

Modeling the Profitability of the Rocket Pool Smoothing Pool

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Abstract

This Monte Carlo based analysis attempts to quantify the likely real-world performance of a Rocket Pool (RP) minipool (i.e., an ethereum validator) that joins the smoothing pool (SP) versus remaining a *solitarius* minipool. The SP is an opt-in feature that will collectively pool the PPV of every member opted into it. *Solitarius* minipools are RP validators whose node operators (NO) have chosen not to opt-in to the SP and remain in their solitary configuration.

We accomplished the modeling using historical Ethereum proof-of-work mining block rewards to calculate future Ethereum staking (beacon chain) proposer payment value (PPV¹) rewards. We used an estimate of the number of beacon chain validators to determine the likelihood that a given minipool would be selected to propose a block post-merge.

Many Monte Carlo tries were performed, predicting the profitability of a set of two cohorts: a collection of solitary minipool(s) and the same set of minipools operated as participants in the SP. Profitability was defined as the amount of PPV earned in a fixed time interval.

We generated a series of plots that displayed the performance of both cohorts over time. We then calculated an average win-loss ratio over all the Monte Carlo trials to determine if joining the SP provided a performance advantage. Finally, we repeated the simulation for four configurations of minipools (1, 3, 10, and 50) operated by a single NO.

We also compiled a series of heat maps looking at a combination of ratios of NO minipools to the number of SP participants to determine how the performance varies based on the relative sizes of the two sets. We calculated an f fraction representing a NO proportion of the minipool in the SP and plotted that against the performance advantage.

Tldr A NO participating in the SP is more likely to receive larger monthly ETH rewards than running solitarius minipools. This depends on two prerequisites. First, the NO joins the SP only when their minipools will not compose the majority of minipools in the SP. Second, the NO will not validate for an indefinite time.

¹ PPV includes both priority fees and inclusion payments that are made payable to block proposers by block builders either via the feeRecipient address or a specific transaction in the pre-built block.

The Smoothing Pool

As part of the planned Q3 of 2022 Redstone² upgrade, Rocket Pool (RP) will add a feature known as the Smoothing Pool (SP). RP describes the new SP as “an opt-in feature that will collectively pool the priority fees of every member opted into it. This is a way to effectively eliminate the randomness associated with block proposals on the Beacon Chain.”

However, this analysis will show that the SP does more than simply reduce the monthly variability of earned PPV. It also has the added benefit of increasing the likelihood of participating minipools outperforming validators that decide not to join the SP. We referred to this later set as *solitarius* minipools to distinguish them from solo node operators that operate independent Ethereum validators that are not part of the RP protocol. *Solitarius* minipools are RP validators whose node operators (NO) have chosen not to opt-in to the SP and remain in their solitary configuration.

Modeling Variables

To determine both the potential returns for a *solitarius* minipool versus an SP participating minipool, we need to model the potential rewards that a minipool can receive. The value of potential rewards that any Ethereum validator, including RP minipools, will receive post-merge depends on two factors. First is the number of block proposals that a validator is randomly selected to produce. This number of expected blocks depends on two inputs: the length of time that the minipool will be validating and the probability of being chosen to propose.

Second is the value of *priority fees* and *inclusion payments* made to the block proposer. Collectively we will refer to the sum of priority fees and inclusion payment rewards as **proposer payment value** or **PPV**.

Validating for a Fixed Interval of Time.

To begin with, we need to acknowledge that a validator will not validate for an indefinite amount of time. This is because a NO will start validating at some time and exit validating later. Since validating is a long-term investment, this period may be in years or decades. We observed in our modeling that the performance effect is most pronounced for shorter periods of validating time. Still, it continues into any reasonable length of time that a NO can be foreseen to be validated for 5, 50, or even 100 years. We will have selected two-time durations for use in our model. The next 28 days corresponds to the RP award claiming period, and five years, which was suggested as a reasonable long-term commitment of investment funds for Ethereum staking of personal funds.

Proposer Opportunities

All validators who actively participate in consensus duties on the beacon chain are randomly selected to block propose. The chance of being selected is a straight probability whose odds are determined by the total number of validators (n) on the Ethereum network. At the writing of this report, the current n of Ethereum validators is 410,335.³ We will use $n = 425,000$ in our models as a reasonable estimate of the number of validators when the Merge is expected to take place.⁴

² <https://medium.com/rocket-pool/rocket-pool-the-merge-redstone-601d9efd6b4>

³ <https://beaconcha.in/> recorded on 7/23/2022.

⁴ Recent conversations among the core devs have suggested the week of September 19th, 2022, pending further testing on Goerli/Prater.

Using that estimate of total beacon chain validators, the median validator will be selected 30 times over a five-year validating period to propose a block. A histogram of the probability density functions for both a 28-day and a 5-year validating period is shown in figures 1 and 2. To model this, we used a standard binomial distribution from the `scipy.stats` library.

It is important to note that the frequency at which a minipool is expected to block-propose will decrease as the total validator count (n) increases. It is anticipated that the number of validators on the Ethereum network will increase as the Merge nears and continue to grow as the post-merge staking APR increases due to the inclusion of PPV for validating duties. A larger number of n can be chosen to model the lower chance of receiving a proposal due to the continued growth of the beacon chain.

Figure 1
Probability Mass Function of Block Proposer Opportunities in 28 days

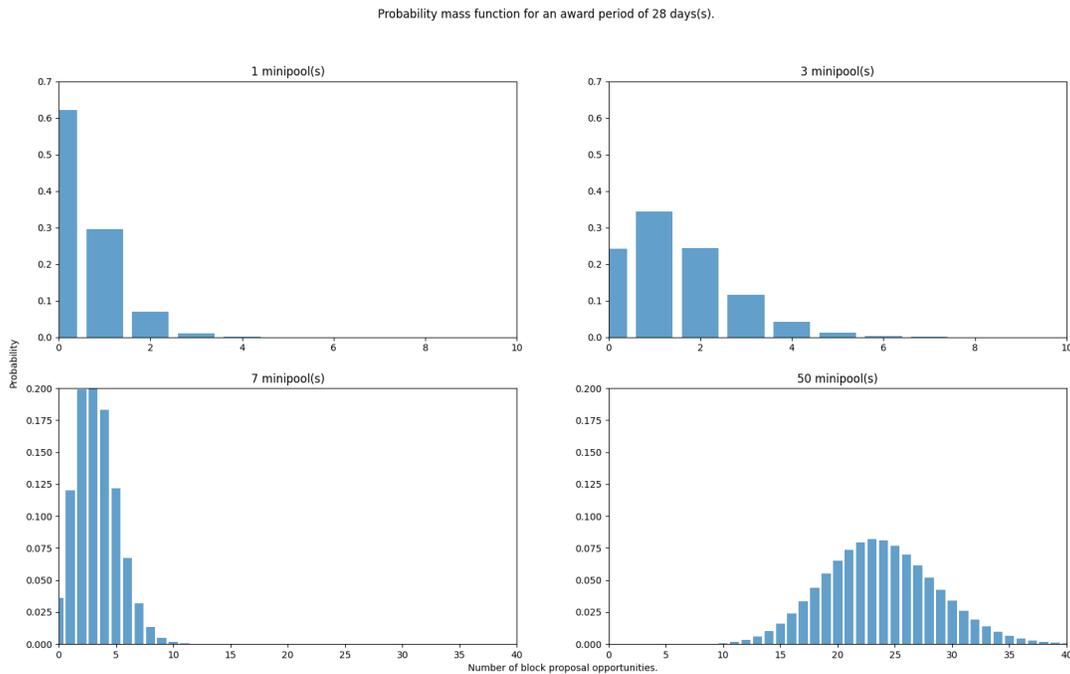
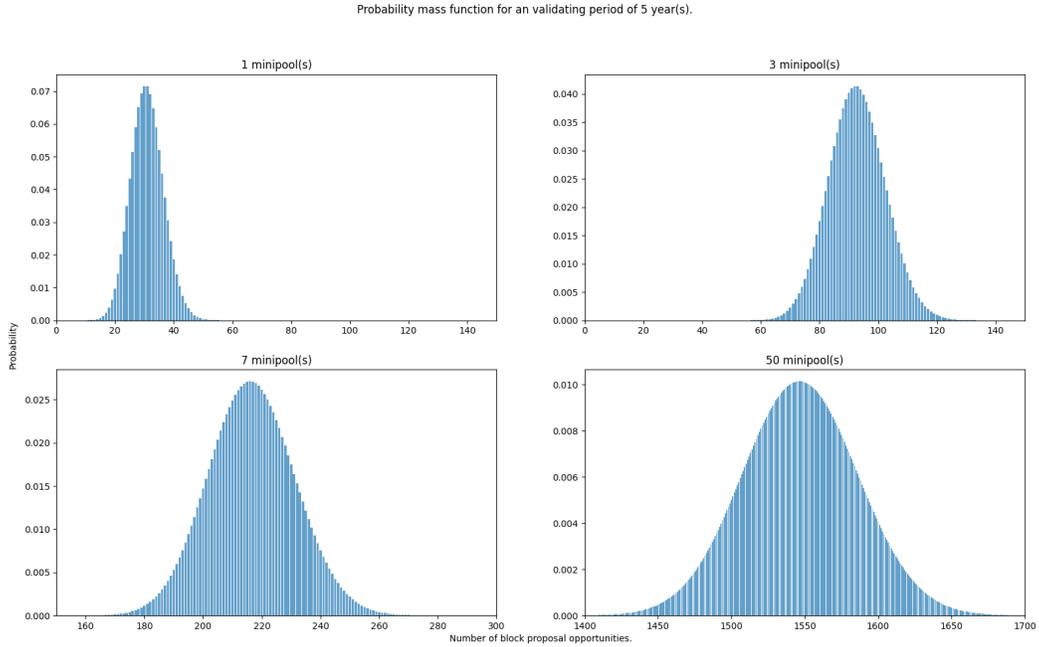
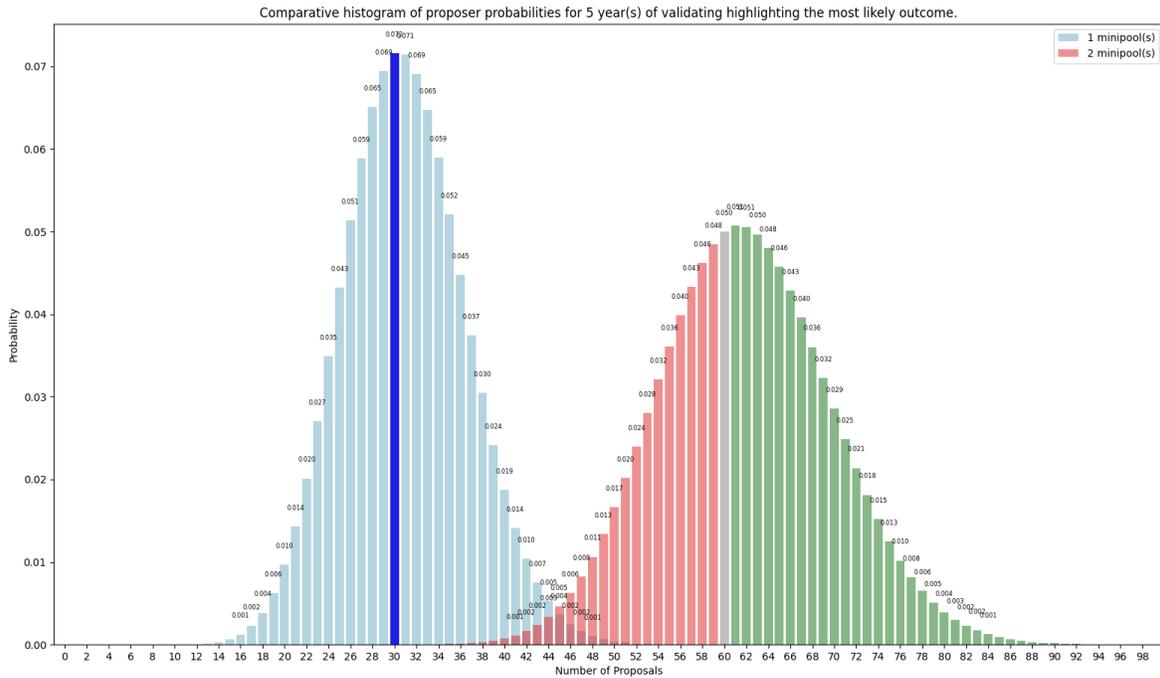


Figure 2
Probability Mass Function of Block Proposer Opportunities in 5 years.



In a given validating period, the number of times a team of 2 minipools will block-propose is expected to be twice that of a single minipool. The odds of any given validator being randomly selected to propose in any slot is simply $1/n$. The probability of a validator working in a team of two that either of the validators will be selected to propose in a slot is $2/n$. Their odds are twice the single minipool odds that at least one of their two teamed minipools will be selected as the block proposer. This comparison is shown in figure 3.

Figure 3
Comparative histogram of proposer probabilities for 5 years of validating, highlighting the most likely outcome.



The astute reader will notice that although the odds of any minipool being selected in a larger set of minipools has improved, the expected number of block proposals, when normalized to a per minipool basis, does not change. For example, the most likely outcome over five years of validating with one minipool is 30 proposals, and with two minipools, the expected outcome is 60 proposals which is still 30 proposals/minipool. We can observe later in our analysis that the improvement in the SP performance is not attributable to an increase in the relative odds of being selected. Instead, it is the fact that the more minipools operated, the more chances (proposals) are obtained in a finite validating period to propose a winning lottery block.

Proposer Payments Value (PPV)

The second prediction we need to perform is estimating the amount of proposer payment value (PPV) expected per minipool. This value is determined by network demand conditions and arbitrage opportunities for value extraction when the minipool is selected to propose a block.

To quantify PPV, we first need to understand the “maximum extractable value” (MEV). MEV refers to the total value that can be extracted from the blockchain by altering the sequence of transactions and injecting bot-generated transactions to take advantage of the mempool and EVM blockchain state. Projects like Flashbots MEV Explore⁵ have attempted to

⁵ <https://explore.flashbots.net/>

quantify and measure a subset⁶ of MEV occurring in the Ethereum network. Most of the transactions observed by Flashbots use timed arbitrage transactions to front-run an automated market maker (AMM) transaction.

We discussed in more detail the composition and overall trends of MEV in a previous report, *A Risk Analysis of Rocket Pool Low Ether Bonded (LEB) Minipools*.⁷ Readers are encouraged to read the section “Proposer Payments Value (PPV)” if they are not already familiar with the subject. Although it is impossible to foresee the actual amount of PPV in the future, we can predict by assuming that the MEV proposer payments obtained in recent transactions are probably representative of proposer payments in the future.

To estimate the amount of PPV in a possible future block, we first need an accurate measurement of priority fees and other inclusion tips made to the block proposer from recent block transactions for use as a surrogate dataset for future MEV possibilities. We downloaded via the etherscan.io API⁸ a record of the last 432,000 blocks worth of data that corresponded from block 14,771,399⁹ to block 14,339,400.¹⁰ These block times varied due to the proof-of-work mining protocol and spanned a total time frame of 67 days, 16 hours, 23 minutes, and 32 seconds. However, we will use this data as a representation covering exactly 60 days as each post-merge as each block will be precisely 12 seconds in duration.

Two ETH (2E18 wei) were subtracted from the `blockReward` field to model only inclusion fees and `coinbase` payments. Even after that correction, it is important to note that this dataset only contains the on-chain payments made to the miner. It does not include any off-chain payments made for block ordering. As such, these payments are not included in the forecasts made by the analytical techniques performed as part of this risk assessment.

The modified etherscan.io dataset will be the basis for Monte Carlo models. The downloaded etherscan dataset was plotted on a semi-log chart in figure 4. Let’s take a moment to understand the distribution and shape of this histogram. It is important to note that there is a tremendously wide distribution of PPV per block. Figure 3, graphed on a semi-log scale, shows an extremely long-tail distribution skewed to the right.

As visualized, most blocks have very little PPV, while some rare blocks in the long-tail distribution have a significant amount of PPV. The most lucrative blocks where the PPV is greater than or equal to the NO deposit will be referred to as *lottery blocks*. The top 1% of blocks (ordered from smallest PPV to largest) contain 45% of the PPV. This means that we can expect that a single minipool in a fixed duration, even if years in length, is highly unlikely to be lucky enough to access these lottery blocks of PPV. However, an SP that is x times larger than that minipool has x times more likely chance of obtaining a lottery PPV block proposal.

This is because most of its value in the SP is obtained from a very small set of the luckiest minipools that receive exceptionally large PPV blocks. As the modeling period lengthens, assuming all conditions are equal, all validators have a random chance of being one of the lucky validators on the network. The expected performance of a single minipool moves from the median toward the average.

⁶ Flashbots inspects only 10 DeFi protocols and tracks profit for single-tx MEV opportunities. It should be considered a lower bound of the total estimate on MEV.

⁷ <https://github.com/htimsk/LEBminipools/blob/main/report/Analysis%20of%20LEB%20Minipools.pdf>

⁸ <https://api.etherscan.io/api>

⁹ Block mined May-14-2022 03:45:52 AM +UTC

¹⁰ Block mined Mar-07-2022 11:22:21 AM +UTC

Figure 4
Probability Mass Function Histogram of Block Proposer Payments in ETH

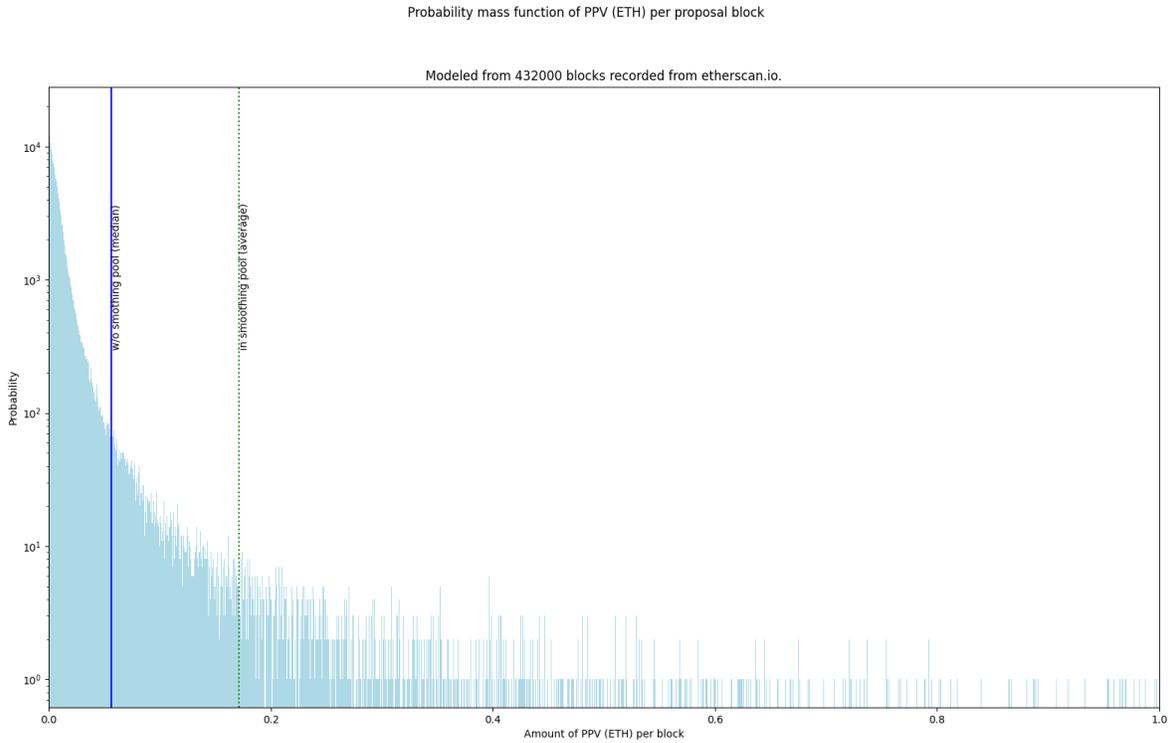


Table 1
Statistical Analysis of blockRewards data form etherscan.io

```

===== RESTART: RP_PPV_blockReward_Histogram.py =====
number of blocks analyzed = 432000
The timespan of block sampled is 60.0 day(s)
  mean      0.1711 ETH in a block
  median    0.0561 ETH in a block
  std       2.2285 ETH
  sum       73900.80 ETH
  
```

Value of PPV reported in ETH						
Cutoff	PPV/blk	lowerSum	lowETH%	upperSum	upETH%	sum
[50.	0.056	5245.174	7.098	68655.627	92.902	73900.801]
[84.1	0.161	19382.128	26.227	54518.673	73.773	73900.801]
[95.	0.363	30207.79	40.876	43693.01	59.124	73900.801]
[97.7	0.646	35710.342	48.322	38190.459	51.678	73900.801]
[99.	1.265	40599.572	54.938	33301.229	45.062	73900.801]
[99.9	12.644	52172.177	70.598	21728.623	29.402	73900.801]]

Foreshadowing the SP Advantage

The entire performance advantage of the SP can be learned from the following statements:

The median is only 0.0561 ETH. A single minipool is best characterized as likely to receive a block proposal represented by the median value of the distribution. Half the single minipools would be expected to receive a PPV above this number, and half would be expected to be below this number. The median is the best value to represent the amount of PPV that a single solitarius minipool would be expected to receive when it is presented with the opportunity to block-propose.

The average PPV was 0.1711 ETH. A large enough set of minipools that collectively shares their PPV rewards is best characterized as likely to receive a block proposer share represented by the mean value of the distribution. The mean value is at the 85% percentile of all PPV block rewards in the historical dataset of proposer payments. The performance enhancement when a solitarius minipool that was likely to receive a median (50%) reward is now likely to receive the mean (85%) award. This large difference in percentiles establishes an upper bound to the performance enhancement experienced by joining the SP.

Assuming that the blockchain events that lead to large MEV blocks are randomly distributed in time, the chance that an individual wins a lottery block is improved only by the number of entries (block proposals) they have in that given window. (i.e., The lottery - You only improve your odds by buying two tickets for the same drawing, not two tickets, one for each week.) Since the chance of receiving a large PPV block is low, the more tries we enter in this defined period, the more likely we will obtain a successful outcome and experience proposing a block with a high PPV award. Since SP participants are in a pool with a higher likelihood of getting more blocks/minipool in that period, so they have better odds of winning a lottery block. Because of the long-tail distribution of lottery proceeds, even after sharing it with all the participants in the SP still, on average, earn more than the median solitarius minipool.

The caveat, of course, is that there is still a chance (albeit not likely) that a solitarius minipool will get a lottery block proposal, and since they are not in the SP, they don't have to share it with anyone. A bit of a winner-keep-all treasure hunt opportunity for those that want to press their luck and are content with likely sub-performance versus joining the SP.

Modeling Method

We performed a Monte Carlo analysis predicting 1,000 tries where each modeled validator was assigned a probability of proposing and then randomly assigned one of the representative 432,000 historic blocks as their estimated PPV reward.

Statistical analysis and visual plots of the aggregate Monte Carlo tries were then performed to make general statements about the probable returns that a typical (the median 50th percentile) NO would receive. We also computed the probabilities for a more lucky validator (the 95th percentile).

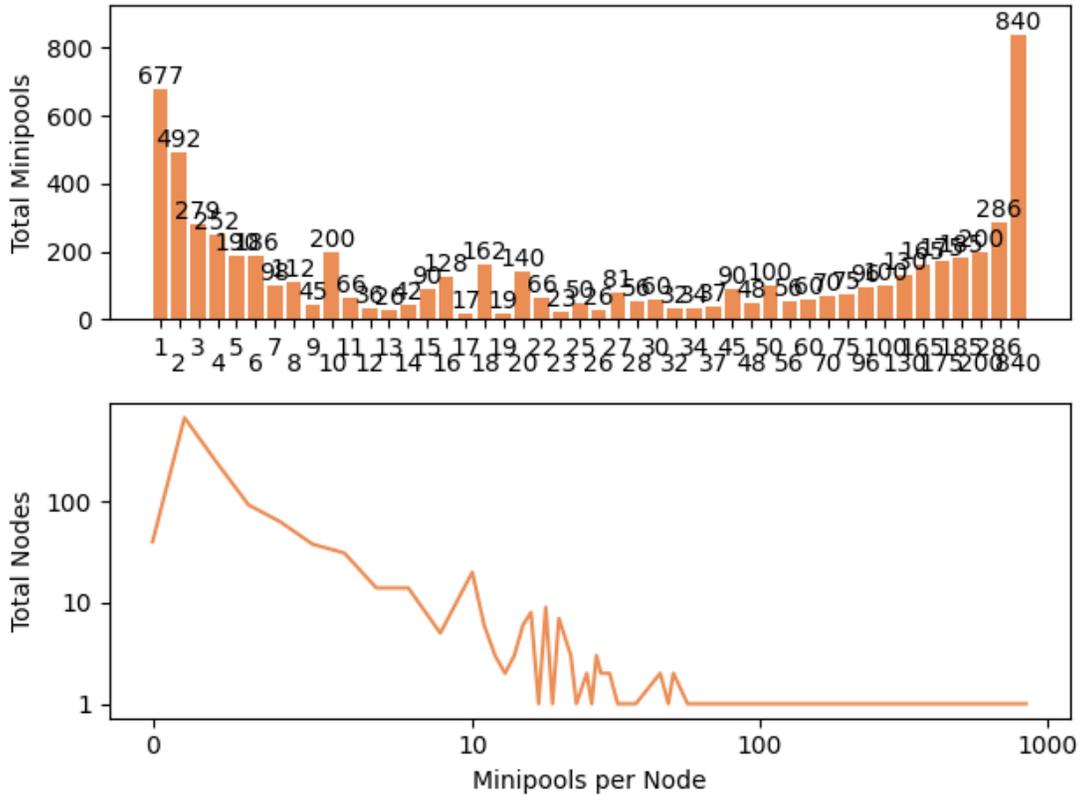
The Monte Carlo modeling algorithm is as follows:

1. Determine some period we wish to model our performance over *a priori*. This duration consisted of a validating duration of 5 years and a periodic smaller duration of 28 days that we used as time points to measure the minipools performance. We used five years as a reasonable period that an individual would commit capital assets in a long-term investment. Twenty-eight days was chosen to match the RP claims period, which is the shortest time an SP participant can receive payments from block proposals. The reader is encouraged to use the provided python code and explore performance curves for longer or shorter investment durations of time.

2. Perform a try. Guess the number of blocks that a solitarius minipool would randomly be selected to propose over that period using a binomial distribution with the probability of being selected based on the estimated number of active validators on the network. ($n = 425,000$ validators). Repeat this for each minipool in the set. We used sets of 1, 3, 7, and 50 minipools operated by a NO which correspond to the percentile of minipools operated by the existing set of RP NOs of 50%, 75%, 90%, and 99%, respectively. <Insert image here from Invis>
3. For each predicted block proposal of a try, randomly choose one of the historically mined blocks from the ehterscan.io dataset containing 432,000 of the most recent blocks. Use this selected modified `blockRewards` value as the surrogate to predict PPV.
4. Use 3,000 as the size of the SP. A number equal to 3000 minus the number of NO minipool will be used to form the SP cohort of minipools that do not belong to the NO (i.e., other people's minipools in the SP). We chose the size of 3,000 for the combined size of the SP as this number is about half the currently active number of RP minipools. Perform steps 1 - 3 above for this set of `smoothie` minipools.
5. To form the alternative configuration (e.g., the NO joined the SP), we will add the performance of the NO's minipools to this set of `smoothie` minipools. Collectively the rewards by each participating SP minipool (the NO minipools and the smoothies) will be summed and the average determined as the SP share of rewards for each participation minipool. This way, the two case scenarios will have been created that use the same try outcomes of the NO minipools. Case A where the NO had run solitarius minipools and case B where the NO had joined the SP.
6. Record information about this try. The try number, the measurement period, the amount of PPV earned in the measurement period, and the cumulative amount of PPV earned up to this measure interval from all the past measurements. These values are recorded both for the solitarius minipools and the SP participants. The share of PPV earned for each SP participant is calculated. Some running statistics about the number of times that the SP PPV earned exceeds the earn of the solitarius case are also recorded.
7. Repeat this Monte Carlo simulation 1,000 times.
8. Finally, display a plot and summary of the analysis of the results.

The use of the Monte Carlo method will allow us to make some generalizations about the performance of both the solitarius and SP case sets. The typical performance of each case is represented by the median value of the tries.

Figure 5
Rocket Watch Bot /minipool_distribution.



50th percentile: 1 minipool per node
 75th percentile: 3 minipools per node
 90th percentile: 7 minipools per node
 99th percentile: 50 minipools per node
 Max: 840 minipools per node
 Total: 6398 minipools

Model Results

In the narrative of this report we will display the plots using a single solitarius minipool for illustrative purposes. The reader can find the other plot outputs for NO minipool sets of 3, 7, and 50 minipool sets in our GitHub repository (<https://github.com/htimsk/SPanalysis>). The reader is also encouraged to download and use the python code to model any combination of NO minipools and SP participants.

The lines of profitability shown on these charts only account for PPV payments. They do not include the consensus layer rewards that a NO earns for performing validating duties.

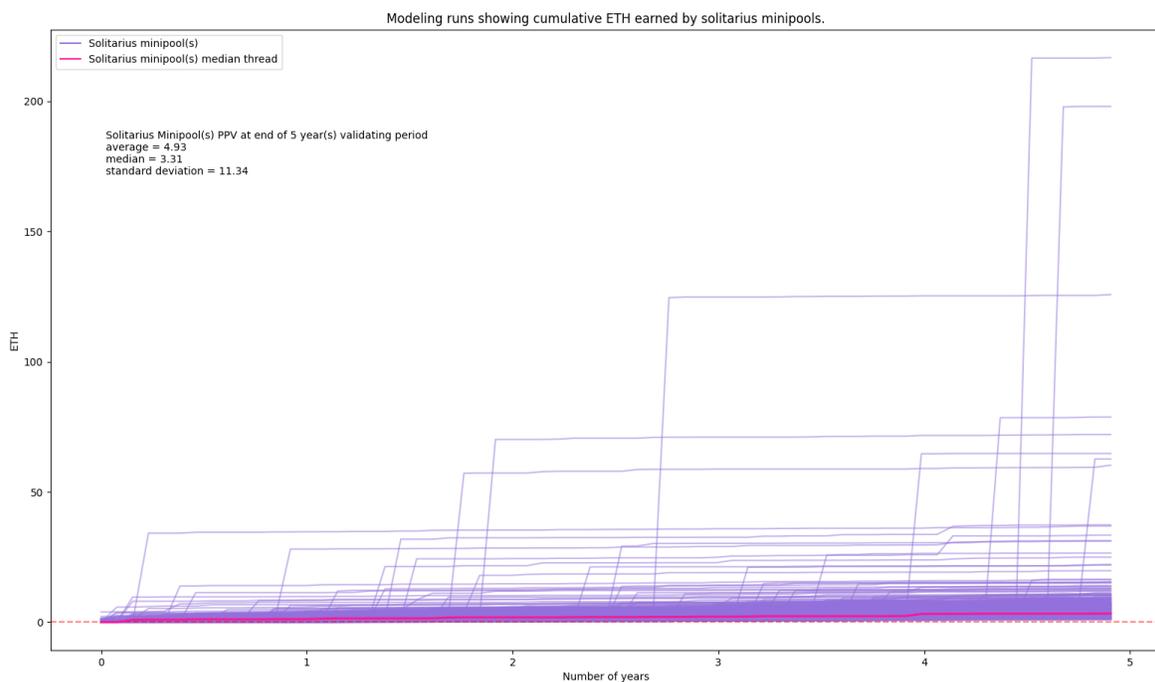
Solitarius Minipool Performance.

In figure 6, we observe a purple line for each of the trials conducted for the case of solitarius minipools. The pink line represents the median track selected at the end of the validating period. You can see here that the density of lines is mainly concentrated at the bottom of the chart. Breaking away from this aggregate are occasional threads that shoot nearly vertically upwards.

Each of these upward lines represents a try when one of the solitarius minipool(s) was lucky and received a lottery block as part of their proposal opportunities. All the line traces terminate after the five-year validating period. We can see specific metrics about the Monte Carlo dataset, including the median, mean and standard deviation displayed in a text box.

Figure 6
Modeling runs showing cumulative ETH earned by solitarius minipools.

Assuming: 425000 beacon validators; NO operating 1 minipools; 3000 SP participants; $f = 0.0$, 28 d reward period; modeled by 1000 tries.



SP Participant Performance.

Next, we modeled the second case where the NO opted into the SP. To model this, we selected a number of an additional set of `smoothie` minipools that, when added to the NO minipools, fill the SP to the desired 3,000 participating minipools model parameter. We repeated the Monte Carlo modeling routine of determining the likelihood of each minipool being selected to propose.

We again selected a random PPVblock from the historical dataset for each proposal. The aggregate earnings of these additional `smoothie` validators are added to our initial NO minipools. The summed PPV earned is calculated. We then redistribute those SP earnings by evenly splitting the PPV by the number of minipools participating in the SP (in our model, 3,000).

Figure 7
Modeling runs showing cumulative ETH earned by solitarius minipools.

Assuming: 425000 beacon validators; NO operating 1 minipools; 3000 SP participants; $f = 0.0$, 28 d reward period; modeled by 1000 tries.

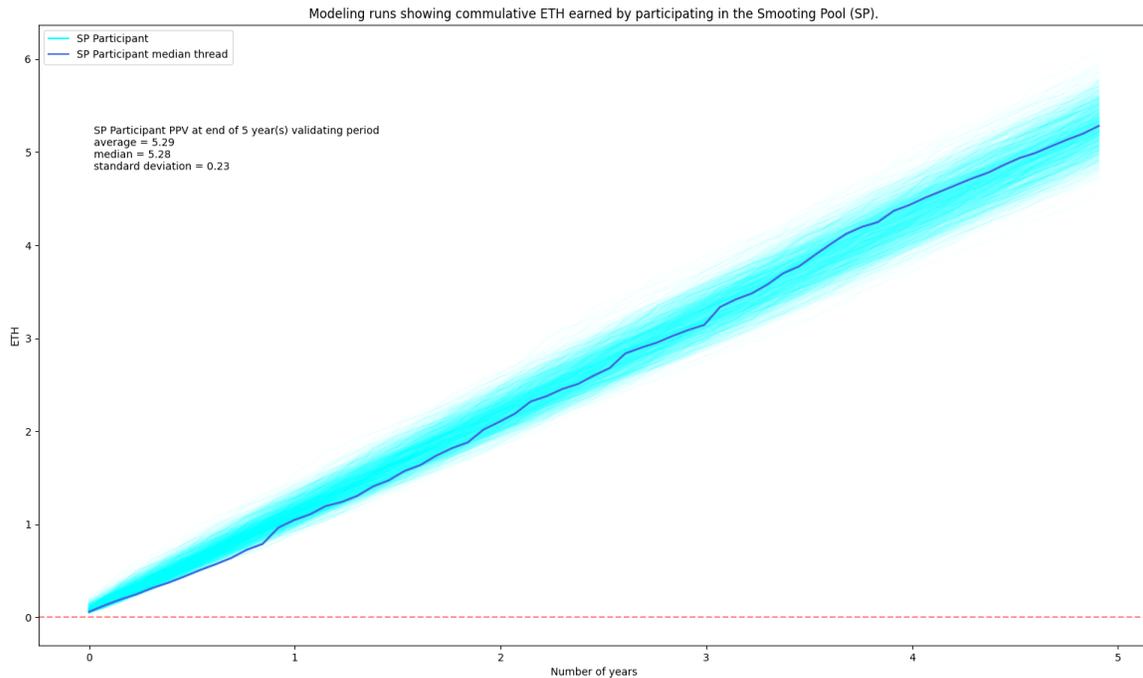


Figure 7 shows the light blue line traces representing the tries modeling this SP. The value shown is the share of SP earnings due to the NO. For example, if the NO had operated only one minipool, they would receive $1 / 3000$ of the earnings, 0.00034%. If the number of minipool operated by the NO is 50, then the NO would receive $50 / 3,000$ or 0.017% of the SP earnings.

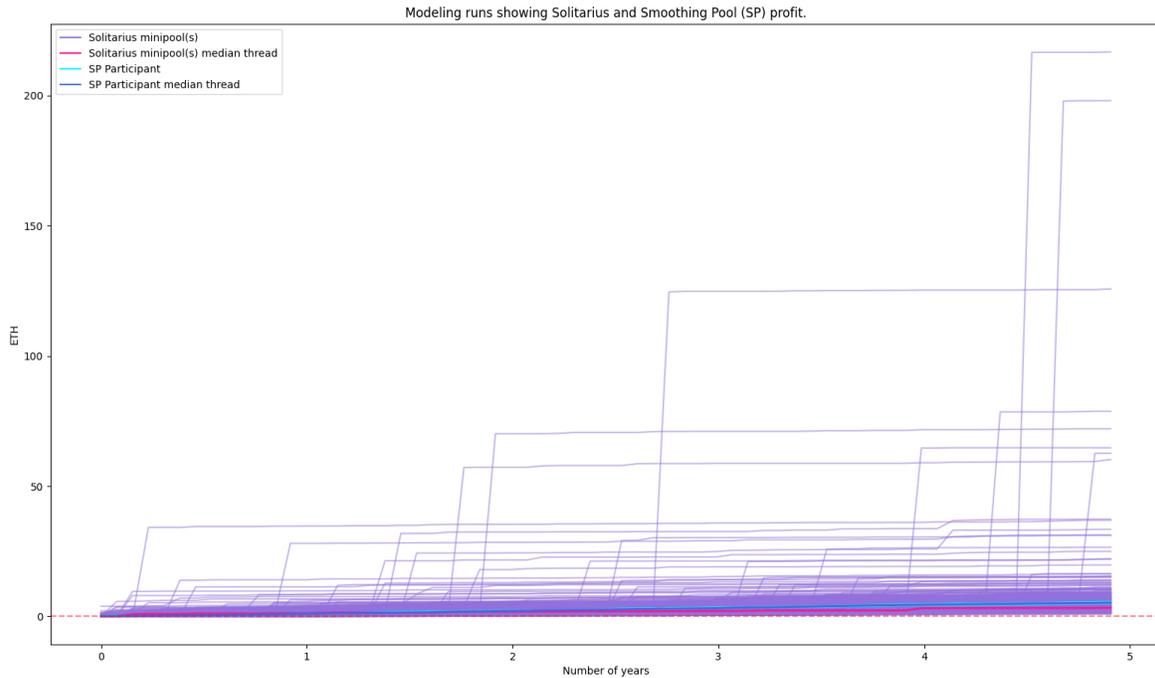
The dark blue line highlighted is the median thread as measured by the amount of PPV earned at the conclusion of the 5-year validating period. Because of the sizable averaging effect, the median (5.29 ETH) is nearly identical to the SP's mean (5.28 ETH), with the standard deviation calculated at 0.23 ETH.

Comparing the Outcomes

In figure 8, we can overlay the two previous images to create a graphical representation of the likely outcomes of both cases, SP vs. solitarius. We can see that the SP shown in light blue outperforms most of the solitarius minipool tries. The solitarius minipools fortunate enough to win lottery blocks are visible as the lines extend upwards rapidly. We highlight in dark pink and blue the median threads for each case as measured by the total PPV earned at the conclusion of the validating period.

Figure 8
Modeling runs showing Solitarius and SP profit.

Assuming: 425000 beacon validators; NO operating 1 minipools; 3000 SP participants; $f = 0.0$, 28 d reward period; modeled by 1000 tries.



Of importance to note is that nearly all of the modeling tries for the SP outperformed (outranked) the median expected performance of a solitarius minipool. Although a lottery-winning solitarius mini pool can outperform the median SP participant, there were very few times in which the solitary case proposed a lottery block such that the solitary case earned more than the median SP try.

To better illustrate this fact, in figure 9, we have removed the SP threads and now show the median thread in dark blue. This demonstrates that most solitarius minipools are predicted to perform less than the median performance number of the SP. We have also overlaid the average performance of the SP with a dotted line, which is nearly identical to the trace of the median thread.

Figure 9
Modeling runs showing Solitarius and averaged SP profit.

Assuming: 425000 beacon validators; NO operating 1 minipools; 3000 SP participants; $f = 0.0$, 28 d reward period; modeled by 1000 tries.

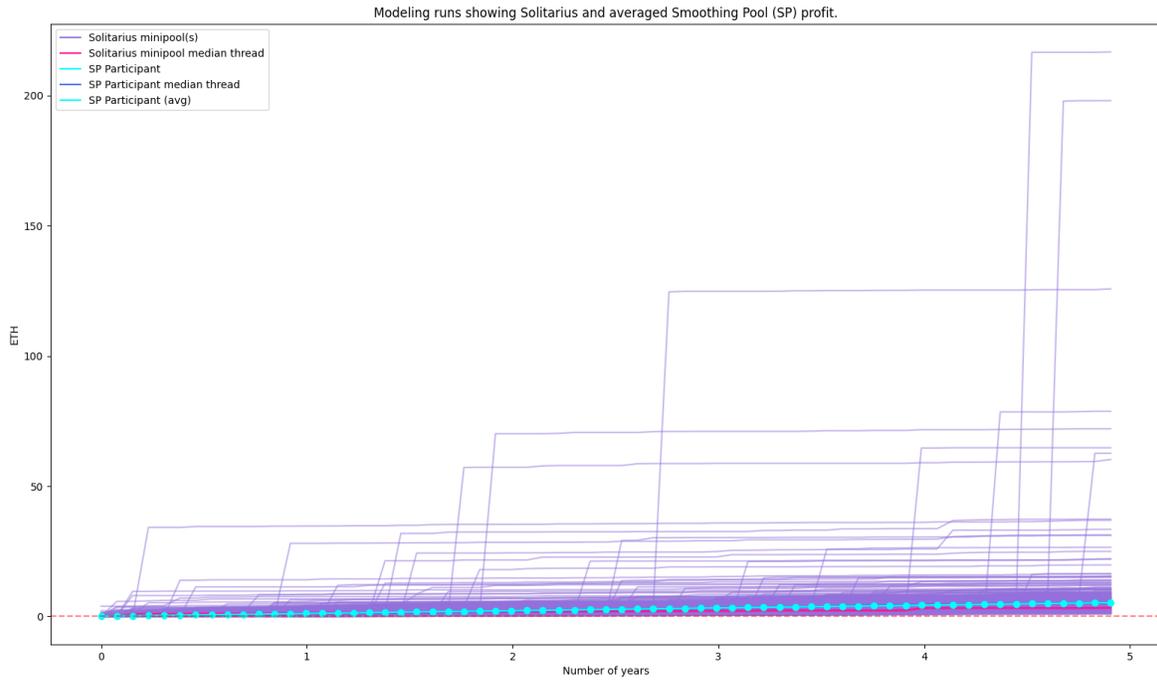
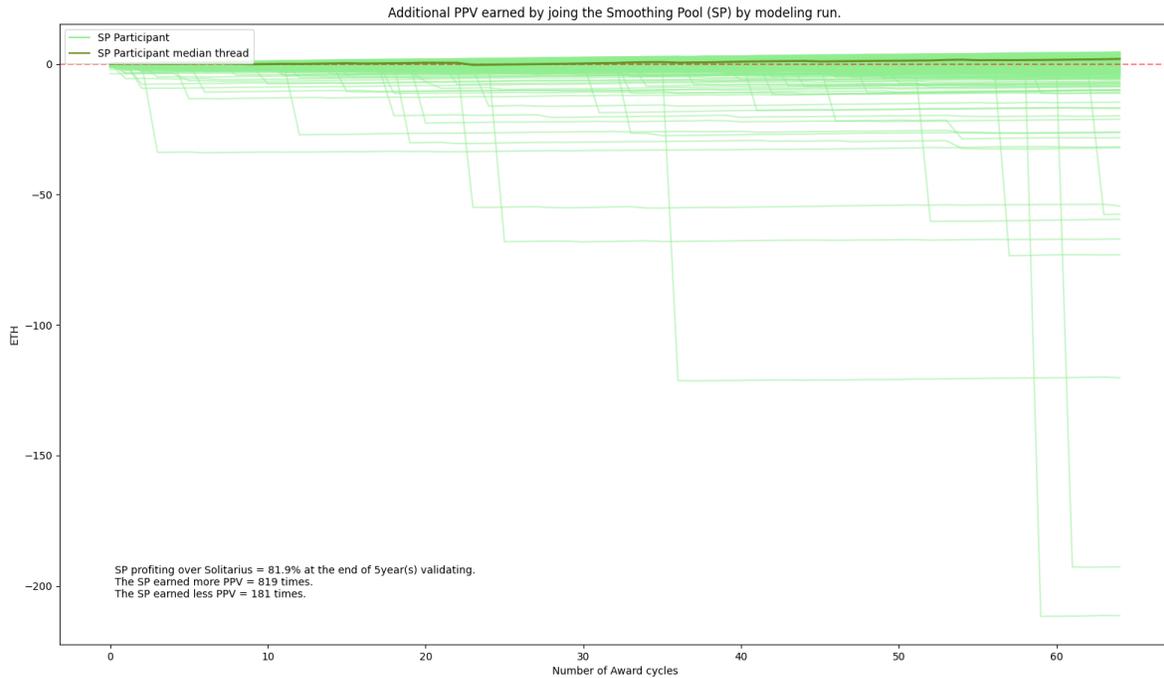


Figure 10
Additional PPV earned by joining the SP by modeling run.

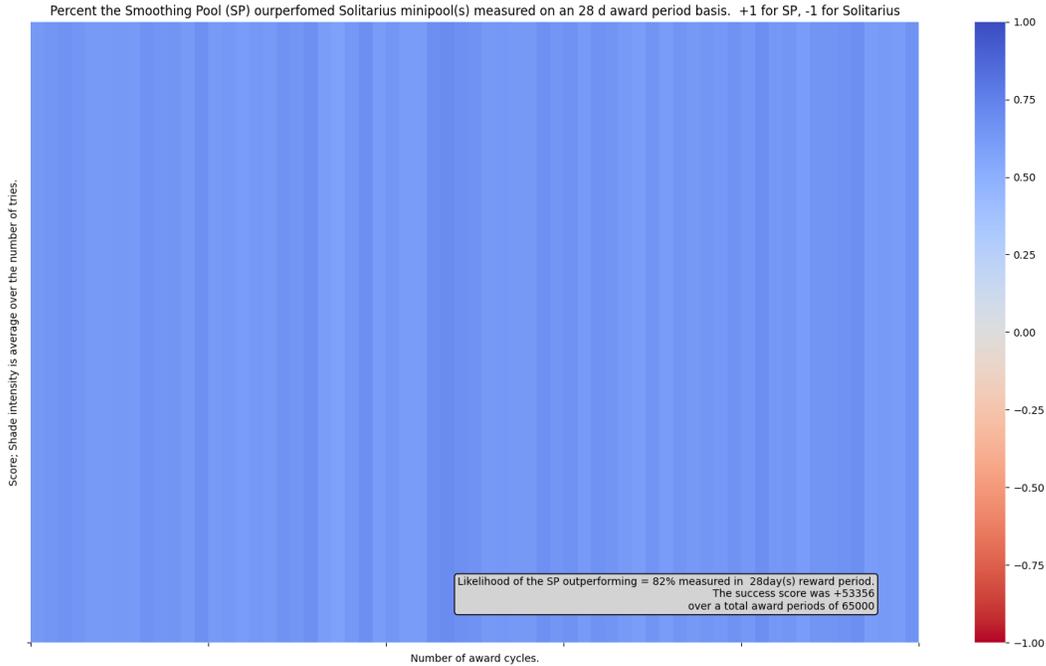
Assuming: 425000 beacon validators; NO operating 1 minipools; 3000 SP participants; $f = 0.0$, 28 d reward period; modeled by 1000 tries.



Another perspective is to compare the PPV profit that a NO makes by participating in the SP vs. operating solitarius minipools. To graph this for each Monte Carlo try, we take the NO's share of the smoothing pool at the end of the validating period and subtract the earned PPV from the corresponding cohort where the NO minipool(s) continued to operate in a solitarius method. If the SP outperforms the solitarius configuration, the difference will be a positive value. If the solitarius minipool was fortunate and proposed a lottery block, the difference will be a negative ETH value. You can see from this graph that the SP was more profitable for most tries at the end of the validating period. For this example of 1,000 trials, the SP outperformed the solitarius mini pools 82% of the time (819 tries). In only 181 cases did the solitarius minipools end the validating period with more ETH in PPV than the SP. The median try (ranked from most profitable to least) is shown in the dark olive color.

Figure 11
Percent the SP outperformed the Solitarius case measured on an award period basis.

Assuming: 425000 beacon validators; NO operating 1 minipools; 3000 SP participants; $f = 0.0$, 28 d reward period; modeled by 1000 tries.



We can further view the propensity for the SP to outperform solitarius minipools by creating a “keyboard” chart that uses a heatmap to illustrate the percent of times that the SP outperformed Solitarius minipools. The keys represent a per reward block basis (every 28 days). Although random chances lead to some blocks having a higher level of performance versus others, over the entire validating period, there is a strong positive correlation for the SP to outperform solitarius minipools. The likelihood of the SP outperforming was 80%. In this model, the SP outperformed the solitarius minipool 5,199 times out of 6500 total award periods.

In our GitHub repository (<https://github.com/htimsk/SPanalysis>), we have included the modeling plots for the other minipool configurations of 3, 7, and 50 minipools.

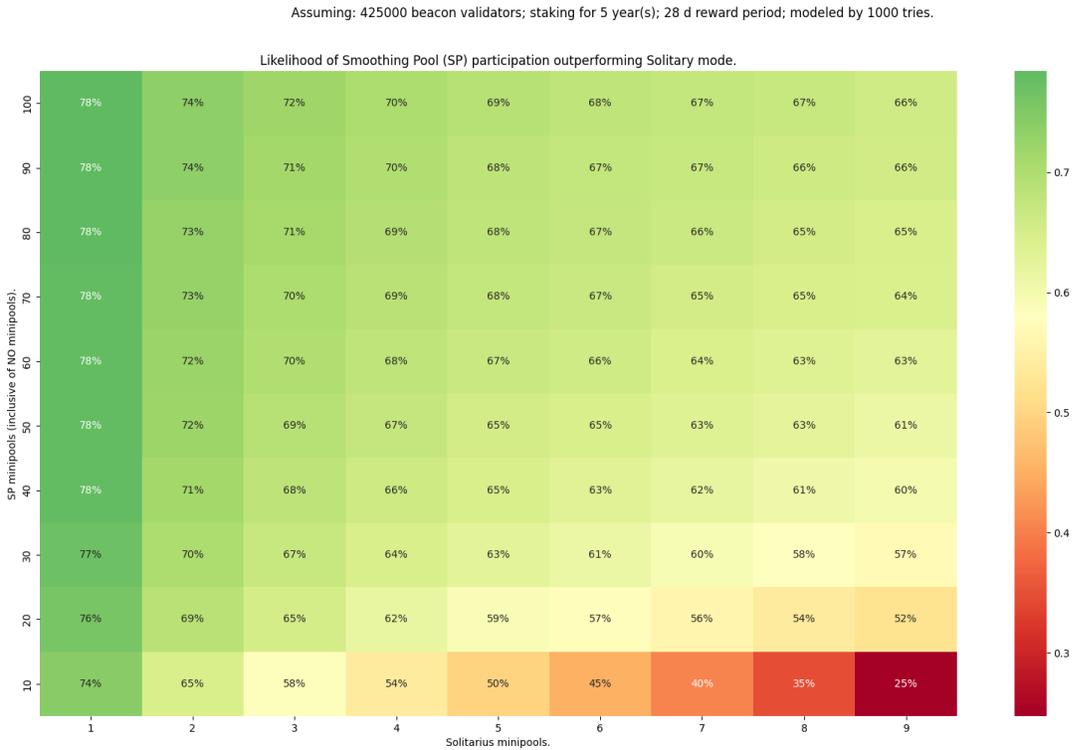
Analysis of SP Performance

Based on these observations, we further expanded our research of the SP and prepared a series of heat maps that look at the SP performance based on the fraction of the SP that a NO contributes. We will refer to this fraction as f , and it is simply the NO minipools / total SP participants.

The performance advantage comes from the fact that as more members participate in the SP, there is a higher likelihood, in a given period, that one of the participating minipools will propose a lottery block. If the NO’s fraction is small, it is most likely that another participant’s minipool will have been the validator to propose the lottery block. Because of the long-tailed distribution of lottery blocks, the amount in the lottery far exceeds the average PPV amount not associated

with large MEV extraction opportunities. The fractional share that a NO received from the SP lottery block will provide a higher profit most of the time than a solitarius configuration.

Figure 12
Likelihood of SP outperforming Solitarius minipool(s) measured at the end of each reward period.



Figures 12 and 13 are similar to each other. Each displayed in a heatmap formation the performance increase of the SP. Along the x-axis Are the number of mini pools operated by the NO. Along the y-axis is the amount of minipools participating in the SP. The value found in each box is the relative performance increase of the SP over a solitarius configuration. For example, a box that indicates 50% means that it is just as likely that a set of validators that operate in the SP will perform equally as well as a set of solitarius minipools.

In figure 12, we can see that a clear pattern emerges. For combinations where f is less than 0.5 (for example, 6 out of 10 participating minipool are owned by a single NO), the observed performance is less than 50%. This means that we expect the SP to underperform vs. solitarius minipools. In these cases, it is recommended that a NO not join the SP if their $f \geq 0.5$.

In cases where the NO f is less than 0.5, for example, 4 out of 10 minipools, we can see that the expected performance is greater than 50%. The scale on the heat map was adjusted manually so that the green color would start to appear at a 2/3rds probability (66%) of outperformance. This was suggested as a confidence level as to when it might be preferred to join the pool. The results show that the SP is expected to outperform so long as $f < 0.5$. This becomes most evident when $f < 0.1$, as illustrated in the first column. The smaller the fraction, the higher the likelihood that participating in the SP will outperform.

Figure 13
Likelihood of SP outperforming Solitarius minipool(s) measured at the end of the validating period.

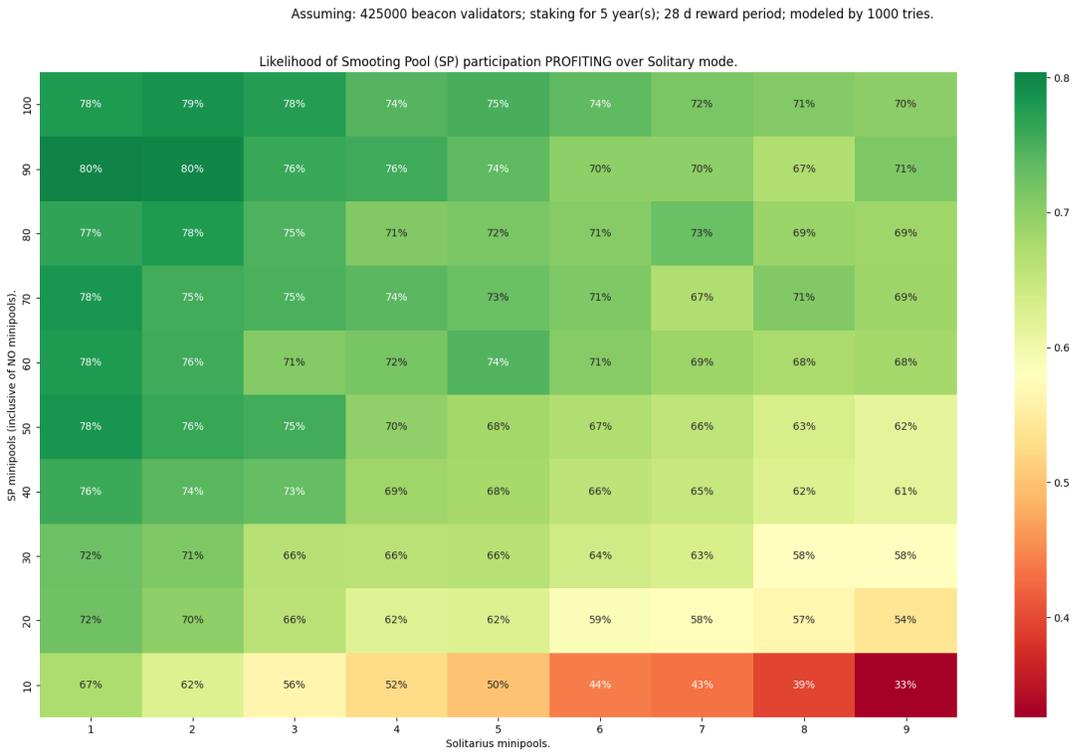
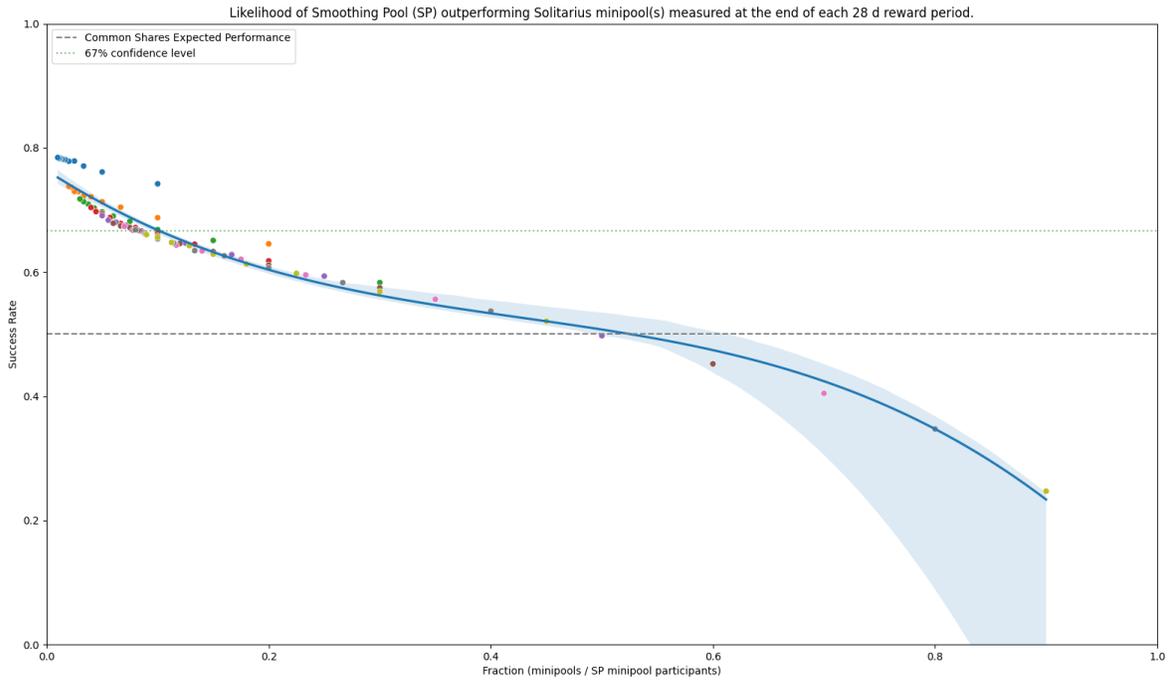


Figure 13 is a similar heatmap, but this time the performance is measured only at the end of the validating period. Here the cumulative sum of each rewards claim period (28 days in our model) is carried forward. Only the final accumulated sums of all PPV earned in the validation period are compared. We can still see a clear pattern that advantages SP participation. The lower the f value, the higher likelihood the SP will earn a solitary configuration.

Figure 14
Modeling runs showing Solitarius and averaged Smoothing Pool (SP) profit.

Assuming: 425000 beacon validators; staking for 5 year(s); 28 d reward period; modeled by 1000 tries.



The increase in SP performance plotted against the fraction value (f) is displayed in figures 14 and 15. Both of these plots can be considered derivatives of figures 12 and 13. In figure 14, we can see that as f decreases, the expected performance increases. The maximum expected performance probability is about 80%. The colors of the individual data points represent the column's from figure 12, which are the number of minipools that the NO has participated in the SP. The blue dots clustered above the best-fit line are from the one minipool column in figure 13. Their separation from the line is most likely attributable to a single minipool's probability of receiving 0 proposals in only 28 days.

Figure 17
Modeling runs showing Solitarius and averaged SP profit.

Assuming: 425000 beacon validators; staking for 5 year(s); 28 d reward period; modeled by 1000 tries.

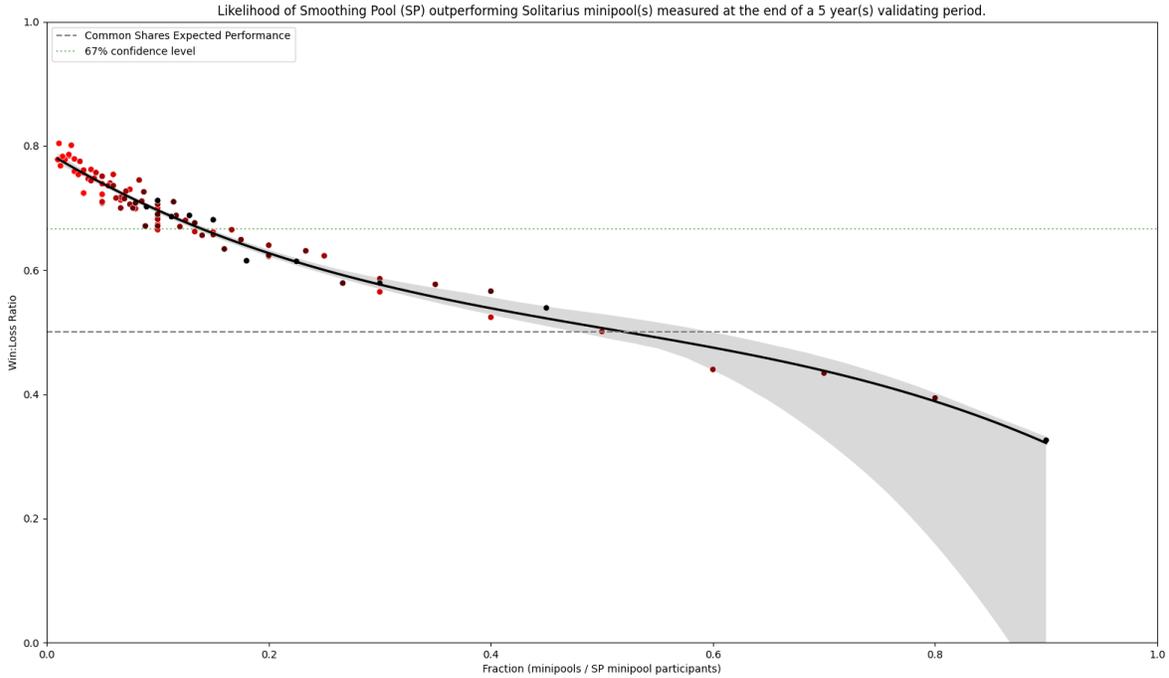


Figure 17 plots the observed performance from the final earnings calculated at the end of the validating period (5 years in our model). We again see a similar trend: the lower the f value, the higher the likelihood that the SP outperforms solitarius minipools. Because of the extended measurement period of 5 years, all minipools sets are likely to propose a non-zero amount of times. Because of that, all the data points cluster around the line of best fit.

Figure 16
Proposal Gain of SP participation over Solitarius minipools.

Assuming: 425000 beacon validators; staking for 5 year(s); 28 d reward period; modeled by 1000 tries.

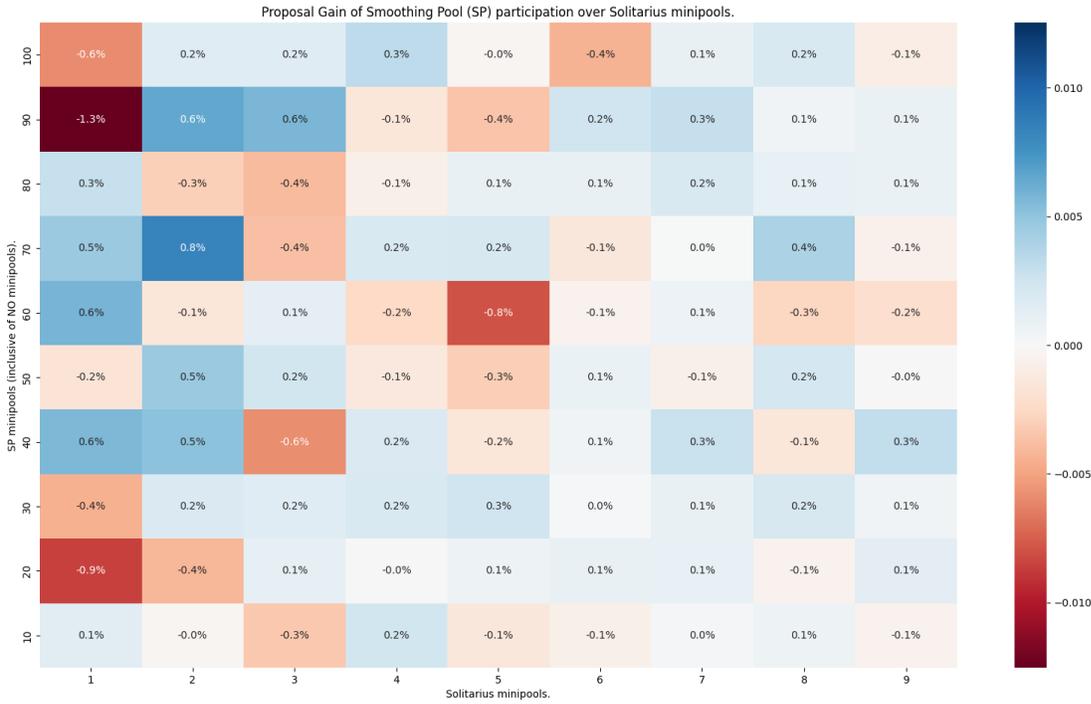


Figure 16 displays the gain of proposals/minipool from participating in the SP divided by the proposals/minipool from the remaining solitarius. Notice there is no pattern, and the observed gain or reduction is random. The results are just a few percentage points above or below zero gains. This means there is no expected advantage in receiving more proposals per minipool by participating in the SP.

Figure 17

Proposal Gain of SP participation vs. Solitarius minipools measured at the end of each reward period.

Assuming: 425000 beacon validators; staking for 5 year(s); 28 d reward period; modeled by 1000 tries.

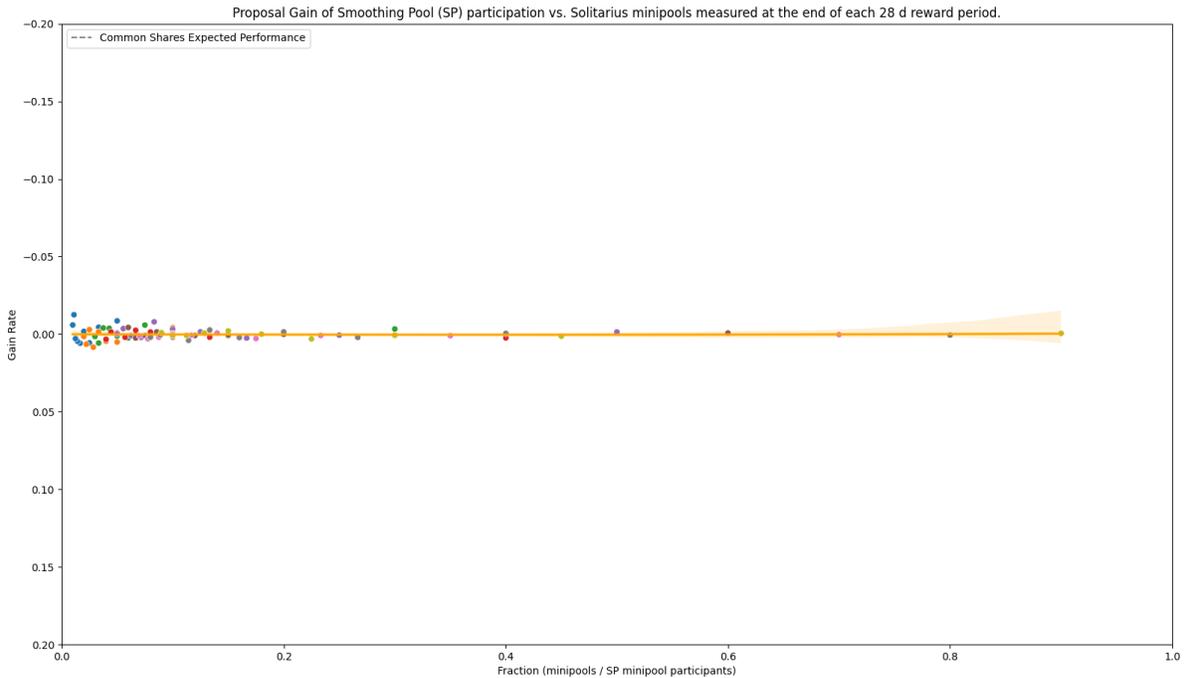


Figure 17 is a derivative of figure 12 that shows no expected increase in the average number of proposals from participating in the SP. There is no observed gain in the number of proposers/minipool.

Contributing Works

Already, derivatives of the above SP models have been shared by Valdorff.¹¹ They have expanded on the modeling results detailed in this report into a single two-plots graphic. In Figure 18, their work is modified to match the same minipools modeled in this report (1, 3, 7, and 50). The expected ETH rewards over a 5-year validating period are plotted on the x-axis and are normalized on a per minipool basis.

In the top plot, we can see a distribution where the curve's height represents the likelihood of a minipool receiving that value of ETH as PPV rewards. In the case of a single solitarius minipool (i.e., the blue line), we see a relatively flat curve peaking at about 3 ETH traversing the range of 0 - 10 ETH. There are probability curves for each of the other solitarius sets of minipools—all their curves peak at values less than 5 ETH.

Finally, in purple is the curve for a minipool participating in the SP. Here we can see a well-defined curve centered just above 5 ETH rapidly dropping to zero within ± 1 ETH of its peak. The curves illustrate the trade-off of a most certain expected earning of 5 ETH from participating in the SP. Although each solitarius curve peaks before the SP curve, they

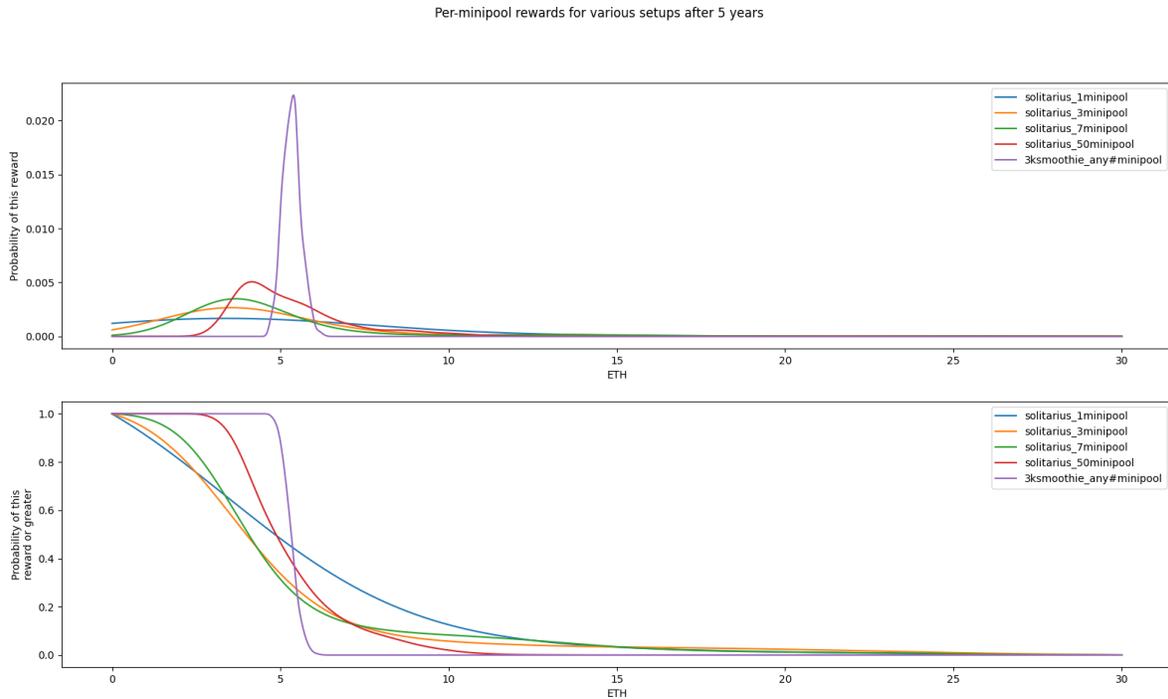
¹¹ <https://github.com/Valdorff/SPanalysis>

all have some trailing tail that extends further than the SP curve. The near certainty of a 5 ETH return is obtained by trading away the low probability of a very fortuitous solitarius opportunity.

The second plot shows the cumulative probability of receiving that amount of ETH or more over the 5-year validating period. You can see that in the case of single-digit minipools, the probability begins to drop almost immediately as the ETH value rises.

On the other hand, the SP continues to deliver a near certain likelihood of delivering at least 5 ETH per minipool over the five-year validating period.

Figure 18
Per-minipool rewards for various setups after 5 years.



Conclusion

This modeling outputs the probabilities of the likely outcome. The outcome experienced may be that the solitarius case may receive more proposals than if it had participated in the SP. Or that the solitarius case would have been lucky and received lottery blocks of PPV. There is no way to be 100 percent certain of the outcome. This modeling technique provides a method for a NO to understand and quantify the probability of the SP succeeding and failing. The NO then can make a more informed choice whether or not they wish to opt-in to the SP and, with moderate certainty, obtain a favorable return. Or, remain in a solitarius configuration and have a more likely outcome of lower earnings but have a small, but not impossible, chance of outperforming SP.

Risks

Because the extraction of MEV involves ordering transactions within a block, payment must be made to the block proposer (e.g., the minipool) to place the transactions in the correct sequence relative to the other transactions. This payment is made on-chain to a specific address referred to as the `feeRecipient`.¹² Because the eth1 address provided for the `feeRecipient` is under the complete control of the NO; there is an opportunity for a NO to steal all the PPV for themselves. This not only proposes the opportunity for the NO to withhold the fair share of the PPV owed to the regular stakers that compose a portion of the minipool, but the NO could also redirect the PPV payment away from the SP and thus not share the reward with the other participants. We will expand upon this attack scenario later in the report.

This modeling assumes that all participants in the SP act honestly. RP is developing a series of watchtower sentries to monitor for cheating and a penalty schema designed to deter a NO from stealing the PPV. A two-strikes policy forgives two occurrences of a PPV redirect before a penalty is assessed to the NO.

Suppose there is a significant amount of cheating by participants. In that case, this will adversely affect the SP's performance, and its performance advantage over the solitarius minipool will be diminished. If the cheating is severe, it may cause the SP to underperform relative to a solitarius configuration.

Further Works

Areas for further model development include:

- An expansion of this modeling could determine the number of selected lottery blocks that can be stolen to affect SP performance adversely.
- Predictive modeling of the SP can be expanded to include the use of CADlabs Ethereum Economic Model.
- Perform a retrospective deconstruction of the Bayesian modeling results, we determine the contributing factors for the observed performance advantage of the SP. The possible combinations of proposer selection and PPV distribution can be modeled to understand what factors best created the observed effects in the model.

General References

1. [Understanding the Validator lifecycle](#). Jim McDonald Jan 23, 2020.
2. [Upgrading Ethereum Edition 0.1: Altair \[WIP\]](#) by Ben Edgington
3. Github: Ethereum / consensus-specs / [slash_validator](#)
4. Github: Ethereum / consensus-specs / [slashings](#)
5. Github: Ethereum / consensus-specs / [Rewards and penalties](#)
6. Investopedia: [Empirical Rule](#)
7. [Rewards and Penalties on Ethereum 2.0 \[Phase 0\]](#) by James Beck March 2, 2020
8. [The Ethereum 2.0 Annotated Specification](#) by Ben Edgington
9. [Validator Rewards in Practice](#) - pintail.xyz

¹² There is a current discussion in the Flashbots discord that the block builder, a third-party separate from the NO who assembles searcher bundles and mempool transactions to compose the most valuable block ordering, will remit payment to the block proposer via a standard transaction included in the custom-built block. For this analysis, it does not matter how the proposer inclusion payment is made. It only matters that when it is made, it is directed to the SP for fair distribution among its participants.