Trigonometric Identities

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I. (Circular) Trigonometric Functions

$$\sin(z) \triangleq \frac{e^{iz} - e^{-iz}}{2i} \qquad \cos(z) \triangleq \frac{e^{iz} + e^{-iz}}{2}$$

A. Circular Identities

$$e^{\pm jx} = \cos x \pm j \sin x$$

$$\sin^2 x = \frac{1 - \cos 2x}{2} \qquad \cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin(2x) = 2 \sin x \cos x \qquad \cos(2x) = \cos^2 x - \sin^2 x$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

II. Hyperbolic Functions

$$\sinh(z) \triangleq \frac{e^{z} - e^{-z}}{2} \qquad \cosh(z) \triangleq \frac{e^{z} + e^{-z}}{2}$$

$$\operatorname{csch}(z) \triangleq \frac{1}{\sinh(z)} \qquad \operatorname{sech}(z) \triangleq \frac{1}{\cosh(z)}$$

$$\tanh(z) \triangleq \frac{\sinh(z)}{\cosh(z)} = \frac{e^{2z} - 1}{e^{2z} + 1}$$

$$\coth(z) \triangleq \frac{\cosh(z)}{\sinh(z)} = \frac{e^{2z} + 1}{e^{2z} - 1}$$

A. Hyperbolic Identities

$$\sinh^2(z) = \frac{\cosh(2z) - 1}{2} \qquad \cosh^2(z) = \frac{\cosh(2z) + 1}{2}$$

$$\cosh^2 z - \sinh^2 z = 1$$
$$\cosh z + \sinh z = e^{z}$$
$$\cosh z - \sinh z = e^{-z}$$

$$\sinh(2z) = 2 \sinh z \cosh z$$
$$\cosh(2z) = 2 \cosh^2 z - 1 = 1 + 2 \sinh^2 z$$

$$\sinh(x + iy) = \sinh x \cos y + i \cosh x \sin y$$
$$\cosh(x + iy) = \cosh x \cos y + i \sinh x \sin y$$

B. Inverse Hyperbolic Functions

$$\sinh^{-1}(z) = \ln(z + \sqrt{1 + z^2})$$
$$\cosh^{-1}(z) = \ln(z + \sqrt{z - 1}\sqrt{z + 1})$$
$$\tanh^{-1}(z) = \frac{1}{2}[\ln(1 + z) - \ln(1 - z)]$$
$$\operatorname{csch}^{-1}(z) = \ln\left(\sqrt{\frac{1}{z^2} + 1} + \frac{1}{z}\right)$$
$$\operatorname{sech}^{-1}(z) = \ln\left(\sqrt{\frac{1}{z} - 1}\sqrt{\frac{1}{z} + 1} + \frac{1}{z}\right)$$
$$\coth^{-1}(z) = \frac{1}{2}\left[\ln\left(1 + \frac{1}{z}\right) - \ln\left(1 - \frac{1}{z}\right)\right]$$