2_planetary_motion

May 28, 2024

1 Planetary Motion

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1.1 Physical Model

From the above defined equation of motion for the spring pendulum, it is only a small step to simulate planetary motion, which you should know well from you mechanics lectures. The equations of motion in angular and radial direction can be obtained very similarly. Here, however, there is no force in the tangential direction as we deal with the central symmetric gravitational potential. The equations of motion read:

$$\ddot{r} = r\dot{\theta}^2 - \frac{G\,M}{r^2} \tag{1}$$

$$\ddot{\theta} = -\frac{1}{r}2\dot{r}\dot{\theta} \tag{2}$$

We know the resulting trajectory of this motion

$$r(\theta) = \frac{p}{1+\epsilon\cos(\theta)}$$

with

$$p = \frac{L^2}{GMm^2} \tag{3}$$

$$\epsilon = \sqrt{1+\frac{2\frac{E}{m}\frac{L^2}{m^2}}{G^2M^2}} \tag{4}$$

The trajectory is therefore determined by p and the excentricity ϵ. For $0 < \epsilon < 1$(E<0) there is a closed orbit with an ellipctical shape. For $\epsilon = 0$ the orbit is circular.

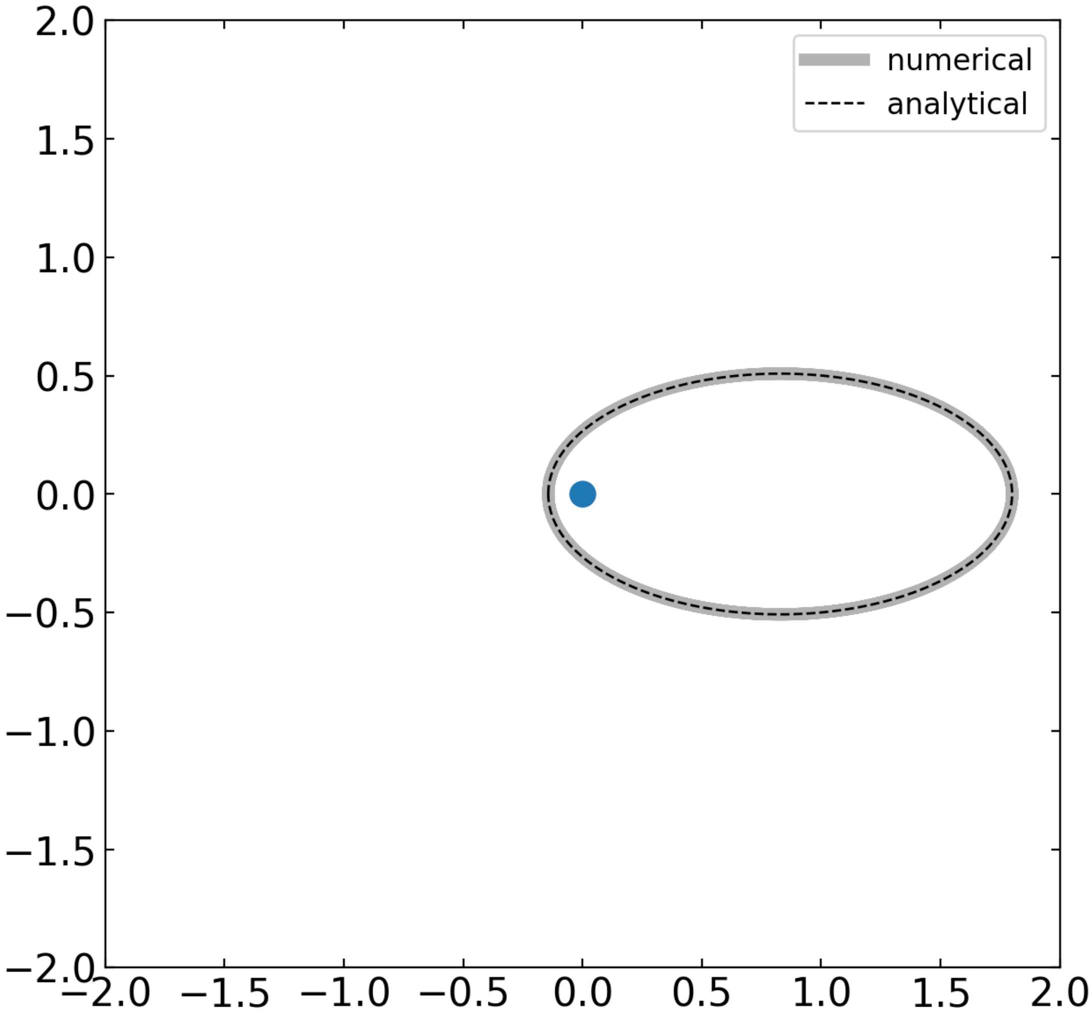
1.2 Numerical Solution

1.2.1 Initial Parameters: Planets

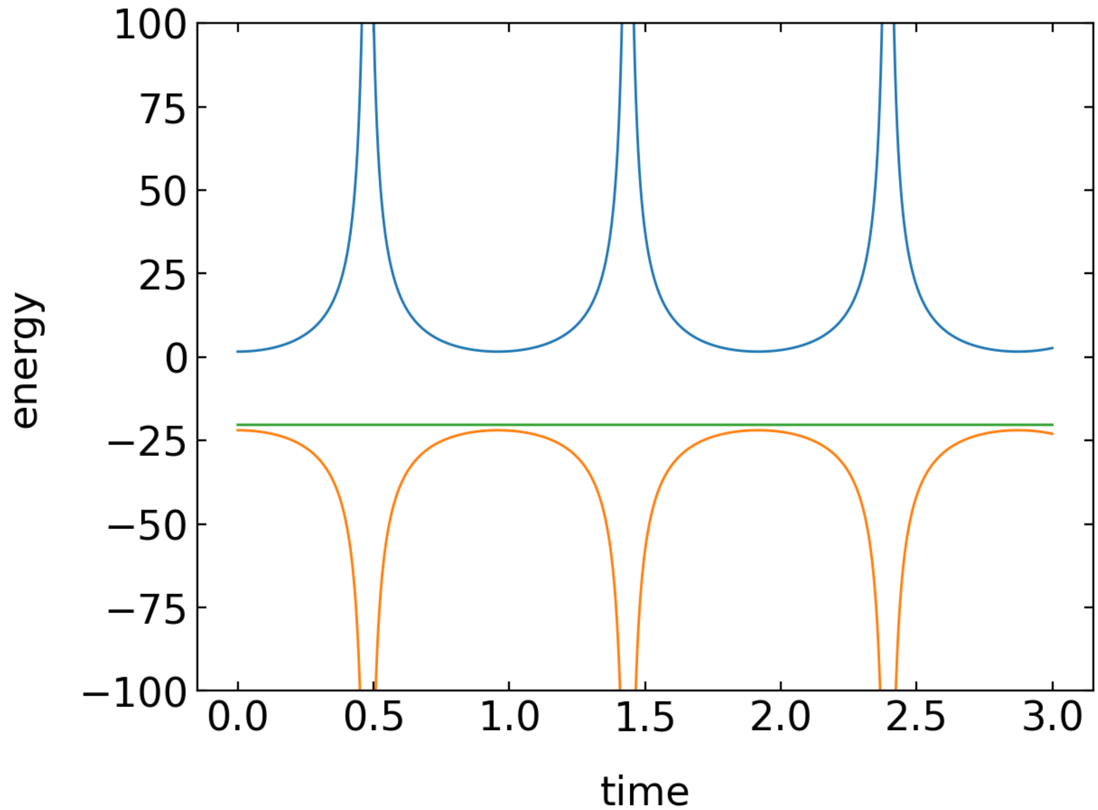
1.2.2 Solution: Planets

1.2.3 Plotting: Planets

Trajectory



Energy



100
75
50
25
0
−25
−50
−75
−100
energy
0.0
0.5
1.0
1.5
2.0
2.5
3.0
time