2_integration

May 28, 2024

1 Numerical Integration

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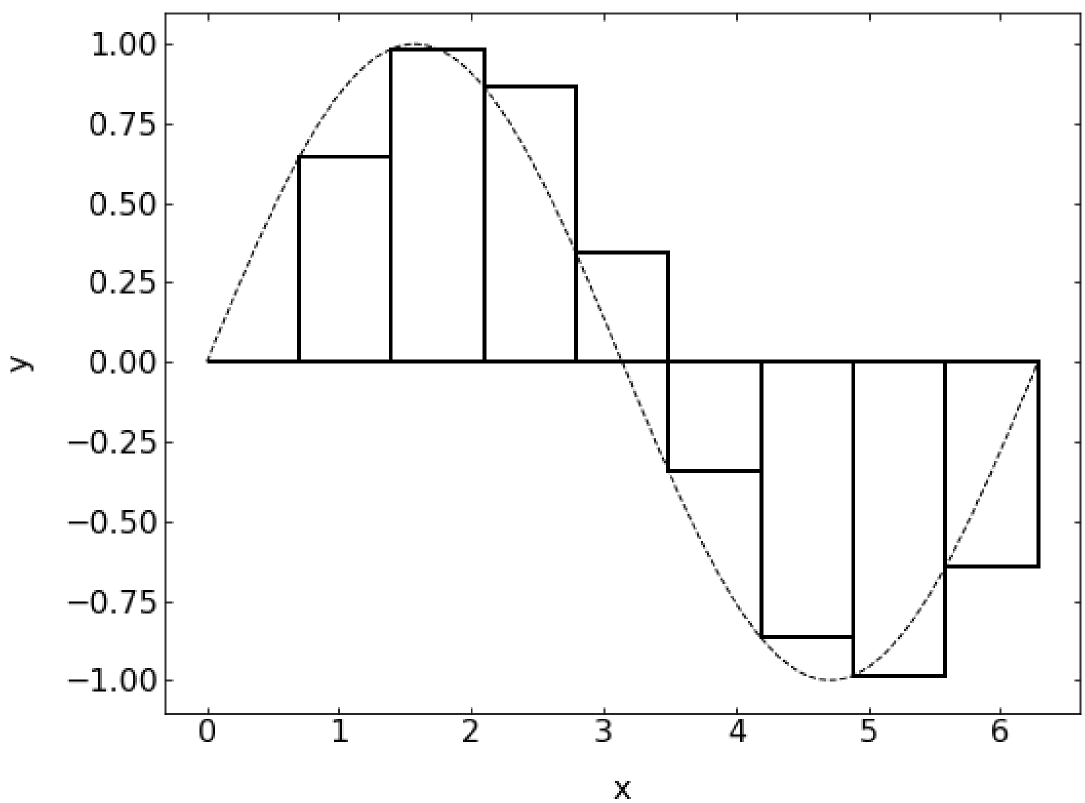
Our second topic today will be about numerical integration, which is useful in determining of course the integrals of functions at certain positions. Here we will only refer to 3 different methods with increasing accuracy.

1.1 Box method



The simplest method for the numerical integration of a function $f(x)$ is the box method. There you approximate the function in a certain intervall Δx by a horizontal line at the function value of the left edge of the intervall for example.

$$\int_a^b f(x) \approx \sum_i f(x_i) \Delta x \tag{1}$$

So lets write a function for that:

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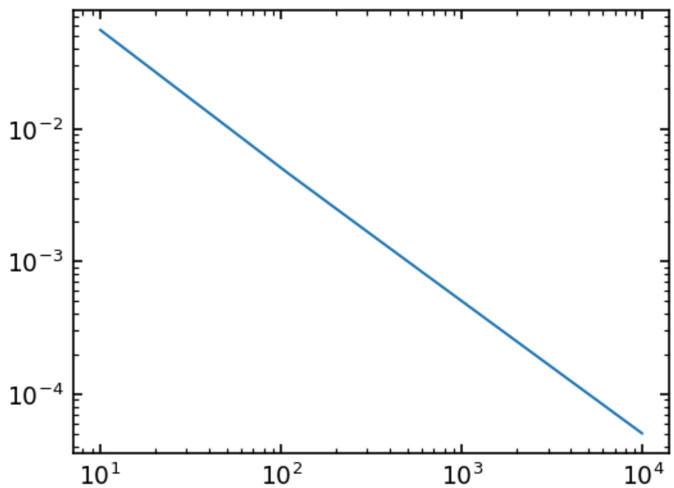
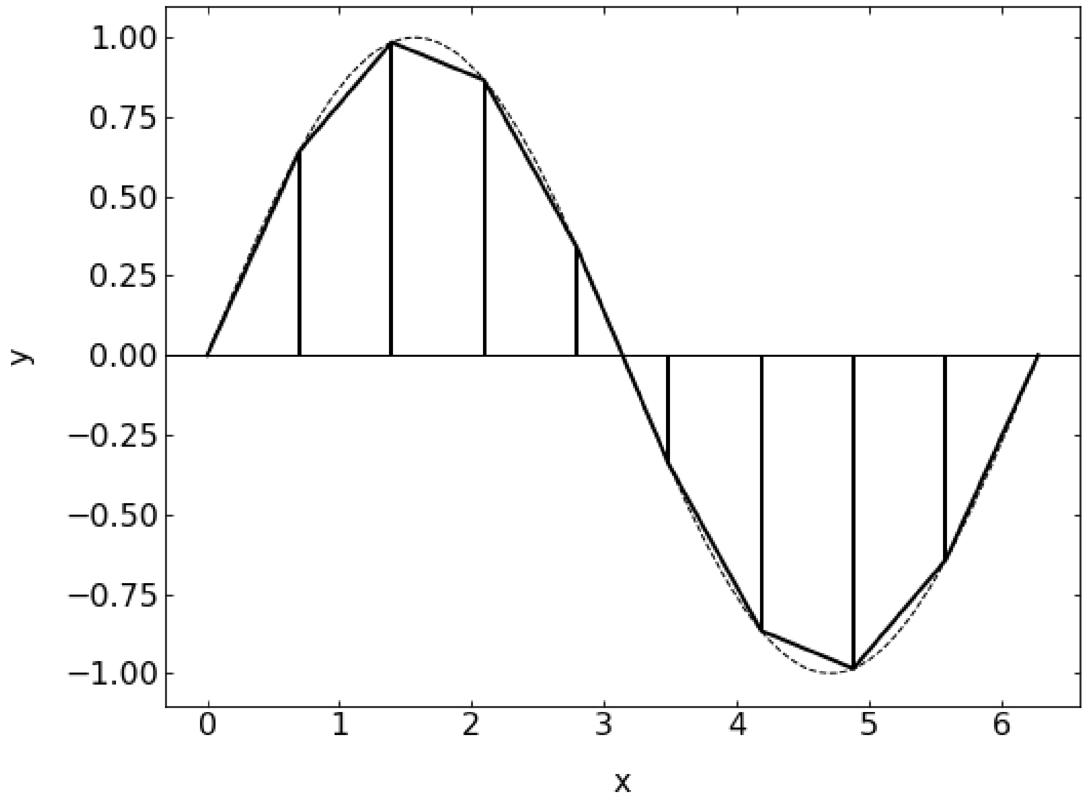
0.5050505050505051
```



1.2 Trapezoid method



The trapezoid method is taking the next step of function approximation in the interval Δx. It is approximating it with a linear function.

$$\int_a^b f(x)dx = \sum_{i=1}^{N} \frac{f(x_i) + f(x_{i-1})}{2} \Delta x \tag{2}$$

which is actually the same as

$$\int_a^b f(x)dx = \left[\frac{f(x_0 + f(x_N))}{2} + \sum_{i=1}^{N-1} f(x_i) \right] \Delta x \tag{3}$$

We will use the first formula for coding it, and you may try the second yourself.

```
0.5050505050505051
```

```
0.5000000000000001
```

The trapez method therefore seems to give a better accuracy than the box method for the same number of steps.

1.3 Simpson method

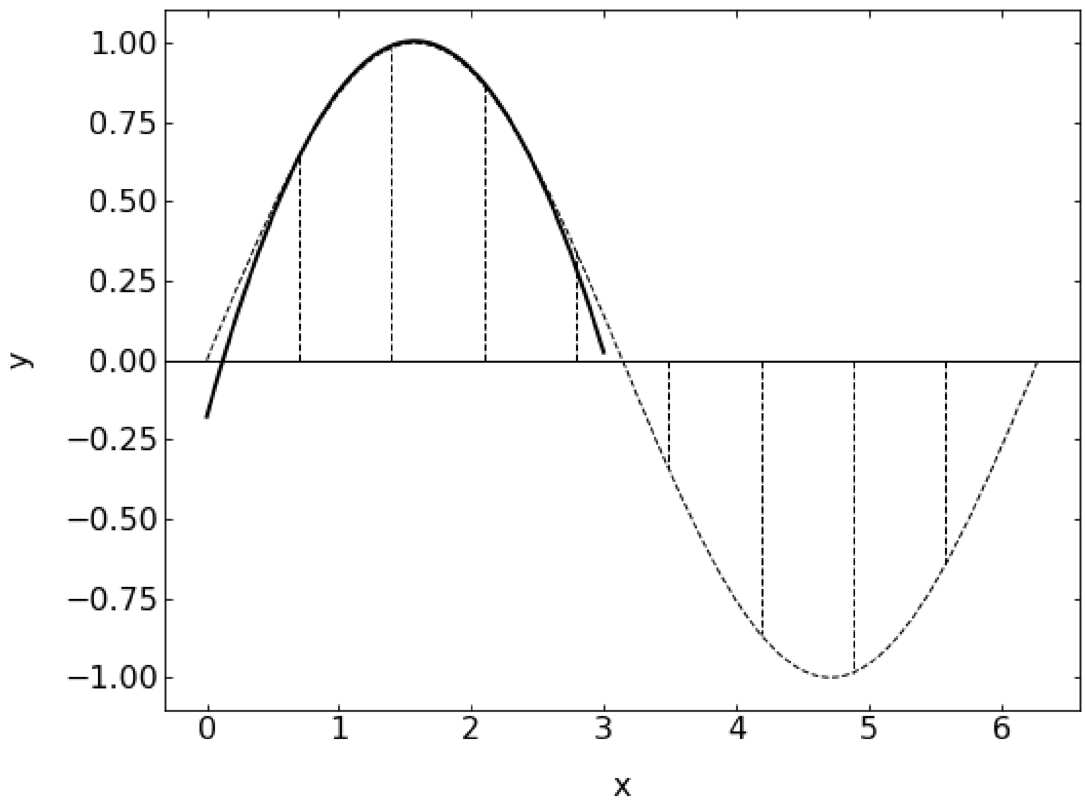
The Simpson method now continues with approximating the function now with a collection of parabolas.

$$\int_a^b f(x)dx \approx \sum_{i=1}^{\frac{N-1}{2}} \int_{x_{2i-1}}^{x_{2i+1}} g_i(x)dx \tag{4}$$

where the function $g_i(x)$ is a parabola

$$g_i(x) = [A]x^2 + [B]x + [C] \tag{5}$$

where the $[A], [B], [C]$ depends only on the function values at the edges of the slice.



After some extensive algebra, which we do not want to do in detail, we arrive at

$$\int_a^b f(x)dx \approx \frac{\Delta x}{3} \sum_{i=\text{odd}}^{N-1} (f(x_{i-1}) + f(x_i) + f(x_{i+1})) \tag{6}$$

as a simple formula on how to calculate the integral of a function using the Simpson method. Note that this method requires N being an odd number, which generates an even number of slices. There is a correction for odd number of slices, which we do not consider here.

```
0.5000000000000001

0.5050505050505051

0.5
```

It turns out, that the Simpson rule is indeed the best among the three methods we have considered. The error is the box method goes as Δx, the one of the trapezoid method as Δx^2, while the simpson method provides and accuracy going with Δx^4. Thus doubling the number of integration points decreases the error by a factor of 16.