0$$

Continued .....



### PROBLEM 2.25 (Cont.)

$$\dot{q} = -2ck = -2\left(-2 \times 10^4 \text{ }^\circ\text{C/m}^2\right) 5 \text{ W/m} \cdot \text{K} = 2 \times 10^5 \text{ W/m}^3 \quad <$$

(c) The heat fluxes at the two boundaries can be determined using Fourier's law and the temperature distribution expression.

$$q_x''(x) = -k \frac{dT}{dx} \quad \text{where} \quad T(x) = a + bx + cx^2$$

$$q_x''(-L) = -k[0 + b + 2cx]_{x=-L} = -[b - 2cL]k$$

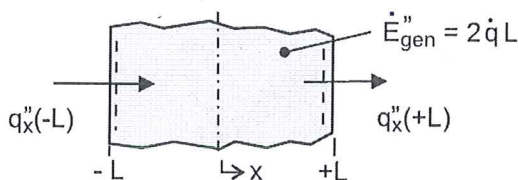
$$q_x''(-L) = -\left[-210^\circ\text{C/m} - 2\left(-2 \times 10^4 \text{ }^\circ\text{C/m}^2\right) 0.020 \text{ m}\right] \times 5 \text{ W/m} \cdot \text{K} = -2950 \text{ W/m}^2 \quad <$$

$$q_x''(+L) = -(b + 2cL)k = +5050 \text{ W/m}^2 \quad <$$

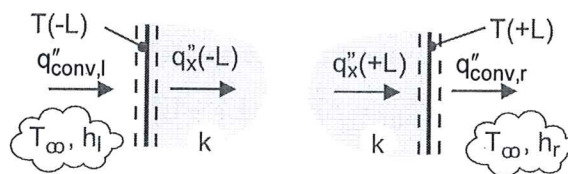
From an overall energy balance on the wall as shown in the sketch below,  $\dot{E}_{\text{in}} - \dot{E}_{\text{out}} + \dot{E}_{\text{gen}} = 0$ ,

$$+q_x''(-L) - q_x''(+L) + 2\dot{q}L = 0 \quad \text{or} \quad -2950 \text{ W/m}^2 - 5050 \text{ W/m}^2 + 8000 \text{ W/m}^2 = 0$$

where  $2\dot{q}L = 2 \times 2 \times 10^5 \text{ W/m}^3 \times 0.020 \text{ m} = 8000 \text{ W/m}^2$ , so the equality is satisfied



Part (c) Overall energy balance



Part (d) Surface energy balances

(d) The convection coefficients,  $h_l$  and  $h_r$ , for the left- and right-hand boundaries ( $x = -L$  and  $x = +L$ , respectively), can be determined from the convection heat fluxes that are equal to the conduction fluxes at the boundaries. See the surface energy balances in the sketch above. See also part (a) result for  $T(-L)$  and  $T(+L)$ .

$$q_{\text{conv},l}'' = q_x''(-L)$$

$$h_l [T_\infty - T(-L)] = h_l [20 - 78.2] \text{ K} = -2950 \text{ W/m}^2 \quad h_l = 51 \text{ W/m}^2 \cdot \text{K} \quad <$$

$$q_{\text{conv},r}'' = q_x''(+L)$$

$$h_r [T(+L) - T_\infty] = h_r [69.8 - 20] \text{ K} = +5050 \text{ W/m}^2 \quad h_r = 101 \text{ W/m}^2 \cdot \text{K} \quad <$$

(e) The expression for the heat flux distribution can be obtained from Fourier's law with the temperature distribution

$$q_x''(x) = -k \frac{dT}{dx} = -k[0 + b + 2cx]$$

$$q_x''(x) = -5 \text{ W/m} \cdot \text{K} \left[ -210^\circ\text{C/m} + 2\left(-2 \times 10^4 \text{ }^\circ\text{C/m}^2\right) x \right] = 1050 + 2 \times 10^5 x \quad <$$

Continued .....

### PROBLEM 2.25 (Cont.)

The distribution is linear with the  $x$ -coordinate. The maximum temperature will occur at the location where  $q_x''(x_{\max}) = 0$ ,

$$x_{\max} = -\frac{1050 \text{ W/m}^2}{2 \times 10^5 \text{ W/m}^3} = -5.25 \times 10^{-3} \text{ m} = -5.25 \text{ mm} \quad <$$

(f) If the source of the heat generation is suddenly deactivated so that  $\dot{q} = 0$ , the appropriate form of the heat diffusion equation for the ensuing transient conduction is

$$k \frac{\partial}{\partial x} \left( \frac{\partial T}{\partial x} \right) = \rho c_p \frac{\partial T}{\partial t}$$

At the instant this occurs, the temperature distribution is still  $T(x) = a + bx + cx^2$ . The right-hand term represents the rate of energy storage per unit volume,

$$\dot{E}_{\text{st}}'' = k \frac{\partial}{\partial x} [0 + b + 2cx] = k [0 + 2c] = 5 \text{ W/m} \cdot \text{K} \times 2 \left( -2 \times 10^4 \text{ }^\circ\text{C/m}^2 \right) = -2 \times 10^5 \text{ W/m}^3 \quad <$$

(g) With no heat generation, the wall will eventually ( $t \rightarrow \infty$ ) come to equilibrium with the fluid,  $T(x, \infty) = T_\infty = 20^\circ\text{C}$ . To determine the energy that must be removed from the wall to reach this state, apply the conservation of energy requirement over an interval basis, Eq. 1.11b. The “initial” state is that corresponding to the steady-state temperature distribution,  $T_i$ , and the “final” state has  $T_f = 20^\circ\text{C}$ . We’ve used  $T_\infty$  as the reference condition for the energy terms.

$$E_{\text{in}}'' - E_{\text{out}}'' = \Delta E_{\text{st}}'' = E_f'' - E_i'' \quad \text{with} \quad E_{\text{in}}'' = 0.$$

$$E_{\text{out}}'' = c_p \int_{-L}^{+L} (T_i - T_\infty) dx$$

$$E_{\text{out}}'' = \rho c_p \int_{-L}^{+L} [a + bx + cx^2 - T_\infty] dx = \rho c_p \left[ ax + bx^2/2 + cx^3/3 - T_\infty x \right]_{-L}^{+L}$$

$$E_{\text{out}}'' = \rho c_p \left[ 2aL + 0 + 2cL^3/3 - 2T_\infty L \right]$$

$$E_{\text{out}}'' = 2600 \text{ kg/m}^3 \times 800 \text{ J/kg} \cdot \text{K} \left[ 2 \times 82^\circ\text{C} \times 0.020 \text{ m} + 2 \left( -2 \times 10^4 \text{ }^\circ\text{C/m}^2 \right) \right. \\ \left. (0.020 \text{ m})^3 / 3 - 2(20^\circ\text{C})0.020 \text{ m} \right]$$

$$E_{\text{out}}'' = 4.94 \times 10^6 \text{ J/m}^2 \quad <$$

**COMMENTS:** (1) In part (a), note that the temperature gradient is larger at  $x = +L$  than at  $x = -L$ . This is consistent with the results of part (c) in which the conduction heat fluxes are evaluated.

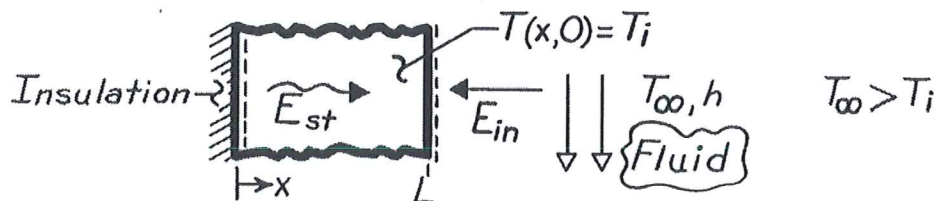
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## PROBLEM 2.46

**KNOWN:** Plane wall, initially at a uniform temperature, is suddenly exposed to convective heating.

**FIND:** (a) Differential equation and initial and boundary conditions which may be used to find the temperature distribution,  $T(x,t)$ ; (b) Sketch  $T(x,t)$  for these conditions: initial ( $t \leq 0$ ), steady-state,  $t \rightarrow \infty$ , and two intermediate times; (c) Sketch heat fluxes as a function of time for surface locations; (d) Expression for total energy transferred to wall per unit volume ( $\text{J/m}^3$ ).

**SCHEMATIC:**



**ASSUMPTIONS:** (1) One-dimensional conduction, (2) Constant properties, (3) No internal heat generation.

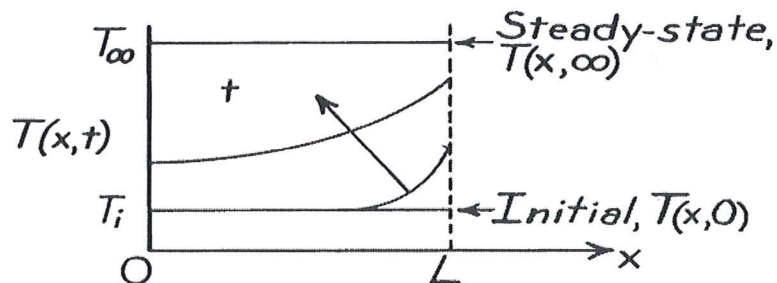
**ANALYSIS:** (a) For one-dimensional conduction with constant properties, the heat equation has the form,

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

and the conditions are:

$$\begin{cases} \text{Initial, } t \leq 0: & T(x,0) = T_i \\ \text{Boundaries: } x=0 & \partial T / \partial x|_0 = 0 \\ & x=L & -k \partial T / \partial x|_L = h [T(L,t) - T_\infty] \end{cases} \quad \begin{matrix} \text{uniform} \\ \text{adiabatic} \\ \text{convection} \end{matrix}$$

(b) The temperature distributions are shown on the sketch.



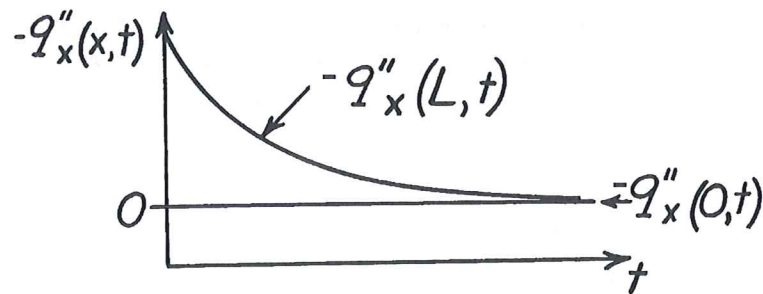
Note that the gradient at  $x = 0$  is always zero, since this boundary is adiabatic. Note also that the gradient at  $x = L$  decreases with time.

(c) The heat flux,  $q_x''(x,t)$ , as a function of time, is shown on the sketch for the surfaces  $x = 0$  and  $x = L$ .

Continued .....



PROBLEM 2.46 (Cont.)



For the surface at  $x = 0$ ,  $q''_x(0, t) = 0$  since it is adiabatic. At  $x = L$  and  $t = 0$ ,  $q''_x(L, 0)$  is a maximum (in magnitude)

$$|q''_x(L, 0)| = h|T(L, 0) - T_\infty|$$

where  $T(L, 0) = T_i$ . The temperature difference, and hence the flux, decreases with time.

(d) The total energy transferred to the wall may be expressed as

$$E_{in} = \int_0^\infty q''_{conv} A_s dt$$

$$E_{in} = h A_s \int_0^\infty (T_\infty - T(L, t)) dt$$

Dividing both sides by  $A_s L$ , the energy transferred per unit volume is

$$\frac{E_{in}}{V} = \frac{h}{L} \int_0^\infty [T_\infty - T(L, t)] dt \quad \left[ \text{J/m}^3 \right]$$

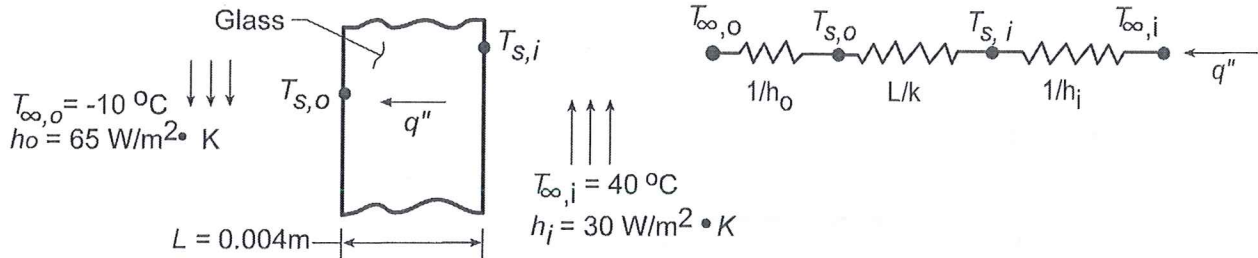
**COMMENTS:** Note that the heat flux at  $x = L$  is into the wall and is hence in the negative  $x$  direction.

### PROBLEM 3.2

**KNOWN:** Temperatures and convection coefficients associated with air at the inner and outer surfaces of a rear window.

**FIND:** (a) Inner and outer window surface temperatures,  $T_{s,i}$  and  $T_{s,o}$ , and (b)  $T_{s,i}$  and  $T_{s,o}$  as a function of the outside air temperature  $T_{\infty,o}$  and for selected values of outer convection coefficient,  $h_o$ .

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible radiation effects, (4) Constant properties.

**PROPERTIES:** Table A-3, Glass (300 K):  $k = 1.4 \text{ W/m}\cdot\text{K}$ .

**ANALYSIS:** (a) The heat flux may be obtained from Eqs. 3.11 and 3.12,

$$q'' = \frac{T_{\infty,i} - T_{\infty,o}}{\frac{1}{h_o} + \frac{L}{k} + \frac{1}{h_i}} = \frac{40^\circ\text{C} - (-10^\circ\text{C})}{\frac{1}{65 \text{ W/m}^2 \cdot \text{K}} + \frac{0.004 \text{ m}}{1.4 \text{ W/m} \cdot \text{K}} + \frac{1}{30 \text{ W/m}^2 \cdot \text{K}}}$$

$$q'' = \frac{50^\circ\text{C}}{(0.0154 + 0.0029 + 0.0333) \text{ m}^2 \cdot \text{K/W}} = 969 \text{ W/m}^2.$$

Hence, with  $q'' = h_i (T_{\infty,i} - T_{s,i})$ , the inner surface temperature is

$$T_{s,i} = T_{\infty,i} - \frac{q''}{h_i} = 40^\circ\text{C} - \frac{969 \text{ W/m}^2}{30 \text{ W/m}^2 \cdot \text{K}} = 7.7^\circ\text{C} \quad <$$

Similarly for the outer surface temperature with  $q'' = h_o (T_{s,o} - T_{\infty,o})$  find

$$T_{s,o} = T_{\infty,o} + \frac{q''}{h_o} = -10^\circ\text{C} + \frac{969 \text{ W/m}^2}{65 \text{ W/m}^2 \cdot \text{K}} = 4.9^\circ\text{C} \quad <$$

(b) Using the same analysis,  $T_{s,i}$  and  $T_{s,o}$  have been computed and plotted as a function of the outside air temperature,  $T_{\infty,o}$ , for outer convection coefficients of  $h_o = 2, 65$ , and  $100 \text{ W/m}^2 \cdot \text{K}$ . As expected,  $T_{s,i}$  and  $T_{s,o}$  are linear with changes in the outside air temperature. The difference between  $T_{s,i}$  and  $T_{s,o}$  increases with increasing convection coefficient, since the heat flux through the window likewise increases. This difference is larger at lower outside air temperatures for the same reason. Note that with  $h_o = 2 \text{ W/m}^2 \cdot \text{K}$ ,  $T_{s,i} - T_{s,o}$ , is too small to show on the plot.

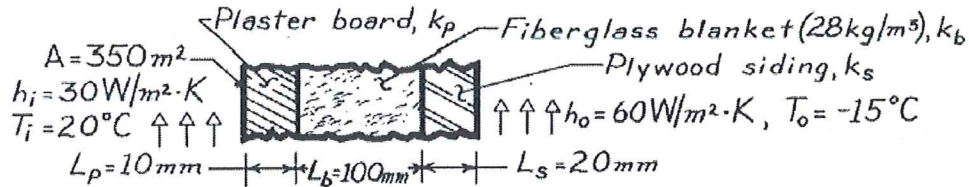
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### PROBLEM 3.13

**KNOWN:** Composite wall of a house with prescribed convection processes at inner and outer surfaces.

**FIND:** (a) Expression for thermal resistance of house wall,  $R_{\text{tot}}$ ; (b) Total heat loss,  $q$  (W); (c) Effect on heat loss due to increase in outside heat transfer convection coefficient,  $h_o$ ; and (d) Controlling resistance for heat loss from house.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) One-dimensional conduction, (2) Steady-state conditions, (3) Negligible contact resistance.

**PROPERTIES:** Table A-3,  $\left(\bar{T} = (T_i + T_o)/2 = (20 - 15)^\circ\text{C}/2 = 2.5^\circ\text{C} \approx 300\text{K}\right)$ : Fiberglass blanket,  $28 \text{ kg/m}^3$ ,  $k_b = 0.038 \text{ W/m}\cdot\text{K}$ ; Plywood siding,  $k_s = 0.12 \text{ W/m}\cdot\text{K}$ ; Plasterboard,  $k_p = 0.17 \text{ W/m}\cdot\text{K}$ .

**ANALYSIS:** (a) The expression for the total thermal resistance of the house wall follows from Eq. 3.18.

$$R_{\text{tot}} = \frac{1}{h_i A} + \frac{L_p}{k_p A} + \frac{L_b}{k_b A} + \frac{L_s}{k_s A} + \frac{1}{h_o A}$$

(b) The total heat loss through the house wall is

$$q = \Delta T / R_{\text{tot}} = (T_i - T_o) / R_{\text{tot}}$$

Substituting numerical values, find

$$R_{\text{tot}} = \frac{1}{30 \text{ W/m}^2 \cdot \text{K} \times 350 \text{ m}^2} + \frac{0.01 \text{ m}}{0.17 \text{ W/m} \cdot \text{K} \times 350 \text{ m}^2} + \frac{0.10 \text{ m}}{0.038 \text{ W/m} \cdot \text{K} \times 350 \text{ m}^2} + \frac{0.02 \text{ m}}{0.12 \text{ W/m} \cdot \text{K} \times 350 \text{ m}^2} + \frac{1}{60 \text{ W/m}^2 \cdot \text{K} \times 350 \text{ m}^2}$$

$$R_{\text{tot}} = [9.52 + 16.8 + 752 + 47.6 + 4.76] \times 10^{-5} \text{ }^\circ\text{C/W} = 831 \times 10^{-5} \text{ }^\circ\text{C/W}$$

The heat loss is then,

$$q = [20 - (-15)]^\circ\text{C} / 831 \times 10^{-5} \text{ }^\circ\text{C/W} = 4.21 \text{ kW}$$

(c) If  $h_o$  changes from 60 to 300  $\text{W/m}^2\cdot\text{K}$ ,  $R_o = 1/h_o A$  changes from  $4.76 \times 10^{-5} \text{ }^\circ\text{C/W}$  to  $0.95 \times 10^{-5} \text{ }^\circ\text{C/W}$ . This reduces  $R_{\text{tot}}$  to  $826 \times 10^{-5} \text{ }^\circ\text{C/W}$ , which is a 0.6% decrease and hence a 0.6% increase in  $q$ .

(d) From the expression for  $R_{\text{tot}}$  in part (b), note that the insulation resistance,  $L_b/k_b A$ , is  $752/830 \approx 90\%$  of the total resistance. Hence, this material layer controls the resistance of the wall. From part (c) note that a 5-fold decrease in the outer convection resistance due to an increase in the wind velocity has a negligible effect on the heat loss.