### 0.1 Static DFE Equalization (LTI Channel)

### 0.1.1 Static Channel Model

Let us consider the general case for transmitting a discrete-time binary signal, $X$, through an LTI channel (Fig. 2):


Figure 1: Channel equalization.

### 0.1.2 DFE Model

The goal of the Rx equalizer will be to re-construct a time-delayed version of $X$ as the discrete-time binary signal, $Y$.


Figure 2: DFE Model.

To achieve this goal, the DFE from Fig. 2 will be used:

$$
\begin{equation*}
Y_{S}=C_{0} B+\sum_{i=1}^{N} C_{i} Y_{L} e^{-j \omega(i T)} \tag{1}
\end{equation*}
$$

For the simple case where the transmit filter $G$ is an ideal pulse generator, $P$ :

$$
\begin{gather*}
B=X P H  \tag{2}\\
P= \begin{cases}1, & 0 \leq t \leq T \\
0, & \text { otherwise. }\end{cases} \tag{3}
\end{gather*}
$$

Break the loop: In an ideal world, the DFE would reconstruct the original signal before sampling it:

$$
\begin{equation*}
Y_{L}=X P e^{-j \omega(D-T)} \tag{4}
\end{equation*}
$$

where $D$ sets the sampling point right before the data is flopped.

Substituting (2), and (4) into (1), the DFE can therefore be expressed as:

$$
\begin{gather*}
Y_{S}=C_{0} X P H+\sum_{i=1}^{N} C_{i} X P e^{-j \omega(i T+D-T)}  \tag{5}\\
\frac{Y_{S}}{X}=C_{0} H P+P \sum_{i=1}^{N} C_{i} e^{-j \omega(i T+D-T)} \tag{6}
\end{gather*}
$$

### 0.1.3 Signal Sampling

$$
\begin{align*}
\mathfrak{s}(t, D) & \triangleq \sum_{-\infty}^{\infty} \delta(t-(n T+D))  \tag{7}\\
\mathfrak{s}(t, D) & =\delta(t-D) * \sum_{-\infty}^{\infty} \delta(t-n T) \tag{8}
\end{align*}
$$

Thus,

$$
\begin{align*}
& \mathfrak{g}(D) \triangleq \mathfrak{F}\{\mathfrak{s}(t, D)\}  \tag{9}\\
& \mathfrak{g}(D)=\frac{2 \pi e^{-j \omega D}}{T} \sum_{k=-\infty}^{\infty} \delta\left(\omega-\frac{2 \pi k}{T}\right) \tag{10}
\end{align*}
$$

### 0.1.4 Amplitude Equalization

$$
\begin{align*}
\frac{Y_{S}}{X} & =e^{-j \omega D}, & \forall t & =i T+D  \tag{11}\\
e^{-j \omega D} & =C_{0} H P+P \sum_{i=1}^{N} C_{i} e^{-j \omega(i T+D-T)}, & \forall t & =i T+D \tag{12}
\end{align*}
$$

Part $1, t=D$ :

$$
\begin{align*}
e^{-j \omega D} & =C_{0} H P, & & t=D  \tag{13}\\
\delta(t-D) & =C_{0} h_{P}(t) & & t=D  \tag{14}\\
C_{0} & =1 / h_{P}(D) & & \tag{15}
\end{align*}
$$

Part 2, $t=i T+D \quad i>0:$

$$
\begin{align*}
& P \sum_{i=1}^{N} C_{i} e^{-j \omega(i T+D-T)}=e^{-j \omega D}-C_{0} H P, \quad t=i T+D \quad i>0  \tag{16}\\
& P \sum_{i=1}^{N} C_{i} e^{-j \omega(i T+D-T)}=e^{-j \omega D}-C_{0} H P, \quad t=i T+D \quad i>0 \tag{17}
\end{align*}
$$

If we want to use arbitrary filter $F$ with feedback coefficients:

$$
\begin{equation*}
C_{i} \Rightarrow F C_{i}, \quad i>0 \tag{18}
\end{equation*}
$$

Thus we get

$$
\begin{equation*}
P \sum_{i=1}^{N} C_{i} e^{-j \omega(i T+D-T)}=\frac{e^{-j \omega D}-C_{0} H P}{F}, \quad t=i T+D \quad i>0 \tag{19}
\end{equation*}
$$

### 0.2 Matrix Test

$$
\begin{align*}
\boldsymbol{V} & =\boldsymbol{Z} \boldsymbol{I}  \tag{20}\\
{\left[\begin{array}{c}
V_{1} \\
V_{2} \\
\vdots \\
V_{N}
\end{array}\right] } & =\left[\begin{array}{cccc}
z_{11} & z_{12} & \cdots & z_{1 n} \\
z_{21} & z_{22} & \cdots & z_{2 n} \\
\vdots & \vdots & \ddots & \vdots \\
z_{n 1} & z_{n 2} & \cdots & z_{n n}
\end{array}\right]\left[\begin{array}{c}
I_{1} \\
I_{2} \\
\vdots \\
I_{N}
\end{array}\right] .
\end{align*}
$$

## Bibliography

