

The Statistical Analysis of Electoral Coalitions

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Abstract

In this paper I derive a statistical model from an expected utility theory of electoral coalition formation in proportional representation systems. The theory predicts that parties are more likely to form a coalition if they are ideologically closer to each other. In addition, there is a party specific threshold that tells us how far the other party can be located and still be offered a coalition agreement. I derive a statistical model that is able to simultaneously estimate the ideological position of parties and the coefficients from the covariates that predict the threshold parameter. These covariates include district magnitude, party size and average education levels in the municipality. The model is estimated using a Bayesian hierarchical model, which I apply to the Brazilian local legislative elections of 2000 and 2004. The results are largely supportive of the hypotheses.

1 Introduction

In the 2000 elections for vereadores (aldermen) in Brazil, 31 parties competed for seats in the more than five thousand municipalities. Vereadores are elected via a proportional representation system in which parties can compete either alone or in cooperation with other parties by forming electoral coalitions. Coalitions are alliances between two or more parties that affect, as I explain below, the allocation of seats among parties in a election. Many studies in the Brazilian political science literature analyze the choice of coalition partners by political parties, mainly by studying the coalitions formed in the elections for Deputados Federais (national lower house legislators). Following a pioneering study conducted by Soares in 1964, the main hypothesis in this literature is that parties choose partners to maximize their electoral chances while minimizing the ideological distance among the members of the coalition. This paper contributes to this literature as follows. First, I present a simple mathematical model that allows a more precise formulation of the hypotheses. Second, I present a statistical model derived from this theory, which enables the empirical test of the hypotheses. Third, the statistical model is able to simultaneously estimate the party ideological positions and the structural components of the model.

The empirical results confirm the expectations from the theoretical model. First, coalitions are more frequent among parties that are closer to each other in the ideological space. A second important result is that coalitions are more frequent in cities where education levels are lower. Finally, the tests show the importance of institutional context. According to Brazil's electoral rules, parties (or, as it may be, coalitions) that gets less than an electoral quotient (total valid votes divided by the number of seats) do not elect candidates in elections. This rule creates an implicit threshold in the electoral formula which, as a proportion of the total number of votes, is higher in municipalities that elect fewer vereadores. The rules create, therefore, a greater incentive to form coalitions in the smaller municipalities (which have in general fewer seats) under the hypothesis of seat maximization. Also, parties under the electoral threshold have more incentives to form coalitions. Theses expectations are confirmed by the statistical results.

The article is structured as follows. First I summarize the institutional features of the elections for vereadores in Brazil and discuss the literature on electoral coalitions in Brazil. I then present a simple model of coalition formation, from which I derive a statistical model to test the hypotheses. A discussion of the empirical results follows, using data from the elections of 2000 and 2004. The

conclusion summarizes the findings of the paper and discusses possible extensions to the theoretical and statistical model.

2 Coalitions in proportional elections in Brazil

Vereadores are elected through a proportional electoral system with open lists,¹ where seats are allocated using a version of the D’Hondt system. In this system, voters can vote for a particular candidate or for a party. The votes are then aggregated up to the level of electoral lists. Only then one calculates the number of seats that each list has won (via the D’Hondt formula). Within each list, candidates are ranked by the number of individual votes received. If, for example, the n seats are assigned to the list, the top n candidates in the list are elected. Electoral lists, in turn, may consist of more than one party. Lists with more than one party are what we call electoral coalitions. It is important to note that the electoral coalitions seats are distributed according to individual votes, without regard to the party affiliation of the candidates.

An important difference between the Brazilian system and the pure D’Hondt is that coalitions that receive less than an electoral quotient (that is, less than the sum of valid votes divided by the number of seats) are not entitled to seats in the Brazilian system. This rule creates an implicit threshold equivalent to $1/\text{number of seats}$, which varies from 11% in small towns (9 seats) and 2% in the largest city (São Paulo, 55 seats). The threshold hinders the election of minor party candidates competing alone, thus producing substantial incentives for these parties to form coalitions. The incentive is greater in small towns, since they elect fewer vereadores.

Figure 1 shows a hypothetical example of coalition between two parties. Party A has a total of 821 votes and party B has 251. Suppose the electoral quotient of this election was 175. The two parties would be entitled to elect candidates. If there is no coalition between the parties, A would win at least four seats, since it got 4.7 electoral quotients.² Competing alone, party B elects at least one candidate, since it got 1.4 electoral quotients.

What happens if coalition AB is formed? The coalition as a whole would gain at least six seats (6.1 electoral quotients.) Prior to the coalition parties had secured a total of five seats. This

¹Cox (1997) is the main text on electoral formulas and their consequences. (Cox calls electoral coalitions “electoral cartels”) For an excellent discussion of the Brazilian system in practice, focusing on the elections for deputies, see Nicolau (2006).

²This is a lower bound. The actual number of seats will depend on the vote distribution across all lists in the election.

suggests that forming a coalition is interesting for both parties. However, the allocation of seats among parties is done through a joint list, ordered by number of votes of individual candidates. This list does not take into account party identification. In this case, a coalition formed by A and B would elect three candidates from Party A and three candidates from Party B. What is interesting to note is that party A has more than three times the number of votes that party B received, but gets the same number of seats! The number of seats it gets is in fact *lower* than what it was *guaranteed* to get by going alone. This example shows that the coalition choices parties make can have important consequences. In the next section I try to estimate the importance of electoral coalitions to electoral results in Brazil.

3 Mechanical impact of the coalitions in the distribution of seats by party

Who benefits from making electoral coalitions? To answer this question, I calculate the distribution of seats across parties in the elections for vereadores under the assumption that coalitions are ignored. The procedure is quite simple: since the votes for candidates and parties by municipality is available, one can calculate the number of seats allocated to each party using the d'Hondt formula while taking into account the electoral quotient of the municipality. The number of seats (which ignores coalitions) can then be compared to the (actual) distribution of seats when coalitions are taken into account. The results are shown in Figure 3. In the y axis we have the number of seats that would be won by each party if coalitions were banned, divided by the actual number of seats won (which takes into account the coalitions.) Thus, when $y = 1$ the party would neither gain nor lose seats (on aggregate) if coalitions were banned. Parties over this line would win more seats if coalitions were not possible. In 2000, for example, only PFL, PMDB and PSDB would be net winners of seats if the seat allocation did not take into account the coalitions. In the x axis we have the percentage of votes for vereadores in the entire country received by each party, represented in the logarithmic scale with base 2. The Figure shows that The smaller the party is, the greater is the benefit it gets from forming coalitions. The only exceptions are the micro-parties PSTU and PAN, which in the 2004 election would not gain or lose seats if coalitions were ignored.

It is important to realize that the “effect” shown in the Figure is purely mechanical. It does

not take into account the strategic effect, that is, the reaction of parties to changes in the rules of electoral competition. It is what we would expect if, for example, it was decided *after* the election (and in a manner not anticipated by the political actors) that coalitions would not count for the allocation of seats among candidates. The important point is that the mechanical effect is demonstrably important. But this does not mean, for example, that individual parties would be better off if they stopped joining coalitions. They might be doing, given the political environment in each election, the best they can.

There is a long scholarship tradition studying the electoral coalition behavior of parties in Brazil, starting with Soares (1964). Based on the spatial theory of voting Downs (1957), Soares proposed two hypotheses. The “cost minimization theory” hypothesis states that parties will form coalitions in order to maximize their seat shares given the number of votes they expect to get in the elections. The “ideological resistance” hypothesis states that parties prefer to form coalitions with parties that have similar ideological positions.

That article ignited a cottage industry of papers studying electoral coalitions, which has been particularly active in the recent period.³ This literature, however, faces a couple of serious shortcomings. First, the ideology measures used are hard to replicate, relying mostly on subjective evaluations. Secondly, the theoretical development seem to have stagnated with Soares article from four decades ago. To wit, “ideological resistance” and “cost minimization” are seen as “either-or” propositions, without an integrated way to think about how they both might affect party behavior. A final problem is the lack of a statistical model, without which the importance of ideology and/or “cost minimization” hypotheses cannot be ascertained.

4 Measuring ideology

The main hypothesis of this paper is that ideology help structure the electoral coalition decisions of Brazilian parties in elections for vereadores. It is therefore important to obtain reasonably replicable and theoretically grounded ideology measurements of the Brazilian political parties.

Ideology in modern political science is a theory of political behavior. One starts by assuming that the policy space is high dimensional. For instance, views about international trade, government support for child care, abortion, taxes, etc, can all vary independently across voters and politicians.

³Miguel and Machado (2007) provide an excellent review of the literature.

However, research has shown that most of the issues governing the day are highly correlated, so that when one analyzes the actions of decision makers, just one or two dimensions might be sufficient to explain most of the behavior. This low dimensional space is labeled “predictive”, “basic” (Poole, 2005, p.13-14) or, most commonly, “ideological” space (Hinich and Munger, 1994).

If one defines ideological space as the predictive dimensions of political behavior, it is not surprising to note that state-of-the-art measurement models of ideology derive ideal point estimates from observed political action. For example, there is a long tradition in political science of measuring ideology of legislators using their roll call votes.(Poole and Rosenthal, 1997, e.g.). Clinton, Jackman and Rivers (2004, p.355) go so far as stating that the “primary use of roll call data is the estimation of ideal points.” As these authors note, roll calls are effective measures for describing the behavior of legislators. There are a many other things that legislators do, of course. But roll calls are both readily available in many legislatures. They are also records of preferences over actual policies, so it is not surprising that they are the main raw material used in the measurement of political preferences. There are, however, alternative sources for the enterprising ideological cartographer. Recent examples include legislators surveys (Saiegh, 2009), analysis of political discourse (Monroe, Colaresi and Quinn, 2008), cosponsorship patterns in the legislative setting (Alemán et al., 2009) and campaign contributions (Bonica, 2009).

What is in fact special about roll call data is the extraordinary quantity and time-span of such data for the United States Congress and many other legislatures across the world. In Brazil a one dimensional model of roll call behavior seems to fit well the data (Leoni, 2002), correctly predicting around 88% of the roll call votes in the 1991-1996 session and 92% in the legislative sessions from 1995 until 2006. However, a serious problem of the measurement of ideology is that the estimated ideal points can only be treated as *ideological* ideal points if roll call behavior is driven predominantly by spatial preferences. Let i index legislators and j index roll calls. A function linking preferences x_i to (roll call) behavior y_{ij} can be written as:

$$y_{ij} = f(\alpha_j + x_i\beta_j + e_{ij}) \tag{1}$$

That is, the model assumes that only preferences drive behavior. More precisely, the errors e_{ij} are assumed to be independent and identically distributed. If, however, there are factors z_{ij} not constant across legislators and correlated with the error term, then estimated ideal points will be

biased. This would be of not much consequence if all we want is a description of roll call behavior, but insufficient if we want to test a theory of behavior.

In the case of Brazil, part of the literature claims there is a *quid pro quo* between Presidents and legislators.(Alston and Mueller, 2006; Alston et al., 2007; Zucco, 2007) Presidents have the authority to withhold expenditures under the Brazilian Constitution. Therefore, according to the argument in these articles, they have leverage when dealing with legislators. Since such vote buying is in general inconsistent with pure spatial voting, the usual estimators of ideal points will lead to misleading estimates of preferences. This argument is more fully developed by Zucco (2009). He combines roll call data with a self-positioning of legislators measured using surveys and finds that the influence of ideology (as measured by the surveys) on roll call behavior of deputies has declined over time in Brazil.

Another way to deal with the issue is to find data where such constraints (e.g. vote buying) are not as relevant. Snyder Jr. and Groseclose (2000) argue that measuring ideology lop-sided roll calls in Congress are closer to the “true” preferences of legislators, and the difference between these preferences measured using lop-sided votes and preferences measured using non lop-sided serve as a measure of party influence on roll call behavior. More generally, we want to find behavior that is driven by political preferences but not overwhelmingly influenced by outside actors (e.g. the President). In this vein, Monroe, Colaresi and Quinn (2008) propose to measure ideal points using political text and speech; Alemán et al. (2009) use cosponsorship, that is, data on which legislators cosponsor each other bills, to estimate ideal points using singular value decomposition techniques; Finally, Saiegh (2009) and Power and Zucco (2009) uses survey data from political elites.

Most of these measures, however, are restricted in their ability to estimate ideal points of small political parties. If there is only one or two legislators in Congress that are members of a particular political party, for example, there is not sufficient information to estimate an ideal point for that party. The problem is an order of magnitude harder when the interest lies at the sub-national political preferences. The massive amount of data on party coalition behavior in local elections in Brazil, I argue, can help us in this regard.

4.1 State level preferences

Brazil is large federal country, with important regional variations in terms of economic and social structure. Thus, it is likely that there is also regional variation of party preferences across states. In the United States, for example, for a long time there was a noticeable split between southern and northern Democrats on civil rights issues (Poole and Rosenthal, 1997). In the case of Brazil, many scholars argue that state-level actors are not only independent on local matters, but also have a great deal of influence on national level politics. Mainwaring (1997) calls this “robust federalism”:

Brazil has long been the case of most robust federalism in Latin America ... By robust federalism, I mean that, during democratic periods, mayors and governors have been powerful actors with significant autonomy vis-à-vis the federal government and with significant resources. The catchall parties are decentralized, and parties and politicians generally follow a logic of federalism. Many of their actions are determined more by what goes on in their own states than by what goes on in national politics. In fact, the national parties are still to a considerable extent a federation of state parties. (Mainwaring, 1997, p.83)

Samuels (2003) and Abrucio (1998) argue that the legislative institutions at the national assembly, particularly the budgetary committees, are structured in order to privilege state-level actors. They argue that in elections for congress (which are based on state-wide districts), governors are more important for the electoral success of candidates than the national parties or the President.

Even if the governors themselves had little influence on roll call behavior, the extensive regional differences across Brazil likely affect the politicians preferences, even if they are members of the same party. Using roll call data both from the Senate and from the Câmara, Desposato (2003) found that national party cohesion is in fact lower than state-party cohesion. The effect estimated was, however, quite small.

As show in the right panel of Figure 2, the coalition data set is much richer than the Câmara dos Deputados roll call data set, or for that matter the Power legislative surveys, for estimating district level preferences of the Brazilian parties. The parties and states in the figure are ordered by the number of coalitions present in each party/state. Note that only for one of the parties (PMDB) there is roll call information from every state. More worryingly, only about 6 or 7 parties have

representatives from more than half the Brazilian states. The surveys reported in Power and Zucco (2009) have an even smaller coverage, given the modest response rate. The coalitions data, on the other hand, covers every state for the main political parties in Brazil.

In the next section I present a model of coalition formation. I derive from it a statistical model that is able to simultaneously estimate party ideal points and, as importantly, structural parameters that relate political context and structure to the likelihood of coalition formation. I argue local coalition decisions are useful because they reflect decisions by the rank-and-file of the party at the lowest level of elected office in Brazil, with arguably little interference from the nation-wide party leadership. That is, they are decisions of not major consequence, allowing one to infer more cleanly the ideological motivations of party behavior. (McCarty, Poole and Rosenthal, 2001)

5 A statistical model of electoral coalition formation

The main hypothesis of this article is the following. The greater the ideological distance between parties, the less likely it is that a coalition will be formed. This follows from the assumption that coalitions are formed in order to maximize the number of seats given the amount of votes won by the party. And that, as argued by Soares (1964), the main cost of building coalitions is the likelihood of electing candidates of other parties with political preferences very different from one's own. Therefore, the costs of coalescing increase with the ideological distance between the two parties. The purpose of this section is to formalize these arguments. The model will, in turn, motivate the working hypotheses and the statistical model.

There are N individual parties competing in M elections. Elections are held across S states. I assume that parties first choose the largest set of parties that they are willing to form a coalition with. Parties that choose the same set S_m form a coalition S'_m . Note that this can be different from S_m , since it is not necessary that all parties in S_m accept to join the coalition for a subset of that coalition to form.⁴ If a coalition involving party i and party j is observed at election m , we know that: $S_{im} = S_{jm}$, $U_{im}(j \notin S_{im}) < U_{im}(j \in S_{im})$ and $U_{jm}(i \notin S_{jm}) < U_{jm}(i \in S_{jm})$. Conversely, if parties i and j are not in the same coalition in election m , we know that $S_{im} \neq S_{jm}$ and either $U_{im}(j \notin S_{im}) > U_{im}(j \in S_{im})$ or $U_{jm}(i \notin S_{jm}) > U_{jm}(i \in S_{jm})$ or both.

⁴I adopt the assumptions of the game named as Δ by Hart and Kurz (1983). Note, however, that in our model parties do not behave strategically. This assumption is similar to the independence of irrelevant alternatives in multinomial logit models. See Train (2003) for a discussion.

I now introduce a parametric form of the model, which is necessary for the derivation of the statistical model. I define the utility for party i of adding a party j to the coalition as follows. Start by assuming that each party i in state s can be located at point x_{is} in a one-dimensional policy space. The utility function of each party has two additive components. The first is the policy cost of forming a coalition with a party that has, in general, different policy positions from one's own. This cost is modeled as $-|x_{is} - x_{js}|$. The second component is the non-spatial threshold parameter v_{im} for party i of adding party j to the coalition in election m . Party i offers party j a coalition proposal if:

$$-|x_{is} - x_{js}| + v_{im} > 0 \quad (2)$$

In plain English, party i will extend an offer to party j if and only if the expected spatial cost is less than the threshold parameter v_{im} . This term represents the costs and benefits of party i in joining a coalition in election m net of the effects of spatial proximity. As I explain below, although this term cannot be measured directly, it is assumed to vary as a linear function of characteristics of party i and the political system at election m . This completes the deterministic component of the parties' utilities.

A concrete example is presented in Figure 10. There are two parties. Party a is in the center-left of the ideological space with ideology set as 3.5. Party b is in the center of the ideological space, in position 5.5. When can we expect the two parties form a coalition? The answer will depend on the parameter v . We can define the *acceptance region* of a party i in election j as the region extending from $x_i - v_{im}$ to $x_i + v_{im}$. Party i would like to form a coalition with any party who is in this region. In panel (a) of the figure, the parties a and b both have small acceptance regions. In that case they would not form a coalition. In the panel (b) the parties have wide acceptance regions. Note that party a is in the acceptance region of b and party b is in the acceptance region of a . In this case, the coalition ab will be formed.

In order to build the statistical model, I now cast this decision in terms of a random utility model, adding error terms e_{ijm}^1 and e_{ijm}^0 to $U_{im}(j \in S_{im})$ and $U_{im}(j \notin S_{im})$ respectively. These are the stochastic components of the utility functions. The probability that party i extends an offer to party j in election m can then be written as:

$$P(z_{ijm}) = P(-|x_{is} - x_{js}| + v_{im} + e_{ijm} > 0) \quad (3)$$

$$= P(-|x_{is} - x_{js}| + v_{im} > -e_{ijm}) \quad (4)$$

This is in general different from the probability that party j extends an offer to party i :

$$P(z_{jim}) = P(-|x_{is} - x_{js}| + v_{jm} > -e_{jim}) \quad (5)$$

Recall that we do not observe the offers being made. We only observe the joint outcome that parties i and j accept offers from each other. Let y_{ijm} be a binary variable reflecting the observation that parties i and j formed a coalition in election m . Given the assumptions this occurs only if the inequalities in equations 3 and 5 are simultaneously satisfied. For tractability, I further simplify the problem by assuming that e_{ijm} and e_{jim} are independent and have each variance 1 and mean 0. That is, I assume that there are no unmeasured factors that affect the joint decision to form a coalition.⁵

With this assumption we now have:

$$P(y_{ijm}) = P(U_i(j \in S_{im}) > U_i(j \notin S_{im})) \cdot P(U_j(i \in S_{jm}) > U_j(i \notin S_{jm})) \quad (6)$$

$$= P(v_{im} - |x_{is} - x_{js}| + e_{ijm} > 0) \cdot P(v_{jm} - |x_{is} - x_{js}| + e_{ijm} > 0) \quad (7)$$

$$= \Phi(v_{im} - |x_{is} - x_{js}|) \cdot \Phi(v_{jm} - |x_{is} - x_{js}|) \quad (8)$$

To sum up, my main hypothesis is that *ideological distance decreases the probability of coalition*. The remaining hypotheses concern the size of the acceptance region, which is driven by the components of the non-spatial benefits/costs denoted by parameter v . First, I argue that *the probability of coalition formation should be higher when the electoral threshold is high*. In other words, political

⁵The estimation of the correlation between the two latent errors is possible (Poirier, 1980, see) when exclusion restrictions are available (i.e. factors that influence the decision of one but not the other actor). On the absence of those, the correlation is estimated based entirely on the distributional assumptions. For a related model, Sartori (2003) shows using Monte Carlo experiments that fixing the correlation to a fixed value outperforms estimating the correlation under these circumstances. The preliminary sensitivity analysis I've performed shows that the model assuming independent errors outperforms using dyads (estimating the joint decision directly) or assuming the correlation is 1 (as suggested by Sartori).

institutions that make it harder for parties to elect their candidates when acting alone should have a smaller acceptance region. However, parties that expect to have less votes than the minimal threshold are more in need of forming coalitions. Thus, *the probability of coalition for small parties should be higher*.

Finally, I argue that the acceptance region also depends on the educational level of the electorate. Huber, Kernell and Leoni (2005), using data from the Comparative Study of Electoral Systems, showed that the higher cognitive resources (measured as years schooling) of the voter, the greater is the probability this voter feels close to any political party. In other words, the differences between party labels are more clearly perceived by individuals with more schooling. In Brazil, Desposato (2006, p.73) shows that deputies whose electorate is on average less educated are more “pork oriented” than those whose electorate has higher levels of schooling. I argue, therefore, that *higher levels of education in the municipality lead to a lower probability of coalitions formation*. After all, the ideological costs are best perceived by voters with higher cognitive ability.

6 Measurement and Estimation

I estimate the model using data from the 2000 and 2004 elections for vereadores in Brazil. Only the 11 parties that were included in the legislators surveys conducted by Power and collaborators Power and Zucco (2009): PT, PFL, PMDB, PSDB, PP, PTB, PL, PDT, PPS, PSB, PCdoB. These parties account for over 80% of the votes and 90% of the seats in the 2000 and 2004 elections.

The data is reshaped⁶ so that, for each municipality, all possible party dyads (pairs) between the parties that competed in that particular municipality/election are listed. The dependent variable is marked as one when a coalition between the two parties in the dyad is observed, zero otherwise.

The education level of the municipality is measured using the education component of the Human Development Index (HDIeduc), available from the United Nations Development Program in Brazil.⁷

The electoral threshold is equal to the inverse of the number of seats available in the election. Party size is measured as a dichotomy. This variable ($small_{im}$) is equal one if party i is below the electoral threshold in the election, zero otherwise.

⁶I used the `reshape` package, by Wickham (2007).

⁷<http://www.pnud.org.br/atlas/tabelas/index.php>

The model I propose is the following⁸:

$$P(y_{ijm}) = \Phi(v_{im} - |x_{is} - x_{js}|) \cdot \Phi(v_{jm} - |x_{is} - x_{js}|) \quad (9)$$

$$v_{im} = \gamma_1 + \gamma_2 \text{small}_{im} + \gamma_3 \frac{1}{z_m} + \gamma_4 \text{HDIeduc}_m + g_m \quad (10)$$

$$v_{jm} = \gamma_1 + \gamma_2 \text{small}_{jm} + \gamma_3 \frac{1}{z_m} + \gamma_4 \text{HDIeduc}_m + g_m \quad (11)$$

Parameter γ_1 is the overall intercept of the statistical model. We expect γ_4 to be negative and both γ_3 and γ_2 to be positive. The error term g_m is assumed to have a normal distribution with variance σ_{city}^2 .

Parties are observed forming coalitions in M separate elections, nested within the 26 Brazilian states.⁹ The parameters x , g and γ are unobserved quantities and have to be estimated. This creates a difficult maximization problem. For each additional party i in state s one has to estimate an additional parameter x_{is} . And for each additional election m one has a new g_m to estimate. Thus, as the sample size increases, so does the number of parameters in the model. This causes problems to the maximum likelihood estimation. (Clinton, Jackman and Rivers, 2004, p.358) Fortunately, this problem is of not very much consequence for Bayesian estimation, since the Bayesian approach holds the data fixed and approximations, when necessary, depend only on the number of simulations performed and not on the size of the data.

The likelihood of the model is:

$$L(x, \gamma | Y) = \quad (12)$$

$$\prod_{m=1}^M \prod_{i=2}^N \prod_{j=1}^{i-1} \{\Phi(v_{im} - |x_{is} - x_{js}| + g_m) \Phi(v_{jm} - |x_{is} - x_{js}| + g_m)\}^{y_{ijm}} \quad (13)$$

$$\times \{(1 - \Phi(v_{im} - |x_{is} - x_{js}| + g_m))(1 - \Phi(v_{jm} - |x_{is} - x_{js}| + g_m))\}^{(1-y_{ijm})} \quad (14)$$

Estimating state-specific ideal points for the parties introduces comparability problems, since parties in any given state interact only with other parties in the same state. Thus, to compare party

⁸I exclude parties with zero votes from the estimation.

⁹The Federal District is excluded, since no proper local elections are held there.

ideal point locations across states and to the national parties' positions we need further assumptions. I follow Martin and Quinn (2002) in using a hierarchical model. Martin and Quinn use a dynamic hierarchical prior on ideal points, more specifically a random walk prior that is conditional on the ideal point in the last period.(p. 135) In the electoral coalitions case, the relevant grouping structure is state, not time. Since states are unordered, the model is not dynamic (and therefore much simpler to estimate.) The prior for the ideal point of party i in state s is:

$$x_{is} \sim N(\mu_i, \sigma_x^2) \quad (15)$$

Bayesian estimation requires us to specify prior distributions for all unknown parameters. What do we know about the ideological positions of Brazilian parties? Based on the literature and knowledge about Brazilian politics, it is safe to say that the ideological position of the Worker's Party (PT) is always to the left of the Liberal Front Party (PFL, now Democratas-DEM). This will solve the reflection invariance problem.¹⁰ We can formalize this restriction as:

$$\mu_{pfl} > v \quad (16)$$

$$\mu_{pt} < v \quad (17)$$

$$x_{pfl,s} > v, \forall s \quad (18)$$

$$x_{pt,s} < v, \forall s \quad (19)$$

Due to the inherent additive aliasing in ideal point models, the value of v can be arbitrarily set. I chose $v = 0$. Thus, all ideological coordinates for the PT are strictly negative, while all the ideological coordinates for the PFL are strictly positive. The ideal points are not yet identified. The additive aliasing (shift invariance) and multiplicative aliasing (scale invariance) problems persist. I solve them by post-processing the simulation results. That is, MCMC ideal points draws are rescaled so that prior for the PT lies at -1 and the prior for the PFL lies at $+1$ of the ideological space. I apply the same rescaling to the γ parameters in order to make them comparable.

For the remaining parameters of the model I chose the following weakly informative priors¹¹

¹⁰For useful discussion and recommendations on implementing Bayesian ideal point models, see Bafumi et al. (2005)

¹¹Note that I follow the advice of Gelman (2004) on the estimation of the variance parameters.

$\gamma_k \sim N(0, 100^2)$; $g_m \sim N(0, \sigma_{city}^2)$; $\sigma_{city} \sim U(0, 100)$; $\sigma_x \sim U(0, 100)$. These priors were chosen to be flexible enough in order to allow the data to drive the posterior (inferences) from the model, but strong enough to regularize the estimates when there is little information in the data. (Gelman et al., 2008)

I estimate the model using the general-purpose **JAGS** (Plummer, 2008) software for Bayesian posterior simulation.¹²

7 Descriptive Analysis

Figure 5 displays coalitions decisions for the 31 parties competing in the 2000 elections. In the diagonal is the number of elections each party was present. Each row displays the count of elections where the row-party formed a coalition with the column party. The parties are ordered according to the sum total of votes in the elections for vereador in the entire country. The color scale displays the proportion of the coalitions that includes the row-party that also include the column-party. For instance, more than 40% of the coalitions that the PCdoB formed also include the PT. On the other hand, only about 25% of the coalitions formed by the PT also include the PCdoB. This follows from the fact that the PT competed in a much larger number of municipalities than the PCdoB.

A noticeable feature of the coalition matrix is its low frequency of zero entries. This implies that (national) ideological preferences are far from being the overwhelming drivers of coalition decisions. The main left wing party (PT) formed coalitions with the main right-wing party (PFL) in about 2% of the elections it participated in 2000. Nevertheless, it is immediately obvious that parties have preferred partners, with PT-PCdoB being just one example. Coalition decisions, therefore, do seem to be at least partly driven by ideology.

Figure 6 presents data from 2004. Comparing Figures 5 and 6, one can notice several contrasting patterns. For example, note that while in 2000 coalitions involving both the PT and PFL occurred in only 92 municipalities, in 2004 it happened in 366 municipalities. The PP, which is part of the government coalition of President Lula da Silva, formed coalitions with the PT in 598 municipalities in 2004 but in only 105 municipalities in 2000. It is also clear that there coalitions among the main parties were more frequent in 2004 than in 2000.

¹²JAGS is a compiler for code written using the BUGS language. The code is available in the appendix.

8 Results: Structural parameters

The model was computed separately for each election (2000, 2004). I plot the structural parameters of the model in Figure 7. The dots show the mean of the MCMC draws of the parameter and the vertical lines show the 95% Bayesian confidence intervals. The empirical results are consistent with the hypotheses of the theoretical model: all coefficients have confidence intervals that do not include zero.

Despite the nonlinear relationship between the estimated coefficients and the likelihood of coalition, the direction of effect on the expected probability is the same as the coefficients displayed.

For example, the higher the inverse of the number of seats, the greater the likelihood of coalition. That is, the statistical model predicts coalitions more frequently in municipalities with fewer number of seats. The coefficient in 2004 is double the estimated using the 2000 elections.

The educational level of the municipality coefficient (as measured by HDIeduc) shows that in municipalities where the education level is high, the likelihood of coalition is low. Contrary to what occurred with the ratio of the number of seats, however, the coefficient of the variable HDIeduc was higher (in absolute terms) in 2000 than in 2004.

To assist in the interpretation of the model, I plot the predicted probability of coalition for combinations of the covariates in Figures 8, 9 and 10.

The predicted probability for each unique number of seats in Brazilian municipalities is plotted in Figure 8. The number of seats vary between 9 and 55. The plot shows that the predicted probability falls precipitously as the number of seats increases. The predicted impact of ideology is also quite high and statistically significant, particularly when the number of seats is small. In 2004, the predicted probability of coalition for larger cities (over 20 seats) is essentially zero. However, for cities with a small number of seats, the probability of coalition in 2004 is higher than that of 2000.

The large and significant effect of district magnitude on the probability of coalitions helps to account for many of the changes in the different patterns in the electoral coalitions in the 2000 and 2004 elections. To understand why, it is important to know that judicial decisions reduced the number of councilors in most Brazilian cities between 2000 and 2004. The courts decided to enforce the criterion of proportionality between population and the number of seats set forth by the

federal constitution.¹³ Table 1 displays the frequency of the number of vereadores per municipality in 2000 and 2004. In 2000, the 3110 (about 58%) Brazilian municipalities had the constitutional minimum of 9 vereadores, while in 2004 no less than 4970 municipalities (about 93%) elected 9 vereadores. The increase in the number of coalitions between 2000 and 2004 can, therefore, be largely attributed to changes in the electoral institutions.

In Figure 9 I show that the predictive effect of the electorate education level is also quite high. When the electorate’s average level of education is high, the probability of coalition formation is much lower than when the education is low. The coalition probability, therefore, varies across municipalities

Finally, Figure 10 shows the predictive effect of party size. As mentioned earlier, parties (or coalitions) have to obtain at least $(1/\text{number of seats})$ votes (as a proportion of valid votes in the municipality) to be eligible for seats. Being a large party (over the threshold), as expected, leads to a much smaller probability of coalition.

In summary, according to the statistical model results, ideological distance has a large negative effect on the probability of coalition, particularly in municipalities with few seats or low levels of education. Cities with a larger number of seats have much lower predicted probability of coalitions, which explains at least in part the large increase in the number of coalitions in 2004 compared to 2000. Finally, parties that are above the electoral threshold are much less likely to form coalitions.

9 Results: Ideological Estimates

A key feature of the statistical model is its ability to recover the parties ideal points, both at the national and at the state level. Do these recovered estimates agree with other measures of party ideology? This question is important, since if they do one would have greater confidence that parties’ behavior is in fact at least partly ideological. After all, if party members have very different preferences when voting in roll calls than when choosing electoral coalition partners, one could not characterize their preferences as “ideological”.

¹³Resolution number 21,702 of the Superior Electoral Tribunal, which follows the decision in the Supreme Court case number 197,717.

9.1 National level (mean party) ideology

Figure 11 shows the correlation between the party ideal points calculated using coalition data and those calculated using roll call data and legislators surveys. Roll call party ideal points were estimated every two years from 1992 to 2004 using the `MCMCirtHier1d` function in the `MCMCpack` R package (Martin, Quinn and Park, 2009). `MCMCirtHier1d` is a Bayesian hierarchical ideal point estimation procedure that allows ideal point covariates to be included in the model. In this case, it allows me to use party affiliation as a categorical predictor of roll call behavior. Legislators survey data come from Power and Zucco (2009). Legislators were asked about where they place the parties in the left-right spectrum in a scale from 1 to 10. Power and Zucco applied a rescaling procedure similar to Aldrich and McKelvey's (1977) in order to allow for separate shift and stretch factors for each respondent.

All ideal points (be they roll call, survey, or coalition based) are rescaled so that PT lies at -1 and PFL lies at $+1$ of the ideological space. This puts them all in the same scale, enhancing comparability. I performed the same post-processing on the MCMC draws of the coalition model, constraining μ_i (the prior mean of party i 's positions at the state level) for PT and PFL to be -1 and $+1$ respectively. The correlations are calculated excluding these two parties.

The left panel of Figure 11 shows the correlations between roll call and coalition estimates are very high (around .9) until 2002, using both 2000 and 2004 elections data. However, neither the 2000 nor the 2004 coalition data predict well the estimated roll call positions in 2003-2004. For example, the correlation between the 2000 coalition estimates and the 2005-2006 roll call estimates is essentially zero! The correlations between the coalition model estimates and the legislative surveys estimates are much more stable. For the 2000 coalition model, the correlations are very high, almost equal one in 2001. The correlations between the 2004 coalition estimates and the survey estimates are lower, but still over .8 for most of the period.

In sum, coalition based estimates are highly correlated with ideological estimates derived from legislative surveys. They are also highly correlated with roll call estimates, but only until 2002. We can take a better look at the fit between roll call derived estimates and coalition derived estimates in Figure 12. In the right panels I compare the coalition base estimates to the 2003-2004 roll call data. It is visible that most parties are estimated to be much closer to the PT than using coalition data. In addition, there is no difference (in the roll call estimates) between the position of the PP

(a traditional right wing party) and the PDT (a traditional left of center party), while they are still quite far apart in the coalition derived estimates.

One possible reason for the lack of fit is a change in the government coalition. Lula da Silva's government coalition included not only parties located on the left during the Cardoso years (e.g. PT, PCdoB) but also parties that were on the right wing of the space (PP and PTB). Sure enough, the latter two parties are responsible for a big part of the lack fit between coalition estimates and roll call estimates during the Lula years. Combining survey based estimates with roll call based estimates, Zucco and Lauderdale (2009) argue that the main dimension of conflict in the Brazilian legislature is not strictly ideological, but a reflection of the disputes between government and opposition. The patterns presented here are consistent with their story.

9.2 State level ideology

The state level party ideal points derived from the coalition model are displayed in Figures 13 and 14 for the 2000 and 2004 elections respectively. The colored circles show the positions of the four largest parties, while party labels shows the position of the remaining parties. There are unfortunately no other estimates of state-level preferences of political parties in Brazil to compare these two. As an alternative, I collected from the Brazilian politics literature and local news a set of stylized facts/hypotheses about regional party preferences in Brazil that one should be able to find using the coalition data. Note that most of the hypotheses refer to the PMDB, which is the largest party in Brazil, but also notoriously fractionalized.

1. **PMDB should be on the far right in Mato Grosso do Sul (MS)** A former governor of the state declared: "We understand the importance of a national alliance with the PMDB. However, here (in MS) we have a very strong polarization with the PMDB since 1996."¹⁴
2. **PSDB in Bahia (BA) should be far to the left from the PFL.** The main leader of PSDB in Bahia, Jutahy Magalhães Junior, was a long time foe of the deceased head of the PFL in the state, Antonio Carlos Magalhães (no relation.) The dissension is so extreme that the PSDB in Bahia supported the PT candidate in the 1994 presidential race *against* the PSDB candidate, Fernando Henrique Cardoso, who was ultimately elected along with Marco Marciel, a PFL politician, as vice-president.

¹⁴Zeca do PT, <http://r1.sharedcopy.com/4jgali10>

3. **PMDB in Pernambuco (PE) should be on the right.** The main leader of the PMDB in Pernambuco, Jarbas Vasconcelos, is known to be on the right wing of the eternally fractured PMDB. The current Senate president and big-wig of the party, José Sarney, claimed in 2009¹⁵ “Jarbas is closer to the PSDB than to the PMDB”. (The PSDB is usually placed to the right of PMDB in Brazil.)
4. **PMDB in Rio de Janeiro (RJ) should be on the left.** The governor of Rio de Janeiro, Sérgio Cabral, is a staunch supporter of President Lula da Silva. His campaign in 2006 was in fact based on a “partnership” with the federal government.
5. **PMDB in Rio Grande do Sul (RS) should be closer to PSDB than the PT** A leader of the PMDB in RS claims that “almost 100%” of the PMDB in the state prefers the opposition candidate Serra to the government candidate Rousseff in the 2010 elections.¹⁶

The reported state level estimates concur, at least at first glance, with these expectations. For example, the PMDB in PE is on the right, but RJ is on the left. The PSDB in BA is very far from the PFL, but they are much closer in RS.

10 Conclusion

The PT was too stuck-up and did not like to form coalitions.

Lula da Silva, <http://bit.ly/bEolHv>

This article shows how ideology, political institutions and social environment affect the electoral coalition decisions of parties in the elections for vereador (alderman) in Brazil. A simple model of electoral coalition choice in proportional elections was presented, in which parties are represented by ideal points in a unidimensional policy space. By forming coalitions parties seek to maximize the number of seats obtained in the election while minimizing the ideological heterogeneity of parties inside the coalition.

The incentives for the formation of coalitions depends not only on the ideological preferences of potential partners but also on the constraints imposed by institutional (the number of seats in

¹⁵<http://r2.sharedcopy.com/1bad6>, <http://r3.sharedcopy.com/7o6ieu>

¹⁶<http://terramagazine.terra.com.br/interna/0,,0I4378918-EI6578,00-Deputado+PMDB+nao+dara+palanque+a+Dilma+no+RS.html>

dispute) and social (education level of the voters) context combined with party specific characteristics (the size of the party in the municipality.) The model was tested on the 2000 and 2004 elections, with results largely supportive of the hypotheses: ideological distance, number of seats (magnitude), party size and education level of the electorate all decrease the probability of coalition.

The empirical results lend credibility to a more nuanced view of Brazilian politics, where both ideology and political expediency play key roles. By constructing better measurements, tightly coupled with a well specified theory, I hope to have escaped from the either/or dichotomy that permeates the current literature on Brazil.

Finally, I hope that the statistical model developed here will also be useful to model coalition data from other sources, in particular coalition data in comparative politics and alliance data in international relations.

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```

model.biprobit.state <- function() {
  ## scale for the priors
  sdp <- 100
  ## loop over dyads
  for (i in 1:n.rows) {
    y[i]~dbern(p[i])
    xb1[i] <- -abs(x[party.i[i],state.i[i]]-x[party.j[i],state.i[i]])+ ##spatial
      b[1]+b[2]*inverse.seats[i]+b[3]*hdi[i]+g[group[i]]+ ## mun level covariates
      b[4]*th.i[i] ## party specific covariates
    xb2[i] <- -abs(x[party.i[i],state.i[i]]-x[party.j[i],state.i[i]])+ ##spatial
      b[1]+b[2]*inverse.seats[i]+b[3]*hdi[i]+g[group[i]]+ ## mun level covariates
      b[4]*th.j[i] ## party specific covariates
    p[i] <- phi(xb1[i])*phi(xb2[i])
  }
  ## priors
  sigma.x ~ dunif(0,sdp)
  tau.x <- pow(sigma.x,-2)
  for (i in 1:4) {
    b[i] ~ dnorm(0,pow(sdp,-2))
  }
  ## normal priors for parties 3:n.parties
  for (i in 3:n.parties) {
    mu.x[i] ~ dnorm(theta[i], pow(sdp,-2))
  }
  mu.x[1] ~ dnorm(theta[1], pow(sdp,-2))%_T(,0)
  mu.x[2] ~ dnorm(theta[2], pow(sdp,-2))%_T(0,)
  ## normal priors for parties 3:n.parties
  for (i in 3:n.parties) {
    for (j in 1:n.states) {
      x[i,j] ~ dnorm(mu.x[i],tau.x)
    }
  }
  ## truncated priors (so 1 is negative and 2 is positive) for parties 1 and 2
  for (j in 1:n.states) {
    x[1,j] ~ dnorm(mu.x[1],tau.x)%_T(,0)
    x[2,j] ~ dnorm(mu.x[2],tau.x)%_T(0,)
  }
  for (i in 1:n.groups) {
    g[i] ~ dnorm(0,tau.g)
  }
}

```

```
}  
sigma.g ~ dunif(0,sdp)  
tau.g <- pow(sigma.g,-2)  
}
```

Seats	2000	2004
9	3110	4970
10	66	319
11	1117	98
12	17	46
13	566	33
14	13	21
15	306	13
16	2	14
17	157	6
18	1	7
19	63	4
20	0	5
21	124	13
33	5	1
34	0	2
35	2	1
36	0	1
37	1	1
38	0	1
41	2	4
42	1	0
50	0	1
55	1	1

Table 1: *Number of vereadores elected by municipality in 2000 and 2004*

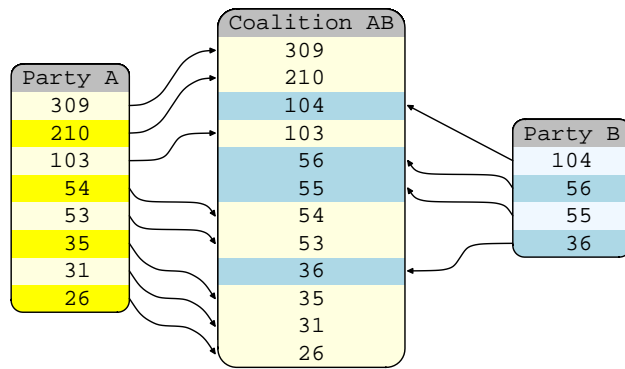


Figure 1: *Hypothetical example of coalition choice.*

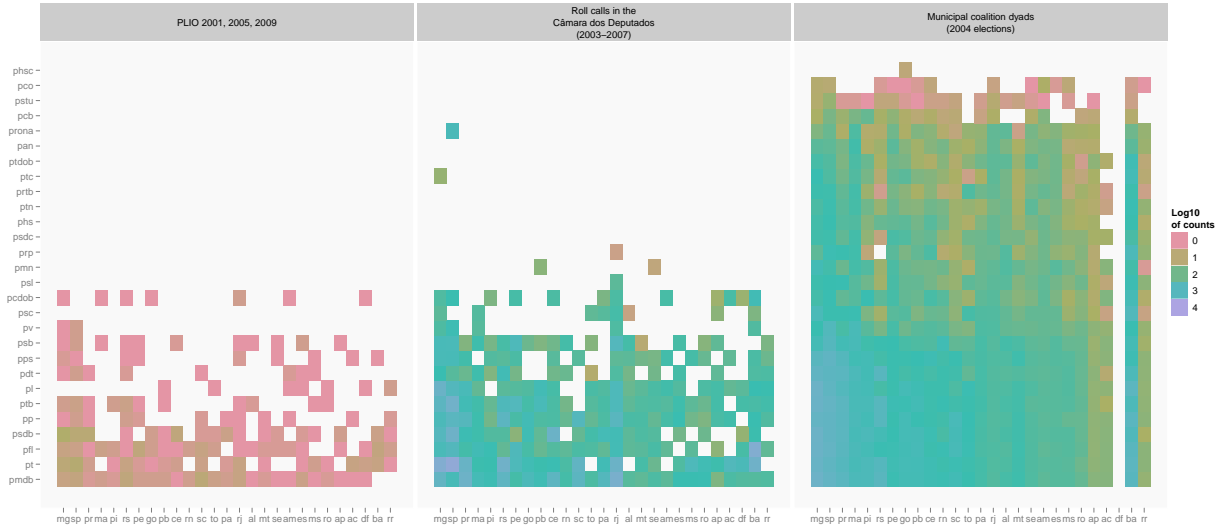


Figure 2: Coalition dyads, roll calls in the Câmara dos Deputados and surveys of Deputados Federais by state.

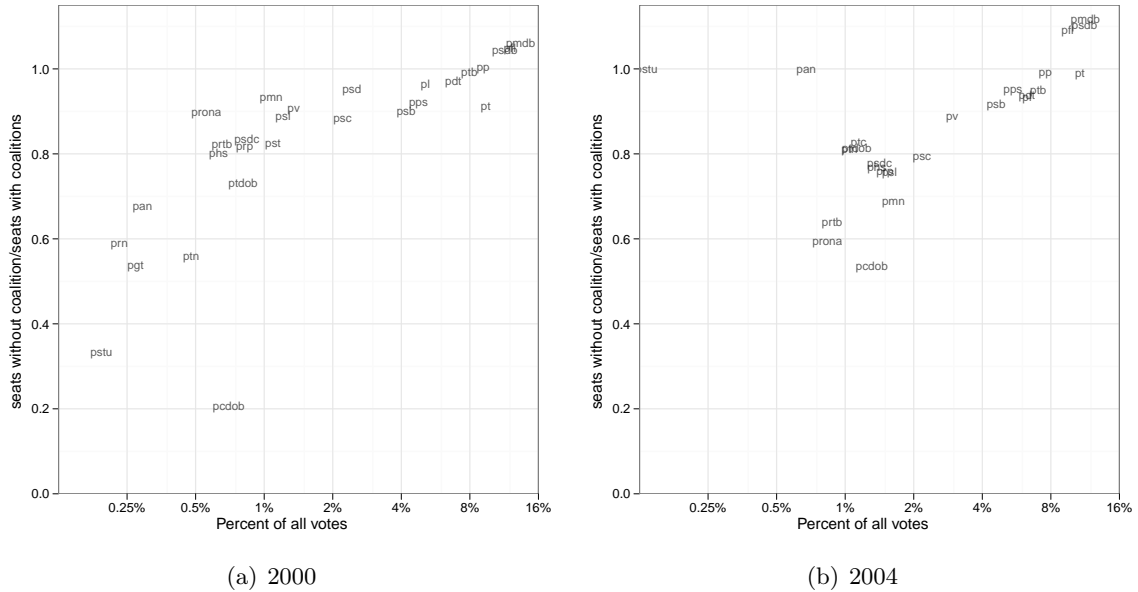
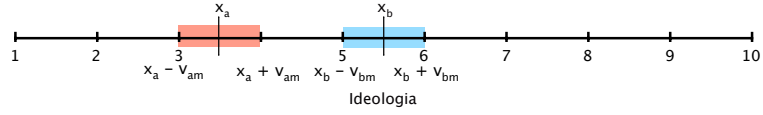
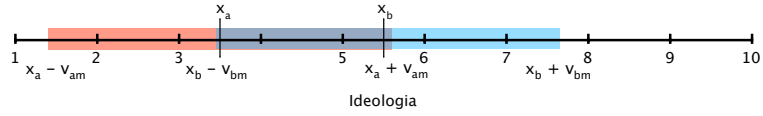


Figure 3: Mechanical effect of coalitions on the elections for vereadores in 2000 and 2004. In the y axis is the ratio of seats without coalitions to seats with coalitions. In both years only three parties (PSDB, PMDB and PFL) were net losers of seats due to the mechanical effect of electoral coalitions. In the x axis are the votes each party got as a percentage of total valid votes for vereador in a logarithmic scale of base 2. The relationship between (log of) votes and the mechanical effect of coalitions is approximately linear: the smaller the party the larger is the expected benefit from coalitions.



(a) Acceptance region is small. Only parties that are close to each other want to form a coalition.



(b) Acceptance region is wide, thus even parties far from each other would like to form a coalition.

Figure 4: *Visualization of the theoretical model.*

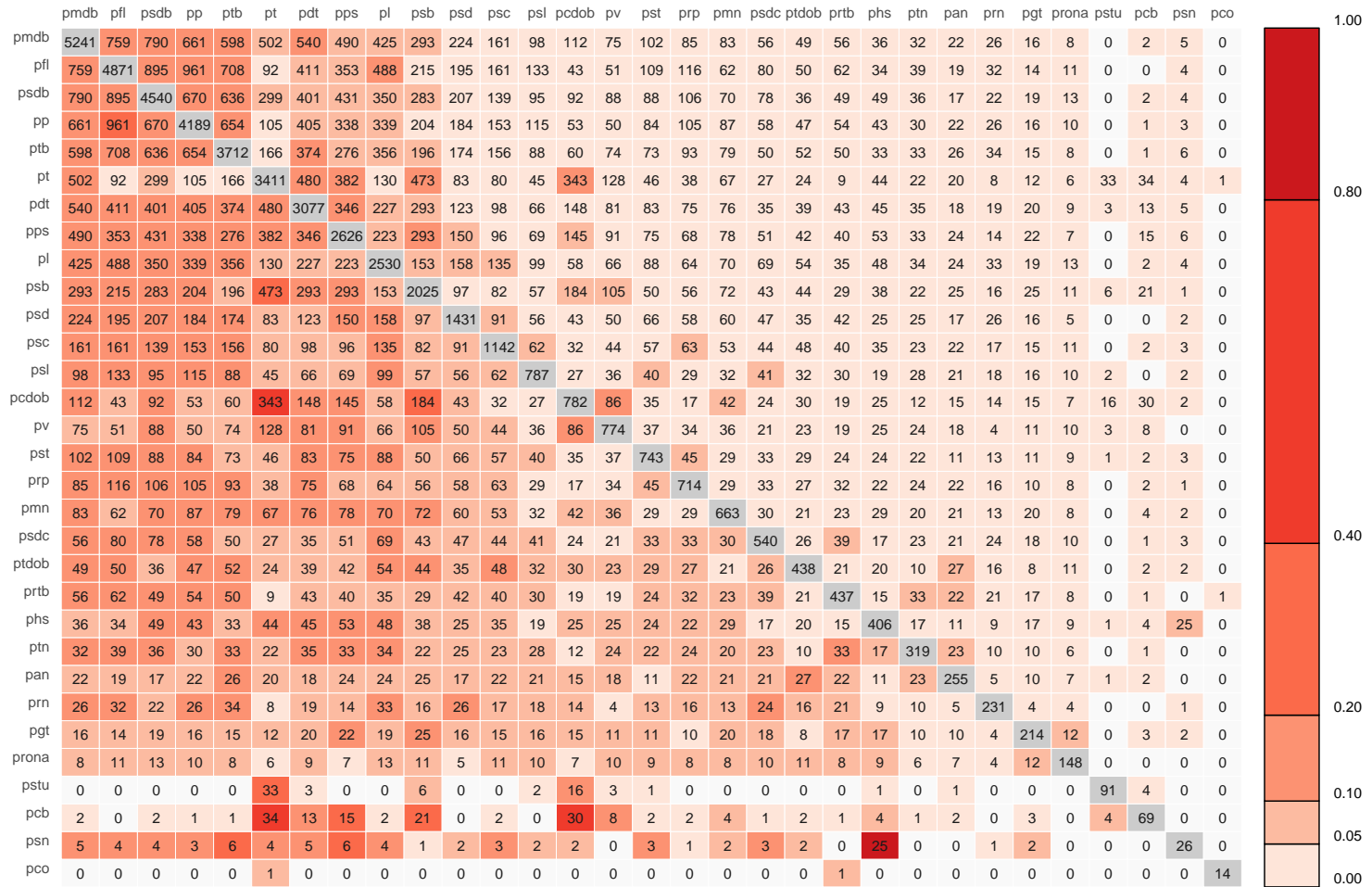


Figure 5: Counts of coalition decisions in the Brazilian local legislative elections of 2000. Each cell displays the counts of coalitions that include the row-party and the column-party. The number of elections that each party was present is in the diagonal of the matrix (in grey.) The cell colors indicate the proportion of the elections that the row party was present that also included the column party.

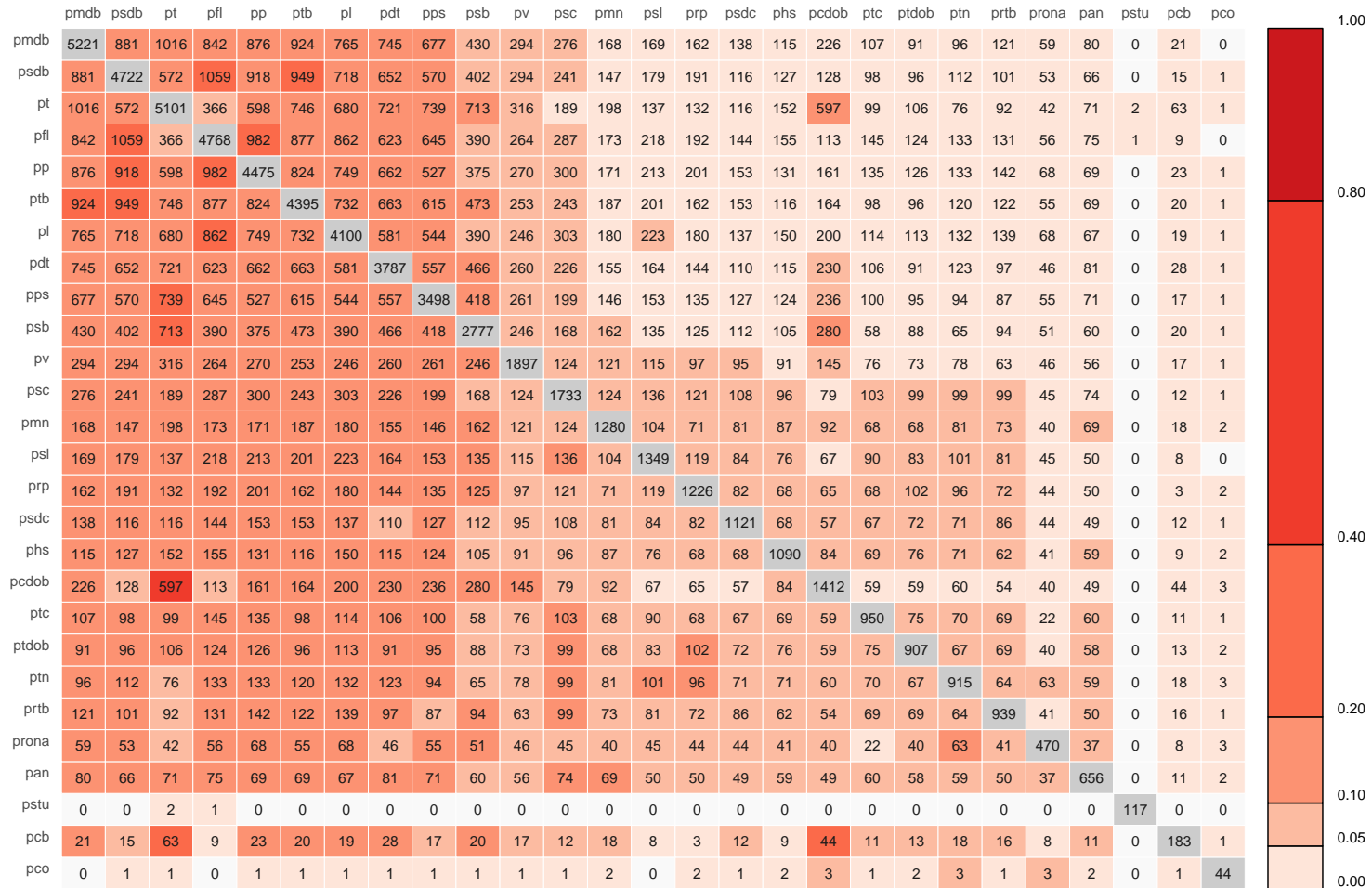


Figure 6: Counts of coalition decisions in the Brazilian local legislative elections of 2004. See explanation in the caption of Figure 5.

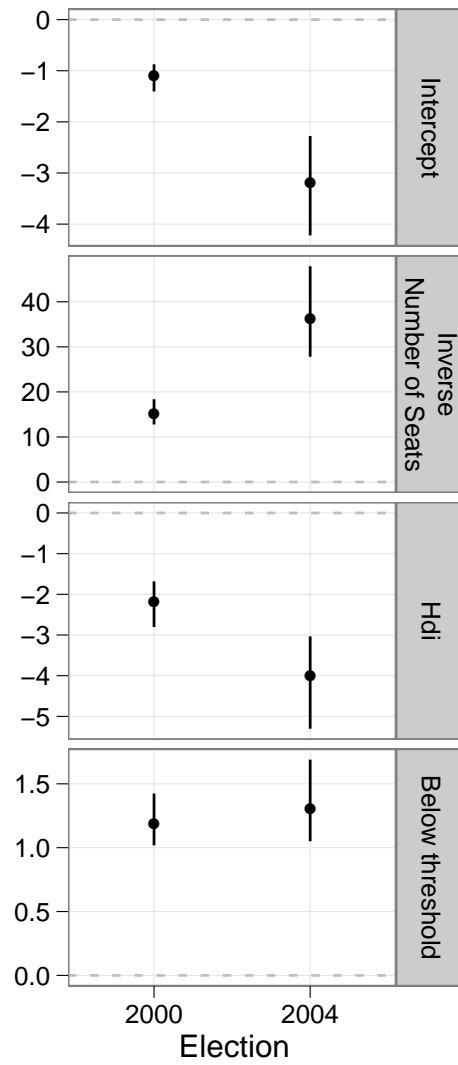


Figure 7: *Structural parameters of the model.*

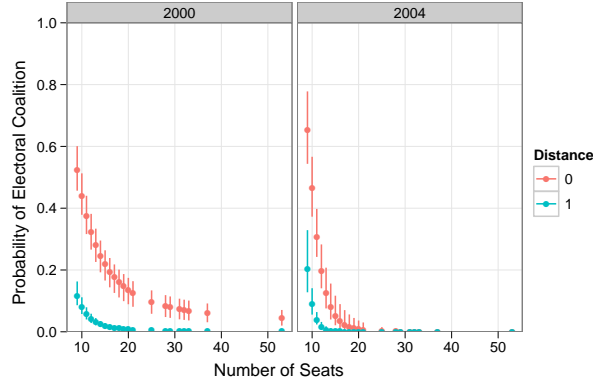


Figure 8: *Predicted probability of coalition formation given ideology and the number of seats. (HDI-education held at 0.8; both parties above the vote threshold.)*

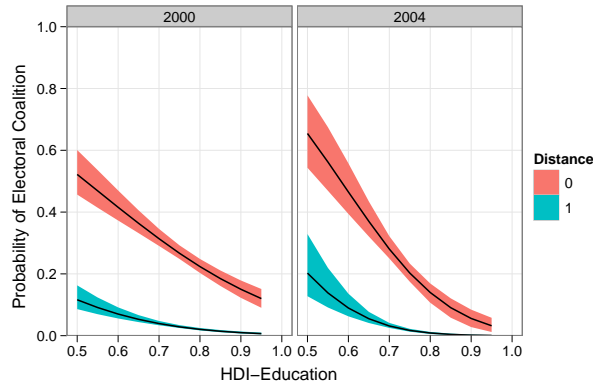


Figure 9: *Predicted probability of coalition formation given ideology and the education level of the municipality. (Number of seats held at 9; both parties above the vote threshold.)*

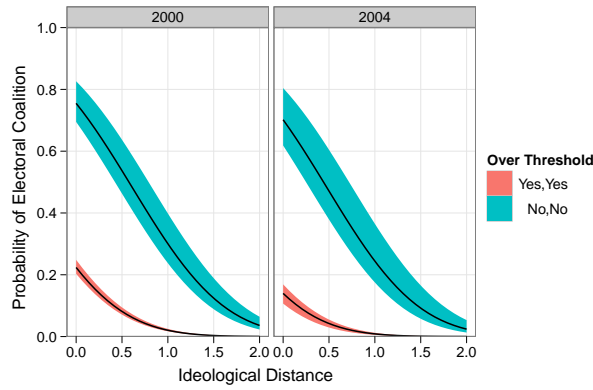


Figure 10: *Predicted probability of coalition formation given ideology and party threshold status. When both parties are below the minimum electoral threshold the probability of coalition is much higher than when the both are above it. (Number of seats held at 9; HDI-education held at 0.8)*

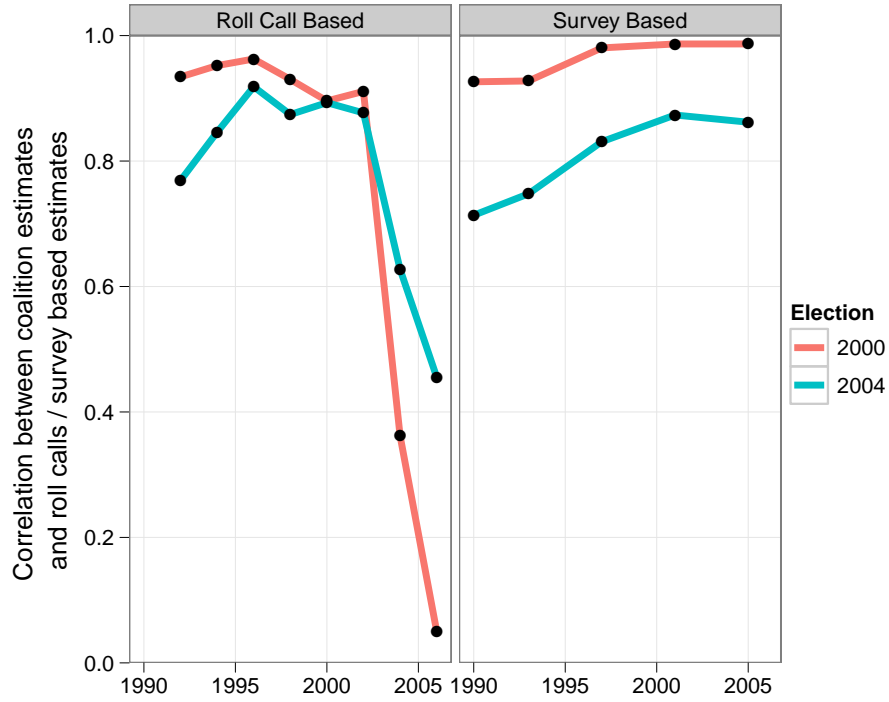


Figure 11: *Correlation between mean party ideal points calculated using roll call or survey data and coalition data. For legislative sessions until 2002, all correlations between roll call estimates and coalition based estimates are above .9. Correlations for 2003-2004 and 2005-2006 legislative sessions are much lower. For survey based estimates, the correlations with 2000 coalition data are all above .9, reaching a high of .99. The 2004 coalition data correlations are lower, reaching a high of .87.*

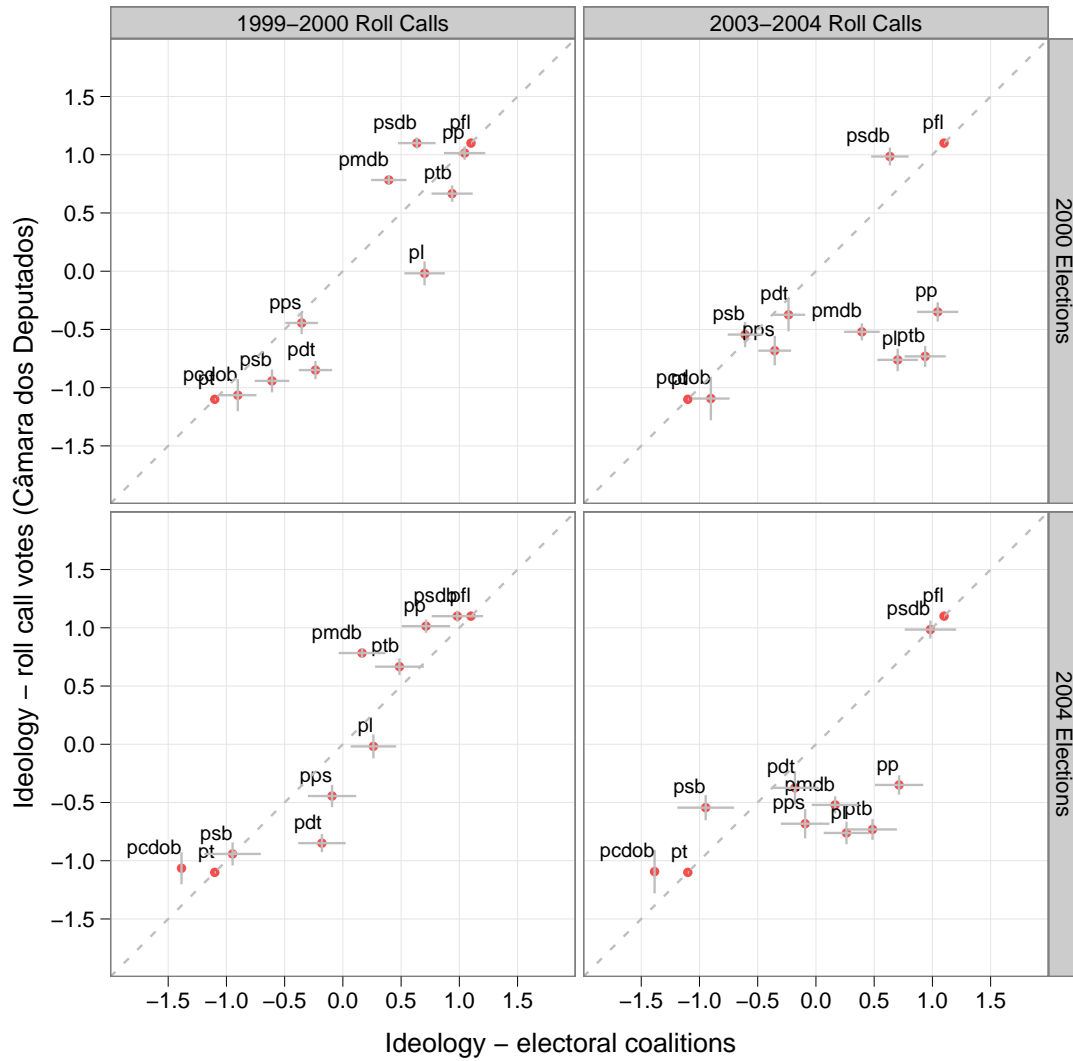


Figure 12: Coalition ideal points versus roll call ideal points. The cross hairs plot +1 and -1 standard deviations of the MCMC estimates. The size of the circles follow the size of the parties, as measured using the total votes they got in the election for vereadores.

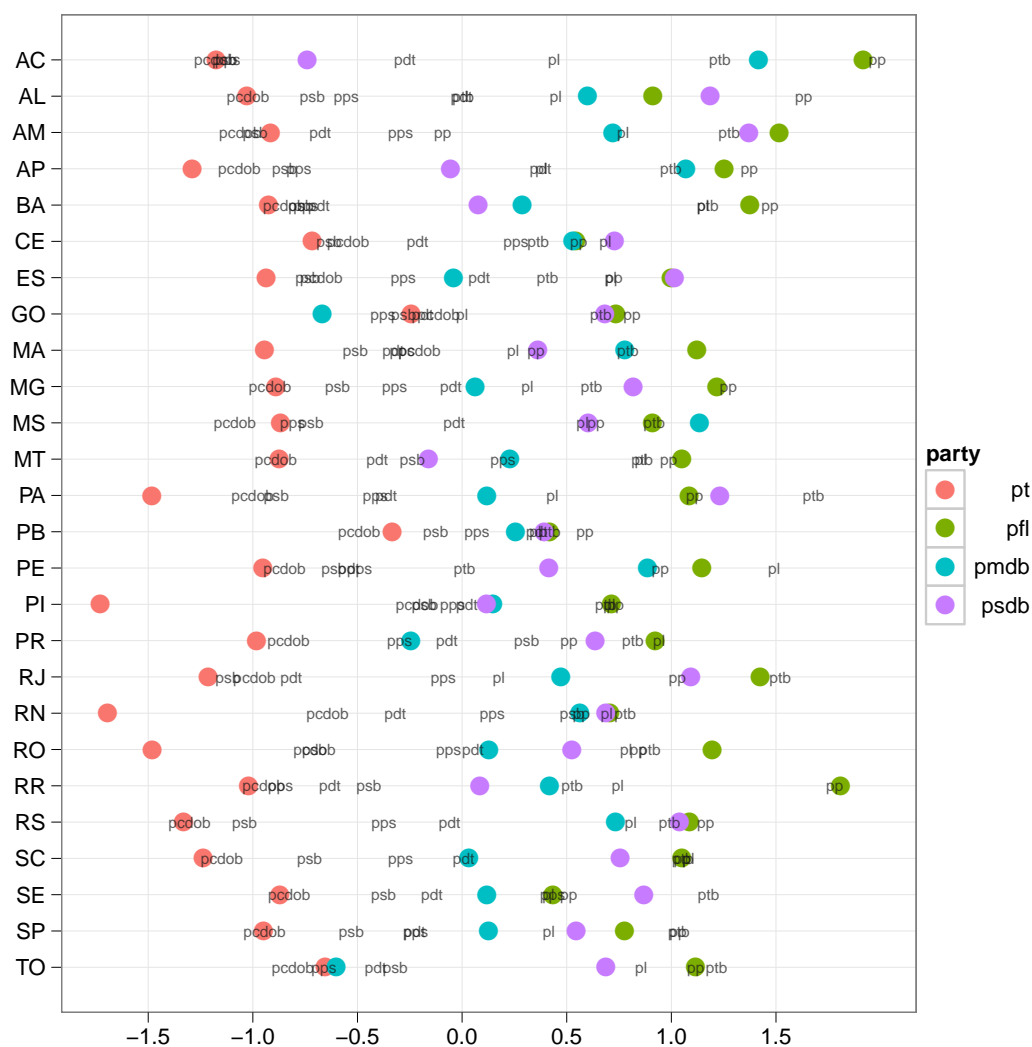


Figure 13: *State level party ideal points in 2000.*

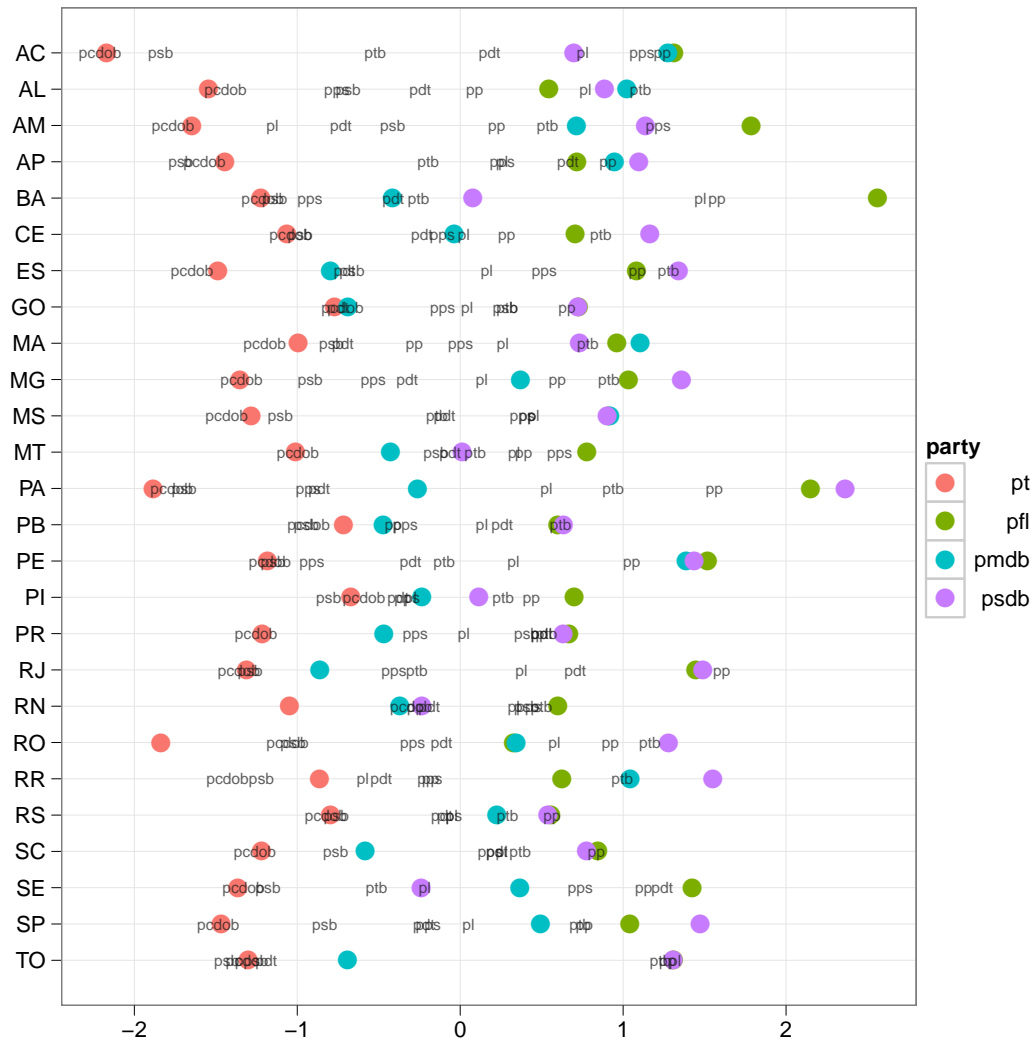


Figure 14: State level party ideal points in 2004.