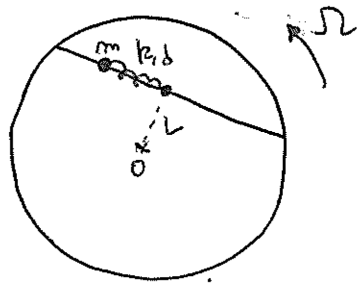
Auxiliar 24

P2 |



a) Fuerzas y pseudo-fuerzas

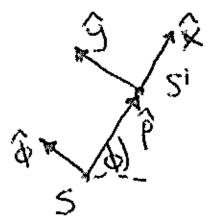
b) Distinguir mov. dependiendo de k/m v/s Ω^2

c) Si $\frac{k}{m} > \Omega^2$ ¿ pos. de eq. relativo?

d) Si parte del reposo respecto al sist. no inercial y a una distancia ε del pto. de equilib encontrar trayectoria

e) Fza. de la barra sobre la part

Sol: a)



$$\left\{ \begin{array}{l} \vec{R} = L\hat{\rho} \\ \dot{\vec{R}} = L\Omega\hat{\phi} \\ \ddot{\vec{R}} = -L\Omega^2\hat{\rho} \\ \vec{\Omega} = \Omega\hat{k} \end{array} \right. \qquad \begin{array}{l} \vec{r}\,' = y\hat{y} \\ \dot{\vec{r}}\,' = \dot{y}\hat{y} \\ \ddot{\vec{r}}\,' = \ddot{y}\hat{y} \end{array}$$

Se ve que $\hat{\rho} = \hat{x}$, $\hat{\phi} = \hat{y}$

DCL:

$$\Big\} \; \vec{F} = -N\hat{\rho} - k(y-d)\hat{\phi}$$

Los pseudo-fuerzas:

- $m\ddot{\vec{R}} = -mL\Omega^2\hat{\rho}$
- $m\vec{\Omega}\times(\vec{\Omega}\times\vec{r}\,') = m\Omega^2\hat{k}\times\underbrace{(\hat{k}\times y\hat{y})}_{}$ con $\hat{k}\times\hat{y} = -\hat{x}$, $\hat{k}\times(-\hat{x}) = \hat{y}$: $= -m\Omega^2 y\hat{\phi}$
- $2m\vec{\Omega}\times\vec{v}\,' = 2m\Omega\underbrace{\hat{k}\times\dot{y}\hat{y}}_{-\hat{x}} = -2m\Omega\dot{y}\hat{\rho}$

b) Escribimos la ec. de mov:

$$m\ddot{y}\hat{\phi} = -N\hat{\rho} - k(y-d)\hat{\phi} + mL\Omega^2\hat{\rho} + m\Omega^2 y\hat{\phi} + 2m\Omega\dot{y}\hat{\rho}$$

$\hat{\phi}$: $m\ddot{y} = -k(y-d) + m\Omega^2 y \Rightarrow \ddot{y} + \left(\frac{k}{m} - \Omega^2\right)y = \frac{k}{m}d$

$\Rightarrow$ $\frac{k}{m} - \Omega^2 > 0$; M.A.S.

$\frac{k}{m} - \Omega^2 = 0$: Mov. uniformemente acelerado

$\frac{k}{m} - \Omega^2 < 0$: Mov. exponencial $\rightarrow$ $y = Ae^{\lambda t} + Be^{-\lambda t}$

c) Para $\frac{k}{m} > \Omega^2$:

$$\ddot{y} + \left(\frac{k}{m} - \Omega^2\right) y = \frac{k}{m} d$$

en pto. de eq. $\ddot{y} = 0$:

$$\left(\frac{k}{m} - \Omega^2\right) y_{eq} = \frac{k}{m} d \Rightarrow \boxed{y_{eq} = \frac{\frac{kd}{m}}{\frac{k}{m} - \Omega^2}}$$

d) Llamando $\omega_0^2 = \frac{k}{m} - \Omega^2$:

$$\ddot{y} + \omega_0^2 y = \frac{k}{m} d \quad \Rightarrow y = y_h + y_p$$

- $y_h = A\cos(\omega_0 t + \delta)$ • $y_p = \frac{\frac{kd}{m}}{\omega_0^2} = y_{eq}$

$$\Rightarrow \quad y = A\cos(\omega_0 t + \delta) + y_{eq}$$

Las condiciones iniciales son $y(0) = y_{eq} + \varepsilon$, $\dot{y}(0) = 0$:

$$\dot{y}(t) = -A\omega_0 \operatorname{sen}(\omega_0 t + \delta)$$

$$\rightarrow \dot{y}(0) = 0 = -A\omega_0 \operatorname{sen}(\delta) \Rightarrow \operatorname{sen}\delta = 0 \Rightarrow \delta = 0$$

$$y(t) = A\cos(\omega_0 t) + y_{eq}$$

$$y(0) = A + y_{eq} = y_{eq} + \varepsilon \quad \Rightarrow A = \varepsilon$$

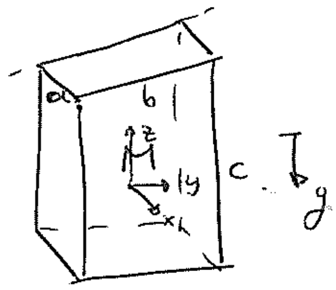
$$\Rightarrow \boxed{y(t) = \varepsilon\cos(\omega_0 t) + y_{eq}}$$

e) Vemos la ec. en $\hat{\rho}$:

$$N = mL\Omega^2 + 2m\Omega\dot{y}$$

$$\Rightarrow \boxed{N = mL\Omega^2 - 2m\Omega\,\varepsilon\omega_0 \operatorname{sen}(\omega_0 t)}$$

P3



a) Determinar I^G

b) Frecuencia pequeñas oscilaciones en que el sólido oscila en torno a cada una de las aristas

Sol: a) Tenemos que,

$$I^G = \int \begin{pmatrix} \rho_y^2+\rho_z^2 & -\rho_x\rho_y & -\rho_x\rho_z \\ -\rho_x\rho_y & \rho_z^2+\rho_x^2 & -\rho_y\rho_z \\ -\rho_x\rho_z & -\rho_y\rho_z & \rho_x^2+\rho_y^2 \end{pmatrix} dm$$

en nuestro caso, $dm = \rho\, dV$, con $\rho = \frac{M}{abc}$, $dV = dx\,dy\,dz$

$$\rightarrow I^G = \frac{M}{abc}\int \begin{pmatrix} y^2+z^2 & -xy & -xz \\ -xy & x^2+z^2 & -yz \\ -xz & -yz & x^2+y^2 \end{pmatrix} dx\,dy\,dz$$

$$\int \longrightarrow \int_{-a/2}^{a/2} dx \int_{-b/2}^{b/2} dy \int_{-c/2}^{c/2} dz$$

- $\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\int_{-c/2}^{c/2} (y^2+z^2)\,dx\,dy\,dz = a\left(c\frac{b^3}{12} + b\frac{c^3}{12}\right)$
- $\iiint (x^2+z^2)\,dV = b\left(c\frac{a^3}{12} + a\frac{c^3}{12}\right)$
- $\iiint (y^2+\cancel{z}x^2)\,dV = c\left(b\frac{a^3}{12} + a\frac{b^3}{12}\right)$
- $\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\int_{-c/2}^{c/2} (-xy)\,dx\,dy\,dz = -c\left(\frac{x^2}{2}\right)\Big|_{-a/2}^{a/2}\left(\frac{y^2}{2}\right)\Big|_{-b/2}^{b/2} = 0$

$$\Rightarrow I^G = \frac{M}{abc}\begin{pmatrix} ac\frac{b^3}{12}+ab\frac{c^3}{12} & 0 & 0 \\ 0 & cb\frac{a^3}{12}+ba\frac{c^3}{12} & 0 \\ 0 & 0 & cb\frac{a^3}{12}+ca\frac{b^3}{12} \end{pmatrix}$$

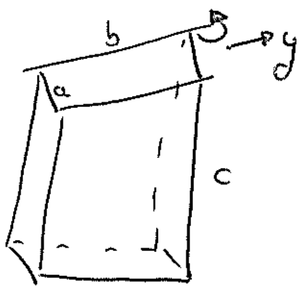
$$\Rightarrow \boxed{I^G = \frac{M}{12}\begin{pmatrix} b^2+c^2 & 0 & 0 \\ 0 & a^2+c^2 & 0 \\ 0 & 0 & a^2+b^2 \end{pmatrix}}$$

b) Teo. de Steiner:

$$I^{O}_{ij} = I^{G}_{ij} + M\left(R_G^2 \delta_{ij} - R_{Gi} R_{Gj}\right)$$

$$\rightarrow I^{O} = I^{G} + M \begin{pmatrix} Y^2+Z^2 & -XY & -XZ \\ -XY & Z^2+X^2 & -YZ \\ -XZ & -YZ & X^2+Y^2 \end{pmatrix}$$

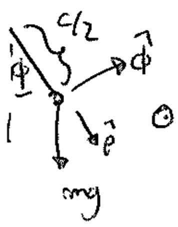
Ya tenemos I^G: $\vec{R}_G = \frac{a}{2}\hat{x} - \frac{c}{2}\hat{z}$



$$\Rightarrow I^{y} = \begin{pmatrix} \frac{b^2}{12}+\frac{c^2}{3} & 0 & 0 \\ 0 & \frac{a^2}{3}+\frac{c^2}{3} & 0 \\ 0 & 0 & \frac{a^2}{3}+\frac{b^2}{12} \end{pmatrix} M$$

el torque:

$$\vec{\tau} = \vec{r} \times \vec{F} = \frac{c}{2}\hat{\rho} \times Mg(\cos\phi\,\hat{\rho} - \operatorname{sen}\phi\,\hat{\phi}) = -\frac{c}{2}Mg\operatorname{sen}\phi\,\hat{k} = \frac{c}{2}Mg\operatorname{sen}\phi\,\hat{y}$$



ó $\hat{k} = -\hat{y}$

usando que $\quad \vec{\tau} = I^{y}\dot{\vec{\Omega}} \quad$ con $\vec{\Omega} = \dot{\phi}\,\hat{k} = -\dot{\phi}\,\hat{y}$

$$\Rightarrow \frac{c}{2}Mg\operatorname{sen}\phi = -\frac{M}{3}(a^2+c^2)\ddot{\phi} \qquad \Rightarrow \ddot{\phi} + \frac{3cg}{2(a^2+c^2)}\underbrace{\operatorname{sen}\phi}_{\sim\phi} = 0$$

$$\Rightarrow \boxed{\omega_{0,b}^2 = \frac{3cg}{2(a^2+c^2)}}$$

de forma análoga:

$$\boxed{\omega_{0,a}^2 = \frac{3cg}{2(b^2+c^2)}} \qquad \boxed{\omega_{0,c}^2 = \frac{3bg}{2(a^2+b^2)}}$$