

# Logic-Based Parsing with Neural Networks

## Compositional Models of Vector-based Semantics

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☸ Certified

no military applications/funding

# The agenda

- ▶ exploring  $\mathcal{A}ethel$
- ▶ parsing with graph learning machinery

# Types

$\text{ILL}_{\multimap}$  plus  $\diamond, \square$  modalities for *dependency domain demarkation*.

$\mathbb{T}$  inductively defined as:

$$\mathbb{T} := A \mid T \multimap T \mid \diamond^d T \mid \square^c T \quad A \in \mathbb{A}, T \in \mathbb{T}$$

$\mathbb{A}$  – closed set of base types

$\multimap$  – linear function builder

$\diamond$  – reserved for “necessary arguments”, i.e. complements

$\square$  – reserved for “optional functions”, i.e. adjuncts

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# Rules & Terms

☺ function/argument structures

$$\frac{}{c : T \vdash c : \bar{T}} \textit{Lex} \qquad \frac{\Gamma \vdash s : T_1 \multimap T_2 \quad \Delta \vdash t : T_1}{\Gamma, \Delta \vdash s t : T_2} \multimap E$$



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☺ simple dependency demarkation

$$\frac{\Gamma \vdash t : T}{\langle \Gamma \rangle^d \vdash \Delta^d t : \diamond^d T} \diamond^d I \qquad \frac{\Gamma \vdash s : \square^d T}{\langle \Gamma \rangle^d \vdash \blacktriangledown^d s : T} \square^d E$$

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$$\frac{\Gamma \vdash t : T}{\langle \Gamma \rangle^d \vdash \Delta^d t : \Diamond^d T} \Diamond^d I \qquad \frac{\Gamma \vdash s : \Box^d T}{\langle \Gamma \rangle^d \vdash \blacktriangledown^d s : T} \Box^d E$$

☺ hypothetical reasoning

$$\frac{}{x : T \vdash x : T} Ax \qquad \frac{\Gamma, x : T_1 \vdash s : T_2}{\Gamma \vdash \lambda x. s : T_1 \multimap T_2} \multimap I$$

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👁️ cosmic horror from the great beyond

$$\frac{\langle \Gamma \rangle^d \vdash s : T}{\Gamma \vdash \blacktriangle^d s : \square^d T} \square^d I \qquad \frac{\Gamma[\langle x : T_1 \rangle^d] \vdash t : T_2 \quad \Delta \vdash s : \diamond^d T_1}{\Gamma[\Delta] \vdash t[x \mapsto \nabla^d s] : T_2} \diamond^d E$$

# Rules & Terms

Bonus:

ad-hoc extraction

$$\frac{\langle \Gamma, \langle \mathbf{x} : T_1 \rangle^X, \Delta \rangle^d \vdash \mathbf{s} : T_2}{\langle \Gamma, \Delta \rangle^d, \langle \mathbf{x} : T_1 \rangle^X \vdash \mathbf{s} : T_2} \times$$

# Today's Example

alt-tab

# Formula Decomposition: Types $\equiv$ Trees

The type assignment of “*where*”:

$$\diamond^{relcl}(\diamond^x \square^x \diamond^{mod} \square^{mod}(ppart \multimap ppart) \multimap ssub) \multimap \square^{mod}(np \multimap np)$$

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$$!(\_) := \diamond \square(\_)$$

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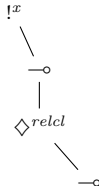
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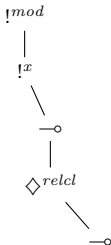
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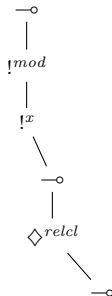
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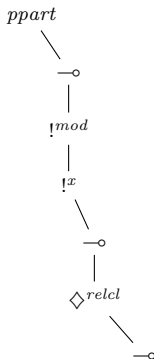
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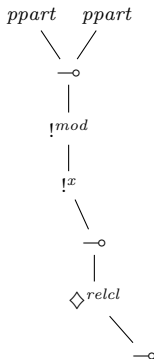
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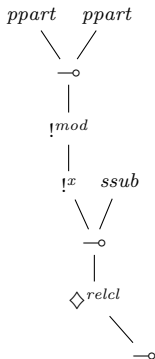
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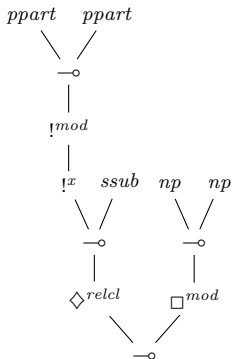




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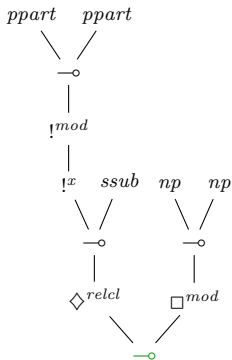


# Formula Decomposition: Polarity Induction

subtree polarity (*preserved* to the right, *inverted* to the left)

+ we **have**

- we **miss**

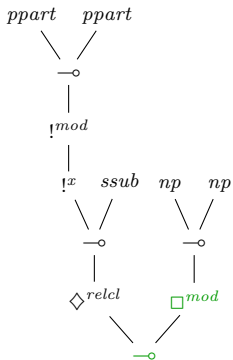


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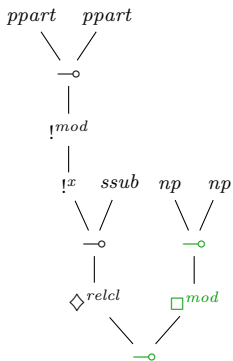


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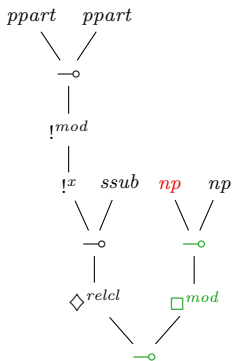


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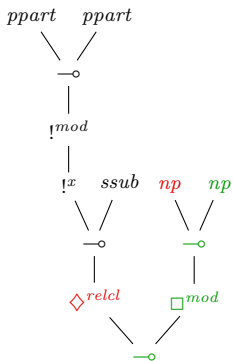


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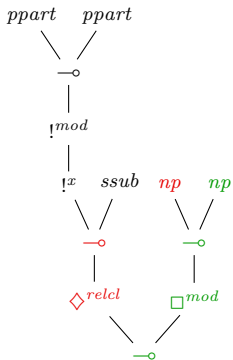


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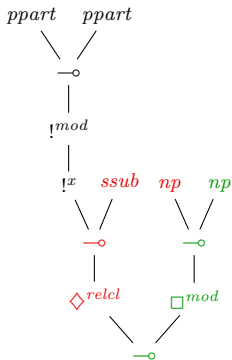


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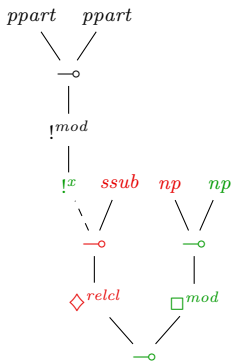


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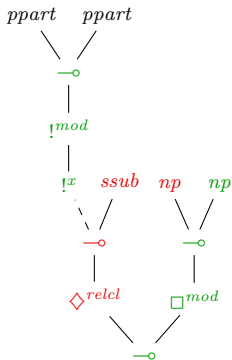


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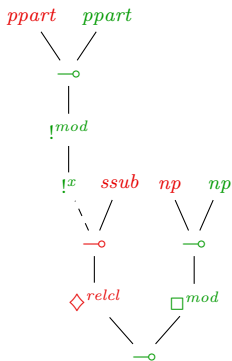
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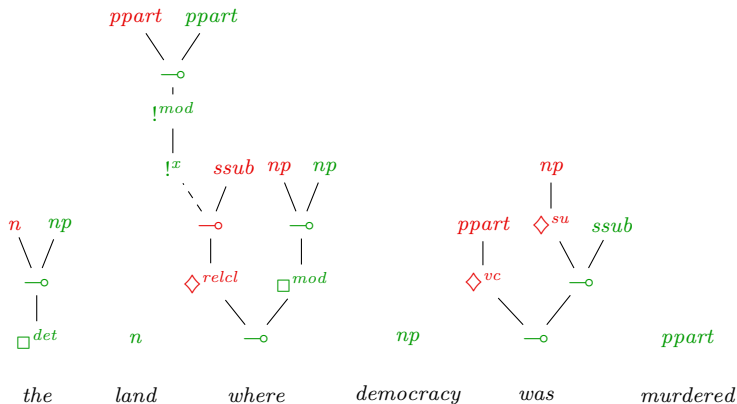
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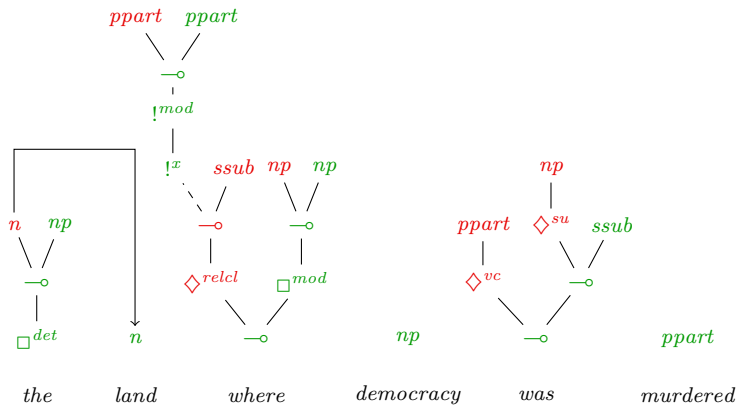
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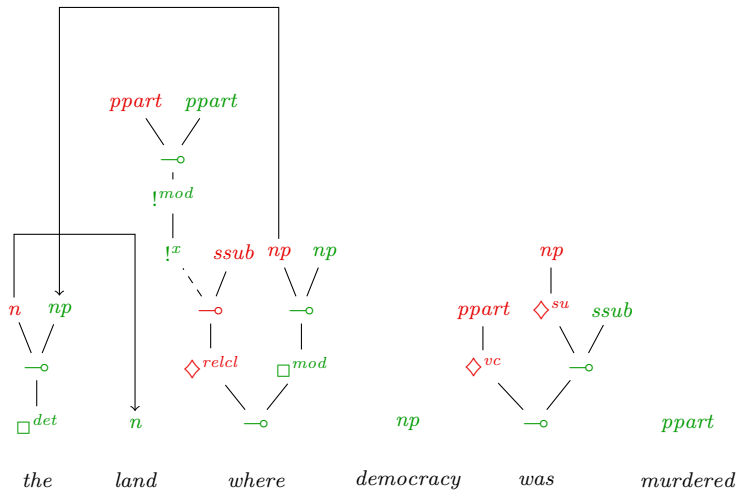
# Proof Frames



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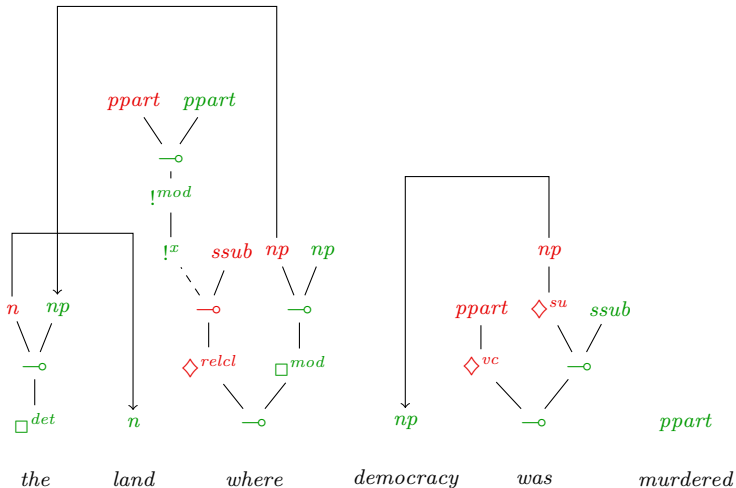


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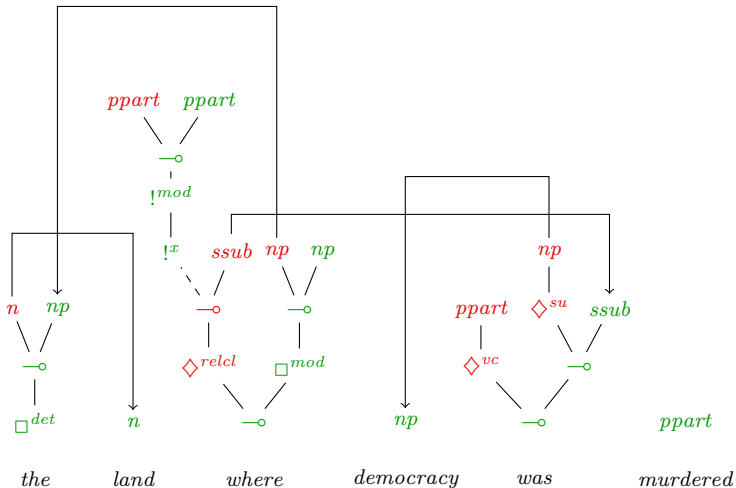




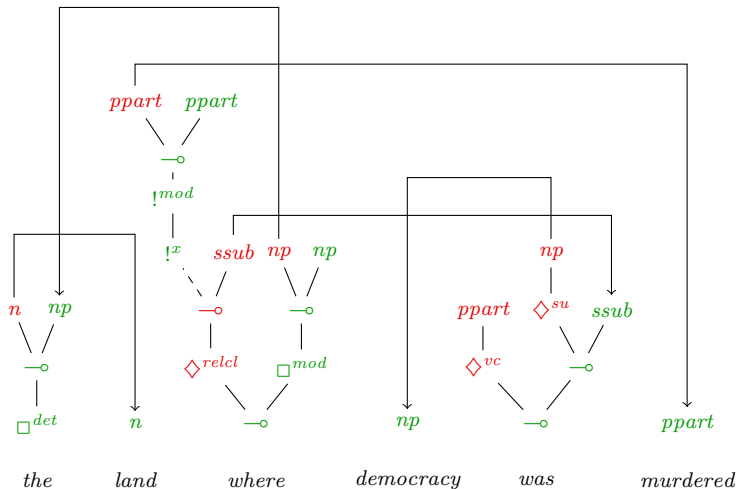
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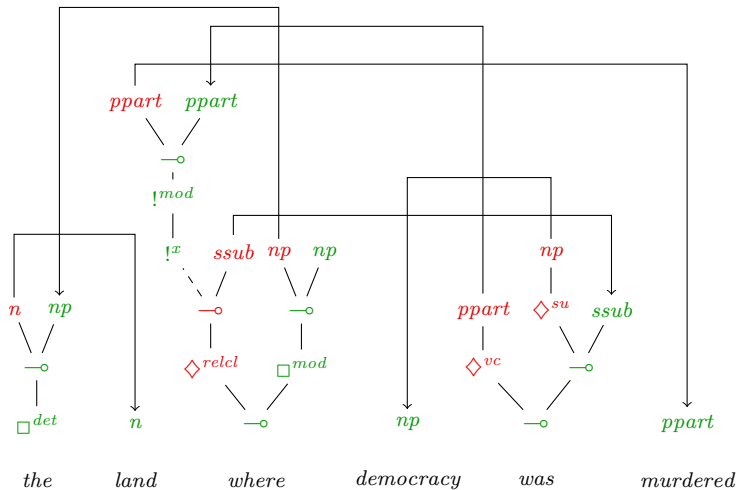
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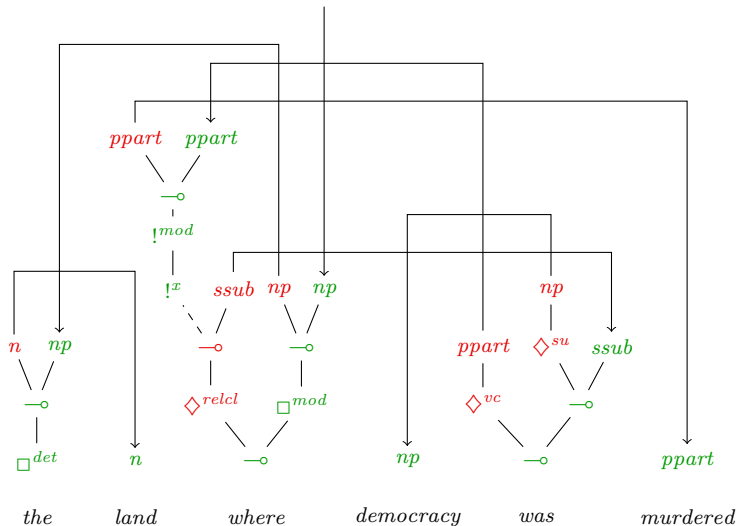
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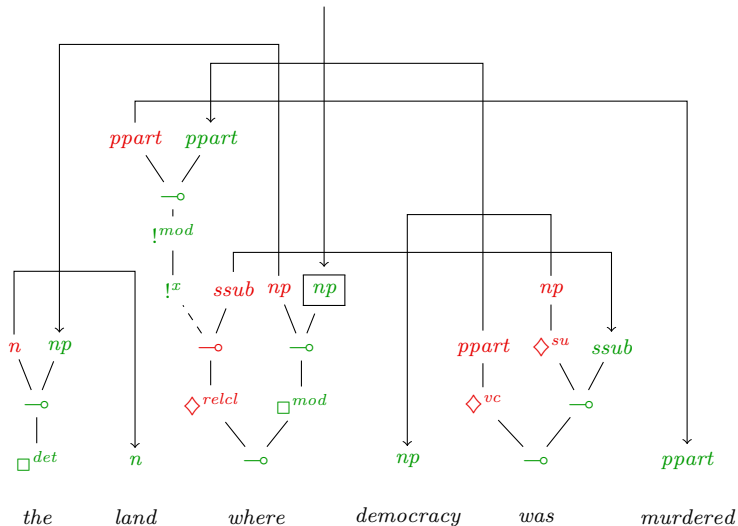
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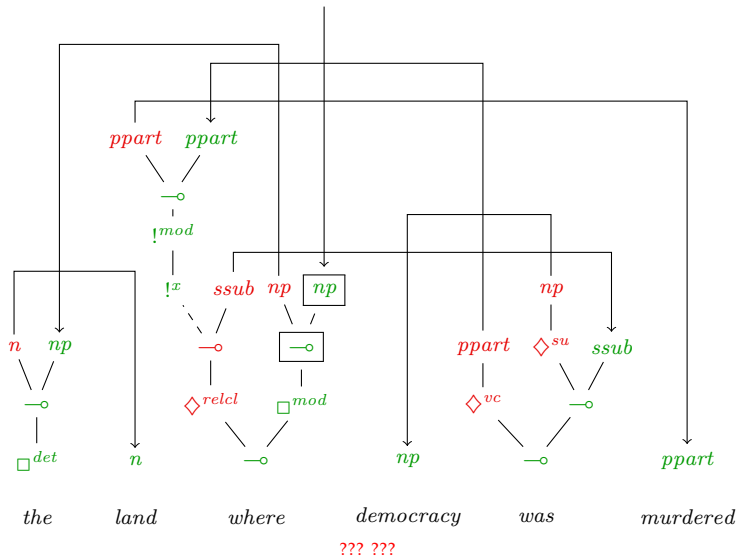
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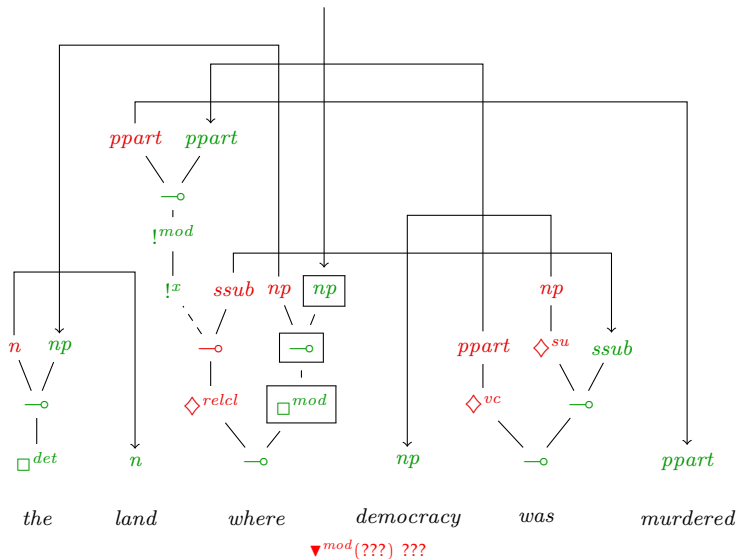
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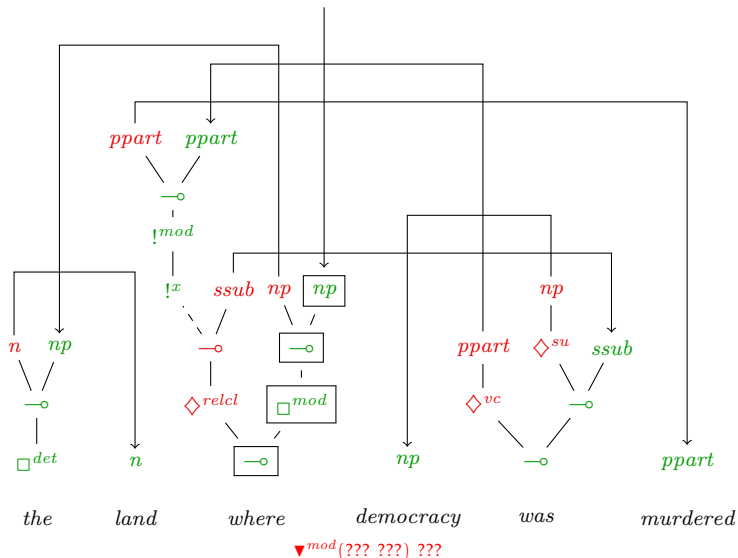


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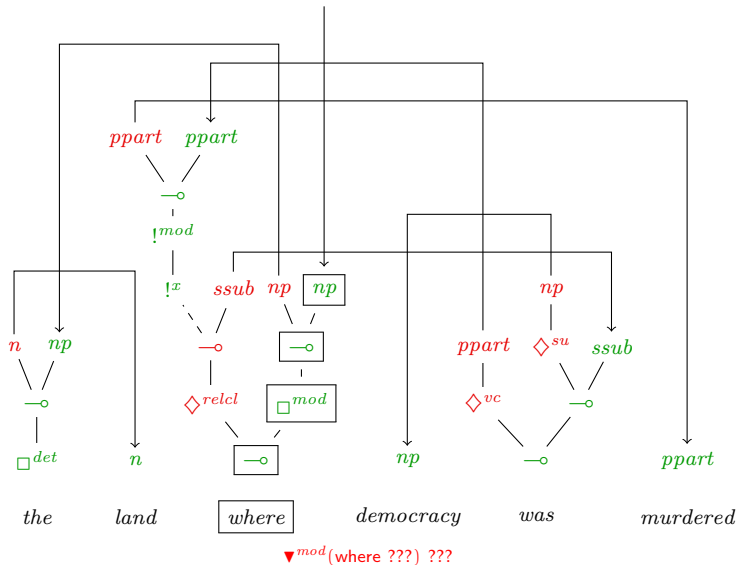




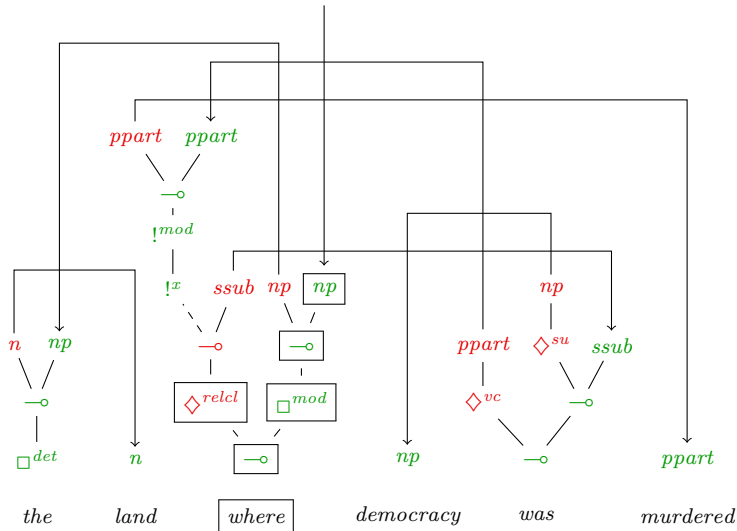
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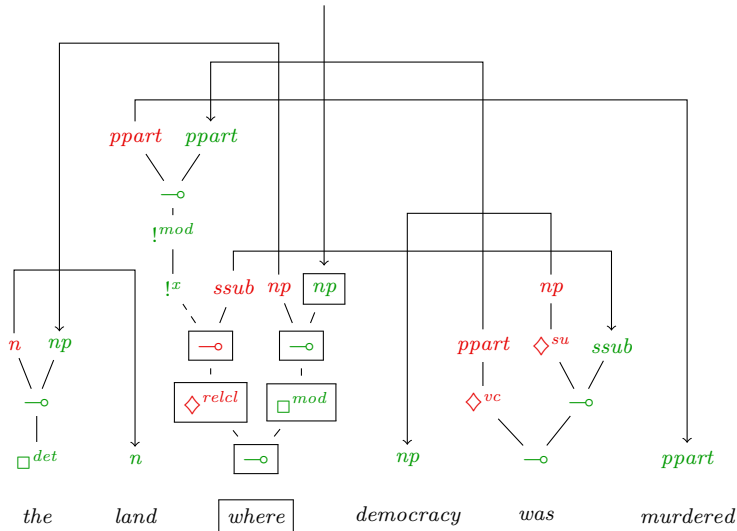


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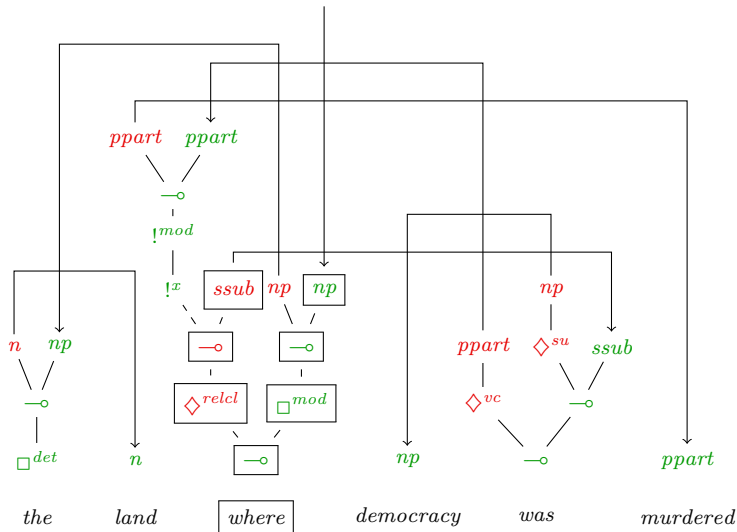
▼<sup>mod</sup>(where Δ<sup>relcl</sup>(??)) ???

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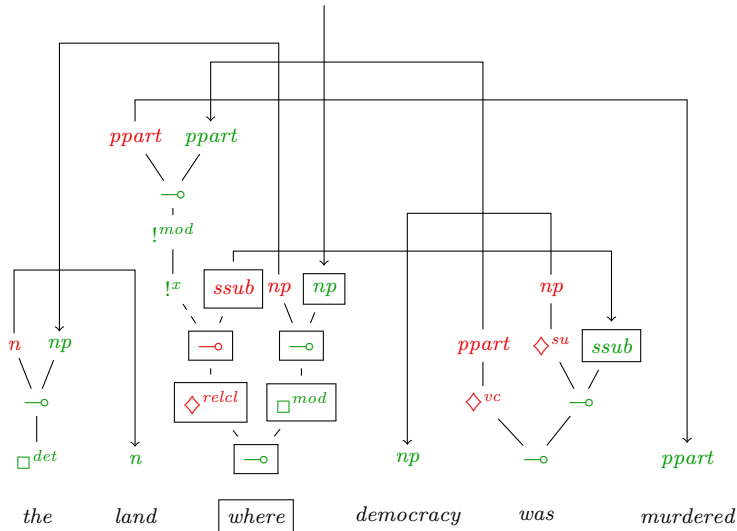
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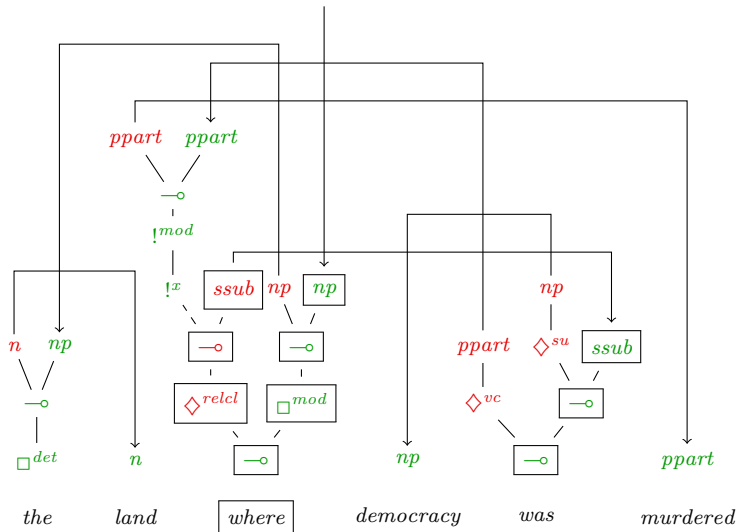
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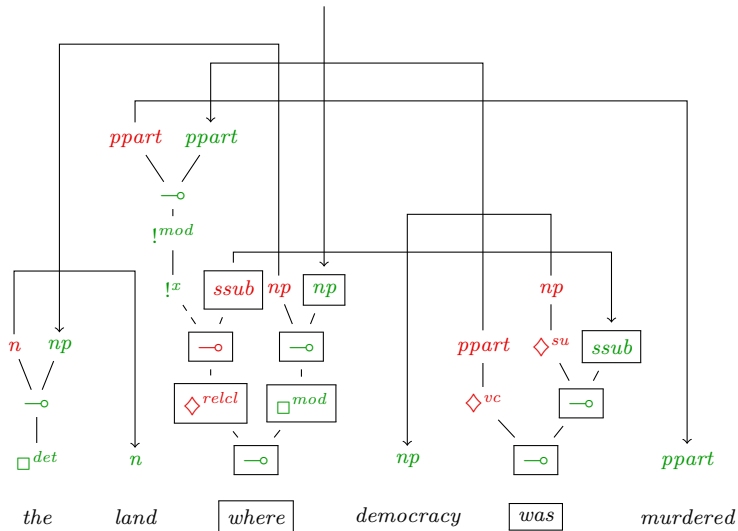
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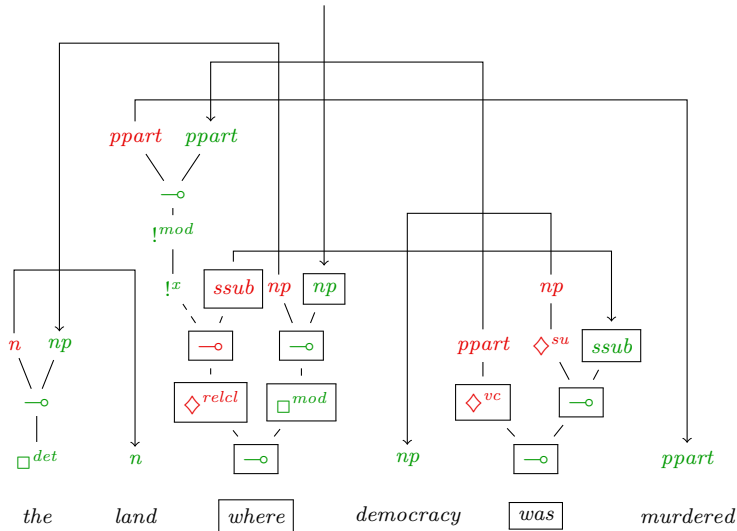


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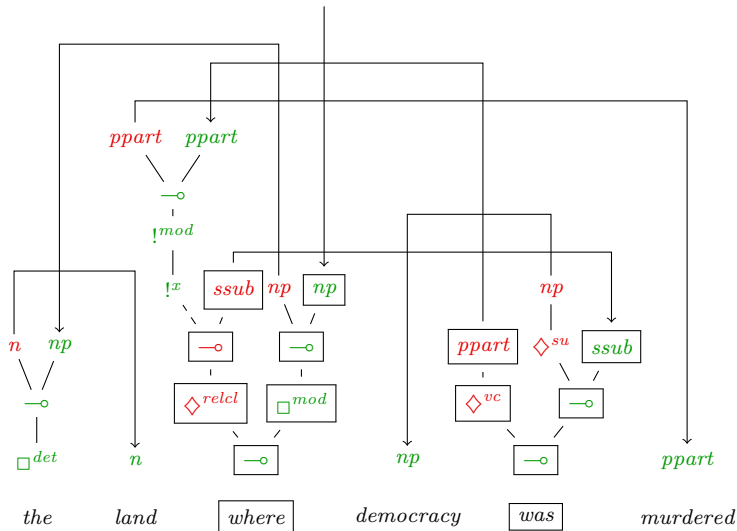
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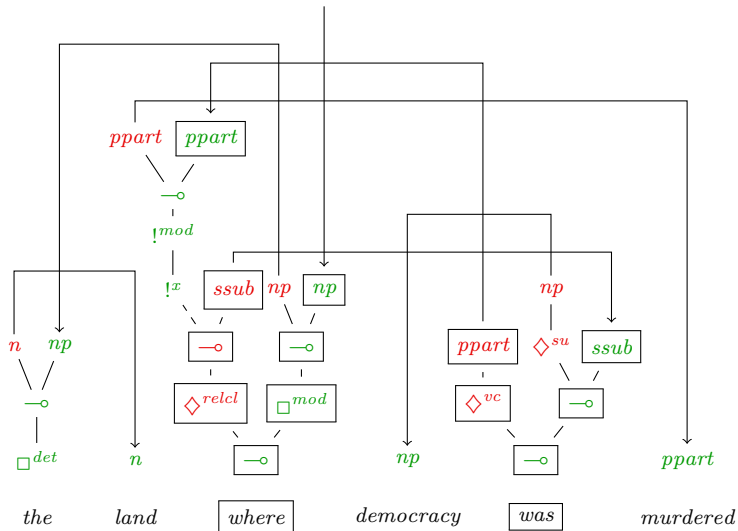
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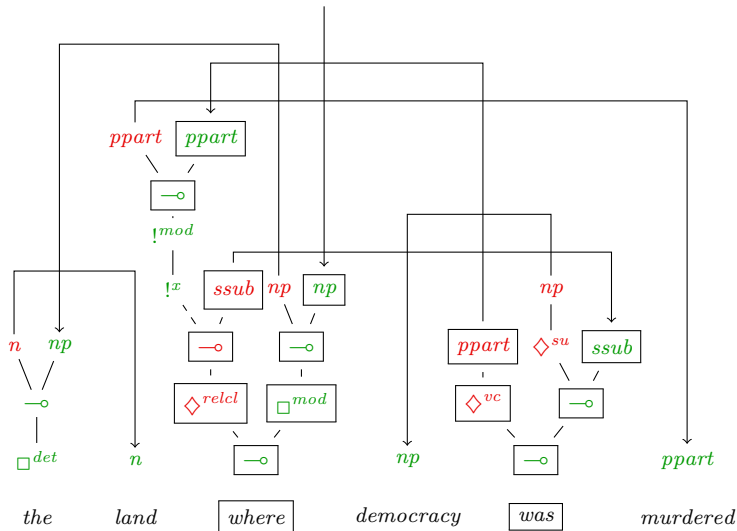
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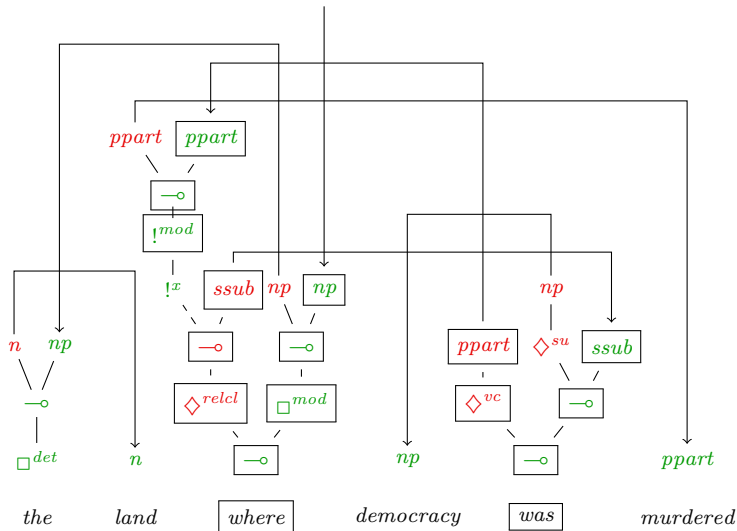
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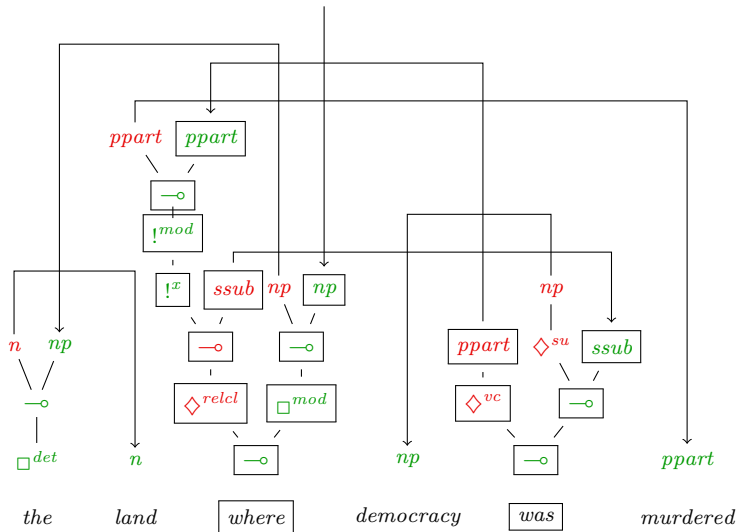
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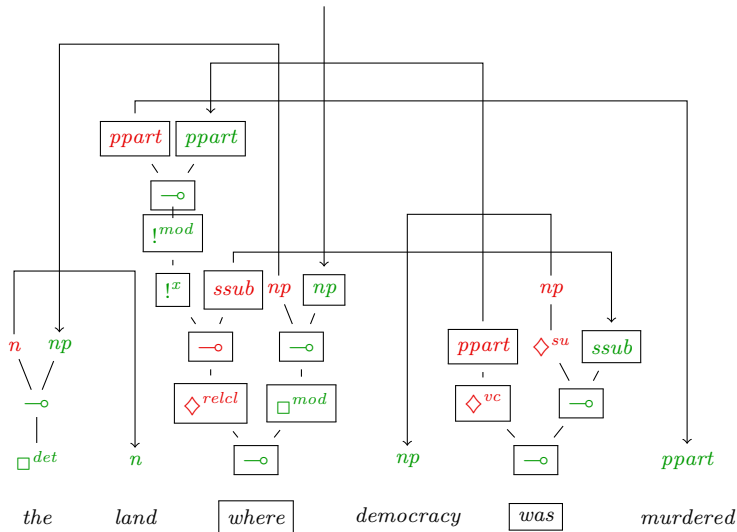
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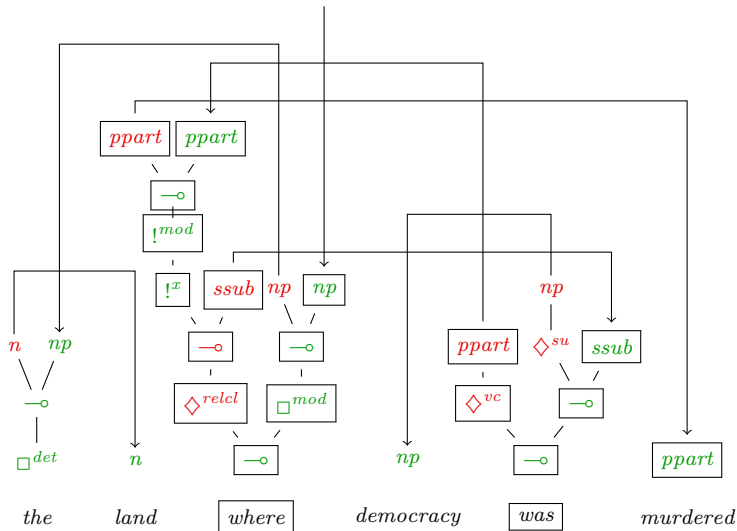
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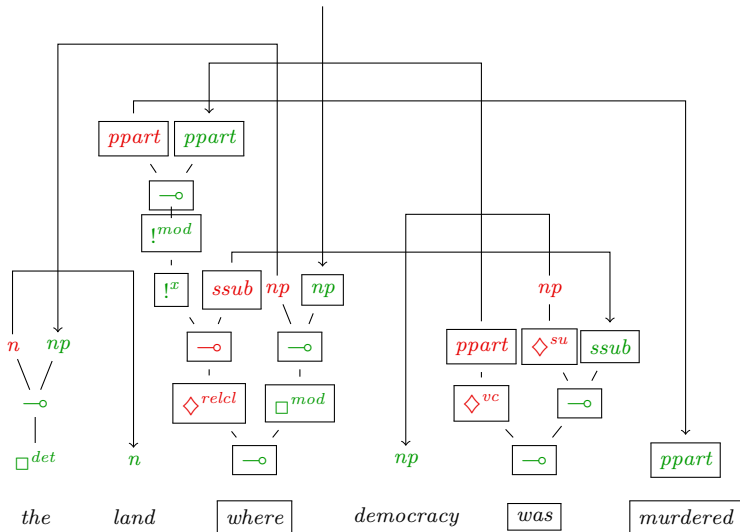
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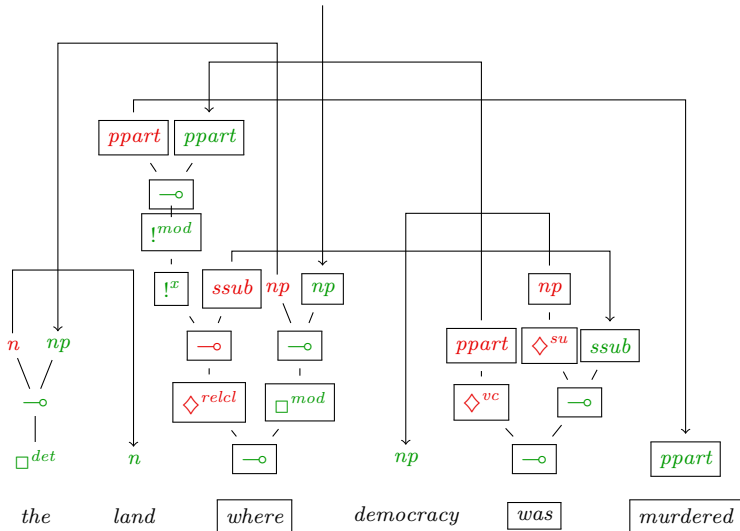


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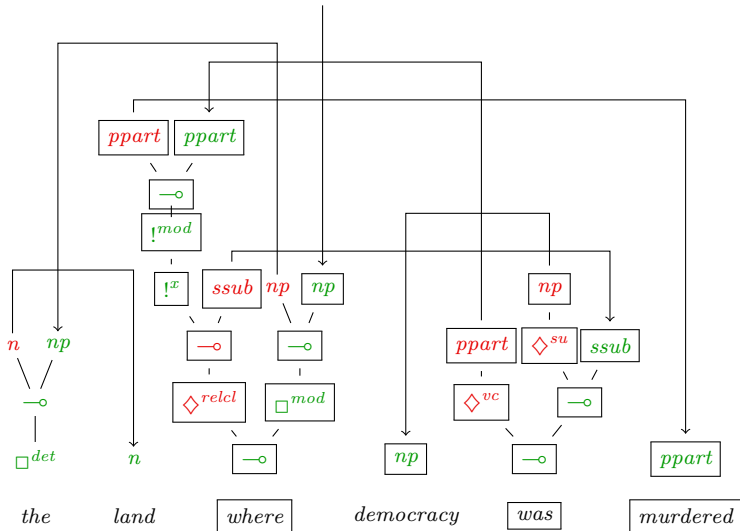
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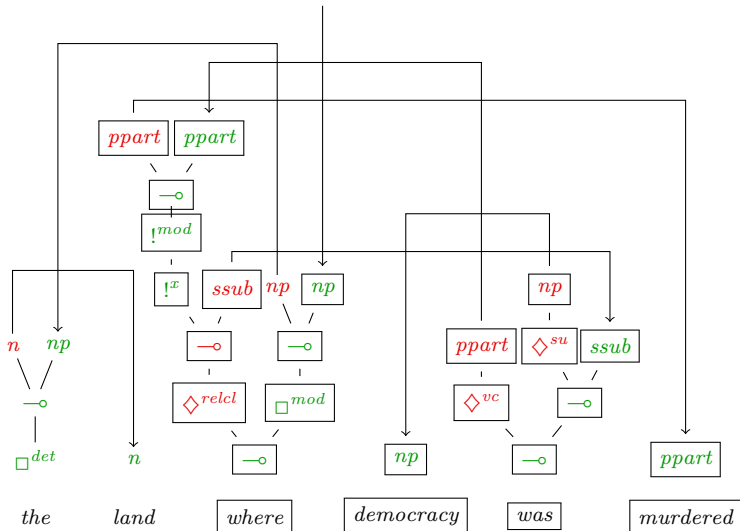
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# Proof Frames Structures



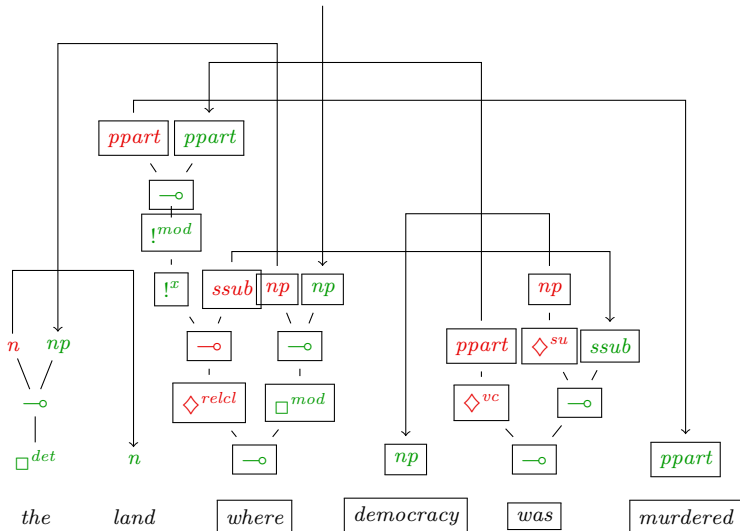
$\nabla^{mod}(\text{where } \Delta^{relcl}(\lambda x_0.(\text{was } \Delta^{vc}(\nabla^{mod}\nabla^{mod}\nabla^x\nabla^x x_0 \text{ murdered}) \Delta^{su}??))) ???$

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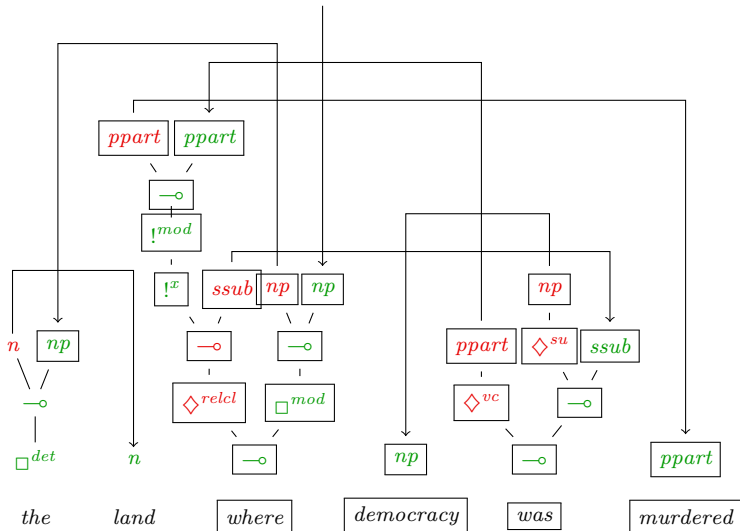
$\nabla^{mod}(\text{where } \Delta^{relcl} (\lambda x_0. (\text{was } \Delta^{vc} (\nabla^{mod} \nabla^{mod} \nabla^x \nabla^x x_0 \text{ murdered}) \Delta^{su} \text{ democracy}))) ???$

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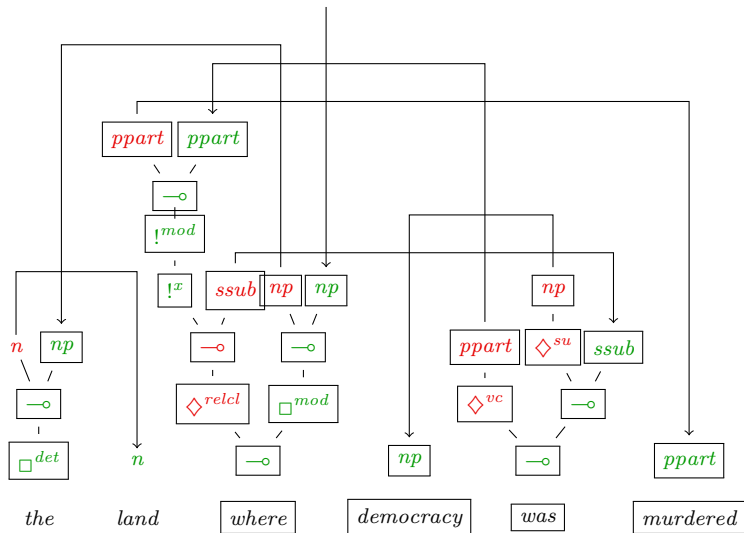


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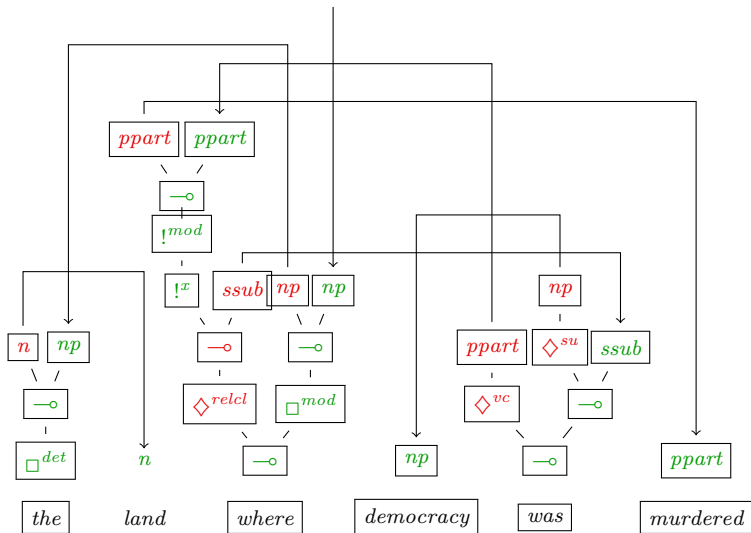
# Proof Frames Structures



$\blacktriangledown^{mod}(\text{where } \Delta^{relcl}(\lambda x_0.(\text{was } \Delta^{vc}(\blacktriangledown^{mod}\blacktriangledown^{mod}\blacktriangledown^x\blacktriangledown^x x_0 \text{ murdered}) \Delta^{su} \text{ democracy}))) (\blacktriangledown^{det} ??? \ ???)$

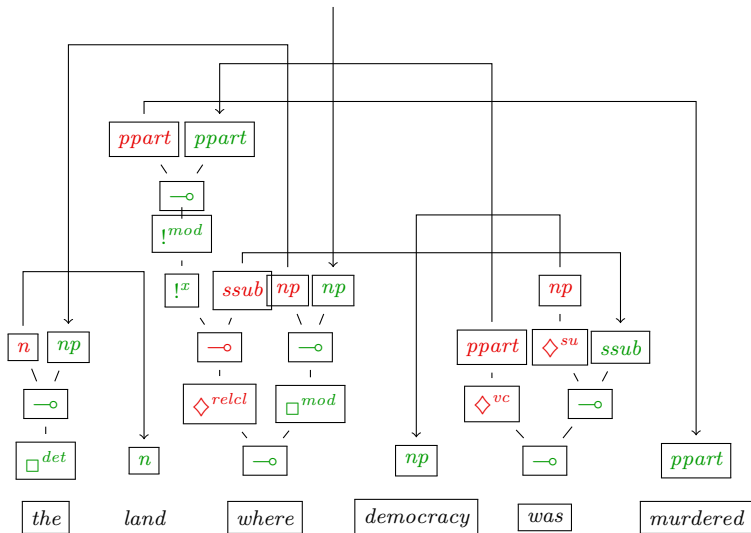


# Proof Frames Structures



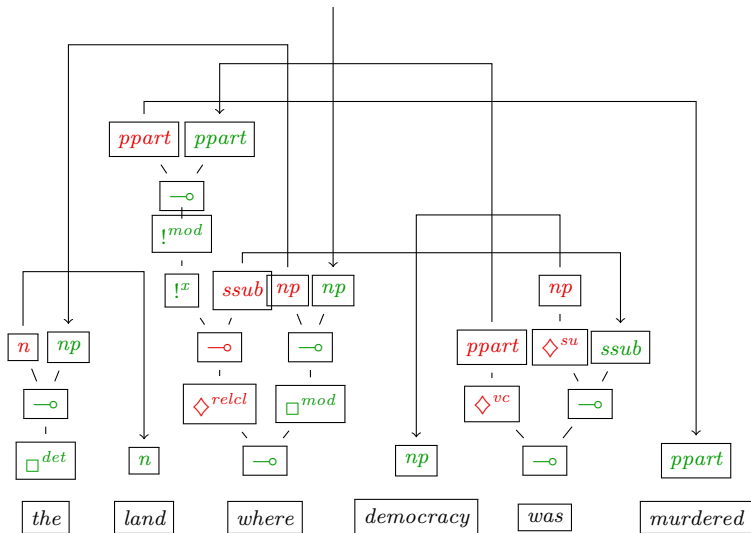
▼<sup>mod</sup>(where Δ<sup>relcl</sup> (λx<sub>0</sub>.(was Δ<sup>vc</sup> (▼<sup>mod</sup>▼<sup>mod</sup>▼<sup>x</sup>▼<sup>x</sup>x<sub>0</sub> murdered) Δ<sup>su</sup> democracy))) (▼<sup>det</sup>the ???)

# Proof Frames Structures



▼<sup>mod</sup>(where Δ<sup>relcl</sup> (λx<sub>0</sub>.(was Δ<sup>vc</sup> (▼<sup>mod</sup>▼<sup>mod</sup>▼<sup>x</sup>▼<sup>x</sup>x<sub>0</sub> murdered) Δ<sup>su</sup> democracy))) (▼<sup>det</sup>the ???)

# Proof Frames Structures Nets



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written and directed by  
KOKOS

*“Lets neural sh!t up”*  
– Alonzo Church, 1973



*“Lets neural sh!t up”*  
– Alonzo Church, 1973

*disclaimer: not an actual quote*

# Main ingredients

- ▶ Type assignment (supertagging)  
parallel tree decoding with dynamic graph convolutions
- ▶ Axiom linking (neural bijections)  
optimal transport with Sinkhorn iterations
- ▶ Formal verification  
proof net traversal

# Supertagging 101

goal

maximize  $p(T_1, \dots, T_n)$  conditional on some input  $(w_1, \dots, w_n)$

the catch

types are sparse  $\implies$  fixed vocabulary classification = no good

# Supertagging 101

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# Supertagging as parallel graph completion

???

???

???

???

???

???

the

land

where

democracy

was

murdered

# Supertagging as parallel graph completion

???

???

???

???

???

???

the

land

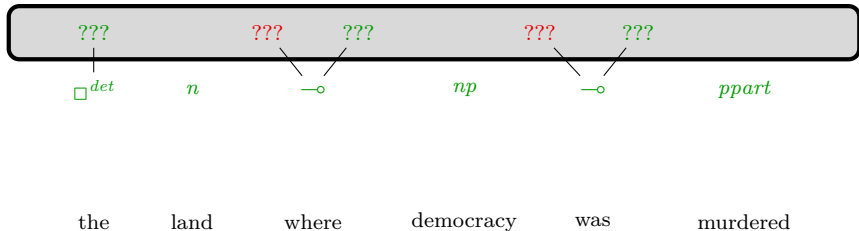
where

democracy

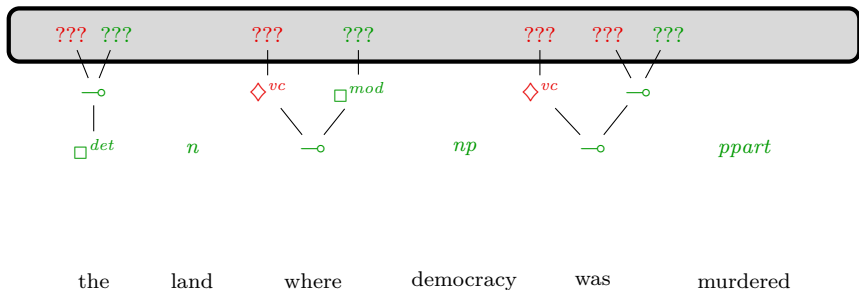
was

murdered

# Supertagging as parallel graph completion

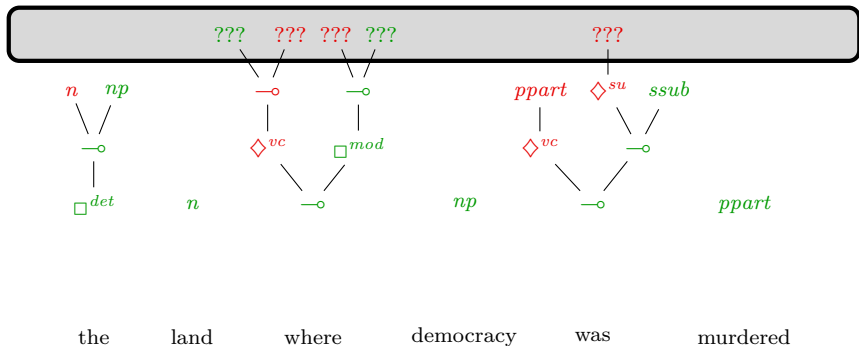


# Supertagging as parallel graph completion

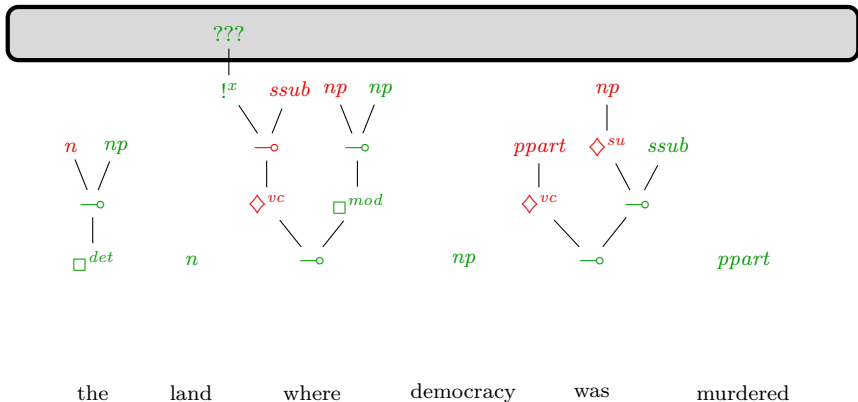




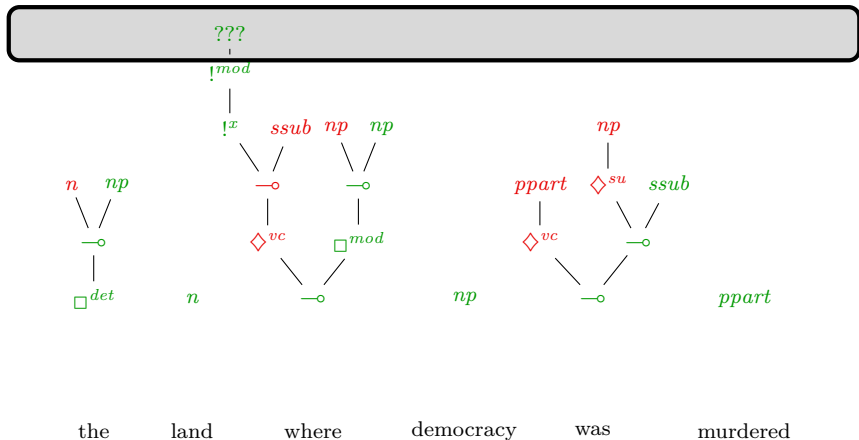
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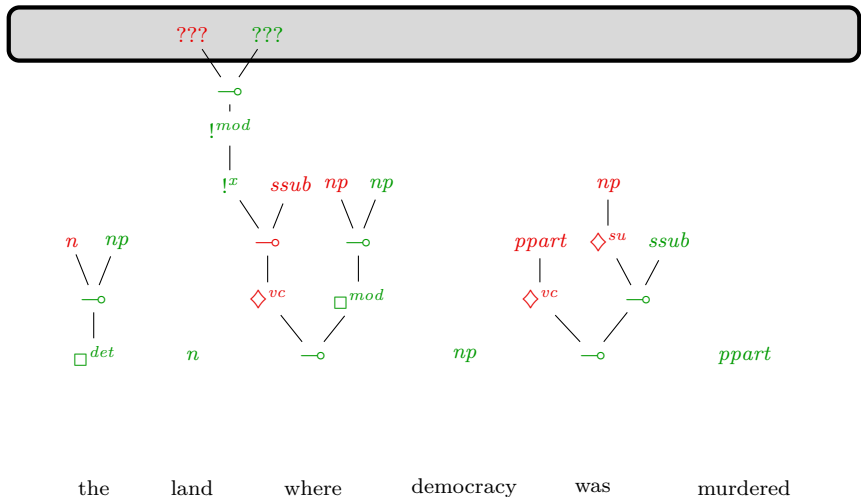
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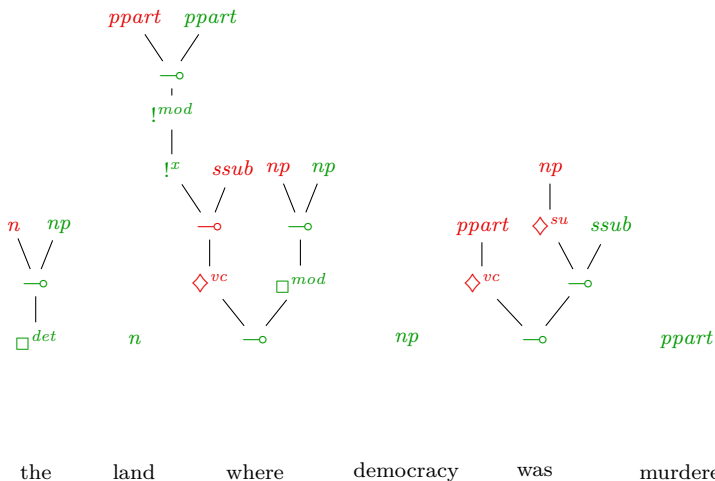
# Supertagging as parallel graph completion



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# Supertagging as parallel graph completion



# Not just a gray rectangle!

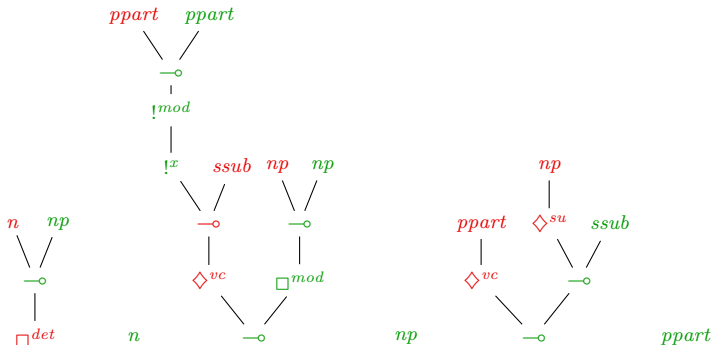
1 decoding step per tree depth; 3 message-passing rounds per step

- ▶ *contextualize: states*  $\rightarrow$  *states*  
universal transformer encoder w/ relative weights  
(many-to-many, update states with neighborhood context)
- ▶ *predict: state*  $\rightarrow$  *nodes*  
token classification w/ dynamic tree embeddings  
(one-to-many, predict fringe nodes from current state)
- ▶ *feedback: nodes*  $\rightarrow$  *state*  
heterogeneous graph attention  
(many-to-one, update state with last predicted nodes)

[Kogkalidis & Moortgat, ???]

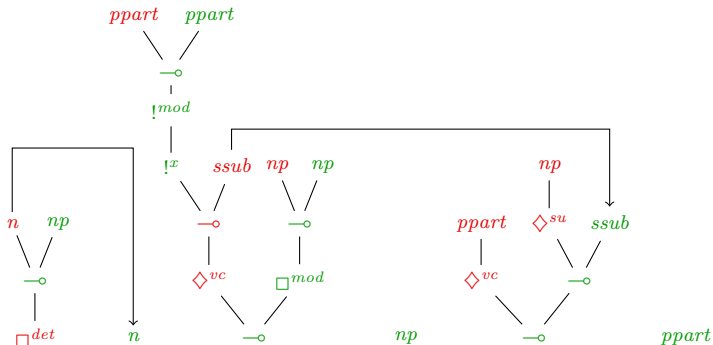
# Axiom Linking

- ! only consider edges between atoms of the *same* sign and *different* polarity
- ! each atom can only be used *once*



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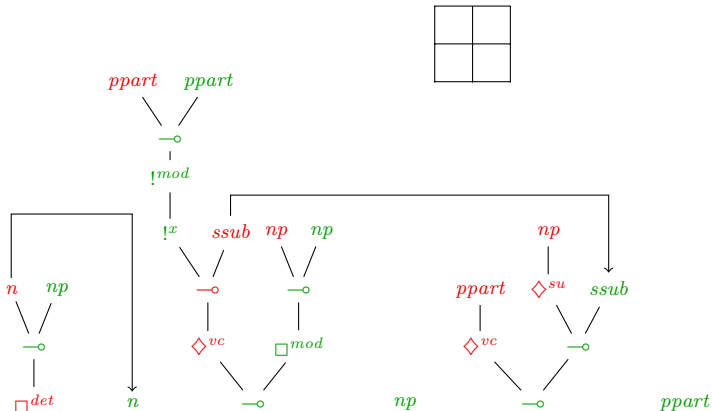


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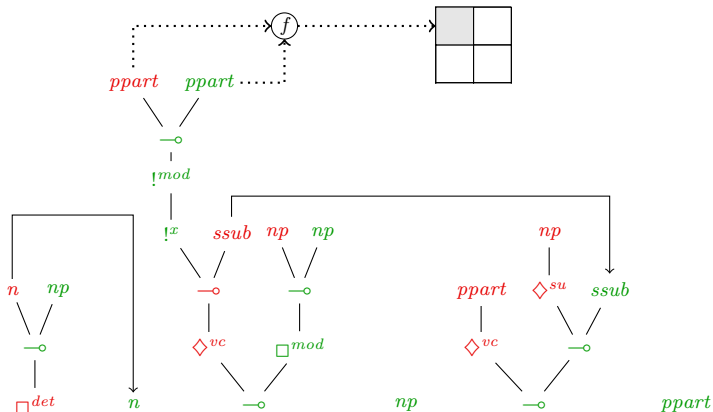


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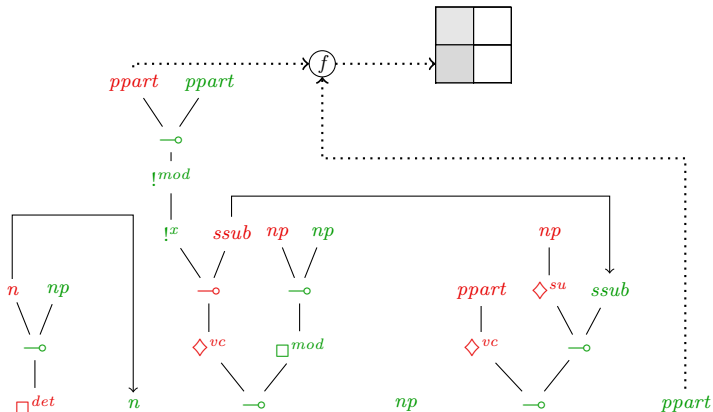


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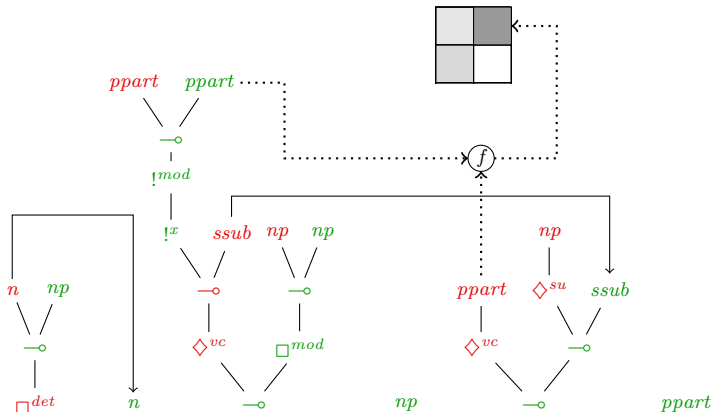
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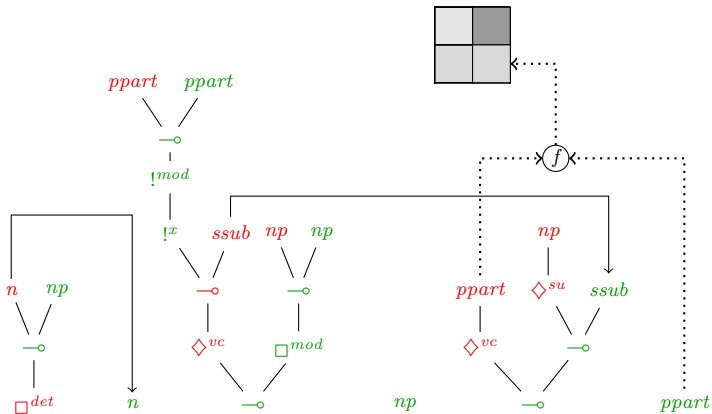
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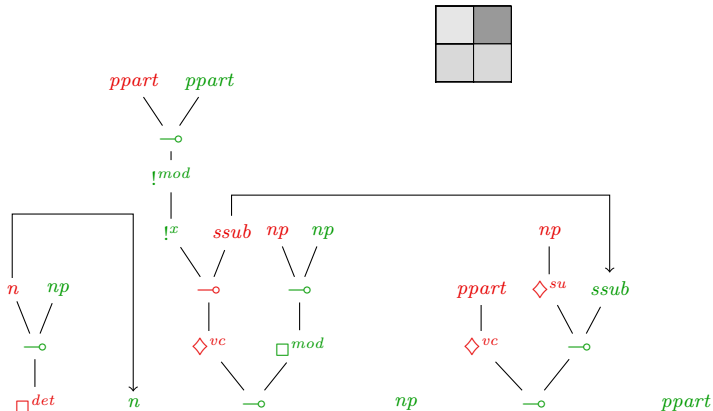


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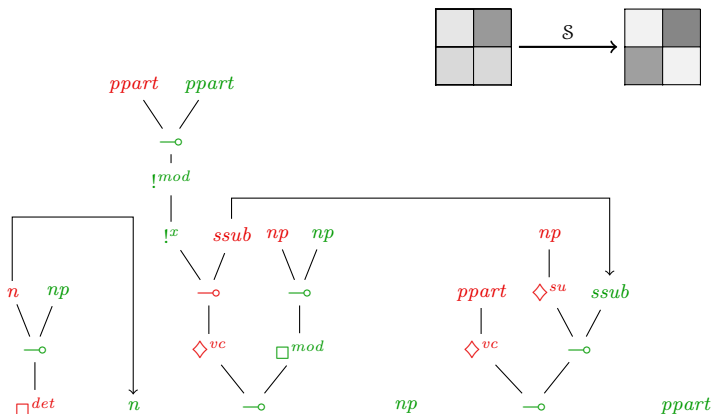


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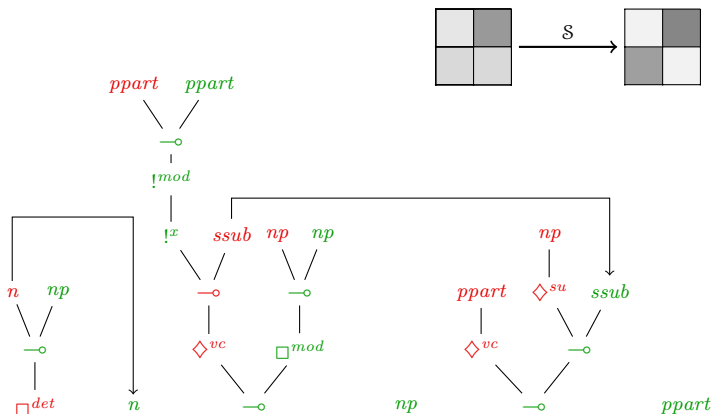


# Axiom Linking

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your turn

break the parser!

# Reality check

- ▶ tagger: applicable to any CG (**applied**)
- ▶ parser: adaptable to any flavor of linear logic (**adapted**)
- ▶ proofs: plug your own semantics / apply your own morphisms (**todo**)

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