Functional renormalization for Bose-Bose mixtures

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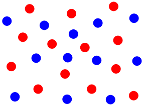
Bose-Bose mixtures

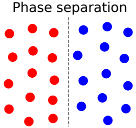


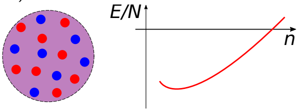
- Gases with two species of bosons have attracted attention in recent years
- Interplay between the two components leads to rich physics:
  - Superfluid drag

    A. F. Andreev and E. P. Bashkin, Sov. Phys. JETP **42**,164 (1975)
  - Phase separation (repulsive inter-species interaction)

    P. Ao and S. T. Chui, PRA **58**, 4836 (1998)
  - Droplet phase (attractive inter-species interaction)

    D. Petrov, PRL **115**, 155302 (2015)





Bose-Bose mixtures

- Bose-Bose mixtures have been achieved experimentally with cold atom gases

  T.-L. Ho and V. B. Shenoy, PRL **77**, 3276 (1996)

- There is currently a great effort on producing droplet phases

  L. Chomaz *et al.*, PRX **6**, 041039 (2016). C. R. Cabrera *et al.*, Science **359**, 301 (2018)

- Theoretically, Bose-Bose mixtures have mostly been studied with perturbative approaches

  D. Larsen, Ann. Phys. **24**, 89 (1963)

  P. Konietin and V. Pastukhov, J. Low Temp. Phys. **190**, 256 (2018). E. Chiquillo, PRA **97**,063605 (2018)

- Building upon FRG works on one-component gases, we study weakly-repulsive Bose-Bose mixtures at zero temperature

  S. Floerchinger and C. Wetterich, PRA **77**, 053603 (2008)

  S. Floerchinger and C. Wetterich, PRA **79**, 063602 (2008)

  A. Rançon and N. Dupuis, PRA **85**, 063607 (2012)

Effective action

We propose the following ansatz

$$\Gamma_k[\varphi] = \int_x \left[\sum_{a=A,B} \psi_a^\dagger \left(S\partial_\tau - \frac{Z}{2m}\nabla^2 - V\partial_\tau^2 \right) \psi_a + U(\rho_A, \rho_B) \right]$$

where $\rho_a = \psi_a^\dagger \psi_a$ and

$$U = \sum_{a,b=A,B} \frac{\lambda_{ab}}{2} (\rho_a - \rho_0)(\rho_b - \rho_0)$$

- We study the balanced mixture $m_A = m_B = m$, $\mu_A = \mu_B = \mu$, $\lambda_{AA} = \lambda_{BB} = \lambda$
- We study the two- and three-dimensional gas
- Physical inputs: a, a_{AB} and μ

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Wetterich equation

$$\partial_k \Gamma_k = \frac{1}{2} \operatorname{tr} \left[\partial_k \mathrm{R} (\mathbf{\Gamma}_k^{(2)} - \mathrm{R})^{-1} \right]$$

Wetterich, Phys. Lett. B (1993)

We use the Litim regulator

$$R_k(\mathrm{q}) = \frac{Z}{2m} (k^2 - \mathrm{q}^2) \Theta(k^2 - \mathrm{q}^2)$$

Effective action

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where $\rho_a = \psi_a^\dagger \psi_a$ and

$$U = \sum_{a,b=A,B} \frac{\lambda_{ab}}{2} (\rho_a - \rho_0)(\rho_b - \rho_0)$$

- We do not include the effect of the relative phase between the two condensates: no superfluid drag
- Superfluid drag not too important in two and three dimensions at zero temperature

Density and spin modes

The determinant of G_k^{-1}

$$\det(\mathrm{G}_k^{-1}) = \Big(S^2\omega^2 + (V\omega^2 + E_{1,k}^{(+)})(V\omega^2 + E_{2,k})\Big) \times \Big(S^2\omega^2 + (V\omega^2 + E_{1,k}^{(-)})(V\omega^2 + E_{2,k})\Big),$$

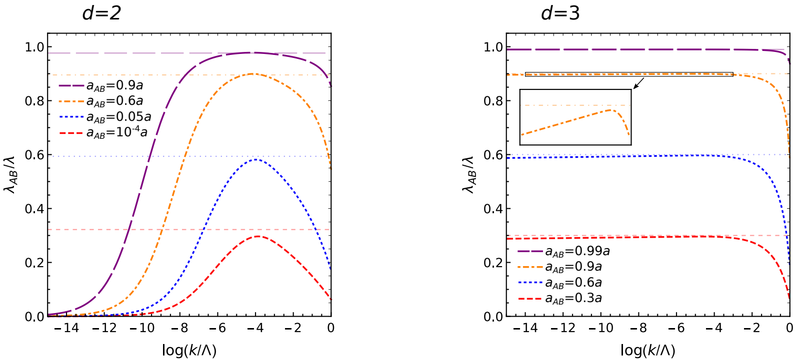
where $E_{1,k}^{(\pm)}(\mathrm{q}) = Z\frac{\mathrm{q}^2}{2m} + 2(\lambda \pm \lambda_{AB})\rho_0 + R_k(\mathrm{q})$, $E_{2,k}(\mathrm{q}) = Z\frac{\mathrm{q}^2}{2m} + R_k(\mathrm{q})$

- Positive and negative branches correspond to density (in-phase) and spin (out-of-phase) modes
- We can define characteristic scales from the dimensionless parameter

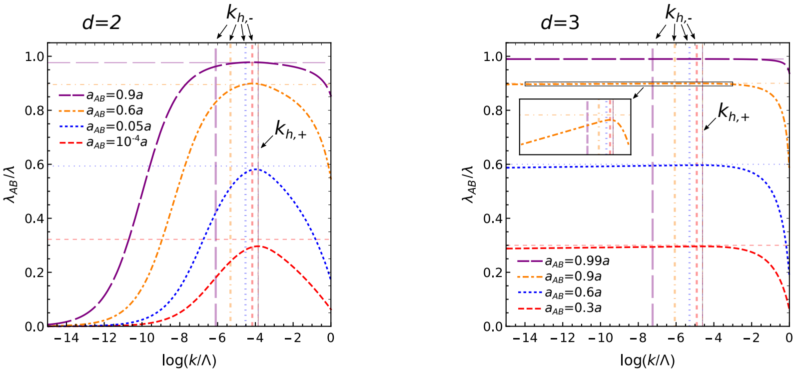
$$\omega_{h,\pm} = \frac{Z^2k^2/2m}{2(\lambda \pm \lambda_{AB})\rho_0}$$

The scales where $\omega_{h,\pm} = 1$ define the density and spin healing scales $k_{h,\pm}$.

Flow of λ_{AB}/λ



Flow of λ_{AB}/λ

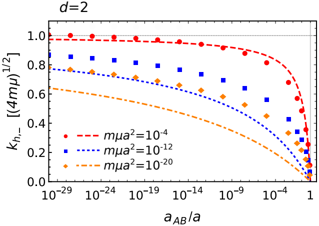


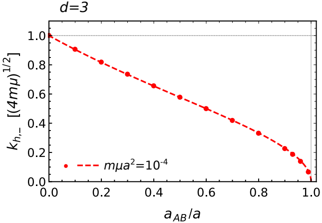
- $k_{h,+}$ remains constant, whereas $k_{k,-}$ decreases as a_{AB} increases
- Condition for phase separation: $\lambda_{AB} = \lambda$

  A. K. Kolezhuk, PRA **81**, 013601(2010)

- At phase separation point $\omega_{h,-} = \frac{Z^2 k^2/2m}{2(\lambda-\lambda_{AB})\rho_0} \to \infty$: $k_{h,-} \to 0$

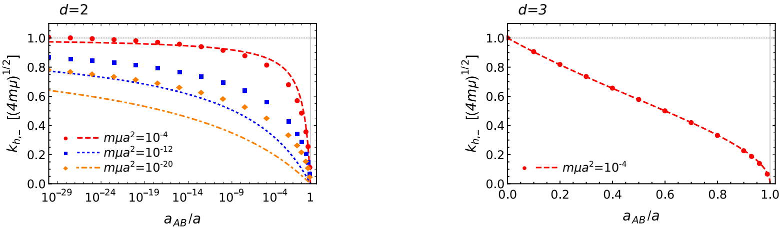
Healing scale and phase separation





Markers: FRG
Lines: Bogoliubov healing scales

Healing scale and phase separation



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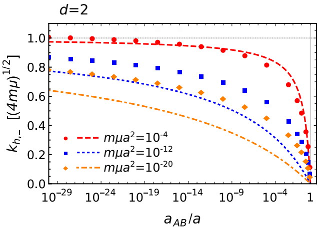
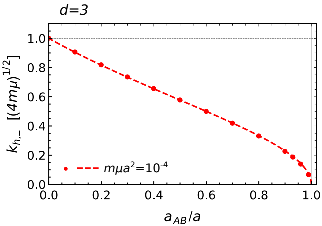
- Within the range of parameters explored, we find that the point of phase separation is $a_{AB} = a$ (MF result)

Healing scale and phase separation





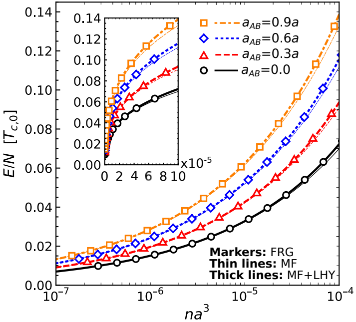
Markers: FRG
Lines: Bogoliubov healing scales

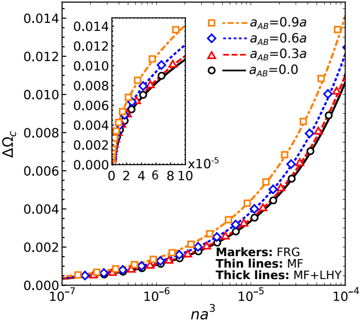
- Within the range of parameters explored, we find that the point of phase separation is $a_{AB} = a$ (MF result)
- RG studies suggest that the phase separation point in two dimensions occurs for $a_{AB} < a$ at logarithmically small densities

  A. K. Kolezhuk, PRA **81**, 013601(2010)
- A study of the mixture around the quantum critical point is required

  One-component FRG study: C. Wetterich, PRB **77**, 064504 (2008)

E/N and condensate depletion Ω_c $(d = 3)$





$T_{c,0} = \frac{2\pi}{m}(n/2\zeta(3/2))^{2/3}$

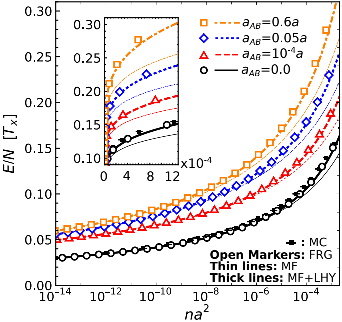
Perturbative expressions:

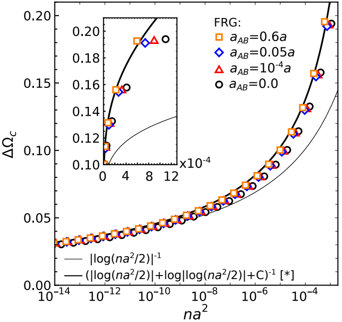
B. Oleś and K. Sacha, J. Phys. A **41**,145005 (2008)

$$\frac{E}{N} = \frac{\pi n}{m}(a + a_{AB}) + \frac{32\sqrt{2\pi}}{15}\frac{n^{3/2}a^{5/2}}{m}f(a_{AB}/a), \qquad f(x) = (1+x)^{5/2} + (1-x)^{5/2}$$

$$\Delta\Omega_c = \frac{4}{3\sqrt{2\pi}}(na^2)^{3/2}h(a_{AB}/a), \qquad h(x) = (1+x)^{3/2} + (1-x)^{3/2}$$

E/N and condensate depletion Ω_c ($d=2$)





MF: G. E. Astrakharchik *et al.*, PRA **79**, 051602 (2009) [*] V. Pastukhov, J. Low Temp. Phys. **194**, 197 (2019)

$T_x = \pi n/m$

Perturbative expression: P. Konietin and V. Pastukhov, J. Low Temp. Phys. **190**, 256 (2018)

$$\frac{E}{N} = \frac{\pi n}{m}\zeta_+(n,a,a_{AB}) + \frac{\pi n}{2m}\sum_{\pm}\zeta_\pm^2(n,a,a_{AB}) \times \left(2\gamma_E + 1/2 + \log\left(\pi\zeta_\pm(n,a,a_{AB})\right)\right)$$

where $\zeta_\pm(n,a,a_{AB}) = 1/|\log(na^2/2)| \pm \Theta(a_{AB})/|\log(na_{AB}^2/2)|$.

Conclusions

- We have studied repulsive and balanced Bose-Bose mixtures with the FRG
- The phase separation point is identified by the vanishing of the spin healing scale
- Macroscopic results compare favorably with other approaches

  More details on: arXiv:2011.00487

- Future work:
  - Implement superfluid drag. Calculations at finite temperatures
  - Study quantum critical point and phase separation point
  - Attractive Bose-Bose mixture: Droplet phase, dimerization and strongly-interacting regime (BCS-BEC crossover)

    H. Hu *et al.*, PRA **102**,043301 (2020)

    I. Morera *et al.*, arXiv:2007.01786 (2020)

RG Flows

- Couplings have the same scaling behavior as in a one-component gas
- Flow affected by the inter-species interaction