Spatial Linear Models

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Goal:
- Which connectivity model (IBD or IBR) better explains genetic diversity?
- Can the same pattern be explained by patch size as a proxy for population size?
- Do the same factors explain genetic diversity across the study area?

Methodological Challenges:

Video 1: Neighbourhood-level analysis
- Define connectivity hypotheses as distance matrices
- Neighborhood approach: connectivity index Si
- Explain genetic diversity by patch connectivity Si

Video 2: Autocorrelation in linear models
- Generalized Least Squares regression (GLS)
- Spatial Regression with SAR
- Spatial Filtering with MEM
- Model selection (see also Week 12)
- Geographically-weighted Regression

Dianthus carthusianorum
(www.schmetterlingswiesen.de)

Refreshers: regression (Week 4), spatial statistics (Week 5), linear mixed models (Week 6)

Diantus Dataset

Rico et al. 2013, Molecular Ecology. See dataset video by Yessica Rico!

Basic Dataset

Patches: 106
Populations: 65
Individuals: 1602
Microsatellites: 11

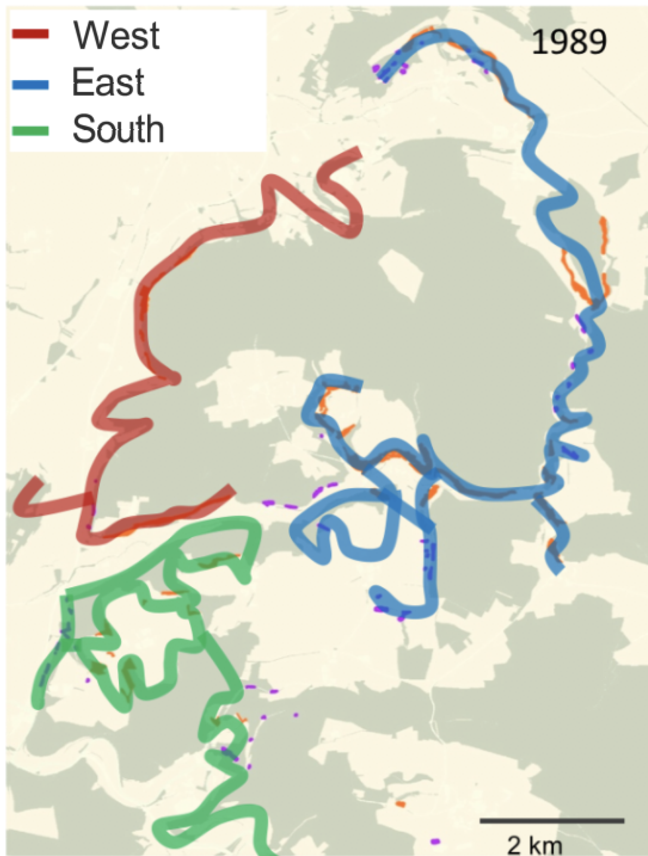
Population size:
< 4, < 40, < 100, 100+

Patch size: ha

Grazing intensity:
- ungrazed
- intermittently
- consistently

www.neuburg-schrobenhausen.de

Connectivity by Shepherding



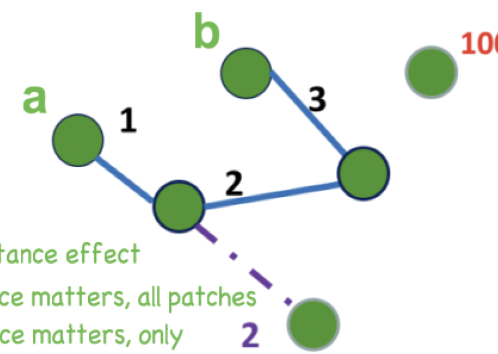
Figures and map: Y. Rico

Alternative Hyptheses

Geographic distance (IBD)

Forest as barrier (IBB)

Connectivity by shepherding (IBR)



- No distance effect
- Distance matters, all patches
- Distance matters, only consistently grazed patches

Connectivity Index Si

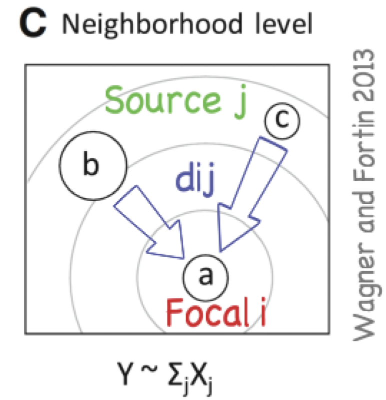
See Week 7 Bonus Material

Sum over all neighbours j

Connectivity Si of focal patch i

$$Si = \sum_{j \neq i} \exp(-\alpha d_{ij}) p_j$$

Source patch: pj, Aj, Nj



Connectivity model:
- ecological distance dij
- dispersal parameter alpha

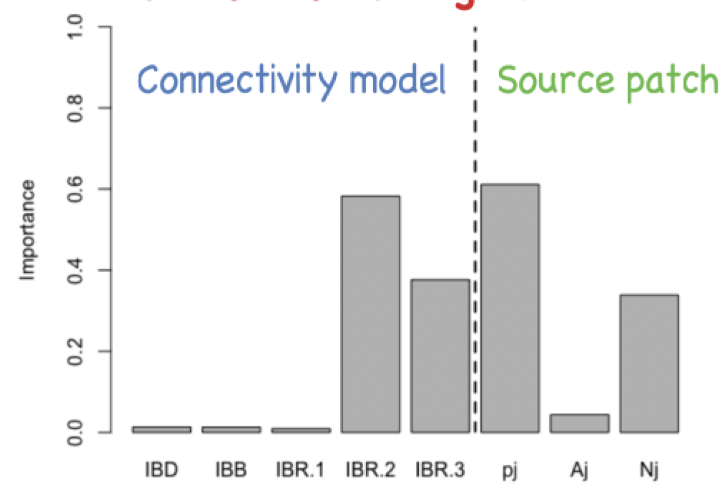
Correlation wit Allelic Richness

	pj	Aj	Nj
IBD	0.02	0.21	0.10
IBB	0.14	0.17	0.14
IBR.1	0.13	-0.05	0.11
IBR.2	0.40	0.20	0.37
IBR.3	0.37	0.27	0.37

MuMIn:
Multi-Model Inference

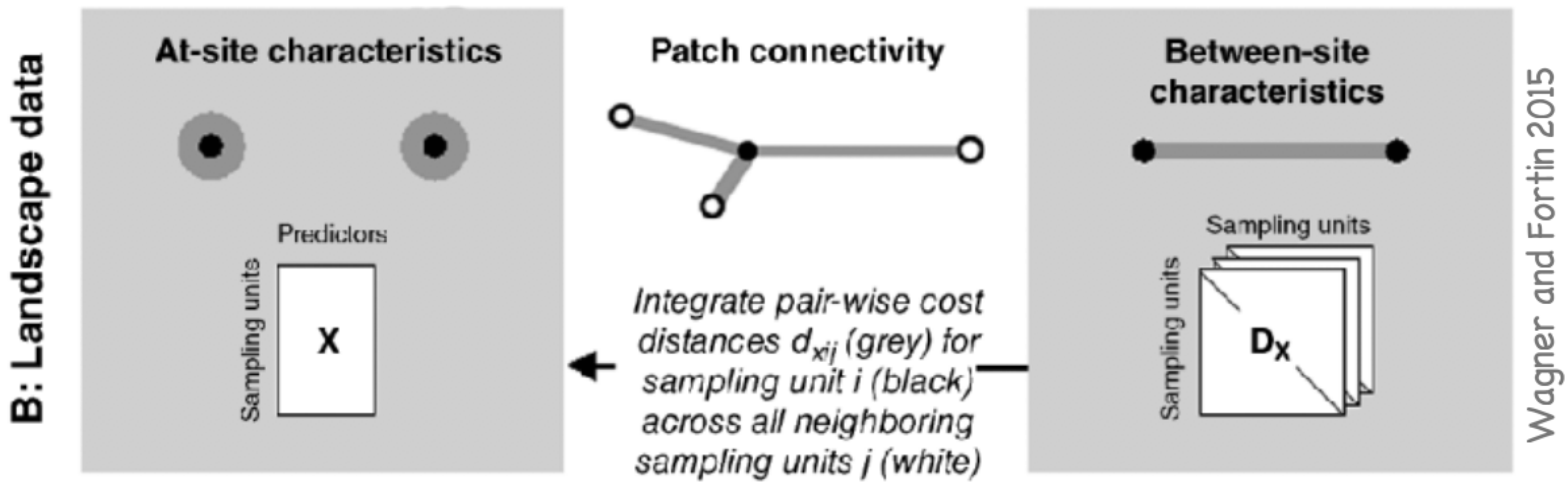
Calculate AIC
Compare models

Sum of model weights w



Back to Regression

Node Level ⇦ **Neighbourhood Level** ⇦ **Link Level**



Site-level data table

Connectivity Index Si

Distance matrices Dx

Multiple Regression etc.:

Allelic richness ~ Connectivity Si + Patch size

Mantel test etc.

Regression on distance matrices

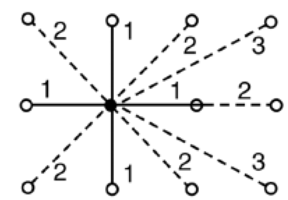
Generalized Least Squares (GLS)

Y = intercept + b * X + residuals

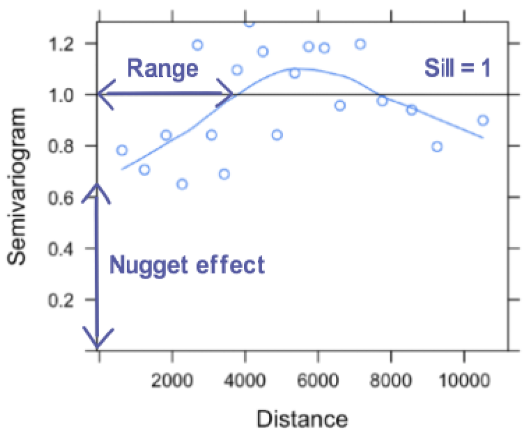
```
model.lm = lm ( Y ~ X, data)
lm.morantest ( residuals ( model.lm ))
```

Direction:	Sign of Moran's I?
Size:	Value of Moran's I?
Significance:	P-value of Moran's I?

Geostatistical model



Empirical Variogram



Fit Theoretical Variogram

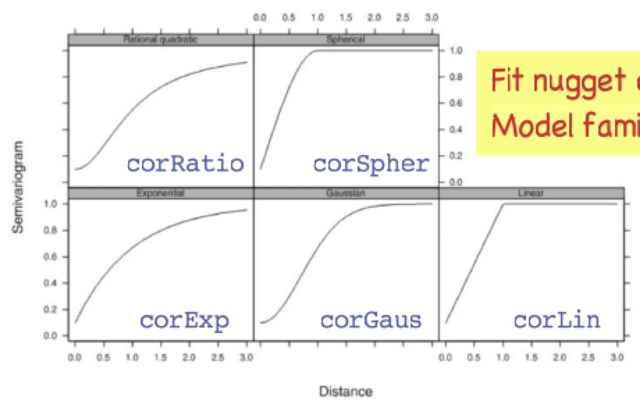


Figure: Pinheiro and Bates 2004, p. 233

Fit nugget effect?
Model family?

```
Variogram( model.lm, form = ~ x  + y )
gls( formula, data, correlation = corExp( form = ~ x + y, nugget=T ))
model.sel ( model.lm, mod.corRatio, mod.corSper, mod.corExp, mod.corGaus, mod.corLin )
```

Which Model Fits Best?

See also Week 12

Ranked models (best to worst)

Models penalized for df used

Criterion used: AICc

delta = AICc – AICc(best)

	(Intrc)	IBR	PtchS	correlation	df	logLik	AICc	delta	weight
mod.corExp	4.044	0.08468	0.04269	n::cE(x+y,T)	6	6.283	1.1	0.00	0.336
mod.corRatio	4.032	0.09704	0.04129	n::cR(x+y,T)	6	6.229	1.2	0.11	0.319
model.lm	4.052	0.11340	0.04266		4	3.702	1.3	0.29	0.291
mod.corGaus	4.047	0.11840	0.04247	n::cG(x+y,T)	6	3.763	6.1	5.04	0.027
mod.corSpher	4.048	0.11710	0.04242	n::cS(x+y,T)	6	3.760	6.1	5.04	0.027

Do estimates vary between models?

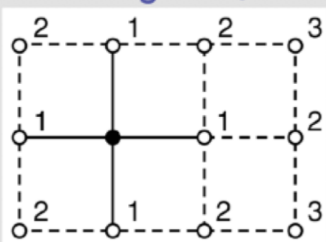
Model weight: Support, given all models in set

Spatial Regression with SAR

Simultaneous Auto-Regressive

SAR

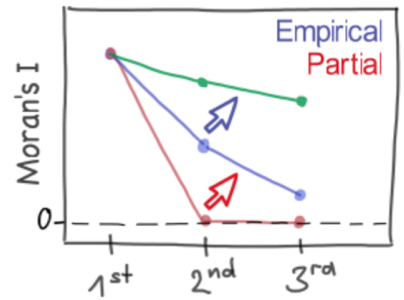
Weight wij



Error covariance = f(wij)

	Ind 1	Ind 2	Ind 3	Ind 4
Ind 1	1	0.7	0.2	0
Ind 2	0.7	1	0.3	0.1
Ind 3	0.2	0.3	1	0.8
Ind 4	0	0.1	0.8	1

Autocorrelation Function (ACF)

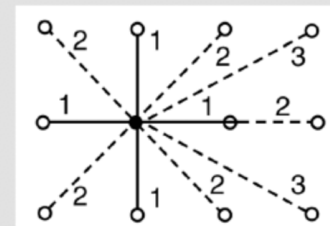


Weights reflect partial ACF

Variogram models empirical ACF

GLS

Distance dij



Error covariance = f(dij)

	Ind 1	Ind 2	Ind 3	Ind 4
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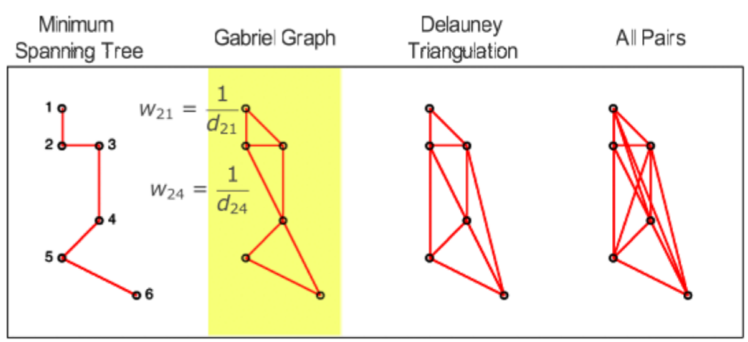
Ind 3	0.2	0.3	1	0.8
Ind 4	0	0.1	0.8	1

How to Choose Neighbours and Weights?



$$\sum_j w_{ij} = 1$$

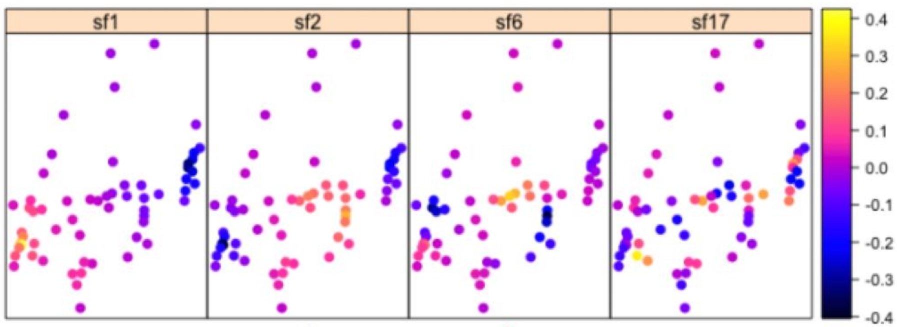
```
errorsarlm ( formula, data, listw )
model.sel ( model.lm, sar.Bin, sar.Inv.d1, sar.Inv.d2 )
```

Spatial Filtering with MEM

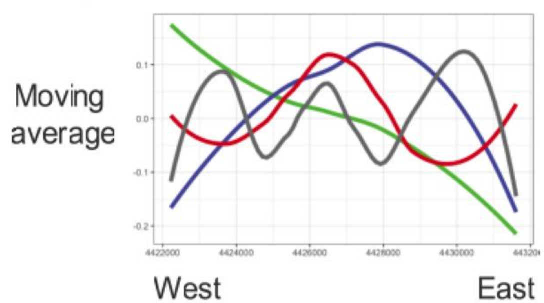
Moran
Eigenvector
Maps

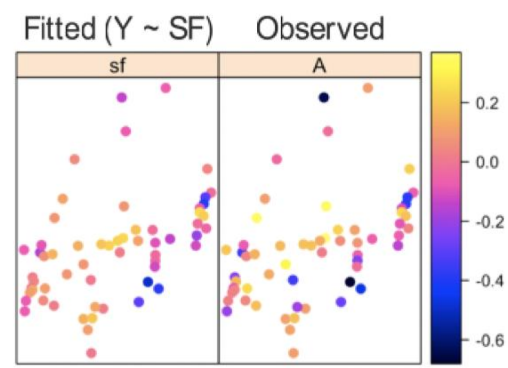
Goal: control for spatial variation when assessing 'lm (Y ~ X)'

Y = intercept + b.x * X + b.sf * SF + residuals



1. Define spatial weights matrix
2. Extract spatial eigenvectors (MEM)
3. Stepwise selection of significant MEM
4. Use as additional predictors SF
5. Effect of predictors X assessed from remaining non-spatial variation in Y

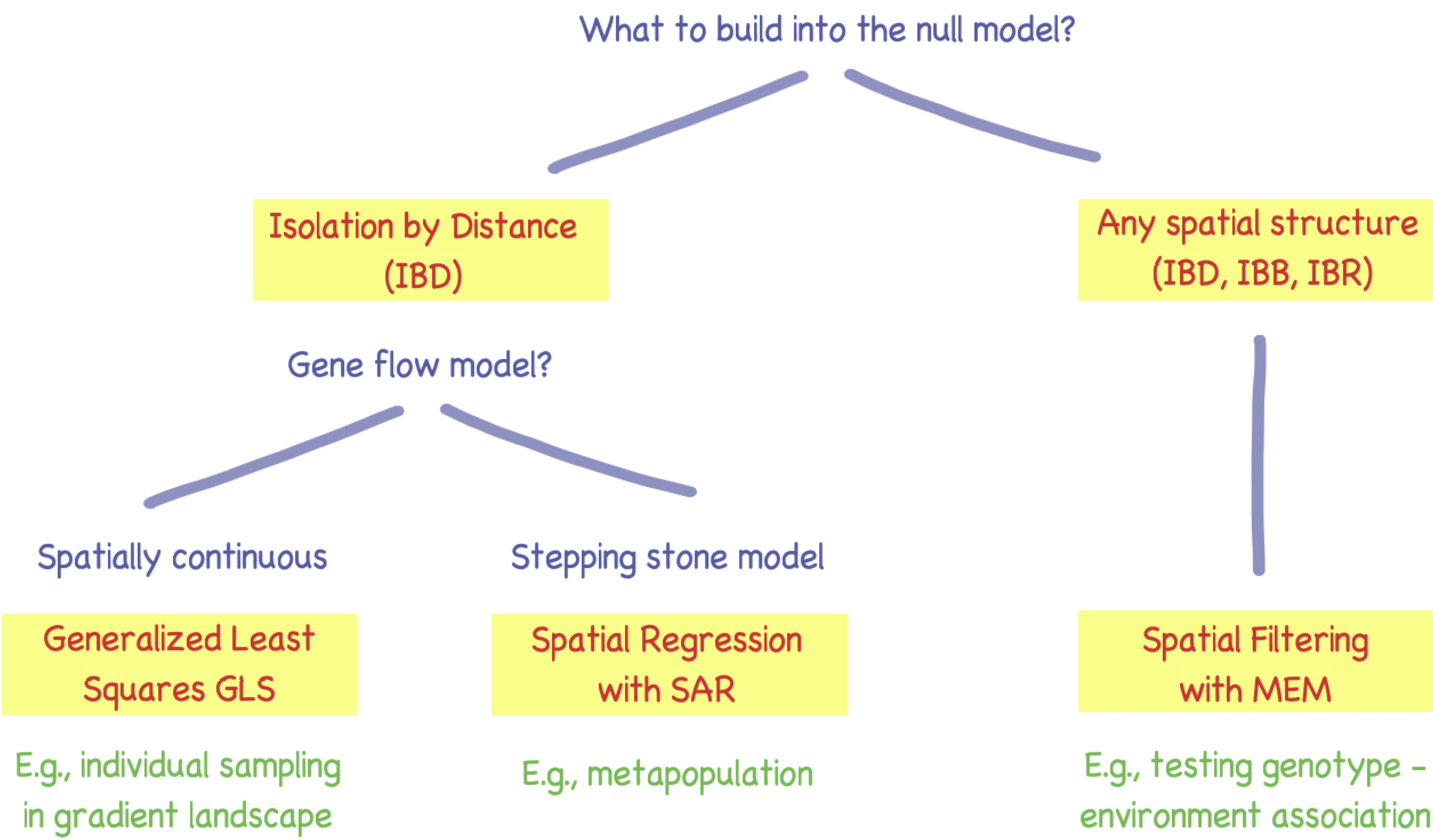





```
meig <- spmoran :: meigen ( coords, cmat )
spmoran :: esf ( y, x, meig, fn = "r2" )
spmoran :: resf ( y, x, meig, fn = "r2" )
```

cmat: 'connectivity matrix' = spatial weights
meig: spatial eigenvectors, eigenvalues
fn: method for stepwise selection
'esf': fixed effects, 'resf': random effects (REML)

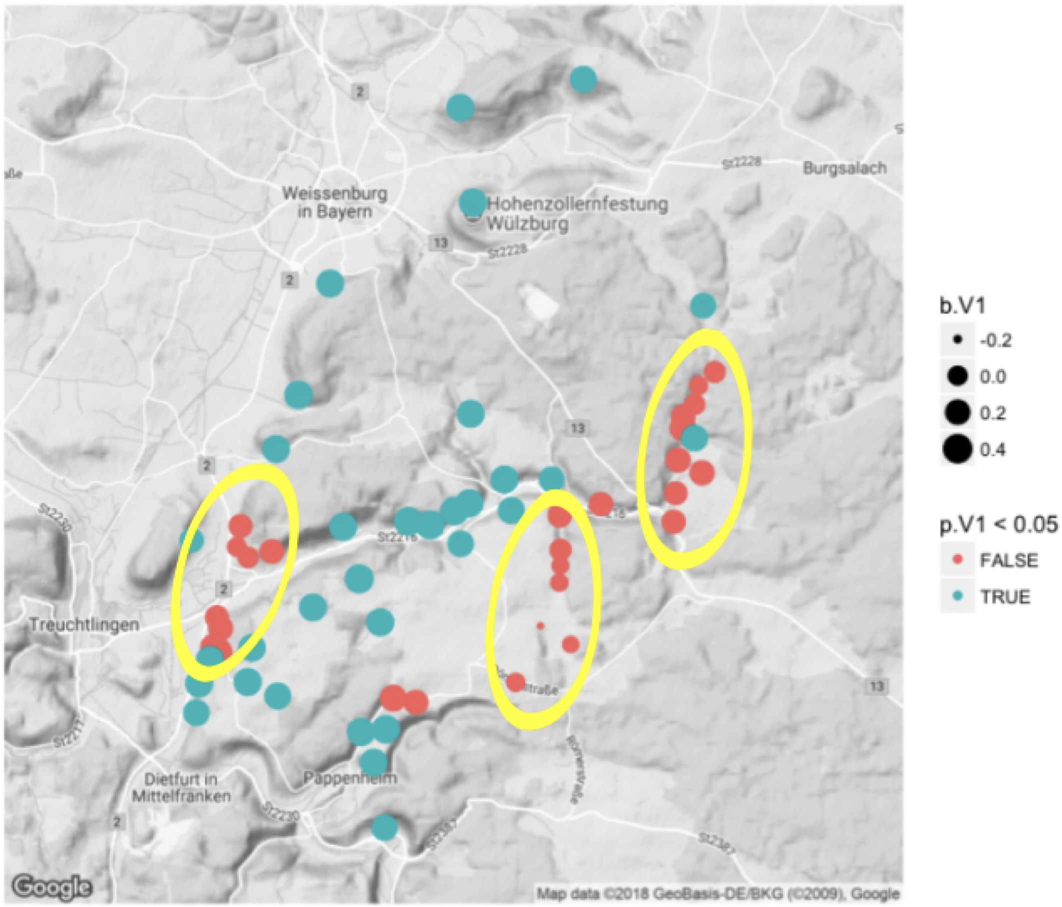
Which Method to Choose?

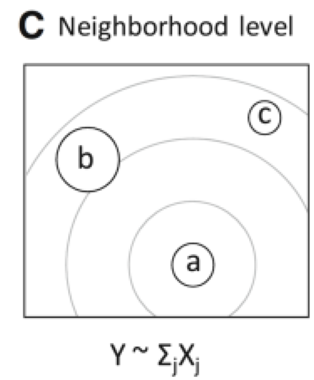


Spatially Varying Coefficients

Similar to Geographically Weighted Regression (GWR)

Does relationship hold across study area?





Slope estimate for Si.IBR varies across study area

Statistical significance varies across study area

(Intercept)		V1 = Si.IBR	
Min.	:3.801	Min.	:-0.2020
1st Qu.	:3.919	1st Qu.	: 0.1545
Median	:3.941	Median	: 0.2549
Mean	:3.941	Mean	: 0.2101
3rd Qu.	:3.969	3rd Qu.	: 0.3113
Max.	:4.060	Max.	: 0.4180

```
spmoran :: resf_vc( y, x, xconst = NULL, meig, method = "reml" )
```