S9 — Graph Theory Closing Report

Systemic Risk Propagation Across DeFi Synthetic-Asset Protocols

Derivalink R&D

August 6, 2026

Abstract

This report closes S9, the graph-theoretic study of systemic risk propagation across DeFi synthetic-asset protocols (spec. §4.4). It covers the full arc of the work: a graph-construction pipeline built from real on-chain data (nodes for protocols, synthetic assets, collateral, oracles and DEX pools; weighted, time-varying edges for collateralization, oracle provision and pool hosting); centrality, community-detection and propagation metrics computed on that graph, including a from-scratch implementation of DebtRank (Battiston et al., 2012); three independent historical replays that calibrate the model against real collapses — DAI/March-2023, UST/May-2022 and Iron Finance/June-2021 — comparing an ex-ante graph-predicted impact to the real ex-post observed loss; and a final multi-protocol generalization pass that resolves a node-identity ambiguity, builds one graph spanning 59 tracked assets, and adds the Iron Finance replay as a third, independent test of the methodology.

The headline result is consistent across all three historical replays: the model is directionally correct in every case — it always flags the right node as seriously at risk — but its precision differs systematically with the shocked protocol's collateral structure. DAI (31 diversified collateral legs) and Iron Finance (2 legs, one of them algorithmic) both understate the real loss by a factor of roughly 2.7 to 4.7, despite very different structures; UST (a single, undiversified collateral edge) overstates it by 1.3×. The multi-protocol snapshot adds a structural finding: the real on-chain graph connecting this project's 59 tracked assets is overwhelmingly a disconnected forest of per-protocol stars, not a densely interlinked system — a genuine, data-grounded answer to how much of DeFi's systemic risk is visible through on-chain collateral, oracle and pool-liquidity edges alone.

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1 Introduction and Scope

1.1 Why graph theory

Every synthetic asset studied elsewhere in this project is embedded in a network of dependencies: it is backed by collateral, priced by one or more oracles, traded in DEX pools, and sometimes shares a token with another protocol entirely. A synthetic's own price history can show that it depegged, but it cannot by itself show *why the shock reached it* or *how far it could have gone*. This project's internal research specification — cited throughout this report by section number only, as §4.4, since the document itself is private — frames this as a graph problem: build the dependency network explicitly, measure which nodes are structurally central, and test whether centrality predicts real-world fragility.

S9 is the part of this project that builds and tests that graph. It was worked in six stages, matching the spec's own subsections:

1. **Graph construction** (§4.4.1): nodes for protocols, synthetic assets, collateral, oracles and DEX pools, built from real on-chain tables.
2. **Edge types** (§4.4.1): weighted, time-varying edges for collateralization, oracle provision, and pool hosting.
3. **Centrality metrics** (§4.4.2): degree, betweenness, eigenvector/PageRank, Katz, closeness.
4. **Community detection** (§4.4.3): Louvain, Leiden, spectral clustering.

5. **Propagation metrics** (§4.4.4): shortest paths, max-flow, spectral radius, and DebtRank.
6. **Historical replay** (§4.4.5): comparing an ex-ante graph-predicted impact to a real ex-post collapse, for the two events spec names by name (DAI/March-2023, UST/May-2022), then extended to a third (Iron Finance/June-2021) and generalized to a real multi-protocol graph.

Stages 1–5 were deliberately built and validated on a single, well-covered protocol first (MakerDAO/DAI), mirroring the same single-asset-pilot-before-generalizing discipline this project used for every prior modeling track (machine learning, reinforcement learning, econometrics). Stage 6 is where this report's real substance sits: three independent historical tests of the whole pipeline, plus a final pass that lifts the single-protocol pilot into a genuine multi-protocol graph.

1.2 What this report does and does not claim

Every number in this report is either read directly from real on-chain or exchange data, or is the output of an algorithm implemented and unit tested in this project's own codebase (64 tests across the graph module at the time of writing). Nothing here is a synthetic demonstration fixture presented as a real result. Where a result is weaker than hoped — and two of the three historical replays understate the real collapse by a factor of three to four — that is reported plainly, with the most plausible structural explanation, rather than reframed to look more successful than it is. This mirrors the reporting discipline used throughout this project's other closing reports.

1.3 Structure of this report

Section 2 describes how the graph itself is built: what a node and an edge are, where their weights come from, and a node-identity ambiguity that had to be resolved before a genuine multi-protocol graph was possible. Section 3 covers the centrality, community-detection and propagation algorithms, including the DebtRank implementation. Sections 5 through 7 present the three historical replays in turn. Section 8 covers the multi-protocol generalization pass proper: the node-identity fix in practice, and a real 59-asset current-state snapshot. Section 9 closes with a cross-replay comparison, a direct answer to spec §4.4.5's own question about whether centrally positioned protocols suffer more, and an honest account of what remains open.

2 Graph Construction

2.1 Nodes and edges

The graph has five node types, each anchored to a real on-disk table: protocols (V_p), synthetic assets (V_a), collateral (V_c), oracles (V_o) and DEX pools (V_d). A sixth type spec names, arbitrageur clusters (V_{arb}), was not built: the raw event table it would need (`mev_events`) is schema-defined but has zero populated partitions anywhere in this project, so there is nothing on disk to cluster.

Three of spec's five edge types have real, on-disk data behind them and were implemented:

- **Collateralizes** (E_{collat}): from a collateral-composition table, weight = the collateral leg's USD value.
- **Provides_oracle** (E_{oracle}): from an oracle-updates table, weight = the number of updates in a trailing 30-day window (a simplified proxy for spec's "update frequency $\times$ volume" framing).
- **Hosted_in_pool** (E_{pool}): from a DEX pool-state table, weight = the pool's total USD liquidity.

Two more, *synth-flows-to-arbitrageur* and *synth-composes-synth*, were not built for the same reason as V_{arb}: no protocol in this project has a confirmed synth-backs-synth relationship on disk, and the arbitrageur-cluster table has no data. Both are documented gaps, not silently patched with an invented alternative.

Every edge is a snapshot at a point in time, not a running average: for a given snapshot timestamp, each edge source takes the latest row at-or-before that timestamp, per its own key, then drops that key entirely if the row is older than a staleness window (90 days by default). This is an explicit choice so that a collateral leg or oracle source that stopped reporting years before a snapshot date does not silently resurrect a phantom edge.

2.2 A gap in the schema, not a bug in the code

Spec's own edge-type enumeration has no protocol-to-asset edge. Protocol nodes are created and catalogued correctly, but nothing points into or out of them in this schema — a genuine gap in the underlying formalization, documented here rather than patched by inventing a sixth edge type outside the agreed schema.

2.3 The node-identity problem

The same real-world token frequently plays two roles across this project's data. USDC, for example, is both a synthetic-adjacent asset this project tracks in its own right (its own oracle history, its own depeg analysis) *and* a named collateral leg backing another protocol (MakerDAO's PSM module). Naively, this produces two disconnected graph nodes for the same real asset: one under its "asset" identity, one under its "collateral" identity, with no edge between them.

This was flagged as an open question the first time DAI's graph was built, and deliberately left unresolved while the graph covered a single protocol — with one tracked asset in scope, the collision never arose. It became unavoidable once a genuine multi-protocol graph was attempted (Section 8), and was resolved as follows: a collateral leg whose identifier is *also* a member of the snapshot's own set of independently tracked assets is routed to that asset's existing node instead of a second, disconnected stub. A leg with no such match — the common case, since most collateral (wrapped BTC, various governance tokens, real-world-asset vaults) is never itself a V_a in this project's scope — is unaffected, and stays a standalone node exactly as before. The match is an exact string comparison on the asset identifier, not a fuzzy or economic-equivalence heuristic: two different on-chain deployments of "the same" token (for instance USDC bridged to two different chains) remain two different nodes unless their identifiers are literally identical, by design.

Confirmed directly on real data before writing the fix: three collateral legs in MakerDAO's own 52-leg collateral history are also independently tracked assets elsewhere in this project — USDC, USDT, and ETH. A fourth collision, TITAN backing Iron Finance's IRON while also being an independently tracked pool-state asset, surfaced naturally while building the third historical replay (Section 7), and became the first real end-to-end exercise of the merge logic.

3 Centrality and Community Detection

3.1 Two different graph projections, for two different questions

Edge weights in this graph are economic magnitudes: USD collateral value, an oracle update count, USD liquidity. Centrality algorithms that ask "how influential is this node" (eigenvector centrality, PageRank) treat a larger weight as more influence, directly. Algorithms that ask "how far is this node from that one" (betweenness, closeness) need a path-length *distance*, which is the opposite of a magnitude: a $1.95B collateral edge should read as short and significant, not long and costly. The implementation inverts weight into distance (distance = 1/weight) for the path-based metrics and uses raw weight directly for the influence-based ones, applied on a single directed graph where parallel edges (same source, same target, different edge type) are summed.

Community detection asks a different kind of question — cohesion, not direction — and runs on a separate, undirected projection of the same graph. This was a deliberate, verified choice: running the same community algorithms on the directed graph, where every edge points *into* the asset node by construction, produces a degenerate "every node is its own singleton" answer, because there is no reciprocal structure for a modularity-based algorithm to reward grouping around. The undirected projection was checked against a synthetic two-triangle test graph with a known community structure before being trusted on real data, confirming all three algorithms (Louvain, Leiden, spectral clustering) genuinely separate real structure when it exists, not merely pass on the single trivial real case available at the time.

3.2 Results on the DAI pilot (2023-03-10, eve of the SVB/USDC depeg)

On DAI's real 35-node, 33-edge graph (31 collateralization edges, 1 oracle edge, 1 pool edge), the asset node dominates every influence metric — PageRank 0.46, Katz centrality 1.0, against near-zero for every leaf node — and betweenness is exactly zero everywhere. Zero betweenness is the mathematically correct answer for a pure directed star topology, not a bug: no leaf collateral node can reach another leaf node without an edge running back *out of* the asset node, which this schema does not define (Section 2). Closeness's absolute scale was flagged as sensitive to raw-dollar edge weights feeding the distance transform unnormalized; a log-scale or min-max normalization pass before inverting weights into distances is a natural refinement not implemented in this pass, to avoid scope creep on a computation whose primary goal was validating the pipeline end-to-end.

Louvain, Leiden and spectral clustering all agree on exactly two communities on this same graph: the connected 34-node star (DAI plus every collateral, oracle and pool node) and the isolated `protocol:makerdao` node — the correct, honest answer for a single-protocol graph, and not yet informative about spec's own framing of "clusters of correlated-risk protocols", which needs more than one protocol sharing real structure to produce a non-trivial finding. That richer test is taken up in Section 8.

4 Propagation Metrics and DebtRank

4.1 A third graph projection, for a stricter reason

Shortest paths, max-flow, spectral radius and DebtRank all ask how a shock or a unit of value propagates along real economic dependencies, which imposes a stricter requirement than either centrality or community detection: every edge weight compared against another must be in the same unit. Collateralization and pool-hosting edges are both USD magnitudes; an oracle-provision edge's weight is an update count over a trailing window — a different unit entirely. Mixing them into one metric would silently distort every one of the four algorithms in this section, so they run on a third projection restricted to the two USD-denominated edge types, kept directed (unlike the community projection), since propagation is inherently about the direction value or distress actually flows.

Dijkstra shortest paths and max-flow (networkx's default push-relabel implementation, one of spec's two explicitly acceptable algorithms) are deliberately not persisted: they answer pairwise questions ("from this collateral leg to that one"), not a per-node, per-snapshot fact, and were demonstrated on real node pairs via direct console output rather than written to disk. On the DAI pilot's pure star topology, both are honestly degenerate between any two leaf nodes — unreachable, zero flow — for the same structural reason betweenness is zero (Section 3): there is no edge running back out of the asset node.

Spectral radius is exactly 0.0 on the DAI pilot graph, on both the directed-collateral-only and the full value subgraph. This is mathematically correct, not a computation failure: a directed acyclic graph's weighted adjacency matrix is nilpotent, so every eigenvalue is zero by construction.

4.2 DebtRank: implementation and verification

DebtRank is the one propagation metric that stays genuinely informative even on a single-protocol star, because it is value-weighted rather than path-existence-based. The implementation follows Battiston et al. (2012)'s original formulation directly: a normalized impact matrix $\mathrm{impact}(i, j) = \mathrm{weight}(i, j)/\mathrm{total\_incoming\_weight}(j)$ — the fraction of j's economic backing that depends on i — and a synchronous-round distress propagation where every node propagates its current distress level exactly once, via a strict undistressed→active→inactive state machine. That single-propagation rule is what guarantees the algorithm terminates within $|V|$ rounds even on a graph containing cycles, rather than risking an infinite reverberation a naive "keep iterating until it stabilizes" approach could hit on cyclic input. Node economic value — the weighting term in the final aggregate DebtRank score — is approximated as total edge weight touching the node (incoming plus outgoing), a documented simplification of Battiston's original balance-sheet-size definition, for which this project's data has no direct analogue.

Before being trusted on real data, the implementation was checked against three hand-computed cases: a two-node full dependency (expected score 0.5), a three-node chain cascade (expected 0.75), and a three-node *cycle* (expected 0.6667) — the cycle case specifically proving the state machine terminates correctly and does not double-count a shock that loops back to the node that originated it.

4.3 A worst-case demonstration on the DAI pilot

Before any calibrated historical replay, DebtRank was exercised as a worst-case capability demonstration: shocking DAI's real largest collateral leg at the time (its PSM-USDC module, approximately \$1.95B, the same leg central to the March-2023 replay in Section 5) to a full 100% wipeout yields a DebtRank score of 0.173, meaning DAI itself reaches 34.55% distress. Shocking the smallest real leg on the same graph (a Loopring LRC position worth approximately \$1,214) gives an appropriately near-zero score, 1.07×10^{-7}. Both numbers are proportional and sanity-checked against real collateral values, not synthetic fixtures — this demonstration is explicitly a worst-case capability check, not yet the calibrated ex-ante prediction against an *actual* shock magnitude that the historical replays in the next three sections compute.

5 Historical Replay I — DAI, March 2023

5.1 Method

Spec §4.4.5 asks for a direct test: build the graph as of the eve of a real historical shock, propagate a shock calibrated to what actually happened (not an arbitrary worst case), and compare the model's prediction to the real, observed outcome. The snapshot used here is dated 2023-03-10, the eve of the SVB-driven USDC depeg — the literal event spec names, not a date chosen after the fact to fit the result. It is the same 35-node, 33-edge graph described in Section 3.

5.2 The real outcome

Read directly from the raw oracle-update table, not a coarser derived daily aggregate (this project's derived DAI tables are daily only, too coarse for an event that peaked and partially recovered within hours):

The secondary-market (Curve pool) trough is materially worse than the oracle-level trough — DAI's traded price dislocated further than its own price reference ever showed, consistent with

Series	Trough price	Deviation	Timestamp (UTC)
USDC (Chainlink oracle)	0.8800	−1200 bp	2023-03-11 07:51
DAI (Chainlink oracle)	0.8888	−1112 bp	2023-03-11 08:10
DAI (Curve 3pool)	0.8283	−1717 bp	(secondary market, cited from an already-audited case study)

Table 1: Real observed troughs, DAI/USDC, March 2023.

liquidity withdrawal and panic amplifying the shock beyond what a static oracle reading alone captures.

5.3 The calibrated prediction

USDC's real oracle trough (0.8800) implies a real fractional loss of 12.00%, applied as the initial DebtRank distress level on DAI's PSM-USDC collateral leg — its single largest leg at the time, at approximately $1.95B — and propagated through the same value subgraph described in Section 4. The result: DebtRank predicts DAI reaches 4.15% distress (−415 bp-equivalent).

	Predicted (DebtRank)	Observed
vs. oracle trough (−1112 bp)	−415 bp	37.3% of observed
vs. Curve-pool trough (−1717 bp)	−415 bp	24.1% of observed

Table 2: Predicted vs. real DAI distress, March 2023.

5.4 Reading the result

The model gets the *direction* right: it correctly identifies that DAI would suffer real, non-trivial stress from a USDC depeg of this size, not nothing, and lands in the right order of magnitude (hundreds of basis points, not single digits, not thousands). It understates the real magnitude by roughly 2.7 to 4.1×. The most plausible structural reasons, in order of likely weight: DebtRank here is a *static* collateral-value-share model, so it only propagates the fraction of DAI's backing that the one shocked leg represents, while the real March-2023 event was not a USDC-only shock — other crypto collateral moved in the same macro flash-crash window, a correlated stress a single-leg model structurally cannot see. Market panic and liquidity withdrawal, which the worse Curve-pool trough already hints at directly, amplify real price impact beyond what a collateral-accounting model reflects. DAI's own PSM instant-redemption mechanics, documented elsewhere in this project as the reason the depeg was survivable and short-lived, are a dynamic recovery channel a static single-round snapshot cannot model either.

This is reported as an honest finding about the limits of this model class, not a failure of the pipeline: a simple, auditable, value-weighted propagation model correctly flags DAI as at risk from its largest collateral dependency and lands in the right order of magnitude, without needing to model market microstructure or dynamic redemption mechanics — a reasonable directional screening tool, not a precise stress-test replacement. Whether this specific undershoot ratio is a general property of the model class or particular to DAI's own diversified collateral structure is tested directly in Section 7.

6 Historical Replay II — UST, May 2022

6.1 Why this needed new work, not a configuration change

Spec §4.4.5 names UST/May-2022 as its second replay target. The collateral-composition table the DAI replay reads from has zero rows for any Terra, UST or Mirror asset — confirmed by a full scan of the whole table, not assumed. This is not a coverage gap to backfill: UST's peg mechanism is algorithmic (LUNA burned to mint UST and back, via Terra's native market module), not CDP-style locked collateral, so it was never going to fit a schema built around per-leg collateral balances.

A new loader synthesizes a collateral-composition-*shaped* table from Terra's own raw swap-event log (LUNA burned via the market-to-UST swap path), so that the entire existing construction and metrics pipeline runs unchanged on Terra data — only the loader differs. This introduces one genuine methodological difference from DAI's edge, disclosed rather than hidden: DAI's collateral weight is a point-in-time USD *balance* (how much is locked right now); UST's algorithmic mechanism has no such balance to read, so its edge weight is instead a *flow* — the total USD value of LUNA burned over the week preceding the snapshot, as a proxy for backing capacity engaged. This distinction affects interpretation but not this replay's headline ratio, since LUNA is UST's only modeled incoming edge and the impact-matrix normalization is exactly 1.0 regardless of the specific dollar figure used for the edge weight.

A real operational constraint surfaced while building this loader: the underlying Terra event table is approximately 48 GB for the UST asset alone, with fourteen distinct event types mixed into the same partitioned files. An initial, unscoped scan exhausted the working machine's memory; the fix scopes every scan to the exact monthly sub-partitions a given date window touches, using a streaming query engine throughout.

6.2 The ex-ante graph and the real outcome

The snapshot is dated 2022-05-08, the eve of the acute break, chosen by checking day by day: UST held within 50 bp of peg through 2022-05-06, cracked to −137 bp on 2022-05-07, and collapsed 24.6% in a single day on 2022-05-09. The resulting graph has exactly three nodes and one edge — UST, its LUNA collateral, and an isolated protocol node — with the LUNA→UST edge weighted at $482.7M (7,511,464 LUNA burned in the pre-snapshot week, at LUNA's own 2022-05-08 closing price of $64.26).

Series	Reference	Trough	Loss
LUNA (exchange spot)	$64.26	$0.00004	99.9999%
UST (exchange spot)	$1.00 (peg)	$0.2346	76.54%

Table 3: Real observed troughs, LUNA/UST, May 2022.

6.3 The calibrated prediction and why it overshoots

LUNA has no $1 peg to calibrate a shock against, so its own pre-crash closing price is used as the reference instead. LUNA's real collapse is, for DebtRank purposes, essentially a full 1.0 shock. Propagated through the single LUNA→UST edge — an impact ratio of exactly 1.0, since it is UST's only modeled incoming edge — DebtRank predicts UST reaches 100.0% distress, against a real observed loss of 76.5%: the prediction is 130.7% of what actually happened.

This is the opposite bias from the DAI replay, and for a legible reason. DAI's real backing is diversified across 31 collateral legs, so a single-leg shock is structurally diluted by the model even though the real event's correlated, multi-collateral stress still exceeded the diluted prediction

— an undershoot. UST's modeled backing has exactly one mechanism, so a near-total collapse of that one mechanism propagates at essentially full strength, with nothing left to dilute it — while the real UST still retained 23.5 cents of value at its exact trough, because of dynamics a static, single-round propagation model cannot see: the Luna Foundation Guard's attempted Bitcoin-reserve deployment, exchanges pausing or adjusting withdrawals, and the simple fact that even a "death spiral" plays out over several days rather than instantaneously.

Read together with the DAI replay, the two results make the same point from opposite directions: a static, single-round DebtRank propagation is a reasonable directional screening tool — both replays correctly flag the shocked node's downstream dependency as seriously at risk, in roughly the right order of magnitude — but it is not a precise stress-test replacement, because real markets have dynamics (panic amplification for a diversified system, partial cushioning for a thin one) that a one-shot linear model cannot capture in either direction.

7 Historical Replay III — Iron Finance, June 2021

7.1 A third, structurally distinct case

Spec names only two replay targets; this project's own S9 work added a third, deliberately chosen to sit structurally between the other two. Iron Finance's IRON was a *partial*-collateral stablecoin, backed by a mix of hard exogenous collateral (USDC) and an endogenous, protocol-issued share token (TITAN), in proportions governed by a moving collateral ratio — neither DAI's fully collateralized model nor UST's zero-collateral algorithmic one. This project already had a complete, independently audited forensic reconstruction of Iron Finance's June-2021 collapse to calibrate against and compare results with, so this replay required no new data acquisition, only new graph-edge synthesis.

The collateral-composition table has zero rows for Iron Finance either, for a reason distinct from UST's: partial-collateral backing here is a single, aggregate two-way split (collateral ratio percentage times total supply), not itemized per leg the way DAI's fifty-two collateral positions are. A new loader synthesizes a two-row collateral-composition-shaped table from Iron Finance's own on-chain collateral-ratio state: one row for the hard USDC leg, one for the residual, algorithmic TITAN leg — reconstructing, independently, a collateral ratio of 75.12%, within 0.02 percentage points of the 75.15% recorded directly on-chain at the same timestamp.

This replay is also the first real end-to-end exercise of the node-identity merge described in Section 2: TITAN backs IRON *and* is independently tracked via its own pool-liquidity data, so the two roles resolve into a single node with a real economic meaning, letting TITAN's actual price collapse propagate to IRON through an ordinary graph edge rather than a narrative link asserted outside the model.

7.2 The ex-ante graph and the real outcome

The snapshot is dated 2021-06-15, the eve of the run (TITAN peaked the following morning). The resulting graph has 18 nodes and 16 edges: the two collateral legs (USDC \$562.8M, TITAN \$186.4M, summing to approximately \$749.2M, matching the total IRON supply value at the snapshot), plus fourteen pool-liquidity edges from IRON's and TITAN's own DEX pools, two of which are large (approximately \$679.7M and \$680.7M, both into IRON) and twelve of which are small or effectively zero.

IRON has two distinct real outcomes worth reporting side by side, the same pattern as DAI's oracle-vs-pool duality: a brief intraday panic trough, and a settled level once redemptions froze and the price found its true collateral-backed floor.

Series	Reference	Outcome	Loss
TITAN	$57.77 (pre-crash)	$0.000008	99.99999%
IRON, intraday trough	$1.00 (peg)	$0.5857	41.4%
IRON, settled level	$1.00 (peg)	≈$0.76	24.0%

Table 4: Real observed troughs, TITAN/IRON, June 2021, cited from an already-audited case study.

7.3 Two calibrated cuts, and why they disagree

TITAN's real collapse is, for DebtRank purposes, a full 1.0 shock. Propagated through the value subgraph using the same convention as the DAI and UST replays (collateral *and* pool-liquidity edges together), DebtRank predicts IRON reaches only 8.8% distress. A second, collateral-only cut — excluding the two large pool-liquidity edges — predicts 24.9% instead.

Value subgraph	**Predicted**	**vs. intraday (41.4%)**	**vs. settled (24.0%)**
Collateral + pool liquidity (DAI/UST-consistent)	8.8%	21.3% of observed	36.8% of observed
Collateral only	24.9%	60.0% of observed	**103.6% of observed**

Table 5: Predicted vs. real IRON distress, two value-subgraph cuts.

The collateral-only cut is, within four percent, an exact match against IRON's real settled loss — the single closest result of the three replays in this report. This is not a coincidence: the algorithmic leg's share of total backing (186.4M / 749.2M $\approx$ 24.9%) is, by construction, close to the algorithmic share implied by the collateral ratio itself (100% − 75.15% $\approx$ 24.85%), and IRON settling near its effective collateral ratio is exactly the mechanism the source forensic report independently describes. DebtRank reconstructs the same fact a completely different, block-level analysis arrived at separately.

7.4 Why the two cuts disagree so much: a real methodological finding

The value-subgraph convention used unchanged across all three replays lumps collateralization and pool-hosting edges into one USD-magnitude projection, because both are dollar-denominated and, on DAI's graph, the single Curve-pool edge was small relative to the 31-leg collateral, so the distinction barely mattered there. Iron Finance breaks that assumption outright: its two DEX pools' combined liquidity ($1.36B) is nearly double its total collateral ($749M). Pool liquidity is a measure of trading depth — how much can move without slippage — which is economically different from collateral backing — what is actually pledged against the peg — and the unmodified value-subgraph convention silently dilutes the shock across both when it should not, for a protocol whose pool liquidity happens to rival its collateral. This was invisible with one or two protocols in scope; it only surfaced once a third, structurally different protocol ran through the same unmodified pipeline, which is the entire point of a generalization pass: it is a result of testing the methodology further, not a defect discovered too late to matter.

The existing convention is left unchanged in the propagation module itself, since it remains correct for DAI and UST and this ambiguity would need to be resolved generically, not patched for one protocol; the collateral-only cut here is computed as a documented, reusable ad hoc analysis reported alongside the primary result, not silently substituted for it.

7.5 Cross-replay comparison

Using the same convention throughout (collateral and pool-liquidity edges together), Iron Finance's undershoot ratio lands in almost the same range as DAI's despite a completely different collateral structure — two legs, one of them algorithmic, versus thirty-one diversified legs:

Replay	Structure	Predicted / observed	Bias
DAI, March 2023	31 diversified legs	24.1–37.3%	Undershoot 2.7–4.1×
UST, May 2022	1 undiversified edge	130.7%	Overshoot 1.3×
Iron Finance, June 2021	2 legs + 2 large pools	21.3–36.8%	Undershoot 2.7–4.7×

Table 6: Cross-replay comparison, primary value-subgraph convention.

This is suggestive, from only three data points, that the roughly three-to-four-times undershoot seen twice is less a diversification-specific effect than a general property of static, single-round DebtRank propagation against a real, multi-hour or multi-day collapse — neither dynamic panic amplification nor a correlated shock across several channels at once is something a one-shot linear model can see by construction, regardless of how diversified the shocked system's backing happens to be.

8 Multi-Protocol Generalization

8.1 Building one graph across every tracked protocol

Every prior section works on a single protocol's own graph, built by naming its assets explicitly. Genuine generalization required discovering the full universe of tracked assets directly from disk — the union of every asset identifier that appears as the primary key of any of the three edge-source tables (collateral composition, oracle updates, pool state) — rather than continuing to hardcode a per-protocol list. This discovery finds 59 distinct tracked assets: MakerDAO/DAI, GMX/GLP, ten Synthetix equity synthetics, Ethena, Resolv, Circle/USDC, Tether/USDT, Lido/stETH, eleven Mirror Protocol synthetics, two bridged-UST variants, Iron Finance, FRAX, and a number of reference price feeds spanning several chains.

Loading fifty-nine assets' full multi-year history eagerly risked exhausting the working machine's memory — Iron Finance's own pool-liquidity history alone is close to six million rows. Since the underlying selection logic only ever needs data within a bounded staleness window of the chosen snapshot date, every load was scoped to that window before materializing, which is lossless for the final result: anything outside the window would have been discarded by the staleness filter regardless, at greater memory cost.

The snapshot date itself is data-driven rather than chosen by hand: the global maximum timestamp across all three edge-source tables, so "current" means "as recent as the data genuinely gets", not a convenient date picked to produce a nicer-looking result. The resulting graph has 110 nodes and 39 edges (4 collateralization, 16 pool-hosting, 19 oracle-provision).

8.2 The node-identity merge, validated on real data

The fix described in Section 2 fires correctly on this real, messy, multi-protocol graph: a reference-price node for ETH shows as a single node carrying both an incoming oracle-provision edge (its own independent price tracking) *and* an outgoing collateralization edge into DAI — a genuine cross-protocol bridge, confirmed directly rather than assumed. Centrality, community-detection and propagation metrics all ran on this graph with *zero* code changes to the algorithms themselves, the intended payoff of building one protocol-agnostic pipeline from the start rather than a bespoke one per protocol.

8.3 A mostly disconnected forest

Louvain, Leiden and spectral clustering all agree on 93 communities across 110 nodes — the real multi-protocol graph is overwhelmingly a collection of singletons and pairs, not a densely connected web. Exactly two clusters of meaningful size exist: a six-node cluster around DAI (DAI itself, ETH, wrapped staked ETH, one real-world-asset collateral position, and one DEX pool) and a nine-node cluster around bridged UST (two wrapped-UST variants plus seven residual DEX pools). Every other tracked protocol — Iron Finance, FRAX, all eleven Mirror synthetics, all ten Synthetix equity synthetics, Ethena, Resolv, GLP — is its own fully isolated star, sharing zero real on-chain edge with any other protocol in this dataset.

This is a genuine, data-grounded finding, not an artifact of an incomplete graph: on-chain collateralization, oracle-provision and pool-hosting edges, as this project's data actually records them, mostly do not connect one synthetic-asset protocol to another. Where real systemic risk does propagate across protocol boundaries in practice — through shared holders, correlated market sentiment, or common infrastructure failures rather than a direct on-chain dependency — this graph schema, built strictly from the edge types spec defines, structurally cannot see it.

8.4 Centrality does not mean economic health

A concrete illustration surfaced directly in this snapshot: bridged UST ties DAI for the highest in-degree of any node in the entire graph (six incoming edges each), despite the Terra peg having been dead since May 2022 (Section 6). The reason is mundane and telling: seven different DEX pools still report fresh residual liquidity for the wrapped token, years after the underlying asset's collapse. Measured this way, graph centrality tracks "how many venues still quote this asset", which is not the same question as "is this asset economically healthy" — a caution against reading structural centrality alone as a safety signal, and a concrete case for pairing it with the kind of ex-post historical replay this report's three central sections provide.

Separately, DAI's own currently-fresh collateral has consolidated to just four legs at this snapshot date, down from the thirty-one seen at the March-2023 replay date — consistent with, though not conclusively proving, the real restructuring MakerDAO's collateral base underwent in the years since. Whether this fully reflects genuine on-chain consolidation or, in part, a narrower window of continued data coverage for a few specific legs is not resolved here and is flagged as an open question rather than asserted either way.

9 Synthesis and Conclusion

9.1 Did centrally positioned protocols suffer more?

Spec §4.4.5 asks this question directly, and this report can now answer it with three independent historical tests rather than one. **Directionally, yes, within each protocol's own local graph**: in every one of the three replays, the node DebtRank identifies as central to the shocked system is exactly the node that, in reality, suffered real, serious, measurable distress — DAI, UST and IRON each moved hundreds to thousands of basis points off peg, in every case in the direction and rough order of magnitude the model predicted.

The fullest, cross-protocol form of the question — do protocols that are more central *relative to other protocols* suffer more than peripheral ones — is a different, harder claim, and this project's own real on-chain data cannot yet support it. Section 8 showed directly that the real multi-protocol graph is overwhelmingly a disconnected forest: there is rarely a genuine edge for a cross-protocol centrality comparison to run along in the first place. This is reported as an honest data-availability limit, not a task quietly left undone: the question was tested as far as this project's real, on-chain collateral, oracle and pool-liquidity data actually allows, and no further.

9.2 What the three replays say about the model class as a whole

Replay	Structure	Predicted / observed	Bias
DAI, March 2023	31 diversified legs	24.1–37.3%	Undershoot 2.7–4.1×
UST, May 2022	1 undiversified edge	130.7%	Overshoot 1.3×
Iron Finance, June 2021	2 legs + 2 large pools	21.3–36.8%	Undershoot 2.7–4.7×

Table 7: All three historical replays, primary value-subgraph convention.

Every replay is directionally correct. The two undershoots land in almost the same range despite structurally very different collateral — thirty-one diversified legs versus two, one of them algorithmic — which is suggestive, though only three data points, that the roughly three-to-four-times understatement is a general property of static, single-round DebtRank against a real, dynamic multi-hour or multi-day collapse, rather than something specific to how diversified the shocked protocol happens to be. UST's overshoot remains the clear outlier of the three, explained cleanly by having a single, undiversified edge with nothing to dilute a near-total collapse, unlike either of the other two. Stated as a single, consistent lesson: this graph schema and this DebtRank implementation are a sound directional, early-warning screening tool — every replay flags real risk in roughly the right order of magnitude — but never a precise stress-test replacement, because real collapses have dynamics (panic amplification, partial circuit-breaker cushioning, multi-day unwinds, and, as Iron Finance's replay specifically revealed, the difference between collateral backing and pool-liquidity depth) that a one-shot linear propagation model cannot see by construction, in either direction.

9.3 What remains open

Three items are explicitly unresolved, and are named here rather than left implicit. Arbitrageur-cluster nodes and two of spec's five edge types (synth-flows-to-arbitrageur, synth-composes-synth) still have no ready data anywhere in this project, unchanged since the very first pilot graph was built — not a coding gap, a data gap. Genuine cross-protocol centrality-versus-suffering correlation, discussed above, would need multiple protocols sharing real edges at a common historical date, which this project's on-chain data mostly does not provide. Whether DAI's real 2026 collateral consolidation to four fresh legs reflects genuine on-chain restructuring or a narrower window of continued data coverage for a few specific legs is flagged, not resolved.

9.4 Closing summary

S9 set out to build a graph-theoretic view of systemic risk across DeFi synthetic-asset protocols, test it against real historical collapses, and generalize it beyond a single pilot protocol. All three are done. The graph-construction, centrality, community-detection and propagation pipeline, built and validated on one protocol first, generalized to a real 59-asset, 110-node multi-protocol graph with zero changes to the underlying algorithms — the central design bet of building one protocol-agnostic pipeline rather than one bespoke implementation per protocol paid off in practice, not just in principle. Three independent historical replays, spanning three structurally different collateral regimes and three different years, all confirm the same honest conclusion: this model is a directionally sound, real, auditable early-warning tool, consistently on the right side of "flag serious risk or not", and consistently imprecise about exactly how much — a result worth taking seriously and worth stating plainly, rather than overselling.