

I. Background

The experimental realisation of **ultracold atomic mixtures** has revitalised the interest in studying **impurities** immersed in **quantum mediums** [1]. Amongst them, **Bose polarons**, i.e. impurities immersed in **bosonic baths**, have attracted increased attention [2]. In this direction, the study of ultracold atom impurities confined in **optical lattices** [3] has emerged as an exciting new platform for studying novel polaron physics [4-6].

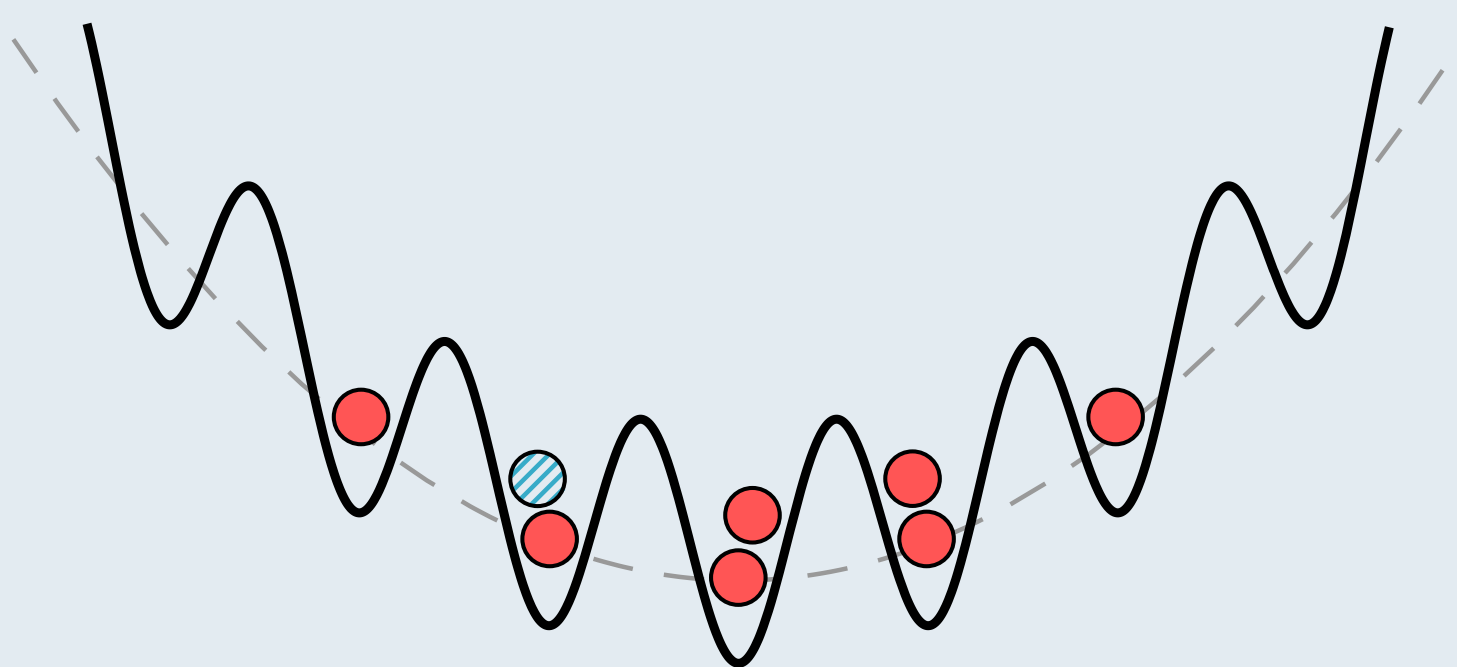


Illustration of the system in consideration. A one-dimensional harmonically confined lattice is filled with a bath of bosons (red circles) and one impurity (blue hatched circle).

In this work, we study an **impurity** interacting with a **bosonic bath** trapped in a **one-dimensional harmonically trapped optical lattice**. We examine the system's **phase diagram** and the polaron properties across the **superfluid-to-Mott insulator** transition.

II. Model

We consider a **two-component Bose-Hubbard Hamiltonian** [7]

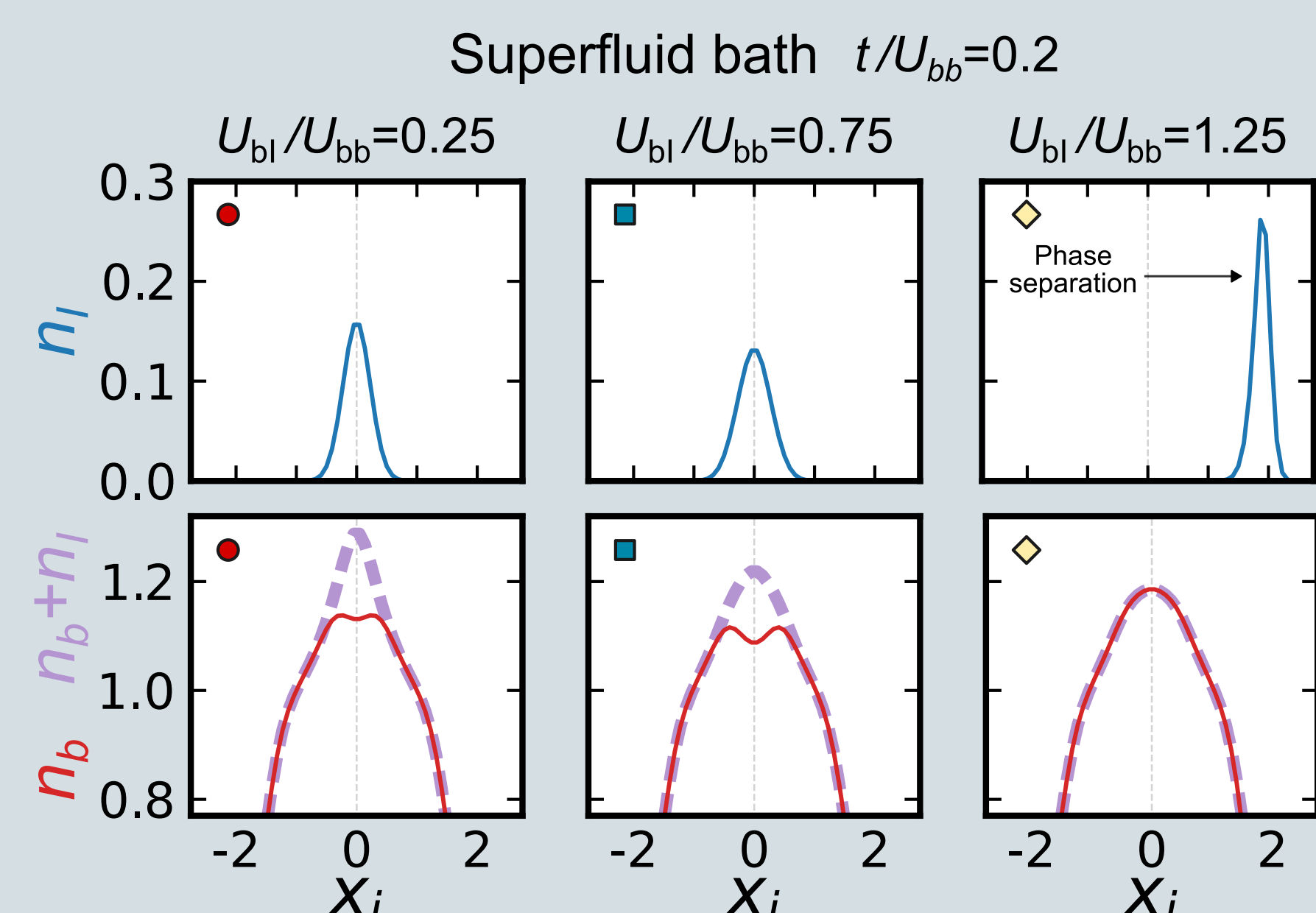
$$\hat{H} = -t \sum_{\sigma=b,I} \sum_{i=1}^M (\hat{a}_{i,\sigma}^\dagger \hat{a}_{i+1,\sigma} + \text{h.c.}) + V_{\text{ho}} \sum_{\sigma=b,I} \sum_{i=1}^M (i - i_0)^2 \hat{n}_{i,\sigma} + \frac{U_{bb}}{2} \sum_{\sigma=b,I} \sum_{i=1}^M \hat{n}_{i,\sigma} (\hat{n}_{i,\sigma} - 1) + U_{bI} \sum_{i=1}^M \hat{n}_{i,b} \hat{n}_{i,I},$$

where b and I denote the bath's bosons and impurities, respectively, M is the number of sites, and i_0 is the central site. We perform numerical simulations for large systems using the **DMRG** technique [8]. We perform simulations for $M=60$ sites and $N_b=40$ bosons in the bath.

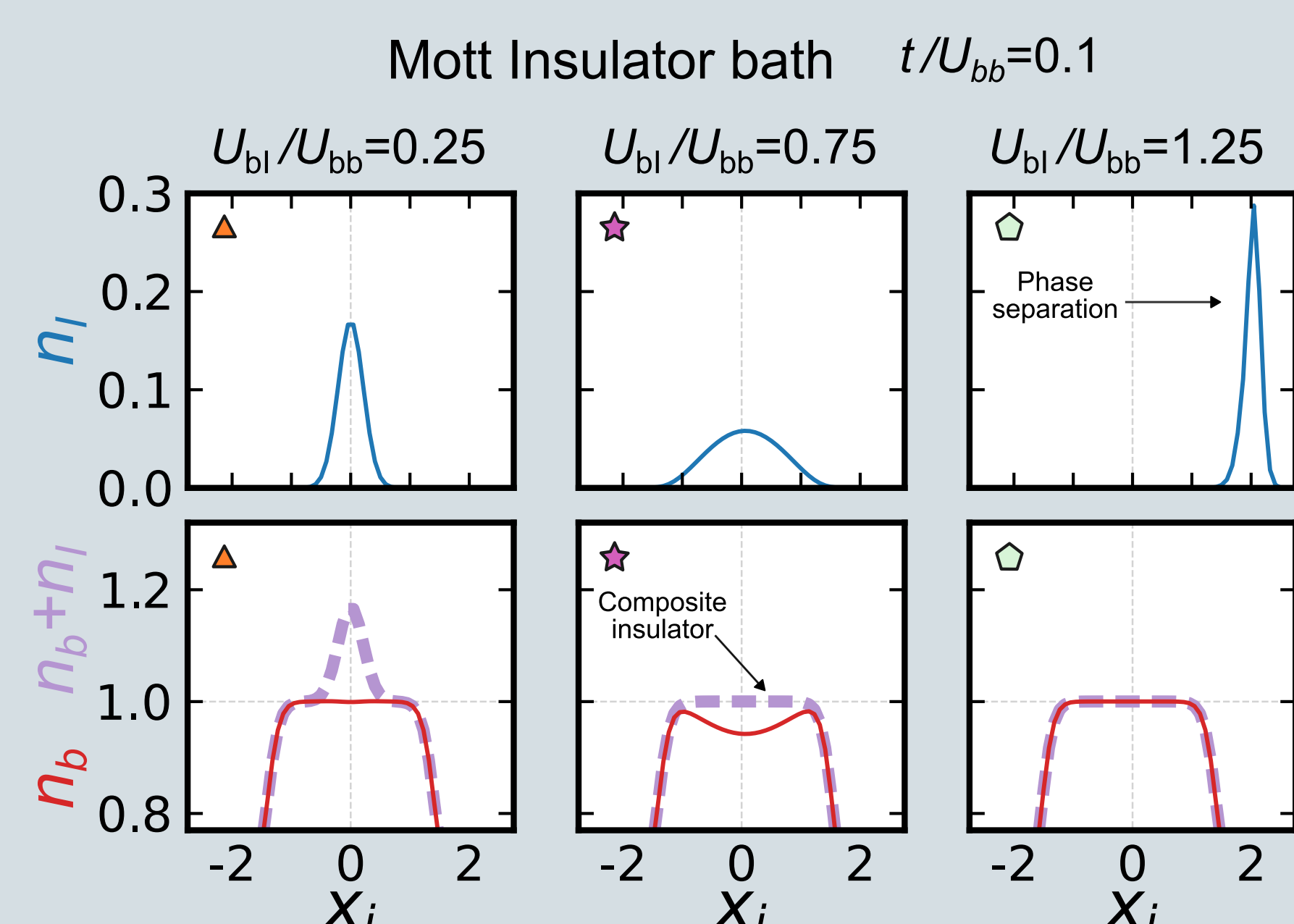
III. Density profiles

We study the **density profiles** $n_\sigma(i) = \langle \hat{n}_{i,\sigma} \rangle$ of each species. We rescale them in terms of the **characteristic length** [7]

$$x_i = (i - i_0)/\xi, \quad \xi = \sqrt{t/V_{\text{ho}}}.$$



In a **superfluid** bath, the impurity is localised at the centre of the trap (**miscible**) for $U_{bI} < U_{bb}$, while it **phase-separates** for $U_{bI} > U_{bb}$.

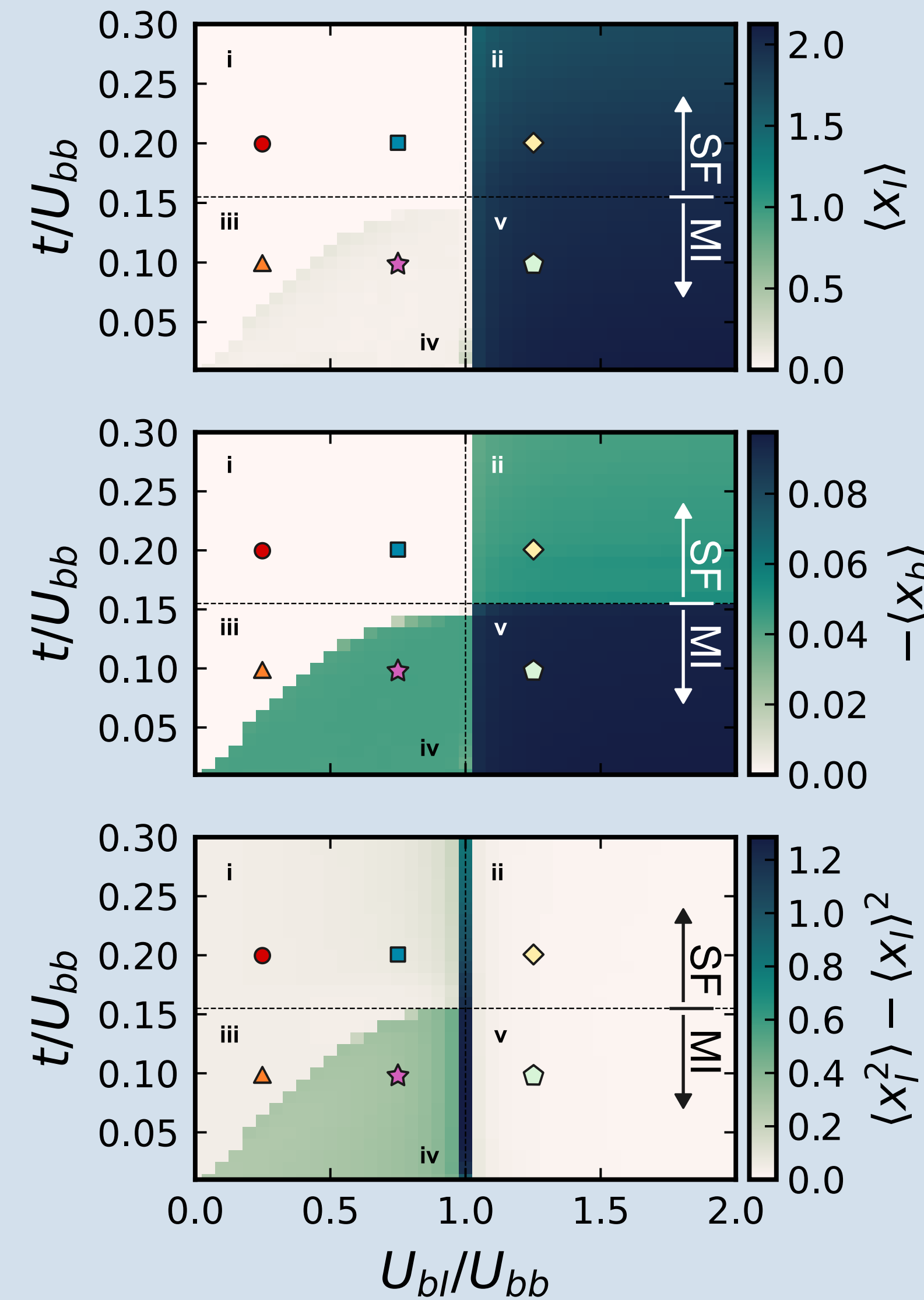


In an **insulating** bath, the impurity is localised at the centre of the trap (miscible) for small U_{bI} and **phase-separates** for $U_{bI} > U_{bb}$. However, for an intermediate U_{bI} the impurity and bath form a **composite Mott insulator**.

IV. Phase diagram

To examine the phase diagram we compute the **average position** of each species and the size of the **impurity cloud**

$$\langle x_\sigma \rangle = \frac{1}{N_\sigma} \sum_{i=1}^M x_i n_\sigma(i), \quad \langle x_I^2 \rangle = \sum_{i=1}^M x_i^2 n_I(i).$$



These quantities enable us to identify **five phases**: (i) **Miscible** and (ii) **phase-separated superfluids**, (iii) **miscible** (iii), **composite** (iv) and **phase-separated** (v) **Mott-insulators**.

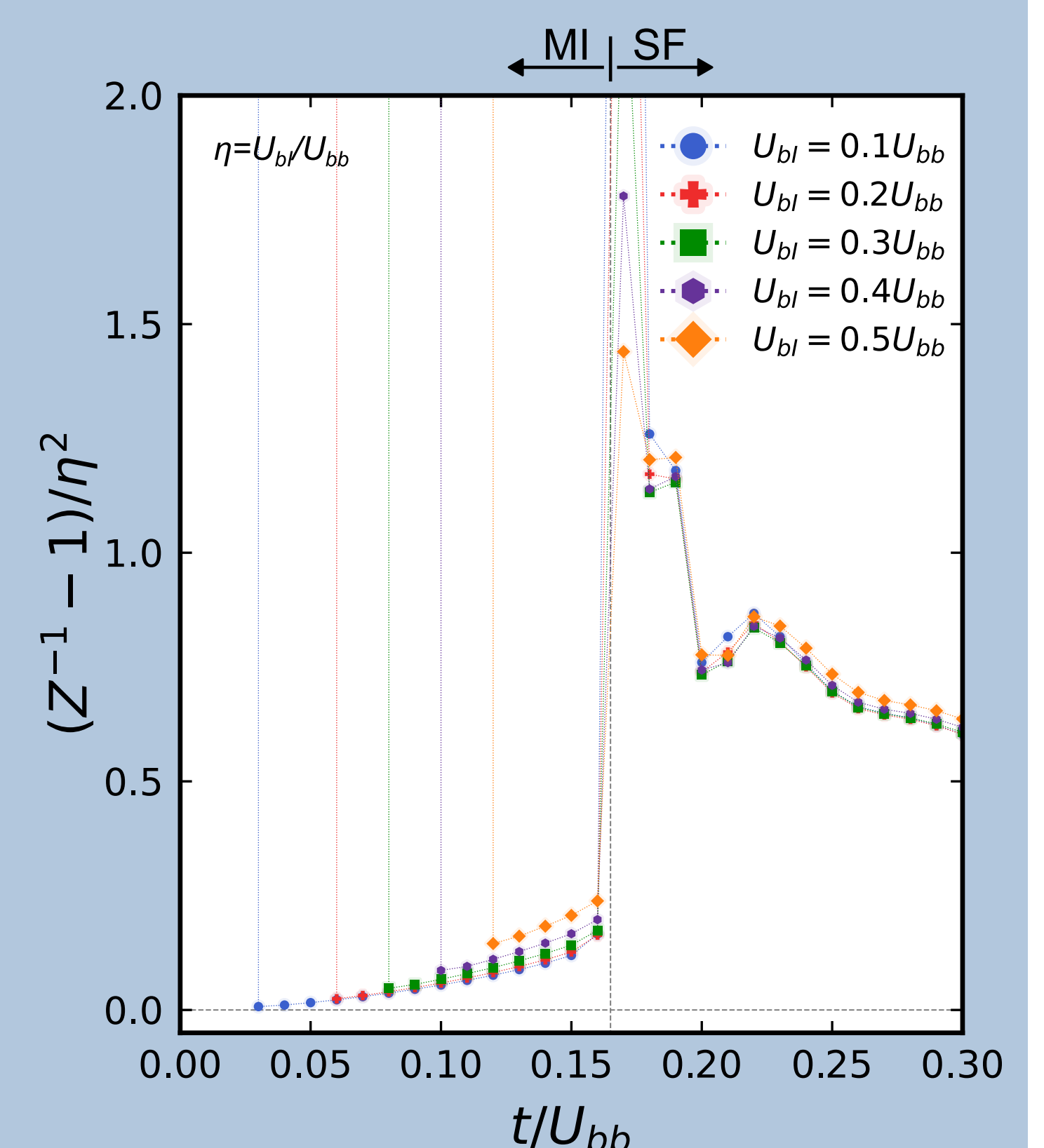
The composite insulator phase shows a **non-trivial phase transition**. It is characterised by a **large spreading of the impurity density** and a small movement of the bath and impurity to **opposite sides** of the lattice.

V. Polaron residue

We examine the **polaron residue** across the bath's phase transition:

$$Z(U_{bI}) = |\langle \Psi(U_{bI} = 0) | \Psi(U_{bI}) \rangle|^2$$

We find that the residue changes its behaviour abruptly **across the transition**. Moreover, the residue shows a **near-universal behaviour** in the rescaled form of the figure. The behaviour of Z is in agreement with previous perturbative calculations in square lattices [4].



VI. Conclusions and outlook

We have studied **one impurity** immersed in a **bosonic bath**, both confined in a **harmonically confined one-dimensional optical lattice**. We find that the system shows a non-trivial phase where both species form a **composite Mott-insulator**.

We have also studied **polaron properties** across the superfluid-to-Mott insulator transition, obtaining agreement with previous works. Future studies will include the consideration of **fermionic baths** and of **two impurities** to study **bipolaron properties**.

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- [1] C. Baroni, G. Lamporesi, and M. Zaccanti, Nat. Rev. Phys. (2024).
- [2] F. Grusdt, N. Mostaan, E. Demler, L. A. Peña Ardila, arXiv:2410.09413 (2024).
- [3] I. Bloch, Nat. Phys. **1**, 23 (2005).
- [4] V. E. Colussi, F. Caleffi, C. Menotti, and A. Recati, Phys. Rev. Lett **130**, 173002 (2023).
- [5] V. R. Yordanov and F. Isaule, J. Phys. B **56**, 045301 (2023).
- [6] F. Isaule, A. Rojo-Francàs, B. Juliá-Díaz, SciPost Phys. Core **7**, 049 (2024).
- [7] G. G. Batrouni *et al.*, Phys. Rev. A **78**, 023627 (2008)
- [8] U. Schollwöck, Rev. Mod. Phys. **77**, 259 (2005).