Unifying the Measurement of Variation in Electoral Support

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Abstract

This paper unifies the measurement of variation in electoral support in time and over territories into a single framework. The framework generalizes the Pedersen volatilty index, provides new indices of electoral continuity and static and dynamic nationalization, and allows to define additional new measures. All measures within the framework are population fractions, and are thus easy to interpret and compare across concepts and polities. The framework rests on comparing the data to substantively interesting baselines, such as complete voter loyalty or fully nationalized competition, by computing the smallest fraction of votes that need to be changed for the observed vote to be identical to the baseline. The framework is demonstrated on a set of over a thousand elections from over a hundred polities spanning 1789–2015.

Keywords: electoral volatility · electoral continuituty · party nationalization · residential segregation · the dissimilarity index · log-linear models

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1 Introduction

Party and election scholars measure a variety of concepts concerned with temporal and territorial variation in electoral support, such as electoral volatility or party nationalization. Different measures are used for different concepts, and some concepts are measured with multiple competing indices. This paper offers a unified framework for such measurement that prioritizes clear and easy substantive interpretation and comparability across polities and concepts. It rests on the idea of measuring concepts by inspecting how far are the data from substantively relevant baselines. Generalizing the highly successful Pedersen (1979) index of electoral volatility, the framework measures such divergence as the smallest fraction of the votes that would need to be different for the data to conform to the baseline.

The Pedersen index dominates electoral volatility measurement thanks to its conceptual clarity and easy interpretation. The framework brings such clarity to the measurement of party nationalization. In stark contrast with electoral volatility, and despite its growing popularity (see Caramani and Kollman, 2017), there is no consensus on its measurement (see e.g. Bochsler, 2010; Golosov, 2016; Lago and Montero, 2014). Nationalization usually denotes an attribute of a party or a party system, but sometimes also a process by which the attribute comes about (Lago and Montero, 2014). The literature agrees that nationalization as an attribute means that something is constant across territories, but discusses at least three options of what it is: (i) party support, (ii) change in support, and (iii) effects on support (Caramani, 1996). This paper focuses on descriptive measurement, and deals with the first two options, known also as *static* and *dynamic* nationalization, respectively.

Disagreement persists on how to measure static nationalization. More than a dozen measures have been proposed, each motivated by a partly different set of desired properties. This paper advances the debate with two independent contributions. First, it clarifies several fundamental issues behind the disagreement by rethinking the problem as one of association in categorical data. From this perspective, static nationalization means that party and territory are not associated, clarifying that some of the properties desired of nationalization indices are counterproductive or impossible. Second, it proposes to measure static nationalization with the fraction of votes that would need to be different for all territories to register the same result in vote shares. This yields a direct counterpart of the Pedersen index, and is equivalent to a well-known residential segregation index, the generalized Duncan index (O.D. Duncan and B. Duncan, 1955; Reardon and Firebaugh, 2002), providing a link to the broad literature on segregation. This index gives the same substantive answers as Bochsler's (2010) highly popular nationalization score, correlating with it practically perfectly in a set of 1,495 elections from 119 polities from 1789 to 2013, while at the same time being considerably easier to interpret in substantive terms.

In contrast to static nationalization, research on the measurement of dynamic nationalization is more cumulative. Since Stokes (1965) first tackled the issue statistically, his approach has been gradually advanced, most recently by Morgenstern and Potthoff (2005) and Alemán and Kellam (2008). All of these approaches are based on linear regression models, and as their assumptions are not always met, they can lead to erroneous substantive inferences. This paper introduces an alternative approach based on the dissimilarity index and log-linear models, generalizing the Pedersen volatility index and the Duncan segregation index. In contrast to the existing approaches, it does not assume any model to be true, and instead looks how far the data is from substantively interesting baselines. The paper gives log-linear models which correspond to substantive baselines of static nationalization, dynamic nationalization, electoral continuity understood as stable local patterns of support (Bartels, 1998; Wittenberg, 2006; 2013), and their logically possible combinations. Applying the dissimilarity index to these models gives rise to new indices, which retain the easy interpretation of the Pedersen index as the smallest fraction of the votes that need to differ for the baseline to fit perfectly. Consequently, the indices allow comparisons not only across polities, but across concepts. Furthermore, it allows to answer substantively interesting questions on the components of variation in electoral support by contrasting the values of the dissimilarity index for pairs of nested models. The approach is shown on the on Canadian General Elections of 2006, 2008, and 2011.

The paper is accompanied by an `R` package for convenient computation of over a score of measures computed in party research from the joint and marginal distributions of votes by party and territory, described in the online supplementary information.

2 Generalizing the Pedersen Volatility Index

Perhaps the most widely used concept measured from electoral data is electoral volatility, and it is almost always measured with the Pedersen index. The index's success rests on its conceptual simplicity and clear interpretation. It captures the smallest fraction of voters that had to change parties provided the same voters voted in the inspected elections. More generally, it is the smallest fraction of votes that would need to be cast differently in order for the inspected elections to have the same percentual results. Formally, for a pair of elections the index is

$$\mathrm{PI} = \frac{1}{2} \sum_{p} \left| s_{p(e=1)} - s_{p(e=2)} \right|,$$

where s_{pe} is the vote share of party p in election e.

Closely related to electoral volatility is the concept of electoral regionalization, which in this context means correspondence between national and regional elections (Jeffery and Hough, 2003; 2009; Johnston, 1980; Pallarés and Keating, 2003; Schakel, 2013). It is usually measured as volatility from a national election to a regional one, or vice versa. In this context the Pedersen index was recognized by Johnston (1980) as a special case of the dissimilarity index (Gini, 1914), a goodness of fit measure for models of contingency table data known also as the Total Variation Distance (Markatou and Chen, 2018). The dissimilarity index equals the smallest fraction of the observations that would need to be changed in order for the model to describe the data perfectly. Formally, it can be defined as

$$D = \frac{\sum_c |o_c - \hat{o}_c|}{2\sum_c o_c}, \tag{1}$$

where the data is a contingency table with C cells, $\{o_1, \ldots, o_C\}$ the observed values and $\{\hat{o}_1, \ldots, \hat{o}_C\}$ those expected under the model.

The dissimilarity index applies to any number of categories. Consequently, it can be applied to measure volatility over more than two elections, as

$$\text{VI} = \frac{\sum_p \sum_e |v_{pe} - \hat{v}_{pe}|}{2\sum_p \sum_e v_{pe}}, \tag{2}$$

where v_{pe} is the observed number of votes for party p in election e and $\hat{v}_{pe}$ the expected one if the parties' relative support was perfectly stable. The expected values can be computed with a variety of procedures. The simplest is for each cell to multiply its row and column marginal and divide the product with the total sum,

$$\hat{v}_{(p=i)(e=j)} = \frac{\left(\sum_p v_{p(e=j)}\right)\left(\sum_e v_{(p=i)e}\right)}{\sum_p \sum_e v_{pe}}.$$

An alternative that scales better is a log-linear model. Log-linear models are a model family for the analysis of associations in categorical data used across a variety of domains (see e.g. Fienberg and Rinaldo, 2007). They are highly flexible and apply to any number of categorical variables with any number of categories. Maximum likelihood estimation of log-linear models (see e.g. Agresti, 2002, pp.314–56) minimizes the dissimilarity index, and is thus an appealing choice in this context.

The substantive baseline of no change in parties' relative support implies that the support is independent of the elections, i.e., that election and party do not interact. This baseline leads to the log-linear independence model

$$
\begin{aligned}
v_{pe} &\sim Poisson\left(\hat{v}_{pe}\right), \\
\ln \hat{v}_{pe} &= \lambda^0 + \lambda_p^P + \lambda_e^E,
\end{aligned}
\tag{3}
$$

where λ^0 is the main intercept, λ^P party and λ^E election coefficients. The quantities of interest are the fitted values $\hat{\boldsymbol{v}} = \{\hat{v}_{pe}\}$, needed to compute the value of the index by plugging them into equation (2). The values of the coefficients are not interesting in themselves in this application, and any of the standard identifying restrictions will deliver the same fitted values. The model can be considered as a Poisson GLM with log link function,

$$\ln \hat{\boldsymbol{v}} = \mathbf{X}\boldsymbol{\lambda},$$

where $\mathbf{X}$ is the design matrix and $\boldsymbol{\lambda} = \{\lambda^0, \lambda_p^P, \lambda_e^E\}$ the coefficient vector. Any conventional design matrix codings, such as dummy, effect, or polynomial (see e.g. von Eye and Mun, 2014), will serve the purpose.

In some cases, a new party emerges, or an existing one collapses. If the emergence or collapse are complete, than the party will receive in one election from a subsequent pair zero votes. If the process is more gradual, the party will receive a very small vote share in one of the elections. Powell and Tucker (2014) find it useful to disaggregate the value of the Pedersen index into a component that belongs to entering and exiting parties (Type A volatility) and a component that belongs to stable parties (Type B volatility), such that PI = PA + PB. The framework presented here allows to generalize this decomposition to any number of elections, so that VI = VA + VB. Type B volatility becomes

$$\mathrm{VB} = \frac{\sum_p \sum_e s_p \left|v_{pe} - \hat{v}_{pe}\right|}{2 \sum_p \sum_e v_{pe}},$$

where s_p is an indicator that takes the value of one if party p is stable and zero otherwise. Type A volatility by definition equals

$$\mathrm{VA} = \mathrm{VI} - \mathrm{VB} = \frac{\sum_p \sum_e \left(1 - s_p\right) \left|v_{pe} - \hat{v}_{pe}\right|}{2 \sum_p \sum_e v_{pe}}.$$

It is easy to see that the D index can be decomposed in this way for any mutually exclusive subsets of parties.

3 Measuring Static Nationalization

After volatility, perhaps the second most common concept concerned with variation in electoral support is nationalization. Introduced by Schattschneider (1960), it has generated a large and growing literature (see Bochsler, 2010; Caramani, 1996; 2004; Caramani and Kollman, 2017; Golosov, 2016; Lago and Montero, 2014; Morgenstern, Swindle, and Castagnola, 2009). The term is used for multiple concepts that are not always fully compatible. Some conceptualize nationalization as an *attribute* of a party, a party system, or a polity's electoral politics, and others as a *process* by which the attribute came about (Lago and Montero, 2014). Furthermore, although the literature agrees that nationalization as an attribute means that something is homogeneous over a polity's territorial divisions, there are different views of what it is. The three main options are (1) party support, (2) change in support, (3) effects on support (Caramani, 1996). The first kind of nationalization is also known as *static* and the second as *dynamic* (Morgenstern, Polga-Hecimovich, and Siavelis, 2014; Morgenstern, Swindle, and Castagnola, 2009). Caramani and Kollman (2017) add a fourth kind of nationalization, understood as correspondence of sub-national party systems to the national one, which matches Johnston's (1980) notion of regionalization discussed above. These four concepts of nationalization focus on electoral support. There are also others that focus on party organizations and actions (Caramani, 2000; Lago and Montero, 2014).

Given the different meanings electoral and party nationalization take in different contexts, it does not come as surprise that multiple nationalization indices are in use. This is especially the case of static nationalization, where more than a score of indices coexist (see Tables A.2 and A.3 in the Supporting Information). This proliferation exists despite a broad agreement on the observable implications of static nationalization, according to which a nationalized party receives the same share of the local vote within each territory, and in a nationalized party system this is true of all parties. The reasons for this apparent paradox lie in several non-intuitive statistical properties of the problem, which can be clarified by rethinking it in terms of association in categorical data.

Since static nationalization means territorially homogeneous support, it implies that party and territory are independent. To fix some ideas, consider the vote counts shown in Table 1, which reports returns from an election in which two parties competed across four territories. Overall, party A gained 51% and party B 49% of the vote. Table 2 shows a distribution of the votes where both parties get the same shares in all four territories, while preserving the number of votes cast in each territory. In fact, it is the only such distribution. It uniquely describes party-territory independence for the given marginals. This is because under static nationalization party and territory are independent, and for a given set of marginals there is

Table 1: Example of electoral returns from a fictitious election.

	Party		
Territory	A	B	
1	111	96	207
2	111	107	218
3	81	98	179
4	93	87	180
	396	388	784

Table 2: Hypothetical vote distribution under static nationalization (values rounded).

	Party		
Territory	A	B	
1	105	102	207
2	110	108	218
3	90	89	179
4	91	89	180
	396	388	784

only one distributions that corresponds to independence.

But what distribution corresponds to the opposite of nationalization? This question might seem easy, but deceivingly so. The reason is related to the concept of association. If nationalization is party-territory independence, then denationalization is party-territory association. However, association lacks a statistical definition equally constraining as that of independence. Consider the two-by-two cross-classification

	$y = 0$	$y = 1$
$x = 0$	a	b
$x = 1$	c	d

Perhaps the most intuitive answer is that association would be perfect if $x = y$ or $x = 1 - y$, i.e., if $b = c = 0$ or $a = d = 0$, respectively. However, for instance under a well-known measure of association in dichotomous cross-classifications, the odds ratio ($or = \frac{ad}{bc}$), association is perfect if any of the four cells is zero which gives the odds ratio its lowest or highest possible value (see e.g. Rudas, 1998). This does not mean that one is wrong and the other right. Simply, there are many measures of association based on different concepts of it (see e.g. Goodman and Kruskal, 1954; 1959). These concepts are not perfectly compatible. Although all prescribe the same distribution for independence, they do not necessarily agree on what association is, and what distributions correspond to perfect association. Choosing between them inevitably means choosing between concepts of association. The implication of this for the measurement of static nationalization is that the existing indices differ not because some or all of them would be biased or invalid, but because they measure different concepts of nationalization.

Further insights come from considering nationalization measures in terms of a divergence d from the observed distribution $\mathbb{O}$ of votes over parties and territories to the hypothetical distribution under nationalization $\mathcal{N}$,

$$d(\mathbb{O}, \mathcal{N}).$$

This corresponds to measurement of model fit, taking $\mathcal{N}$ as the distribution under the model, i.e., fitted values, and $\mathbb{O}$ as the observed distribution. A more general statement is

$$d(\mathbb{O}, \mathcal{M}),$$

where $\mathcal{M}$ is a distribution that belongs to the model, and is either fixed ex ante, or selected to optimize d. In the context of volatility measurement as discussed in the previous section d is computed with equation (1), with the observed votes $\{v_{pe}\}$ taking the role of $\mathbb{O}$ and the fitted votes $\{\hat{v}_{pe}\}$ the role of $\mathcal{M}$.

Conventionally, the larger the d's value is the farther the model is from the data. In the case of volatility, this results in a straightforward interpretation, as the baseline model is perfect stability. This is not the case of static nationalization, as the baseline is perfect nationalization, and the higher the value the further the data are from it. However, the interpretation of a static nationalization index is easier if perfectly nationalized results are assigned a value of zero. For measures that are constrained to the unit interval, this can be achieved with

$$c(\mathbb{O}, \mathcal{N}) = 1 - d(\mathbb{O}, \mathcal{N}),$$

where c is a measure with the desired property. This rescaling is used in the recent work on static nationalization measurement (e.g. Bochsler, 2010; Golosov, 2016; Jones and Mainwaring, 2003), and applies just as easily to the dissimilarity index. For example, if D is used to measure volatility, it takes the value of the smallest fraction of the votes that would need to be cast differently for perfect stability to occur, and its complement $1 - D$ takes the value of the largest fraction of the votes that are described by perfect stability, which is a stability index.

In sum, static nationalization is equivalent to independence in bivariate categorical setting, and its absence to association. Any measure of association can be thought of as a measure of divergence of the data to the baseline of independence. There is only one distribution that corresponds to independence for a given set of marginals, but there are many different kinds of divergence from it, i.e., many different concepts of association. To measure association one thus needs to clarify first which of its concepts is best suited to the substantive questions at hand. This issue has been successfully resolved in the case of volatility, defining it as a kind of association–minimum possible change to arrive at stability. Although there is only one concept of static nationalization there are many of denationalization, which is the concept researchers in fact want to measure.

Recent research has made advances in resolving this issue by defining the opposite of nationalization as concentration, which has lead to measures such as the Party Nationalization Score (PNS) (Jones and Mainwaring, 2003), weighted PNS (WPNS) and standardized PNS (SPNS) (Bochsler, 2010), normalized PNS (NPNS), or indices of party nationalization and party system nationalization (IPN and IPSN) (Golosov, 2016). All of these indices measure the divergence of the data from perfect concentration. Considerable progress has been made in sensitising these measures to different unit sizes and scaling them onto the unit interval. However, one issue hinders their universal adoption. Just as the Gini (1921) coefficient of inequality, which they build on, these indices take perfect concentration to mean that all the resources belong to a single unit. In electoral context, this means that a party with a perfectly concentrated vote receives all its votes from a single territory. By extension, in a party system with perfect concentration each party receives all its votes from one territory only. In practice, there are many more territories than parties, and the maximum possible concentration is thus far from perfect in this sense. Consequently, the usefulness of perfect concentration as a substantive baseline is limited by the fact that in contrast with static nationalization or stability in most cases it is not possible even in principle. Finally, although these indices (PNS, WPNS, SPNS, NPNS, IPN, and IPSN) have many appealing properties, they share one substantial drawback. Namely, their numeric values are not easy to interpret substantively.

An index free of these drawbacks that retains the clarity and easy substantive interpretation of the Pedersen index is available by applying the proposed framework to measure the divergence of electoral support from static nationalization. Here, the baseline is party-territory independence, which in log-linear terms is

$$\ln \hat{v}_{tp} = \lambda^0 + \lambda_t^T + \lambda_p^P,$$

where $\hat{v}_{tp}$ are the fitted votes of party p from territory t, λ^0 the intercept, and λ^T and λ^P the territory and party coefficients. Applying the dissimilarity index to this model following equation (1) gives the smallest fraction of votes that would need to change to achieve complete static nationalization. If party and territory are independent, all territorial party vote shares are the same as the national ones, the party system has full static nationalization, and the value of the index is zero. As the index increases with the vote diverging from static nationalization, it gives an index of denationalization (territorialization). Subtracting its value from one gives an index of nationalization. Taking into account the whole table gives the value of the index for the party system. Values for any party can be computed using the formula in equation (1) subsetting v_{tp} and $\hat{v}_{tp}$ on p. In the same way, values for any territory are available by subsetting on t.

The D index has a long history in *segregation* research, where it is known as the Duncan index (O.D. Duncan and B. Duncan, 1955; Massey and Denton, 1988; Morgan, 1975; Reardon and Firebaugh, 2002; Sakoda, 1981), and applied to tables that cross-classify individuals by place (residence, school etc.) and group (ethnicity, gender etc.). The index equals zero if all places have the same group proportions, and reaches its maximum if each place contains only one group. Massey and Denton (1988) identify the index as the best of the 20 surveyed measures in terms of capturing 'evenness,' one of their five dimensions of residential segregation together with 'exposure,' 'concentration,' 'centralization,' and 'clustering,' and recommended it as the measure of first choice due to its ease of interpretation and strong correlation with the other measures.

The proposed indices of territorialization and nationalization meet the overwhelming majority of the requirements given by Bochsler (2010) and Golosov (2016), as shown in Table 3. Specifically, they have the properties of directness of measurement, decomposability, zero-to-one limit, scale invariance, computational simplicity, easy understanding, which are five out of the seven properties required by Golosov (2016). In addition, they take into account both party and territory sizes, and is insensitive to the number of territories and the local number of parties, as required by Bochsler (2010).

The proposed indices lack the property of *normalization* required by Golosov (2016). It would be easy to endow them with it. However, this would hamper rather than enhance their usefulness. The lower bound of the territorialization index is always zero (and thus the upper bound of $1 - D$ is one), when the vote completely conforms to static nationalization. This holds regardless of the number of parties and territories. On the other hand, its upper bound is always below one (and the lower bound of $1 - D$ above zero), because it depends on the numbers of parties and territories. Thus, the index is easy to rescale so that for any number of parties or territories its upper bound will be one (and the value of its complement will be zero). This would strip the index of its easy substantive interpretation and make it less rather than more comparable across polities. The value of the index is simply the smallest fraction of the votes that would need to change to fully conform with the baseline. Thus, the values of the index in two polities can be directly compared even if they have different numbers of territories or countries. The dependence of the bound on the number of categories captures substantively relevant characteristics. Suppose polity A has T_A constituencies and P_A parties such that $P_A < T_A$ and polity B has $T_B = T_A$ constituencies and P_B parties such that $P_B = T_B$. In B, the upper bound will be larger than in A. This reflects the fact that in B the worst case is each constituency completely taken over by its own party, whereas in A even in the worst case at least one party will be present in multiple territories. These two situations are substantively different. It is not clear what would be the value of assigning them the same numeric value. Suppose further that there

Table 3: Properties required of static nationalization measures and by Bochsler (2010) 1, 4–6, 8–11 and by Golosov (2016) 1–7 and the proposed territorialization (segregation) index and its complement the nationalization index.

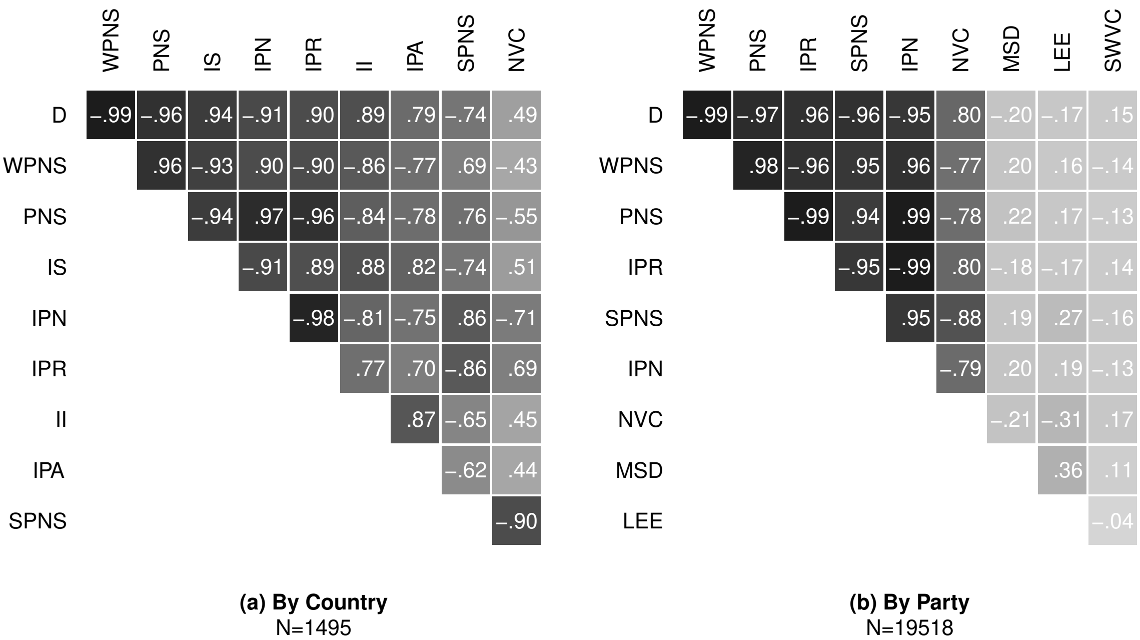
1.	**Directness of measurement**	✓	Both indices are computed from vote counts.
2.	**Decomposability**	✓	Both indices can be computed for any subsets of parties and or territories.
3.	**Normalization**	?	The indices can be rescaled onto the unit interval for any given number of parties and territories, but just like in the case of the existing measures it severely diminishes their interpretability.
4.	**Zero-to-one limit**	✓	As a fraction of the data, the indices always stay on the unit interval.
5.	**Scale invariance**	✓	The indices do not change if all vote counts are multiplied by the same number.
6.	**Sensitivity to transfers**	?	The indices are sensitive to transfers only under the weak version of Dalton's principle, or under Merschrod's (1981) alternative strong version of the principle.
7.	**Computational simplicity and easy interpretability**	✓	The indices are easy to compute and interpret.
8.	**Accounting for party and territory sizes**	✓	Thanks to the log-linear model of party-territory independence, the indices of territorialization and nationalization account for party and territory sizes.
9.	**Insensitivity to the number of territories**	✓	The value of D does not change if each territory is cloned the same number of times. Suppose a party records the results $\{25, 75\}$, which gives a D of 0.25. Cloning each territory once gives $\{25, 25, 75, 75\}$ and $D = 0.25$ again.
10.	**Insensitivity to the local party system size**	✓	The indices do not take into account the local numbers of parties.
11.	**Insensitivity to the level of territorial aggregation**	?	Depending on the interpretation of this property, the D index either has it, or the property is undesirable due to its relation to ecological fallacy.

is a polity C in which $P_C = T_C$, and $T_C > T_B$. In C, just like in B, the worst case is each territory taken over by its own party. However, since in C there are more territories, this in substantive terms means a greater fragmentation. Again, it is not clear what is the benefit of giving both worst cases the same value. Furthermore, Golosov (2016) justifies the requirement of normalization as necessary for decomposability. This does not hold as the proposed indices already possess that property. In short, not normalizing the index helps its substantive interpretation.

The D index also lack strong *insensitivity to transfers* required by Bochsler (2010) and Golosov (2016). This requirement is adopted from research on wealth inequality and is also known as Dalton's (1920) principle of transfers. Its strong version requires inequality indices to decrease if some assets are moved from a richer unit to a poorer one, and its weak version requires that they do not increase. Merschrod (1981) argues that such strong version is a faulty generalization of Dalton's principle, and the correct one is that only transfers from units with more than the mean wealth to units with less than the mean wealth decrease inequality. The dissimilarity index meets this version of strong insensitivity to transfers as well as the weak version. To illustrate this, suppose 12 votes distributed into four categories as $\{1, 2, 3, 6\}$. For uniformity, $\{3, 3, 3, 3\}$, at least four votes have to move, so the D index equals $0.\overline{33}$. Moving one vote so that $\{1, 2, 4, 5\}$ does not affect D. Moving one vote into any of the bottom two categories decreases it to 0.25. Just as in the case of independence and association, there is only one concept of equality, but many concepts of inequality. Ultimately, the question is not which of the two strong versions of the principle follows Dalton's intention, but rather which one is more useful for studying electoral nationalization. The D index lacks one property that researchers might find useful, but unlike other measurements that have it, it has a straightforward substantive interpretation. Furthermore, as shown below, it also correlates nearly perfectly with the most advanced measure that has this property, the WPNS.

The property of *insensitivity to the level of territorial aggregation* can be interpreted in two ways. The first interpretation is that if we split a territory into multiple territories with which the relative proportions of parties remain the same, or if we merge territories within which the relative proportions are identical, then the index should not change its value. Under this interpretation, the property is identical to that of *insensitivity to the number of territories* as stated in Table 3, and the D index (and its complement $1 - D$) possesses it. The second interpretation is that the index should not change if we merge any territories or split any territories. This interpretation follow recent literature (Bochsler, 2010; Morgenstern, Polga-Hecimovich, and Siavelis, 2014) that theorizes nationalization as independent of the specific territorial divisions. This interpretation relates to *ecological fallacy*, which rests on attributing an association observed at a higher level of aggregation to a lower level of aggregation (see e.g. Freedman et al., 1991). As discussed above, any

Figure 1: Correlations between measures used in party nationalization research. Order and color by the absolute value of Pearson correlation with the segregation index (D).



Note: Weighted Party Nationalization Score (WPNS), Party Nationalization Score (PNS), Inflation Score (IS), Index of Party Nationalization (IPN), Index adjusted for Party size and number of Regions (IPR), Inflation Index (II), Indicator of Party Aggregation (IPA), Standardized Party Nationalization Score (PNS), Normalized Variability Coefficient (NVC), Mean Standard Deviation (MSD), Lee index (LEE), Standardized and Weighted Variability Coefficient (SWVC). Computed from vote data by Kollman et al. (2014).

index of party system static nationalization can be considered in terms of association between party and territory. From that perspective, observing an association between party and region and attributing it to lower level such as counties equates to ecological fallacy. Consider the example of a polity with wards nested in constituencies that has two parties and two classes and voting is purely class-driven. It has two parties and two classes, and voting is entirely class-driven. All constituencies have the same class proportions, but the classes are perfectly segregated into wards. Consequently, party is independent of constituency, but not of ward. An index that gives the same value on both levels would be substantively useless. This shows a fault in theories that conceptualize nationalization as independent of the level of territorial aggregation. Fortunately, this fault can be corrected by rethinking the issue in the following way. If nationalization at the lowest relevant level of territorial aggregation is complete, then any upward aggregation of the territories will retain this. If nationalization at the lowest level is not complete, than upward aggregation may change the divergence from nationalization. The D index (and its complement $1 - D$) has this property, and avoids ecological fallacy.

How does the new territorialization index compare to the existing measures in practice? As shown in Figure 1, for a large inspected set of constituency-level results from 1,495 elections which took place in

119 polities between 1789 and 2013 (see the SI, Section B), it correlates strongly with some of the measures. The correlations with some are negative, as these measures increase with decreasing distance from the nationalized distribution. Regardless of whether parties or party systems are inspected, the correlation is especially strong with the WPNS and PNS, both of which are based on the Gini coefficient of inequality, and meet the strong version of Dalton's principle. Compared to these measures, the D index is considerably easier to interpret. In short, despite meeting only the weak version of Dalton's principle the D index leads with real data to the same substantive conclusions, but in contrast also has substantively meaningful numerical values. Furthermore, it is defined within a unified framework, and as shown in more detail below, allows also comparisons across concepts.

4 Electoral Continuity and Dynamic Nationalization

Compared to static nationalization, methodological work on dynamic nationalization shows a greater degree of agreement and cumulative advances. Since Stokes (1965) first tackled the issue statistically, advances have been proposed, most recently by Morgenstern and Potthoff (2005) and Alemán and Kellam (2008).[1] All of these approaches are linear in the sense that they are based on linear regression models, and rest on partitioning the variance in vote shares into national and sub-national components.

In this literature, many of the quantities of interest are immediately given causal interpretations. Such causal inferences are not always warranted. For example, if a party's vote is found to be geographically heterogeneous, this is discussed in terms of the importance or effects of local factors (Morgenstern and Potthoff, 2005). However, territorial variation in support can readily result from factors that are not well described as local. For illustration, suppose a polity with two parties, two social classes, and many constituencies, each with different class proportions. If the vote is entirely class-driven, it will vary over constituencies. The cause of this heterogeneity–class voting–can be thought of as both local and national. It is local in the sense that local class proportions explain the local party shares, and it is national in the sense that the effect of class on vote is homogeneous across the constituencies. For simplicity, this paper sticks to a purely descriptive approach also with regards to the interpretation of the quantities computed under the linear approach.

Morgenstern and Potthoff (2005), extend work by Stokes (1965; 1967) and Kawato (1987). It is a linear model

$$s_{tpe} = \mu_p + \beta^T_{tp} + \beta^E_{pe} + \beta^{TE}_{tpe},$$

[1] Interested readers can consult a more detailed lineage of this approach by Alemán and Kellam (2008).

where s_{tpe} is the fraction of votes from territory t for party p in election e, and all the terms β are random coefficients (effects) estimated separately for each party. The territory coefficients $\{\beta_t^T\}$ are intended to capture district heterogeneity, the election coefficients $\{\beta_e^E\}$ volatility, and the territory-election interaction coefficients $\{\beta_{te}^{TE}\}$ labeled as district-time effects. Alemán and Kellam (2008) advance the linear approach by adapting it to the compositional character of electoral data in multi-party systems. First, for a system with P parties, they transform the vote shares of the first $P-1$ parties to the logs of their ratios to the vote share of the P^{th} party,

$$y_{tpe} = \ln \frac{s_{tpe}}{s_{t(p=P)e}}.$$

Next, they model the $\{y_{tpe}\}$ with seemingly unrelated regression (Jackman, 2000; Tomz, Tucker, and Wittenberg, 2002), where

$$\begin{aligned}
\left\{y_{t1e}, \ldots, y_{t(P-1)e}\right\} &\sim MVN\left(\left\{\hat{y}_{t1e}, \ldots, \hat{y}_{t(P-1)e}\right\}, \Sigma\right), \\
\hat{y}_{t1e} &= \alpha_{1e} + \beta_{1e} y_{(t-1)1e} + \cdots + \beta_{(P-1)e} y_{(t-1)(P-1)e}, \\
\Sigma &= \left\{ \begin{matrix} \sigma_{11}, & \cdots & \sigma_{1(P-1)} \\ \vdots & \ddots & \vdots \\ \sigma_{(P-1)1}, & \cdots & \sigma_{(P-1)(P-1)} \end{matrix} \right\},
\end{aligned}$$

where $\{\hat{y}_{tpe}\}$ are systematic and $\{\sigma_{pp}\}$ stochastic components, with the former former interpreted as national and the latter as local. The parameter estimates are used to simulate the vote shares under the model, $\{\tilde{\mathbf{y}}_{pe}\}$. Inferences on static and dynamic nationalization are drawn by subtracting the average vote shares in the previous elections,

$$\tilde{\boldsymbol{\delta}}_{pe} = \tilde{\mathbf{y}}_{pe} - \bar{\tilde{y}}_{p(e-1)}.$$

The mean of $\tilde{\boldsymbol{\delta}}_{pe}$ is attributed to national and its standard deviation to sub-national forces. Finally, a coefficient of relative nationalization is computed as

$$\mathrm{RN}_{pe} = \frac{\left(\bar{\tilde{\delta}}_{pe}\right)^2}{\left(\bar{\tilde{\delta}}_{pe}\right)^2 + \mathrm{Var}\left(\tilde{\boldsymbol{\delta}}_{pe}\right)},$$

which takes the value of zero if none of the change is attributed to national factors.

Several issues limit the usefulness of the MP and the AK approaches. First, they transform vote counts into shares. When territories vary in electorate size, this means leaving out potentially useful information. This information can be re-incorporated with regression weights, but at the price of further complicating substantive interpretation. Second, both MP and AK approaches require the model to be true, which may not always be met, and is at best difficult to test. Third, in modeling the district coefficients as random Morgenstern and Potthoff (2005) rely on the assumption that the districts are drawn from an infinite superpopulation of districts. This and similar superpopulation assumptions with regards to aggregate data in comparative research is controversial (see e.g. Berk, Western, and Weiss, 1995; Western and Jackman, 1994). This assumption is particularly questionable in the case of elections. Fourth, many researchers might find the AK approach perhaps a bit unintuitive.

The unified framework based on the dissimilarity index generates an alternative approach that inputs directly vote counts, does not assume a superpopulation, and delivers easy-to-interpret quantities of interest. The approach is based on log-linear models, and generalizes the Pedersen index and the static nationalization measure proposed in the previous section. It rests on inspecting the joint distribution of votes over parties, territories, and elections by fitting log-linear models that represent substantive baselines and computing how far in terms of the dissimilarity index are they from the data. Furthermore, it allows to measure the concept of electoral continuity understood as stable local patterns of electoral support (Bartels, 1998; Wittenberg, 2006; 2013). Instead of assuming the models to be true it treats them as baselines and draws inferences from their fit.

Table 4 features the log-linear models that serve as the baselines. Each of the models lifts a different set of restrictions on the expected count of votes $\hat{v}_{tpe}$ from territory t for party p in election e. Usually, national party totals differ, as do the totals across constituencies and elections. These three kinds of variability are captured by the independence model m_{ind} with territory, party, and election terms $\{\lambda^T, \lambda^P, \lambda^E\}$. This model assumes that the relative sizes of territorial electorates stay the same over elections. In reality, they can change due to factors such as population movement, redistricting, or differential turnout. Such changes are captures by territory-election interaction λ^{TE}. Including this interaction into the independence model m_{ind} yields model m_{sta}. Both represent the substantive baseline of perfect static nationalization (or de-segregation) with no volatility. In practice, the more complex of the two models is more appealing as the territory-party interaction is rarely substantively interesting.

Lifting the restriction of static nationalization yields the substantive baseline of electoral continuity. Under electoral continuity, party and territory are associated in the same way over a series of elections. In log-linear terms, this is achieved by including the territory-party interaction λ^{TP}, which yields model m_{con}. Under the substantive baseline of perfect static and dynamic nationalization both electoral support and its change are independent of territory. In log-linear terms, dynamic nationalization is captured by the party-election interaction λ^{PE}, which extending the static nationalization model m_{sta} yields model

Table 4: Log-linear models for votes cross-classified by territory, party, and election.

Models		
Independence	m_{ind}	$\ln \hat{v}_{tpe} = \lambda^0 + \lambda^T_t + \lambda^P_p + \lambda^E_e$
Static nationalization	m_{sta}	$\ldots + \lambda^{TE}_{te}$
Static & dynamic nationalization	m_{nat}	$\ldots + \lambda^{PE}_{pe}$
Electoral continuity	m_{con}	$\ldots + \lambda^{TP}_{tp}$
Dynamic nationalization	m_{dyn}	$\ldots + \lambda^{PE}_{pe} + \lambda^{TP}_{tp}$
No nationalization	m_{non}	$\ldots + \lambda^{PE}_{pe} + \lambda^{TP}_{tp} + \lambda^{TPE}_{tpe}$
Terms		
Main intercept	λ^0	
Territory	λ^T	Electorate size can vary over territories.
Party	λ^P	National vote totals can vary over parties.
Election	λ^E	National electorate size can vary over elections.
Territory-Election	λ^{TE}	Territorial electorate sizes can vary over elections.
Territory-Party	λ^{TP}	Party vote can vary over territories–voter segregation.
Party-Election	λ^{PE}	National party totals can vary over elections–dynamic nationalization.
Three-way	λ^{TPE}	Vote count can vary over territory-party-election combinations.

m_{nat}. Including both the dynamic nationalization coefficients λ^{PE} and the segregation coefficients λ^{TP} yields model m_{dyn}, which represents the substantive baseline of a denationalized electoral support with nationalized electoral change. Finally, including the territory-party-election three-way interaction yields the saturated model m_{non} which always fits perfectly.

As in the case of static nationalization, the substantive interpretation of numerical values is easier if we take the dissimilarity index's complement $1 - D$, so that the value of one marks the equivalence of the data and the baseline, and values decrease as the data becomes less similar to the model. This for m_{sta} gives an index of static nationalization with stable support $1 - D(m_{\text{sta}})$, for m_{con} an index of electoral continuity, for m_{nat} an index of statically and dynamically nationalized support, and for m_{dyn} an index of dynamic nationalization. Additional inferential leverage stems from the fact that model m_{sta} is nested both in model m_{nat} and m_{con}, and both models are nested in model m_{dyn}. In other words, m_{sta} is equivalent to m_{nat} or m_{con} with one set of interactions set to zero, and m_{nat} and m_{con} have this relationship to m_{dyn}. Subtracting $D(m_{\text{nat}})$ from $D(m_{\text{nat}})$ gives the fraction of the votes accounted for by the party-election interaction, and $D(m_{\text{sta}}) - D(m_{\text{con}})$ gives the fraction of the votes accounted for by the territory-party interaction.

The log-linear approach is best demonstrated on a set of elections characterized by complex territorial and temporal variability in electoral support. Such an example is provided by early 21th century Canada, where one of the largest parties–the Liberal Party–has declined and the New Democratic Party a rose to the second place, and the support of the Bloc Québécois was strongly regionalized.[2] Table 5 shows the dissimilarity index values for four log-linear models fit to the Canadian data. The elections are far from static nationalization with or without dynamic nationalization. Both would need more than one in four votes to be cast different to describe the returns. On the other hand, less than one in ten votes would need to be cast differently for electoral continuity to fit perfectly. Compared to stable static nationalization dynamic nationalization describes an additional 1% and electoral continuity an additional 18% of the votes, and compared to electoral continuity dynamic nationalization describes an additional 3%. In short, territorial variation was markedly more pronounced than the temporal one.

Partial fits reported in Figure 2 offer a more detailed picture. Static nationalization fails to describe first and foremost the votes of Bloc Québécois and in the Other category. Electoral continuity fits better overall and these two categories in particular–votes for BQ are strongly associated with constituencies. It fits the least to 2011, for all parties except the Conservatives, due to Liberals' losses and NDP gains. This is further evidenced by the fact that dynamic nationalization fits only slightly better overall, but considerably

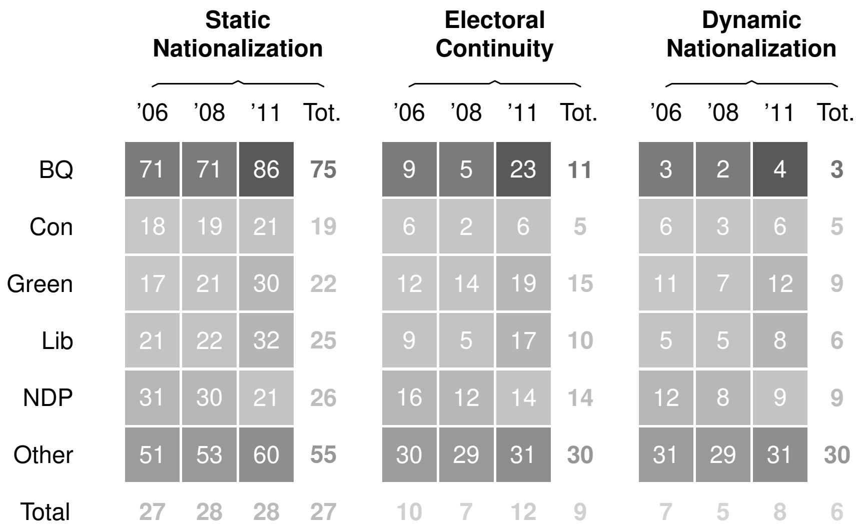
[2] The data analyzed here consists of returns from all 308 constituencies in the 2006, 2008, and 2011 General Elections (Kollman et al., 2014) for five parties with more than 1% of the national vote and an aggregate category for the remaining parties.

Table 5: Dissimilarity index for four log-linear models fit to the Canadian data (in rounded percents).

Baseline	Model	*D*	1 − *D*
Stable static nationalization	T, P, E, T-E	27	73
Static and dynamic nationalization	T, P, E, T-E, P-E	26	74
Electoral continuity	T, P, E, T-E, T-P	9	91
Dynamic nationalization	T, P, E, T-E, T-P, P-E	6	94

Note: Computed from vote data by Kollman et al. (2014). Abbreviations: T - territory, P - party, E - election.

Figure 2: Local, marginal, and total values of the dissimilarity index for three models of the Canadian parliamentary elections of 2006, 2008, and 2011 (in rounded percents).



Note: Computed from vote data by Kollman et al. (2014).

better to the Liberals and NDP. In short, the voters appear markedly segregated, chiefly in the case of BQ, with only weak dynamic nationalization that affects the Liberals and the NDP the most.

5 Adjusting for covariates

The proposed framework allows to adjust for covariates any of the indices that it provides. A hypothetical example will help to fix some intutions. Suppose a country with single-member constituencies and rules that do not allow parties replace candidates who die immediately before the election, nor the citizens to vote for such candidates. Suppose a candidate of a major party passed away in such a manner, resulting in the party receiving zero votes in the constituency. A researcher who wishes to measure static nationalization in such election may wish to adjust for this. Within the framework, this is done by expanding the baseline model

$$\ln \hat{v}_{tp} = \lambda^0 + \lambda_t^T + \lambda_p^P,$$

to include a covariate z_{tp} which takes the value of 1 if t corresponds to the affected constituency and p to the affected party, and equals to zero otherwise. This yields

$$\ln \hat{v}_{tp} = \lambda^0 + \lambda_t^T + \lambda_p^P + \beta_1 z_{tp},$$

where β_1 is a coefficient associated with the covariate z, and assures that in the affected cell the baseline value equals the observed value of zero, which in effect excludes it from the computation of the D index.

This approach applies more generally

$$\ln \hat{v}_{tp} = \lambda^0 + \lambda_t^T + \lambda_p^P + f c,$$

6 Conclusion

The main contribution of this paper is a general formal framework for the measurement of variation in electoral support in time, over territories, or across any other partitioning of the electorate, such as by ethnicity or gender. The framework generalizes the highly successful Pedersen index, and delivers indices of static and dynamic nationalization and electoral continuity, all of which retain the easy substantive interpretation of the Pedersen index. The Pedersen index is simply the *smallest* fraction of the votes that would need to be changed for the relative distributions of votes over parties in a pair of elections to become the same. In short, it gives the *lower bound* on the unobserved share of voters who switched parties. The indices of nationalization and continuity within the framework retain this lower bound property. All of the indices are special cases of the dissimilarity index D, also known as the Total Variation Distance, which has been productively applied across a variety of domains. Its formal and statistical properties are well documented, as is its estimation with sample data, making it attractive also in settings where researchers measure nationalization with survey data.

Another contribution of the paper is a formal clarification of some fundamental issues in the measurement of static nationalization. There, competing indices have proliferated, which may lead one to ask whether it is perhaps not the photographers' fault and Big Foot really is blurry, to borrow Mitch Hedberg's words. That is, whether the problem is not in the indices' deficiencies but rather in the fuzziness of the concept they are designed to measure. This paper clarifies that static nationalization corresponds to independence between party and territory in the vote distribution, and that attempts to make nationalization independent of the level of aggregation relate to ecological fallacy. The index of static nationalization defined within the framework has an easy and direct substantive interpretation, as well as two additional appeals. First, it correlates nearly perfectly with the most advanced existing measure of static nationalization, Bochsler's (2010) WPNS. Thus, it leads to the same substantive answers, while being easier to interpret and compare across concepts. Second, it corresponds to a popular segregation index. In this way, it bridges party nationalization research with segregation research. This author thinks that party research would much benefit from engaging with that field and building on its theoretical and methodological advances. However, a deeper engagement lies out of the scope of this paper.

References

Agresti, A., 2002. *Categorical data analysis, 2nd ed.* John Wiley & Sons.

Alemán, E. and Kellam, M., 2008. The nationalization of electoral change in the Americas. *Electoral Studies*, 27(2), pp.193–212.

Bartels, L.M., 1998. Electoral continuity and change, 1868–1996. *Electoral Studies*, 17(3), pp.301–326.

Berk, R.A., Western, B., and Weiss, R.E., 1995. Statistical inference for apparent populations. *Sociological Methodology*, pp.421–458.

Bochsler, D., 2010. Measuring party nationalisation: a new Gini-based indicator that corrects for the number of units. *Electoral Studies*, 29(1), pp.155–168.

Caramani, D., 1996. The nationalisation of electoral politics: a conceptual reconstruction and review of the literature. *West European Politics*, 19(2), pp.205–224.

Caramani, D., 2000. *Elections in Western Europe since 1815: Electoral results by constituencies*. Macmillan.

Caramani, D., 2004. *The nationalization of politics: the formation of national electorates and party systems in Western Europe*. Cambridge University Press.

Caramani, D. and Kollman, K., 2017. Symposium on "the nationalization of electoral politics: frontiers of research". *Electoral Studies*, 47, pp.51–54.

Dalton, H., 1920. The measurement of the inequality of incomes. *The Economic Journal*, 30(119), pp.348–361.

Duncan, O.D. and Duncan, B., 1955. A methodological analysis of segregation indexes. *American Sociological Review*, 20(2), pp.210–217.

Fienberg, S.E. and Rinaldo, A., 2007. Three centuries of categorical data analysis: log-linear models and maximum likelihood estimation. *Journal of Statistical Planning and Inference*, 137(11), pp.3430–3445.

Freedman, D.A., Klein, S.P., Sacks, J., Smyth, C.A., and Everett, C.G., 1991. Ecological regression and voting rights. *Evaluation review*, 15(6), pp.673–711.

Gini, C., 1914. Di una misura della dissomiglianza tra due gruppi di quantità e delle sue applicazioni allo studio delle relazione statistiche. *Atti del Reale Instituto Veneto di Scienze, Lettere ed Arti (Series 8)*, 74, pp.185–213.

Gini, C., 1921. Measurement of inequality of incomes. *The Economic Journal*, 31(121), pp.124–126.

Golosov, G.V., 2016. Party system nationalization: the problems of measurement with an application to federal states. *Party Politics*, 22(3), pp.278–288.

Goodman, L.A. and Kruskal, W.H., 1954. Measures of association for cross classifications*. *Journal of the American Statistical Association*, 49(268), pp.732–764.

Goodman, L.A. and Kruskal, W.H., 1959. Measures of association for cross classifications. II: further discussion and references. *Journal of the American Statistical Association*, 54(285), pp.123–163.

Jackman, S., 2000. Estimation and inference via Bayesian simulation: An introduction to Markov chain Monte Carlo. *American Journal of Political Science*, pp.375–404.

Jeffery, C. and Hough, D., 2003. Regional elections in multi-level systems. *European Urban and Regional Studies*, 10(3), pp.199–212.

Jeffery, C. and Hough, D., 2009. Understanding post-devolution elections in Scotland and Wales in comparative perspective. *Party Politics*, 15(2), pp.219–240.

Johnston, R., 1980. Federal and provincial voting: contemporary patterns and historical evolution. In: D.J. Elkins and R. Simeon, eds. *Small worlds: provinces and parties in Canadian political life*. Toronto: Methuen, pp.131–78.

Jones, M.P. and Mainwaring, S., 2003. The nationalization of parties and party systems: an empirical measure and an application to the Americas. *Party Politics*, 9(2), pp.139–166.

Kawato, S., 1987. Nationalization and partisan realignment in congressional elections. *American Political Science Review*, 81(4), pp.1235–1250.

Kollman, K., Hicken, A., Caramani, D., Backer, D., and Lublin, D., 2014. *Constituency-level elections archive* [Online].

Lago, I. and Montero, J.R., 2014. Defining and measuring party system nationalization. *European Political Science Review*, 6(2), pp.191–211.

Markatou, M. and Chen, Y., 2018. Non-quadratic distances in model assessment. *Entropy*, 20(6), p.464.

Massey, D.S. and Denton, N.A., 1988. The dimensions of residential segregation. *Social Forces*, 67(2), pp.281–315.

Merschrod, K., 1981. The index of dissimilarity as a measure of inequality. *Quality & Quantity*, 15(4), pp.403–411.

Morgan, B.S., 1975. The segregation of socio-economic groups in urban areas: a comparative analysis. *Urban Studies*, 12(1), pp.47–60.

Morgenstern, S., Polga-Hecimovich, J., and Siavelis, P.M., 2014. Seven imperatives for improving the measurement of party nationalization with evidence from Chile. *Electoral Studies*, 33, pp.186–199.

Morgenstern, S. and Potthoff, R.F., 2005. The components of elections: district heterogeneity, district-time effects, and volatility. *Electoral Studies*, 24(1), pp.17–40.

Morgenstern, S., Swindle, S.M., and Castagnola, A., 2009. Party nationalization and institutions. *Journal of Politics*, 71(4), pp.1322–1341.

Pallarés, F. and Keating, M., 2003. Multi-level electoral competition regional elections and party systems in Spain. *European Urban and Regional Studies*, 10(3), pp.239–255.

Pedersen, M.N., 1979. The dynamics of European party systems: changing patterns of electoral volatility. *European Journal of Political Research*, 7(1), pp.1–26.

Powell, E.N. and Tucker, J.A., 2014. Revisiting electoral volatility in post-communist countries: new data, new results and new approaches. *British Journal of Political Science*, 44(1), pp.123–147.

Reardon, S.F. and Firebaugh, G., 2002. Measures of multigroup segregation. *Sociological Methodology*, 32(1), pp.33–67.

Rudas, T., 1998. *Odds ratios in the analysis of contingency tables*. Sage.

Sakoda, J.M., 1981. A generalized index of dissimilarity. *Demography*, 18(2), pp.245–250.

Schakel, A.H., 2013. Congruence between regional and national elections. *Comparative Political Studies*, 46(5), pp.631–662.

Schattschneider, E.E., 1960. *The semi-sovereign people: A realist's view of democracy in America*. Rinehart & Winston.

Stokes, D.E., 1965. A variance components model of political effects. In: J. Claunch, ed. *Mathematical applications in political science*. Dallas: The Arnold Foundation, pp.61–85.

Stokes, D.E., 1967. Parties and the nationalization of electoral forces. In: W. Chambers and W. Burnham, eds. *The American party systems: stages of political development*. OUP, pp.182–202.

Tomz, M., Tucker, J.A., and Wittenberg, J., 2002. An easy and accurate regression model for multiparty electoral data. *Political Analysis*, 10(1), pp.66–83.

von Eye, A. and Mun, E.-Y., 2014. *Log-linear modeling: concepts, interpretation, and application*. John Wiley & Sons.

Western, B. and Jackman, S., 1994. Bayesian inference for comparative research. *American Political Science Review*, 88(2), pp.412–423.

Wittenberg, J., 2006. *Crucibles of political loyalty: church institutions and electoral continuity in Hungary*. Cambridge University Press.

Wittenberg, J., 2013. How similar are they? Rethinking electoral congruence. *Quality & Quantity*, 47(3), pp.1687–1701.

Supporting Information for

"Unifying the Measurement of Variation in Electoral Support"

Tuesday 17th May, 2022 at 20:15

A Nationalization Measures

Table A.1: Referential conventions for equations in Tables A.2 and A.3 to follow closer the conventions of the party nationalization literature. A different set of conventions is used throughout the paper.

T	The number of territories.
s	The national vote share of a party.
v_i	The local vote count of a party.
l_i	The local vote total.
p_i	The local vote share of a party.
$\bar{p}$	The mean local vote share of a party.
r_i	The rank of a local vote share of a party.
c_i	Party's local vote as a fraction of its national vote.
m_i	The local number of seats.
e_i	An indicator variable showing whether the party did enter the race in the territory.

Table A.2: Measures of party system nationalization available only for the whole party system. The equations use referential conventions reported in Table A.1, which to better follow the practices of the party nationalization literature differ from those used in the rest of this paper.

Measure	Formula
Indicator of Party Aggregation (Chhibber and Kollman, 1998)	$\frac{1}{\Sigma_j s^2} - \frac{1}{T} \sum_i \left(\frac{1}{\Sigma_j p_{i,j}^2} \right)$
Inflation Score (Cox, 1999)	$\frac{100}{\Sigma_j s^2} \left(\frac{1}{\Sigma_j s^2} - \frac{1}{T} \sum_i \left(\frac{1}{\Sigma_j p_{i,j}^2} \right) \right)$
Inflation Index (Moenius and Kasuya, 2004)	$\frac{100}{(1/T)\Sigma_i \left(1/\Sigma_j p_{i,j}^2\right)} \left(\frac{1}{\Sigma_j s^2} - \frac{1}{T} \sum_i \left(\frac{1}{\Sigma_j p_{i,j}^2} \right) \right)$
Weighted Inflation Index (Moenius and Kasuya, 2004)	$100 \left(\frac{(\Sigma_i l_i) / (\Sigma_j s^2)}{\Sigma_i \left(l_i / \Sigma_j p_{i,j}^2 \right)} - 1 \right)$
Local Entrant Measure (Lago and Montero, 2014)	$\sum_j \left(s_j \frac{\Sigma_i (m_i e_{i,j})}{\Sigma_i m_i} \right)$

Table A.3: Measures of party nationalization applicable to individual parties. Party system values can be obtained by unweighted or weighted averages of party values. The referential conventions differ from those used in the rest of the paper to follow closer the party nationalization literature and are reported in Table A.1.

Measure	Formula
Mean Absolute Deviation (Rose and Urwin, 1975)	$\frac{1}{T}\sum_i \lvert p_i - s\rvert$
Mean Squared Deviation	$\frac{1}{T}\sum_i (p_i - s)^2$
Variance	$\frac{1}{T-1}\sum_i (p_i - s)^2$
Lee index (Lee, 1988)	$\frac{1}{2}\sum_i \lvert p_i - s\rvert$
Variability Coefficient (Ersson, Janda, and Lane, 1985)	$\frac{1}{\bar{p}}\sqrt{\frac{1}{T-1}\sum_i (p_i - \bar{p})^2}$
Normalized Variability Coefficient (Golosov, 2016)	$\frac{1}{\bar{p}\sqrt{T}}\sqrt{\frac{1}{T-1}\sum_i (p_i - \bar{p})^2}$
Standardized and Weighted Variability Coefficient (Ersson, Janda, and Lane, 1985)	$\frac{\sqrt{T}}{s}\sqrt{\frac{1}{T-1}\sum_i (p_i - s)^2}$
Index adjusted for Party size and number of Regions (Caramani, 2004)	$\sqrt{\frac{T\sum_i \lvert p_i - \bar{p}\rvert}{2(T-1)\sum_i p_i}}$
Cumulative Regional Inequality (Rose and Urwin, 1975)	$\frac{1}{200}\sum_i \lvert p_i - c_i\rvert$
Territorial Coverage Index (Caramani, 2004)	$\frac{1}{T}\sum_i e_i$
Index of Party Regionalization (Golosov and Ponarin, 1999)	$\sqrt{\frac{T\sum_i \lvert p_i - \bar{p}\rvert}{2(T-1)\sum_i p_i}}$
Coefficient of Party Regionalization (Golosov, 2016)	$\frac{1}{T-1}\left(T - \left((\sum_i p_i)^2/\sum_i p_i^2\right)\right)$
Index of Party Nationalization (Golosov, 2016)	$1 - \frac{1}{T-1}\left(T - \left((\sum_i p_i)^2/\sum_i p_i^2\right)\right)$
Normalized Gini Coefficient (Golosov, 2016)	$\frac{2\sum_i (p_i r)}{(T-1)\sum_i p_i} - \frac{T+1}{T-1}$
Party Nationalization Score (Jones and Mainwaring, 2003)	$\frac{2T+1}{T} - \frac{2\sum_i (p_i r)}{T\sum_i p_i}$
Weighted Party Nationalization Score (Bochsler, 2010) (l and p ordered by p/l)	$2\frac{\sum_i \left(l_i\left(\sum_{k=1}^{i} p_k - p_i/2\right)\right)}{\sum_i l_i \sum_i p_i}$
Scaled Party Nationalization Score (Bochsler, 2010)	$WPNS^{\left(\frac{1}{\log_{10} E}\right)};\ \ E = (\sum_i l_i)^2/\sum_i l_i^2$

B Data Description

B.1 CLEA Data

The CLEA dataset (Kollman et al., 2014) in its version from 12th August 2014 contains 1,781 sets of constituency-level results from 132 countries. First, I have excluded all constituencies which were uncontested and/or had missing data on at least one party. From this data all the 1,495 sets with data on more than one party with a positive national vote count were used. The resulting set contains elections from 119 countries, the earliest being from 1789 and the latest from 2013, and contains vote counts for 19,518 party-election combinations.

Table B.4: List of 1,495 elections from 119 countries sourced from the vote data by Kollman et al. (2014). Number of elections in parentheses.

Albania	(4) 2001–2013	Estonia	(6) 1992–2011	Nicaragua	(5) 1990–2011
Andorra	(4) 1997–2011	Finland	(35) 1907–2007	Nigeria	(1) 2003
Angola	(2) 2008–2012	France	(7) 1978–2002	Norway	(34) 1882–2009
Anguilla	(6) 1989–2010	Gambia	(2) 1997–2007	Pakistan	(1) 2002
Antigua and Barbuda	(13) 1951–2009	Georgia	(1) 2012	Paraguay	(3) 1998–2008
Argentina	(14) 1983–2009	Germany	(38) 1871–2005	Peru	(8) 1963–2011
Armenia	(2) 2007–2012	Ghana	(3) 2000–2008	Philippines	(2) 1998–2007
Australia	(34) 1901–1984	Greece	(22) 1926–2000	Poland	(5) 1991–2005
Austria	(25) 1919–2008	Grenada	(15) 1951–2013	Portugal	(14) 1975–2011
Azerbaijan	(1) 2010	Guatemala	(7) 1984–2011	Puerto Rico	(5) 1992–2008
Bahamas	(4) 1997–2012	Guinea	(1) 2013	Romania	(4) 1990–2000
Bangladesh	(3) 1991–2008	Guinea-Bissau	(2) 1994–2004	Russian Federation	(3) 2003–2011
Barbados	(11) 1966–2013	Guyana	(6) 1953–2006	Saint Kitts and Nevis	(13) 1952–2010
Belgium	(61) 1847–1995	Honduras	(7) 1980–2005	Saint Lucia	(14) 1951–2006
Belize	(14) 1954–2012	Hungary	(6) 1990–2010	Samoa	(1) 2011
Benin	(3) 1991–2011	Iceland	(26) 1916–1995	Seychelles	(1) 2007
Bermuda	(12) 1963–2012	India	(13) 1977–2009	Singapore	(11) 1963–2006
Bhutan	(2) 2008–2013	Indonesia	(2) 1999–2004	Somaliland	(1) 2005
Bolivia	(9) 1979–2009	Iraq	(1) 2010	South Africa	(1) 2009
Bosnia and Herzegovina	(1) 2006	Ireland	(26) 1922–1997	Spain	(8) 1977–2004
Botswana	(9) 1969–2009	Italy	(16) 1919–1996	Sri Lanka	(12) 1952–2010
Brazil	(14) 1945–2010	Jamaica	(14) 1944–2002	St. Vincent and the G.	(16) 1951–2010
British Virgin Islands	(3) 2003–2011	Japan	(21) 1947–2012	Suriname	(3) 2000–2010
Bulgaria	(6) 1991–2009	Kenya	(5) 1961–2013	Sweden	(31) 1911–2006
Cambodia	(1) 2008	Korea	(16) 1958–2012	Switzerland	(45) 1848–1995
Cameroon	(2) 1997–2002	Kosovo	(2) 2007–2010	Taiwan	(6) 1986–2004
Canada	(40) 1867–2011	Latvia	(3) 1998–2006	Tanzania	(1) 2005
Cape Verde	(4) 1995–2011	Lesotho	(7) 1965–2012	Thailand	(8) 1969–1992
Cayman Islands	(2) 2005–2009	Liberia	(1) 2005	Togo	(2) 2007–2013
Colombia	(4) 1998–2010	Liechtenstein	(21) 1945–2013	Trinidad and Tobago	(7) 1991–2010
Costa Rica	(15) 1953–2010	Luxembourg	(18) 1919–1994	Turkey	(16) 1950–2011
Croatia	(1) 2007	Macedonia	(4) 2002–2011	Turks and Caicos Islands	(2) 2007–2012
Cyprus	(3) 2001–2011	Malawi	(2) 1999–2004	UK	(38) 1832–2010
Czech Republic	(6) 1990–2006	Malaysia	(1) 2013	Ukraine	(1) 1998
Denmark	(69) 1849–2011	Mauritius	(9) 1967–2005	Uruguay	(11) 1954–2009
Dominica	(3) 1995–2005	Mexico	(8) 1991–2012	US	(284) 1789–2012
Dominican Republic	(11) 1962–2006	Mozambique	(3) 1999–2009	US	(1) 1980
Ecuador	(9) 1979–2013	Nepal	(1) 2008	Zambia	(5) 1968–2006
El Salvador	(7) 1994–2012	Netherlands	(36) 1888–2012	Zimbabwe	(2) 2005–2013
Equatorial Guinea	(1) 1993	New Zealand	(20) 1946–2011		

B.1.1 Canada

Figure B.1: Vote fractions in rounded percents obtained in three Canadian General Elections.

	'06	'08	'11
Bloc Quebecois	10	10	6
Conservative	36	38	40
Green	4	7	4
Liberal	30	26	19
New Democratic	17	18	31
Other	1	1	1

Note: Computed from vote data by Kollman et al. (2014).

C R Package partynat

partynat is an R package for the computation of several indices used in party research, all of which can be computed from party-by-territory matrices of votes. Where the data comes from a sample rather than from a complete population, the user can obtain confidence intervals for the statistic of interest using several resampling techniques. The main tool in the package is the partynat function, which provides a unified interface to all the implemented indices:

```
partynat(mat,
         statistic = "PNS",
         weight_choice = TRUE,
         weight_territory = TRUE,
         boot = FALSE,
         jack = FALSE,
         subsample = FALSE,
         n_rep = 1e1,
         bias = TRUE,
         size = round(sum(mat)/2),
         confidence_level = 0.95)
```

The function has the following arguments:

- `mat` is the matrix with votes to be inspected in which each row corresponds to a different territory and each column to a different choice (e.g., party).
- `statistic` specifies which index should be computed. The implemented indices and their abbreviations are reported in Table C.5.
- `weight_choice` and `weight_territory` indicate whether for indices where this is an option choice and territory weight should be applied.
- `boot`, `jack`, and `subsample` specify whether nonparametric bootstrap, jackknife, or subsampling (leave-*k* out jackknife) should be applied. In each call of the function at most one of them can be set to `TRUE`.
- `n_rep` specifies the number of replicates if either `boot` or `subsample` are set to `TRUE`.
- `bias` indicates whether bias corrections for the resampling estimates should be applied.
- `size` specifies the size of the sample if `subsample = TRUE`. Defaults to half the number of the observed votes.
- `confidence_level` sets the level for the confidence intervals if a resampling procedure is applied.

The function outputs an object of S3 class `'partynat'`, a list composed of

- `call` The matched call
- `stat` The statistics argument
- `name` The name of the index
- `total` Value of the index for the whole table. If resampling is applied, includes standard errors and confidence intervals.

- `choices` Values of the index for each choice. NA if not defined under the selected index. If resampling is applied, includes standard errors and confidence intervals.
- `confidence_level` The requested confidence level in case a resampling procedure was applied.

Table C.5: Measures implemented in the `partynat` package

Measure	Call
Weighted Party (System) Nationalization Score (Bochsler, 2010)	`"PNSW"`
Standardized Party (System) Nationalization Score (Bochsler, 2010)	`"PNS10"`
Territorial Coverage Index (Caramani, 2004)	`"TCI"`
Index of Party Regionalization (Golosov and Ponarin, 1999)	`"IPR1"`
Coefficient of Party Regionalization (Golosov, 2016)	`"CPR"`
Index of Party (System) Nationalization (Golosov, 2016)	`"IPN"`
Normalized Party (System) Nationalization Score (Golosov, 2016)	`"NPNS"`
Party Nationalization Score (Jones and Mainwaring, 2003)	`"PNS"`
Lee index (Lee, 1988)	`"Lee"`
Index of variation/Mean Absolute Deviation (Rose and Urwin, 1975)	`"MAD"`
Mean Standard Deviation of row shares	`"MSD"`
Variance of row shares	`"var"`
Variability Coefficient (Ersson, Janda, and Lane, 1985)	`"VC"`
Standardized and Weighted Variability Coefficient (Ersson, Janda, and Lane, 1985)	`"SWVC"`
Normalized Coefficient of Variation (Golosov, 2016)	`"NVC"`
Index adjusted for Party size and number of Regions (Caramani, 2004)	`"IPR2"`
Cumulative Regional Inequality (Rose and Urwin, 1975)	`"CRI"`
Indicator of Party Aggregation (Chhibber and Kollman, 1998)	`"IPA1"`
Inflation Score (Cox, 1999)	`"IS"`
Index of Party Aggregation (Allik, 2006)	`"IPA2"`
Inflation Index (Moenius and Kasuya, 2004)	`"II"`
Gini's Dissimilarity index for party-territory independence	`"Delta"`
Mutual Information (in bits)	`"MI"`

References

Allik, M., 2006. *Parteisüsteemid föderaalriikides*. Master's thesis. University of Tartu.

Bochsler, D., 2010. Measuring party nationalisation: a new Gini-based indicator that corrects for the number of units. *Electoral Studies*, 29(1), pp.155–168.

Caramani, D., 2004. *The nationalization of politics: the formation of national electorates and party systems in Western Europe*. Cambridge University Press.

Chhibber, P. and Kollman, K., 1998. Party aggregation and the number of parties in India and the United States. *American Political Science Review*, 92(2), pp.329–342.

Cox, G., 1999. Electoral rules and electoral coordination. *Annual Review of Political Science*, 2, pp.145–161.

Ersson, S., Janda, K., and Lane, J.-E., 1985. Ecology of party strength in Western Europe a regional analysis. *Comparative Political Studies*, 18(2), pp.170–205.

Golosov, G.V., 2016. Party system nationalization: the problems of measurement with an application to federal states. *Party Politics*, 22(3), pp.278–288.

Golosov, G.V. and Ponarin, E., 1999. Regional bases of party politics: a measure and its implications for the study of party system consolidation in new democracies.

Jones, M.P. and Mainwaring, S., 2003. The nationalization of parties and party systems: an empirical measure and an application to the Americas. *Party Politics*, 9(2), pp.139–166.

Kollman, K., Hicken, A., Caramani, D., Backer, D., and Lublin, D., 2014. *Constituency-level elections archive* [Online].

Lago, I. and Montero, J.R., 2014. Defining and measuring party system nationalization. *European Political Science Review*, 6(2), pp.191–211.

Lee, A., 1988. The persistence of difference: electoral change in Cornwall. *Political studies association conference, Plymouth*.

Moenius, J. and Kasuya, Y., 2004. Measuring party linkage across districts: some party system inflation indices and their properties. *Party Politics*, 10(5), pp.543–564.

Rose, R. and Urwin, D.W., 1975. *Regional differentiation and political unity in Western nations*. Sage.