

A LEAST-COMMON-MULTIPLE BOUND FOR THE ERDŐS–SELFIDGE FUNCTION

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ABSTRACT. For $k \geq 2$, let $g(k)$ be the least integer $n > k + 1$ such that every prime divisor of $\binom{n}{k}$ exceeds k . We prove that $g(k) < \text{lcm}(1, \dots, k)$ for all sufficiently large k , establishing the eventual form of a conjecture of Ecklund, Erdős, and Selfridge. The proof modifies the least common multiple by adding one factor of each small prime and removing the primes in a short proportional interval below k . Lucas’s digit criterion reduces the remaining conditions to a finite sieve. Truncated inclusion–exclusion, with an explicit counting error, produces a witness below the required bound.

1. INTRODUCTION

For an integer $k \geq 2$, define

$$g(k) = \min \left\{ n > k + 1 : p \nmid \binom{n}{k} \text{ for every prime } p \leq k \right\}, \quad L_k = \text{lcm}(1, 2, \dots, k).$$

The restriction $n > k + 1$ excludes the elementary example in which $n = k + 1$ is prime. On the stated domain, $\binom{n}{k} > 1$, so the divisibility condition says exactly that all its prime factors exceed k . The existence of admissible n , and an upper bound for the least one, were established by Ecklund, Erdős, and Selfridge [1]. They proved

$$(1.1) \quad k^{1+c} < g(k) \leq \exp(k + o(k))$$

for some absolute $c > 0$ and all sufficiently large k . At the end of their paper they asked for the sharper comparison $g(k) < L_k$. We establish this comparison for all sufficiently large k .

Theorem 1.1. *There exists an integer $K \geq 2$ such that, for every integer $k \geq K$, there is an integer n with*

$$k + 1 < n < L_k, \quad p \nmid \binom{n}{k} \quad \text{for every prime } p \leq k.$$

In particular, $g(k) < L_k$ for every sufficiently large k .

The distinction from (1.1) is a comparison below the specific quantity L_k . Since $\log L_k \sim k$, the estimate $g(k) \leq \exp(k + o(k))$ alone does not determine whether $g(k)$ lies above or below L_k .

1.1. Earlier work and the expected size of $g(k)$. The lower bound in (1.1) was improved to $g(k) \gg k^2 / \log k$ by Erdős, Lacampagne, and Selfridge [2, Theorem 1]. Granville and Ramaré [3, Theorem 8] proved

$$g(k) > \exp \left(c \sqrt{\frac{(\log k)^3}{\log \log k}} \right)$$

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for an absolute $c > 0$ and sufficiently large k ; this in particular establishes growth faster than any fixed power of k . Konyagin [4, Theorem 1] subsequently obtained

$$g(k) \geq \exp(c(\log k)^2).$$

These lower bounds leave a substantial gap to the upper bound proved here. The conjectural scale is

$$(1.2) \quad \log g(k) \asymp \frac{k}{\log k};$$

see [2, p. 216] for the lower-bound expectation and [4, equation (2)] for the two-sided conjecture.

Sorenson, Sorenson, and Webster [5] give an algorithm for computing $g(k)$ and a heuristic explanation of (1.2). For fixed k , the admissibility conditions describe a periodic set of integers. If its density is $\Delta(k)$, they prove bounds of order $k/\log k$ for $\log \Delta(k)^{-1}$ [5, Theorems 5.2 and 6.1]. Their uniform distribution heuristic then predicts that the first nontrivial admissible integer has roughly the size $\Delta(k)^{-1}$. The distinction between density and the location of the first admissible integer is essential: a density calculation over a full period does not by itself give a witness in a prescribed shorter interval.

Theorem 1.1 concerns the eventual lcm comparison. It does not establish (1.2) or settle the conjectures

$$\limsup_{k \rightarrow \infty} \frac{g(k+1)}{g(k)} = \infty, \quad \liminf_{k \rightarrow \infty} \frac{g(k+1)}{g(k)} = 0$$

posed in [1, equations (1)–(2)].

1.2. The construction and the size of the sieve error. We seek an admissible integer in the progression

$$(1.3) \quad n = Mt - 1, \quad 1 \leq t \leq T.$$

This starts from the upper-bound construction in [1, pp. 648–649], which supplements L_k with small-prime factors and varies a multiplier. The unmodified choice $M = L_k$ has two limitations. If $p^{a_p} \leq k < p^{a_p+1}$, the congruence $n \equiv -1 \pmod{p^{a_p}}$ fixes the lowest a_p digits but leaves the leading digit of k uncontrolled. Also, the condition $L_k t - 1 < L_k$ permits only $t = 1$, leaving no interval of multipliers in which to impose the remaining digit conditions. Adding one extra factor of a prime p to the power already present in L_k makes the relevant base- p digits of $Mt - 1$ all equal to $p - 1$. Lucas's theorem then excludes p from the binomial coefficient, independently of t . However, supplementing L_k in this way makes the modulus larger than the desired upper bound.

We therefore also remove every prime in $((1 - \varepsilon)k, k]$, for a fixed small $\varepsilon > 0$, and supplement only the primes at most $\varepsilon^2 k$. The resulting modulus is

$$(1.4) \quad M = \frac{L_k}{\prod_{(1-\varepsilon)k < p \leq k} p} \prod_{p \leq \varepsilon^2 k} p.$$

Products over p always run over primes. The prime number theorem gives

$$\log(M/L_k) = (\varepsilon^2 - \varepsilon)k + o(k).$$

There is consequently room to let t range up to $\exp(\varepsilon k/2)$ while keeping every candidate in (1.3) below L_k .

Why remove primes near k ? For such a prime, write $k = p + v$. The digit $v = k - p$ is small relative to p . The fraction of residues for which Lucas's criterion fails is therefore $O(\varepsilon)$. There are $O(\varepsilon k / \log k)$ removed primes, so their total forbidden density is $O(\varepsilon^2 k / \log k)$. The middle primes contribute a smaller amount. One factor of ε comes from the number of removed primes; the other comes from the forbidden proportion for each of them. Thus the logarithmic space created by removing the primes is of order εk , whereas the sum of their forbidden densities is of order $\varepsilon^2 k / \log k$.

The remaining issue is to turn these local densities into a count on the interval $[1, T]$. A union bound cannot do this with the estimates available here: its lower bound becomes vacuous once the sum of the densities exceeds one. Nor is counting over the full product of the moduli sufficient for such a short interval. We use inclusion–exclusion through an odd degree $J = O(\varepsilon^2 k / \log k)$. This makes the truncation error smaller than the product of the local survival densities. More crucially, it keeps the accumulated residue-class counting error at most $\exp(O(\varepsilon^2 k))$, which can be absorbed by the interval length $T = \exp(\varepsilon k / 2 + o(1))$ when ε is small enough. Section 5 carries out this comparison with explicit constants.

1.3. Notation and inputs. All asymptotic statements concern integer $k \rightarrow \infty$. The parameter ε will be fixed at $1/1000$; implicit constants may depend on this fixed value. We write

$$\vartheta(x) = \sum_{p \leq x} \log p, \quad \pi(x) = \#\{p \leq x : p \text{ prime}\},$$

and use the prime number theorem in the forms $\vartheta(x) \sim x$ and $\pi(x) \sim x / \log x$. All intervals of primes used in these estimates have endpoints at fixed positive multiples of k ; no prime number theorem for intervals of length $o(k)$ is required. The other arithmetic input is Lucas’s digit criterion, stated in Section 3; a version with proof appears in [2, Lemma 1]. The finite sieve estimate needed here is proved in Section 4.

2. THE MODIFIED LEAST COMMON MULTIPLE

Fix

$$(2.1) \quad \varepsilon = \frac{1}{1000}, \quad y = \varepsilon^2 k, \quad T = \lfloor e^{\varepsilon k / 2} \rfloor.$$

Let

$$\mathcal{R}_k = \{p : (1 - \varepsilon)k < p \leq k\}, \quad \mathcal{M}_k = \{p : y < p \leq (1 - \varepsilon)k\},$$

where both sets consist of primes, and define M by (1.4). The denominator in that formula is a product of distinct prime divisors of L_k , so M is an integer.

Lemma 2.1. *For all sufficiently large k , the following statements hold. For each prime $p \leq k$, let a_p be the integer satisfying $p^{a_p} \leq k < p^{a_p+1}$. Then*

$$(2.2) \quad v_p(M) = \begin{cases} a_p + 1, & p \leq y, \\ 1, & p \in \mathcal{M}_k, \\ 0, & p \in \mathcal{R}_k. \end{cases}$$

Moreover,

$$(2.3) \quad M > k + 2, \quad MT < L_k.$$

Consequently every integer $t \in [1, T]$ gives a candidate $n = Mt - 1$ with $k + 1 < n < L_k$.

Proof. Take k sufficiently large that $y^2 > 2k$. For $p > y$ we then have $p^2 > 2k$, so $v_p(L_k) = 1$. For smaller primes, $v_p(L_k) = a_p$. The small and removed ranges are disjoint, and (2.2) follows directly from the definition of M .

The prime number theorem gives

$$(2.4) \quad \begin{aligned} \log(M/L_k) &= \vartheta(\varepsilon^2 k) - \vartheta(k) + \vartheta((1 - \varepsilon)k) \\ &= (\varepsilon^2 - \varepsilon)k + o(k). \end{aligned}$$

Since $\log T \leq \varepsilon k / 2$ and $\varepsilon^2 < \varepsilon / 2$, we obtain

$$\log(MT/L_k) \leq (\varepsilon^2 - \varepsilon/2)k + o(k) < 0$$

eventually. This proves the upper bound in (2.3). For the lower bound, the product of all primes at most $(1 - \varepsilon)k$ divides M , and hence

$$\log M \geq \vartheta((1 - \varepsilon)k) > k/2$$

for large k . Since $e^{k/2} > k + 2$ eventually, $M > k + 2$. Finally, for $1 \leq t \leq T$,

$$k + 1 < M - 1 \leq Mt - 1 < MT < L_k.$$

□

3. THE LOCAL CONDITIONS

If $n = \sum_j n_j p^j$ and $k = \sum_j k_j p^j$ are the base- p expansions, Lucas's theorem gives

$$\binom{n}{k} \equiv \prod_j \binom{n_j}{k_j} \pmod{p}.$$

Each factor with $0 \leq k_j \leq n_j < p$ is nonzero modulo p . Thus the criterion we need is

$$(3.1) \quad p \nmid \binom{n}{k} \iff k_j \leq n_j \text{ for every } j.$$

This criterion also appears in [2, Lemma 1] and [5, Theorem 1.1].

For the rest of the proof, take k large enough for Lemma 2.1 to apply. We associate to each prime $y < p \leq k$ a set Ω_p of forbidden residues for t modulo p^2 , and put

$$b_p = |\Omega_p|, \quad \delta_p = b_p/p^2.$$

The primes $p \leq y$ impose no condition on t .

Lemma 3.1. *For every positive integer t and $n = Mt - 1$, the following hold.*

- (1) *If $p \leq y$, then $p \nmid \binom{n}{k}$.*
- (2) *If $p \in \mathcal{M}_k$, write $k = up + v$, with $0 \leq v < p$. There is a set $\Omega_p \subseteq \mathbb{Z}/p^2\mathbb{Z}$ of size pu such that $p \nmid \binom{n}{k}$ if and only if $t \bmod p^2 \notin \Omega_p$. In particular,*

$$(3.2) \quad \delta_p = \frac{\lfloor k/p \rfloor}{p} \leq \frac{k}{p^2}.$$

- (3) *If $p \in \mathcal{R}_k$, put $v = k - p$. There is a set $\Omega_p \subseteq \mathbb{Z}/p^2\mathbb{Z}$ of size $pv + p - v$ with the same avoidance property, and*

$$(3.3) \quad \delta_p = \frac{v}{p} + \frac{1}{p} - \frac{v}{p^2} \leq \frac{v+1}{p} \leq 2\varepsilon$$

for sufficiently large k .

Proof. For $p \leq y$, (2.2) shows that $p^{a_p+1} \mid M$. Hence $n \equiv -1 \pmod{p^{a_p+1}}$, and the digits $n_0, \dots, n_{a_p}$ are all $p - 1$. The higher digits of k are zero. Criterion (3.1) therefore holds for every t .

Now let $p \in \mathcal{M}_k$. Since $k < p^2$, the only possibly nonzero digits of k are its units digit v and its next digit u . Write $M = pc$, where $p \nmid c$, using (2.2). Then

$$n = p(ct - 1) + (p - 1).$$

The units digit of n is $p - 1$ and already satisfies the criterion. Its next digit is $(ct - 1) \bmod p$. This digit is smaller than u precisely when

$$(3.4) \quad ct \bmod p \in \{1, \dots, u\}.$$

Multiplication by c is a bijection modulo p , so these are exactly u forbidden classes for t modulo p . Define Ω_p by taking their full inverse image modulo p^2 . Each class has p lifts, giving $b_p = pu$

and (3.2). Using p^2 here lets all active primes have the same type of modulus; the lift leaves the density unchanged.

Finally let $p \in \mathcal{R}_k$. We have $p > k/2$ and $k = p + v$, so the two digits of k are v and 1. Since $p \nmid M$, the map

$$t \mapsto Mt - 1 \pmod{p^2}$$

is a bijection. An acceptable residue for n has units digit in $\{v, \dots, p-1\}$ and next digit in $\{1, \dots, p-1\}$. There are exactly $(p-v)(p-1)$ such residues. Their complement, pulled back to residues for t , has size

$$p^2 - (p-v)(p-1) = pv + p - v.$$

This proves the formula in (3.3). For its final inequality, $v < \varepsilon k$ and $p > (1-\varepsilon)k$, so

$$\frac{v+1}{p} < \frac{\varepsilon k + 1}{(1-\varepsilon)k} \leq 2\varepsilon$$

eventually. □

The three cases are summarized in Table 1. For a removed prime the contribution v/p comes from the excluded units digits; the remaining contribution $(p-v)/p^2$ comes from the zero next digit among residues whose units digit is acceptable. This explains why a prime close to k , for which $v = k - p$ is small, can be removed at a small cost in density.

Prime range	Change from L_k	Modulus for t	Forbidden density
$p \leq \varepsilon^2 k$	Add one factor of p	None	0
$\varepsilon^2 k < p \leq (1-\varepsilon)k$	Retain p	p^2	$\lfloor k/p \rfloor / p$
$(1-\varepsilon)k < p \leq k$	Remove p	p^2	$\leq 2\varepsilon$

TABLE 1. The roles of the three prime ranges in the construction. All assertions hold for sufficiently large k .

Proposition 3.2. *Let*

$$\lambda = \sum_{y < p \leq k} \delta_p, \quad A = \frac{5\varepsilon^2 k}{\log k}.$$

For all sufficiently large k ,

$$(3.5) \quad 0 \leq \delta_p \leq \frac{1}{2} \quad (y < p \leq k), \quad 0 \leq \lambda \leq A.$$

There are at most k active primes, and $b_p \leq k^2$ for each one.

Proof. For middle primes, $p^2 > 2k$ and (3.2) give $\delta_p < 1/2$. For removed primes use (3.3) and $2\varepsilon < 1/2$.

If $p > y = \varepsilon^2 k$, then $k/p^2 \leq 1/(\varepsilon^4 k)$. The bound $\pi(k) \leq 2k/\log k$, valid for large k , implies

$$(3.6) \quad \sum_{p \in \mathcal{M}_k} \delta_p \leq \frac{2}{\varepsilon^4 \log k} \leq \frac{\varepsilon^2 k}{\log k}$$

eventually. This last estimate is deliberately coarse; its constant is harmless because ε is fixed before k tends to infinity. At the two fixed proportional scales, the prime number theorem yields

$$|\mathcal{R}_k| = \pi(k) - \pi((1-\varepsilon)k) = (\varepsilon + o(1)) \frac{k}{\log k} \leq \frac{2\varepsilon k}{\log k}.$$

Consequently,

$$(3.7) \quad \sum_{p \in \mathcal{R}_k} \delta_p \leq \frac{4\epsilon^2 k}{\log k}.$$

Adding (3.6) and (3.7) proves the bound for λ . Finally, there are at most k primes in the range and $b_p \leq p^2 \leq k^2$. $\square$

4. A FINITE SIEVE ESTIMATE

We now give a standard truncated inclusion–exclusion estimate, with its elementary proof, to control the finite interval in (1.3). Its error depends on the number of residue classes occurring in the truncated expansion. No distribution hypothesis about the surviving integers is used.

Lemma 4.1. *Let $q_1, \dots, q_r \geq 2$ be pairwise coprime integers, and let $\Omega_i \subseteq \mathbb{Z}/q_i\mathbb{Z}$. Write*

$$b_i = |\Omega_i|, \quad \delta_i = b_i/q_i, \quad \lambda = \sum_i \delta_i, \quad \mathcal{P} = \prod_i (1 - \delta_i).$$

For a positive integer T , let $N(T)$ count the integers $1 \leq t \leq T$ that avoid all the Ω_i . For every positive odd integer J ,

$$(4.1) \quad N(T) \geq T \left(\mathcal{P} - \frac{\lambda^{J+1}}{(J+1)!} \right) - E_J, \quad E_J = \sum_{\substack{S \subseteq \{1, \dots, r\} \\ |S| \leq J}} \prod_{i \in S} b_i.$$

If also $\delta_i \leq 1/2$, then

$$(4.2) \quad \mathcal{P} \geq e^{-2\lambda}.$$

If $r \leq k$, $b_i \leq k^2$, and $k \geq 2$, then

$$(4.3) \quad E_J \leq (J+1)k^{3J}.$$

Proof. For $S \subseteq \{1, \dots, r\}$, let N_S be the number of $t \in [1, T]$ that lie in every forbidden set indexed by S . The Chinese remainder theorem gives exactly $\prod_{i \in S} b_i$ simultaneous residue classes modulo $\prod_{i \in S} q_i$. Each class contributes within one of its expected count, whence

$$(4.4) \quad \left| N_S - T \prod_{i \in S} \delta_i \right| \leq \prod_{i \in S} b_i.$$

For $S = \emptyset$, the count is exactly T and the same bound holds. The lower Bonferroni inequality, truncated at the odd degree J , gives

$$N(T) \geq \sum_{|S| \leq J} (-1)^{|S|} N_S \geq T \sum_{j=0}^J (-1)^j e_j - E_J,$$

where $e_j = \sum_{|S|=j} \prod_{i \in S} \delta_i$ and $e_j = 0$ for $j > r$. Apply the odd and even Bonferroni inequalities to independent events with probabilities $\delta_1, \dots, \delta_r$. They give

$$\sum_{j=0}^J (-1)^j e_j \leq \mathcal{P} \leq \sum_{j=0}^{J+1} (-1)^j e_j,$$

and therefore

$$0 \leq \mathcal{P} - \sum_{j=0}^J (-1)^j e_j \leq e_{J+1} \leq \frac{\lambda^{J+1}}{(J+1)!}.$$

The final inequality follows by expanding λ^{J+1} : each product of $J+1$ distinct factors occurs $(J+1)!$ times, and all other terms are nonnegative. This proves (4.1).

The elementary inequality $\log(1-x) \geq -2x$ for $0 \leq x \leq 1/2$ gives (4.2). For (4.3), the sum at degree j is at most

$$\binom{r}{j} k^{2j} \leq k^{3j}.$$

There are at most $J+1$ degrees, all at most J . $\square$

Remark 4.2. The independent events in this proof are an auxiliary finite probability model used to bound a polynomial remainder. The actual residue conditions on $[1, T]$ need not be independent; their counting error is accounted for explicitly by (4.4) and E_J .

5. A SURVIVING MULTIPLIER

We apply Lemma 4.1 to the forbidden sets of Lemma 3.1, with moduli $q_p = p^2$ for $y < p \leq k$. These moduli are pairwise coprime. Proposition 3.2 supplies all the remaining hypotheses for (4.2) and (4.3).

Let J be the least positive odd integer with $J \geq 10A$. Then

$$(5.1) \quad 10A \leq J < 10A + 2.$$

This depth is large enough to make the truncation error much smaller than e^{-2A} . Indeed, the elementary bound $s! \geq (s/e)^s$ gives

$$(5.2) \quad \frac{\lambda^{J+1}}{(J+1)!} \leq \left(\frac{e\lambda}{J+1} \right)^{J+1} \leq \left(\frac{e}{10} \right)^{J+1} \leq e^{-10A},$$

where we used $\lambda \leq A$, $J+1 \geq 10A$, and $e/10 \leq e^{-1}$. Thus the number of surviving multipliers satisfies

$$(5.3) \quad N(T) \geq T(e^{-2A} - e^{-10A}) - (J+1)k^{3J}.$$

It remains to compare the two terms on the right. Since $A \rightarrow \infty$ and $T \geq \frac{1}{2}e^{\varepsilon k/2}$ eventually, the first is at least

$$(5.4) \quad \frac{1}{4} \exp(\varepsilon k/2 - 2A).$$

Also $J+1 \leq k$ for large k , so (5.1) gives

$$(5.5) \quad \begin{aligned} \log((J+1)k^{3J}) &\leq \log k + 3J \log k \\ &< 150\varepsilon^2 k + 7 \log k. \end{aligned}$$

The crucial comparison is now explicit:

$$150\varepsilon^2 < \frac{\varepsilon}{2} \quad \left(\varepsilon = \frac{1}{1000} \right).$$

Since $A = o(k)$ and $\log k = o(k)$,

$$\left(\frac{\varepsilon}{2} - 150\varepsilon^2 \right) k - 2A - 7 \log k - \log 4 \longrightarrow \infty.$$

Consequently (5.4) exceeds the error in (5.5) for all sufficiently large k . Equation (5.3) therefore yields $N(T) > 0$.

Choose a surviving integer $t \in [1, T]$ and set $n = Mt - 1$. Lemma 2.1 gives $k+1 < n < L_k$. Lemma 3.1 excludes the small primes automatically and excludes every other prime at most k because t survives its forbidden set. Hence

$$p \nmid \binom{n}{k} \quad \text{for every prime } p \leq k.$$

This n is admissible in the definition of $g(k)$, and $g(k) \leq n < L_k$. Each estimate used above holds for every k beyond a fixed threshold; taking their maximum proves Theorem 1.1. $\square$

6. FURTHER REMARKS

6.1. The role of the parameters. The fixed value $\varepsilon = 1/1000$ leaves ample room in the last comparison. The proof does not seek a small numerical threshold for k . In particular, absorbing the middle-prime contribution in (3.6) uses k large relative to a negative power of ε . The purpose of the cutoff $y = \varepsilon^2 k$ is to protect small primes at a logarithmic cost of order $\varepsilon^2 k$, while removing the top band saves order εk .

The sieve comparison also indicates a limitation of the argument as presented. Making J larger improves the tail estimate (5.2), but worsens the discrepancy (4.3). The proof balances these terms on an exponential-in- k interval. It does not place a witness on the conjectural scale $\exp(O(k/\log k))$.

6.2. Formalization. The accompanying Lean development uses the same construction and proves an explicit witness statement before deducing the bound for g . In its notation, the two final declarations are

`Erdos1095.erdos1095_target` and `Erdos1095.erdos1095_main`.

The finite sieve, digit conditions, and estimates above correspond to the same mathematical steps in that development. In particular, the middle-prime forbidden classes are lifted to modulus p^2 , as in Lemma 3.1.

The development uses Lean 4.27.0 and pinned dependencies. The verifier builds the proof, checks the statement correspondence and imports, and checks that the final theorem depends only on `propext`, `Classical.choice`, and `Quot.sound`. Two unfinished declarations in the pinned prime-number-theorem dependency are outside this theorem’s dependency closure. The Lean source, verification instructions, and manuscript sources are maintained together at

<https://github.com/ethn-y/erdos-1095-lean>.

Tool and computational resource disclosure. The mathematical proof was generated by GPT-6 Astra using Codex. Separate GPT-6 Astra sessions were used to review the argument and reconstruct the formal statement; GPT-5.6 Sol was used for the Lean proof implementation. GPT-6 Astra was also used to prepare this manuscript and its literature review. The author’s contributions were problem selection, direction of these workflows, provision of literature, and editorial decisions. The author did not supply the mathematical argument or independently check the proof by hand. The reviews by AI systems and the Lean verification are distinct from human expert review.

The result was selected from a broader exploration of several problems. No claim about the success rate or comparative performance of the systems is made here.

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