Auxiliar 1

P1 $\vec{a} = \overset{\text{cte}}{k}\sqrt{x}\,\hat{x}$ Obtener $a(t), v(t)$ y $x(t)$.

$v(t=0)=0$, $x(t=0)=0$

Sol: Tenemos $\vec{a}(x)$, la idea es obtener $x(t)$ y luego derivar

$$\underbrace{a}_{} = k\sqrt{x}$$

$$\frac{dv}{dt} = \frac{d\dot{x}}{dt} \underset{\text{"truco"}}{=} \underbrace{\frac{dx}{dt}}_{\dot{x}}\frac{d\dot{x}}{dx} \longrightarrow \dot{x}\frac{d\dot{x}}{dx} = k\sqrt{x} \rightarrow \dot{x}\,d\dot{x} = k\sqrt{x}\,dx$$

integramos:

$$\int_{\dot{x}(t=0)=0}^{\dot{x}(t)} \dot{x}\,d\dot{x} = k\int_{x(t=0)=0}^{x(t)} \sqrt{x}\,dx \Rightarrow \frac{\dot{x}^2}{2} = k\cdot\frac{2}{3}x^{3/2} \quad /\sqrt{()}$$

$$\underbrace{\dot{x}}_{\frac{dx}{dt}} = 2\sqrt{\frac{k}{3}}\,x^{3/4}$$

queda:

$$\int_{x(t=0)=0}^{x(t)} dx\,x^{-3/4} = 2\sqrt{\frac{k}{3}}\int_{t=0}^{t} dt$$

$$4x^{1/4} = 2\sqrt{\frac{k}{3}}\,t \Rightarrow \boxed{x(t) = \left(\sqrt{\frac{k}{3}}\,\frac{t}{2}\right)^4} = \frac{k^2}{144}t^4$$

ahora la velocidad:

$$v = \frac{dx}{dt} \Rightarrow \boxed{v(t) = \frac{1}{36}k^2t^3}$$

y la aceleración:

$$a = \frac{dv}{dt} \Rightarrow \boxed{a(t) = \frac{1}{12}k^2t^2}$$

P2 $\vec{a} = -kv^2\hat{x}$

$x(t=0)=0 \quad , \quad v(t=0)=v_0$

a) rapidez en función de x

b) rapidez en fn. del tiempo

Sol:

a) Buscamos $v(x)$:

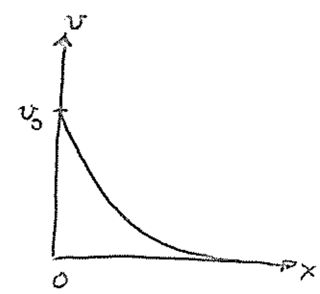
$$a(x) = \underbrace{\frac{dv}{dt}}_{\frac{dv}{dx}\underbrace{\frac{dx}{dt}}_{v}} = -kv^2 \quad \Rightarrow \quad v\frac{dv}{dx} = -kv^2$$

$$\frac{dv}{v} = -k\,dx$$

integramos: $$\int_{v_0}^{v} \frac{dv}{v} = -k\int_0^x dx$$

$$\ln\left(\frac{v}{v_0}\right) = -kx \qquad / \exp(\,)$$

$$\frac{v}{v_0} = e^{-kx} \quad \Rightarrow \quad \boxed{v(x) = v_0\, e^{-kx}}$$



b) Ahora buscamos $v(t)$:

Partimos de la ecuación inicial:

$$a = \frac{dv}{dt} = -kv^2$$

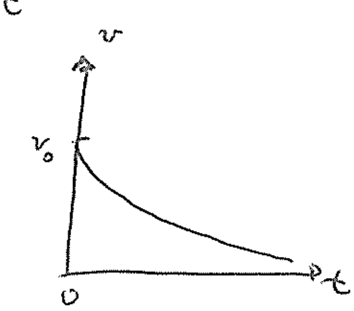
$$\frac{dv}{v^2} = -k\,dt$$

integramos:

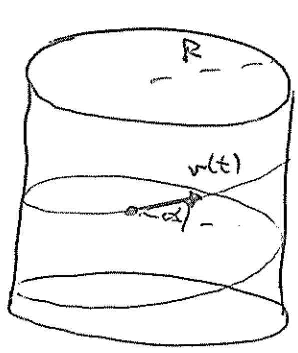
$$\int_{v_0}^{v} \frac{dv}{v^2} = -k\int_0^t dt \quad \Rightarrow \quad -\frac{1}{v} + \frac{1}{v_0} = -kt$$

$$\frac{1}{v} = \frac{1}{v_0} + kt = \frac{1+v_0kt}{v_0}$$

$$\Rightarrow \boxed{v(t) = \frac{v_0}{1+v_0kt}}$$

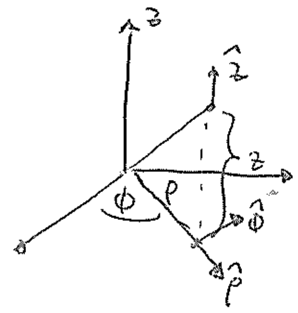


P3



Buscamos la velocidad y aceleración en cilíndricas

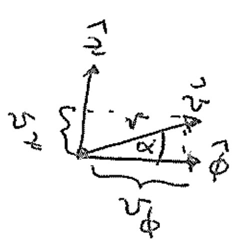
Sol: La velocidad en cilíndricas:



$$\vec{v} = \dot{\rho}\hat{\rho} + \rho\dot{\phi}\hat{\phi} + \dot{z}\hat{k} = v_\rho\hat{\rho} + v_\phi\hat{\phi} + v_z\hat{k}$$

Aquí $\rho = R \Rightarrow \dot{\rho} = 0 \Rightarrow v_\rho = 0$

Debemos determinar $\dot{\phi}$ y $\dot{z}$:



$$\begin{cases} v_\phi = v\cos\alpha \\ v_z = v\,\mathrm{sen}\,\alpha \end{cases}$$

$$\Rightarrow \boxed{\vec{v} = v(t)\cos\alpha\,\hat{\phi} + v(t)\,\mathrm{sen}\,\alpha\,\hat{k}}$$

Aprovecharemos de sacar la aceleración:

$$\vec{a} = (\ddot{\rho} - \rho\dot{\phi}^2)\hat{\rho} + (2\dot{\rho}\dot{\phi} + \rho\ddot{\phi})\hat{\phi} + \ddot{z}\hat{k}$$

Ya sabemos que $\dot{\rho} = \ddot{\rho} = 0$, nos falta $\dot{\phi}, \ddot{\phi}$ y $\ddot{z}$:

$$v_\phi = \rho\dot{\phi} = R\dot{\phi} = v\cos\alpha \Rightarrow \dot{\phi} = \frac{v}{R}\cos\alpha$$

derivando: $\frac{d\dot{\phi}}{dt} = \ddot{\phi} = \frac{\cos\alpha}{R}\frac{dv}{dt}$

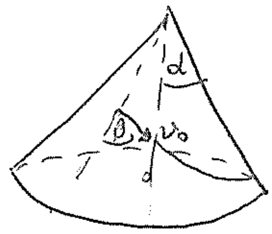
de forma análoga:

$$v_z = \dot{z} = v\,\mathrm{sen}\,\alpha \Rightarrow \ddot{z} = \mathrm{sen}\,\alpha\frac{dv}{dt}$$

entonces la aceleración:

$$\boxed{\vec{a} = -\frac{v^2}{R}\cos^2\alpha\,\hat{\rho} + \cos\alpha\frac{dv}{dt}\hat{\phi} + \mathrm{sen}\,\alpha\frac{dv}{dt}\hat{k}}$$

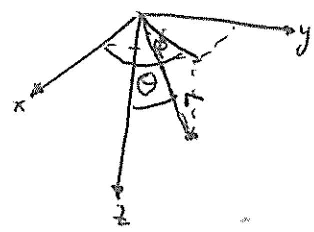
P4)



Una partícula se mueve en la superficie de un cono a rapidez constante v_0

Determinar ecuación de la trayectoria

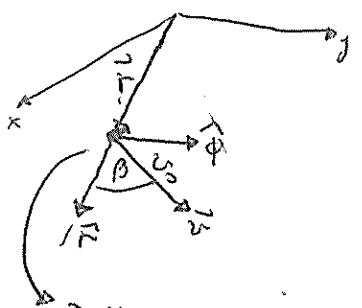
Sol: En esféricos:



$$\vec{v} = \dot{r}\hat{r} + r\dot{\phi}\hat{\phi}\,\mathrm{sen}\,\theta + r\dot{\theta}\hat{\theta}$$

Si observamos bien, nos damos cuenta que:

$$\theta = \alpha = cte \Rightarrow \dot{\theta} = 0$$



$$v_r = \dot{r} = v_0 \cos\beta$$
$$v_\phi = r\dot{\phi}\,\mathrm{sen}\,\alpha = v_0\,\mathrm{sen}\,\beta$$

$$\Rightarrow \boxed{\vec{v} = v_0\cos\beta\,\hat{r} + v_0\,\mathrm{sen}\,\beta\,\hat{\phi}}$$

lo que define la trayectoria, pero podemos obtenerla:

$$\dot{r} = \frac{dr}{dt} = v_0\cos\beta$$

$$\int_{r_0}^{r} dr = v_0\cos\beta\int_0^t dt \Rightarrow \boxed{r(t) = v_0\cos\beta\, t + r_0}$$

$$v_\phi = r\dot{\phi}\,\mathrm{sen}\,\alpha = v_0\,\mathrm{sen}\,\beta$$

$$(v_0\cos\beta t + r_0)\frac{d\phi}{dt}\mathrm{sen}\,\alpha = v_0\,\mathrm{sen}\,\beta$$

$$\int_{\phi_0}^{\phi} d\phi = \frac{v_0\,\mathrm{sen}\,\beta}{\mathrm{sen}\,\alpha}\int_0^t \frac{dt}{v_0\cos\beta t + r_0} = \frac{\cancel{v_0}\,\mathrm{sen}\,\beta}{\mathrm{sen}\,\alpha}\frac{1}{\cancel{v_0}\cos\beta}\ln(v_0\cos\beta t + r_0)\Big|_0^t$$

$$\Rightarrow \phi(t) = \frac{\mathrm{tg}\,\beta}{\mathrm{sen}\,\alpha}\ln\left(\frac{v_0\cos\beta t + r_0}{r_0}\right)$$