MATLAB FOR BIOMEDICAL ENGINEERS

Modeling Molecular Biophysics

MOLECULAR TRANSPORT

DIFFUSION THROUGH SOLUTION

- Fick's First Law states that flux is proportional to the concentration gradient at steady state
  - Where J is flux, D is the diffusion coefficient, C is concentration, and x is location

$$J = -D\frac{\delta C}{\delta x}$$

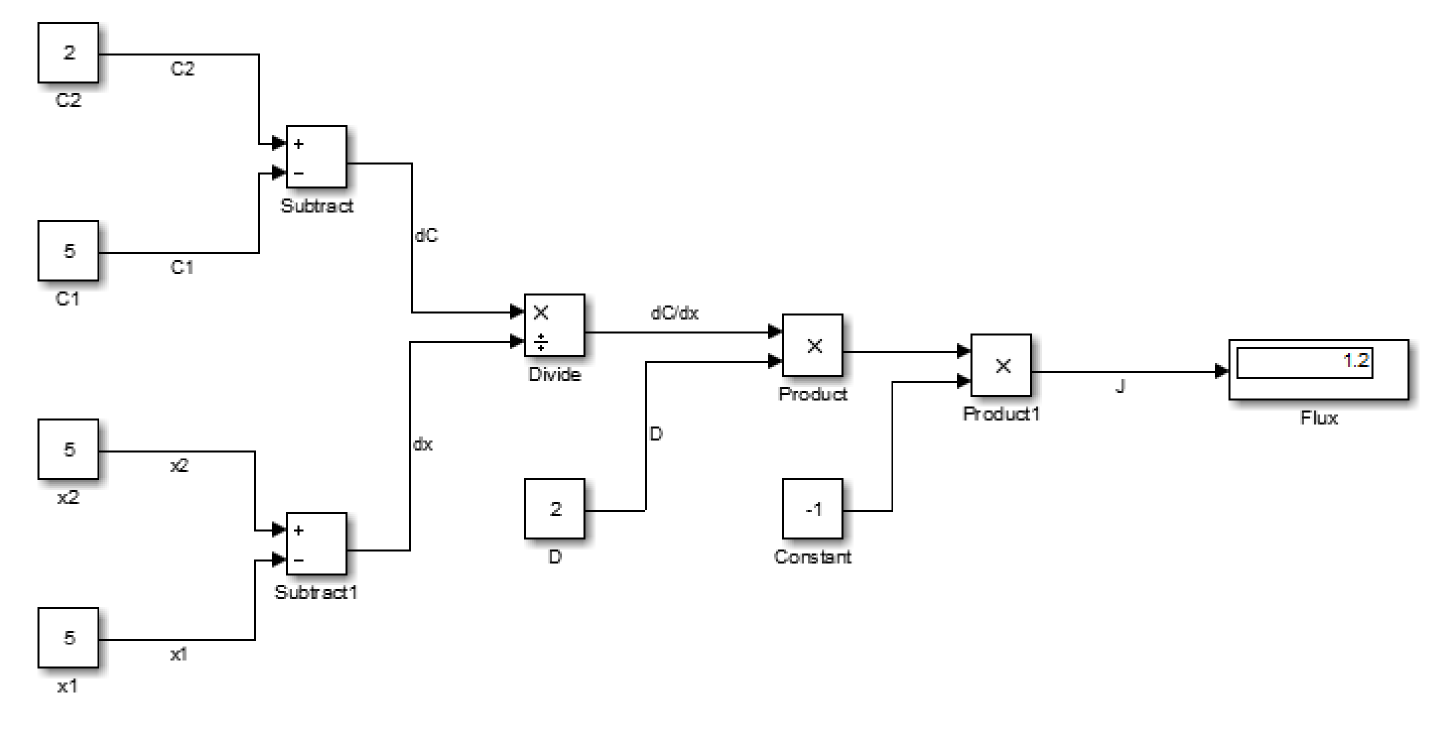
- For the simplest way to solve for flux, rewrite the equation so

$$J = -D\frac{C_2 - C_1}{x_2 - x_1}$$

  - This allows the block diagram to only consist of basic operations

DIFFUSION THROUGH SOLUTION

- Simulink model will look something like this



DIFFUSION THROUGH A MEMBRANE

- Diffusion through the cell membrane uses different models
  - Depends on type of solute
- Cell membrane is impermeable to ions and large uncharged polar molecules
  - These substances use channels or pores in the membrane
  - Use the Microporous Membrane model
  - Some of these pores or channels are not always open/active – must use Facilitated Diffusion model
- Hydrophobic and small uncharged polar molecules diffuse directly through the membrane
  - Use the Lipid Bilayer model

MICROPOROUS MEMBRANE MODEL

- For the microporous membrane model at steady state
  - Flux is proportional to the concentration gradient and pore size
  - Where n is the number of pores and a is the pore radius

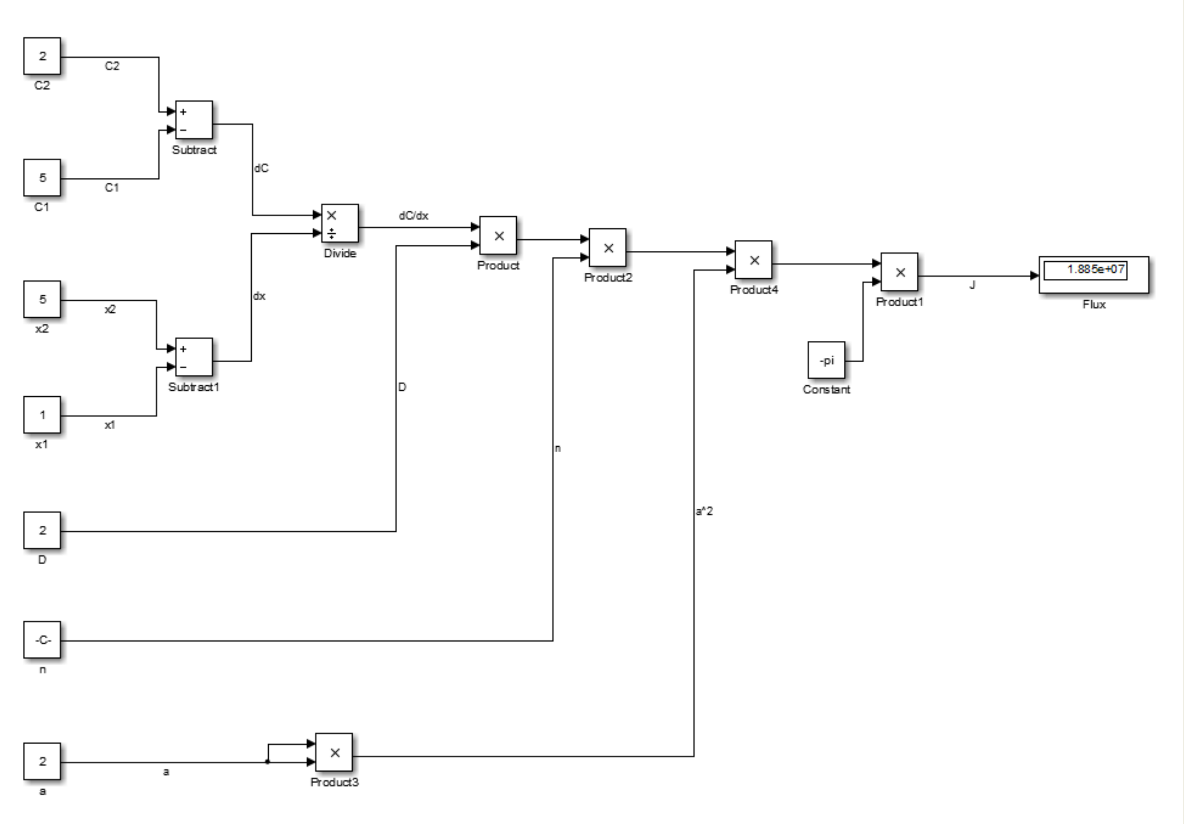
$$J_s = n\pi a^2 D \frac{\delta C}{\delta x}$$

  - Again, to make modeling simpler, the equation can be rewritten as

$$J_s = n\pi a^2 D \frac{C_2 - C_1}{x_2 - x_1}$$

MICROPOROUS MEMBRANE MODEL

- Now the Simulink model becomes a bit more complicated, but still only uses simple operations



LIPID BILAYER MEMBRANE MODEL

- For the lipid bilayer membrane model at steady state
  - p is the permeability coefficient
  - c'_{in} is the concentration just inside the membrane on the interior side
  - c_{in} is the concentration on the inside of the cell
  - D_m is the diffusion coefficient of the solute in the membrane
  - k is the partition coefficient
  - x is the membrane thickness

$$J = p\Delta C = p(c_{in} - c'_{in})$$

$$p = \frac{D_m k}{x}$$

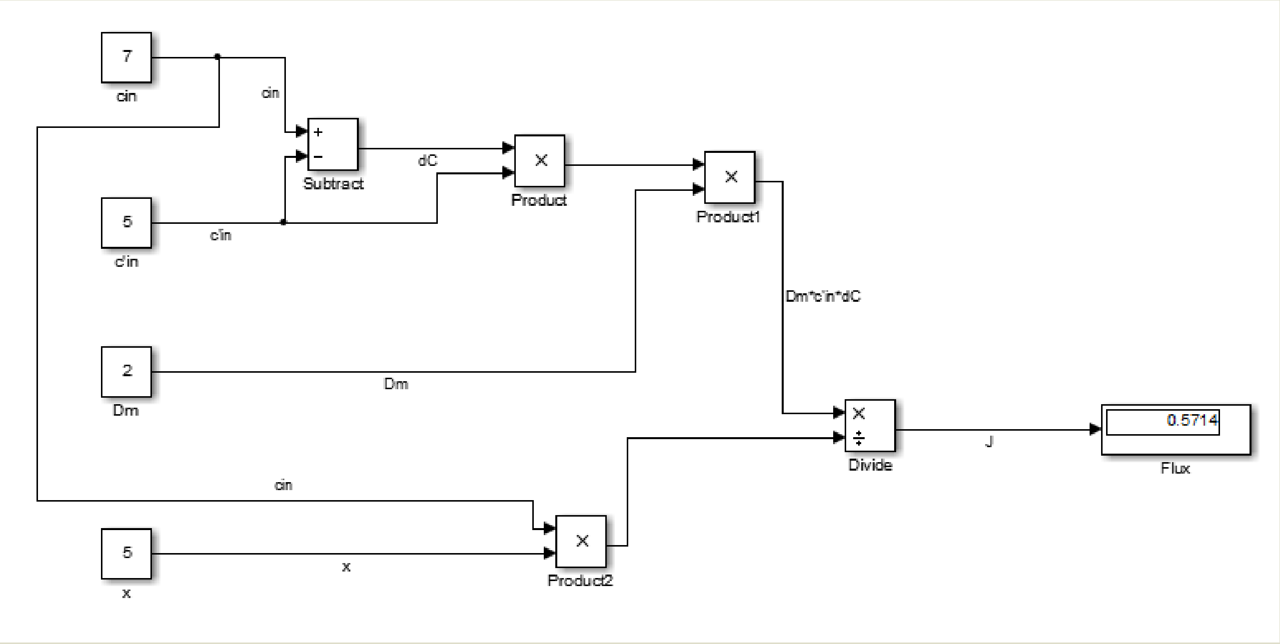
$$k = \frac{c'_{in}}{c_{in}}$$

LIPID BILAYER MEMBRANE MODEL

- So, expanded out, the new equation for flux through a lipid bilayer membrane is

$$J = \frac{D_m c'_{in}}{x c_{in}}(c_{in} - c'_{in})$$

- The Simulink model will be



CHANNEL DIFFUSION

CHANNEL DIFFUSION

- Includes facilitated diffusion and active transport
- Very similar, except active transport goes against the concentration gradient
  - Requires ATP hydrolysis
- Other than ATP usage, can be modeled very similarly

CHANNEL DIFFUSION

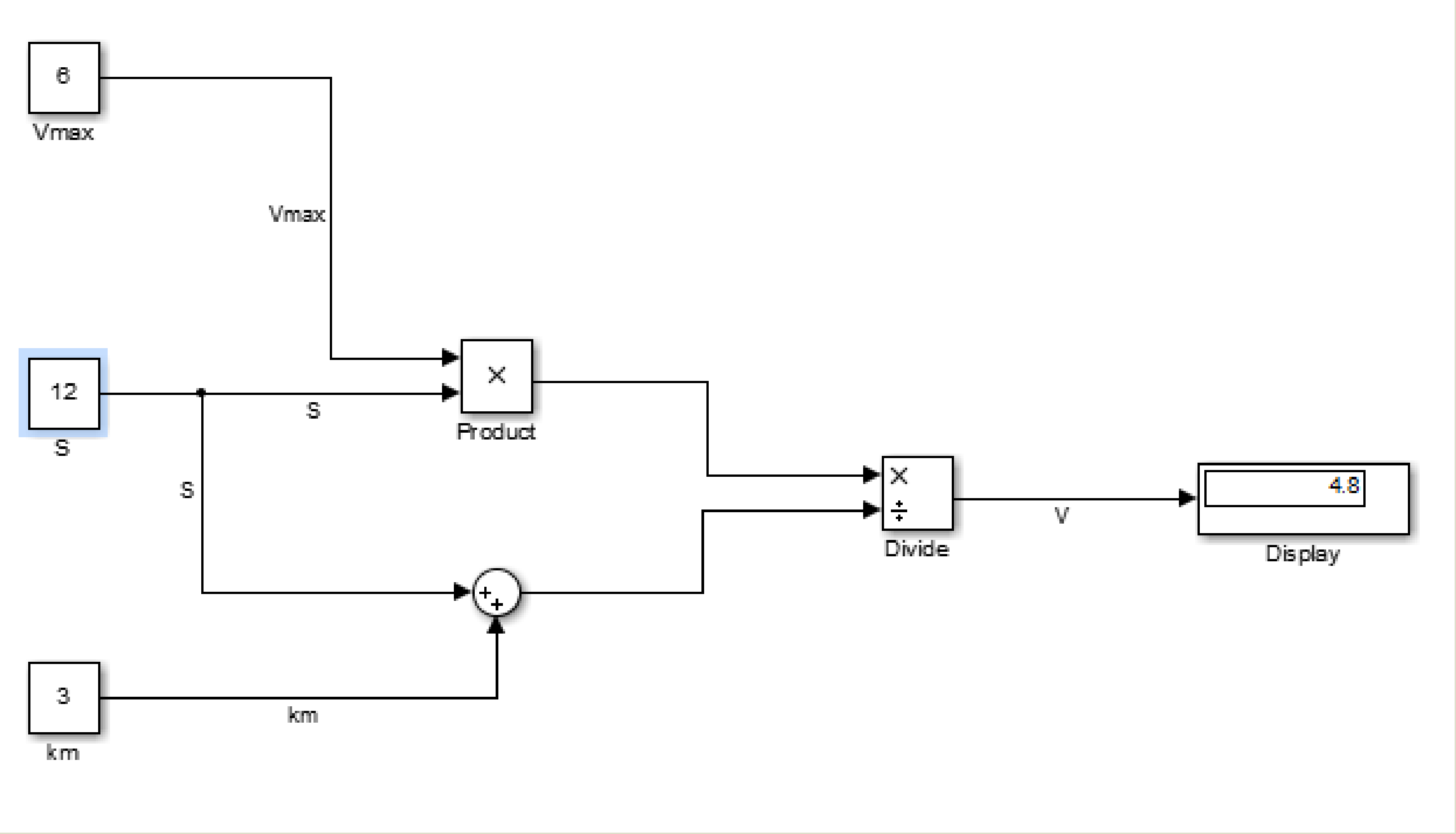
- Used when solutes require designated transport proteins
- Can be modeled using Henri-Michaelis-Menten Kinetics
  - Where V_{max} is the maximum rate, [S] is the concentration of solute, and k_m is the concentration of solute at ½V_{max}

$$V = \frac{V_{max}[S]}{k_m + [S]}$$

- Simplest modeling assumes all variables are known

CHANNEL DIFFUSION

- Simplest model



OSMOSIS

OSMOSIS

- Osmosis is the flow of water through a membrane in response to concentration differences or external forces
- Important concept – osmolarity
  - Measure of total solute concentration in solution
  - Ionic compounds will dissociate, increasing osmolarity
    - ex) 1M glucose → 1 osmolar solution
      1M NaCl → 2 osmolar solution
  - Osmolarity is used to calculate osmotic pressure, the driving force behind osmotic flux

OSMOTIC PRESSURE

- Osmotic pressure π is proportional to the osmolarity of the solution

$$\pi = RTC_{solute}$$

- Where R is the gas constant, T is the temperature in Kelvin, and C_{solute} is the osmolarity
- Difference in osmotic pressure across the membrane is the driving force of osmosis

$$\Delta\pi = RT\Delta C_{solute}$$

- However, if the membrane is semipermeable to solute, $\Delta\pi = \sigma RT\Delta C_{solute}$ where $\sigma = 1 - \frac{P_{solute}}{P_{water}}$ and P values are permeability

OSMOTIC FLUX

- Osmotic flux is proportional to the osmotic pressure and external applied force
  - Where L_p is hydraulic permability, $\Delta\pi$ is osmotic pressure difference, and ΔP is external pressure difference (essentially $\Delta F/A$)

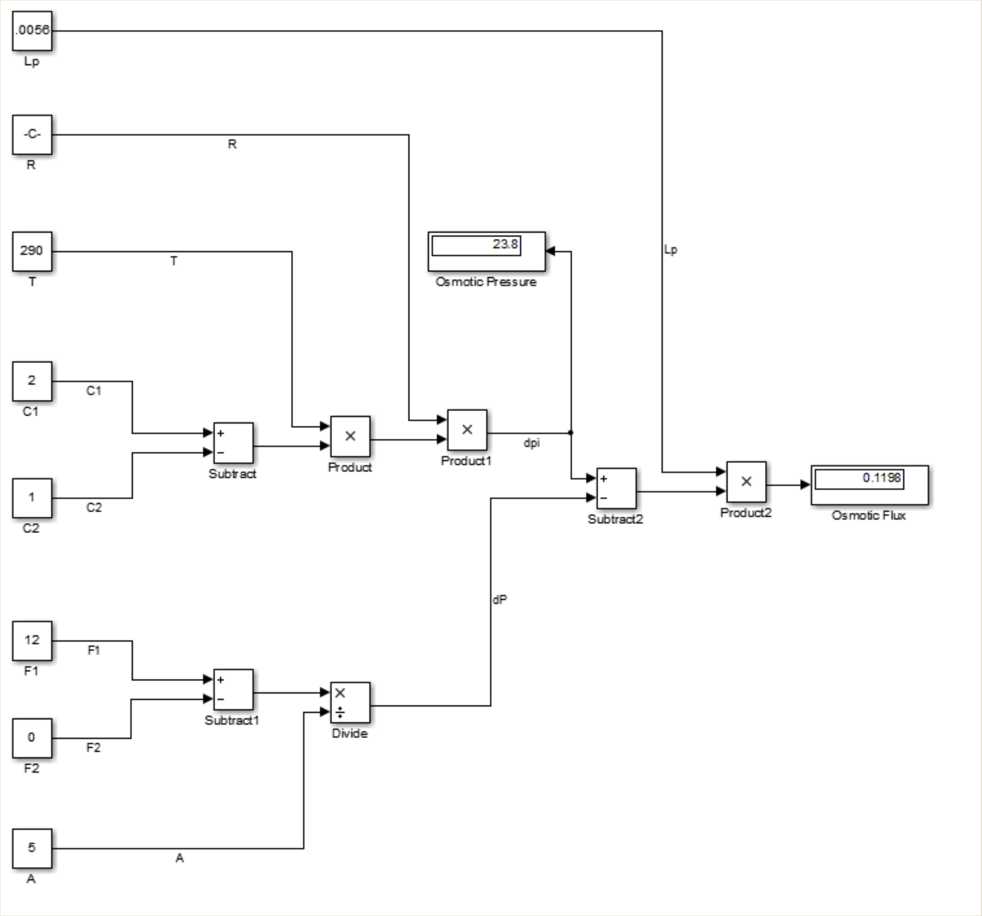
$$J = L_p(\Delta\pi - \Delta P)$$

- Expanded form of osmotic flux (A = membrane surface area)

$$J = L_p(RT\Delta C_{solute} - \frac{\Delta F}{A})$$

OSMOTIC FLUX

- Osmotic Flux model



ELECTROCHEMICAL POTENTIAL

ELECTROCHEMICAL POTENTIAL

- Potential of solution due to electrical and chemical components
  - Electrical due to ions – *z* is ion charge, *F* is Faraday's constant, *Φ* is the electrical potential
  - Chemical due to solute concentration gradients
  - Has a baseline component μ_0

$$\mu = \mu_0 + RT \ln C + zF\varphi$$

- Derivative is the electrochemical force

TRANSMEMBRANE POTENTIAL

- Measure of the electrical potential for a specific ion across a membrane
- Derived from the electrochemical potential difference across the membrane
- Transmembrane potentials calculated at equilibrium for each species
  - ΔG = 0 → Δμ = 0 → $-RT \ln \Delta C + zF\Delta\varphi = 0$
  - V is ΔΦ, the electrical potential (voltage)

$$V = \frac{RT}{zF} \ln \frac{C_{out}}{C_{in}}$$

TRANSMEMBRANE POTENTIAL

- Simple model