Auxiliar 22

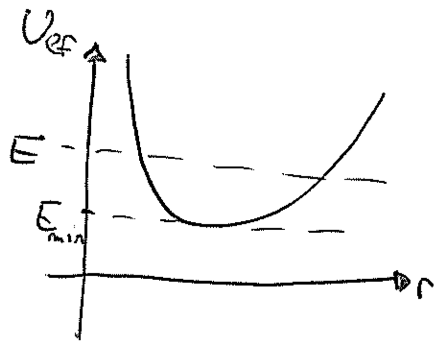
P1 $U_{ef} = \frac{\ell^2}{2mr^2} + Cr^3$

a) ¿$F(r)$?

b) Radio r_0 de órbita circular

c) Cuando $E > E_{min}$, encontrar $\frac{dr}{dt}$ en $r = r_0$. Expresar en $E, E_{min}, \ell, C, m, r_0$

d) K en $r = r_{max}$



Sol: a) El potencial: $U(r) = Cr^3$

$$\Rightarrow F(r) = -\frac{d}{dr}U \Rightarrow \boxed{F(r) = -3Cr^2} \Big\} \text{atractiva}$$

b) Para que sea órbita circular:

$$\left.\frac{dU_{ef}}{dr}\right|_{r_0} = 0 = \frac{-\ell^2}{mr_0^3} + 3Cr_0^2 \rightarrow 0 = -\ell^2 + 3Cmr_0^5$$

$$\Rightarrow \boxed{r_0 = \left(\frac{\ell^2}{3Cm}\right)^{1/5}}$$

c) La energía cinética:

$$K = \frac{\ell^2}{2mr^2} + \frac{m}{2}\left(\frac{dr}{dt}\right)^2$$

en r_0 $\rightarrow$ $$E = \frac{\ell^2}{2mr_0^2} + \frac{m}{2}\left.\left(\frac{dr}{dt}\right)^2\right|_{r_0} + U_{ef}(r_0)$$

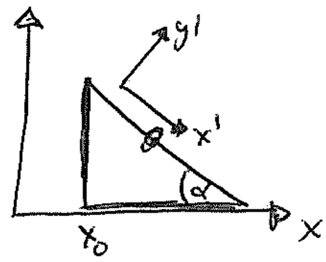
$$E_{min} = \frac{\ell^2}{2mr_0^2} + U_{ef}(r_0)$$

$$\Rightarrow E - E_{min} = \frac{m}{2}\left.\left(\frac{dr}{dt}\right)^2\right|_{r_0} \Rightarrow \boxed{\left|\frac{dr}{dt}\right| = \sqrt{\frac{2}{m}(E - E_{min})}}$$

d) En r_{max}, $\left.\frac{dr}{dt}\right|_{r_{max}} = 0$

$$\Rightarrow \boxed{K = \frac{\ell^2}{2mr_{max}^2}}$$

P2)



En $t=0$ el anillo está en reposo.

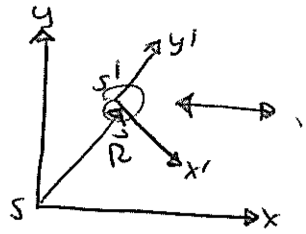
La estructura oscila con:

$$x(t) = A(1-\cos(\omega t))$$

a) Ec. en coor. x' b) Normal en y'

c) $x'(t)$ con $x'(t=0)=0$

Sol: a)



$$\left\{ \vec{R} = y_0\hat{y} + A(1-\cos(\omega t))\hat{x} \right. \qquad \vec{\Omega} = \vec{0}$$

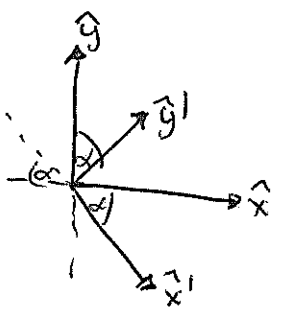
$$\Rightarrow \dot{\vec{R}} = A\omega\,\mathrm{sen}(\omega t)\hat{x}$$

$$\ddot{\vec{R}} = A\omega^2\cos(\omega t)\hat{x}$$

En el sist. no inercial:

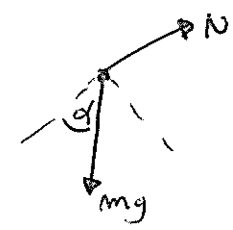
$$\vec{r}' = x'\hat{x}' \ , \ \vec{v}' = \dot{x}'\hat{x}' \ , \ \vec{a}' = \ddot{x}'\hat{x}'$$

Relaciono los vectores unitarios:



$$\begin{cases} \hat{x} = \cos\alpha\,\hat{x}' + \mathrm{sen}\,\alpha\,\hat{y}' \\ \hat{y} = -\mathrm{sen}\,\alpha\,\hat{x}' + \cos\alpha\,\hat{y}' \end{cases}$$

DCL



$$\left\{ \vec{F} = N\hat{y}' + mg\,\mathrm{sen}\,\alpha\,\hat{x}' - mg\cos\alpha\,\hat{y}' \right.$$

$$\Rightarrow m\ddot{x}'\hat{x}' = mg(\mathrm{sen}\,\alpha\,\hat{x}' - \cos\alpha\,\hat{y}') + N\hat{y}' - Am\omega^2\cos(\omega t)(\cos\alpha\,\hat{x}' + \mathrm{sen}\,\alpha\,\hat{y}')$$

en $\hat{x}'$: $\boxed{\ddot{x}' = g\,\mathrm{sen}\,\alpha - A\omega^2\cos(\omega t)\cos\alpha}$ (1)

b) en $\hat{y}'$: $0 = -mg\cos\alpha + N - Am\omega^2\cos(\omega t)\,\mathrm{sen}\,\alpha$

$$\Rightarrow \boxed{N = m(g\cos\alpha + A\omega^2\cos(\omega t)\,\mathrm{sen}\,\alpha)}$$

c) Resolvemos (1): $\ddot{x}' = \frac{d\dot{x}'}{dt}$

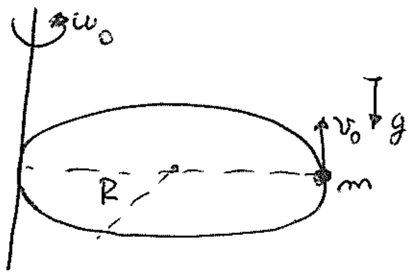
$$\int_0^{\dot{x}'} d\dot{x}' = \int_0^t (g\,\mathrm{sen}\,\alpha - A\omega^2\cos(\omega t)\cos\alpha)\,dt$$

$$\dot{x}' = g\,\mathrm{sen}\,\alpha\, t - A\omega\cos\alpha\,\mathrm{sen}(\omega t)\Big|_0^t = g\,\mathrm{sen}\,\alpha\,t - A\omega\cos\alpha\,\mathrm{sen}(\omega t)$$

$$\rightarrow \dot{x}' = \frac{dx'}{dt} \Rightarrow \int_0^{x'} dx' = \int_0^t (g\,\mathrm{sen}\,\alpha\,t - A\omega\cos\alpha\,\mathrm{sen}(\omega t))\,dt$$

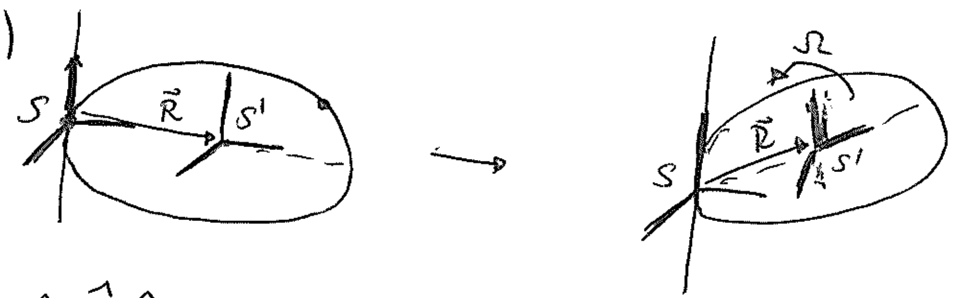
$$\Rightarrow \boxed{x'(t) = \frac{g}{2}\mathrm{sen}\,\alpha\,t^2 + A\cos\alpha(\cos(\omega t) - 1)}$$

P3)



¿Mínimo valor de v_0 para que anillo llegue al eje?

Sol: 1)



$\vec R = R\hat\rho$
$\dot{\vec R} = R\omega_0\hat\phi$
$\ddot{\vec R} = -R\omega_0^2\hat\rho$
$\vec\Omega = \omega_0\hat k$

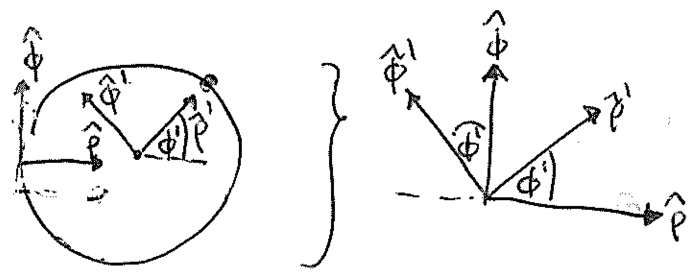
2) S: $\hat\rho, \hat\phi, \hat k$; S': $\hat\rho', \hat\phi', \hat k$

3)

Para el sistema no inercial:

$$\vec r' = R\hat\rho' \qquad \vec v' = R\dot\phi'\hat\phi' \qquad \vec a' = -R\dot\phi'^2\hat\rho' + R\ddot\phi'\hat\phi'$$

4) Relaciono los ejes:



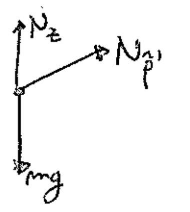
$$\hat\rho = \cos\phi'\hat\rho' - \text{sen}\,\phi'\hat\phi'$$
$$\hat\phi = \text{sen}\,\phi'\hat\rho' + \cos\phi'\hat\phi'$$

5) Centrífuga: $m\vec\Omega\times(\vec\Omega\times\vec r') = \underbrace{m\omega_0\hat k\times(\omega_0\underbrace{\hat k\times R\hat\rho'}_{\hat\phi'})}_{-\hat\rho'} = -mR\omega_0^2\hat\rho'$

Coriolis: $2m\vec\Omega\times\vec v' = 2m\omega_0\hat k\times R\dot\phi'\hat\phi' = -2m\omega_0R\hat\rho'$

Transversal: $\vec 0$

6) DCL:



$\vec F = N_z\hat k - mg\hat k + N_\rho\hat\rho'$

7) $m(-R\dot\phi'^2\hat\rho' + R\ddot\phi'\hat\phi') = N_z\hat k - mg\hat k + N_\rho\hat\rho' + R\omega_0^2(\cos\phi'\hat\rho' - \text{sen}\,\phi'\hat\phi')m$
$+ mR\omega_0^2\hat\rho' + 2m\omega_0R\hat\rho'$

La condición para que llegue al eje sale de la ecuación en $\hat\phi'$, al integrar hasta π:

$\hat\phi'$: $\cancel{m}R\ddot\phi' = -R\omega_0^2\,\text{sen}\,\phi'\,\cancel{m}$

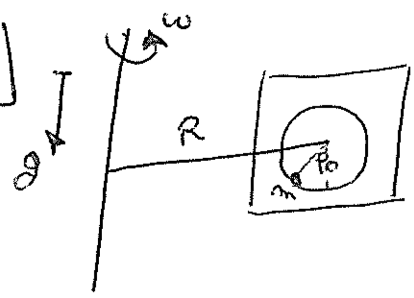
$$\ddot{\phi}' = -\omega_0^2 \operatorname{sen}\phi'$$

$$\int_{v_0/R}^{0} \dot{\phi}' \, d\dot{\phi}' = -\omega_0^2 \int_0^{\pi} \operatorname{sen}\phi' \, d\phi'$$

$$-\frac{1}{2}\frac{v_0^2}{R^2} = +\omega_0^2 \cos\phi' \Big|_0^{\pi} = \omega_0^2(-1-1) = -2\omega_0^2$$

$$\Rightarrow \boxed{v_{0,min}^2 = 4R^2\omega_0^2}$$

P4)

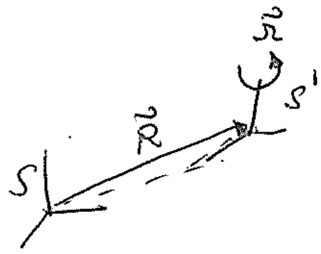


a) Definir S, S' y calcular pseudofuerzas

b) Obtener ec. de mov. y una ec. para el ángulo $\ddot{\phi} = f(\phi)$

c) Bajo qué condiciones $\phi = 0$ es estable, y calcule frec. de pequeñas oscilaciones

Sol: a) 1)

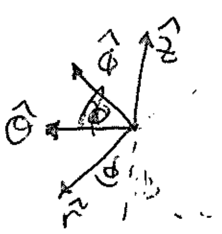


$$\left\{ \begin{array}{l} \vec{R} = R\hat{\rho} \rightarrow \ddot{\vec{R}} = -R\omega^2\hat{\rho} \\ \vec{\Omega} = \omega\hat{z} \end{array} \right.$$

2) S: $\hat{\rho}, \hat{\theta}, \hat{z}$ $\quad S'$: $\hat{r}, \hat{\phi}, \hat{z}'$

3) $\vec{r}' = \rho_0\hat{r}$ $\quad \vec{v}' = \rho_0\dot{\phi}\hat{\phi}$ $\quad \vec{a}' = -\rho_0\dot{\phi}^2\hat{r} + \rho_0\ddot{\phi}\hat{\phi}$

4) Ahora relaciono las coordenadas:



$$\left\{ \begin{array}{l} \hat{\phi} = \cos\phi\,\hat{\phi} + \operatorname{sen}\phi\,\hat{r} \\ \hat{z} = \operatorname{sen}\phi\,\hat{\phi} - \cos\phi\,\hat{r} \end{array} \right.$$

además: $\hat{z}' = \hat{\rho}$

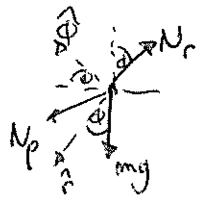
5) Pseudo fuerzas:

- $m\ddot{\vec{R}} = -mR\omega^2\hat{z}'$
- $m\vec{\Omega}\times(\vec{\Omega}\times\vec{r}') = m\omega(\operatorname{sen}\phi\hat{\phi} - \cos\phi\hat{r}) \times (\omega(\underbrace{\operatorname{sen}\phi\hat{\phi} - \underbrace{\cos\phi\,\hat{r}) \times \rho_0\hat{r}}_{0}}_{-\hat{z}'}))=$

$$= m\omega^2\rho_0(\operatorname{sen}\phi\hat{\phi} - \cos\phi\hat{r}) \times -\operatorname{sen}\phi\,\hat{z}' = m\omega^2\rho_0(-\operatorname{sen}^2\phi\,\hat{r} - \operatorname{sen}\phi\cos\phi\,\hat{\phi})$$

- $2m\vec{\Omega}\times\vec{v}' = 2m\omega(\operatorname{sen}\phi\hat{\phi} - \cos\phi\hat{r}) \times \rho_0\dot{\phi}\hat{\phi} = -2m\omega\rho_0\dot{\phi}\cos\phi\,\hat{z}'$
- $m\dot{\vec{\Omega}}\times\vec{r}' = 0$

b) 6) DCL:



$$\vec{F} = -N_\rho \vec{z}\,' - N_r \hat{r} + mg\cos\phi\,\hat{r} - mg\,\text{sen}\,\phi\,\hat{\phi}$$

7) La ec. de mov:

$$\boxed{\begin{aligned} m(-\rho_0\dot{\phi}^2\hat{r} + \rho_0\ddot{\phi}\hat{\phi}) = {} & -N_\rho\hat{z} - N_r\hat{r} + mg\cos\phi\,\hat{r} - mg\,\text{sen}\,\phi\,\hat{\phi} \\ & + mR\omega^2\vec{z}\,' + m\omega^2\rho_0(\text{sen}^2\phi\,\hat{r} + \text{sen}\,\phi\cos\phi\,\hat{\phi}) \\ & + 2m\omega\rho_0\dot{\phi}\cos\phi\,\vec{z}\,' \end{aligned}}$$

Para $\hat{\phi}$: $\boxed{m\rho_0\ddot{\phi} = -mg\,\text{sen}\,\phi + m\omega^2\rho_0\,\text{sen}\,\phi\cos\phi}$.

c) Ordenando la ec. anterior:

$$\ddot{\phi} = -\frac{g}{\rho_0}\text{sen}\,\phi + \omega^2\,\text{sen}\,\phi\cos\phi$$

Para ángulos pequeños: $\text{sen}\,\phi \sim \phi$
$\cos\phi \sim 1$

$$\Rightarrow \ddot{\phi} = -\frac{g}{\rho_0}\phi + \omega^2\phi \Rightarrow \ddot{\phi} + \left(\frac{g}{\rho_0} - \omega^2\right)\phi = 0$$

Para que sea estable: $\frac{g}{\rho_0} - \omega^2 > 0 \Rightarrow \boxed{\omega_0^2 = \frac{g}{\rho_0} - \omega^2}$

inestable: $\frac{g}{\rho_0} - \omega^2 < 0$