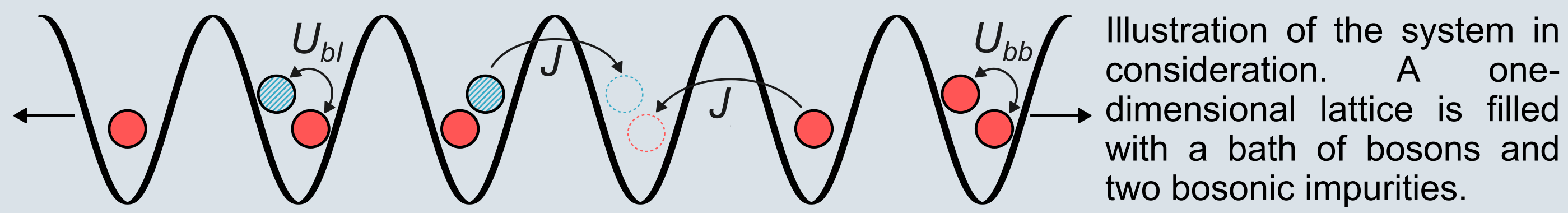




I. Background

The study of **optical lattices** with **two impurities** and a **bosonic bath** has received increasing theoretical interest [1-4]. Firstly, the consideration of two impurities enables the formation of **bipolarons**, which have been predicted to form in BECs [5]. Secondly, optical lattices offer a unique setting to study impurities due to the onset of **Mott insulator** phases [6].



We study **two mobile bosonic impurities** immersed in a **one-dimensional lattice** and interacting with a **bosonic bath**. We examine the formation of **bound dimers** of impurities and study the **dynamics** after a **quench** of the interactions.

II. Model

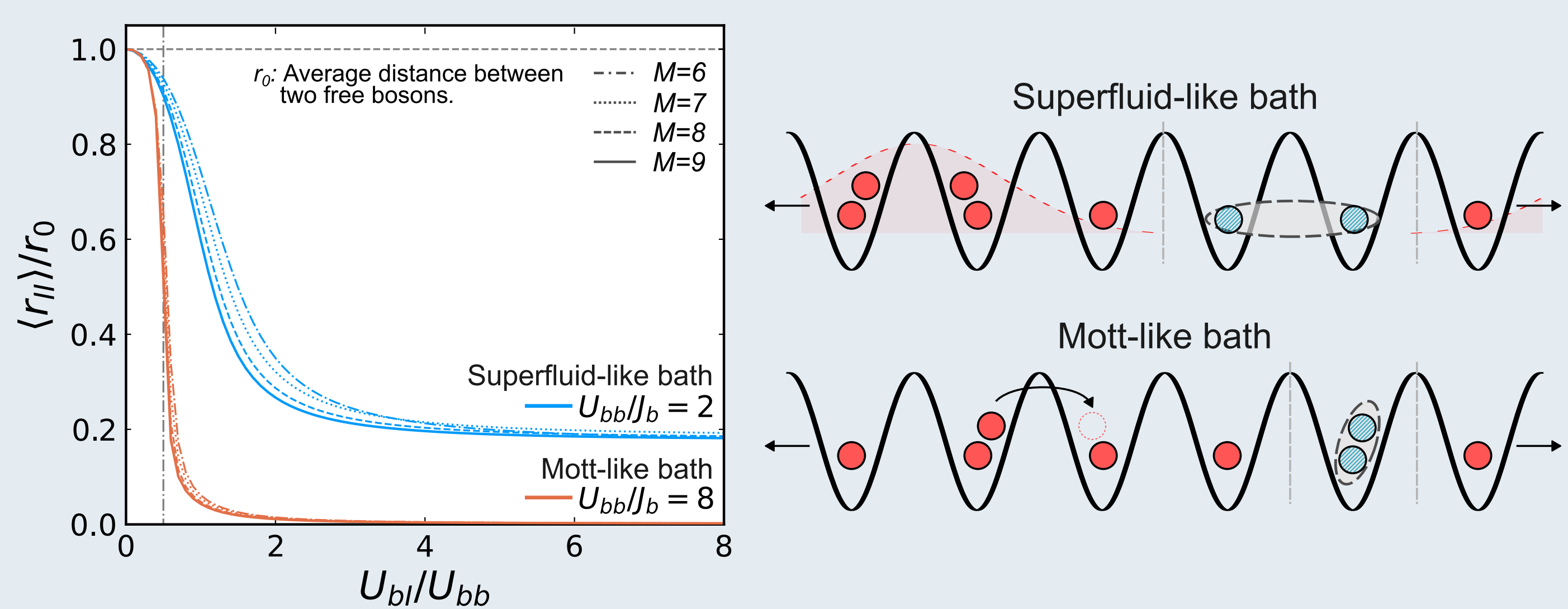
We consider a **two-component Bose-Hubbard Hamiltonian**

$$\hat{H} = -J \sum_{\sigma=b,I} \sum_i \left(\hat{a}_{i,\sigma}^\dagger \hat{a}_{i+1,\sigma} + \text{h.c.} \right) + \frac{U_{bb}}{2} \sum_i \hat{n}_{i,b} (\hat{n}_{i,b} - 1) + U_{bI} \sum_i \hat{n}_{i,b} \hat{n}_{i,I} \quad U_{bb}, U_{bI} > 0$$

where b and I denote the bath's bosons and impurities, respectively. To study stationary properties we employ the **exact diagonalisation** method [7], whereas for the dynamics as a function of time t , we **exponentiate** the Hamiltonian [8]. We consider small **periodic lattices** with M sites and loaded with **two impurities** and a bath with M bosons (**unity filling**).

III. Bound Impurities

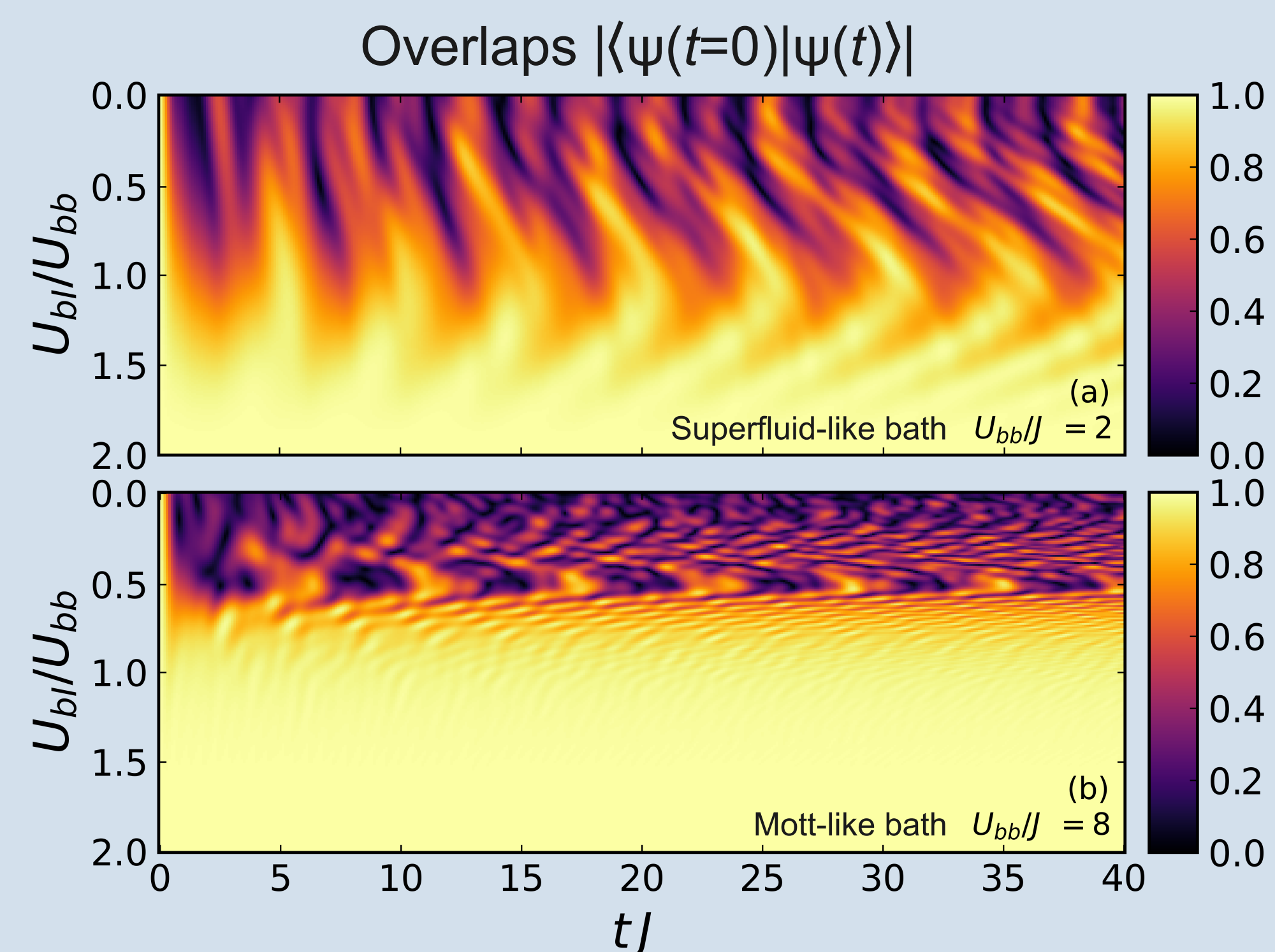
For large U_{bI}/U_{bb} , the bath and impurities **phase separate**, inducing the formation of a **bound dimer** of impurities [4]. This is nicely captured by the **average distance** between impurities.



A small **tightly-bound dimer** is formed in Mott-like baths, while a large **shallow dimer** is formed in superfluid-like baths.

IV. Quench-dynamics of the overlaps

We prepare an **initial state** with a **dimer** formed for $U_{bI}/U_{bb}=2$, and perform a **quench** to a **smaller** value of U_{bI} at $t=0$. We examine the **time-evolution** of the **overlaps** between the initial and evolved state.

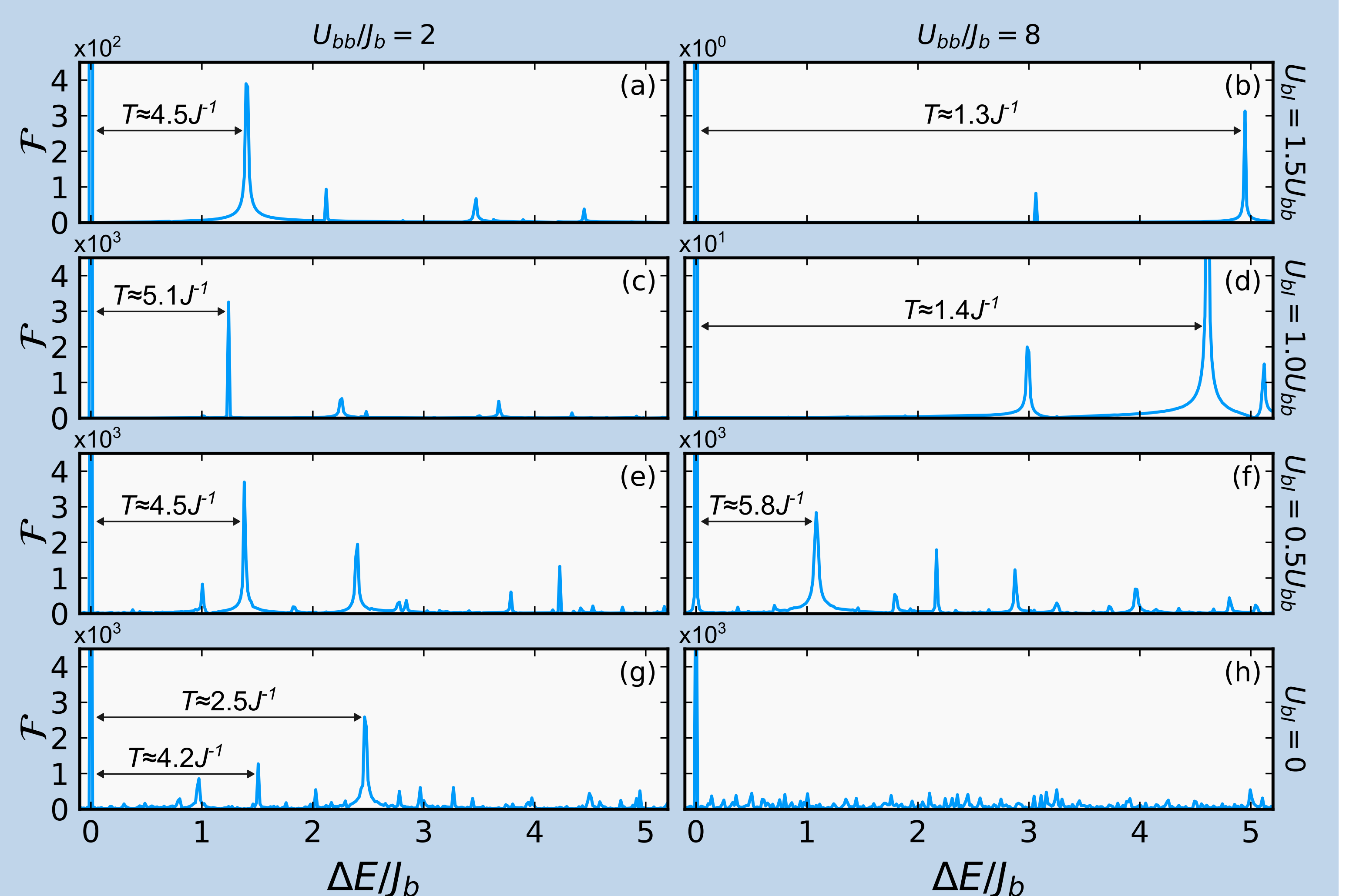


We observe a **periodic** onset of **collapses** and **revivals** of the dimer state. Also, in superfluid-like baths the dimers still show revivals for **large quenches** of U_{bI} to zero, while these disappear in Mott-like baths.

Interestingly, the **period of oscillations** reaches a **maximum** around the **phase separation strength** $U_{bI}/U_{bb} \approx 0.5$.

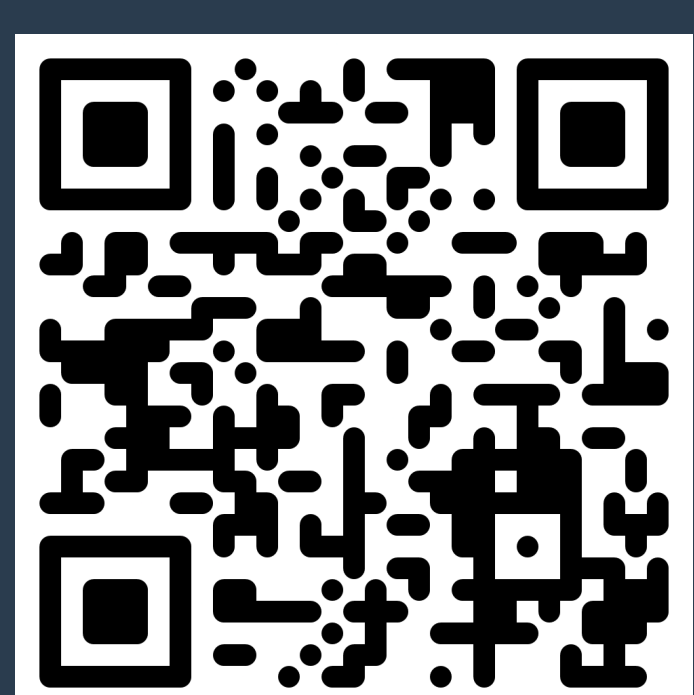
V. Fourier Analysis

To understand the oscillation patterns, we perform a **Fourier analysis** of the time evolution. For $M=7$ we obtain:



The peaks provide the **observed periods** of oscillations, including the maxima [panels (c) and (f)]. We find that the peaks correspond to **phase-separated excitations** where the dimers have a **larger size**. These excitations drive the oscillations after the interaction quenches.

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VI. Outlook

We have studied **two bosonic impurities** immersed in a small **one-dimensional lattice Bose bath**. We have examined the **dynamics** after an **interaction quench** and found an intriguing onset of **collapses** and **revivals** of the **bound impurities**.

The studied model could be produced experimentally with a **few-body** [9] and **highly-imbalanced mixture** [10] in **ring configurations** [11], while the revivals could be observed with measurements of atomic **correlations** [12,13].