A curious case of fractal geometry in popular culture. Studying self-similarity and Minskowski-Bouligand dimensions in Esporte Clube Bahia's crest

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Abstract

Esporte Clube Bahia (Bahia Sport Club) is the most popular and valuable football club in Northeast Brazil. Its badge was designed in the 1930s and presents self-similar fractal patterns. This text characterizes its geometry. The motive is composed by squares and circles alternately inscribed in each other. Self-similarity dimension and empirical measures of Minkowski-Bouligand box-counting dimension were calculated. A recursive algorithm is provided to plot the crest.

Keywords Self-similarity dimension Fractals Minkowski-Bouligand dimension Julia

1 Introduction

Esporte Clube Bahia, ECB, (Bahia Sport Club) is a Brazilian professional football club, based in Salvador, capital city of the Brazilian state of Bahia. It is also the most popular and valuable football club in Northeast Brazil [2]. . The club's mascot is inspired by Superman (DC Comics) and it is called Super-Homem Tricolor (Tricolor Superman). ECB won national titles in 1959 and 1988. The crest (Figure 1) was designed in the 1930s: 'Under the slogan of "Born to Win", Esporte Clube Bahia emerged in 1931. Historically, it was Raimundo Magalhães who created the tricolor badge in the late 1930s.' The design is heavily inspired by Esporte Clube Corinthians flag. However, a very special detail was added: the flag is composed by an infinite number of copies of itself. [4]

In its self-similar pattern, squares and circles are alternately inscribed in each other. The design starts with an inner square presenting red horizontal stripes. This square is inscribed in a circle, which is also inscribed in an intermediate blue square. These 3 shapes compose the left upper quarter in a new red striped square, forming the loop.

1.1 Self-similarity dimension

Self-similarity dimension for exactly self-similar objects with different scaling factors is formally defined[1, 3].

Let n be the number of scaled down pieces in the construction of an exactly self-similar fractal and let $s_1, ..., s_n$ be the scaling factors (some of them can be equal). Then, the self-similarity dimension Ds is the solution to:

$$\sum_{i}^{n} S_i^D = 1$$

2 Materials and Methods

2.1 Methods for empirical measures

A home-brewed recursive function was written to plot the crest using Julia (v1.6.3) with packages Cairo.jl(v1.0.5) and Compose.jl(v0.9.3). Code is available in the **Appendix** section and also in a public repository (fargolo/ecb_fractal) .

Black and white contour only plots with resolution ranging from 32x32 to 8192x8192px were written to PNG (Portable Network Graphics) files. Then, open-source packages were used to calculate box-counting dimension from the 2D images. Namely, BoxCount and Fractal-Dimension Python(v3.8.10) libs.

3 Results

3.1 Crest's proportions

We do not consider the outer circle bearing the club's name for calculations. It circumscribes the repeating motive, which is described as follows.

A (1) square flag of side L_0, whose (2) upper left quarter is a square of side $L_{top} = L_0/2$, containing one (3) inscribed circle of radius R_1, which in turn circumscribes (4) a second square flag of side L_1.

Then, motives (2) and (3) are repeated in the new square of side L_1.

We can use basic Euclidian geometry to show that the side of the second square L_1 is given by $L_0 \frac{\sqrt{2}}{4}$:

First, notice that circle R_1 is inscribed in the L_{top} square. Therefore, R_1 corresponds to

$$R_1 = L_{top}/2 = L_0/4$$

.

The diagonal ($\text{Diag}_1 = L_1\sqrt{2}$) of the flag inscribed in the inner circle corresponds to the diameter: ($2R_1$):

$$2R_1 = \text{Diag}_1 = L_1\sqrt{2} \implies R_1 = \frac{L_1\sqrt{2}}{2}$$

.

Then, we may find the ratio between L_0 and L_1.

$$\frac{L_1\sqrt{2}}{2} = L_0/4$$

$$L_1 = \frac{2L_0}{4\sqrt{2}} = \frac{L_0}{2\sqrt{2}} = \frac{L_0\sqrt{2}}{4} \implies \frac{L_1}{L_0} = \frac{\sqrt{2}}{4}$$

Therefore, the side L_0 is decreased by the factor $\frac{1}{2\sqrt{2}}$(or $\frac{\sqrt{2}}{4}$) every two copies. And the area A_1 is $(\frac{L_0}{2\sqrt{2}})^2 = \frac{A_0}{8}$.

3.2 Self-similarity dimension

Solving the equation for D:

$$\sum_{i}^{n} S_i^D = 1$$

The outer square has 4 line segments with length L, 1 large internal horizontal stripe (length L) and 2 smaller internal horizontal stripes ($L/2$). Closing a square in the top left section requires 2 more segments ($L/2$). Finally, the inscribed circle has 1 segment ($2\pi L/4$)

$$4(\frac{L_1}{L_0})^D + 1(\frac{L_1}{L_0})^D + 2\frac{1}{2}(\frac{L_1}{L_0})^D + 2\frac{1}{2}(\frac{L_1}{L_0})^D + 2\pi\frac{1}{4}(\frac{L_1}{L_0})^D = 1$$

$$7(\frac{L_1}{L_0})^D + \frac{\pi}{2}(\frac{L_1}{L_0})^D = 1$$

$$(7 + \frac{\pi}{2})(\frac{L_1}{L_0})^D = 1$$

For $D \in \mathbb{R}$ and $L_1/L_0 = \sqrt{2}/4$.

$$D = -(log(4) - 2log(14 + \pi))/log(8) \sim 2.066$$

3.3 Computer assisted calculation of Minkowski-Bouligand dimension in ECB's crest

Empirical values for computer assisted calculation of Minkowski-Bouligand dimension$d$ are available at Table 1.

4 Discussion

The recursive design in ECB's crest was brought up by a naive artist, decades before Mandelbrot started the formal investigation of fractals.

Altough ECB remains the most popular team in Northeast Brazil, the fractal pattern in its crest went unnoticed by mathematicians for almost a century.

The self-similarity dimension in ECB's crest ($2 < d < 3; d \sim 2.067$) reflects the notion of depth (3D like) presented in the design. Further investigations might clarify the meaning of empirical Minkowski-Bouligand measures based on contour-only figures with different resolutions.

5 Acknowledgments

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6 Declaration of interest statement

The author declares that there is no conflict of interest

References

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7 Appendix

Code for plotting the colored crest, performing empirical Minkowski-Bouligand calculations and plotting scattered values is available in a public repository (fargolo/ecb_fractal)

The following Julia code plots the contour-only version.

```
using Cairo, Compose

function plot_ecb_crest(n)
    L_0 = 1
    L_1 = L_0*1/2
    R_1 = L_1*1/2
    L_2 = L_0/(2*sqrt(2))
    xy_new = R_1 - L_2/2

    if n == 0
        compose(context(),
        (context(0,0,1/2,1/2), #Top-left section
            Compose.rectangle(),
            Compose.circle(1/2,1/2,1/2),
            Compose.fill("white"),fillopacity(0.1),Compose.stroke("black")),
        (context(), # Outward rectangles (red filled in original)
            Compose.rectangle(0,3/4,1,1/4),Compose.rectangle(1/2,1/4,1/2,1/4),
            Compose.stroke("black"),Compose.fill("white")),
        (context(), # Outer square
            Compose.rectangle(),
        fill("white"),Compose.stroke("black")))
    else
        t = plot_ecb_crest(n-1)
        compose(context(),
            (context(xy_new,xy_new,L_2,L_2),t), # Repeat motive in smaller section
            plot_ecb_crest(0)) # Plot motive in outer space
    end
end
```

8 Tables and Figures

Resolution |	BoxCount	Software Fractal-Dimension
32px	1.89	1.91
64px	1.79	1.89
128px	1.69	1.77
256px	1.60	1.66
512px	1.51	1.57
1024px	1.45	1.49
2048px	1.39	1.43
4096px	1.35	1.38
8192px	NA	1.34

Figure 1: Official crest.

Figure 2: Bahia fans.

Figure 3: Bahia's 'bandeirão' (big flag) with self-similar pattern being held by fans during a match.

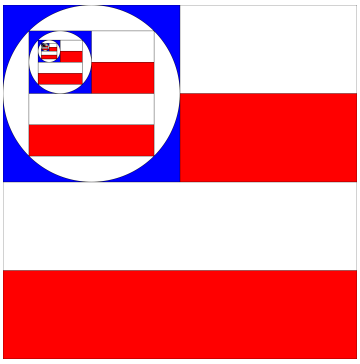

Figure 4: Computer generated crest.

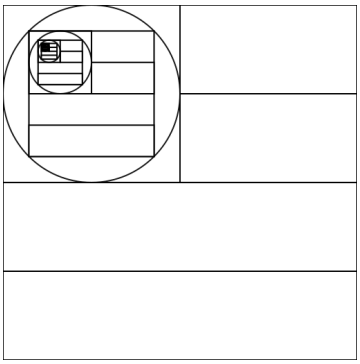

Figure 5: Computer generated crest (countours only).

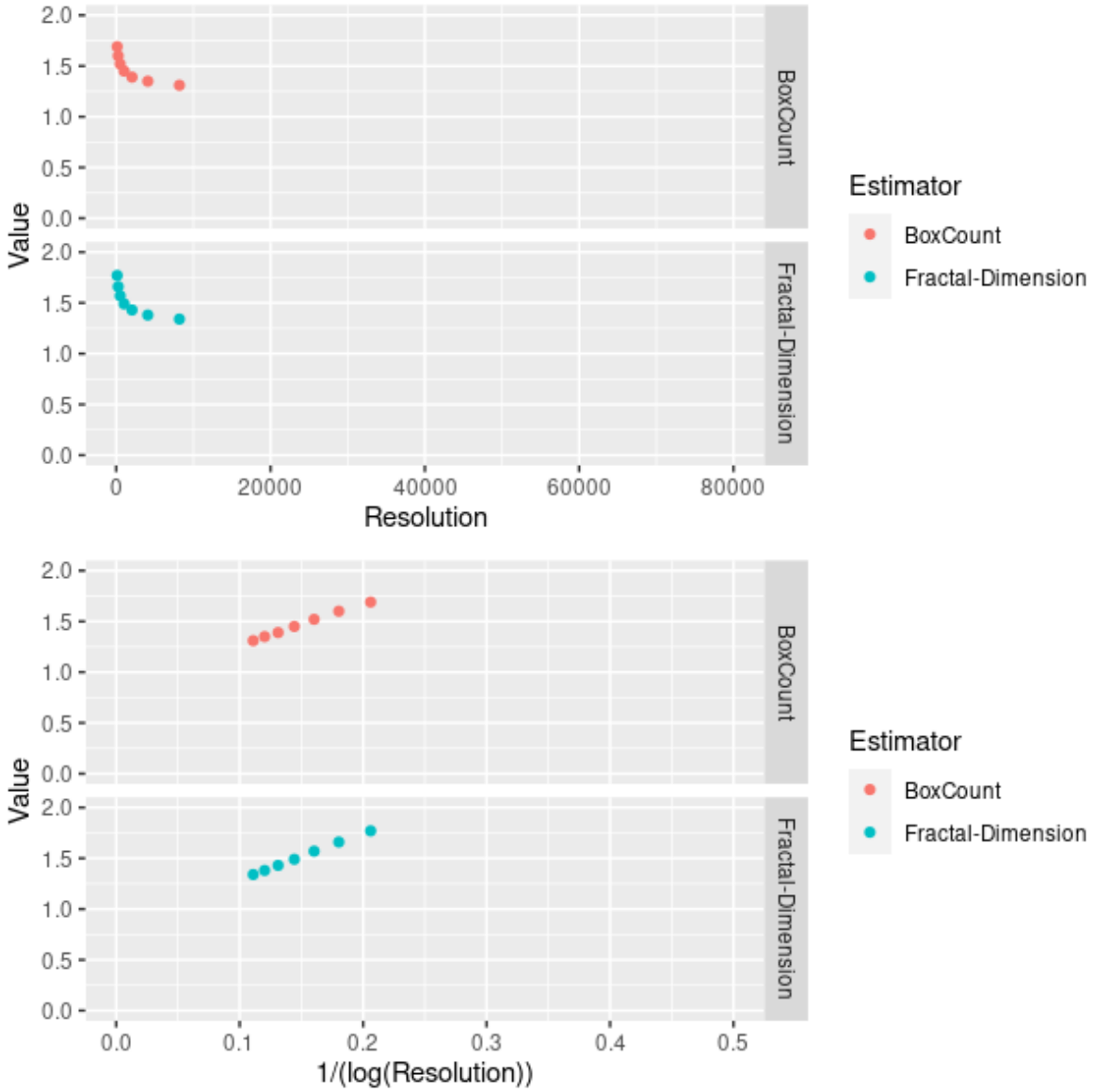

Figure 6: Resolution vs. Empirical MB dimension