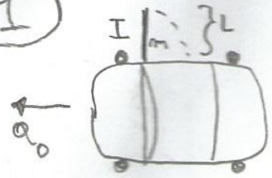
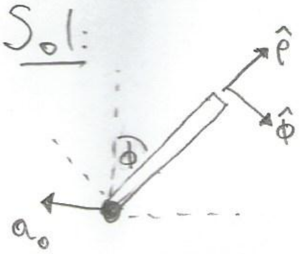
P1) a_o constante.



¿Tiempo en que se cierra la puerta?

Sol:



$\otimes \hat{z}$

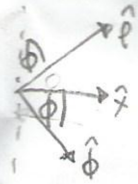
Sabemos que:

$$m\frac{d^2\vec{R}_G}{dt^2} = \vec{F}$$

$$\text{mov. relativo} \rightarrow \ddot{\vec{R}}_G = \vec{a}_o + \cancel{\vec{a}'_G}_{0} + \vec{\Omega}\times\cancel{\vec{v}'_G}_{0} + \vec{\Omega}\times(\vec{\Omega}\times\underbrace{\vec{r}\,'_G}_{\frac{L}{2}\hat{\rho}}) + \dot{\vec{\Omega}}\times\vec{r}\,'_G =$$

$$= \vec{a}_o + \vec{\Omega}\times(\vec{\Omega}\times\tfrac{L}{2}\hat{r}) + \dot{\vec{\Omega}}\times\tfrac{L}{2}\hat{\rho}$$

Veré los vectores:



• $\Big\}\ \hat{x} = \hat{\rho}\,\text{sen}\phi + \hat{\phi}\cos\phi \Rightarrow \vec{a}_o = -a_o\hat{x} = -a_o(\hat{\rho}\,\text{sen}\phi + \hat{\phi}\cos\phi)$

• $\vec{\Omega} = \Omega\hat{z} = \dot{\phi}\hat{z} \Rightarrow \dot{\vec{\Omega}} = \ddot{\phi}\hat{z}$

Entonces:

$$\ddot{\vec{R}}_G = -a_o(\hat{\rho}\,\text{sen}\phi + \hat{\phi}\cos\phi) + \dot{\phi}\hat{z}\times\overbrace{(\dot{\phi}\hat{z}\times\tfrac{L}{2}\hat{\rho})}^{\dot{\phi}\frac{L}{2}\hat{\phi}} + \ddot{\phi}\hat{z}\times\tfrac{L}{2}\hat{\rho} =$$

$$= -a_0(\hat{\rho}\,\mathrm{sen}\,\phi + \hat{\phi}\cos\phi) - \frac{L}{2}\dot{\phi}^2\hat{\rho} + \frac{L}{2}\ddot{\phi}\hat{\phi} =$$

$$= \left(-a_0\,\mathrm{sen}\,\phi - \frac{L}{2}\dot{\phi}^2\right)\hat{\rho} + \left(-a_0\cos\phi + \frac{L}{2}\ddot{\phi}\right)\hat{\phi}$$

y la fuerza que actúa sobre la visagra: $\vec{F} = m\ddot{\vec{R}}_G$

El torque:

$$\vec{\tau} = \underbrace{\left(-\frac{L}{2}\hat{\rho}\right)}_{\text{el (-) es porque actúa en la visagra}} \times \vec{F} = -m\frac{L}{2}\left(a_0\cos\phi + \frac{L}{2}\ddot{\phi}\right)\hat{z}$$

Sabemos que: $\vec{l}_G = I^G\vec{\Omega}$

$$\Rightarrow \vec{\tau} = \frac{d\vec{l}_G}{dt} = I^G\ddot{\phi}\hat{z} \qquad \text{con} \quad I = I^G + m\left(\frac{L}{2}\right)^2 \Big\}\ \text{Steiner}$$

$$\Rightarrow -m\frac{L}{2}\left(-a_0\cos\phi + \cancel{\frac{L}{2}\ddot{\phi}}\right) = \left(I - \cancel{m\frac{L^2}{4}}\right)\ddot{\phi}$$

la ec. de movimiento: $\ddot{\phi} - \frac{mLa_0}{2I}\cos\phi = 0$

multiplicando por $\dot{\phi}$:

$$\ddot{\phi}\dot{\phi} - \frac{mLa_0}{2I}\dot{\phi}\cos\phi = 0 \qquad / \int (\,) dt$$

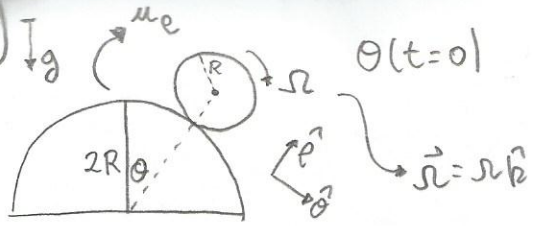
$$\int_0^{\dot{\phi}} \frac{d\dot{\phi}}{dt}\dot{\phi}\, dt - \frac{mLa_0}{2I}\int_0^{\phi} \frac{d\phi}{dt}\cos\phi\, dt = 0$$

$$\frac{\dot{\phi}^2}{\cancel{2}} - \frac{mLa_0}{\cancel{2}I}\operatorname{sen}\phi = 0$$

despejando:

$$\dot{\phi} = \sqrt{\frac{mLa_0}{I}\operatorname{sen}\phi} = \frac{d\phi}{dt}$$

$$\Rightarrow \int_0^t dt = \sqrt{\frac{I}{mLa_0}}\int_0^{\pi/2}\frac{d\phi}{\sqrt{\operatorname{sen}\phi}} \Rightarrow \boxed{t = \sqrt{\frac{I}{mLa_0}}\int_0^{\pi/2}\frac{d\phi}{\sqrt{\operatorname{sen}\phi}}}$$

P2) 

$\Theta(t=0)$

$\vec{\Omega} = \Omega \hat{k}$

$$I^{G} = \frac{1}{2} m R^2$$

a) Demuestre que mientras el disco rueda sin resbalar $\Omega = 3\dot{\theta}$.

Sol: La vel. del centro de masa del disco:

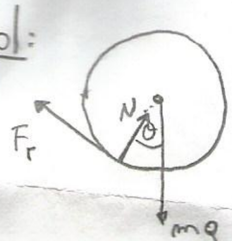
$$\vec{v}_G = 3R\dot{\theta}\hat{\theta}$$

como rueda sin resbalar:

$$\vec{v}_G = \vec{\Omega} \times R\hat{\rho}$$

$$3R\dot{\theta}\hat{\theta} = \Omega\hat{k} \times R\hat{\rho} = \Omega R \hat{\theta} \Rightarrow \boxed{\Omega = 3\dot{\theta}}$$

b) Ec. de mov. centro de masa del disco y la ecuación de momento angular c/r al C.M.

Sol: 

Newton:

$$\hat{\rho}: N - mg\cos\theta = -m\underbrace{\rho}_{3R}\dot{\theta}^2 = -3mR\dot{\theta}^2$$

$$\hat{\theta}: mg\sin\theta - F_r = m\ddot{\theta} = 3mR\ddot{\theta}$$

...emos: $\vec{L}_G = I^G \vec{\Omega} = I^G \cdot 3\dot{\theta}\hat{k}$

$\rightarrow \dfrac{d\vec{L}_G}{dt} = 3I^G\ddot{\theta}\hat{k} = \vec{\tau}_G = (-R\hat{\rho}) \times (-F_r\hat{\theta}) = RF_r\hat{k}$

$\Rightarrow \boxed{3I^G\ddot{\theta} = RF_r}$ $(*)$ $\Big\}$ ec. de mov.

de $\hat{\theta}$: $\quad F_r = m(g\,\mathrm{sen}\,\theta - 3R\ddot{\theta})$

remplazando en $(*)$:

$$3I^G\ddot{\theta} = mR(g\,\mathrm{sen}\,\theta - 3R\ddot{\theta})$$

$\rightarrow 3(I^G + mR^2)\ddot{\theta} = mRg\,\mathrm{sen}\,\theta \quad \Rightarrow \boxed{\ddot{\theta} = \dfrac{mRg}{3(I^G + mR^2)}\,\mathrm{sen}\,\theta}$

c) Si $\mu_e = \frac{1}{2}$, encontrar ec. para θ_d donde comienza a deslizar el cilindro.

Sol: Usando que $I^g = \frac{mR^2}{2}$:

$$\ddot{\theta} = \frac{mgR}{3(I^g + mR^2)} \operatorname{sen}\theta = \frac{\cancel{m}gR}{3 \cdot \frac{3}{2}\cancel{m}R^{\cancel{2}}} \operatorname{sen}\theta = \frac{2}{9}\frac{g}{R}\operatorname{sen}\theta \qquad (**)$$

integro:

$$\frac{d\dot{\theta}}{dt} = \frac{d\dot{\theta}}{d\theta}\frac{d\theta}{dt} = \dot{\theta}\frac{d\dot{\theta}}{d\theta} = \frac{2}{9}\frac{g}{R}\operatorname{sen}\theta$$

$$\int_0^{\dot{\theta}} \dot{\theta}\, d\dot{\theta} = \frac{2}{9}\frac{g}{R}\int_0^{\theta} \operatorname{sen}\theta\, d\theta = \frac{2}{9}\frac{g}{R}(-\cos\theta)\Big|_0^{\theta}$$

$$\Rightarrow \frac{\dot{\theta}^2}{2} = \frac{2}{9}\frac{g}{R}(1-\cos\theta)$$

Ahora puedo sacar N de la ec. para $\hat{\rho}$:

$$N - mg\cos\theta = -3mR\dot{\theta}^2 = -\cancel{3}mR \cdot \frac{4}{\cancel{9}_3}\frac{g}{R}(1-\cos\theta)$$

$$N - mg\cos\theta = -\frac{4}{3}mg(1-\cos\theta)$$

$$\Rightarrow \quad N = mg\left(\frac{7}{3}\cos\theta - \frac{4}{3}\right)$$

El límite para deslizar: $F_r = \mu_e N$

$\hat{\theta}: \longrightarrow \; mg\operatorname{sen}\theta - F_r = \cancel{3}mR\,\underbrace{\left(\frac{2}{3}\frac{g}{R}\operatorname{sen}\theta\right)}_{\ddot{\theta}}$

$$\Rightarrow F_r = \frac{mg\operatorname{sen}\theta}{3} = \mu_e N = \mu_e\, mg\left(\frac{7}{3}\cos\theta - \frac{4}{3}\right)$$

despejando:

$$\frac{\cancel{mg}\operatorname{sen}\theta_d}{\cancel{3}} = \mu_e \cancel{mg}\left(\frac{7}{\cancel{3}}\cos\theta_d - \frac{4}{\cancel{3}}\right)$$

$$\mu_e = \frac{1}{2} \longrightarrow \boxed{\operatorname{sen}\theta_d - \frac{7}{2}\cos\theta_d + 2 = 0}$$