ME 231

Fluids

Banerjee

Some common established nondimensional parameters or Π's encountered in fluid mechanics and heat transfer*

Name	Definition	Ratio of Significance
Archimedes number	$\mathrm{Ar} = \frac{\rho_s g L^3}{\mu^2}(\rho_s - \rho)$	$\frac{\text{Gravitational force}}{\text{Viscous force}}$
Aspect ratio	$\mathrm{AR} = \frac{L}{W}$ or $\frac{L}{D}$	$\frac{\text{Length}}{\text{Width}}$ or $\frac{\text{Length}}{\text{Diameter}}$
Biot number	$\mathrm{Bi} = \frac{hL}{k}$	$\frac{\text{Surface thermal resistance}}{\text{Internal thermal resistance}}$
Bond number	$\mathrm{Bo} = \frac{g(\rho_f - \rho_v)L^2}{\sigma_s}$	$\frac{\text{Gravitational force}}{\text{Surface tension force}}$
Cavitation number	$\mathrm{Ca}\ (\text{sometimes } \sigma_c) = \frac{P - P_v}{\rho V^2}$ $\left(\text{sometimes } \frac{2(P - P_v)}{\rho V^2}\right)$	$\frac{\text{Pressure} - \text{Vapor pressure}}{\text{Inertial pressure}}$
Darcy friction factor	$f = \frac{8\tau_w}{\rho V^2}$	$\frac{\text{Wall friction force}}{\text{Inertial force}}$
Drag coefficient	$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$	$\frac{\text{Drag force}}{\text{Dynamic force}}$
Eckert number	$\mathrm{Ec} = \frac{V^2}{c_P T}$	$\frac{\text{Kinetic energy}}{\text{Enthalpy}}$
Euler number	$\mathrm{Eu} = \frac{\Delta P}{\rho V^2} \left(\text{sometimes } \frac{\Delta P}{\frac{1}{2}\rho V^2}\right)$	$\frac{\text{Pressure difference}}{\text{Dynamic pressure}}$
Fanning friction factor	$C_f = \frac{2\tau_w}{\rho V^2}$	$\frac{\text{Wall friction force}}{\text{Inertial force}}$
Fourier number	$\mathrm{Fo}\ (\text{sometimes } \tau) = \frac{\alpha t}{L^2}$	$\frac{\text{Physical time}}{\text{Thermal diffusion time}}$
Froude number	$\mathrm{Fr} = \frac{V}{\sqrt{gL}} \left(\text{sometimes } \frac{V^2}{gL}\right)$	$\frac{\text{Inertial force}}{\text{Gravitational force}}$
Grashof number	$\mathrm{Gr} = \frac{g\beta \lvert\Delta T\rvert L^3 \rho^2}{\mu^2}$	$\frac{\text{Buoyancy force}}{\text{Viscous force}}$
Jakob number	$\mathrm{Ja} = \frac{c_p(T - T_{sat})}{h_{fg}}$	$\frac{\text{Sensible energy}}{\text{Latent energy}}$
Knudsen number	$\mathrm{Kn} = \frac{\lambda}{L}$	$\frac{\text{Mean free path length}}{\text{Characteristic length}}$
Lewis number	$\mathrm{Le} = \frac{k}{\rho c_p D_{AB}} = \frac{\alpha}{D_{AB}}$	$\frac{\text{Thermal diffusion}}{\text{Species diffusion}}$
Lift coefficient	$C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A}$	$\frac{\text{Lift force}}{\text{Dynamic force}}$
Mach number	$\mathrm{Ma}\ (\text{sometimes } M) = \frac{V}{c}$	$\frac{\text{Flow speed}}{\text{Speed of sound}}$
Nusselt number	$\mathrm{Nu} = \frac{Lh}{k}$	$\frac{\text{Convection heat transfer}}{\text{Conduction heat transfer}}$
Peclet number	$\mathrm{Pe} = \frac{\rho L V c_p}{k} = \frac{LV}{\alpha}$	$\frac{\text{Bulk heat transfer}}{\text{Conduction heat transfer}}$
Power number	$N_P = \frac{\dot{W}}{\rho D^5 \omega^3}$	$\frac{\text{Power}}{\text{Rotational inertia}}$
Prandtl number	$\mathrm{Pr} = \frac{\nu}{\alpha} = \frac{\mu c_p}{k}$	$\frac{\text{Viscous diffusion}}{\text{Thermal diffusion}}$
Pressure coefficient	$C_p = \frac{P - P_\infty}{\frac{1}{2}\rho V^2}$	$\frac{\text{Static pressure difference}}{\text{Dynamic pressure}}$
Rayleigh number	$\mathrm{Ra} = \frac{g\beta \lvert\Delta T\rvert L^3 \rho^2 c_p}{k\mu}$	$\frac{\text{Buoyancy force}}{\text{Viscous force}}$
Reynolds number	$\mathrm{Re} = \frac{\rho V L}{\mu} = \frac{VL}{\nu}$	$\frac{\text{Inertial force}}{\text{Viscous force}}$
Richardson number	$\mathrm{Ri} = \frac{L^5 g\, \Delta\rho}{\rho \dot{V}^2}$	$\frac{\text{Buoyancy force}}{\text{Inertial force}}$
Schmidt number	$\mathrm{Sc} = \frac{\mu}{\rho D_{AB}} = \frac{\nu}{D_{AB}}$	$\frac{\text{Viscous diffusion}}{\text{Species diffusion}}$
Sherwood number	$\mathrm{Sh} = \frac{VL}{D_{AB}}$	$\frac{\text{Overall mass diffusion}}{\text{Species diffusion}}$
Specific heat ratio	$k\ (\text{sometimes } \gamma) = \frac{c_p}{c_v}$	$\frac{\text{Enthalpy}}{\text{Internal energy}}$
Stanton number	$\mathrm{St} = \frac{h}{\rho c_p V}$	$\frac{\text{Heat transfer}}{\text{Thermal capacity}}$
Stokes number	$\mathrm{Stk}\ (\text{sometimes St}) = \frac{\rho_p D_p^2 V}{18 \mu L}$	$\frac{\text{Particle relaxation time}}{\text{Characteristic flow time}}$
Strouhal number	$\mathrm{St}\ (\text{sometimes S or Sr}) = \frac{fL}{V}$	$\frac{\text{Characteristic flow time}}{\text{Period of oscillation}}$
Weber number	$\mathrm{We} = \frac{\rho V^2 L}{\sigma_s}$	$\frac{\text{Inertial force}}{\text{Surface tension force}}$

* A is a characteristic area, D is a characteristic diameter, f is a characteristic frequency (Hz), L is a characteristic length, t is a characteristic time, T is a characteristic (absolute) temperature, V is a characteristic velocity, W is a characteristic width, $\dot{W}$ is a characteristic power, ω is a characteristic angular velocity (rad/s). Other parameters and fluid properties in these Π's include: c = speed of sound, c_p, c_v = specific heats, D_p = particle diameter, D_{AB} = species diffusion coefficient, h = convective heat transfer coefficient, h_{fg} = latent heat of evaporation, k = thermal conductivity, P = pressure, T_{sat} = saturation temperature, $\dot{V}$ = volume flow rate, α = thermal diffusivity, β = coefficient of thermal expansion, λ = mean free path length, μ = viscosity, ν = kinematic viscosity, ρ = fluid density, ρ_f = liquid density, ρ_p = particle density, ρ_s = solid density, ρ_v = vapor density, σ_s = surface tension, and τ_w = shear stress along a wall.

DIMENSIONS OF FLUID-MECHANICS PROPERTIES

Quantity	Symbol	Dimensions $\{MLT\Theta\}$	Dimensions $\{FLT\Theta\}$
Length	L	L	L
Area	A	L^2	L^2
Volume	$\mathcal{V}$	L^3	L^3
Velocity	V	LT^{-1}	LT^{-1}
Speed of sound	a	LT^{-1}	LT^{-1}
Volume flow	Q	L^3T^{-1}	L^3T^{-1}
Mass flow	$\dot{m}$	MT^{-1}	FTL^{-1}
Pressure, stress	p, σ	$ML^{-1}T^{-2}$	FL^{-2}
Strain rate	$\dot{\varepsilon}$	T^{-1}	T^{-1}
Angle	θ	None	None
Angular velocity	ω	T^{-1}	T^{-1}
Viscosity	μ	$ML^{-1}T^{-1}$	FTL^{-2}
Kinematic viscosity	ν	L^2T^{-1}	L^2T^{-1}
Surface tension	Υ	MT^{-2}	FL^{-1}
Force	F	MLT^{-2}	F
Moment, torque	M	ML^2T^{-2}	FL
Power	P	ML^2T^{-3}	FLT^{-1}
Work, energy	W, E	ML^2T^{-2}	FL
Density	ρ	ML^{-3}	FT^2L^{-4}
Temperature	T	Θ	Θ
Specific heat	c_p, c_v	$L^2T^{-2}\Theta^{-1}$	$L^2T^{-2}\Theta^{-1}$
Thermal conductivity	k	$MLT^{-3}\Theta^{-1}$	$FT^{-1}\Theta^{-1}$
Expansion coefficient	β	Θ^{-1}	Θ^{-1}

Exams

Name David Malawey

Section Number: 1A

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Prof. A. Banerjee : ME231: Thermo-Fluid Mechanics I

Mid-Term 3: November 18, 2010

75 minutes, DO ALL THE PROBLEMS

Reminder:

1) **Read the problem carefully**, and draw a diagram. Check if your diagram agrees with the problem.
2) State all your assumptions.
3) Write your solution, including working.
4) Check the problem again; have you answered all of it?
5) The time listed in brackets is the suggested approximate time you should spend doing each problem.
6) Good Luck!

Problem #	Points
1	6 /10
2	26/30
3	29 /30
4	27 /30
Total	88/100
Course Grade	A 86.6%

6/10

Problem#1. (10 points) Answer <u>T</u>rue or <u>F</u>alse in the space provided to the following. (Estimated time: 5 minutes)

(i)	For a circular tube the hydraulic diameter is twice the tube diameter	**(..F..)**	
(ii)	The entrance length is only a function of the Reynolds number of the flow	**(..T..)**	f
(iii)	The Darcy-Weisbach Equation which predicts the friction factor for laminar flow in a circular pipe is based on an exact analysis (i.e. solving the Navier-Stokes Equation)	**(..F..)**	f
(iv)	Major and minor losses are indicative of the amount of the losses.	**(..F..)**	
(v)	Stream Functions and 2-D Velocity Potentials are orthogonal to each other except @ stagnation point	**(..T..)**	
(vi)	A small Reynolds number indicates that viscous forces are not important in the flow.	**(..T..)**	f
(vii)	Pi Terms are sometimes dimensional.	**(..F..)**	
(viii)	A venturi meter has a smaller head loss as compared to a orifice meter.	**(..T..)**	
(ix)	No friction factor data is available in the Moody chart for the transitional flow regime	**(..F..)**	f
(x)	Dynamic similarity ensures that the forces (both in the model and the prototype) are in the same ratio and have equivalent directions)	**(..T..)**	

$\frac{26}{30}$

Problem 2: (30 points) The power P generated by a certain wind-turbine design depends upon its diameter D, the air density ρ, the wind velocity V, the rotation rate Ω, and the number of blades n. (a) Write this relationship in dimensionless form. A model windmill, of diameter 50 cm, develops 2.7 kW at sea level when V = 40 m/s and when rotating at 4800 rev/min. (b) What power will be developed by a geometrically and dynamically similar prototype, of diameter 5 m, in winds of 12 m/s at 2000 m standard altitude? (c) What is the appropriate rotation rate of the prototype? Please use table A.6 from appendix for properties of air. **(Estimated time: 20 minutes)**

	P	D ✓	ρ ✓	V ✓	Ω	n
	ML^2T^{-3}	L	ML^{-3}	LT^{-1}	T^{-1}	$M^0L^0T^0$
model	2.7kW	.5m	1.2255	40 m/s	4800rpm	
proto	?	5m	1.0067	12 m/s		

$\Pi_1 = P\, D^a \rho^b V^c$

M $0 = 1 + 0 + b + 0$
L $0 = 2 + a - 3b + c$
T $0 = -3 + 0 + 0 - c$

$b = -1$
$a = -2$
$c = -3$

$\Pi_1 = \dfrac{P}{D^2 \rho V^3}$ ✓

$$\frac{2.7kW}{(.5)^2(1.226)(40)^3} = \frac{P_{proto}}{5^2(1.007)(12)^3} \Rightarrow \boxed{P_p = 5.988\,kW}$$ ✓

$\Pi_2 = \Omega\, D^a \rho^b V^c$

M $0 = 0 + b$
L $0 = 0 + a - 3b$
T $0 = -1 - c$

$b = 0$
$a = 0$
$c = -1$

$\Pi_2 = \dfrac{\Omega}{V}$ ✗ (-2)

$\Pi_3 = ?$ (-1)

$$\frac{4800\ rpm}{40\ m/s} = \frac{\Omega_p}{12\ m/s}$$

$\boxed{\Omega_p = 1440\ rpm}$ ✗

(-1)

$\frac{29}{30}$

Problem 3. (30 points) A two-dimensional incompressible flow has the velocity potential:

$$\phi = K(x^2 - y^2) + C\ln(x^2 + y^2)$$

where K, C are constants. In this discussion, avoid the origin, which is a singularity (infinite velocity).
(a) Find the sole stagnation point of this flow, which is somewhere in the upper half plane.
(b) Prove that a velocity potential exists for this flow; (c) Prove that a stream function exists and then find $\psi(x, y)$. [Hint: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$] **(Estimated time: 20 minutes)**

$V=0$ @ stagnation pt.

$u = \frac{\partial \phi}{\partial x} \quad v = \frac{\partial \phi}{\partial y}$

$u = 2kx + \frac{2cx}{x^2+y^2}$

Continuity: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$2k - \frac{2(x^2-y^2)}{(x^2+y)^2} + -2k + \frac{-2(y^2-x^2)}{(x^2+y^2} = 0$

+5

$v = -2ky + 2c\frac{y}{x^2+y^2}$

Ok, velocity potential exists,

+5

stag. pt. $= u = v = 0$

$u=0 \left\{ \cancel{2}kx - \frac{\cancel{2}cx}{x^2+y^2} = 0 \right.$

$kx = \frac{cx}{x^2+y^2} \quad kx^3 + kxy^2 = cx$

$kx^2 + ky^2 = c$

$x^2+y^2 = \frac{c}{k}$

$x^2+y^2 = -(x^2+y^2)$

+5

$v=0 \left\{ -\cancel{2}ky(x^2+y^2) = \cancel{2}cy \right.$

$x^2+y^2 = -\frac{c}{k}$

stag. point

$x=0 \quad y=0$

avoid origin?

$x=0, y=\sqrt{\frac{c}{k}}$

(-1)

$x^2 = y^2 + \frac{c}{k} \quad x^2 = -x^2 - \frac{c}{k} + \frac{c}{k}$

$y^2 = -\frac{c}{k} - x^2$

find $\psi \Rightarrow$ must be irrotational $\Rightarrow \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0 \quad \frac{\partial}{\partial x}\left(\frac{y}{x^2+y^2}\right) - \frac{\partial}{\partial y}\frac{x}{x^2+y^2} \neq 0$

+5

$= \frac{-xy}{(x^2+y^2)^2} - \frac{-xy}{(x^2+y^2)^2} = 0$

ok, S.F. exists

$u = \frac{\partial \psi}{\partial y} \quad v = -\frac{\partial \psi}{\partial x}$

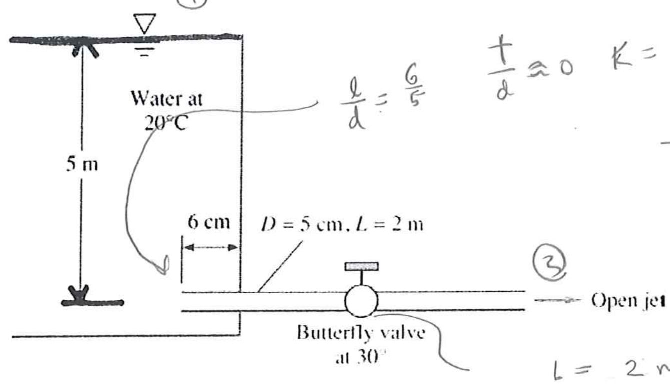
$\psi = \int u\,dy = 2kxy + \frac{2cx}{x}\tan^{-1}\left(\frac{y}{x}\right) + f(x)$

+10

$-\frac{\partial \psi}{\partial x} = v = -2ky + \frac{2cy}{(x^2+y^2)} = \frac{\partial}{\partial x}\left(2kxy + 2c\tan^{-1}\left(\frac{y}{x}\right) + f(x)\right)$

$\frac{27}{30}$

Problem 4. (30 points) In the figure below, the pipe is galvanized iron. Estimate the percentage increase in flow rate if : (a) the pipe entrance is cut off flush with the wall, and (b) if the butterfly valve is opened wide. Assume f = 0.0284. [Hint: Please use 6.19 and 6.21a for finding the friction factor and the minor loss coefficients – use your engineering skills for choosing values of minor loss coefficients when reading these charts]. **(Estimated Time: 20 minutes)**



$\frac{l}{d} = \frac{6}{5}$ $\quad \frac{t}{d} \approx 0$ $\quad K = 1.0$ or 0.5 flush

Valve

$30°: K = 60$

open $K = 0.3$

$L = 2$ m cut off

2.06 regular

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 - h_f$$

+15

$$h_f = z_1 - z_2 + \frac{V_2^2}{2g}$$

$$= 5 + \frac{V_2^2}{2g} = \frac{V^2}{2g}\left(f\frac{L}{d} + \Sigma K\right)$$

$Q = VA$

$= V\left(\frac{\pi (.06)^2}{4}\right)$

$$\frac{V_2^2}{2g}\left(f\frac{L}{d} + \Sigma K - 1\right) = 5$$

+10

$$V_2^2 = 10g\left(f\frac{L}{d} + \Sigma K - 1\right)$$

ΔQ
0
$1.7(10^{-5})\,m^3/s$
$1.95\,(10^{-1})\,m^3/s$

① $V_2^2 = 98.1\left(.0284\left(\frac{2.06}{.05}\right) + (1+60) - 1\right)^{-1} \Rightarrow V = 1.266$

③ cutoff $= 9.81\left(.0284\left(\frac{2}{.05}\right) + (0.5+60) - 1\right)^{-1} \Rightarrow V = 1.272$

③ open $= 9.81\left(.0284\left(\frac{2.06}{.05}\right) + (1 + 0.3) - 1\right)^{-1} \Rightarrow V = 8.169$

+2

(-1)

(-2)

Formulas to know

$$y: \underbrace{F_{s,y}}_{\text{surface forces}} + \underbrace{F_{b,y}}_{\text{body forces}} = \underbrace{\frac{d}{dt}\int_{\forall} V_{xyz}\, \rho\, d\forall}_{\text{change in momentum}} + \underbrace{\int_A V_{xyz}\, \rho\, \vec{V}_{xyz} \cdot dA}_{\text{momentum flux}}$$

using V_{rel} for moving control volume

what is $F_{s,y}$?

what is $F_{B,y}$? $\rho \cdot \forall \cdot g$

in 2D, $\nabla \cdot \vec{V} = 0$ means $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

∇, free surface, does not move

inviscid means: no friction, &

Cons. mom. p155 (3.37)

Cons. mass 151

Bernoulli 171

Navier Stokes 243

Name David Malawey **Section Number: 1A**

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Prof. A. Banerjee : ME231: Thermo-Fluid Mechanics I

Mid-Term 2: October 21, 2010

75 minutes, DO ALL THE PROBLEMS

Reminder:

1) **Read the problem carefully**, and draw a diagram. Check if your diagram agrees with the problem.
2) State all your assumptions.
3) Write your solution, including working.
4) Check the problem again; have you answered all of it?
5) The time listed in brackets is the suggested approximate time you should spend doing each problem.
6) Good Luck!

Problem #	Points
1	6 /10
2	30 /30
3	26 /30
4	29 /30
Total	91 /100
Course Grade	A.

Very Good!

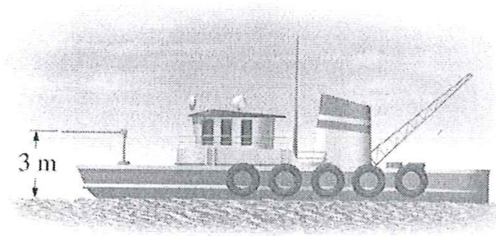
84.904 %

Problem#1. (10 points) Answer **T**rue or **F**alse in the space provided to the following. (Estimated time: 5 minutes)

A	The conservation of mass equation reduces to the form: $\int_{CS} \vec{V} \cdot d\vec{A} = 0$ only when the flow is steady.	(…T…)
B	The Reynolds Transport Theorem relates Eulerian and Lagrangian viewpoints in fluid mechanics.	(…T…)
C	Bernoulli's equation is valid for ~~unsteady~~, frictionless flow along a streamline.	(…F…)
D	A stream-function can be defined for a steady, incompressible, ~~three-dimensional~~ flow. 2D	(…F…)
E	In a wall driven flow, the pressure gradient along the direction of the flow can be neglected.	(…T…)

0

Problem 2: (30 points) A fireboat is fighting fires by drawing sea water (ρ=1030 kg/m^3) through a 20-cm diameter pipe at a rate of 0.1 m^3/s and discharging it through a nozzle of exit diameter 5-cm. The total irreversible head loss of the system is 3 m, and the position of the nozzle is 3 m above the sea level. For a pump efficiency of 70%, determine the required shaft power input to the pump and the water discharge velocity. (Estimated time: 20 minutes)



$\dot{V}_{in} = .1\ m^3/s$ $\quad A_1 \quad (.2m)^2 \frac{\pi}{4} = .03142\ m^2$ $\quad A_2 = (.05m)^2 \frac{\pi}{4} = .001963$

$V_{in} = \dot{V}/A_1 = 3.183\ m/s$

$\boxed{V_{out} = \dot{V}/A_2 = 50.92\ m/s}$

$$\cancel{\frac{P_1}{\gamma}}^{0} + \frac{V_1^2}{2g} + \cancel{z_1}^{0} = \cancel{\frac{P_2}{\gamma}}^{0} + \frac{V_2^2}{2g} + z_2$$

$$\dot{W}_{in} = \dot{m}(\text{head}_2 - \text{head}_1) = \frac{V_2^2 - V_1^2}{2g} + z_2 = 131.75m + 6m =$$

$\gamma = \rho g = 9.81(1030)$

$\gamma = 10100\ N/m^3$

$\dot{m} = \dot{V}\rho = .1 \frac{m^3}{s} (1030\ kg/m^3) =$

$\dot{W}_{in} = 103\ kg/s\ (137.75m)$

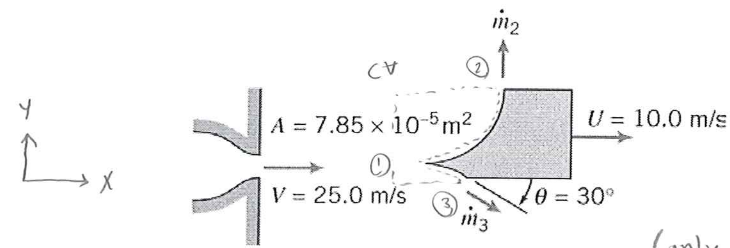
$\dot{W}_{in} = 14188\ \frac{kg\ m}{s}$

make this newtons

$\dot{W}_{actual} = \frac{\dot{W}}{.70} = 20269\ \frac{kg \cdot m}{s} \Rightarrow 198839\ \frac{N \cdot m}{s}$

$= \boxed{198.4\ kW}$

Problem 3. (30 points) A plane jet of water strikes a splitter plate and divides into two flat streams as shown. Find the mass flow ratio, $\frac{\dot{m}_2}{\dot{m}_3}$, required to produce zero net vertical force on the splitter vane. Determine the horizontal force that must be applied under these conditions to maintain the vane motion at steady speed. (Estimated time: 20 minutes)



(only for solving ratio)

(replace dA magnitude with $\dot{m}$ because constant ρ)

$$F_{S,y} + \cancel{F_{B,y}}^{0} = \cancel{\frac{d}{dt}\int_{CV} V_y \rho \, dV}^{0} + \int_{CS} V_y \rho \vec{V} \cdot d\vec{A}$$

$$F_{S,y} = 0 = 0 + (25\,m/s)(\rho)(25\,m/s)(\dot{m}_2) - 25\,m/s(\sin 30)\rho(25\,m/s)(\dot{m}_3)$$

$$\dot{m}_3(312.5\rho) = \dot{m}_2(625)\rho$$

$$\boxed{\frac{\dot{m}_2}{\dot{m}_3} = 0.5}$$

-4

$$F_{S,x} = \int_{CS} V_x \rho \vec{V} \cdot d\vec{A}$$

$$= \int_1 25\,m/s \overset{(+)}{(\rho)} \overset{(-)}{(25\,m/s)}(7.85 \times 10^{-5}\,m^2) + 0 + \int_2 25\,m/s(\cos 30)\rho \overset{(+)}{25\,m/s} \overset{(+)}{\left(\frac{\dot{m}_2}{\dot{m}_2+\dot{m}_3}\right)(7.85E^{-5})}$$

$$= -48.96\text{ N} + 14.13\text{ N}$$

$$F_{S,x} = -34.83\text{ N}$$ Now multiply by $\frac{V_{rel}}{V}$ because I forgot to use V_{rel}

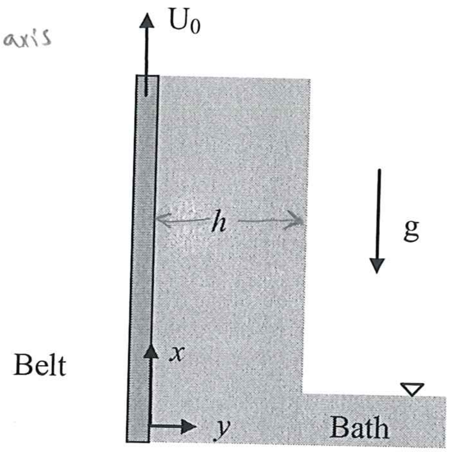
$$-34.83\left(\frac{25\,m/s - 10\,m/s}{25\,m/s}\right)$$

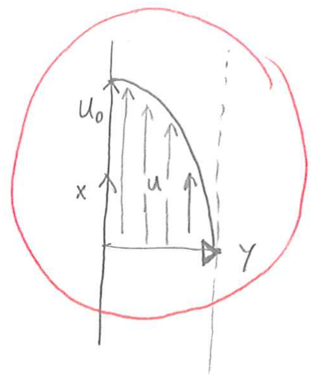
$$\boxed{F_{S,x} = -20.9\cancel{0}\text{ N } \hat{\imath}}$$

I did not use V_{rel} for 1st equation but it does not effect the ratio.

Problem 4. (30 points) A continuous belt, moving upwards through a chemical bath at speed U_0, picks up a liquid film of thickness h, density, ρ, and viscosity μ. Gravity tends to make the liquid drain down, but the movement of the belt keeps the liquid from running off completely. Assuming that the flow is steady and fully developed, and that the atmosphere produces no shear stress at the outer surface of the film; obtain an expression for the velocity profile. Sketch a representative velocity profile. (Estimated time: 20 minutes)

assume: 2d flow in x-y axis





?

continuity: $\cancel{\frac{\partial u}{\partial x}} + \frac{\partial v}{\partial y} = 0 \Rightarrow v \neq v(y)$ & $v \neq v(z \text{ or } x)$ so $v = 0$

fully developed, steady

(-1)

x-mom: $\rho \cancel{\frac{du}{dt}}^{0} = -\cancel{\frac{\partial p}{\partial x}}^{0} + \rho g_x + \mu\left(\cancel{\frac{\partial^2 u}{\partial x^2}}^{0} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}^{0}\right)$

$-\rho(-g) = \mu\left(\frac{\partial^2 u}{\partial y^2}\right)$ $\qquad \frac{\partial u}{\partial y} = \frac{+\rho g}{\mu} y + C_1$ $\qquad u(y) = \frac{+\rho g}{\mu}\frac{y^2}{2} + C_1 y + C_2$

Boundary conditions:

@ $y = 0$, $u(y) = U_0$ $\qquad U_0 = 0 + 0 + C_2 \Rightarrow C_2 = U_0$

@ $y = h$, shear $= 0 \Rightarrow \mu \frac{\partial u}{\partial y} = 0 \Rightarrow \frac{\partial u}{\partial y} = 0 \Rightarrow 0 = -\frac{\rho g h}{\mu} + C_1 \Rightarrow C_1 = \frac{\rho g h}{\mu}$

$u(y) = -\frac{\rho g y^2}{2\mu} + \frac{\rho g h y}{\mu} + U_0 = \boxed{\frac{+\rho g y^2}{2\mu}\left[1 - \frac{2h}{y}\right] + U_0}$

Name David Malawey **Section Number: 1A**

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Prof. A. Banerjee : ME231: Thermo-Fluid Mechanics I

Mid-Term 1: September 16, 2010

75 minutes, DO ALL THE PROBLEMS

Reminder:

1) **Read the problem carefully**, and draw a diagram. Check if your diagram agrees with the problem.
2) State all your assumptions.
3) Write your solution, including working.
4) Check the problem again; have you answered all of it?
5) The time listed in brackets is the suggested approximate time you should spend doing each problem.
6) Good Luck!

Problem #	Points
1	08 /20
2	30 /30
3	10/25
4	25 /25
Total	73/100
Course Grade	B - 25.25

Problem#1. (20 points) Answer <u>T</u>rue or <u>F</u>alse in the space provided to the following. (Estimated time: 10 minutes)

A	An example of a pseudo-plastic is tooth-paste.	(……) T
B	The continuum hypothesis for fluids relates velocity gradients to shear stress.	(…T…)
C	The Eulerian method description is more appropriate for solid mechanics.	(…T…)
D	In an inviscid fluid, the viscosity is non-zero.	(…T…)
E	Atmospheric temperature increases linearly with altitude.	(…F…)
F	If the contact angle is less than 90 the liquid is termed as non-wetting.	(…F…)
G	The unit of mass in BG system is lbm (pound mass).	(…F…)
H	For a transient (non-steady) velocity field, the convective acceleration is the sum of material and local accelerations.	(…T…)
I	A body immersed in a liquid experiences a vertical buoyant force equal to its own weight.	(…F…)
J	Mass (M), Length (L) and Time (T) are the three fundamental dimensions.	(…T…)

Problem 2: (30 points – 5 points for each question) – Please circle the correct answer and show work. No points will be awarded if you DO NOT show work. (Estimated time: 15 minutes)

1 2

A. If the density of glycerin is 1260 kg/m^3, the specific weight (KN/m^3), specific volume and specific gravity of glycerin in SI units are:

(i) 12.36 ; 7.94×10^{-4}; 1.26 (ii) 1.26 ; 7.94×10^{-4}; 12.36 (iii) 12.36; 1.26 ; 7.94×10^{-4}

(iv) 1.26; 12.36 ; 7.94×10^{-4} (v) 7.94×10^{-4} ;1.26; 12.36

B. A tank of water (SG=1.0) has a gate in its vertical wall 5 m high and 3 m wide. The top edge of the gate is 2 m below the surface. What is the hydrostatic force on the gate?

(i) 147 kN (ii) 367 kN (iii) 490 kN

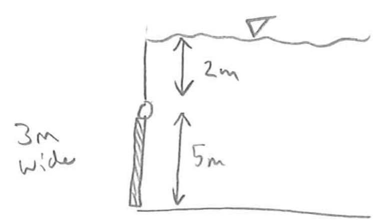
(iv) 661 kN (v) 1028 kN

$F = \gamma\, h_{CG}\, A$

kg/m^3 m/s^2 (m) m m

$= 998(9.81)\left(2+\frac{5}{2}\right)(5\times3)$

$= 660851\,N$



C. Heavy cream is a non-Newtonian fluid whose shear stress can be approximated by

$\tau \approx 0.12\left(\frac{du}{dy}\right)^{\frac{2}{3}}$; where all quantities are expressed in SI units. The effective viscosity μ_e such that $\tau = \mu_e\left(\frac{du}{dy}\right)$ is:

$\mu\left(\frac{du}{dy}\right) = .12\left(\frac{du}{dy}\right)^{\frac{2}{3}}$

$\mu = .12\left(\frac{du}{dy}\right)^{\left(\frac{2}{3}-1\right)}$

(i) $0.12\left(\frac{du}{dy}\right)^{-\frac{1}{3}}$ (ii) $0.12\left(\frac{du}{dy}\right)^{\frac{1}{3}}$ (iii) $0.12\left(\frac{du}{dy}\right)^{-\frac{2}{3}}$

$\mu = .12\left(\frac{du}{dy}\right)^{-1/3}$

(iv) 0.12 (v) None of the above

D. The tank of water in figure 1 accelerates uniformly by freely rolling down a 30° incline. If the wheels are frictionless, what is the angle θ? assumptions: no sloshing, smooth accel

(i) 15°   (ii) 10°   (iii) 30°

(iv) 0°   (v) -30°

$g_x = 9.81 \sin 30 = 4.905$ $\theta = \tan^{-1}\left(\frac{4.905}{8.495}\right) = 30$

$g_y = 9.81 \cos 30 = 8.4957$

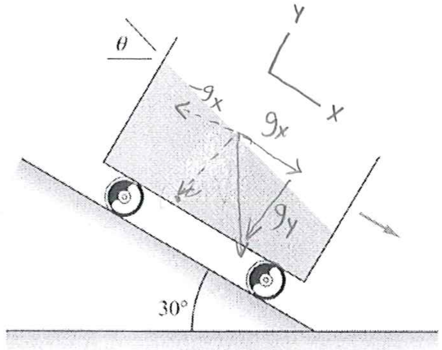


Figure 1

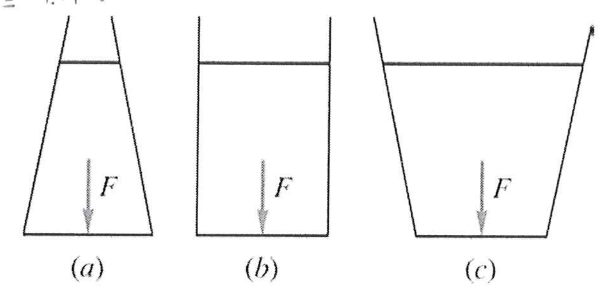


Figure 2

E. Figure 2 shows three containers which have the *same bottom area but different shapes*. Clearly, the weight of liquid in each tank is different due to differing volumes. The maximum pressure occurs at the bottom of which container?

(i) A   (ii) B   (iii) c

(iv) All of the above   (v) None of the above

assumption: ~~liquid is~~ the same liquid is in each cup

F. In Figure 3, both fluids are at 20˚C. If the surface tension effects are negligible, what is the density of oil in SI Units?

(i) 7357.5 kg/m^3 (ii) 750 kg/m^3 (iii) 1333.33 kg/m^3

(iv) 981 kg/m^3 (v) None of the above

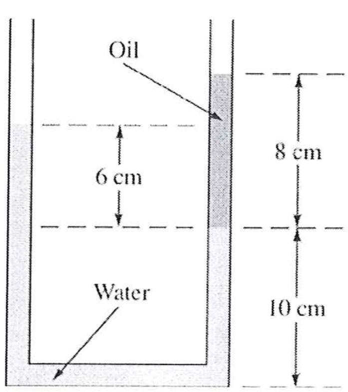


Figure 3

$\rho_{H_2O}(.16\,m) = \rho_{oil}(.08\,m) + \rho_{H_2O}(.1\,m)$

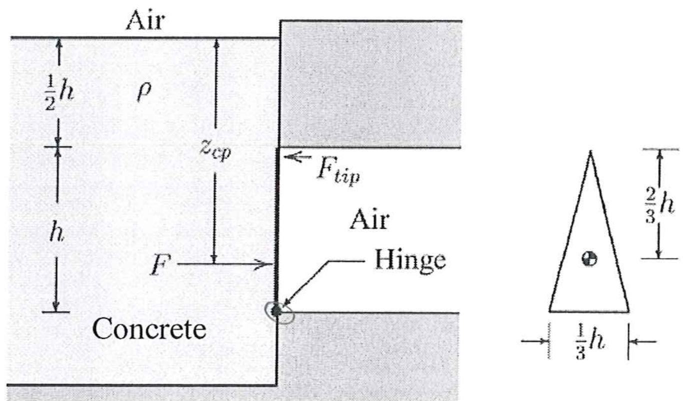
$\rho_{oil}(.08\,m) = \rho_{H_2O}(.06\,m)$

$\rho_{oil} = .75\,\rho_{H_2O}$

$= .75\,(998\ kg/m^3)$

Problem 3. (25 points) A triangular access port, hinged as shown below, is provided in the side of a form containing liquid concrete. Determine the hydrodynamic force on the access port by the concrete, which has a density ρ. Also, compute the force needed at the upper tip of the access port to prevent it from opening. **(Estimated time: 20 minutes)**

assumptions:
- constant density ρ
- neglect friction of hinge
- homogeneous concrete



-15

$F = \gamma_c z_{cp} A$

$= \rho g \left(\frac{1}{2} + \frac{2}{3}\right)h \left(\frac{2}{3}h \times \frac{1}{3}h\right)\left(\frac{1}{2}\right)$

$\boxed{F_{concrete} = \rho g h^3 (.1295)}$

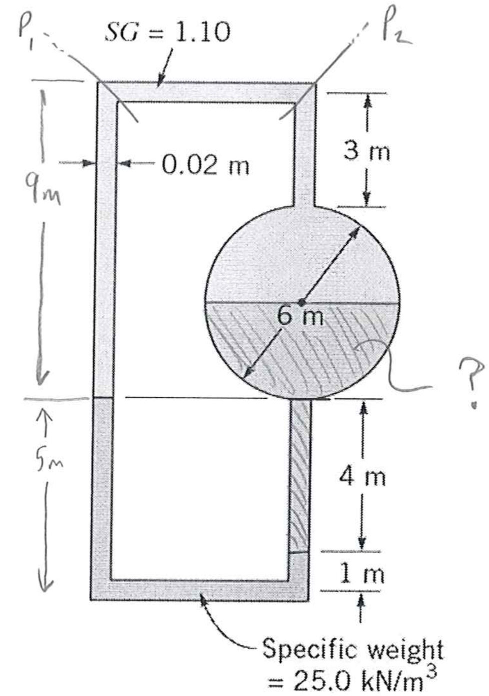
$\Sigma M_{hinge} = 0$

$F_{tip}(h) - F_{concrete}\left(\frac{1}{3}h\right) = 0$

$F_{tip} = \frac{1}{3} F_{concrete}$

$\boxed{F_{tip} = \rho g h^3 (.0288)}$

Problem 4. (25 points) A 0.02 m-diameter tube is connected to a 6-m diameter full tank as shown in the figure. Determine the density of the unknown fluid in the tank. Sp. Wt. of water is 9.8 kN/m^3. **(Estimated time: 15 minutes)**



assumption:
- fluids do not move
- fluids at 20 °c
- surface tension can be ignored

$25000/9790 = SG = 2.554$

$P_1 = P_2$

$$1.10\,\rho_{water}\,(\cancel{9m})^{3m} + 2.554\,\rho_{water}\,(\cancel{5m})^{4m} = \cancel{1.10\,\rho_{water}\,(6m)} + SG_?\,\rho_{water}\,(7m)$$
$$\cancel{+\ 2.554\,\rho_{water}\,(1m)}$$

$$\cancel{\rho_{water}}\,(13.516\ m) = SG_?\,\cancel{\rho_{water}}\,(7m)$$

$$SG_{water}\,(1.931) = SG_?$$

$$SG_? = 1.931$$

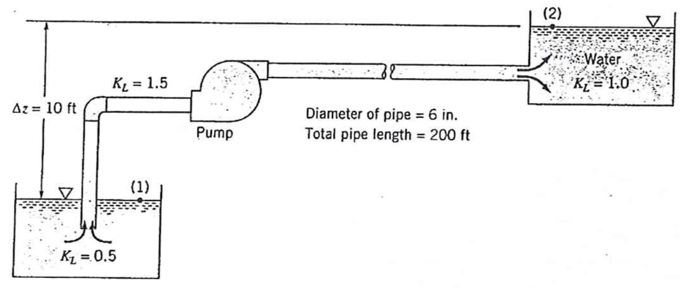
$$\rho_? = 998\,\frac{kg}{m^3}\,(1.931) = 1927\ kg/m^3$$

$$\boxed{\rho_{unknown} = 1927\,\frac{kg}{m^3}}$$

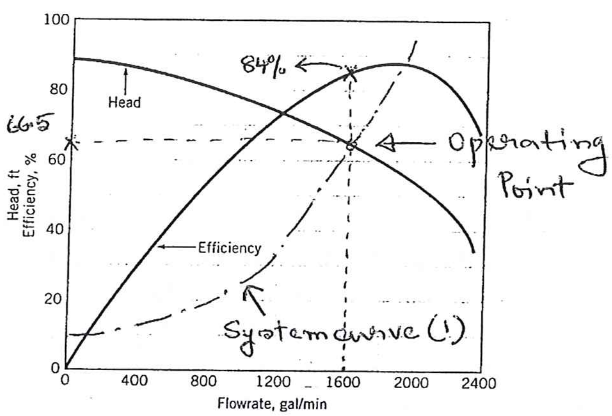
In-class problem
sets

ME 231 Thermo-Fluid Mechanics I
In class problems-Set 19-supplement
Prof. A. Banerjee

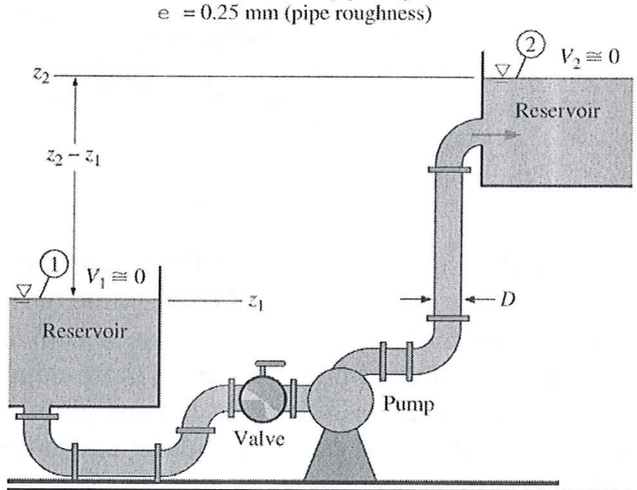
1. Water is to be pumped from one large, open tank to a second large, open tank as shown in the figure. The pipe diameter throughout is 6 in. and the total length of the pipe between the pipe entrance and the exit is 200 ft. Minor loss coefficients for the entrance, exit, and elbow are shown, and the friction factor for the pipe can be assumed constant and equal to 0.02. A certain centrifugal pump having the performance characteristics shown in the plot is suggested as a good pump for this system. With this pump, what would be the flow-rate between the tanks? Do you think that this pump would be a good choice?



(a)

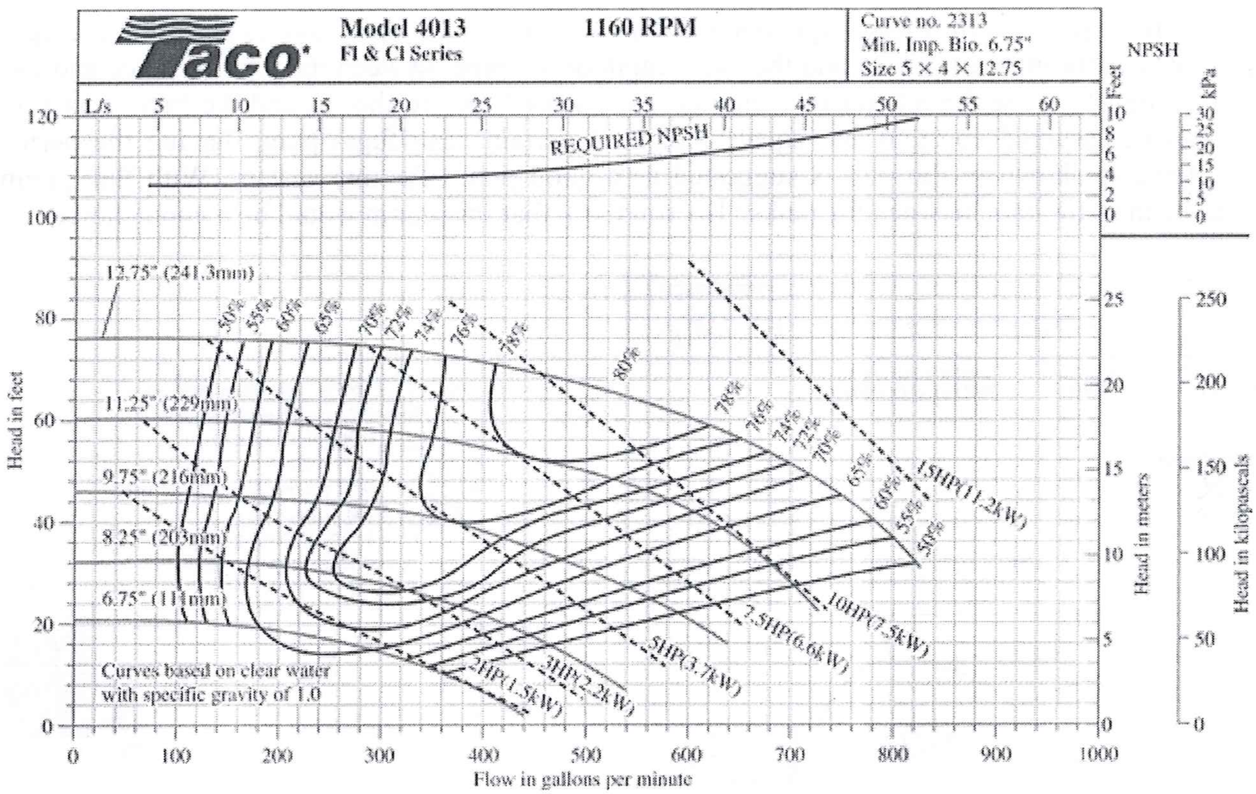


$z_2 - z_1$ = 7.85 m (elevation difference)
D = 2.03 cm (pipe diameter)
$K_{L,\ entrance}$ = 0.50 (pipe entrance)
$K_{L,\ valve}$ = 17.5 (valve)
$K_{L,\ elbow}$ = 0.92 (each elbow—there are 5)
$K_{L,\ exit}$ = 1.05 (pipe exit)
L = 176.5 m (total pipe length)
e = 0.25 mm (pipe roughness)

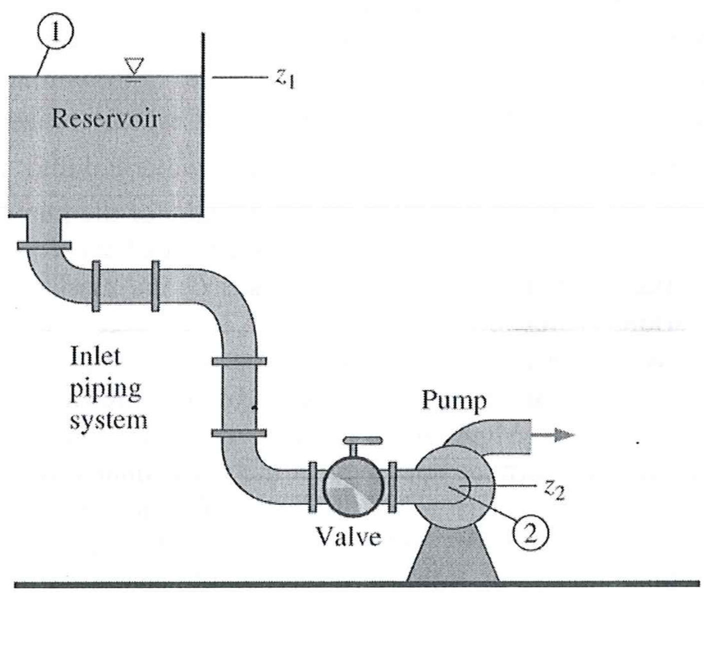


2. A water pump is used to pump water from one large reservoir to another that is at a higher elevation. The free surface of both reservoirs is exposed to the atmosphere. The dimensions and minor loss coefficients is provided in the figure. The pump's performance is approximated by the expression: $H_{available} = H_0 - aQ^2$, where the shutoff head H_0= 24.4 m of water column, coefficient a = 0.0678 m/Lpm^2, available pump head $H_{available}$ is in units of meters of water column, and capacity Q is in units of liters/min (Lpm). Estimate the capacity delivered by the pump. For the pump and piping system given in the figure, plot the required pump head (H_{required}) as a function of the volume flow rate (Q). On the same plot, compare the available pump head ($H_{\text{available}}$) versus Q and mark the operating point. Discuss design considerations for various changes.
3. A washing operation at a power plant requires 370 gallons per minute of water. The required net head is about 24 ft at this flow rate. A newly hired engineer looks through some catalogs and decides to purchase the 8.25 in impeller option of the Taco Model 4013 FI series centrifugal pump (see figure attached at the back). If the pump operates at 1160 rpm, as specified in the performance plot, the engineer reasons, its performance curve intersects 370 gpm at H = 24 ft. The chief engineer, who is very concerned about efficiency, glances at the performance curves and notes that the efficiency of this pump at the operating point is only 70%. He sees that the 9.75 in. impeller option would achieve a higher efficiency (76.5%) at

the same flow rate. He notes that a throttle valve can be installed downstream of the pump to increase the required net head so that the pump operates at this higher efficiency. He asks the junior engineer to justify this choice of impeller diameter. Perform the comparison and discuss.



4. The 11.25 in impeller option of the pump in problem 1 is used to pump water at 25°C from a reservoir whose surface is 4.0 ft above the centerline of the pump inlet. The piping system from the reservoir to the pump consists of 10.5 ft of cast iron pipe with an ID of 4.0 in and average inner roughness height of 0.02 in. There are several minor losses: a sharp-edged inlet (K_L=0.5), three flanged smooth 90 degree elbows (K_L=0.3 each), and a fully open flanged globe valve (K_L=6.0). Estimate the maximum volume flow rate (in gpm) that can be pumped without cavitation. If the water were warmer, would this maximum flow rate increase or decrease? Discuss how you might increase the maximum flow rate while still avoiding cavitation.



ME 231 Thermo-Fluid Mechanics I
In class problems-Set19
Prof. A. Banerjee

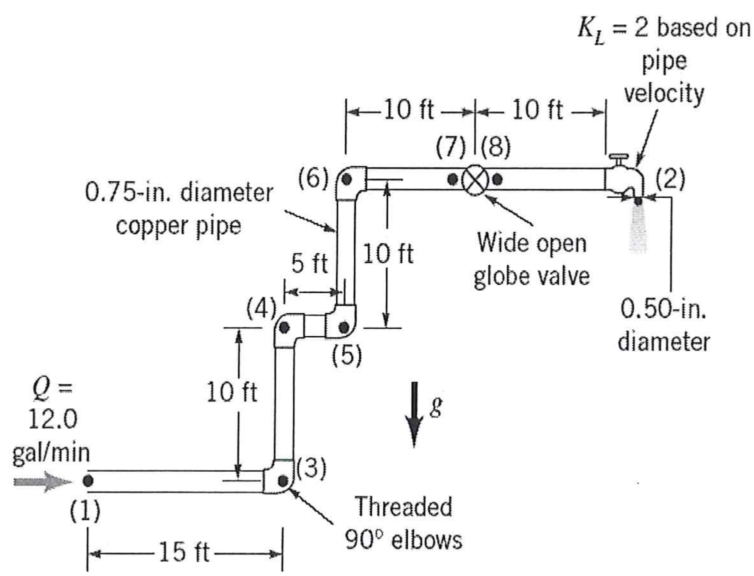
1. A compressed air drill requires an air supply of 0.25 kg/sec at a gage pressure of 650 kPa at the drill. The hose from the air compressor to the drill is 40 mm inside diameter. The maximum compressor discharge gage pressure is 690 kPa. Neglect changes in density and any effects due to hose curvature. Air leaves the compressor at 40 C. Calculate the longest hose that may be used.

2. Water at 60 F flows from the basement to the 2nd floor through the 0.75-in. diameter copper pipe at a rate of Q = 12.0 gal/min = 0.0267 ft^3/s and exits through a faucet of diameter 0.50 in as shown in the figure. Determine the pressure at point (1) if:

   (a) all losses are neglected,

   (b) the only losses included are the major losses, or

   (c) all losses are included.



ME 231 Thermo-Fluid Mechanics I
In class problems-Set ~~18~~ 17
Prof. A. Banerjee

1. Consider steady flow of water through a smooth-walled, horizontal, circular pipe of diameter, D, length, L. If the fluid density is ρ, and the fluid viscosity, μ, obtain a functional relationship using Buckingham's pi theorem to determine the pressure drop per unit length, Δp in the pipe.
2. The drag force F, on a smooth sphere depends on the relative velocity, V, the sphere diameter, D, the fluid density, ρ, and the fluid viscosity, μ. Obtain a set of dimensionless groups that can be used to correlate experimental data.

$F \rightarrow V, D, \rho, \mu$ $n =$

Prereq's for ICE

Thermo II   Math 4-6
Thermo I

Phys II
Phys I

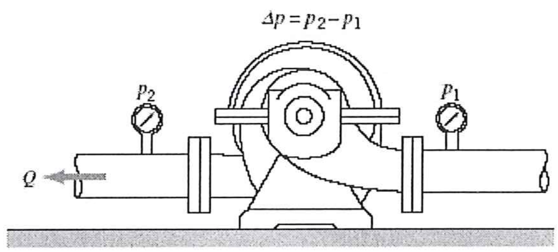
Diff Eq
Calc III
Calc II
Calc I

ME 231 Thermo-Fluid Mechanics I
In class problems-Set18
Prof. A. Banerjee

1. The drag of a sonar transducer is to be predicted, based on a wind tunnel test data. The prototype, a 1ft diameter sphere, is to be towed at 5 knots in sea-water at 5C. The model is 6 in. in diameter. Determine the required test speed in air. If the drag of the model test conditions is 5.58 lbf, estimate the drag of the prototype.
2. The pressure rise, Δp, across a centrifugal pump as shown in the figure can be expressed as: $\Delta p = f(D, \omega, \rho, Q)$, where D is the impeller diameter, ω the angular velocity of the impeller, ρ the density of the fluid, and Q the volumetric flow rate through the pump. A model pump having a diameter of 8 in. is tested in the laboratory using water. When operated at an angular velocity of 40π rad/s, the model pressure rises as a function of Q as shown in the figure. Use this curve to predict the pressure rise across a geometrically similar pump (prototype) for a prototype flow-rate of 6 ft^3/s. The prototype has a diameter of 12in. and operates at an angular velocity of 60π rad/s. Water is used in both the cases.

find ΔP_p

	M	P
D	8in	12 in
ω	40π rad/s	60π rad/s
Q	f (ΔP)	6 ft^3/s
ρ		



Centrifugal pump

(a)

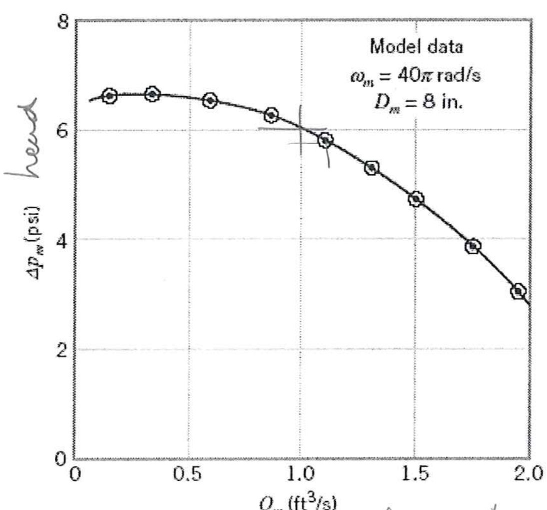
$Q_p = 6\text{ ft}^3/s$

$Q_m =$

n = 5

D	ω	Q	ρ	ΔP
L	$M^0L^0T^{-1}$	L^3T^{-1}	ML^{-3}	MT^{-2}

head



(b)

flow rate

$\frac{\omega D^3}{Q} = f\left(\frac{\Delta P D^3}{\rho Q^2}\right)$

repeating parameters j = 3

Geometry: D

Mat. prop: ρ

Ext. effects: Q

$\Pi_1 = \omega D^a \rho^b Q^c$

$T^{-1}[L]^a[ML^{-3}]^b[L^3T^{-1}]^c$

M: $1(b) = 0 \Rightarrow \boxed{b=0}$

L: $a + -3(b) + 3(c) = 0 \Rightarrow \boxed{a=+3}$

T: $-1 + -1c = 0 \Rightarrow \boxed{c=-1}$

$\Pi_1 = \frac{\omega D^3}{Q}$

$\Pi_2 = \Delta P D^a \rho^b Q^c$

$= MT^{-2}[L]^a[ML^{-3}]^b[L^3T^{-1}]^c$

M: $1 + B = 0 \Rightarrow B = -1$

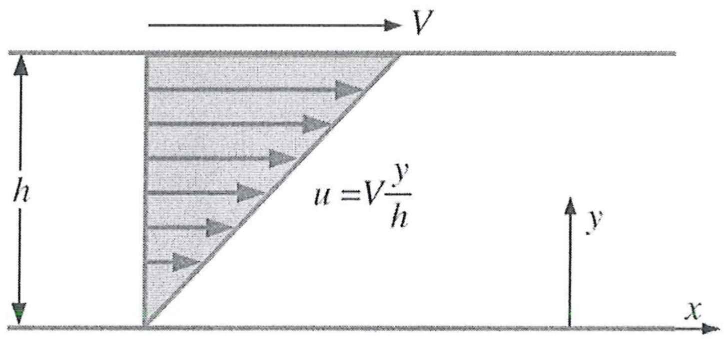
L: $A + -3b + 3C = 0$ $\quad A = 3$

T: $-2 + -1C = 0 \Rightarrow C = -2$

$\Pi_2 = \frac{\Delta P D^3}{\rho Q^2}$

ME 231 Thermo-Fluid Mechanics I
In class problems-Set16
Prof. A. Banerjee

1. Consider fully developed Couette Flow between two infinite parallel plates separated by distance h, with the top plate moving and the bottom plate fixed. The flow is steady, incompressible and two-dimensional in the x-y plane. The velocity field is give by: $\vec{V} = V\frac{y}{h}\hat{i} + 0\hat{j}$.



   a. Is this flow rotational or irrotational?
   b. If it is irrotational, calculate the vorticity component in the z-direction.
   c. Do fluid particles in this flow rotate clockwise or counter-clockwise?
   d. Calculate the linear strain rates in the x- and y- directions.
   e. Calculate the shear strain rate ε_{xy}
   f. Combine your results to form the 2D strain rate tensor ε_{ij} - are the x- and y-axes principal axes?

2. The velocity potential for a flow field is: $\phi = 4xy$. Determine the corresponding stream-function.

3. A 2D flow field is formed by adding a source at the origin of the co-ordinate system to the velocity potential: $\phi = r^2 \cos 2\theta$. Locate any stagnation points in the upper half of the co-ordinate plane ($0 \le \theta \le \pi$).

David Malaway

ME 231 Thermo-Fluid Mechanics I (Section 1A)
Prof. A. Banerjee
Recitation for Mid-term 2

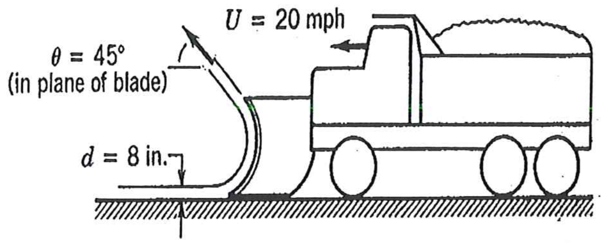
Probs from previous exams

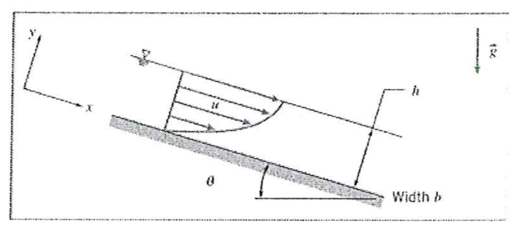
Problem 1. A two-dimensional flow has the velocity components:

$$u = ax + \frac{b}{xy^2} \quad \text{and} \quad v = -\left(ay + \frac{b}{x^2 y}\right)$$

where, a and b are constants. Find the equation of stream lines at (1,1) with a=b=1. Sketch the equation of the streamline that satisfies the above condition (label your axis).

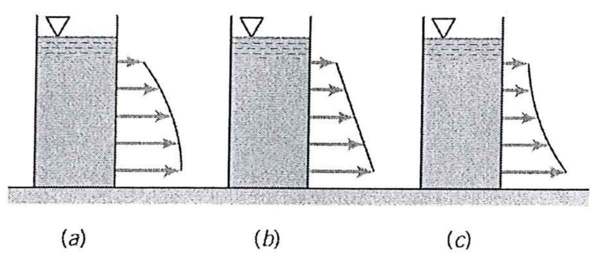
Problem 2. A snow plow mounted on a truck clears a path 12 ft wide through heavy wet snow. The snow is 8 in. deep and its density is 10 lbm/ft^3. The truck travels at 20 mph. Snow is discharged at an angle of 45° from the travel direction and 45° above the horizontal. Evaluate the force required to push the plow.

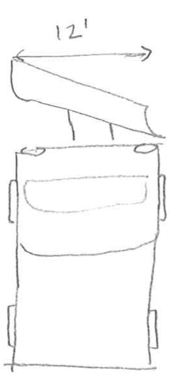




Problem 3. A liquid flows down an inclined plane surface in a steady, fully developed laminar film of thickness h. Simplify the continuity and Navier-Stokes equations to model this flow field. Obtain expressions for the liquid velocity profile, the shear stress distribution, the volume flow rate, and the average velocity.

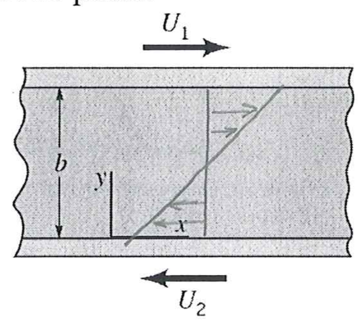
Problem4. Several holes are punched into a tin can as shown in the figure. Which of the figures represents the variation of the water velocity as it leaves the holes? Justify your choice mathematically. Note: No points will be awarded for the correct choice if a mathematical proof is not provided.



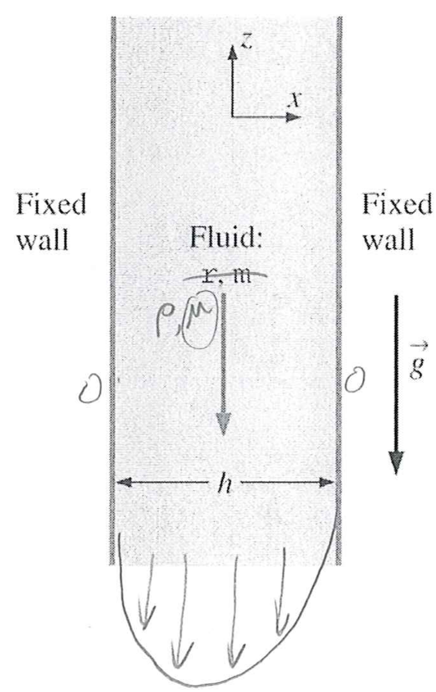


ME 231 Thermo-Fluid Mechanics I
In class problems-Set14
Prof. A. Banerjee

1. An incompressible, viscous fluid is placed between horizontal, infinite parallel plates as shown in the figure. The two plates move in opposite directions with constant velocities, U_1 and U_2 as shown. The pressure gradient in the x-direction is zero, and the only body force is due to the fluid weight. Use the Navier-Stokes Equations to derive an expression for the velocity distribution between the two plates.

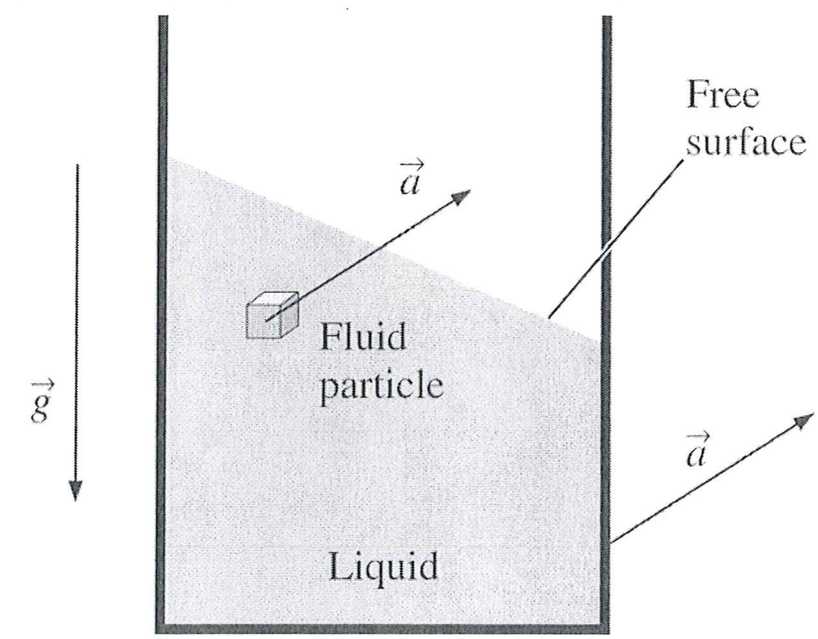


2. Consider steady, incompressible, parallel, laminar flow of a viscous fluid falling between two infinite vertical walls. The distance between the walls is h, and gravity acts in the downward direction (negative z direction). There is no applied (forced) pressure driving the flow and the fluid falls by gravity alone. The pressure is constant everywhere in the flow field. Calculate the velocity field and sketch the velocity profile using appropriate non-dimensionalized variables.



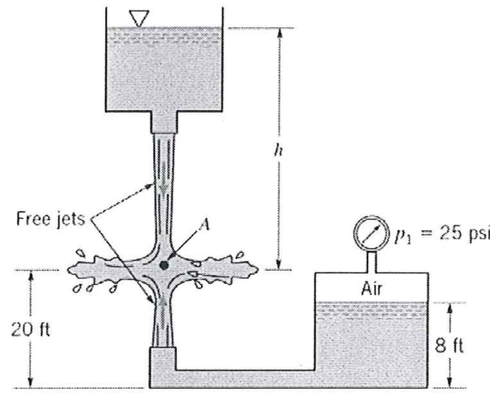
ME 231 Thermo-Fluid Mechanics I
In class problems-Set13
Prof. A. Banerjee

1. Given a velocity field: $\vec{V} = ax\hat{i} - ay\hat{j}$; x and y are in metres; $a = 0.1$ sec^{-1}. Find:
   a) Equation of the streamline in the x-y plane.
   b) Plot the streamline through point (2,8,0).
   c) Velocity of particle at point (2,8,0).
2. For a flow in the xy plane, the x-component of velocity is given by: $u = x^3 - 3xy^2$. Determine a possible y component for steady, incompressible flow. Is it also valid for unsteady, incompressible flow? Why? How many possible y components are there?
3. Consider the following steady, two dimensional, incompressible velocity field: $V = (ax + b)\,\hat{i} + (-ay + c)\,\hat{j}$, where a, b and c are constants. Calculate the pressure as a function of x & y.
4. Simplify the Navier-Stokes equation as much as possible for the case of an incompressible liquid being accelerated as a rigid body in an arbitrary direction (see figure 1 below). Gravity acts in the –z direction. Begin with the incompressible vector form of the Navier-Stokes Equation, explain how and why some terms can be simplified, and give your final results as a vector equation.

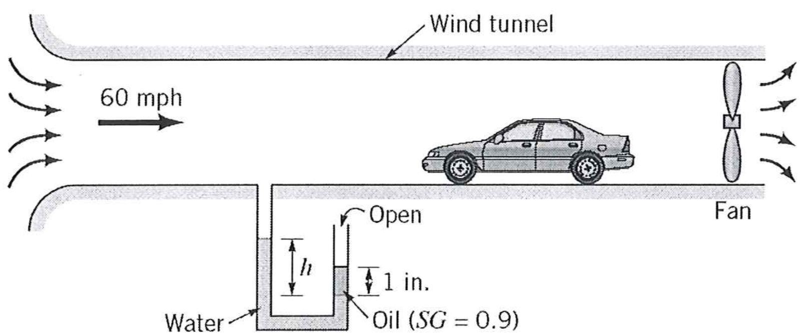


ME 231 Thermo-Fluid Mechanics I
In class problems-Set11
Prof. A. Banerjee

1. Streams of water from two tanks impinge upon each other as shown in the figure below. If viscous effects are negligible and point A is the stagnation point, determine the height A.

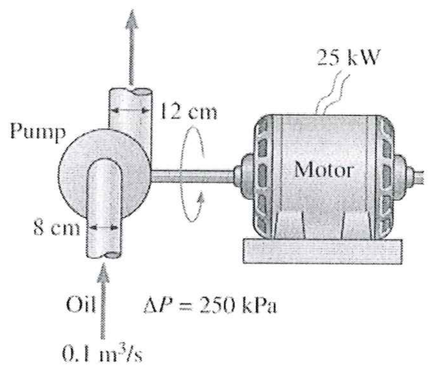


2. Air is drawn into a wind-tunnel for testing automobiles as shown in the figure. Determine: (a) the manometer reading, h, when the velocity in the test section is 60 mph (Note that there is a 1-in. column of oil of water in the manometer); (b) Determine the difference between the stagnation pressure on the front of the automobile and the pressure in the test section.

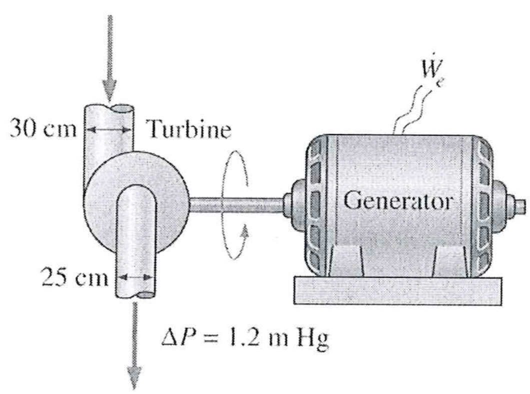


ME 231 Thermo-Fluid Mechanics I
In class problems-Set10
Prof. A. Banerjee

1. An oil pump is drawing 25kW of electric power while pumping oil with $\rho = 860\ kg/m^3$ at a rate of $0.1\ m^3/s$. The inlet and outlet diameters of the pipe are 8 cm and 12 cm respectively. If the pressure rise of the oil in the pump is measured to be 250 kPa and the motor efficiency is 90%, determine the mechanical efficiency of the pump. Take the kinetic energy correction factor to be 1.05.

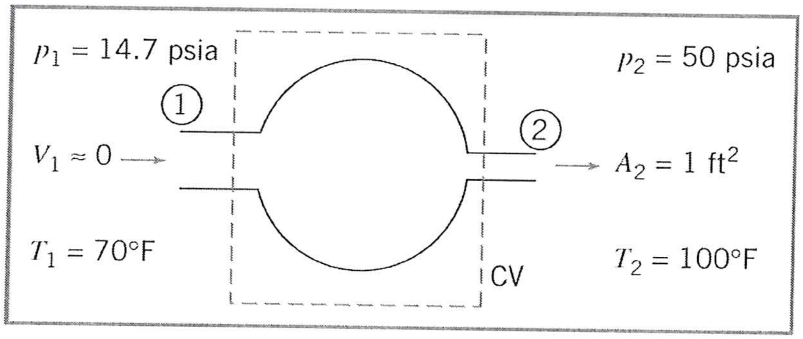


2. Water enters a hydraulic turbine through a 30-cm-diameter pipe at a rate of $0.6\ m^3/s$ and exits through a 25-cm diameter pipe. The pressure drop in the turbine is measured by a mercury manometer to be 1.2 m. For a combined turbine-generator efficiency of 83%, determine the net electric power output. Disregard the effect of the kinetic energy correction factors.

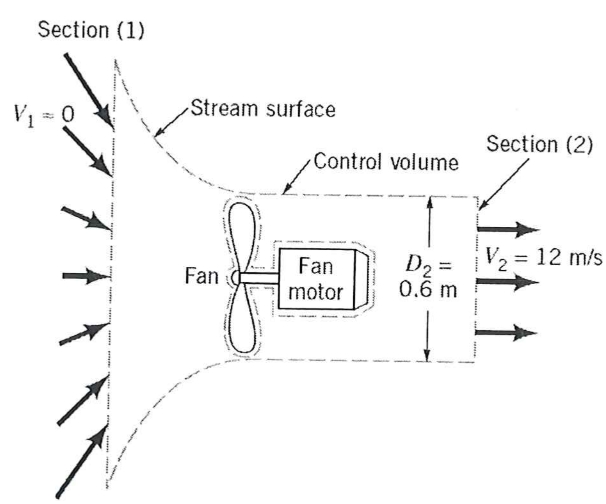


ME 231 Thermo-Fluid Mechanics I
In class problems-Set 9
Prof. A. Banerjee

1. Air at 14.7 psia, 70F, enters a compressor with negligible velocity and is discharged at 50 psia, 100F through a pipe with 1 ft^2 area. The flow rate is 20 lbm/s. The power input to the compressor is 600 hp. Determine the rate of heat transfer.



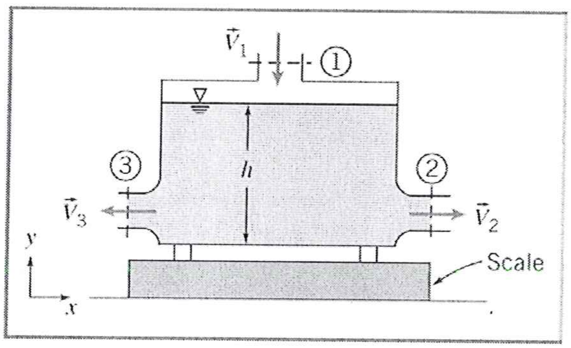
2. An axial flow ventilating fan driven by a motor that delivers 0.4 kW of power to the fan blades produces a 0.6-m-diameter axial stream of air having a speed of 12 m/s. The flow upstream of the fan involves negligible speed. Determine how much of the work of the air actually produces a useful effect, that is, a rise in available energy, and estimate the fluid mechanical efficiency of this fan.

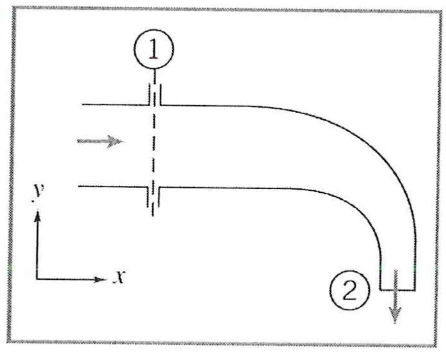


ME 231 Thermo-Fluid Mechanics I
In class problems-Set8
Prof. A. Banerjee

given 9-21-10

1. A metal container 2 ft. high, with an inside cross-sectional area of $1 ft^2$, weighs 5lbf when empty. The container is placed on a scale and water flows in through an opening in the top and out through two equal area openings in the sides, as shown in the diagram. Under steady flow conditions, the height of the water in the tank is 1.9 ft. Determine the reading on the scale.





9-23

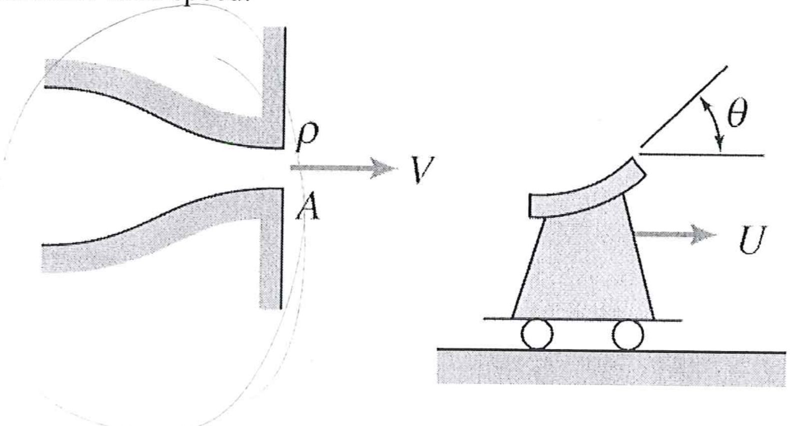
9-23

2. Water flows steadily through the 90° reducing elbow as shown in the diagram. At the inlet to the elbow, the absolute pressure is 221 kPa and the cross sectional area is $0.01 m^2$. At the outlet, the cross-sectional area is $0.0025 m^2$ and the velocity is 16 m/sec. The pressure at the outlet is atmospheric. Determine the force required to hold the elbow in position.

-101.3

9-23

3. The sketch shows a vane with a turning angle of 60°. The vane moves at a constant speed, U = 10m/sec, and receives a jet of water that leaves a stationary nozzle with speed V = 30 exit m/sec. The nozzle has an exit area of $0.003 m^2$. Determine the force that must be applied to maintain a constant vane speed.



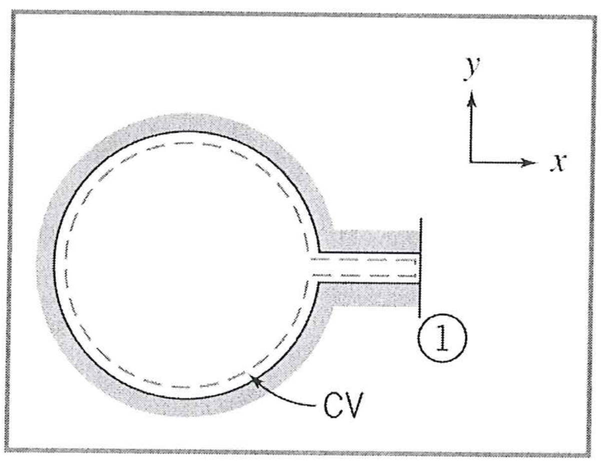
9-14-10

ME 231 Thermo-Fluid Mechanics I
In class problems-Set7
Prof. A. Banerjee

1. Students are running out of the classroom after their ME231 fluid mechanics exam at a rate of 4 students/sec (mostly because they had a great exam). Including space between students, each occupies 40 ft^3. Using RTT (with number density replacing mass density), determine the speed of the students as they leave the room. The door is 7.5 ft by 3.5 ft.

2. Consider steady flow of water through the device shown in the diagram. The areas are A_1=0.2 ft^2, A_2=0.5 ft^2, and A_3=A_4=0.4ft^2. The mass flow rate out though section 3 is given as 3.88 slug/sec. The volume flow rate in through section 4 is given as 1ft^3/sec, and $\vec{V}_1 = 10\hat{i}$ ft/sec. If the properties are assumed uniform across all the inlet and outlet flow sections, determine the flow velocity at section 2. (figure to be provided in class)

3. A tank of 0.05 m^3 volume contains air at 800 kPa (absolute) and 15 C. At t = 0, air escapes from the tank through a valve with a flow area 65 mm^2. The air passing though the valve has a speed of 311 m/sec and a density of 6.13 kg/m^3. Properties in the rest of the tank may be assumed to be uniform at each instant. Determine the instantaneous rate of change of density at t = 0.



#1 $\left.\frac{dB}{dt}\right)_{system} = \frac{\partial}{\partial t}\int_{\forall} \overset{1}{\cancel{b}}\rho \, d\forall + \int_A \overset{1}{\cancel{b}}\rho\vec{V}\cdot d\vec{A}$ $N_s = B = \#$ of students $b = \frac{B}{N} = 1$

$\left.\frac{dB}{dt}\right|_{sys} = 0 = \frac{\partial N_s}{\partial t} + \int \rho\vec{V}\cdot dt$

$= -\frac{4 \text{ students}}{sec} + \int_1^2 \rho\vec{V}\cdot d\vec{A}$ $\rho = \#$ density $= \frac{1 \text{ student}}{40 \text{ ft}^3}$ $\dot{B}_{net} = \dot{B}_{out} - \cancel{\dot{B}_{in}}^{\,0}$

$\hookrightarrow = \frac{1 \text{ stu}}{40 \text{ ft}^3}\cdot(\vec{V}\cdot d\vec{A}) = \frac{1}{40}(VA) = \frac{1 \text{ stud.}}{40 \text{ ft}^3}\times(V\times 7.5\times 3.5 \text{ ft})$

$\vec{V}$
dA

dA = area normal vector, pointing outwards $= A\cdot\hat{n}$

$V = 6.1$ ft/s

Tuesday @ 5pm

ME 231 Thermo-Fluid Mechanics I
In class problems-Set 6
Prof. A. Banerjee

1. The figure below shows an iceberg floating on sea-water. Quantify the statement that only the tip of an iceberg is visible. (Hint: Find % volume over water: SG: for ice = 0.917, for sea-water =1.025).

$W = F_B$

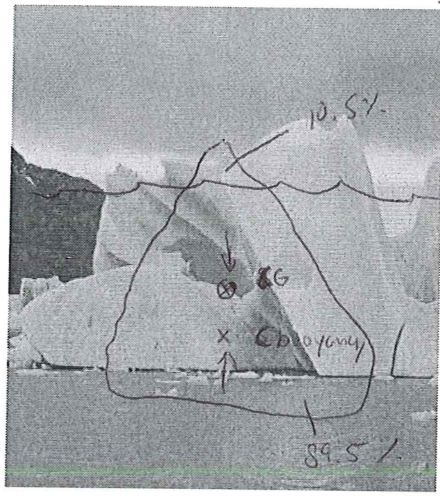
$F_B = \rho_{sw} V_{submerged} \cdot g$

$W = \rho_{ice} V_{iceberg\ total} \cdot g$

$\rho_{sw} V_{sub} = \rho_{ice} V_{ice\ total}$

$V_{sub} = \frac{\rho_{ice}}{\rho_{sw}} V_{total}$

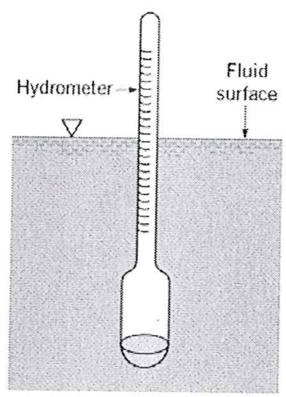
$V_{sub} = \frac{SG_{ice}}{SG_{sw}} V_{total}$



$SG = \frac{\rho}{\rho_{water}}$

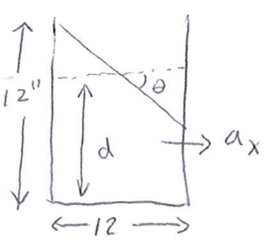
$\frac{V_{sub}}{V_{total}} = .895$

2. A hydrometer is a specific gravity indicator, the value being indicated by the level at which the free surface intersects the stem when floating in a liquid. The 1.0 mark is the level when in distilled water. For the unit shown, the immersed volume in distilled water is 15 cm^3. The stem is 6 mm in diameter. Find the distance, *h*, from the 1.0 mark to the surface when the hydrometer is placed in a nitric acid solution of specific gravity 1.5.



3. A fish tank 12 in. × 24 in. × 12in. partially filled with water is to be transported in an automobile. Find the allowable depth of water to ensure that the water will not spill during the trip.

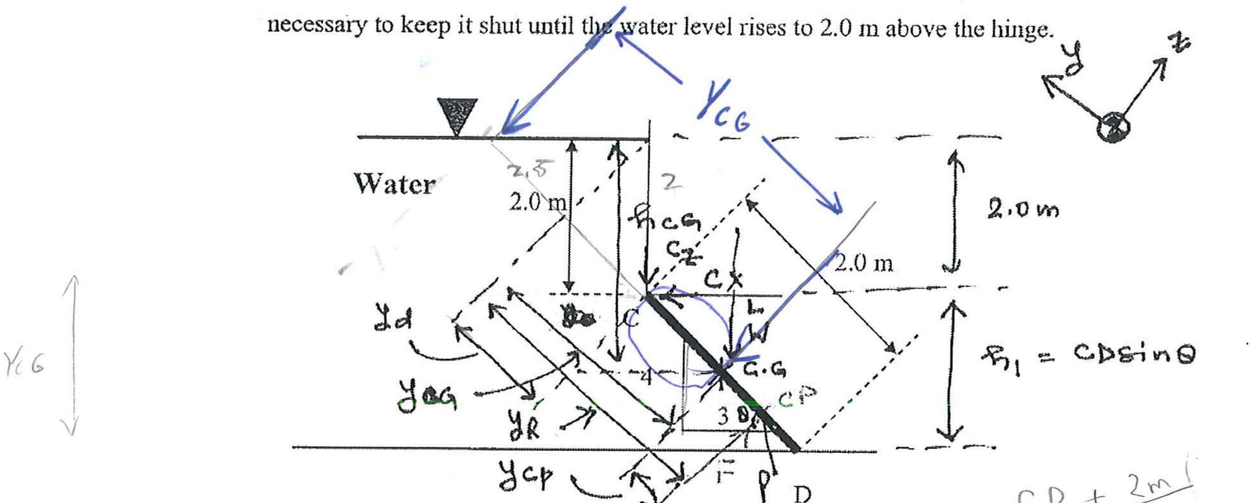
$accel_x = 5\ ft/s^2$



Y_{CG} is drawn incorrectly

ME 231 Thermo-Fluid Mechanics I
In class problems-Set 5
Prof. A. Banerjee

1. The rectangular gate CD of Figure 1 is 1.8 m wide and 2.0 m long. Assuming the material of the gate to be homogeneous and neglecting friction at the hinge C, determine the weight of the gate necessary to keep it shut until the water level rises to 2.0 m above the hinge.



$\tan\theta = \frac{4}{3} \Rightarrow \sin\theta = \frac{4}{5}$

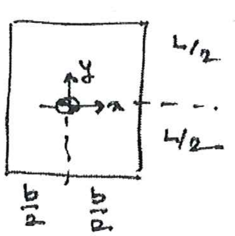
$\Rightarrow h_1 = \overline{CD}\sin\theta = 2.0 \times \frac{4}{5} = \frac{8}{5}$ m $= 1.6$ m

$h_{cg} = 2.0 + \frac{1}{2} \times h_1 = 2.8$ m

Reaction Force : $F = \gamma_{water} \cdot h_c \cdot A = \left(9.8 \frac{kN}{m^3}\right)(2.8\,m)(2.0 \times 1.8)m^2$

$\Rightarrow \boxed{F = 98.784\,kN}$

$y_{CG} = \frac{2.0}{\sin\theta} + \frac{CD}{2} = 2 \times \left(\frac{5}{4}\right) + 1 = 3.5$ m



$A = bL$

$I_{xx} = \frac{bL^3}{12}$

$I_{xy} = 0$

$$y_{cp} = -\frac{I_{xx}\sin\theta}{h_{CG} \cdot A} = -\frac{\frac{1}{12}(1.8m)(2m)^3 \cdot 4/5}{(2.8m)(1.8 \times 2.0)m^2} = -0.0952\,m$$

$$\Rightarrow \quad y_R = y_{CG} - y_{cp} = 3.5 - (-0.0952)\ m$$

$$= 3.595\ m .$$

For equilibrium : $\quad \Sigma M_c = 0$

$$\Rightarrow \quad W \cdot \overline{CL} - F \cdot \overline{CP} = 0$$

where $\quad \overline{CL} = \frac{CD}{2} \cos\theta = \frac{1}{2}(2m)\left(\frac{3}{5}\right) = \left(\frac{3}{5}\right) m$

$$\overline{CP} = y_R - y_{\overset{c}{\cancel{d}}} = 3.595\ m - \frac{2.0\ m}{\sin\theta}$$

$$= 3.595 m - 2.5\ m = 1.095\ m$$

$$\Rightarrow \quad W \times \left(\frac{3}{5}\right) - (98.784\ kN) \times (1.095 m) = 0$$

$$\Rightarrow \quad \boxed{W = 180\ kN}$$

ME 231 Thermo-Fluid Mechanics I
In class problems-Set 5
Prof. A. Banerjee

1. The rectangular gate CD of Figure 1 is 1.8 m wide and 2.0 m long. Assuming the material of the gate to be homogeneous and neglecting friction at the hinge C, determine the weight of the gate necessary to keep it shut until the water level rises to 2.0 m above the hinge.

$\Sigma M_C = 0$

$F_{hydro} = F_R$ (reaction force)

$= \gamma_{water} \cdot h_{CG} \cdot A$

$= 9.8 \frac{kN}{m^3}(2.8m)(1.8 \times 2.0 m^2)$

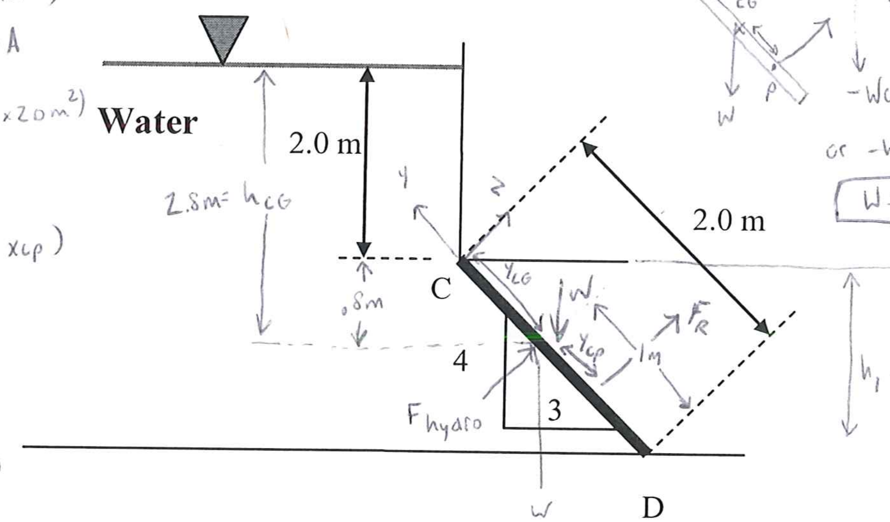
$F_R = 98.874$ KN

Center of pressure (y_{cp}, x_{cp})

$y_{cp} = \frac{I_{xx} \sin\theta}{h_{CG} \cdot A}$

$= \frac{\frac{1}{12}(1.8)(2m)^3 \cdot \frac{4}{5}}{2.8m \times (1.8 \times 2 m^2)}$

$= -0.0952$ m



$\overline{CP} = (.0952 + 1.0m)$
$= 1.0952$ m

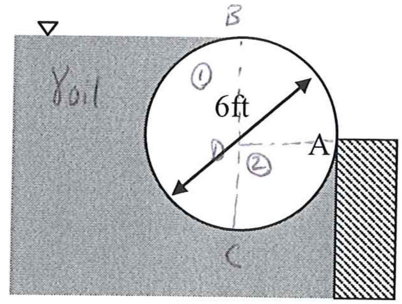
$-W\cos\theta \times 1 + F_R \times CP = 0$

or $-W\left(\frac{3}{5}\right) + 98.784 kN \times 1.0952 m = 0$

$\boxed{W = 180 KN}$

$h_1 = CD \sin\theta$
$= \frac{4}{5}(2.0m) = 1.6m$

2. A 9 ft long cylinder as shown in the figure floats in oil (γ_{oil} = 57 lbf/ft³) and rests against a wall. Determine the horizontal force the cylinder exerts on the wall at the point of contact A.



Projection of $\widehat{AC} = \overline{AD}$

" of $\widehat{BPC} = \overline{BC}$

$F \rightarrow$ ①

$F = \gamma_{oil} h_{CG} (A_p) =$

$= \gamma_{oil} \times \left(\frac{D}{2}\right)(D \times W)$

$= 57 \frac{lbf}{ft^3}(3 ft)(6 \cdot 9 ft)$

$= 9234$ lbf

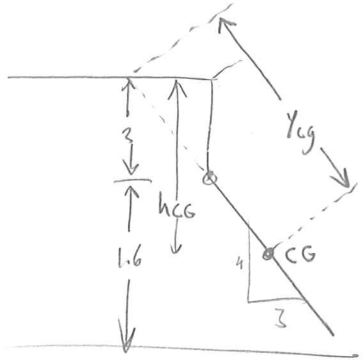
$D/2$ ② $\leftarrow F_2$

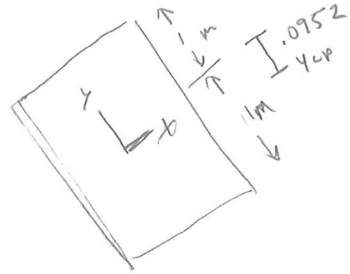
$F_2 = \gamma_{oil} \times \left(\frac{D}{2} + \frac{1}{2}D_2\right) \times \frac{1}{2} DW$

$= 57 \times 4.5 \times 3 \times 9$

$= 6926$

Force on pt. A = 2934 − 6926





$H_{CG} = 2 + .8 = 2.8\text{m}$

Reaction Force $= \gamma h_{CG} A_{\cancel{proj}} = 9790\ \text{N/m}^3\,(2.8\text{m})(1.8 \times 2.0) = \boxed{98.68\text{kN}}$

$$Y_{cp} = -\frac{I_{xx}\sin\alpha}{h_{CG}A} = \frac{\frac{\cancel{1.8}(2)^3}{12}\left(\frac{4}{5}\right)}{2.8(\cancel{1.8})(2)} = \boxed{-.0952}$$

$X_{cp} = \text{N/A}$ $\qquad Y_{CG} = \frac{2.0}{\sin\theta} + \frac{CD}{2} = 2.0\left(\frac{5}{4}\right) + \left(\frac{2}{2}\right) = 3.5$

$Y_R = Y_{CG} - Y_{cp}$

$= 3.5 + .0952$

$= 3.595$

$M_A =$

$\overline{CP} = Y_R - Y_0$

$W \cdot \overline{CL} = F \cdot \overline{CP}$

$= 98.68\text{ kN}\,(3.595 - 2.5)$

$$W = \frac{98.68\text{kN}\,(1.095)}{\frac{3}{5}\left(\frac{2}{2}\right)}$$

$\boxed{W = 180\text{ kN}}$

9-7

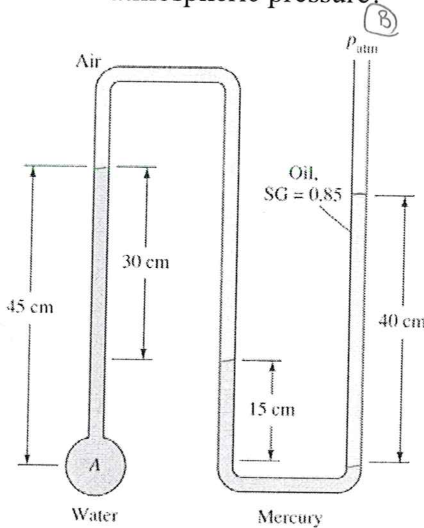
ME 231 Thermo-Fluid Mechanics I
In class problems-Set 4
Prof. A. Banerjee

1. If sea-level pressure is 101.3 kPa, compute the standard pressure at an altitude of 5000 m by using:

   a. The exact formula: $p = p_a\left[1-\dfrac{Bz}{T_0}\right]^{\frac{g}{RB}}$, where $\dfrac{g}{RB} = 5.26$ for air; B is the lapse rate.

   b. An isothermal assumption at a standard sea-level temperature of 15°C.

2. In the figure below, determine the gage pressure at point A in Pa. Is it higher or lower than the atmospheric pressure?



$\gamma_{water} = 9790\ N/m^3$

$\gamma_{Hg} = 133100\ N/m^3$

$\gamma_{oil} = SG \cdot \gamma_{water} = 8321.5\ N/m^3$

Start at B $P_{atm} + \gamma_{oil}(.4m)$

$- \gamma_{Hg}(.15m) - \gamma_{air}(.3m)$

$+ \gamma_{water}(.45m) = P_A$

$P_A = P_{atm} - 12234.63\ P_A$

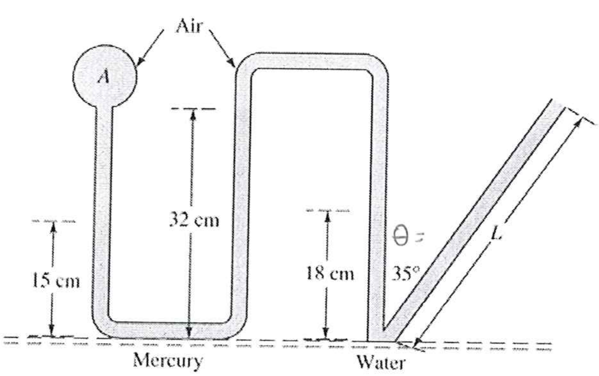
$= -12.334\ kPa$ (Vacuum)

1a) $= 101.3kPa\left[1 - \dfrac{.0065\ k/m \times 5000\ m}{288.16\ K}\right]^{5.26}$

$= 54\ kPa$

1b) $P_2 = P_{atm} \cdot \exp\left[-\dfrac{gz}{RT}\right] = 56\ kPa$

3. The system in the figure below is open to 1 atm on the right side. (a) If L = 120 cm, what is the air pressure inside the container A? (b) Conversely, if p_A = 135 kPa, what is the length L?



Problem: to find the hydrostatic force on a "plane" or a "curved" surface that is submerged

ME 231 Thermo-Fluid Mechanics I
In class problems-Set3
Prof. A. Banerjee

1. Given the Eulerian velocity vector field: $\vec{V} = 3t\,\hat{i} + xz\,\hat{j} + ty^2\,\hat{k}$. Find the total, local and convective acceleration of the particle.

2. For the velocity fields (i) – (ii) given below, determine:

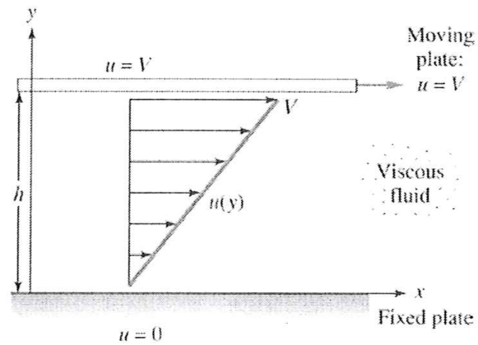
a. whether the flow is one-, two- or three-dimensional and why?
b. whether the flow is steady or unsteady, and why?

(i) $\vec{V} = ae^{-bx}\hat{i} + bt^2\hat{j}$ 1-D transient $\vec{v}(x,t)$

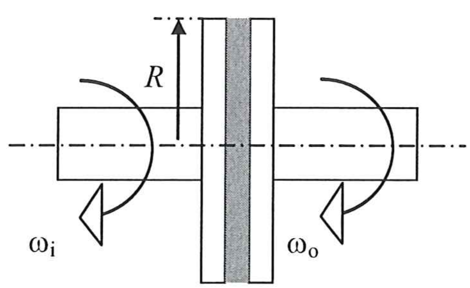
(ii) $\vec{V} = ax^2\hat{i} + bxz\hat{j} + cy\hat{k}$ 3D steady state

ME 231 Thermo-Fluid Mechanics I
In class problems-Set2
Prof. A. Banerjee

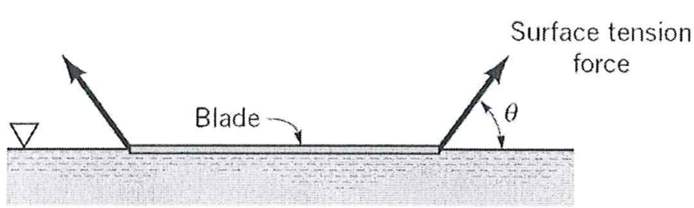
1. A classic problem is the flow induced between a fixed lower plate and an upper plate moving steadily at velocity V. Suppose the fluid being sheared is SAE30 oil at 20°C. Compute the shear stress if $V = 3$ m/s and $h = 2$cm.



2. A viscous clutch is to be made from a pair of closely spaced parallel disks enclosing a thin layer of viscous fluid. Develop algebraic expressions for the torque and the power transmitted by the disk pair, in terms of liquid viscosity, μ, disk radius, R, disk spacing, a, and the angular speeds: ω_i of the input disk and ω_o of the output disk. Also develop expressions for the slip ratio, $s=\Delta\omega/\omega_i$, in terms of ω_i and the torque transmitted.



3. Surface tension forces can be strong enough to allow a double-edge steel razor blade to "float" on water. Assume that the surface tension forces act at an angle θ relative to the water surface as shown in the figure. The mass of the blade is 0.64×10^{-3} kg, and the total length of the sides is 206 mm. Determine the angle θ required to maintain equilibrium between the blade weight and the resultant surface tension force.



ME 231 Thermo-Fluid Mechanics I
In class problems-Set1
Prof. A. Banerjee

1. A body weighs 1000 lbf when exposed to a standard earth gravity g = 32.174 ft/s^2.
   a. What is its mass in kg?
   b. What will the weight of this body be in N if exposed to the moon's standard acceleration?

2. In 1890, Robert Manning, an Irish Engineer, proposed the following empirical formula for the average velocity V in uniform flow due to gravity down an open channel (in BG Units): $V = \frac{1.49}{n} R^{\frac{2}{3}} S^{\frac{1}{2}}$, where R = hydraulic radius of channel (m), S= channel slope (tan θ) and n = Manning's roughness factor (dimensionless) that is given for a surface condition of the walls and the bottom of the channel.. Find:
   a. The dimensions for Manning's constant for formula to be dimensionally homogeneous, and
   b. Rewrite Manning's equation to SI system.

3. The air in an automobile tire with a volume of 0.53 ft^3 is at 90°F and 20 psig. Determine the amount of air that must be added to raise the pressure to the recommended value of 30 psig. Assume that the atmospheric pressure is 14.6 psia and the temperature and volume remain constant.

Suggested Reading

ME 231 Thermo-Fluid Mechanics I
Mechanical & Aerospace Engineering Department
Prof. A. Banerjee
Suggested Readings from Text Book

Chapter 7: Flow passed immersed bodies (external flows)

Objectives:

1. Identify and understand various characteristics of external flow
2. Discuss the fundamental properties of boundary layers, including laminar, transitional and turbulent boundary layers
3. Calculate boundary layer parameters for flow past a flat plate
4. Understand the concept of flow separation
5. Calculate lift and drag forces for various objects

Suggested readings from text:

Section	Topic	Page #	Sample Problem #
7.1	Reynolds Number & Geometry Effects	457-460	7.1
7.2	Momentum Integral Estimates	461-464	7.2
7.3	Boundary Layer Equations	464-467	
7.4	Flat Plate Boundary Layer	467-470	7.3
7.5	Effect of Pressure Gradient – concept of flow separation	476-482	
7.4	Experimental External Flows Concept of Lift & Drag	Class Notes 482-509	7.6-7.10

ME 231 Thermo-Fluid Mechanics I
Mechanical & Aerospace Engineering Department
Prof. A. Banerjee
Suggested Readings from Text Book

Chapter 11: Turbomachinery

Objectives:

1. Design analysis of a piping system including centrifugal pumps – understand pump performance curves and system matching

Suggested readings from text:

Chapter 11:

Section	Topic	Page #	Sample Problem #
11.1	Introduction and Classification	759-762	NA
11.2-11.3	Centrifugal Pump, Pump Performance Curves	768-771 + Class Notes	Set19-supplement

ME 231 Thermo-Fluid Mechanics I
Mechanical & Aerospace Engineering Department
Prof. A. Banerjee
Suggested Readings from Text Book

Chapter 6: Viscous Flow in Ducts

Objectives:

1. Identify and understand various characteristics of flow in a pipe
2. Discuss the main properties of laminar and turbulent pipe flow and appreciate their difference
3. Calculate losses in pipes – differentiate between major and minor losses in a system
4. Predict flow-rate in a pipe using Bernoulli Obstruction Theory

Suggested readings from text:

Chapter 6:

Section	Topic	Page #	Sample Problem #
6.1-6.2	Reynolds Number Regime, Internal vs. External Flow	347-355	6.1, 6.2
6.3 - 6.4 (Omit 6.5)	Head Loss, Friction factor, Laminar fully developed flow	Class Notes, 355-359	6.3, 6.4
6.8	Flow in non-circular ducts (Please omit section on turbulent flow solution – pg 376)	379-382	6.13-6.15
6.9	Major & Minor Losses	388-397	6.16
6.10	Multiple-Pipe systems (Reading Assignment)	397-403	6.17-6.19
6.12	Fluid Meters (Reading Assignment)	408-429 X	6.21

won't talk about this in class

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Suggested Readings from Text Book

Chapter 5: Dimensional Analysis and Similarity

Objectives:

1. Apply the Buckingham pi theorem
2. Develop a set of dimensionless variables for a given flow situation
3. Discuss the use of dimensionless variables in data analysis
4. Apply the concepts of modeling and similitude to develop equations for predicting various flow scenarios

Suggested readings from text:

Section	Topic	Page #	Sample Problem #
5.1	Introduction	293-296	5.1
5.2	Principle of Dimensional Homogeneity	296-302	
5.3	Buckingham pi-theorem	302-312	5.2-5.4
5.4	Dimensionless parameters	312-321	5.7, 5.8
5.5	Modeling and Pitfalls	321-333	5.9- 5.11

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Chapter 4: Differential Relations for Fluid Flow

Objectives:

1. Discuss the differences between Eulerian and Lagrangian descriptions of fluid motion.
2. Identify various flow characteristics based on velocity field. Determine the streamline pattern and acceleration field given a velocity field.
3. Determine various kinematic elements of the flow given the velocity field.
4. Apply the concepts of conservation of mass, momentum and energy on a fluid element and discuss how it is different from the control volume approach of chapter 3.
5. Apply boundary conditions to various flow problems.
6. Simplified solutions to Navier Stokes Equation (conservation of momentum).
7. Apply the concepts of stream-function, velocity potential and vorticity.

Suggested readings:

Section	Topic	Page #	Example Problem #
4.1	The Acceleration Field	230-231	4.1
4.2	Conservation of Mass	232-237	4.3
4.3	Conservation of Linear Momentum	238-244	4.5
4.5	Conservation of Energy	246-248	
4.6	Boundary Condition	249-253	4.6
4.7	Stream Function	253-260	4.7, 4.8
4.8	Vorticity and Irrotationality	261-263	
4.9	Frictionless Irrotational Flows (Velocity Potential, etc.)	263-268	4.9
4.10	Illustrative Cases of Viscous Flows (incompressible)	268-275	4.10

Note:

- All page numbers and sample problems are from Fluid Mechanics by Frank White (7th Ed.)

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Chapter 3: Integral relations for a Control Volume

Objectives:

1. Discuss the differences between a system and control volume
2. Apply Reynolds Transport Theorem and the material derivative
3. Identify an appropriate control volume
4. Apply conservation of mass and conservation of linear momentum equations
5. Use the energy equation to account for losses (due to friction) as well as effects of fluid machinery (pumps and turbines)
6. Explain the *development*, *uses* and *limitations* of Bernoulli equation
7. Apply concept of static, stagnation, dynamic and total pressures/head
8. Calculate various flow properties using the energy grade line (EGL) and the hydraulic grade line (HGL).

Suggested readings:

Section	Topic	Page #
3.1	Basic Laws of Fluid Mechanics	139-143
3.2	Reynolds Transport Theorem	143-150
3.3	Conservation of Mass	150-155
3.4	Conservation of Linear Momentum	155-165
3.5	Bernoulli Equation (Frictionless flow), EGL and HGL, Restrictions on the use of Bernoulli Equations	169-178
3.7	The Energy Equation	184-194

Note:

Section 3.6 – Conservation of Angular Momentum is beyond the scope of ME231.

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Suggested Readings from Text Book

Chapter 2: Pressure Distribution in a Fluid

Objectives:

1. Determine pressure at various locations in a fluid at rest
2. Explain the concept of manometers and apply appropriate equations to determine pressures
3. Calculate the hydrostatic force on a plane and submerged surface
4. Analyze fluid undergoing rigid body motion
5. Calculate buoyant force on a body and discuss stability of floating objects.

Suggested readings from text:

Section	Topic	Page #
2.1-2.2	Pressure and Pressure Gradient	65-68
2.3	Hydrostatic Pressure Distribution	68-74
2.4	Application to Manometry	75-78
2.5	Hydrostatic Force on a Plane Surface	78-85
2.6	Hydrostatic Force on a Curved Surface	86-89
2.8	Buoyancy	91-94
2.9	Pressure distribution in rigid body motion	97-103
2.10	Pressure Measurement Devices (Self-reading)	105-109

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Suggested Readings from Text Book

Chapter 1: Introduction

Objectives:

1. Define a fluid and analyze fluid behavior
2. Determine dimensions and units of physical quantities
3. Identify key properties used in analysis of fluid behavior
4. Use concepts of secondary fluid properties : viscosity and surface tension
5. Learn basic flow analysis techniques.

Suggested readings from text:

Section	Topic	Page #
1.1-1.2	History and scope of fluid mechanics	3 - 5
1.4-1.5	The concept of a fluid, continuum hypothesis	6 - 9
1.6	Dimensions and Units	9 – 16
1.8-1.9	Properties of a fluid (Thermodynamic properties, viscosity, surface tension)	18 - 40
1.7, 1.10	Properties of the flow field, basic flow analysis techniques	17-18, 40-41

Fluid Mechanics on the web:

http://www-math.mit.edu/~dhu/Striderweb/striderweb.html

http://www.sciencedaily.com/videos/2006/0710-robot_walks_on_water.htm

http://library.thinkquest.org/CR0215471/oil_spills.htm

http://en.wikipedia.org/wiki/Deepwater_Horizon_oil_spill

http://www.bp.com/extendedsectiongenericarticle.do?categoryId=40&contentId=7061813

Study prob. set 5

ME 231 Thermo-Fluid Mechanics I
Prof. A. Banerjee
Study Problem Set 3

****

Few notes about study problems:

(a) These problems are similar to the Home-work problems. So please spend some time reviewing these problems (and my solutions) before you start your home-work.
(b) It is a good idea to also read the corresponding book chapter before solving any problem. Refer to the suggested reading document for sections in your book to read.

****

1. Consider steady, incompressible flow through the device shown in figure 1. Determine the magnitude and direction of the volume flow rate through port 3.
2. Consider steady incompressible flow of air in a boundary layer on the length of the porous plate as shown in figure 2. Assume that the boundary layer at the downstream end of the surface has an approximate velocity profile (i.e. at section *cd*) of: $\dfrac{u}{U_\infty} = 2\left(\dfrac{y}{\delta}\right) - \left(\dfrac{y}{\delta}\right)^2$.
Uniform suction is applied along the porous surface, as shown. Calculate the volume flow rate across surface *cd*, through the porous suction surface *bc*, and across surface *bc*.
3. The velocity profile for laminar flow in an annulus as shown in figure 3 is given by: $u(r) = -\dfrac{\Delta p}{4\mu L}\left[R_0^2 - r^2 + \dfrac{R_0^2 - R_i^2}{\ln\left(R_i/R_0\right)}\ln\left(\dfrac{R_0}{r}\right)\right]$, where $\Delta p/L$=-10kPa/m is the pressure gradient, μ is the viscosity (SAE10 oil at 20C) and R_0 = 5mm and R_i = 1 mm are the outer and inner radii. Find the volume flow rate, the average velocity and the maximum velocity. Plot the velocity distribution using a plotting software.

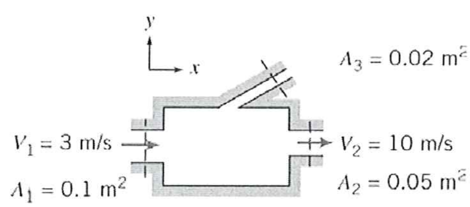


Figure 1

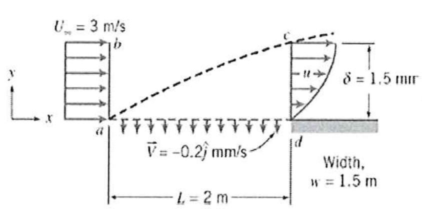


Figure 2

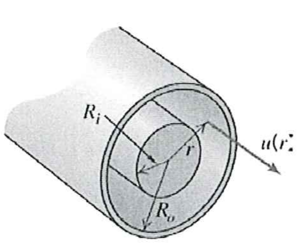


Figure 3

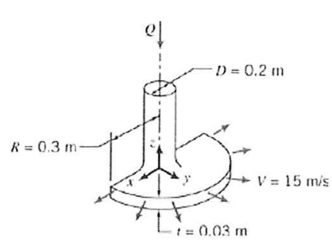


Figure 4

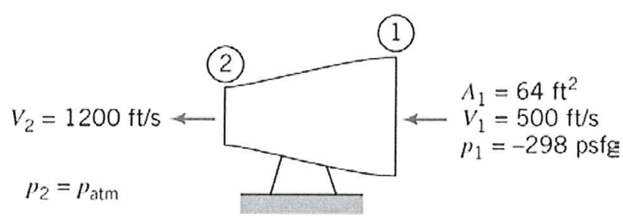


Figure 5

4. The nozzle shown in figure 4 discharges a sheet of water through a 180° arc. The water speed is 15 m/s and the jet thickness is 30 mm at a radial distance of 0.3 m from the centerline of the supply pipe. Find (a) the volume flow rate of water in the jet sheet, and, (b) the *y* component of force required to hold the nozzle in place.
5. At rated thrust, a liquid-fuelled rocket motor consumes 80 kg/s of nitric acid as oxidizer and 32 kg/s of aniline as fuel (see figure 5). Flow leaves axially at 180 m/s relative to the nozzle and at 110 kPa. The nozzle exit diameter is D = 0.6 m. Calculate the thrust produced by the motor on a test stand at standard sea-level pressure.

6. A reducer in a piping system is shown in figure 6. The internal volume of the reducer is 0.2 m^2 and it mass is 25 kg. Evaluate the total force that must be provided by the surrounding pipes to support the reducer. The fluid is gasoline.

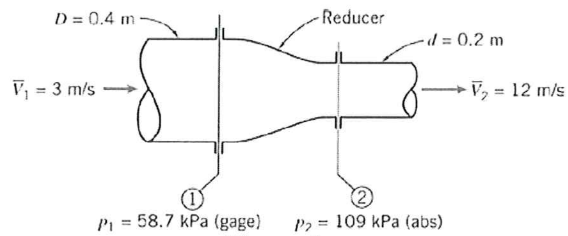


Figure 6

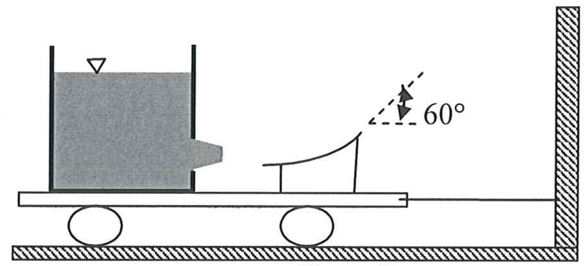
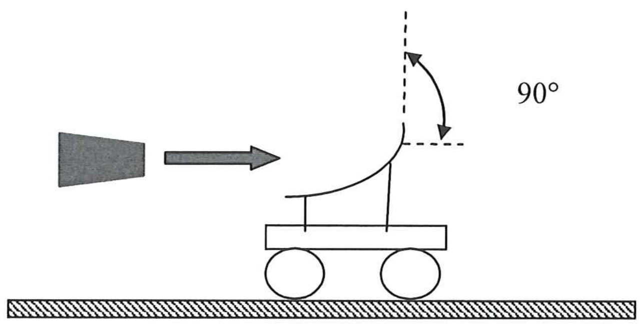


Figure 7

7. A turning vane, which deflects water through 60°, is attached to the cart. Water issues from the tank through a 600 mm^2 nozzle at a speed of 10 m/s. The water level in the tank is maintained constant by adding water through a vertical pipe. Determine the tension in the wire holding the cart stationary and the force of the vane on the cart.

8. A water jet issuing from a stationary nozzle, encounters a vane curved through an angle of 90° that is moving away from the nozzle at a constant speed of 15 m/s. The jet has a cross sectional area of 600 mm^2 and a speed of 30 m/s. Determine the force that must be applied to maintain a constant vane speed.



9. Solution P#1:

Given: Data on flow through device

Find: Volume flow rate at port 3

Solution:

Basic equation $\sum_{CS} \left(\vec{V} \cdot \vec{A}\right) = 0$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Uniform flow

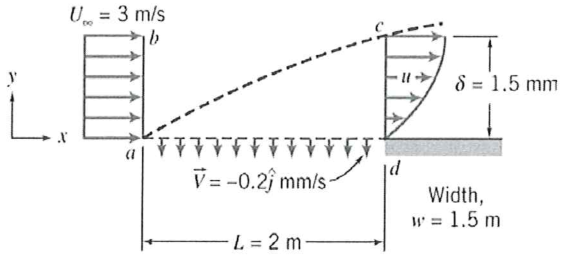
Then for the box $\sum_{CS} \left(\vec{V} \cdot \vec{A}\right) = -V_1 \cdot A_1 + V_2 \cdot A_2 + V_3 \cdot A_3 = -V_1 \cdot A_1 + V_2 \cdot A_2 + Q_3$

Note we assume outflow at port 3

Hence $Q_3 = V_1 \cdot A_1 - V_2 \cdot A_2$ $\quad Q_3 = 3 \cdot \frac{m}{s} \times 0.1 \cdot m^2 - 10 \cdot \frac{m}{s} \times 0.05 \cdot m^2$ $\quad Q_3 = -0.2 \cdot \frac{m^3}{s}$

The negative sign indicates the flow at port 3 is inwards. Flow rate at port 3 is 0.2 m^3/s inwards

Solution P#2:

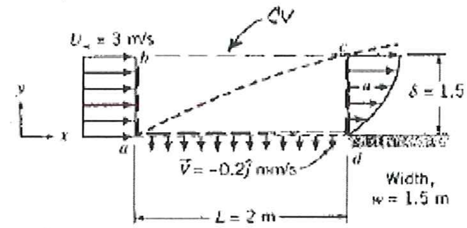


Find: (a) Volume flow rate across cd.

(b) Volume flow rate through porous surface (ad).

(c) Volume flow rate across bc.

<u>Solution:</u> Apply conservation of mass to CV shown.



Basic equation:

$$0 = \underbrace{\frac{\partial}{\partial t}\int_{CV} \rho\, dV}_{=0(1)} + \int_{CS} \rho \vec{V}\cdot d\vec{A}$$

Assumptions: (1) Incompressible flow

(2) Parabolic profile at section cd: $\frac{u}{U_\infty} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$

Then $\quad 0 = \int_{CS} \vec{V}\cdot d\vec{A} = Q_{ab} + Q_{bc} + Q_{cd} + Q_{da} \qquad (1)$

$$Q_{cd} = \int_{cd} \vec{V}\cdot d\vec{A} = \int_0^\delta u w\, dy = wU_\infty \delta \int_0^1 \frac{u}{U}\, d\left(\frac{y}{\delta}\right) = wU_\infty\delta \int_0^1 \left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right] d\left(\frac{y}{\delta}\right)$$

$$= wU_\infty\delta\left[\left(\frac{y}{\delta}\right)^2 - \frac{1}{3}\left(\frac{y}{\delta}\right)^3\right]_0^1 = \frac{2}{3} w\delta U_\infty$$

$Q_{cd} = \frac{2}{3} \times 1.5\,m \times 0.0015\,m \times 3\,\frac{m}{s} = 4.50\times10^{-3}\,m^3/s$ (out of CV) $\qquad Q_{cd}$

Flow across ad is uniform, so

$$Q_{ad} = \vec{V}\cdot\vec{A} = v\hat{j}\cdot wL(-\hat{j}) = -vwL$$

$Q_{ad} = -0.2\,\frac{mm}{s} \times 1.5\,m \times 2\,m \times \frac{m}{1000\,mm} = 6.00\times10^{-4}\,m/s$ (out of CV) $\qquad Q_{ad}$

Finally, from Eq. 1,

$$Q_{bc} = -Q_{ab} - Q_{cd} - Q_{da} \qquad (2)$$

But $Q_{ab} = \vec{U}_\infty \cdot \vec{A}_{ab} = U_\infty\hat{i}\cdot w\delta(-\hat{i}) = -w\delta U_\infty$

$Q_{ab} = -1.5\,m \times 0.0015\,m \times 3\,\frac{m}{s} = -6.75\times10^{-3}\,m^3/s$ (into CV)

Substituting into Eq. 2,

$$Q_{bc} = \left[-(-6.75\times10^{-3}) - 4.50\times10^{-3} - 6.00\times10^{-4}\right] m^3/s$$

$Q_{bc} = 1.65\times10^{-3}\,m^3/s$ (out of CV) $\qquad Q_{bc}$

Solution P#3:

Solution:

Governing equation For the flow rate (Eq. 4.14a) and average velocity (Eq. 4.14b) $Q = \int \vec{V}\, d\vec{A}$ $V_{av} = \frac{Q}{A}$

The given data is $R_o = 5 \cdot mm$ $R_i = 1 \cdot mm$ $\frac{\Delta p}{L} = -10 \cdot \frac{kPa}{m}$ $\mu = 0.1 \cdot \frac{N \cdot s}{m^2}$ (From Fig. A.2)

$$u(r) = \frac{-\Delta p}{4 \cdot \mu \cdot L} \cdot \left(R_o^2 - r^2 + \frac{R_o^2 - R_i^2}{\ln\left(\frac{R_i}{R_o}\right)} \cdot \ln\left(\frac{R_o}{r}\right) \right)$$

The flow rate is $Q = \int_{R_i}^{R_o} u(r) \cdot 2 \cdot \pi \cdot r\, dr$

Considerable mathematical manipulation leads to $$Q = \frac{\Delta p \cdot \pi}{8 \cdot \mu \cdot L} \cdot \left(R_o^2 - R_i^2\right) \cdot \left[\frac{\left(R_o^2 - R_i^2\right)}{\ln\left(\frac{R_o}{R_i}\right)} - \left(R_i^2 + R_o^2\right) \right]$$

Substituting values $$Q = \frac{\pi}{8} \cdot \left(-10 \cdot 10^3\right) \cdot \frac{N}{m^2 \cdot m} \cdot \frac{m^2}{0.1 \cdot N \cdot s} \cdot \left(5^2 - 1^2\right) \cdot \left(\frac{m}{1000}\right)^2 \left[\frac{5^2 - 1^2}{\ln\left(\frac{5}{1}\right)} - \left(5^2 + 1^2\right) \right] \cdot \left(\frac{m}{1000}\right)^2$$

$Q = 1.045 \times 10^{-5} \frac{m^3}{s}$ $Q = 10.45 \cdot \frac{mL}{s}$

The average velocity is $V_{av} = \frac{Q}{A} = \frac{Q}{\pi \cdot \left(R_o^2 - R_i^2\right)}$ $V_{av} = \frac{1}{\pi} \times 1.045 \times 10^{-5} \cdot \frac{m^3}{s} \times \frac{1}{5^2 - 1^2} \cdot \left(\frac{1000}{m}\right)^2$ $V_{av} = 0.139 \frac{m}{s}$

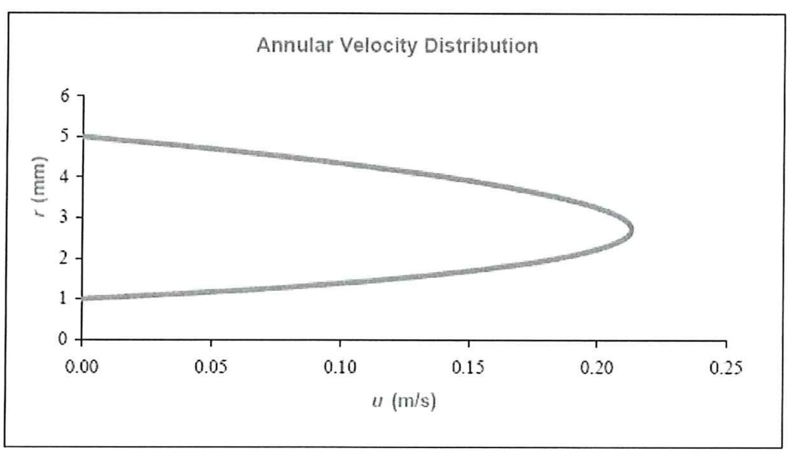
The maximum velocity occurs when $$\frac{du}{dr} = 0 = \frac{d}{dx}\left[\frac{-\Delta p}{4 \cdot \mu \cdot L} \cdot \left(R_o^2 - r^2 + \frac{R_o^2 - R_i^2}{\ln\left(\frac{R_i}{R_o}\right)} \cdot \ln\left(\frac{R_o}{r}\right) \right) \right] = -\frac{\Delta p}{4 \cdot \mu \cdot L} \cdot \left[-2 \cdot r - \frac{\left(R_o^2 - R_i^2\right)}{\ln\left(\frac{R_i}{R_o}\right) \cdot r} \right]$$

$r = \sqrt{\frac{R_i^2 - R_o^2}{2 \cdot \ln\left(\frac{R_i}{R_o}\right)}}$ $r = 2.73 \cdot mm$ Substituting in u(r) $u_{max} = u(2.73 \cdot mm) = 0.213 \cdot \frac{m}{s}$

The maximum velocity using Solver instead, and the plot, are also shown in the corresponding *Excel* workbook

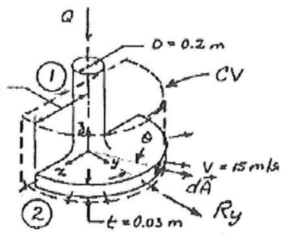
R_o =	5	mm
R_i =	1	mm
$\Delta p/L$ =	-10	kPa/m
μ =	0.1	N.s/m²

r (mm)	*u* (m/s)
1.00	0.000
1.25	0.069
1.50	0.120
1.75	0.157
2.00	0.183
2.25	0.201
2.50	0.210
2.75	0.213
3.00	0.210
3.25	0.200
3.50	0.186
3.75	0.166
4.00	0.142
4.25	0.113
4.50	0.079
4.75	0.042
5.00	0.000



Solution P#4:

Find: (a) Volume flow rate
(b) y-component of force required to hold in place



Solution: Choose CV and coordinates shown. Apply continuity and momentum equation in y-direction.

Basic equations: $Q = \int_A \vec{V} \cdot d\vec{A}$

$$F_{S_y} + \cancel{F_{B_y}} = \underbrace{\cancel{\frac{\partial}{\partial t}}}_{=0(3)}\int_{CV} v \rho \, d\forall + \int_{CS} v \rho \vec{V} \cdot d\vec{A}$$

(F_{B_y} = 0(2))

Assumptions: (1) Flow uniform across exit section
(2) $F_{By} = 0$
(3) Steady flow

At section (2), $\vec{V} \cdot d\vec{A} = VRt\,d\theta$, since flow out of CV. Then

$$Q = \int_{-\pi/2}^{\pi/2} VRt\,d\theta = VRt\left[\theta\right]_{-\pi/2}^{\pi/2} = VRt\pi$$

$$Q = 15\frac{m}{s} \times 0.3\,m \times 0.03\,m \times \pi = 0.424\ m^3/s$$ Q

From momentum

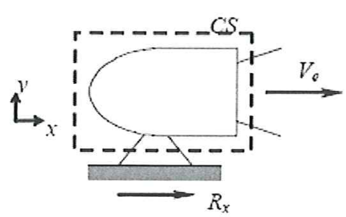
$$R_y = \int_{CS} v \rho \vec{V} \cdot d\vec{A} = \int_{A_1} v_1 \{-|\rho V_1 dA_1|\} + \int_{A_2} v_2 \{+|\rho V_2 dA_2|\}$$

with $v_1 = 0$ $v_2 = V\cos\theta$

$$R_y = \int_{-\pi/2}^{\pi/2} V\cos\theta\, \rho V R t\, d\theta = \rho V^2 R t\left[\sin\theta\right]_{-\pi/2}^{\pi/2} = 2\rho V^2 R t$$

$$R_y = 2 \times 999\frac{kg}{m^3} \times (15)^2\frac{m^2}{s^2} \times 0.3\,m \times 0.03\,m \times \frac{N \cdot s^2}{kg \cdot m} = 4.05\ kN$$ Ry

Solution P#5:



Given: Data on rocket motor

Find: Thrust produced

Solution:

Basic equation: Momentum flux in x direction for the elbow
$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u \rho \, d\forall + \int_{CS} u \rho \vec{V} \cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Neglect change of momentum within CV 3) Uniform flow

Hence $R_x - p_{eg} \cdot A_e = V_e \cdot (\rho_e \cdot V_e \cdot A_e) = m_e \cdot V_e$ $R_x = p_{eg} \cdot A_e + m_e \cdot V_e$

where p_{eg} is the exit pressure (gage), m_e is the mass flow rate at the exit (software cannot render dot over m!) and V_e is the xit velocity

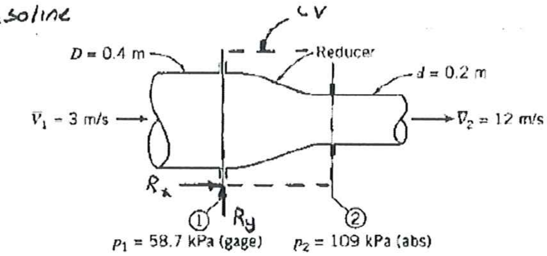
For the mass flow rate $m_e = m_{nitricacid} + m_{aniline} = 80 \cdot \frac{kg}{s} + 32 \cdot \frac{kg}{s}$ $m_e = 112 \cdot \frac{kg}{s}$

Hence $R_x = (110 - 101) \times 10^3 \cdot \frac{N}{m^2} \times \frac{\pi \cdot (0.6\,m)^2}{4} + 112 \cdot \frac{kg}{s} \times 180 \cdot \frac{m}{s} \times \frac{N \cdot s^2}{kg \cdot m}$ $R_x = 22.7kN$

Solution P#6:

Given: Flow through reducer in gasoline piping system, as shown.

$M = 25\ kg \quad \forall = 0.2\ m^3$



Find: Force needed to hold reducer in place.

Solution: Apply the x and y components of the momentum equation, using the CV and coordinates shown. Use gage pressures to cancel p_{atm}.

Basic equations:

$$F_{Sx} + \cancel{F_{Bx}}^{=0(1)} = \cancel{\frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall}^{=0(2)} + \int_{CS} u\rho\vec{V}\cdot d\vec{A}$$

$$F_{Sy} + F_{By} = \cancel{\frac{\partial}{\partial t}\int_{CV} v\rho\, d\forall}^{=0(2)} + \int_{CS} v\rho\vec{V}\cdot d\vec{A}$$

Assumptions: (1) $F_{Bx} = 0$
(2) Steady flow
(3) Uniform flow at each section
(4) Incompressible flow, $SG = 0.72$ {Table A.2, Appendix A}

From the x component of momentum,

$$R_x + p_{1g}A_1 - p_{2g}A_2 = u_1\{-|\rho V_1 A_1|\} + u_2\{+|\rho V_2 A_2|\} = (V_2 - V_1)\rho V_1 A_1$$

$$u_1 = V_1 \qquad u_2 = V_2$$

$$R_x = p_{2g}A_2 - p_{1g}A_1 + (\bar{V}_2 - \bar{V}_1)\rho \bar{V}_1 A_1 \qquad \text{Note } \rho = SG\,\rho_{H_2O}$$

$$= (109-101)10^3\frac{N}{m^2}\times\frac{\pi}{4}(0.2)^2 m^2 - 58.7\times10^3\frac{N}{m^2}\times\frac{\pi}{4}(0.4)^2 m^2$$
$$+ (12-3)\frac{m}{s}\times(0.72)1000\frac{kg}{m^3}\times 3\frac{m}{s}\times\frac{\pi}{4}(0.4)^2 m^2 \times \frac{N\cdot s^2}{kg\cdot m}$$

$R_x = -4.68\ kN$ (force must be applied to left) $\qquad R_x$

From the y component of momentum,

$$R_y - Mg - \rho g\forall = \cancel{v_1}^{=0}\{-|\rho V_1 A_1|\} + \cancel{v_2}^{=0}\{+|\rho V_2 A_2|\}$$

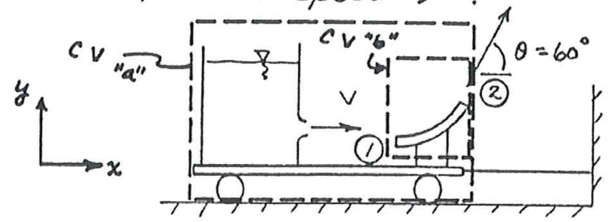
$$R_y = Mg + \rho g \forall$$

$$= 25\ kg \times 9.81\frac{m}{s^2}\times\frac{N\cdot s^2}{kg\cdot m} + (0.72)1000\frac{kg}{m^3}\times 9.81\frac{m}{s^2}\times 0.2\ m^3\times\frac{N\cdot s^2}{kg\cdot m}$$

$R_y = 1.66\ kN$ (force must be applied up) $\qquad R_y$

Solution P#7:

Given: Cart and jet with turning vane as shown. (The water level is held constant by flow from a vertical spout.)



$V = 10\ m/sec,\ A = 600\ mm^2$

Find: (a) the tension in the wire holding the cart stationary
(b) the force of the vane on the cart

Solution: Apply the x component of the momentum equation. To find the force in the wire, use CV "a", which cuts it and encloses the cart.

Basic equation: $F_{Sx} + \underbrace{F_{Bx}}_{=0(2)} = \underbrace{\frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall}_{=0(3)} + \int_{CS} u\rho \vec{V}\cdot d\vec{A}$

Assumptions: (1) No pressure forces act, so $F_{Sx} = R_x$
(2) $F_{Bx} = 0$
(3) Steady flow; $\frac{\partial}{\partial t} = 0$
(4) Uniform flow at each section of jet
(5) Jet area and velocity constant across vane

Then

$$R_{xa} = u_2\{|\rho_2 V_2 A_2|\} = V_2 \cos 60° \{|\rho_2 V_2 A_2|\} = \rho V^2 A \cos 60°$$

$$R_{xa} = 999\frac{kg}{m^3} \times (10)^2\frac{m^2}{sec^2} \times 600\ mm^2 \times \cos 60° \times \frac{m^2}{10^6\ mm^2} \times \frac{N\cdot sec^2}{kg\cdot m} = 30.0\ N \qquad R_{xa}$$

{The wire is in tension.}

To find force of vane on cart, use CV "b" which encloses the vane. Using the previous assumptions,

$$R_{xb} = u_1\{-|\rho_1 V_1 A_1|\} + u_2\{|\rho_2 V_2 A_2|\} = -\rho V^2 A + \rho V^2 A\cos\theta = \rho V^2 A(\cos\theta - 1)$$

$$u_1 = V \qquad u_2 = V\cos\theta$$

$$K_{xb} = -R_{xb} = \rho V^2 A(1-\cos\theta) = \text{force of CV "b" } \underline{\text{on}} \text{ cart}$$

$$K_{xb} = 999\frac{kg}{m^3} \times (10)^2\frac{m^2}{sec^2} \times 600\ mm^2(1-\cos 60°)\frac{m^2}{10^6\ mm^2} \times \frac{N\cdot sec^2}{kg\cdot m} = 30.0\ N \qquad K_{xb}$$

and

$$R_{yb} = v_1\{-|\rho_1 V_1 A_1|\} + v_2\{|\rho_2 V_2 A_2|\} = \rho V^2 A\sin\theta$$

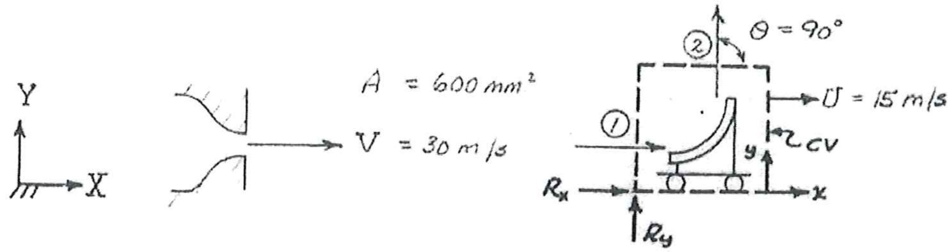
$$v_1 = 0 \qquad v_2 = V\sin\theta$$

$$K_{yb} = -\rho V^2 A\sin\theta = -51.9\ N \qquad K_{yb}$$

{Vertical force of vane $\underline{\text{on}}$ cart is $\underline{\text{down}}$.}

Solution P#8:

Given: Jet of water striking a moving vane as shown.



Find: Force needed to maintain the vane speed constant.

Solution: Apply momentum equation using moving CV shown.

Basic Equations: $F_{Sx} + \cancel{F_{Bx}} = \cancel{\frac{\partial}{\partial t}\int_{CV} u_{xyz}\,\rho\, d\forall} + \int_{CS} u_{xyz}\,\rho \vec{V}_{xyz}\cdot d\vec{A}$ $\quad$ (F_{Bx} =0(2), $\frac{\partial}{\partial t}$ =0(3))

$F_{Sy} + \cancel{F_{By}} = \cancel{\frac{\partial}{\partial t}\int_{CV} v_{xyz}\,\rho\, d\forall} + \int_{CS} v_{xyz}\,\rho \vec{V}_{xyz}\cdot d\vec{A}$ $\quad$ (F_{By} =0(2), $\frac{\partial}{\partial t}$ =0(3))

Assumptions: (1) No net pressure forces on CV; $F_{Sx} = R_x$, $F_{Sy} = R_y$
(2) $F_{Bx} = 0$; neglect F_{By}
(3) steady flow relative to vane
(4) Uniform flow at each section
(5) Jet area and speed relative to vane are constant

All velocities must be relative to CV. Then

$$R_x = u_1\{-|\rho(V-U)A|\} + u_2\{|\rho(V-U)A|\}$$

$$u_1 = V - U \qquad u_2 = (V-U)\cos\theta$$

and

$$R_x = \rho(V-U)^2 A\,(\cos\theta - 1)$$

$$= 999\,\frac{kg}{m^3}\,(30-15)^2\frac{m^2}{s^2}\times 600\,mm^2\times\frac{m^2}{10^6\,mm^2}\,(\cos 90° - 1)\,\frac{N\cdot s^2}{kg\cdot m}$$

$R_x = -135$ N (to the left) $\qquad R_x$

Also

$$R_y = v_1\{-|\rho(V-U)A|\} + v_2\{|\rho(V-U)A|\}$$

$$v_1 = 0 \qquad v_2 = (V-U)\sin\theta$$

$$R_y = \rho(V-U)^2 A\sin\theta$$

$$= 999\,\frac{kg}{m^3}\,(30-15)^2\frac{m^2}{s^2}\times 600\,mm^2\times\frac{m^2}{10^6\,mm^2}\times \sin 90°\times\frac{N\cdot s^2}{kg\cdot m} = 135\,N$$

$R_y = 135$ N (force must be up) $\qquad R_y$

ME 231 Thermo-Fluid Mechanics I
Prof. A. Banerjee
Study Problem Set 4

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****

1. Consider a river flowing toward a lake at an average speed of 4 m/s at a rate of 500 m^3/s. The river bed is at a location of 70 m above the lake surface (figure 1). Determine the total mechanical energy of the river water per unit mass and the power generation potential of the entire river at that location.
2. An oil pump is drawing 25kW of electric power while pumping oil with $\rho = 860$ kg/m^3 at a rate of 0.1 m^3/s. The inlet and outlet diameters of the pipe are 8 cm and 12 cm respectively (fig. 2). If the pressure rise of the oil in the pump is measured to be 250 kPa and the motor efficiency is 90%, determine the mechanical efficiency of the pump. Take the kinetic energy correction factor to be 1.05.
3. Water enters a hydraulic turbine through a 30-cm-diameter pipe at a rate of 0.6 m^3/s and exits through a 25-cm diameter pipe (see fig.3). The pressure drop in the turbine is measured by a mercury manometer to be 1.2 m. For a combined turbine-generator efficiency of 83%, determine the net electric power output. Disregard the effect of the kinetic energy correction factors.

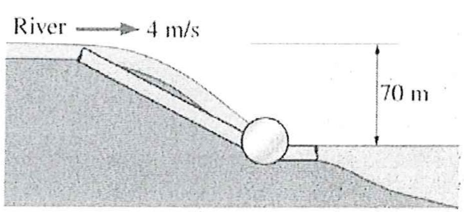


Figure 1

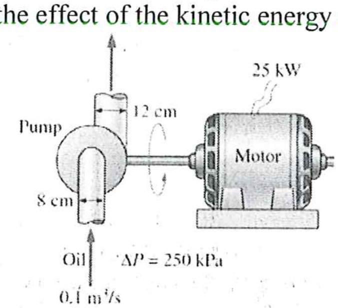


Figure 2

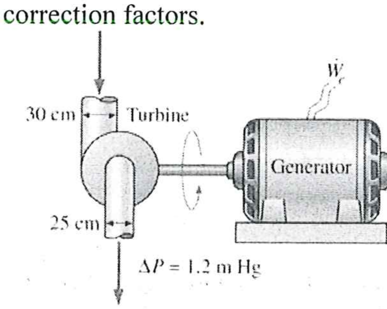


Figure 3

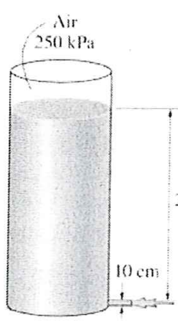


Figure 4

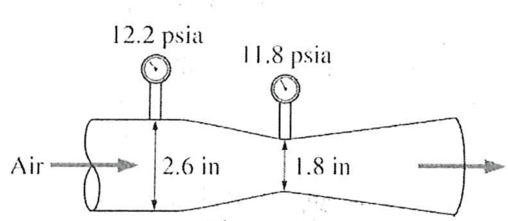


Figure 5

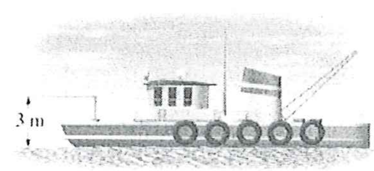


Figure 6

4. A pressurized tank of water has a 10-cm diameter orifice at the bottom, where water discharges to the atmosphere (see figure 4). The water level is 2.5 m above the outlet. The tank air pressure above the water level is 250 kPa (absolute) while the atmospheric pressure is 100 kPa. Neglecting frictional effects, determine the initial discharge rate of water from the tank.
5. Air is flowing through a venturi meter whose diameter is 2.6 in at the inlet and 1.8 in at the throat. The gage pressures are measured as shown in figure 5. Neglecting frictional effects, determine the volume flow rate of air. Take air density to be 0.075 lbm/ft^3.
6. A fireboat is fighting fires by drawing sea water (ρ=1030 kg/m^3) through a 20-cm diameter pipe at a rate of 0.1 m^3/s and discharging it through a nozzle of exit diameter 5-cm. The total irreversible head loss of the system is 3 m, and the position of the nozzle is 3 m above the sea level. For a pump efficiency of 70%, determine the required shaft power input to the pump and the water discharge velocity.

Solution P#1: A river is flowing at a specified velocity, flow rate, and elevation. The total mechanical energy of the river water per unit mass, and the power generation potential of the entire river are to be determined.

Assumptions **1** The elevation given is the elevation of the free surface of the river. **2** The velocity given is the average velocity. **3** The mechanical energy of water at the turbine exit is negligible.

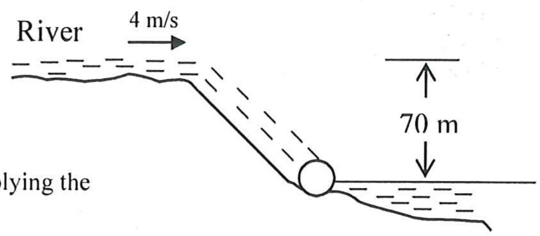
Properties We take the density of water to be $\rho = 1000\ \text{kg/m}^3$.

Analysis Noting that the sum of the flow energy and the potential energy is constant for a given fluid body, we can take the elevation of the entire river water to be the elevation of the free surface, and ignore the flow energy. Then the total mechanical energy of the river water per unit mass becomes

$$e_{\text{mech}} = pe + ke = gh + \frac{V^2}{2}$$

$$= \left((9.81\ \text{m/s}^2)(70\ \text{m}) + \frac{(4\ \text{m/s})^2}{2} \right) \left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2} \right)$$

$$= 0.695\ \text{kJ/kg}$$



The power generation potential of the river water is obtained by multiplying the total mechanical energy by the mass flow rate,

$$\dot{m} = \rho\dot{V} = (1000\ \text{kg/m}^3)(500\ \text{m}^3/\text{s}) = 500{,}000\ \text{kg/s}$$

$$\dot{W}_{\text{max}} = \dot{E}_{\text{mech}} = \dot{m}e_{\text{mech}} = (500{,}000\ \text{kg/s})(0.695\ \text{kg/s}) = 347{,}350\ \text{kW} \cong \mathbf{347\ MW}$$

Therefore, 347 MW of power can be generated from this river as it discharges into the lake if its power potential can be recovered completely.

Discussion Note that the kinetic energy of water is negligible compared to the potential energy, and it can be ignored in the analysis. Also, the power output of an actual turbine will be less than 347 MW because of losses and inefficiencies.

Solution P#2: A pump is pumping oil at a specified rate. The pressure rise of oil in the pump is measured, and the motor efficiency is specified. The mechanical efficiency of the pump is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The elevation difference across the pump is negligible. **3** All the losses in the pump are accounted for by the pump efficiency and thus $h_L = 0$. **4** The kinetic energy correction factors are given to be $\alpha_1 = \alpha_2 = \alpha = 1.05$.

Properties The density of oil is given to be $\rho = 860\ \text{kg/m}^3$.

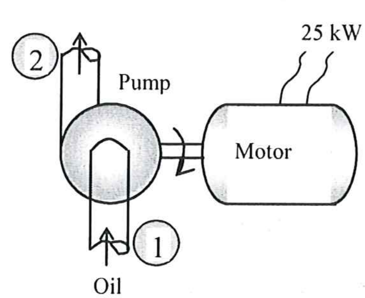
Analysis We take points 1 and 2 at the inlet and the exit of the pump, respectively. Noting that $z_1 = z_2$, the energy equation for the pump reduces to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \quad \rightarrow \quad h_{\text{pump, u}} = \frac{P_2 - P_1}{\rho g} + \frac{\alpha(V_2^2 - V_1^2)}{2g}$$

where

$$V_1 = \frac{\dot{V}}{A_1} = \frac{\dot{V}}{\pi D_1^2/4} = \frac{0.1\ \text{m}^3/\text{s}}{\pi (0.08\ \text{m})^2/4} = 19.9\ \text{m/s}$$

$$V_2 = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{\pi D_2^2/4} = \frac{0.1\ \text{m}^3/\text{s}}{\pi (0.12\ \text{m})^2/4} = 8.84\ \text{m/s}$$



Substituting, the useful pump head and the corresponding useful pumping power are determined to be

ME 231 Thermo-Fluid Mechanics I
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Study Problem Set 4

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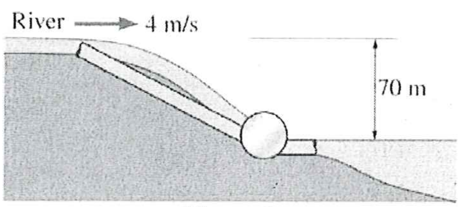


Figure 1

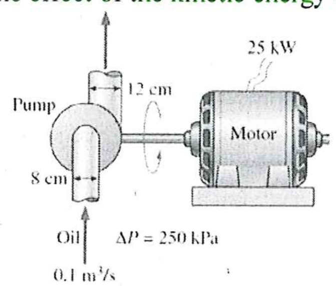


Figure 2

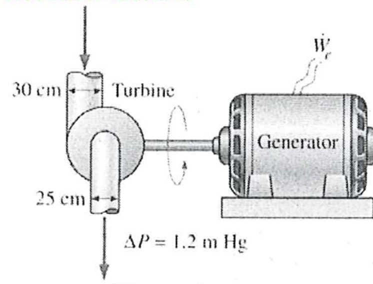


Figure 3

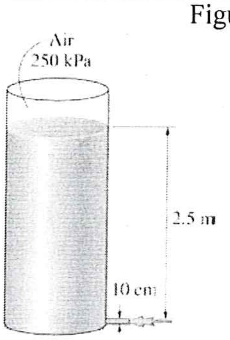


Figure 4

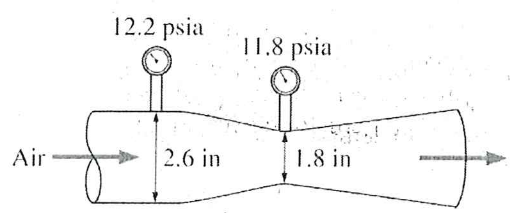
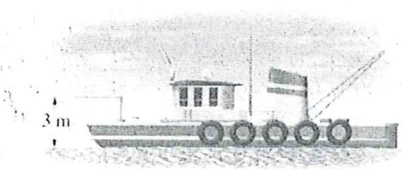


Figure 5



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4. A pressurized tank of water has a 10-cm diameter orifice at the bottom, where water discharges to the atmosphere (see figure 4). The water level is 2.5 m above the outlet. The tank air pressure above the water level is 250 kPa (absolute) while the atmospheric pressure is 100 kPa. Neglecting frictional effects, determine the initial discharge rate of water from the tank.
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$$h_{\text{pump, u}} = \frac{250{,}000\ \text{N/m}^2}{(860\ \text{kg/m}^3)(9.81\ \text{m/s}^2)}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) + \frac{1.05[(8.84\ \text{m/s})^2 - (19.9\ \text{m/s})^2]}{2(9.81\ \text{m/s}^2)} = 29.6 - 17.0 = 12.6\ \text{m}$$

$$\dot{W}_{\text{pump,u}} = \rho\dot{V}gh_{\text{pump, u}} = (860\ \text{kg/m}^3)(0.1\ \text{m}^3/\text{s})(9.81\ \text{m/s}^2)(12.6\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kW}}{1\ \text{kN}\cdot\text{m/s}}\right) = 10.6\ \text{kW}$$

Then the shaft pumping power and the mechanical efficiency of the pump become

$$\dot{W}_{\text{pump,shaft}} = \eta_{\text{motor}}\dot{W}_{\text{electric}} = (0.90)(25\ \text{kW}) = 22.5\ \text{kW}$$

$$\eta_{\text{pump}} = \frac{\dot{W}_{\text{pump, u}}}{\dot{W}_{\text{pump, shaft}}} = \frac{10.6\ \text{kW}}{22.5\ \text{kW}} = 0.471 = \mathbf{47.1\%}$$

Discussion The overall efficiency of this pump/motor unit is the product of the mechanical and motor efficiencies, which is 0.9 × 0.471 = 0.42.

Solution P#3: Water enters a hydraulic turbine-generator system with a known flow rate, pressure drop, and efficiency. The net electric power output is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** All losses in the turbine are accounted for by turbine efficiency and thus $h_L = 0$. **3** The elevation difference across the turbine is negligible. **4** The effect of the kinetic energy correction factors is negligible, $\alpha_1 = \alpha_2 = \alpha = 1$.

Properties We take the density of water to be 1000 kg/m^3 and the density of mercury to be 13,560 kg/m^3.

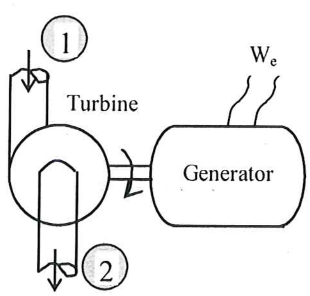
Analysis We choose points 1 and 2 at the inlet and the exit of the turbine, respectively. Noting that the elevation effects are negligible, the energy equation in terms of heads for the turbine reduces to

$$\frac{P_1}{\rho g} + \alpha_1\frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \quad \rightarrow \quad h_{\text{turbine, e}} = \frac{P_1 - P_2}{\rho_{\text{water}}\, g} + \frac{\alpha(V_1^2 - V_2^2)}{2g} \qquad (1)$$

where

$$V_1 = \frac{\dot{V}}{A_1} = \frac{\dot{V}}{\pi D_1^2/4} = \frac{0.6\ \text{m}^3/\text{s}}{\pi(0.30\ \text{m})^2/4} = 8.49\ \text{m/s}$$

$$V_2 = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{\pi D_2^2/4} = \frac{0.6\ \text{m}^3/\text{s}}{\pi(0.25\ \text{m})^2/4} = 12.2\ \text{m/s}$$



The pressure drop corresponding to a differential height of 1.2 m in the mercury manometer is

$$P_1 - P_2 = (\rho_{\text{Hg}} - \rho_{\text{water}})gh$$

$$= [(13{,}560 - 1000)\ \text{kg/m}^3](9.81\ \text{m/s}^2)(1.2\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 148\ \text{kN/m}^2 = 148\ \text{kPa}$$

Substituting into Eq. (1), the turbine head is determined to be

$$h_{\text{turbine, e}} = \frac{148\ \text{kN/m}^2}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)}\left(\frac{1000\ \text{kg}\cdot\text{m/s}^2}{1\ \text{kN}}\right) + (1.0)\frac{(8.49\ \text{m/s})^2 - (12.2\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = 15.1 - 3.9 = 11.2\ \text{m}$$

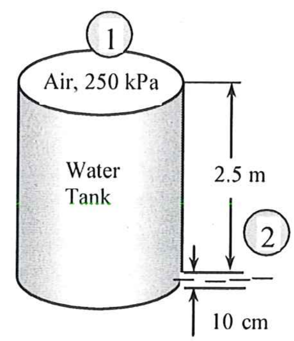
Then the net electric power output of this hydroelectric turbine becomes

$$\dot{W}_{\text{turbine}} = \eta_{\text{turbine-gen}} \dot{m} g h_{\text{turbine, e}} = \eta_{\text{turbine-gen}} \rho \dot{V} g h_{\text{turbine, e}}$$

$$= 0.83(1000 \text{ kg/m}^3)(0.6 \text{ m}^3/\text{s})(9.81 \text{ m/s}^2)(11.2 \text{ m})\left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right)\left(\frac{1 \text{ kW}}{1000 \text{ N} \cdot \text{m/s}}\right) = \mathbf{55 \text{ kW}}$$

Discussion It appears that this hydroelectric turbine will generate 55 kW of electric power under given conditions. Note that almost half of the available pressure head is discarded as kinetic energy. This demonstrates the need for a larger turbine exit area and better recovery. For example, the power output can be increased to 74 kW by redesigning the turbine and making the exit diameter of the pipe equal to the inlet diameter, $D_2 = D_1$. Further, if a much larger exit diameter is used and the exit velocity is reduced to a very low level, the power generation can increase to as much as 92 kW.

Solution P#4: Water discharges to the atmosphere from the orifice at the bottom of a pressurized tank. Assuming frictionless flow, the discharge rate of water from the tank is to be determined.



Assumptions **1** The orifice has a smooth entrance, and thus the frictional losses are negligible. **2** The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable).

Properties We take the density of water to be 1000 kg/m^3.

Analysis We take point 1 at the free surface of the tank, and point 2 at the exit of orifice, which is also taken to be the reference level ($z_2 = 0$). Noting that the fluid velocity at the free surface is very low ($V_1 \cong 0$) and water discharges into the atmosphere (and thus $P_2 = P_{atm}$), the Bernoulli equation simplifies to

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad \rightarrow \quad \frac{V_2^2}{2g} = \frac{P_1 - P_2}{\rho g} + z_1$$

Solving for V_2 and substituting, the discharge velocity is determined to

$$V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho} + 2gz_1} = \sqrt{\frac{2(250 - 100) \text{ kPa}}{1000 \text{ kg/m}^3}\left(\frac{1000 \text{ N/m}^2}{1 \text{ kPa}}\right)\left(\frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}}\right) + 2(9.81 \text{ m/s}^2)(2.5 \text{ m})} = 18.7 \text{ m/s}$$

Then the initial rate of discharge of water becomes

$$\dot{V} = A_{\text{orifice}} V_2 = \frac{\pi D^2}{4} V_2 = \frac{\pi (0.10 \text{ m})^2}{4}(18.7 \text{ m/s}) = \mathbf{0.147 \text{ m}^3/\text{s}}$$

Discussion Note that this is the maximum flow rate since the frictional effects are ignored. Also, the velocity and the flow rate will decrease as the water level in the tank decreases.

Solution P#5: Air is flowing through a venturi meter with known diameters and measured pressures. A relation for the flow rate is to be obtained, and its numerical value is to be determined.

Assumptions **1**The flow through the venturi is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** The effect of air column on the pressure change is negligible because of its low density, and thus the pressure can be assumed to be uniform at a given cross-section of the venturi meter (independent of elevation change). **3** The flow is horizontal (this assumption is usually unnecessary for gas flow.).

Properties The density of air is given to be $\rho = 0.075$ lbm/ft^3.

Analysis We take point 1 at the main flow section and point 2 at the throat along the centerline of the venturi meter. Noting that $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

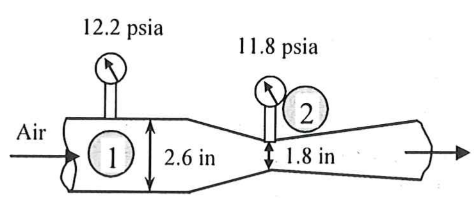
$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad \rightarrow \quad P_1 - P_2 = \rho \frac{V_2^2 - V_1^2}{2} \qquad (1)$$

The flow is assumed to be incompressible and thus the density is constant. Then the conservation of mass relation for this single stream steady flow device can be expressed as

$$\dot{V}_1 = \dot{V}_2 = \dot{V} \quad \rightarrow \quad A_1 V_1 = A_2 V_2 = \dot{V} \quad \rightarrow \quad V_1 = \frac{\dot{V}}{A_1} \quad \text{and} \quad V_2 = \frac{\dot{V}}{A_2} \qquad (2)$$

Substituting into Eq. (1),

$$P_1 - P_2 = \rho \frac{(\dot{V}/A_2)^2 - (\dot{V}/A_1)^2}{2} = \frac{\rho \dot{V}^2}{2A_2^2}\left(1 - \frac{A_2^2}{A_1^2}\right)$$



Solving for $\dot{V}$ gives the desired relation for the flow rate,

$$\dot{V} = A_2 \sqrt{\frac{2(P_1 - P_2)}{\rho[1 - (A_2/A_1)^2]}} \qquad (3)$$

The flow rate for the given case can be determined by substituting the given values into this relation to be

$$\dot{V} = \frac{\pi D_2^2}{4} \sqrt{\frac{2(P_1 - P_2)}{\rho[1 - (D_2/D_1)^4]}} = \frac{\pi (1.8/12 \text{ ft})^2}{4} \sqrt{\frac{2(12.2 - 11.8) \text{ psi}}{(0.075 \text{ lbm/ft}^3)[1 - (1.8/2.6)^4]} \left(\frac{144 \text{ lbf/ft}^2}{1 \text{ psi}}\right)\left(\frac{32.2 \text{ lbm} \cdot \text{ft/s}^2}{1 \text{ lbf}}\right)}$$

$$= \mathbf{4.48 \ ft^3/s}$$

Discussion Venturi meters are commonly used as flow meters to measure the flow rate of gases and liquids by simply measuring the pressure difference P_1 - P_2 by a manometer or pressure transducers. The actual flow rate will be less than the value obtained from Eq. (3) because of the friction losses along the wall surfaces in actual flow. But this difference can be as little as 1% in a well-designed venturi meter. The effects of deviation from the idealized Bernoulli flow can be accounted for by expressing Eq. (3) as

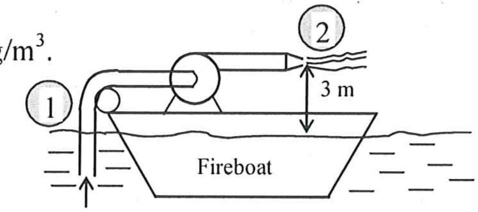
$$\dot{V} = C_c A_2 \sqrt{\frac{2(P_1 - P_2)}{\rho[1 - (A_2/A_1)^2]}}$$

where C_c is the *venturi discharge coefficient* whose value is less than 1 (it is as large as 0.99 for well-designed venturi meters in certain ranges of flow). For Re > 10^5, the value of venturi discharge coefficient is usually greater than 0.96.

Solution P#6: A fireboat is fighting fires by drawing sea water and discharging it through a nozzle. The head loss of the system and the elevation of the nozzle are given. The shaft power input to the pump and the water discharge velocity are to be determined.

Assumptions 1 The flow is steady and incompressible. 2 The effect of the kinetic energy correction factors is negligible, $\alpha = 1$.

Properties The density of sea water is given to be $\rho = 1030\ \text{kg/m}^3$.



Analysis We take point 1 at the free surface of the sea and point 2 at the nozzle exit. Noting that $P_1 = P_2 = P_{atm}$ and $V_1 \cong 0$ (point 1 is at the free surface; not at the pipe inlet), the energy equation for the control volume between 1 and 2 that includes the pump and the piping system reduces to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \quad \rightarrow \quad h_{\text{pump, u}} = z_2 - z_1 + \alpha_2 \frac{V_2^2}{2g} + h_L$$

where the water discharge velocity is

$$V_2 = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{\pi D_2^2/4} = \frac{0.1\ \text{m}^3/\text{s}}{\pi (0.05\ \text{m})^2/4} = 50.9296\ \text{m/s} \cong \mathbf{50.9\ m/s}$$

Substituting, the useful pump head and the corresponding useful pump power are determined to be

$$h_{\text{pump, u}} = (3\ \text{m}) + (1)\frac{(50.9296\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + (3\ \text{m}) = 138.203\ \text{m} \cong 138\ \text{m}$$

$$\dot{W}_{\text{pump,u}} = \rho \dot{V} g h_{\text{pump, u}} = (1030\ \text{kg/m}^3)(0.1\ \text{m}^3/\text{s})(9.81\ \text{m/s}^2)(138.203\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kW}}{1\ \text{kN}\cdot\text{m/s}}\right)$$
$$= 139.644\ \text{kW}$$

Then the required shaft power input to the pump becomes

$$\dot{W}_{\text{pump, shaft}} = \frac{\dot{W}_{\text{pump, u}}}{\eta_{\text{pump}}} = \frac{139.644\ \text{kW}}{0.70} = 199.49\ \text{kW} \cong \mathbf{199\ kW}$$

Discussion Note that the pump power is used primarily to increase the kinetic energy of water.

ME 231 Thermo-Fluid Mechanics I (Section 1A)
Prof. A. Banerjee
Recitation for Mid-term 1

1. **Multiple Choice Problems:**

A. Two parallel plates, one moving at 4 m/s and the other fixed, are separated by a 5-mm thick layer of oil of specific gravity 0.80 and kinematic viscosity 1.25×10^{-4} m^2/s. What is the average shear stress in the oil?

(i) 80 Pa (ii) 100 Pa (iii) 125 Pa

(iv) 160 Pa (v) 200 Pa

$\tau = \mu \frac{V}{h} = 1.25\times10^{-4}\ m^2/s \frac{4\ m/s}{.005\ m} = .1\ kPa$

B. In figure 1 (see below), if oil in region B has SG=0.8 and the absolute pressure at point A is 1 atm, what is the absolute pressure at point B?

(i) 5.6 kPa (ii) 10.9 kPa (iii) 107 kPa

(iv) 112 kPa (v) 157 kPa

$P_B = 100 + .05\ SG_{H_2O}\gamma_{H_2O} + .04\ SG_{Hg}\gamma_{H_2O} - .03\ SG_{oil}\gamma_{H_2O}$

$P_B = 1atm + .05(1)(9790)$
$+(.08-.04)(13.56)(9790)$
$-.03(.8)(9790)$

$= 100 + .5684\ \gamma_{Hg} = 100 + 5.564\ kPa$

$= 106.56\ kPa$

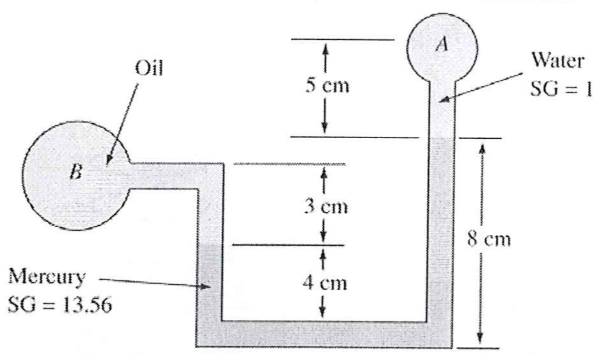


Figure 1

C. In figure 1, if oil in region B has SG=0.8 and the absolute pressure at point B is 14 psia, what is the absolute pressure at point A? 96.53kPa

$96.53 = P_a + .05\ SG_{H_2O}\gamma_{H_2O} + .04\ SG_{HG} - .03\ SG_{oil}\gamma_{H_2O}$

$9790[.05 + .04(13.56) - .03(.8)]$

(i) 11 kPa (ii) 41 kPa (iii) 86 kPa

(iv) 91 kPa (v) 101 kPa

$P_a = 90.97\ kPa$

D. If a uniform solid body weighs 50 N in air and 30 N in water, its specific gravity is

(i) 1.5 (ii) 1.67 (iii) 2.5

(iv) 3.0 (v) 5.0

$\frac{50}{9.81} = 5.096\ kg$

$20N = \rho V g$

$2.039kg = \rho_{H_2O} V_{H_2O\ displaced}$

$\frac{5.096}{2.039} = \boxed{2.50 = SG}$

Only had to say $\frac{50}{20} =$

2) $\tau = \mu \cancel{\frac{V}{h}} \frac{du}{dy}$

$\frac{u}{U} = 2\frac{y}{h} - \frac{y^2}{h^2}$ $\quad u = U\left[\frac{2y}{h} - \frac{y^2}{h^2}\right]$ $\quad \frac{du}{dy} = \frac{U2}{h} - \frac{2Uy}{h^2}$

$\tau = 1.0E^{-3}(2)\left[\frac{3m/s}{.1m} - \frac{3m/s\,\cancel{(y)}}{(.1m)^2}\right] \xrightarrow{y=0} \frac{1.0(10^{-3})\frac{kg}{m\cdot s}(2)(3\, m/s)}{(0.1\, m)}$

0

$\tau = 0.06 \frac{kg}{m\cdot s} =$

3) $P_s = \cancel{P_a} - .4ft - SG_{Hg}\,\gamma_{H_2O} + 1.6 ft\,(1)\,H_2O$

$-.4(8)\gamma + 1.6\gamma$

$= 62.32(1.6 - .4(8))$

$= 99.71\ lbf/ft^2$

$\frac{lbf}{ft^3}/Pa =$

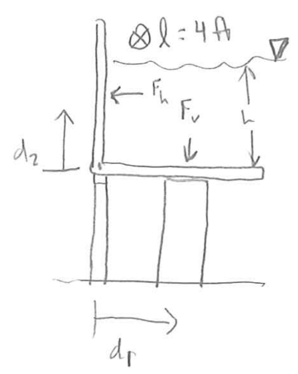
$\gamma = 9790\ N/m^3 = 62.32\ lbf/ft^3$

$F = -99.71 \cdot \pi(.5\,ft)^2$

$= 78.31\ lb$

$998 \frac{kg}{m^3}(9.81) = 9790\ N/m^3$

4)



$\Sigma M_o = 0$ $\quad F_v d_1 = F_h d_2$

$F_v = h\gamma A = \left[h\gamma\pi\frac{D^2}{4}\right]\left[3\,ft\right]$
moment arm

$F_h = \gamma h_{CG} A_{proj} = \left[\gamma\frac{h}{2}h\ell\right]\left[\frac{h}{3}\right]$
moment arm

$= 125h^2$

$F_v = 49.0\cancel{h}(3ft) = 125h^2\left(\frac{\cancel{h}}{3}\right)$

$147 = 41.67h^2$

$h = 1.88$

231 Exam 1 review

5) $\tau_{\text{shear stress}} = \mu \dfrac{du}{dy}$

assume linear velocity dist.

$$\frac{F}{A} = \mu \frac{du}{dy}$$

$\Downarrow$ linear

$$\frac{F}{A} = \mu \frac{V}{h}$$

$\Downarrow$

$\tau_{\text{torque}} = FR$
$= FR_i$

$\omega = \dfrac{V}{R_i}$

$A = \pi 2R(h)$
$= \pi 2 R_i l$

$h = R_o - R_i$

$$\frac{\tau_{\text{torque}}}{R_i(2\pi R_i l)} = \mu\left(\frac{\omega R_i}{R_o - R_i}\right)$$

$$\boxed{\tau_{\text{torque}} = \frac{\mu \omega R_i^3 2\pi l}{(R_o - R_i)}}$$

Great! 1st try

In class problem set 5

2. A layer of water flows down an inclined fixed surface with the velocity profile shown in figure 2. Determine the magnitude and direction of the shearing stresses that water exerts on the fixed surface for $U = 3$ m/s and $h = 0.1$ m.

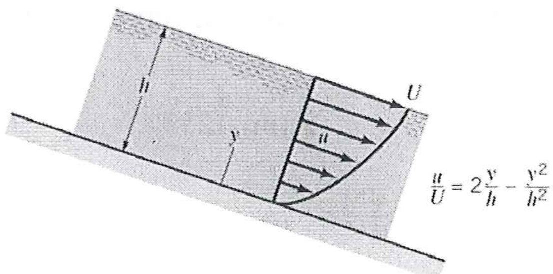


Figure 2

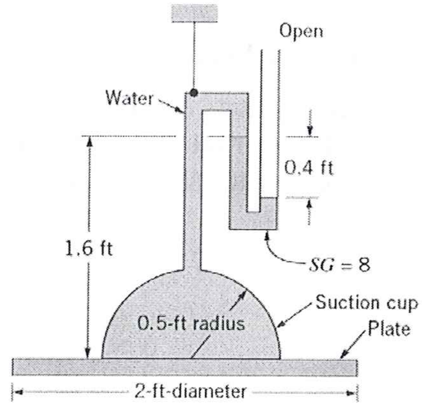
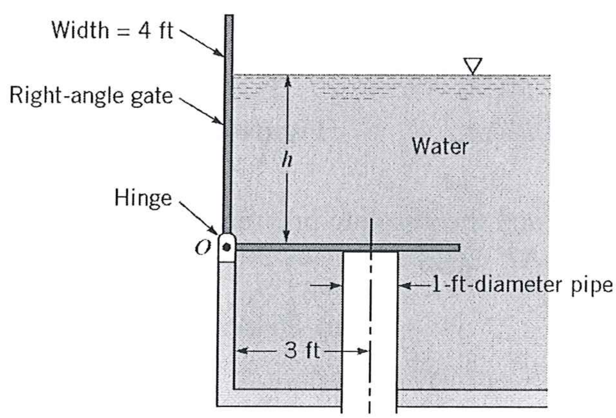


Figure 3

3. A suction cup is used to support a plate of weight W as shown in figure 3. For the conditions shown, determine W.

4. A thin 4-ft wide, right angle gate with negligible mass is free to pivot about a frictionless hinge at point O as shown in figure 4 below. The horizontal portion of the gate covers a 1-ft-diameter drain pipe, which contains air at atmospheric pressure. Determine the minimum water depth, h, at which the gate will pivot to allow water to flow into the pipe.



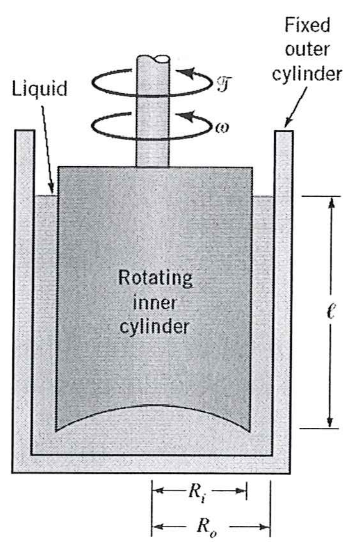
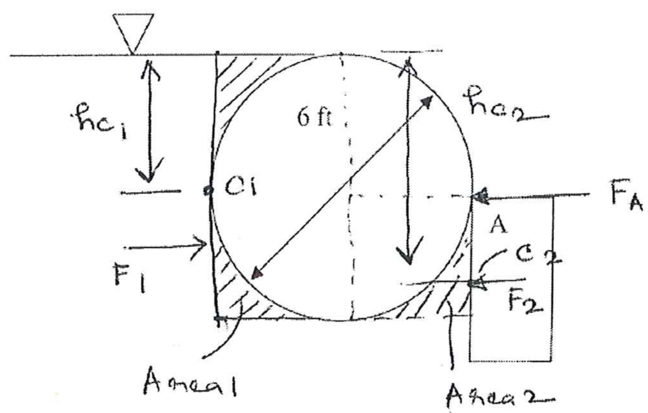


Figure 4 **Figure 5**

5. The viscosity of liquids can be measured through the use of a rotating cylinder viscometer of the type illustrated in figure 5. In the device, the outer cylinder is fixed and the inner cylinder is rotated at an angular velocity, ω. The torque τ, required to develop ω is measured, and the viscosity is calculated from these two measurements. Develop an equation relating all the variables in the figure. Assume that the velocity distribution in the gap is linear and neglect end effects.

Review

5. A 9-ft long cylinder as shown in the figure floats in oil and rests against the wall. Determine the horizontal force the cylinder exerts on the wall at the point of contact A. $\left(\gamma_{oil} = 57 \frac{lb}{ft^3}\right)$



$F_A = F_1 - F_2$

where: $F_1 = \gamma h_{c_1} \cdot A_1 = \left(57.0 \frac{lb}{ft^3}\right)\left(\frac{6ft}{2}\right)(6ft \times 9ft)$

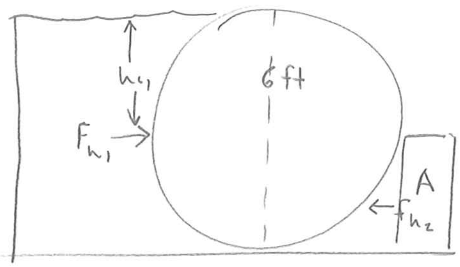
or, $\boxed{F_1 = 9234 lb}$

and, $F_2 = \gamma h_{c_2} A_2 = \left(57.0 \frac{lb}{ft^3}\right)\left(3ft + \frac{3ft}{2}\right)(3ft \times 9ft)$

or, $\boxed{F_2 = 6926 lb}$

Note: F_2 is $\neq \frac{1}{2} F_1$

$\boxed{F_A = F_1 - F_2 = 2308 lb}$ → acting on the wall!



$$F_{h_1} = \gamma_{oil}\, h_{cp_1} A_{proj} = 57 \frac{lb}{ft^3} \left(\frac{6}{2}\right) ft\, (6 \times 9)\, ft^2 = 9234\, lbf$$

$$F_{h_2} = \gamma_{oil}\, h_{cp_2} A_{proj} = 57 \frac{lb}{ft^3} \left(3 + \tfrac{1}{2}(3)\right) ft\, (3 \times 9)\, ft^2 = 6925.5\ lbf$$

$$\boxed{\overset{+}{\rightarrow} F_{horizontal} = 9234 - 6926 = 2308\ lbf}$$

$\downarrow$ ~~easy~~