# Structure and cooling of neutron stars: nuclear pairing and superfluid effects



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## Introduction

Neutron stars are extremely dense and compact objects with mean mass density about  $2 - 3\rho_0$ , where  $\rho_0 = 2.8 \times 10^{14} \text{ g cm}^{-3}$  is the nuclear saturation density. The inner layers of neutron star are governed by the equation of state (EoS) of dense matter, which is poorly known.

Here we report results for structure and cooling properties based on microscopic descriptions of nuclear matter. EoS based on solutions of BHF equations for interacting nucleons. Hidrostatic equilibrium investigated solving Tolman-Oppenheimer-Volkoff equations. Both direct and modified Urca processes are investigated. Superfluid states not included in the cooling process

## The BHF approximation

In infinite nuclear matter, we solved the many-body problem using the BHF approximation. The effective interaction between pairs is described by the matrix *g*, which satisfies [1]:

$$g(\omega) = v + v \frac{Q}{\omega + i\eta - \hat{h}_1 - \hat{h}_2} g(\omega)$$

## **EoS for** $\beta$ **-stable neutron star matter**

Energy density for medium containing protons, neutrons, electrons and muons:

$$\epsilon = (n_n m_n + n_p m_p)c^2 + B/A(n_p, n_n)n + n_\mu m_\mu c^2 + \frac{\hbar^2}{2m_\mu} \frac{(3\pi^2 n_\mu)^{5/3}}{5\pi^2} + \hbar c \frac{(3\pi^2 n_e)^{4/3}}{4\pi^2}$$
(8)

 $n_i$  for particle density of species *i*.  $\beta$ -stable matter :

$$\mu_e = \mu_\mu \qquad \mu_n = \mu_p + \mu_e \qquad n_p = n_e + n_\mu \qquad n = n_p + n_n$$
(9)

where  $\mu = \partial \epsilon / \partial n_i$  is the chemical potential for specie *i*. From Eq. (8) we infer EoS [1],

$$P_i = n_i^2 \frac{\partial}{\partial n_i} \left(\frac{\epsilon}{n_i}\right) \tag{10}$$



where v the bare interaction between nucleons,  $\hat{h}_i$  the singleparticle energy of nucleon i and Q the Pauli blocking operator. The solution enables the evaluation of the mass operator

$$M(k;E) = \sum_{|p| \le k_f} \langle \frac{1}{2}(k-p) | g_K(E+e_p) | \frac{1}{2}(k-p) \rangle$$

where K = k + p is the total momentum, and

$$e_p = \frac{p^2}{2m} + U(p)$$

(2)

(3)

(4)

(5)

(6)

(7)

the sp energy. In the BHF approximation the sp potential is

$$J(k) = ReM(k; e_k)$$

These equations are solved self-consistent.

In the low-density regime large *g*-matrix elements appear in the  ${}^{1}S_{0}$  and  ${}^{3}SD_{1}$  channels due to NN bound states. In this work these bound states are explicitly accounted for, leading to coexisting solutions in symmetric matter [2].

Self-consistent solutions for U(k) at zero temperature were obtained using the Argonne  $v_{18}$  potential. We solved for symmetric and neutronic matter.

The energy per nucleon (B/A) is obtained from the sp potential:

$$B/A = \frac{\int_0^{k_F} \frac{k^2}{2m} k^2 dk + \frac{1}{2} \int_0^{k_F} U(k) k^2 dk}{c^{k_F} 1^{21} c}$$

### Neutron star structure

To obtain the neutron star mass and radius as a function of the central pressure (or density), the EoS for  $\beta$ -stable matter is used in the Tolman-Oppenheimer-Volkoff (TOV) equations [1]

$$\frac{dp}{dr} = -\frac{G\epsilon(p)m(r)}{c^2r^2} \left(1 + \frac{p}{\epsilon(p)}\right) \left(1 + \frac{4\pi r^3 p}{m(r)c^2}\right) \left(1 - \frac{2Gm(r)}{c^2r}\right)^{-1} \qquad \frac{dm}{dr} = 4\pi^2\epsilon(p) \tag{11}$$

We solved these equations for a set of central densities from  $0.5\rho_0$  to  $7\rho_0$ 





[1] M.Baldo, I.Bombaci, G.F.Burgio. Astron. Astrophys. 328, 274-282 (1997).
[2] H. F. Arellano, J.-P. Delaroche and A. Rios, in *Proceedings of Science* PoS(X LASNPA)045.

## Superfluidity

Neutronic matter becomes superfluid in the  ${}^{1}S_{0}$  and  ${}^{3}PF_{2}$  channels. Energy gaps are obtained by solving the BCS gap equation [3]:

$$\Delta(k) = -\sum_{k'} \langle k | U | k' \rangle \frac{\Delta(k')}{2E(k')}$$
$$E(k) = \sqrt{\epsilon(k)^2 + |\Delta(k)|^2} \quad \epsilon(k) = \epsilon(k) - u$$



## Neutrino emissivity

Neutron stars are born with temperatures above  $10^{10}$  K. For  $10^4 - 10^5$  years after birth, neutrino emission is the dominating cooling mechanism. We have been able to calculate the emissivity Q for the modified and the direct Urca processes [4]:

• **Direct Urca**: 
$$n \to p + e^- + \overline{\nu}_e \qquad p + e^- \to n + \nu_e$$

$$Q^{D} = \frac{457\pi}{10080} G_{F}^{2} \cos^{2}\theta_{C} (1 + 3g_{A}^{2}) \frac{m_{n}^{*} m_{p}^{*} m_{e}^{*}}{\hbar^{10} c^{3}} (k_{B}T)^{6} \Theta(k_{F,p} + k_{F,e} - k_{F,n})$$
(12)

• Modified Urca: 
$$b + n \rightarrow b + p + e^- + \overline{\nu}_e$$
  $b + p + e^- \rightarrow b + n + \nu_e$   $b = p, n$ 

Neutron branch : 
$$Q^{Mn} = \frac{11513}{30240} \frac{G_F^2 cos^2 \theta_C g_A^2 (m_n^*)^3 m_p^*}{2\pi} \left(\frac{f^\pi}{m_\pi}\right)^4 \frac{k_{F,p} (k_B T)^8}{\hbar^9 c^8} \alpha_n \beta_n$$
 (13)

Proton branch : 
$$Q^{Mp} = Q^{Mn} \left(\frac{m_p^*}{m_n^*}\right) \frac{(k_{F,e} + 3k_{F,p} - k_{F,n})^2}{8k_{F,e}k_{F,p}} \Theta(3k_{F,p} + k_{F,e} - k_{F,n})$$
 (14)

where the effective masses for baryons (b) and leptons (l) are

$$m_b^* = m_b \left[ 1 + \frac{m}{k} \frac{\partial U(k)}{\partial k} \right] \qquad m_l^* = \frac{\mu}{c}$$

and the Fermi momentum of specie *i* 



where e(k) is the sp energy and  $\mu$  the chemical potential.



[3] Ø.Elgarøy, L.Engvik, M.Hjorth-Jensen and E.Osnes, Nucl. Phys. A **604**, 466-490 (1996).



#### Analog equations for muons.



[4] D.G. Yakovlev, A.D.Kaminker, O.Y.Gnedin and P.Haensel, Phys. Rep. 254, 1-155 (2001).

## **Conclusions and future work**

- Obtained mass-ratio relation and emissivities consistent with current findings.
- Evaluate cooling curves that can be compared with data.
- Ongoing work to include 3-body forces.
- Finite-temperature within BHF (work in progress).

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