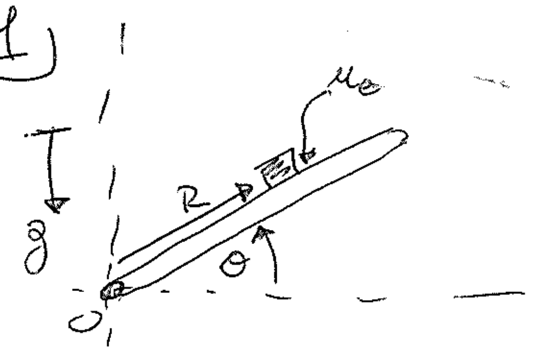
Auxiliar 7

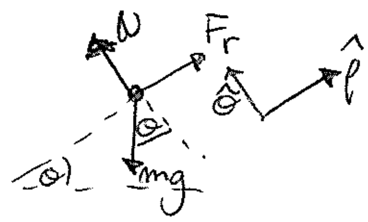
P1)



$$\alpha = \ddot{\theta} = \frac{2g}{R}$$

¿μ_e para que no deslice entre $0 < \theta < \pi$?

Sol: DCL:



$$\hat{\rho}: \; m(\cancel{\ddot{\rho}} - \rho\dot{\theta}^2) = F_r - mg\,\text{sen}\,\theta \qquad (1)$$

$$\hat{\theta}: \; mR\underbrace{\ddot{\theta}}_{\alpha} = N - mg\cos\theta \qquad (2)$$

de (2): $\quad m\cancel{R}\cdot\frac{2g}{\cancel{R}} = N - mg\cos\theta$

$$\Rightarrow N = mg(2+\cos\theta) > 0 \quad \text{para todo } \theta$$

de (1): $\quad F_r = mg\,\text{sen}\,\theta - mR\dot{\theta}^2 \qquad (3)$

pero $\ddot{\theta} = \frac{2g}{R} = \dot{\theta}\frac{d\dot{\theta}}{d\theta}$

$$\Rightarrow \frac{\dot{\theta}^2}{2} = \frac{2g}{R}\theta \Rightarrow \dot{\theta}^2 = \frac{4g}{R}\theta$$

remplazando en (3):

$$F_r = mg\,\text{sen}\,\theta - mg\cdot 4\theta = mg(\text{sen}\,\theta - 4\theta)$$

para que no resbale: $\quad |F_r| \leq \mu_e |N|$

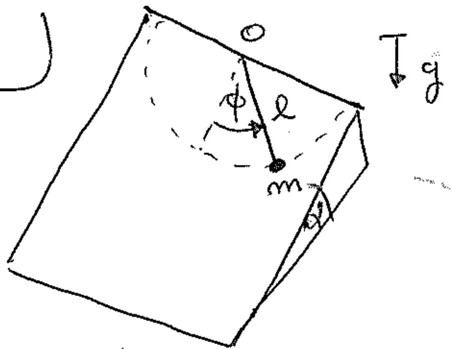
$$\cancel{mg}\,|\text{sen}\,\theta - 4\theta| \leq \mu_e \cancel{mg}\,|2+\cos\theta|$$

$$\Rightarrow \boxed{\mu_e \geq \frac{|\text{sen}\,\theta - 4\theta|}{2+\cos\theta}} \rightarrow \mu_e \geq \frac{4\theta - \text{sen}\,\theta}{2+\cos\theta}$$

Tomando $F_r = mg(4\theta - \text{sen}\,\theta)$, es máximo cuando: $\theta = \pi$

$$\Rightarrow \mu_e \geq \frac{4\pi - \overset{0}{\cancel{\text{sen}\,\pi}}}{2+\underbrace{\cos\pi}_{-1}} \Rightarrow \boxed{\mu_e \geq 4\pi}$$

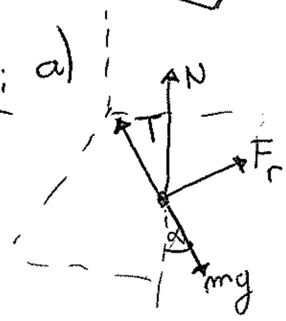
P2)



a) Escribir fuerzas en cilíndricas

b) $\phi_0 = \frac{\pi}{2}$, $\dot\phi_0 = 0$. Encontrar μ_d tal que $\dot\phi(\phi=0)=0$.

Sol: a)



$\vec{T} = -T\hat{\rho}$ $\quad$ $\vec{F}_r = \mu N \hat{\phi}$ } es + porque la masa se mueve en $-\hat{\phi}$

$\vec{N} = N\hat{z}$

$m\vec{g} = mg(\text{sen}\alpha\cos\phi\,\hat{\rho} - \text{sen}\alpha\,\text{sen}\phi\,\hat{\phi} - \cos\alpha\,\hat{z})$

b) $\rho = L$ y $z = 0$ $\Rightarrow$ $\dot\rho = \ddot\rho = \dot z = \ddot z = 0$

La aceleración es: $\vec{a} = -l\dot\phi^2\hat{\rho} + l\ddot\phi\hat{\phi}$

Entonces:

Newton: $\hat{\rho})$: $-l\dot\phi^2 = mg\,\text{sen}\alpha\cos\phi - T$ $\quad$ (1)

$\hat{\phi})$: $l\ddot\phi = \mu N - mg\,\text{sen}\alpha\,\text{sen}\phi$ $\quad$ (2)

$\hat{z})$: $0 = N - mg\cos\alpha$ $\quad \Rightarrow \quad N = mg\cos\alpha$

$$(2)\dot\phi: \quad \underbrace{l\dot\phi\ddot\phi}_{\frac{1}{2}\frac{d\dot\phi^2}{dt}} = \mu N \underbrace{\dot\phi}_{\frac{d\phi}{dt}} - mg\,\text{sen}\alpha\,\underbrace{\text{sen}\phi\,\dot\phi}_{-\frac{d}{dt}\cos\phi}$$

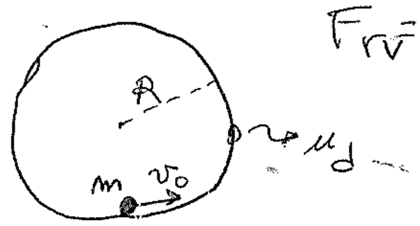
integrando entre $t=0$ y t:

$$\frac{1}{2}\dot\phi^2 = \frac{\mu N}{l}\left(\phi - \frac{\pi}{2}\right) + \frac{mg\,\text{sen}\alpha}{l}\cos\phi$$

imponiendo $\dot\phi(\phi=0)=0$:

$$0 = -\frac{\mu N}{\cancel{l}}\frac{\pi}{2} + \frac{mg\,\text{sen}\alpha}{\cancel{l}} \quad \Rightarrow \quad \boxed{\mu = \frac{2}{\pi}\text{tg}\,\alpha}$$

P3)



$F_{rv} = -kv$

a) Distancia que recorre?

b) $k=0$?

Sol: a) DCL:

$\vec{v} = R\dot{\phi}\hat{\phi}$

$\vec{a} = -R\dot{\phi}^2\hat{\rho} + R\ddot{\phi}\hat{\phi}$

Newton: $\hat{\rho}$: $-mR\dot{\phi}^2 = -N$ (1)

$\hat{\phi}$: $mR\ddot{\phi} = -kR\dot{\phi} - \mu_d N$ (2)

usando (1) en (2):

$$m\cancel{R}\ddot{\phi} = -k\cancel{R}\dot{\phi} - \mu_d m\cancel{R}\dot{\phi}^2$$

$$\ddot{\phi} = \cancel{\dot{\phi}}\frac{d\dot{\phi}}{d\phi} = -\left[\frac{k}{m}\cancel{\dot{\phi}} + \mu_d\dot{\phi}^{\cancel{2}}\right]$$

$$\int_{v_0/R}^{\dot{\phi}} \frac{d\dot{\phi}}{\frac{k}{m} + \mu_d\dot{\phi}} = -\int_0^{\phi} d\phi$$

$$\frac{1}{\mu_d}\ln\left(\frac{\frac{k}{m} + \mu_d\dot{\phi}}{\frac{k}{m} + \mu_d\frac{v_0}{R}}\right) = -\phi$$

el ángulo máximo que recorre ocurre cuando $\dot{\phi} = 0$:

$$\phi_{max} = \frac{1}{\mu_d}\ln\left(\frac{\frac{k}{m} + \mu_d\frac{v_0}{R}}{\frac{k}{m}}\right) = \frac{1}{\mu_d}\ln\left(1 + \frac{\mu_d m v_0}{kR}\right)$$

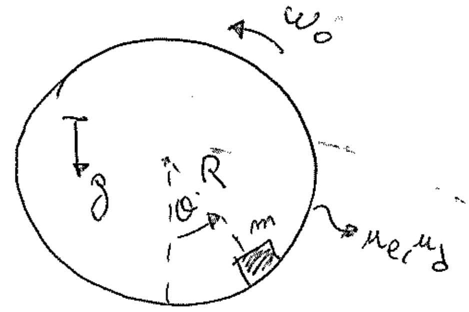
y la distancia que recorre:

$$s = R\phi_{max} \Rightarrow \boxed{s = \frac{R}{\mu_d}\ln\left(1 + \frac{\mu_d m v_0}{kR}\right)}$$

b) Si $k=0$:

$s \longrightarrow \infty \Rightarrow$ no se detiene.

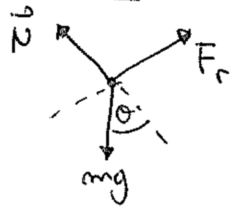
P4)



a) Valor de μ_d para que $\theta = \theta_0$ siempre

b) ω_0 y μ_e para que bloque de una vuelta completa

Sol: a) DCL:



$$\vec{F} = mg\cos\theta\,\hat{\rho} - mg\,\mathrm{sen}\,\theta\,\hat{\theta} - N\hat{\rho} + f_r\hat{\theta}$$

Para que se quede quieto: $\vec{F} = m\vec{a} = 0$

$$\hat{\rho}: \; mg\cos\theta - N = 0 \Rightarrow N = mg\cos\theta \Rightarrow f_r = \mu_d N = \mu_d\, mg\cos\theta$$

$$\hat{\theta}: \; -mg\,\mathrm{sen}\,\theta + \mu_d\, mg\cos\theta = 0 \Rightarrow \boxed{\mu_d = \mathrm{tg}\,\theta}$$

b) El DCL es igual cambiando roce dinámico por estático

la aceleración: $\vec{a} \underset{\rho = R,\ \dot{\theta} = \omega_0}{=} -R\omega_0^2\hat{\rho}$

$$\hat{\rho}: \; mg\cos\theta - N = -mR\omega_0^2 \Rightarrow N = mg\cos\theta + mR\omega_0^2$$

$$\hat{\theta}: \; -mg\,\mathrm{sen}\,\theta + f_r = 0$$

Para que no se despegue:

$$N > 0 \Rightarrow \cancel{m}g\cos\theta + \cancel{m}R\omega_0^2 > 0 \rightarrow \omega_0^2 > \underbrace{\frac{-g}{R}\cos\theta}_{\text{máximo cuando } \theta = -\pi} \Rightarrow \boxed{\omega_0^2 > \frac{g}{R}}$$

Ahora la condición para μ_e:

$$|f_r| \leq \mu_e |N| \Rightarrow \cancel{m}g\,\mathrm{sen}\,\theta \leq \mu_e \cancel{m}(g\cos\theta + R\omega_0^2)$$

$$\Rightarrow \mu_e \geq \frac{g\,\mathrm{sen}\,\theta}{g\cos\theta + R\omega_0^2} = f(\theta)$$

buscamos el máximo:

$$\left.\frac{df}{d\theta}\right|_{\theta^*} = \frac{\cancel{g}\cos\theta^*}{\cancel{g\cos\theta^* + R\omega_0^2}} + \frac{g^{\cancel{2}}\,\mathrm{sen}^2\theta^*}{(g\cos\theta^* + R\omega_0^2)^{\cancel{2}}} = 0 \Rightarrow \cos\theta^* + \frac{g\,\mathrm{sen}^2\theta^*}{g\cos\theta^* + R\omega_0^2} = 0$$

$$\underbrace{g\cos^2\theta^* + g\operatorname{sen}^2\theta^*}_{g} + R\omega_0^2\cos\theta^* = 0$$

$$R\omega_0^2\cos\theta^* + g = 0 \Rightarrow \cos\theta^* = \frac{-g}{R\omega_0^2} \Rightarrow \theta^* \in [\tfrac{\pi}{2}, \tfrac{3\pi}{2}]$$

remplazando:

$$\mu_e \geq \frac{g\operatorname{sen}\theta^*}{g\cos\theta^* + R\omega_0^2} = \frac{g\sqrt{1-\cos^2\theta^*}}{g\cos\theta^* + R\omega_0^2} = \frac{g\sqrt{1-\frac{g^2}{R^2\omega_0^2}}}{\frac{-g^2}{R\omega_0^2} + R\omega_0^2} = \frac{\frac{g}{\cancel{R\omega_0^2}}\sqrt{R^2\omega_0^4 - g^2}}{\frac{-g^2}{\cancel{R\omega_0^2}} + \frac{R^2\omega_0^4}{\cancel{R\omega_0^2}}} =$$

$$= \frac{g\sqrt{R^2\omega_0^4 - g^2}}{R^2\omega_0^4 - g^2} = \frac{g}{\sqrt{R^2\omega_0^4 - g^2}}$$

$$\Rightarrow \boxed{\mu_e \geq \frac{g}{\sqrt{R^2\omega_0^4 - g^2}}}$$