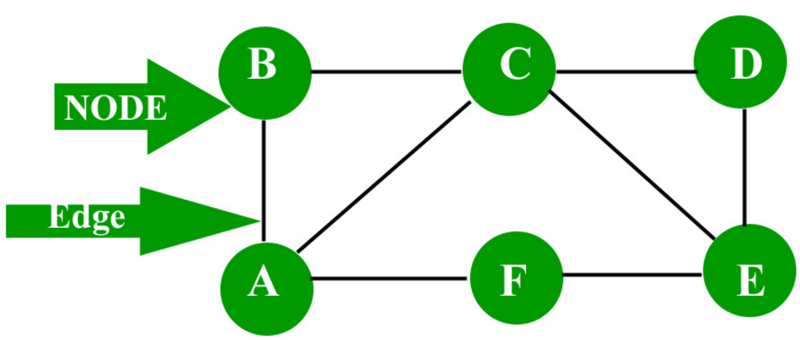
Mathematics | Graph Theory Basics – Set 1

Difficulty Level : Easy • Last Updated : 03 Jan, 2023

A graph is a data structure that is defined by two components :

1. A **node** or a vertex.
2. An edge E or **ordered pair is a connection between two nodes u,v** that is identified by unique pair(u,v). The pair (u,v) is ordered because (u,v) is not same as (v,u) in case of directed graph.The edge may have a weight or is set to one in case of unweighted graph.

Consider the given below graph,



To know about "Graph representation" click here

Applications: Graph is a data structure which i... sed extensively in our real-life.

1. Social Network: Each ... ctivities,suggestion and friend list are represen...
2. Google Maps: Various ... des and the roads are represented as edges ... th between two nodes.

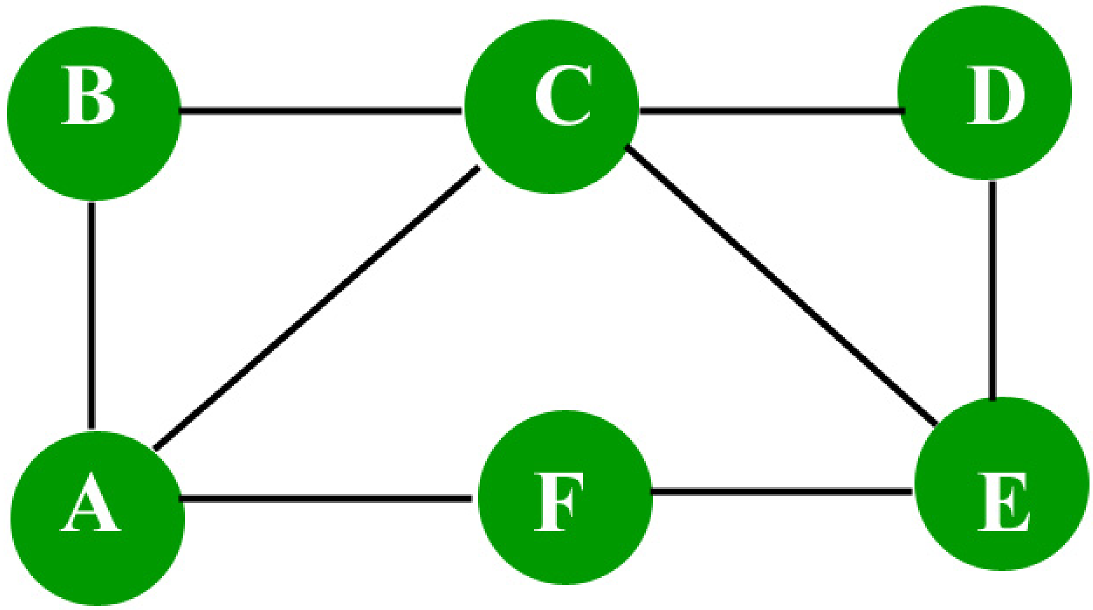
3. Recommendations on e-commerce websites: The "Recommendations for you" section on various e-commerce websites uses graph theory to recommend items of similar type to user's choice.
4. Graph theory is also used to study molecules in chemistry and physics.

More on graphs: **Characteristics of graphs:**

1. Adjacent node: A node 'v' is said to be adjacent node of node 'u' if and only if there exists an edge between 'u' and 'v'.
2. Degree of a node: In an undirected graph the number of nodes incident on a node is the degree of the node. In case of directed graph ,**Indegree** of the node is the **number of arriving edges** to a node. **Outdegree** of the node is the **number of departing edges to a node.**

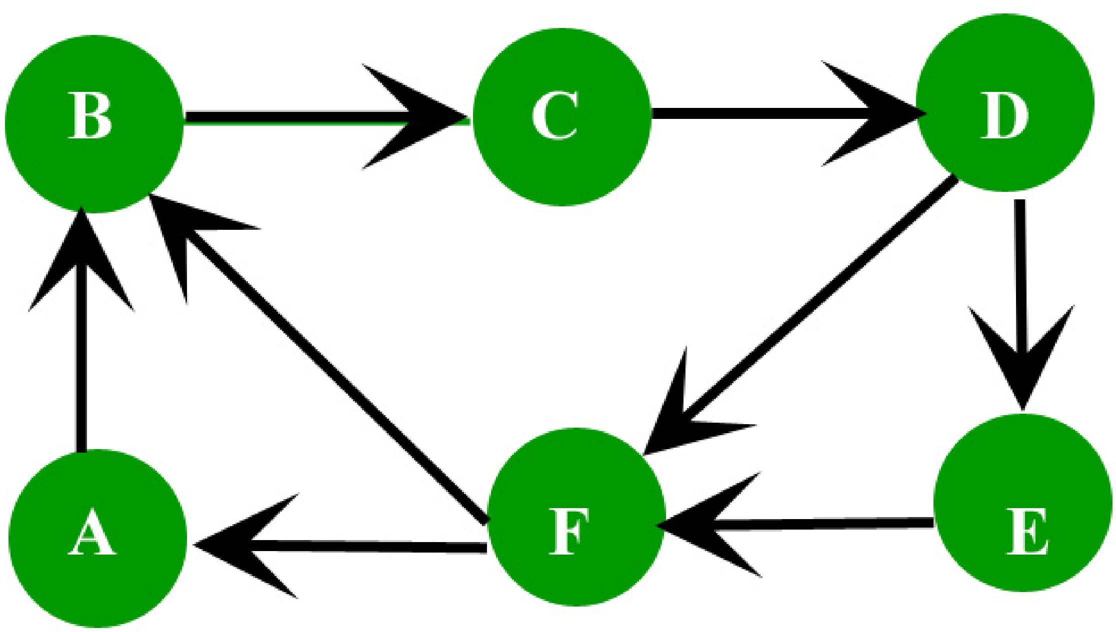
Note: 1 a self-loop is counted twice

2 the sum of degree of all the vertices in a graph G is even.



Un-directed graph

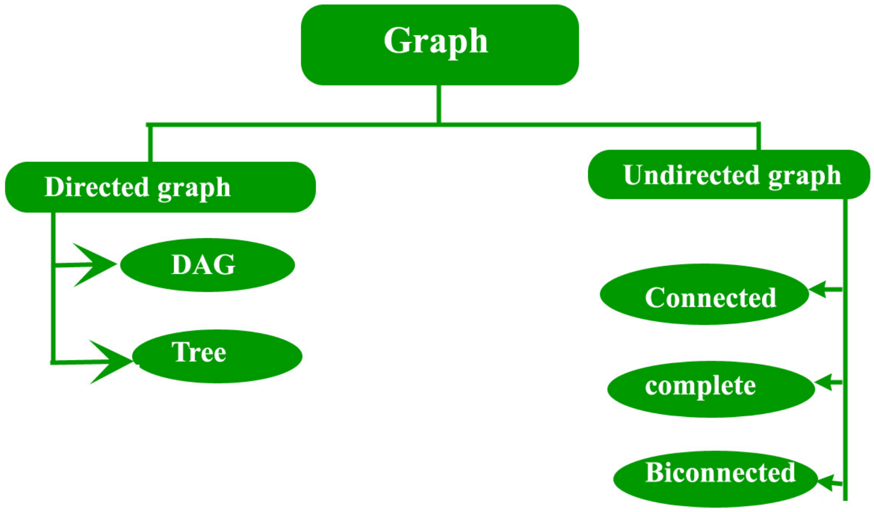
1.



2. Path: A path of length [illegible] quence of n+1 nodes.

P(u,v)=(v0,v1,v2,v3…….vn)

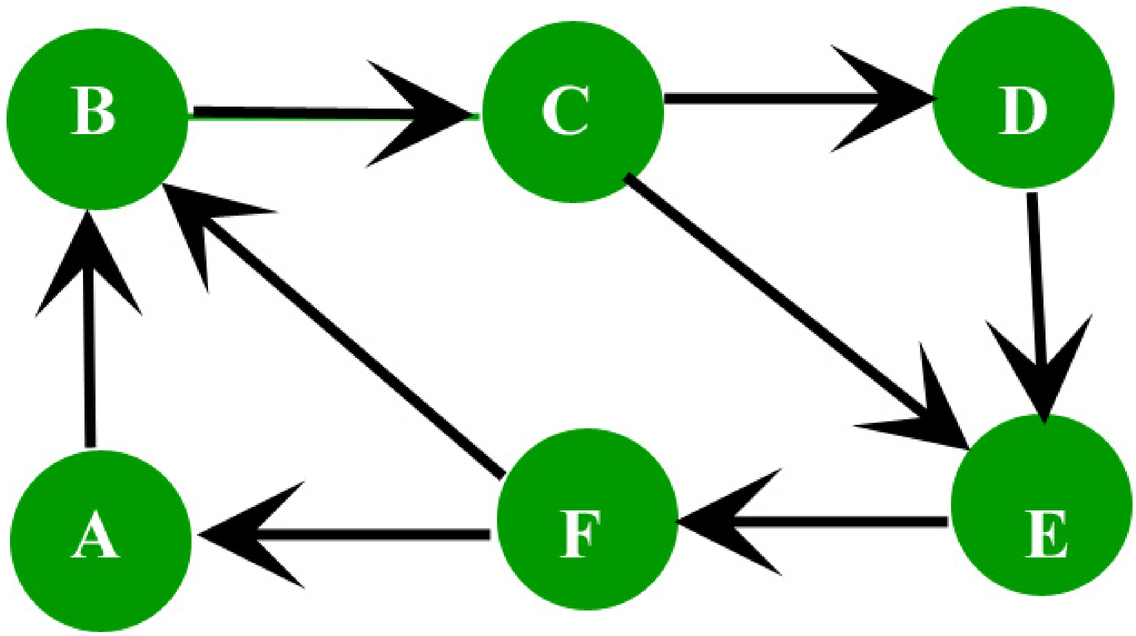
1. A path is simple if all the nodes are distinct,**exception is source and destination are same.**
2. Isolated node: A node with degree 0 is known as isolated node.Isolated node can be found by Breadth first search(BFS). It finds its application in **LAN network** in finding whether a **system is connected or not.**



Types of graphs:

1. Directed graph: A graph in which the direction of the edge is defined to a particular node is a directed graph.
   - Directed Acyclic graph: It is a directed graph with no cycle.For a vertex ‘v’ in DAG there is no directed edge starting and ending with vertex ‘v’. a) Application :Critical game analysis,expression tree evaluation,game evaluation.
   - Tree: A tree is just a restricted form of graph.That is, it is a **DAG with a restriction that a child can have only one parent.**

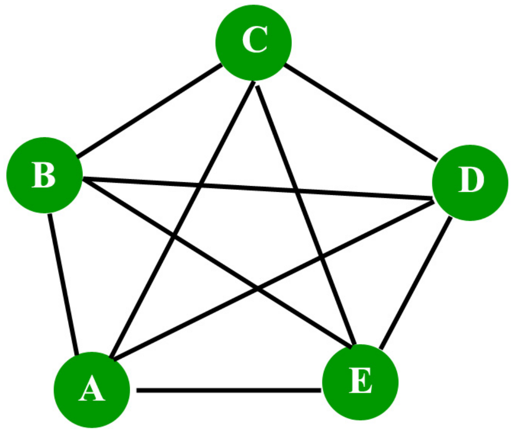
2. Undirected graph: A graph in which the direction of the edge is not defined.So if an edge exists between node ‘u’ and ‘v’,then there is a path from node ‘u’ to ‘v’ and vice versa.
   - Connected graph: A graph is connected when there is a **path between every pair of vertices.**In a connected graph there is no unreachable node.
   - Complete graph: A graph in which each pair of graph vertices is connected by an edge.In other words,every node ‘u’ is adjacent to every other node ‘v’ in graph ‘G’.A complete graph would have **n(n-1)/2** … ee below for proof.
   - Biconnected graph … down into any further pieces by deletion … **n point.**



Connected Graph

Proof for complete graph:

1. Consider a complete graph with n nodes. Each node is connected to other n-1 nodes. Thus it becomes n * (n-1) edges. But this counts each edge twice because this is a undirected graph so divide it by 2.
2. Thus it becomes n(n-1)/2.



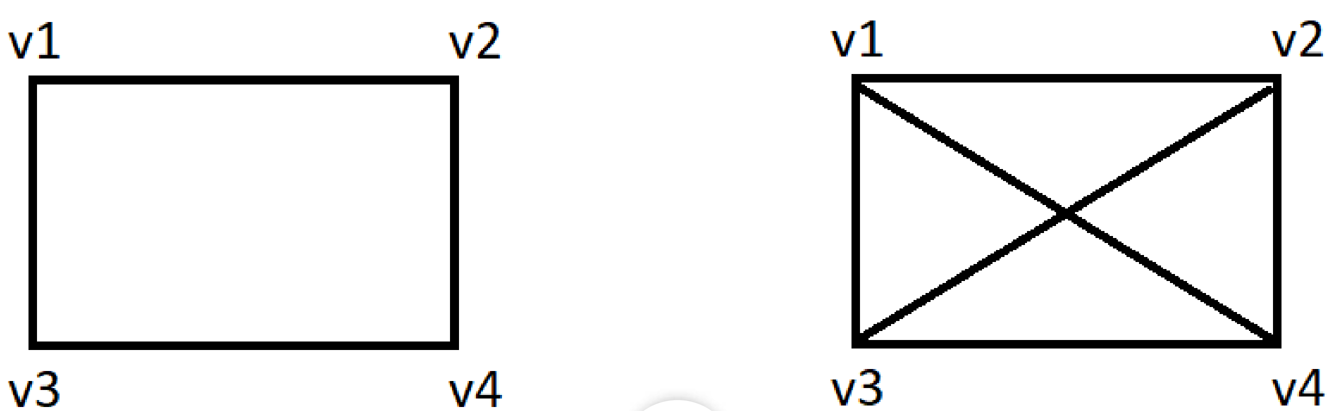
Complete Graph

Consider the given graph, //Omit the repetitive edges Edges on node A = (A,B),(A,C),(A,E),(A,C). Edges on node B = (B,C),(B,D),(B,E). Edges on node C = (C,D),(C,E). Edges on node D = (D,E). Edges on node E = EMPTY.https://en.wikipedia.org/wiki/Graph_theory Total edges = 4+3+2+1+0=10 edges. Number of node = 5. Thus n(n-1)/2=10 edges. Thus proven. Read next set – Graph Theory Basics

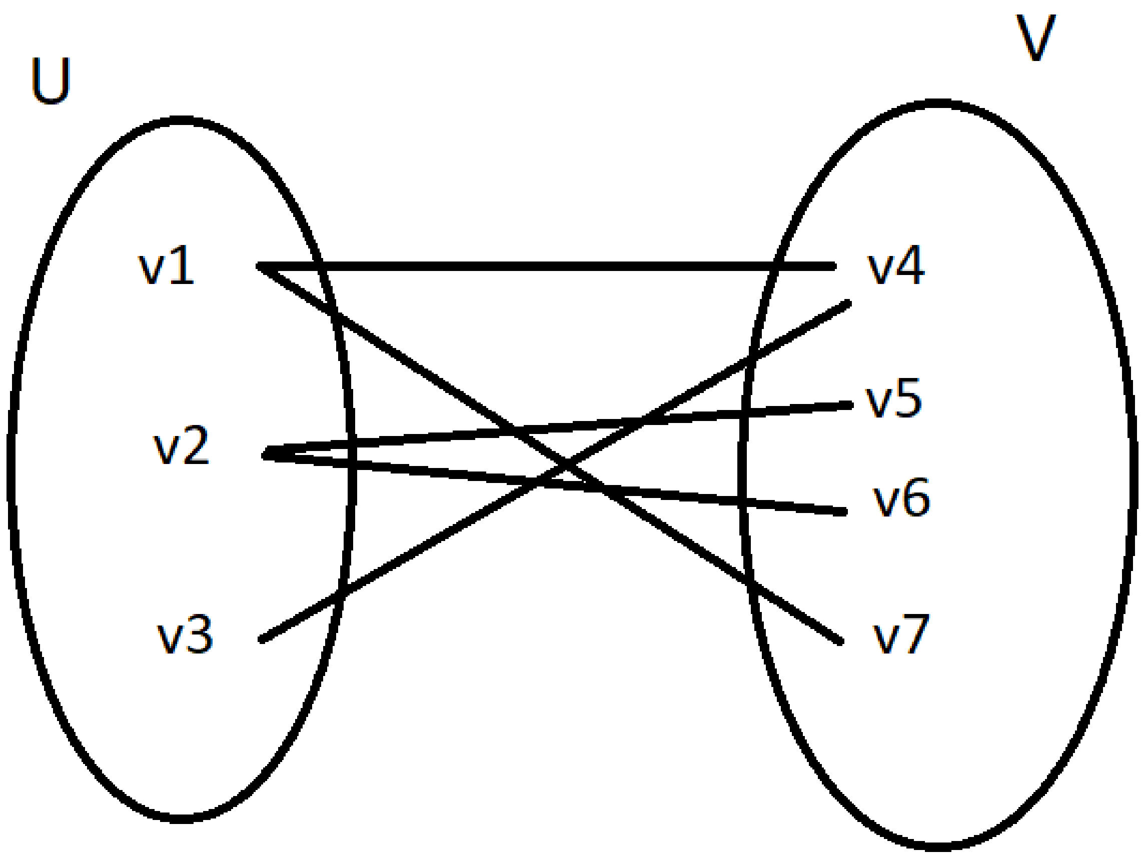
Some more graphs :

1. Regular graph :A graph in which every vertex x has same/equal degree.k-regular graph means every vertex has k degree.

Every complete graph K_n will have (n-1)-regular graph which means degree is n-1.



2. Bipartite graph : It is gr into two subsets U and V such that each edge of oint in V.

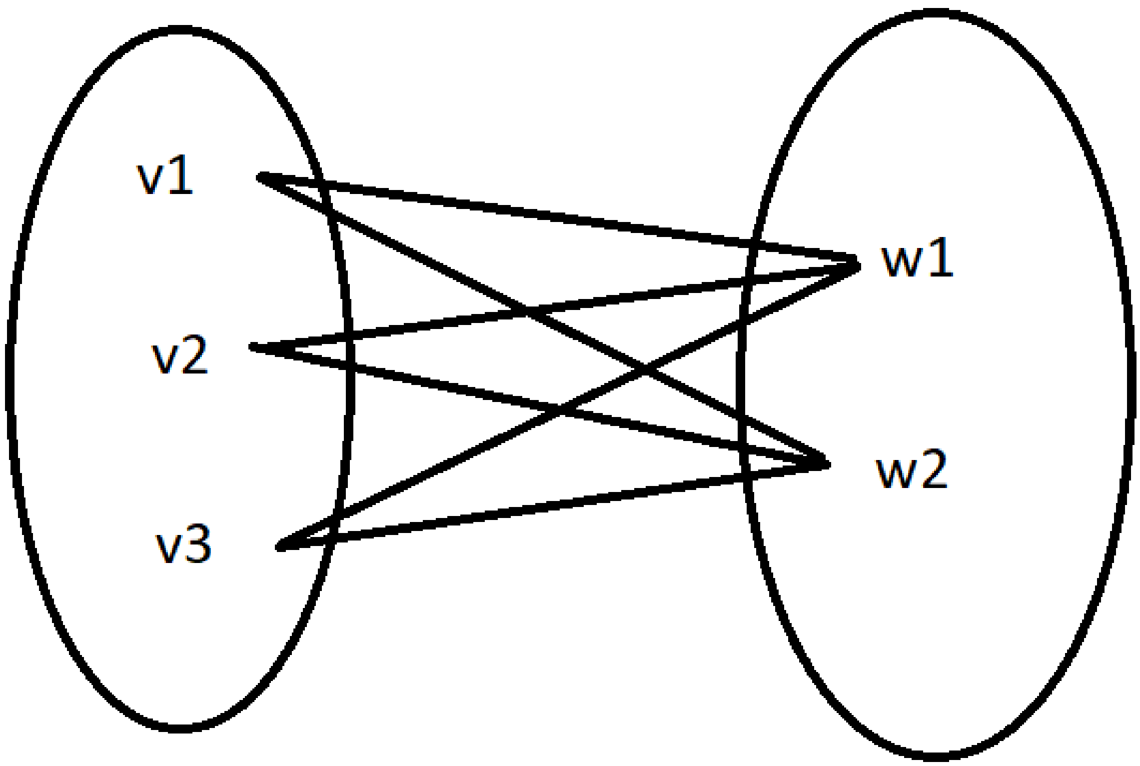


Bipartite graph

3. Complete Bipartite graph : it is a simple graph with vertex set partitioned into two subsets : $U=\{v_1,v_2\ldots\ldots\ldots.v_m\}$ and $W=\{w_1,w_2,\ldots\ldots\ldots.w_n\}$

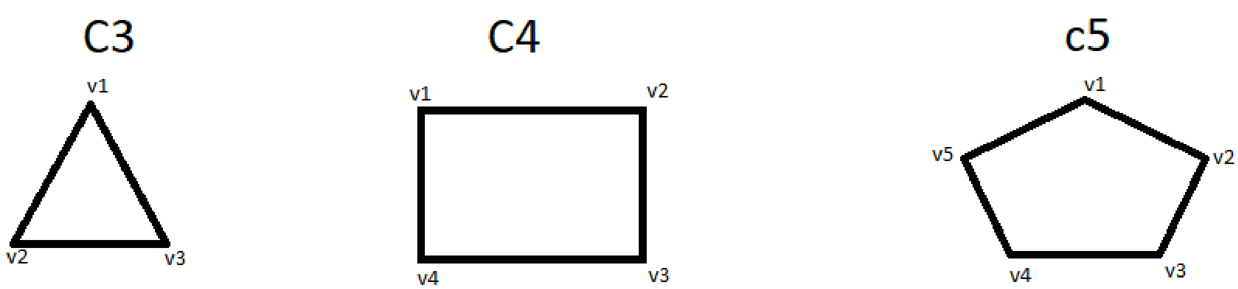
i. There is an edge from each v_i to each w_j.

ii. there is not an selp loop.



Complete Bipartite graph

4. Cycle graph : A graph of n vertices $(n\geq 3)$. $v_1, v_2, \ldots\ldots\ldots\ldots\ldots v_n$ with edges $(v_1,v_2),(v_2,v_3),\ldots\ldots\ldots,(v_{n-1},v_n),(v_n,v_1)$.



Cycle graph

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