Cold atom gases from functional Renormalization

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EXCELENCIA MARÍA DE MAEZTU

Weakly-interacting Bose gases

- Dilute gas of bosons interacting through weak repulsive short-range interactions

$$r_0 \ll n^{-d}$$

r_0: range of the interaction
n : boson density

- Long standing theoretical interest for describing Bose-Einstein condensation (BEC) and superfluidity in many-body systems

N. Bogoliubov, Izv. AN SSSR Ser. Fiz. **11**, 77 (1947)

- Interest greatly increased since the experimental realisation of BEC in cold atom gases

M.H. Anderson *et al.* Science **269**, 198 (1995), K. B. Davis *et al.* PRL **75**, 3969 (1995)

- Nowadays, Bose gases are well understood, but are relevant for building more robust approaches to study related systems

Microscopic model

- Bare action for a non-relativistic weakly-interacting Bose gas

$$\mathcal{S}[\boldsymbol{\varphi}] = \int_0^\beta \mathrm{d}\tau \int \mathrm{d}^d\mathbf{x} \left[\varphi^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu\right)\varphi + \frac{g}{2}(\varphi^\dagger\varphi)^2\right]$$

$\boldsymbol{\varphi} = (\varphi, \varphi^\dagger)$: boson fields

g: weak repulsive contact interaction

$\tau = it$, $\beta = 1/T$

- Gran canonical partition function

$$Z[\boldsymbol{\varphi}] = \int D\boldsymbol{\varphi} e^{-\mathcal{S}[\boldsymbol{\varphi}]} = \mathrm{tr}\left[e^{-\beta(H-\mu N)}\right]$$

- Interaction related to the two-body T-matrix:

$$T^{2B} = \begin{cases} \frac{4\pi a_{3D}}{m} & : d = 3 \\ \frac{4\pi/m}{\log(2/|\mu| a_{2D}^2) - 2\gamma_E} & : d = 2 \end{cases}$$

a: s-wave scattering length

Long-distance behavior ($|\mathbf{x}| \to \infty$) of the correlation function

$$G_n(\mathbf{x}) = \langle \phi^\dagger(\mathbf{x})\phi(0) \rangle$$

Three dimensions:

$$G_n(\mathbf{x}) \to \quad \rho_c > 0 \qquad : T < T_c$$

$$G_n(\mathbf{x}) \propto \begin{cases} |X|^{-\eta^*} & : T = T_c,\ \eta^* > 0 \\ e^{-|X|/\xi} & : T > T_c \end{cases}$$

ρ_c: Condensate density

- System shows long-range-order (LRO) and condensation
- $U(1)$ symmetry is broken, with ρ_c as order parameter
- Gas is superfluid as long is condensed

$$T < T_c: \quad 0 < \rho_c \leq \rho_s \qquad\qquad T > T_c: \quad \rho_s = 0$$

ρ_s: Superfluid density

Two dimensions:

$$G_n(\mathbf{x}) \rightarrow \quad \rho_c > 0 \qquad : T = 0$$

$$G_n(\mathbf{x}) \propto \begin{cases} |X|^{-\eta} & : T < T_c,\ \eta > 0 \\ |X|^{-\eta^*} & : T = T_c,\ \eta^* > 0 \\ e^{-|X|/\xi} & : T > T_c \end{cases}$$

- Condensation only possible at $T = 0$ (Mermin-Wagner theorem)
- System shows quasi-long-range-order (QLRO)
- $\rho_c = 0$ for $0 < T \leq T_c$, but superfluid density $\rho_s > 0$
- Phase transition driven by the unbinding of vortex pairs: Berezinskii-Kosterlitz-Thouless (BKT) transition

One dimension:

$$G_n(\mathbf{x}) \propto \begin{cases} |X|^{-\eta} & : T = 0, \\ e^{-|X|/\xi} & : T > 0, \end{cases}$$

- System shows quasi-long-range-order (QLRO) at $T = 0$
- In the limit of strong interaction the one-dimensional Bose gas behaves as a free Fermi gas (Tonks-Girardeau gas)

- Mean-field theory gives a qualitative description of Bose gases at low temperatures. Fluctuations need to be considered
- Perturbation theory is plagued by IR divergences

$$G_{||} = \frac{1}{\mathbf{q}^2 + q_c^2}, \qquad G_\perp = \frac{1}{\mathbf{q}^2}$$

- Divergences cancel, but cancellations are lost if expansions are truncated
- Bose gases are now generally well described
  - MC simulations
    N. Prokof'ev *et al.*, PRA **66**, 043608 (2002)
    N. Prokof'ev *et al.*, PRA **69**, 053625 (2004)
    S. Pilati *et al.*, PRA **74**, 043621 (2006)
    G. E. Astrakharchik *et al.*, PRA **79** 051602 (2009)
  - RG approaches
    J.-P. Blaizot *et al.*, EPL **72**, 705 (2005)
    S. Pistolesi *et al.*, PRA **74**, 043621 (2006)
    S. Floerchinger *et al.*, PRA **77**, 053603 (2008)
    A. Rançon *et al.*, PRA **85**, 063607 (2012)
  - Beliaev technique
    B. Capogrosso-Sansone *et al.*, New J. Phys. **12**, 043010 (2010)

Generating functional

- Generating functional

$$Z[\mathbf{J}] = e^{W[\mathbf{J}]} = \int D\varphi e^{-\mathcal{S}[\varphi] + \int_x \mathbf{J}\cdot\varphi}$$

- The n-point correlation functions are given by

$$\langle \varphi(x_1) ... \varphi(x_n) \rangle = \frac{\delta^n W[\mathbf{J}]}{\delta \mathbf{J}(x_1) ... \delta \mathbf{J}(x_n)}$$

- Classical fields:

$$\phi(x) = \langle \varphi(x) \rangle = \frac{\delta W[\mathbf{J}]}{\delta \mathbf{J}(x)}$$

- To compute W we need to consider all the different paths

Effective action

- It is convenient to work in terms of an **effective action** Γ that contains the effect of fluctuations

$$\Gamma[\phi] = -W[\mathbf{J}] + \int_x \mathbf{J} \cdot \phi$$

- Γ is the generating functional of the 1PI correlation functions
- The equilibrium state

$$\left.\frac{\delta \Gamma[\phi]}{\delta \phi}\right|_{\phi_0} = 0$$

- The grand canonical potential: $\Omega_G = \frac{1}{\beta}\Gamma[\phi_0]$
- Different ways to compute Γ. For example, in a perturbative expansion

$$\Gamma[\phi] = \mathcal{S}[\phi] + \frac{1}{2}\,\mathrm{Str}\log\left(\mathcal{S}^{(2)}[\phi]\right) + \dots$$

Functional Renormalization Group (FRG)

- Within the FRG the effective action Γ is obtained by solving a RG equation
- A regulator function $\mathbf{R}_k$ is added to the theory which suppresses all fluctuations for momenta $q < k$

$$\Delta\mathcal{S}_k[\varphi] = \frac{1}{2}\int_q \varphi^\dagger(-q)\cdot\mathbf{R}_k(q)\cdot\varphi(q)$$

$$\mathcal{S}_k[\varphi] = \mathcal{S}[\varphi] + \Delta\mathcal{S}_k[\varphi]$$

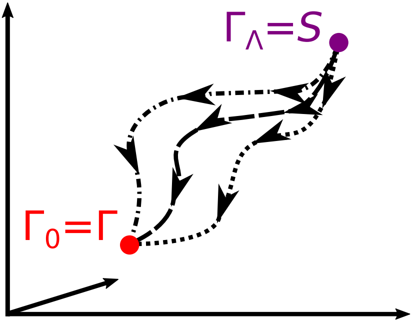
- The elements of $\mathbf{R}_k$ must satisfy

$$\begin{aligned} R_k(q) &\rightarrow \infty \qquad (k\rightarrow\infty),\\ R_k(q) &\approx k^2 \qquad (q\rightarrow 0),\\ R_k(q) &\rightarrow 0 \qquad (k\rightarrow 0). \end{aligned}$$

Functional Renormalization Group (FRG)

- Effective average action Γ_k (k-dependent)

$$\Gamma_k[\phi] = -W_k[\mathbf{J}] + \int_x \mathbf{J} \cdot \phi - \Delta \mathcal{S}_k[\phi]$$

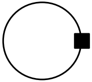


Flow equation (Wetterich equation)

$$\partial_k \Gamma_k = \frac{1}{2}\,\mathrm{Str}\left[\partial_k \mathbf{R}(\boldsymbol{\Gamma}_k^{(2)} - \mathbf{R}_k)^{-1}\right]$$

C. Wetterich, Phys. Lett. B **301**, 90 (1993)

$$\partial_k \Gamma_k = \frac{1}{2}$$



$$\boldsymbol{\Gamma}_k^{(2)} = \left(\frac{\delta \Gamma_k}{\delta \boldsymbol{\phi}}\right) \frac{\overleftarrow{\delta}}{\delta \boldsymbol{\phi}}$$

- This equation is exact. FRG is a non-perturbative framework
- FRG also known as non-perturbative RG and exact RG (ERG)
- Flow equation cannot be solved, so approximations are used: derivative expansion, vertex expansion, BMW approximation, etc

- Encouraging results for cold atom systems
  - Dilute Bose and Fermi gases — I. Boettcher *et al*, Nucl. Phys. B **228**, 63 (2012)
  - Bosons in optical lattices — A. Rançon and N. Dupuis, PRA **85**, 063607 (2012)
  - Efimov Physics — S. Moroz *et al.*, PRA **79**, 042705 (2009)
- Some applications in nuclear physics
  - Nuclear matter — M. Drews and W. Weise, Prog. Part. Nucl. Phys. **93**, 69 (2016)
  - Few-nucleon systems — M. Birse *et al.*, PRC **87**, 054001 (2013)
- Particle physics — H. Gies , Lect. Notes Phys. **852**, 287 (2012).
- Other applications in statistical mechanics (frustrated magnets, out of equilibrium systems, etc) — B. Delamotte *et al.*, PRB **69**, 134413 (2004); L. Canet *et al.*, PRL **95**, 100601 (2005)

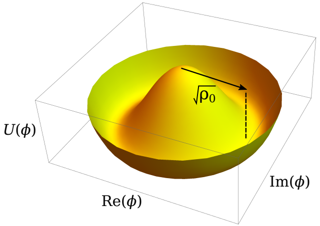
- Ansatz from a derivative expansion

$$\Gamma = \int_x \left[\phi^\dagger \left(S_\phi \partial_\tau - \frac{Z_\phi}{2m} \nabla^2 - V_\phi \partial_\tau^2 \right) \phi + U(\rho) \right]$$

$\rho = \phi^\dagger \phi$. U is the effective potential:

$$U = -p + u_\phi(\rho - \rho_0) + \frac{\lambda_\phi}{2}(\rho - \rho_0)^2 \qquad \rho_0 = \langle \rho \rangle$$

- S_ϕ, Z_ϕ, V_ϕ, p, u_ϕ, λ_ϕ and ρ_0 flow with k
- ρ_0 is the order parameter: Broken phase: $\rho_0 > 0, u_\phi = 0$
  Symmetric phase: $\rho_0 = 0, u_\phi > 0$



- Condensate density: $\rho_c = \rho_0$
- Superfluid stiffness: $\rho_s = Z_\phi \rho_0$
  (superfluid density in $d = 2, 3$)

Thermodynamics

$U(\rho_0)$ corresponds to the density of the grand canonical potential Ω_G

$$d\Omega_G = -PdV - SdT - Nd\mu$$

Thus

$$n = -\frac{\partial U}{\partial \mu}\bigg|_{\rho_0}, \qquad s = -\frac{\partial U}{\partial T}\bigg|_{\rho_0},$$

are the k-dependent boson density and entropy density, respectively.

We can easily extract the energy per particle of the gas through

$$E/N = -P/n + \mu + s\,T/n$$

Initial conditions

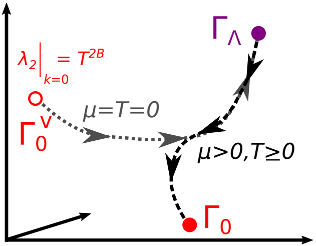
$$\rho_0(\Lambda) = \frac{\mu}{\lambda_\phi(\Lambda)}\Theta(\mu), \quad u_\phi(\Lambda) = -\mu\Theta(-\mu),$$

$$Z_\phi(\Lambda) = S_\phi(\Lambda) = 1, \quad V_\phi(\Lambda) = 0.$$

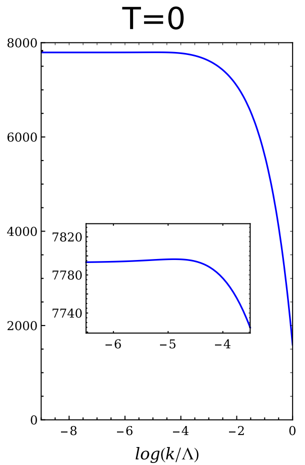
- λ_ϕ needs to be renormalized
- It is imposed that in vacuum

$$\lambda_\phi(k=0) = T^{2B}$$

- The only physical inputs are μ, T and a_s

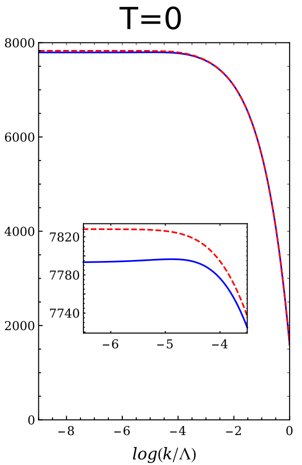


Three dimensions



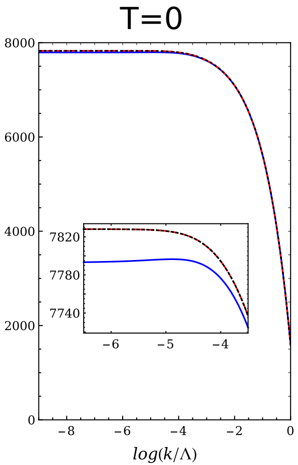
ρ_c: Condensate density

Three dimensions



ρ_c: Condensate density
ρ_s: Superfluid density

Three dimensions

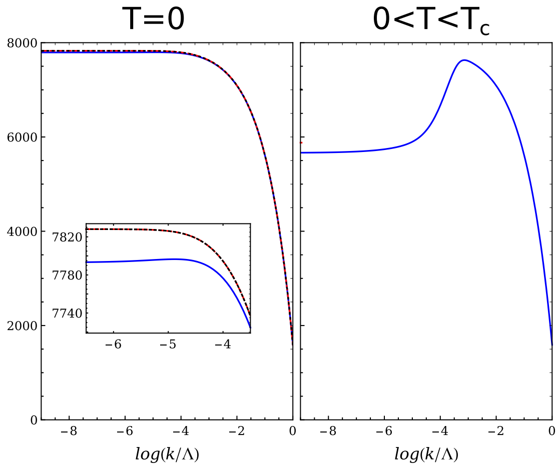


ρ_c: Condensate density

ρ_s: Superfluid density

n : Boson density

Three dimensions



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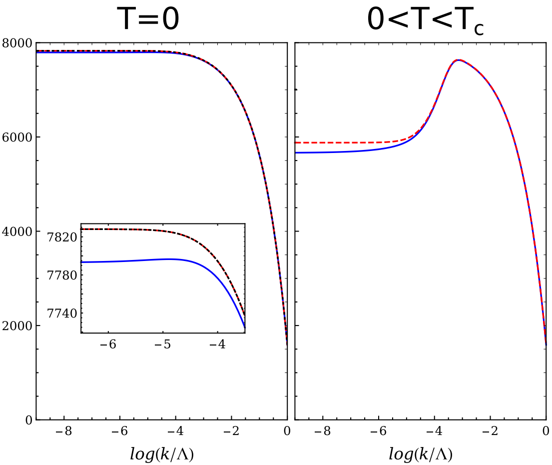

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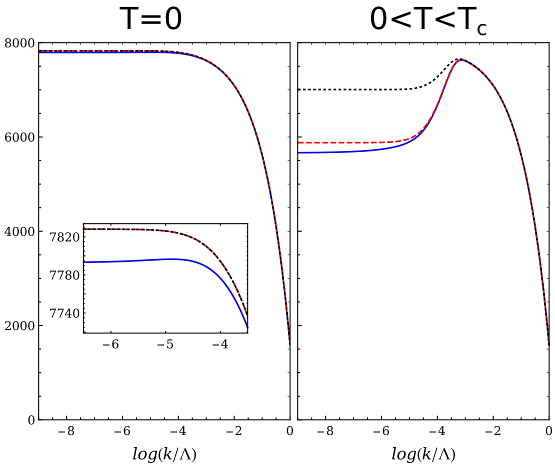

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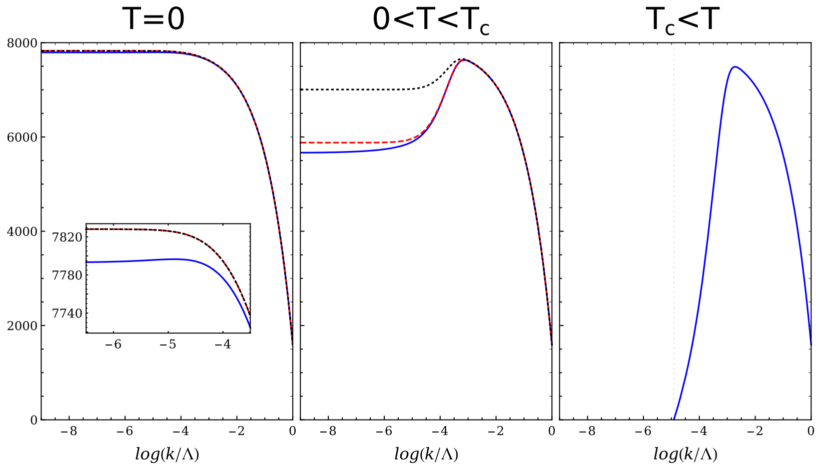

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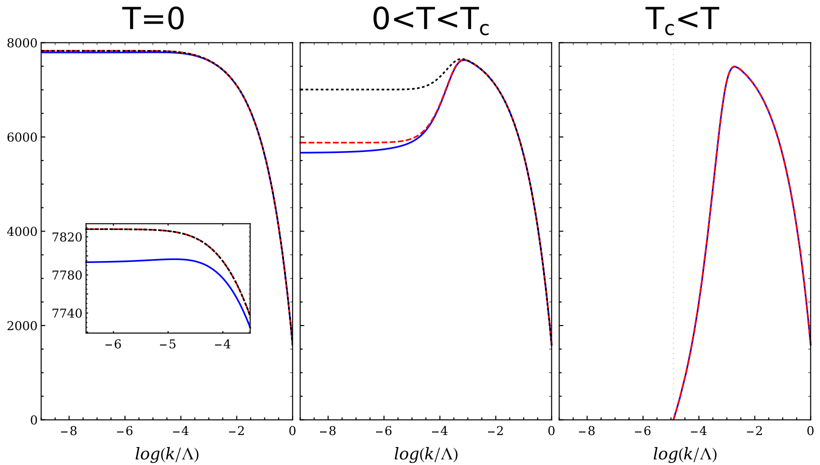

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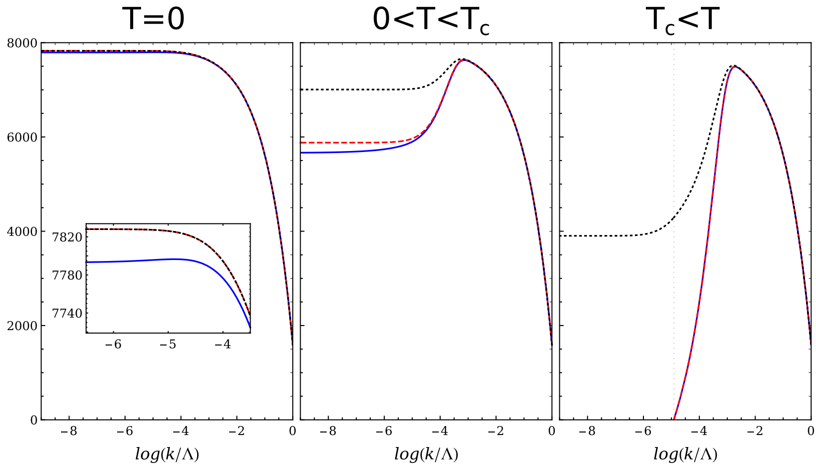
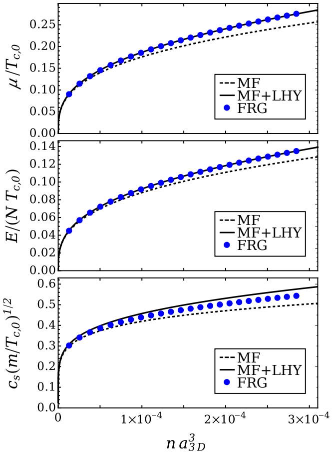
Three dimensions



ρ_c: Condensate density
ρ_s: Superfluid density
n : Boson density

Three dimensions ($T = 0$)

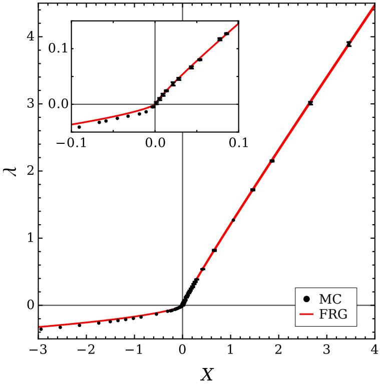


$$T_{c,0} = \frac{2\pi}{m}\left(\frac{n}{\xi(3/2)}\right)$$

LYH: T.D. Lee *et al.*, PR **105**, 1119 (1957)
T.D. Lee *et al.*, PR **106**, 1135 (1957)

Three dimensions ($T > 0$)

$$\lambda = \frac{n - n_c}{m^3 T^2 \lambda_\phi^2(\Lambda)}$$

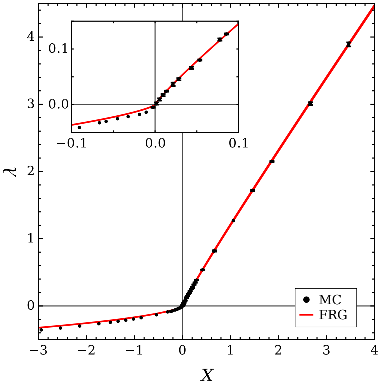


$$X = \frac{\mu - \mu_c}{m^3 T^2 \lambda_\phi^2(\Lambda)}$$

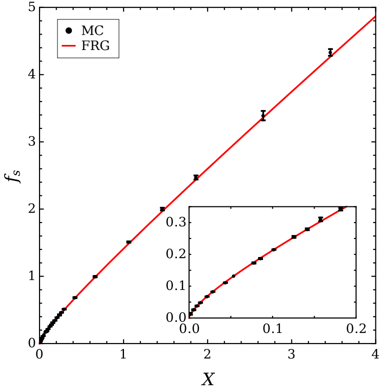
MC: N. Prokof'ev *et al.*, PRA **69**, 053625 (2004)

Three dimensions ($T > 0$)

$$\lambda = \frac{n - n_c}{m^3 T^2 \lambda_\phi^2(\Lambda)}$$



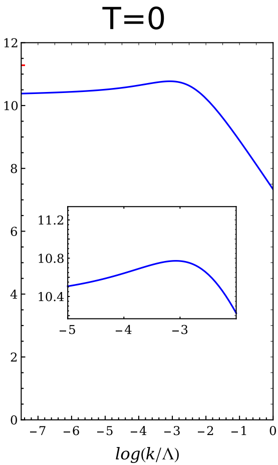
$$f_s = \frac{\rho_s}{m^3 T^2 \lambda_\phi(\Lambda)}$$



$$X = \frac{\mu - \mu_c}{m^3 T^2 \lambda_\phi^2(\Lambda)}$$

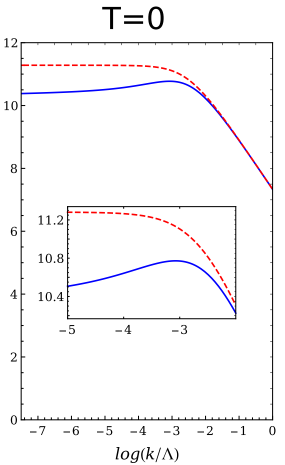
MC: N. Prokof'ev *et al.*, PRA **69**, 053625 (2004)

Two dimensions



ρ_c: Condensate density

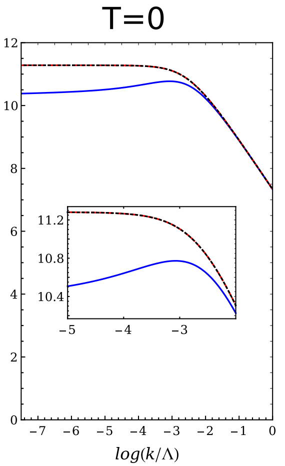
Two dimensions



ρ_c: Condensate density

ρ_s: Superfluid density

Two dimensions

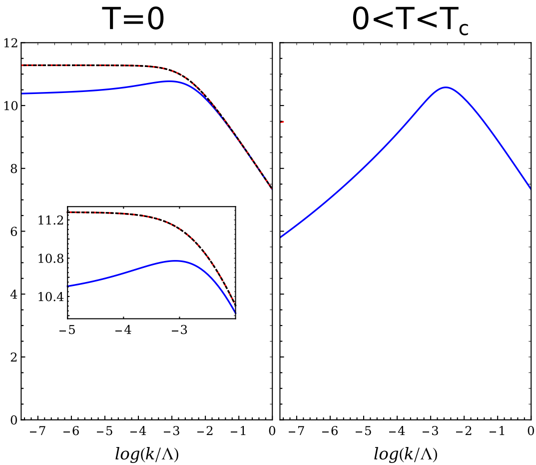


ρ_c: Condensate density

ρ_s: Superfluid density

n : Boson density

Two dimensions



ρ_c: Condensate density
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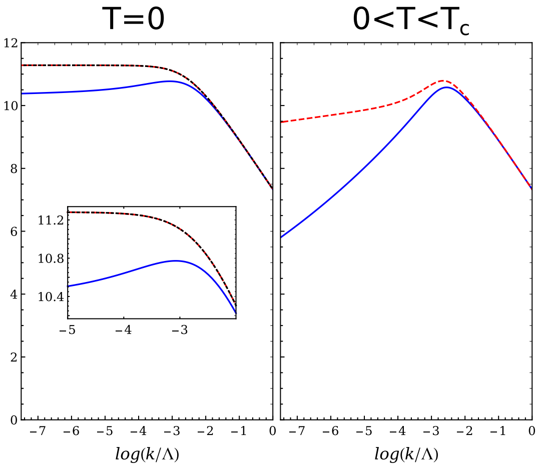
Two dimensions



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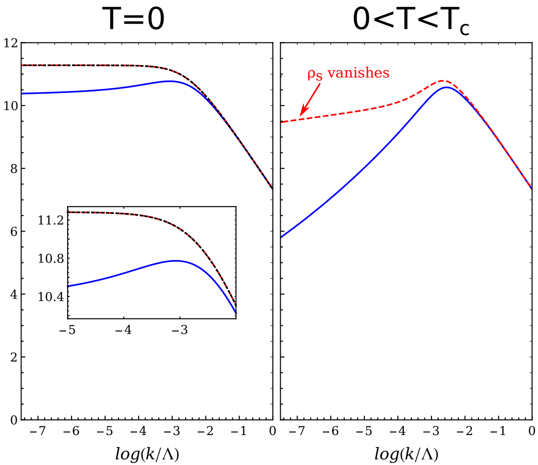
Two dimensions



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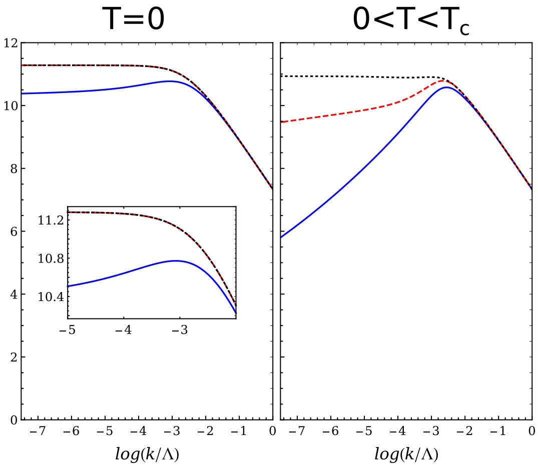
Two dimensions



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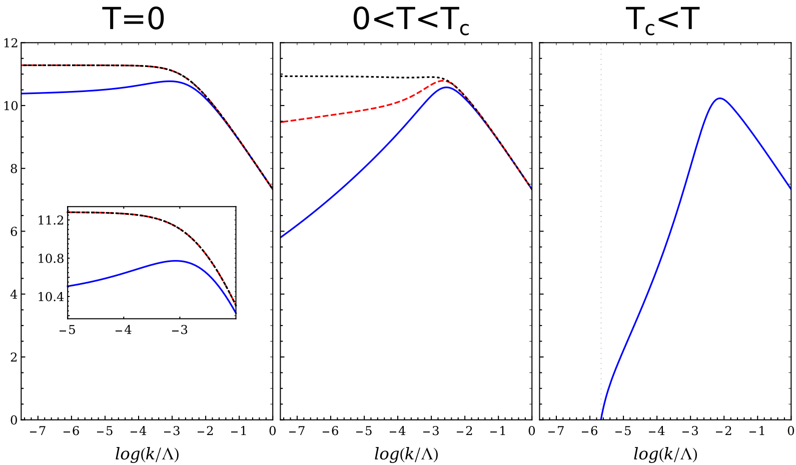

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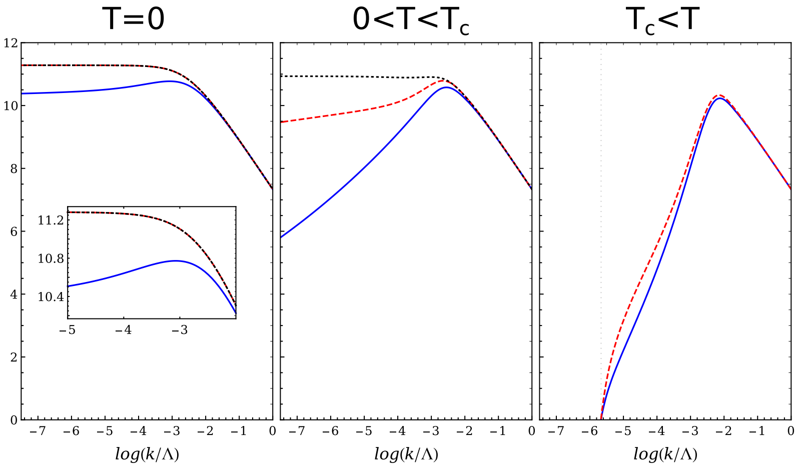
Two dimensions



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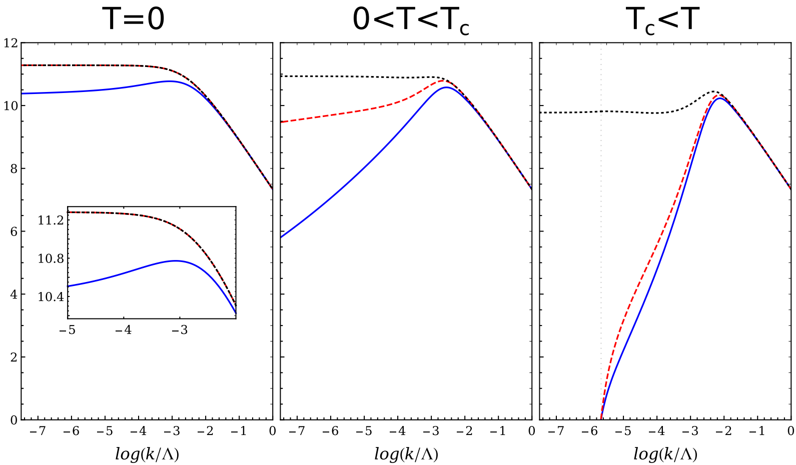
Two dimensions



ρ_c: Condensate density

ρ_s: Superfluid density

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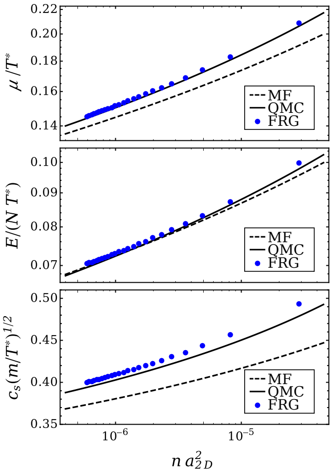
Two dimensions ($T = 0$)



$T^* = 2\pi n/m$

MC: G.E. Astrakharchik *et al.*, PRA **79** 051602 (2009)

Two dimensions ($T > 0$)

- Superfluid phase not recovered. Correlation length is large but not infinite
- Improvements can be achieved with better truncations or by fine-tuning the regulator

  P. Jakubczyk *et al.*, PRE **90**, 062105 (2014)
- Signatures of BKT physics recovered from a line of quasifixed points

Field representations

- Perturbation theory suffer IR divergences if we use a Cartesian representation for the fields

$$\phi(\mathbf{x}) = \sqrt{\rho_0} + \sigma(\mathbf{x}) + i\pi(\mathbf{x})$$

ρ_0: minimum of the action

- These can be avoided by using a convenient field representation
- Similar divergences arise when using a linear sigma model to describe broken chiral symmetry
- These can be solved by using a non-linear sigma model as in chiral-perturbation theory

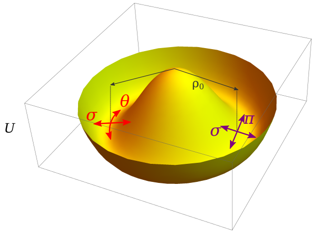
- Bose gases in the IR can be described by the hydrodynamic effective theory introduced by Popov

Popov, *Functional Integrals and Collective Excitations* (1987)

- Popov introduced an Amplitude-Phase (AP) representation:

$$\phi(\mathbf{x}) = (\sqrt{\rho_0} + \sigma(\mathbf{x}))e^{i\theta(\mathbf{x})}$$

- Cartesian representation should be used in the UV, whereas AP representation in the IR



- Hydrodynamic theory is useful to study low-dimensional gases

U. Al Khawaja *et al.*, PRA **66**, 013615 (2002)
C. Mora *et al.*, PRA **67**, 053615 (2003)

Long-distance behaviour of correlation function:

$$\lim_{|\mathbf{x}|\to\infty} G_n(\mathbf{x}) = \begin{cases} \rho_0 & : \textbf{(Cart.)} \qquad \rho_c = \rho_0, \\ \rho_0 e^{\langle(\theta(\mathbf{x})-\theta(0))^2\rangle} & : \textbf{(AP)} \qquad \rho_q = \rho_0, \end{cases}$$

- With AP representation long-distance behavior driven by phase correlations
- ρ_q is the quasi-condensate density
  Al Khawaja *et al*, PRA **66**, 013615 (2002)
- In systems with QLRO $\rho_c = 0$ but $\rho_q > 0$ (quasi-condensate)
  Yu Kagan *et al*, PRA **61**, 043608 (2000)
- Can the AP representation be used within the FRG?
  N. Defenu *et al*, PRB **96**, 174505 (2017)

Interpolating representation

- Following Popov's ideas we use k-dependent fields:

$$\phi = (\sigma + b_k)e^{i\vartheta/b_k} - (b_k - \sqrt{\rho_0}), \quad b_k \in [\sqrt{\rho_0}, \infty)$$

Lamprecht, Diploma thesis, Ruprecht-Karls-Universität Heidelberg (2007)

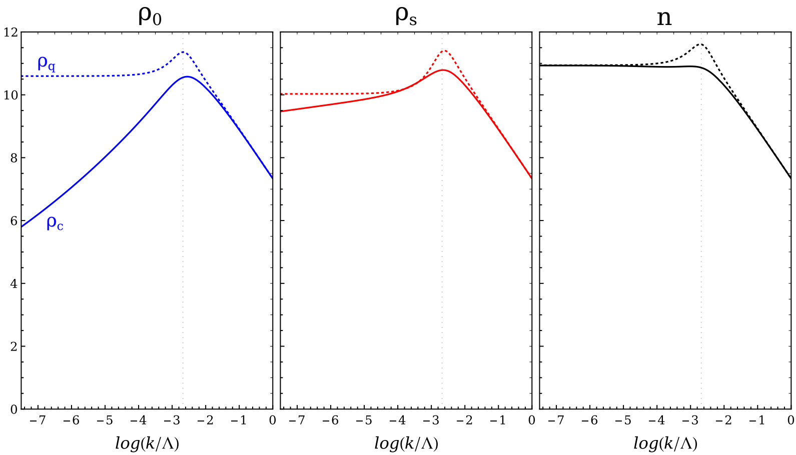
- In the limits ϕ take the forms:

$$\phi = \begin{cases} (\sqrt{\rho_0} + \sigma) + i\vartheta & : b_k \to \infty \quad \textbf{(Cartesian)}, \\ (\sqrt{\rho_0} + \sigma)e^{i\vartheta/\sqrt{\rho_0}} & : b_k = \sqrt{\rho_0} \quad \textbf{(AP)}. \end{cases}$$

- UV and IR regimes are characterized by $w_k = \frac{Z_\phi k^2/2m}{2\lambda_\phi \rho_0}$
- Transition should be made around $w_{k_h} \approx 1$

B. Capogrosso-Sansone *et al*, New J. Phys. **12**, 043010 (2010)

Two dimensions ($0 < T < T_c$)



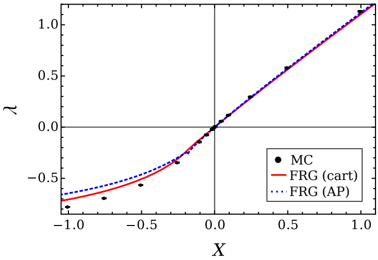
Solid: Cartesian

Dashed: Interpolating

$\rho_s = Z_\phi \rho_0$ is still the superfluid density

Two dimensions ($T > 0$)

$$\lambda = \frac{n - n_c}{mT\lambda_\phi(\Lambda)}$$



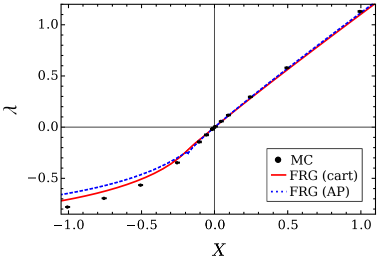
$$X = \frac{\mu - \mu_c}{mT\lambda_\phi(\Lambda)}$$

MC: N. Prokof'ev *et al.*, PRA **66**, 043608 (2002)

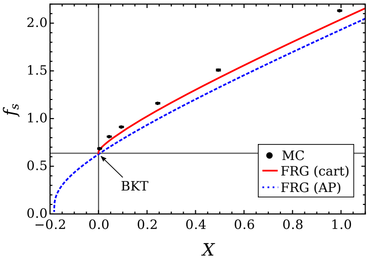
$$\rho_s(X = 0) = 2mT/\pi$$

Two dimensions ($T > 0$)

$$\lambda = \frac{n-n_c}{mT\lambda_\phi(\Lambda)}$$



$$f_s = \frac{\rho_s}{mT}$$

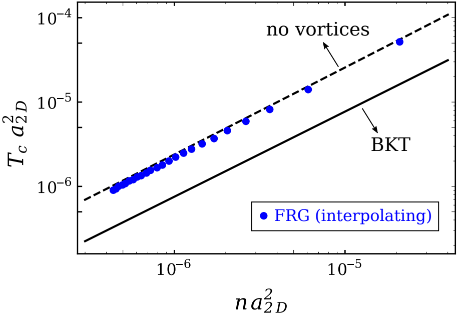


$$X = \frac{\mu-\mu_c}{mT\lambda_\phi(\Lambda)}$$

MC: N. Prokof'ev *et al.*, PRA **66**, 043608 (2002)

$$\rho_s(X=0) = 2mT/\pi$$

Critical temperature



No vortex effects

$$T_c = \frac{2\pi n}{m} \frac{1}{\log(\log(1/na_{2D}^2))}$$

Fisher *et al.*, PRB **37**, 4936 (1988)

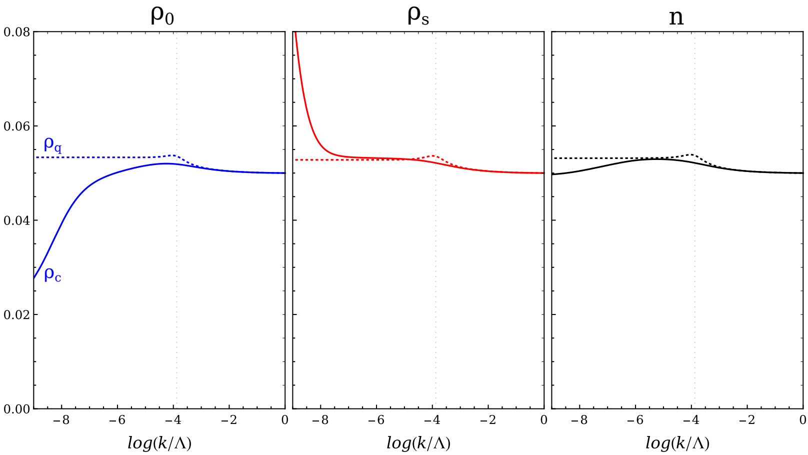
$\longrightarrow$

Including vortex effects

$$T_c = \frac{2\pi n}{m} \frac{1}{\log(\xi/4\pi) + \log(\log(1/na_{2D}^2))}$$

Prokof'ev *et al.* PRL **87**, 270402 (2001)

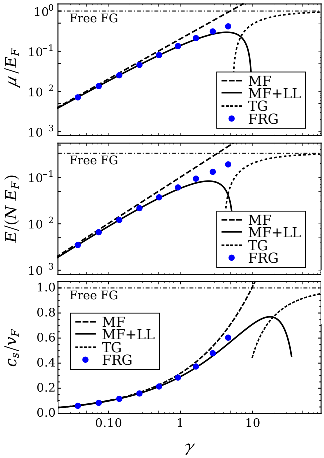
One dimension ($T = 0$)



Solid: Cartesian

Dashed: Interpolating

One dimension ($T = 0$)



$$\gamma = -\frac{2}{na_{1D}}$$

LL: E. H. Lieb and W. Liniger, PR **130**, 1605 (1963)

Fermi gases and the BCS-BEC crossover

- Dilute two-component Fermi gases interacting through attractive short-range interactions

$$\mathcal{S} = \int_x \left[\psi_s^\dagger \left(-\partial_\tau + \frac{\nabla^2}{2m} + \mu \right) \psi_s + \mathfrak{g} \psi_1^\dagger \psi_2^\dagger \psi_2 \psi_1 \right]$$

ψ_s: fermionic *atom* fields

- We introduce *dimer* fields by means of a Hubbard-Stratonovich transformation

$$\mathcal{S} = \int_x \left[\psi_s^\dagger \left(-\partial_\tau + \frac{\nabla^2}{2m} + \mu \right) \psi_s - u_d \phi^\dagger \phi - g \left(\phi^\dagger \psi_1 \psi_2 + \phi \psi_2^\dagger \psi_1^\dagger \right) \right]$$

ϕ: bosonic *dimer* fields $\sim \psi_1 \psi_2$

- They can be studied with cold-atom experiments

C. A. Regal *et al.*, PRL **92**, 040403, (2004). C. Chin, Science **305**, 1128 (2004)

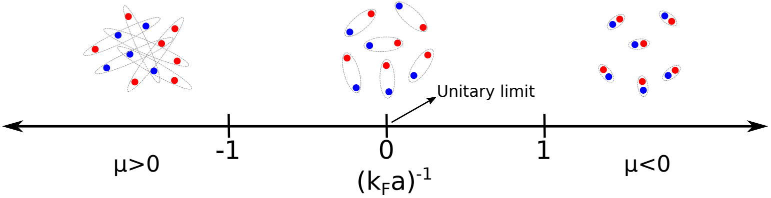
BCS

Condensation of Cooper pairs
Large pair size
Weak coupling

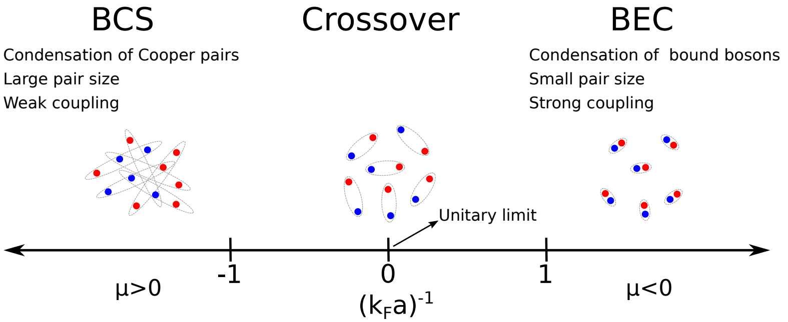
Crossover

BEC

Condensation of bound bosons
Small pair size
Strong coupling



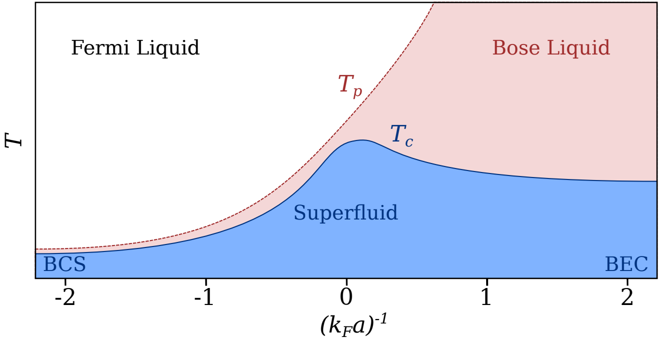
$$k_F = (3\pi^2 n)^{1/3}$$



$$k_F = (3\pi^2 n)^{1/3}$$

- BCS-BEC crossover regime is strongly-interacting
- Relevant in nuclear physics:
  low-density neutron matter, neutron-rich nuclei.

  N. Zinner and A. Jensen, J. Phys. G: Nucl. Part. Phys. **40**, 053101 (2013)



- In the BEC side pairs of fermions can still exist for $T > T_c$
- System can show a *pseudogap*

- BCS-BEC crossover also present in one and two dimensions: High-temperature superconductors, nuclear pastas, etc
- In two dimensions there is no unitary limit and the crossover is characterized by $-\log(k_F a_{2D})$

$$k_F = (2\pi n)^{1/2}$$

- BCS-BEC crossover has been studied with Monte-Carlo simulations, DSE, ϵ-expansion, RG approaches, etc

W. Zwerger, *The BCS-BEC Crossover and the Unitary Fermi Gas* (2012)
M. Randeria and E. Taylor, Annu. Rev. Condens. Matter **5**, 209 (2014)

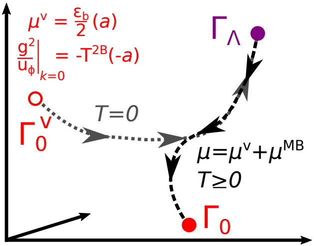
We use the following ansatz based on a derivative expansion:

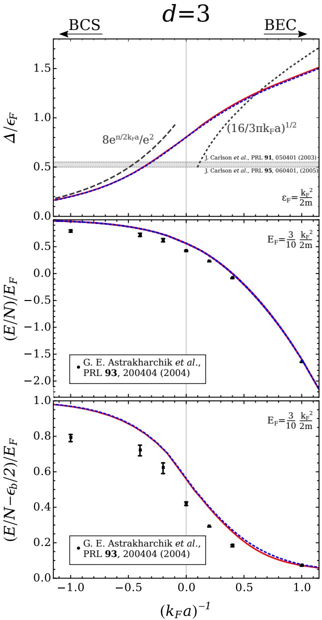
$$\Gamma = \int_x \left[\psi_s^\dagger \left(\partial_\tau - \frac{1}{2m}\nabla^2 + \mu \right) \psi_s + \phi^\dagger \left(S_\phi \partial_\tau - \frac{Z_\phi}{4m}\nabla^2 \right) \phi + g\left(\phi^\dagger \psi_1 \psi_2 + \phi \psi_2^\dagger \psi_1^\dagger \right) + U(\rho,\mu) \right]$$

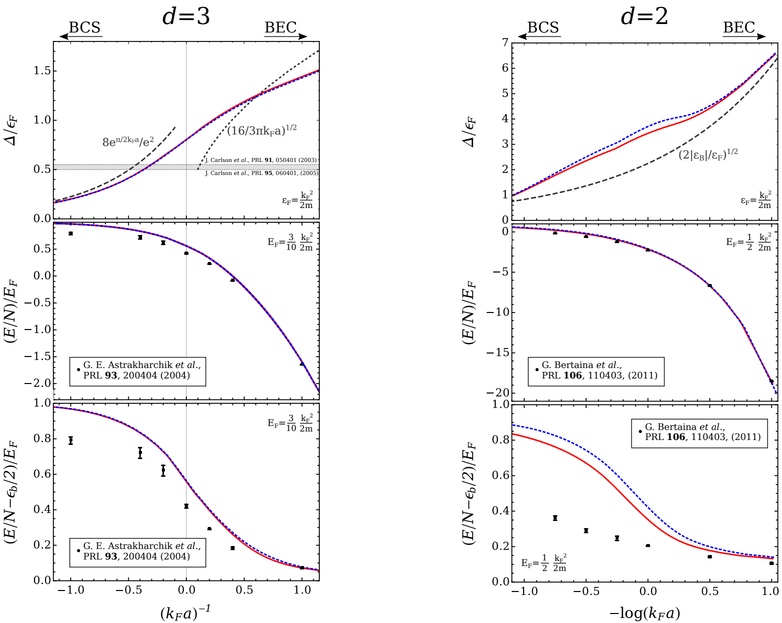
where $\rho = \phi^\dagger \phi$ and $U = u_0 + u_1(\rho - \rho_0) + \frac{u_2}{2}(\rho - \rho_0)^2$. $\rho_0 = \langle \rho \rangle$

- $\Delta = g\rho_0^{1/2}$ is the pairing gap
- $d = 3$: $\epsilon_b = \frac{-1}{m a_{3D}^2}\Theta(a_{3D})$, $T^{2B} = \frac{4\pi a_{3D}}{m}\Theta(-a_{3D})$
- $d = 2$: $\epsilon_b = \frac{-4}{m e^{2\gamma_E} a_{2D}^2}$
- AP representation for dimers fields

  L. Salasnich et al., PRA **88**, 053612 (2013)
- Preliminary study with interpolating representation




d=3
BCS
BEC
Δ/ϵ_F
$8e^{\pi/2k_Fa}/e^2$
$(16/3\pi k_F a)^{1/2}$
J. Carlson et al., PRL 91, 050401 (2003)
J. Carlson et al., PRL 95, 060401, (2005)
$\varepsilon_F=\frac{k_F^2}{2m}$
$(E/N)/E_F$
$E_F=\frac{3}{10}\frac{k_F^2}{2m}$
G. E. Astrakharchik et al., PRL 93, 200404 (2004)
$(E/N-\epsilon_b/2)/E_F$
$E_F=\frac{3}{10}\frac{k_F^2}{2m}$
G. E. Astrakharchik et al., PRL 93, 200404 (2004)
$(k_Fa)^{-1}$


d=3
BCS
BEC
Δ/ε_F
8e^{π/2k_F a}/e^2
(16/3πk_F a)^{1/2}
J. Carlson et al., PRL 91, 050401 (2003)
J. Carlson et al., PRL 95, 060401, (2005)
ε_F = k_F^2/2m
(E/N)/E_F
E_F = 3/10 k_F^2/2m
G. E. Astrakharchik et al., PRL 93, 200404 (2004)
(E/N−ε_b/2)/E_F
(k_F a)^{-1}
d=2
(2|ε_B|/ε_F)^{1/2}
E_F = 1/2 k_F^2/2m
G. Bertaina et al., PRL 106, 110403, (2011)
−log(k_F a)

Conclusions

- The FRG is a powerful yet simple non-perturbative formalism to study quantum gases
- It allows us to give a consistent and unified description of few- and many-body systems in different dimensions
- The amplitude-phase representation stabilizes the flows in systems with QLRO, but current implementation misses important physics

  More details: F. Isaule, M. Birse and N. Walet, PRB **98**, 144502 (2018)
  F. Isaule, M. Birse and N. Walet, Ann. Phys. **412**, 168006 (2020)

- Quantitative accuracy in Fermi gases can be achieved by including missing physics

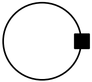
  M. Scherer *et al.*, Phil. Trans. R. Soc. A **369**, 2779 (2011)
  I. Boettcher *et al.*, PRA **89**, 053630 (2014)
  B. Faigle-Cedzich *et al.*, arXiv:1910.07365 (2019)

- Future work: Bose-Bose mixtures, few-body mixture, optical lattices, low-density nuclear matter

Flow equation (Wetterich equation)

$$\partial_k \Gamma_k = \frac{1}{2} \mathrm{Str}\left[\partial_k \mathbf{R} (\boldsymbol{\Gamma}_k^{(2)} - \mathbf{R}_k)^{-1}\right]$$

C. Wetterich, Phys. Lett. B **301**, 90 (1993)

$$\partial_k \Gamma_k = \frac{1}{2}$$ 

$$\boldsymbol{\Gamma}_k^{(2)} = \left(\frac{\delta \Gamma_k}{\delta \phi}\right) \frac{\overleftarrow{\delta}}{\delta \phi}$$

- We work in Fourier space ($q = (\omega_n, \mathbf{q})$)

$$\int_q = T \sum_{n=-\infty}^{n=\infty} \frac{1}{(2\pi)^d} \int \mathrm{d}^d \mathbf{q} \quad \xrightarrow{T \to 0} \quad \frac{1}{2\pi i} \int \mathrm{d}q_0 \int \frac{1}{(2\pi)^d} \mathrm{d}^d \mathbf{q}$$

$$\omega_n = \begin{cases} 2\pi n T & : \text{ bosons} \\ 2\pi (n + 1/2) T & : \text{ fermions} \end{cases}$$

Flow equations

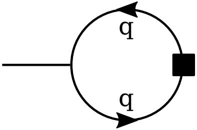
- We use $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$, with $\langle\phi_1\rangle = \sqrt{2\rho_0}$, $\langle\phi_2\rangle = 0$

$$
\begin{aligned}
\dot{u}_0 - u_\phi \dot{\rho}_0 =& \dot{\Gamma}\Big|_{\rho_0}, & \frac{\dot{Z}_\phi}{2m} =& \partial^2_{\mathbf{p}} \left(\dot{\Gamma}^{(2)}_{\phi_2\phi_2}\right)\Big|_{\rho_0, p=0}, \\
\sqrt{2\rho_0}\,(\dot{u}_\phi - \lambda_\phi \dot{\rho}_0) =& \dot{\Gamma}^{(1)}_{\phi_1}\Big|_{\rho_0}, & \dot{S}_\phi =& \partial_{\nu_n} \left(\dot{\Gamma}^{(2)}_{\phi_1\phi_2}\right)\Big|_{\rho_0, p=0}, \\
\dot{u}_\phi + 2\rho_0\dot{\lambda} - \lambda\dot{\rho}_0 =& \dot{\Gamma}^{(2)}_{\phi_1\phi_1}\Big|_{\rho_0}, & \dot{V}_\phi =& \frac{1}{2}\partial^2_{\nu_n} \left(\dot{\Gamma}^{(2)}_{\phi_2\phi_2}\right)\Big|_{\rho_0, p=0}.
\end{aligned}
$$

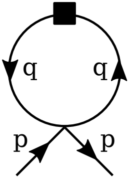
External momentum: $p = (\nu_n, \mathbf{p})$

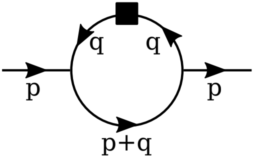
- $\dot{\Gamma} = \frac{1}{2}\,\mathrm{Str}\left[\dot{\mathbf{R}}(q)\,\mathbf{G}(q)\right]$ $\qquad\qquad \mathbf{G}(q) = (\mathbf{\Gamma}^{(2)} - \mathbf{R})^{-1}$

- $\dot{\Gamma}^{(1)}_a = -\frac{1}{2}\,\mathrm{Str}\left[\dot{\mathbf{R}}(q)\,\mathbf{G}(q)\,\mathbf{\Gamma}^{(3)}_a(0,q,-q)\,\mathbf{G}(q)\right]$



- $$\dot{\Gamma}^{(2)}_{ab}(p) = -\frac{1}{2}\,\mathrm{Str}\left[\dot{\mathbf{R}}(q)\,\mathbf{G}(q)\,\mathbf{\Gamma}^{(4)}_{ab}(p,-p,q,-q)\,\mathbf{G}(q)\right] + \mathrm{Str}\left[\dot{\mathbf{R}}(q)\,\mathbf{G}(q)\,\mathbf{\Gamma}^{(3)}_a(p,q,-p-q)\,\mathbf{G}(p+q) \times \mathbf{\Gamma}^{(3)}_a(-p,p+q,-q)\,\mathbf{G}(q)\right]$$



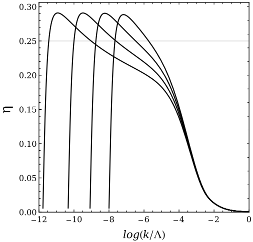


Two dimensions ($T > 0$)

- Superfluid phase not recovered. Correlation length is large but not infinite
- Expected behavior: $\rho_0 \sim k^\eta$, $Z_\phi \sim k^{-\eta}$ $\qquad\qquad \eta > 0$
- Improvements can be achieved with better truncations or by fine-tuning the regulator

  P. Jakubczyk *et al.*, PRE **90**, 062105 (2014)

- Signatures of BKT physics recovered from a line of quasifixed points



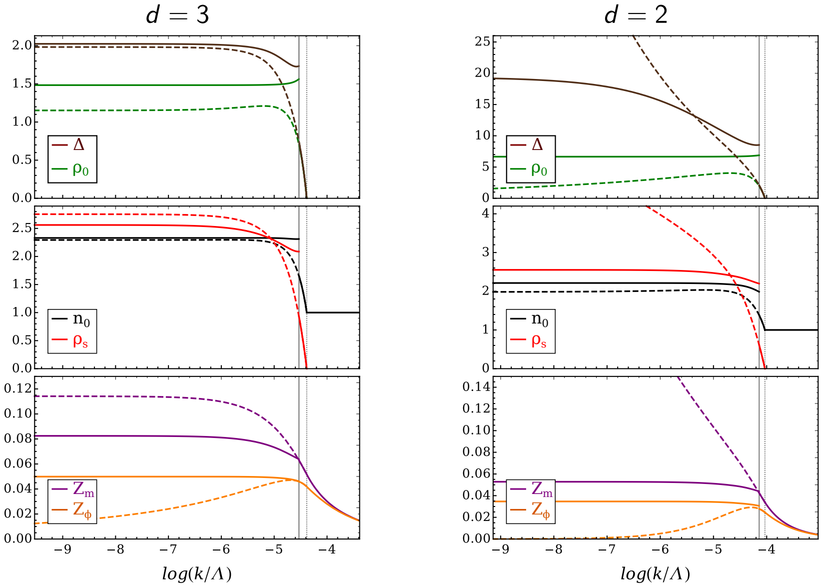
$$\eta = -k\partial_k \log Z_\phi$$

Interpolating representation

Flow equation for k-dependent fields:

$$\partial_k \Gamma + \dot{\boldsymbol{\phi}} \cdot \frac{\delta \Gamma}{\delta \boldsymbol{\phi}} = \frac{1}{2} \operatorname{tr} \left[\dot{\mathbf{R}} (\boldsymbol{\Gamma}^{(2)} - \mathbf{R})^{-1} \right] + \operatorname{tr} \left[\dot{\boldsymbol{\phi}}^{(1)} \mathbf{R} (\boldsymbol{\Gamma}^{(2)} - \mathbf{R})^{-1} \right]$$

Flows Fermi gas $T = 0$



Dashed: Cartesian representation, **Solid:** Interpolating representation