Quick Sort Algorithm

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Outline:

- What is Quick Sort in Data Structure?
- When to Use Quick Sort?
- Explanation of Quick Sort Algorithm
- Example of Quick Sort
- Time Complexity of Quick Sort
- Advantages and Disadvantages of Quick Sort
- Applications of Quick Sort

What is Quick Sort in Data Structure?

- Quick Sort is a sorting algorithm based on the Divide and Conquer approach that picks an element as a pivot and partitions the given array around the picked pivot by placing the pivot in its correct position in the sorted array.

What is Divide and Conquer approach?

This technique can be divided into the following 3 parts:

- **Divide:** This involves dividing the problem into smaller sub-problems.
- **Conquer:** Solve sub-problems by calling recursively until solved.
- **Combine:** Combine the sub-problems to get the final solution of the whole problem.

Here is the three-steps divide-and-conquer process for sorting a typical array A[p….r].

Divide: Partition (rearrange) the array A[p.....r] into two (possibly empty) subarrays A[p.....q - 1] and A[q+1.....r] such that each element of A[p.....q - 1] is less than to A[q], and each element of A[q+1.....r] is greater than or equal to A[q]

Compute the index q as part of this partitioning procedure.

Conquer: Sort the two subarrays A[p....q - 1] and A[q+1.....r] by recursive calls to quicksort.

Combine: Since the subarrays are sorted in place, now work is needed to combine them

So we can define that the entire array A[p....r] is now sorted.

Quick Sort Algorithm

```
QUICKSORT(array A, int p, int r)
1  if (p < r)
2      then q ← PARTITION(A, p, r)
3           QUICKSORT(A, p, q − 1)
4           QUICKSORT(A, q + 1, r)
```

To sort array call QUICKSORT($A, 1, length[A]$).

Quick Sort Partition Algorithm

```
PARTITION(array A, int p, int r)
1  x ← A[r]                                  ▷ Choose pivot
2  i ← p − 1
3  for j ← p to r − 1
4        do if (A[j] ≤ x)
5              then i ← i + 1
6                   exchange A[i] ↔ A[j]
7  exchange A[i + 1] ↔ A[r]
8  return i + 1
```

Example : A=[2, 8, 7, 1, 3, 5, 6, 4]

- Here the value of p=1 and r=8
- As per the Partition Algorithm the value of x=4 (As A[r]=A[8]=4)
- The value of i=0 (i.e p-1= 1-1=0)
- As per step no. 3 for loop will start from (p=1) and it will continue up to (r-1=8-1=7).

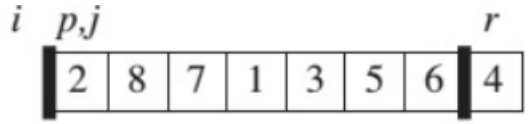
```
PARTITION(array A, int p, int r)
1  x ← A[r]                          ▷ Choose pivot
2  i ← p − 1
3  for j ← p to r − 1
4        do if (A[j] ≤ x)
5              then i ← i + 1
6                   exchange A[i] ↔ A[j]
7  exchange A[i + 1] ↔ A[r]
8  return i + 1
```

Process for j=1

- **A=[2, 8, 7, 1, 3, 5, 6, 4]**
- Here the value of

  x= 4, r=8, i=0, j=1, A[1]=2

  do if (2≤ 4) Ture
- Then the value of iwill be 1 (i. e i=1) and we have to swap :A[1] ↔ A[1]
- Output :



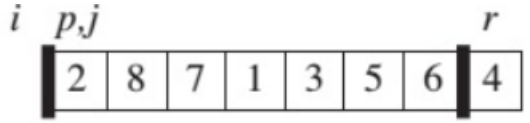

```
PARTITION(array A, int p, int r)
1  x ← A[r]                          ▷ Choose pivot
2  i ← p − 1
3  for j ← p to r − 1
4        do if (A[j] ≤ x)
5              then i ← i + 1
6                   exchange A[i] ↔ A[j]
7  exchange A[i + 1] ↔ A[r]
8  return i + 1
```

Process for j=2

- **A=[2, 8, 7, 1, 3, 5, 6, 4]**
- Here the value of

  x= 4, r=8, i=1, j=2, A[2]=8

  do if (8≤ 4) False
- Then we will skip the steps 5 and 6 and the output after processing j=2 will be define as
- Output :



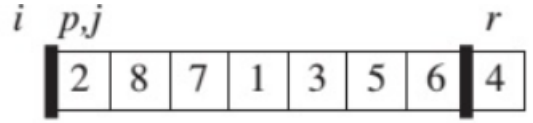

```
PARTITION(array A, int p, int r)
1  x ← A[r]                          ▷ Choose pivot
2  i ← p − 1
3  for j ← p to r − 1
4        do if (A[j] ≤ x)
5              then i ← i + 1
6                   exchange A[i] ↔ A[j]
7  exchange A[i + 1] ↔ A[r]
8  return i + 1
```

Process for j=3

- **A=[2, 8, 7, 1, 3, 5, 6, 4]**
- Here the value of

  x= 4, r=8, i=1, j=3, A[3]=7

  do if (7≤ 4) False
- Then we will skip the steps 5 and 6 and the output after processing j=3 will be define as
- Output :



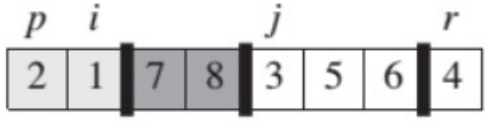

```
PARTITION(array A, int p, int r)
1  x ← A[r]                          ▷ Choose pivot
2  i ← p − 1
3  for j ← p to r − 1
4        do if (A[j] ≤ x)
5              then i ← i + 1
6                   exchange A[i] ↔ A[j]
7  exchange A[i + 1] ↔ A[r]
8  return i + 1
```

Process for j=4

- **A=[2, 8, 7, 1, 3, 5, 6, 4]**
- Here the value of

  x= 4, r=8, i=1, j=4, A[4]=1

  do if (1≤ 4) True
- Then the value of i will be 2 (i. e i=2) and we have to swap :A[2] ↔ A[4]
- Output :




```
PARTITION(array A, int p, int r)
1  x ← A[r]                          ▷ Choose pivot
2  i ← p − 1
3  for j ← p to r − 1
4        do if (A[j] ≤ x)
5              then i ← i + 1
6                   exchange A[i] ↔ A[j]
7  exchange A[i + 1] ↔ A[r]
8  return i + 1
```

Process for j=5

- **A=[2, 1, 7, 8, 3, 5, 6, 4]**
- Here the value of

  x= 4, r=8, i=2, j=5, A[5]=3

  do if (3≤ 4) True
- Then the value of i will be 3 (i. e i =3) and we have to swap :A[3] ↔ A[5]
- Output :


```
PARTITION(array A, int p, int r)
1  x ← A[r]                          ▷ Choose pivot
2  i ← p − 1
3  for j ← p to r − 1
4        do if (A[j] ≤ x)
5              then i ← i + 1
6                   exchange A[i] ↔ A[j]
7  exchange A[i + 1] ↔ A[r]
8  return i + 1
```

Process for j=6

- **A=[2, 1, 3, 8, 7, 5, 6, 4]**
- Here the value of

  x= 4, r=8, i=3, j=6, A[6]=5

  do if (5≤ 4) False
- Then we will skip the steps 5 and 6 and the output after processing j=6 will be define as
- Output


```
PARTITION(array A, int p, int r)
1  x ← A[r]                          ▷ Choose pivot
2  i ← p − 1
3  for j ← p to r − 1
4        do if (A[j] ≤ x)
5              then i ← i + 1
6                   exchange A[i] ↔ A[j]
7  exchange A[i + 1] ↔ A[r]
8  return i + 1
```

Process for j=7

- **A=[2, 1, 3, 8, 7, 5, 6, 4]**
- Here the value of

  x= 4, r=8, i=3, j=7, A[7]=6

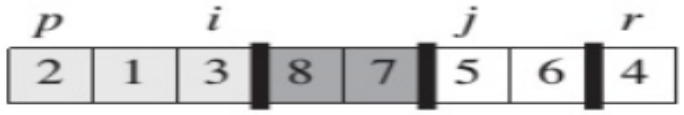
  do if ($6 \leq 4$) False
- Then we will skip the steps 5 and 6 and the output after processing j=7 will be define as
- Output


```
PARTITION(array A, int p, int r)
1  x ← A[r]                          ▷ Choose pivot
2  i ← p − 1
3  for j ← p to r − 1
4        do if (A[j] ≤ x)
5              then i ← i + 1
6                   exchange A[i] ↔ A[j]
7  exchange A[i + 1] ↔ A[r]
8  return i + 1
```

Process Step 7 and 8

- **A=[2, 1, 3, 8, 7, 5, 6, 4]**

or



Here the value of

x= 4, r=8, i=3, j=7, A[7]=6

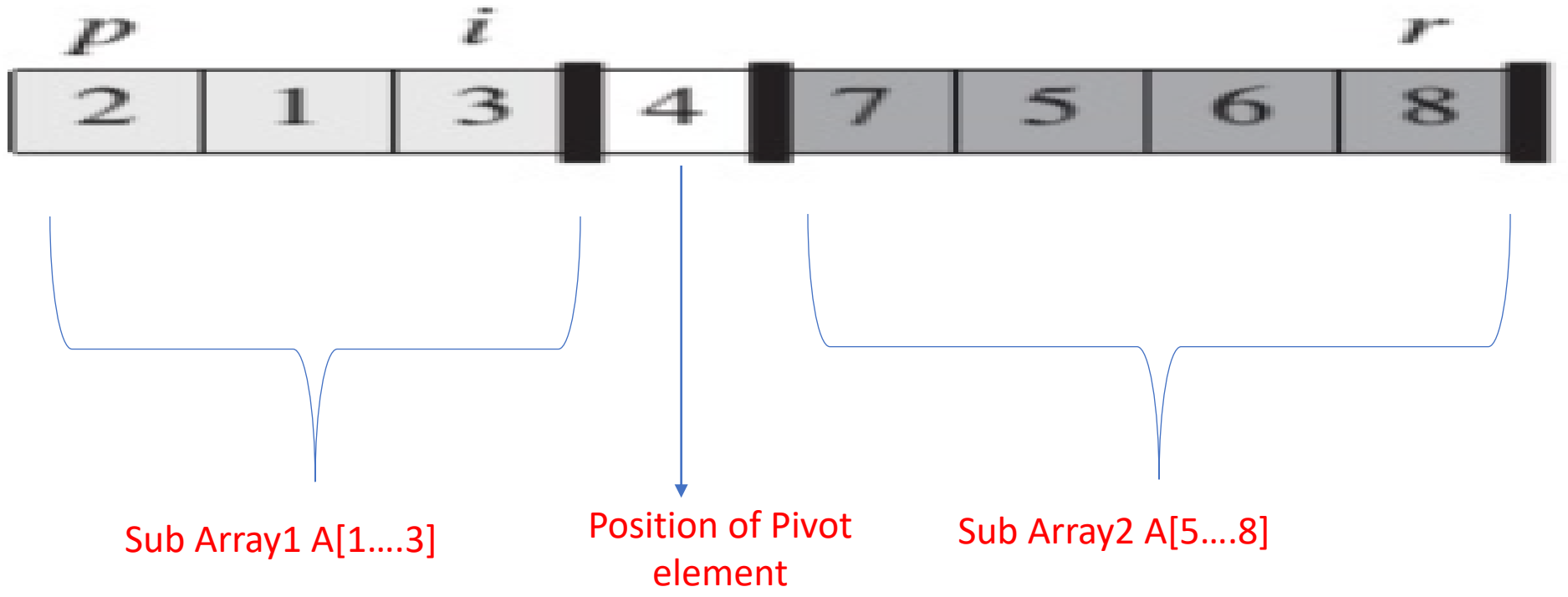
Now using step 7 we have to exchange A[i+1] ↔ A[r] i.e A[4] ↔ A[8]

Output:

Step 8 will return value 4

```
PARTITION(array A, int p, int r)
1  x ← A[r]                          ▷ Choose pivot
2  i ← p − 1
3  for j ← p to r − 1
4        do if (A[j] ≤ x)
5              then i ← i + 1
6                   exchange A[i] ↔ A[j]
7  exchange A[i + 1] ↔ A[r]
8  return i + 1
```

- **Using Quick Sort Partition algorithm return the value 4 and that value will be assign to variable q.**
- **Now we divide the array in two sub arrays A[1…3] and A[5….8] and the position of the Pivot element is 4.**



- **After getting the value of q we will call Quicksort(A, p, q-1) and Quicksort(A, q+1, r)**

- **This process will continue until we satisfy the condition.**

```
QUICKSORT(array A, int p, int r)
1  if (p < r)
2      then q ← PARTITION(A, p, r)
3           QUICKSORT(A, p, q − 1)
4           QUICKSORT(A, q + 1, r)
```

To sort array call QUICKSORT(A, 1, length[A]).

Best-case running time

- **In the most even possible split, PARTITION produces two subproblems, each of size no more than (n/2), since one is of size ⌊n/2⌋ and one of size ⌊(n/2)-1⌋.In this case, quicksort runs much faster.**

- **The recurrence for the running time is then:**

  $$T(n) = 2T(n/2) + (n)$$

Here T(n) =2T(n/2) + (n)

So we can solve the above recurrence equation using Master Method

Master Method Standard equation is T(n) =aT(n/b) + f(n)

If we compare the two equations we will obtained the value of a=2, b=2 and f(n)=n

Now we have to calculate $n^{\log_b a}$

=> $n^{\log_2 2}$

=> n

If we compare the value of f(n) and $n^{\log_b a}$ both values are equal.

So, T(n) =0(f(n).logn)

=>T(n)=0(nlogn)

Best Case running time of Quick sort is 0(nlogn)

Worst-case running time

- **The efficiency of the Quicksort algorithm very much depends on the selection of the pivot element.**

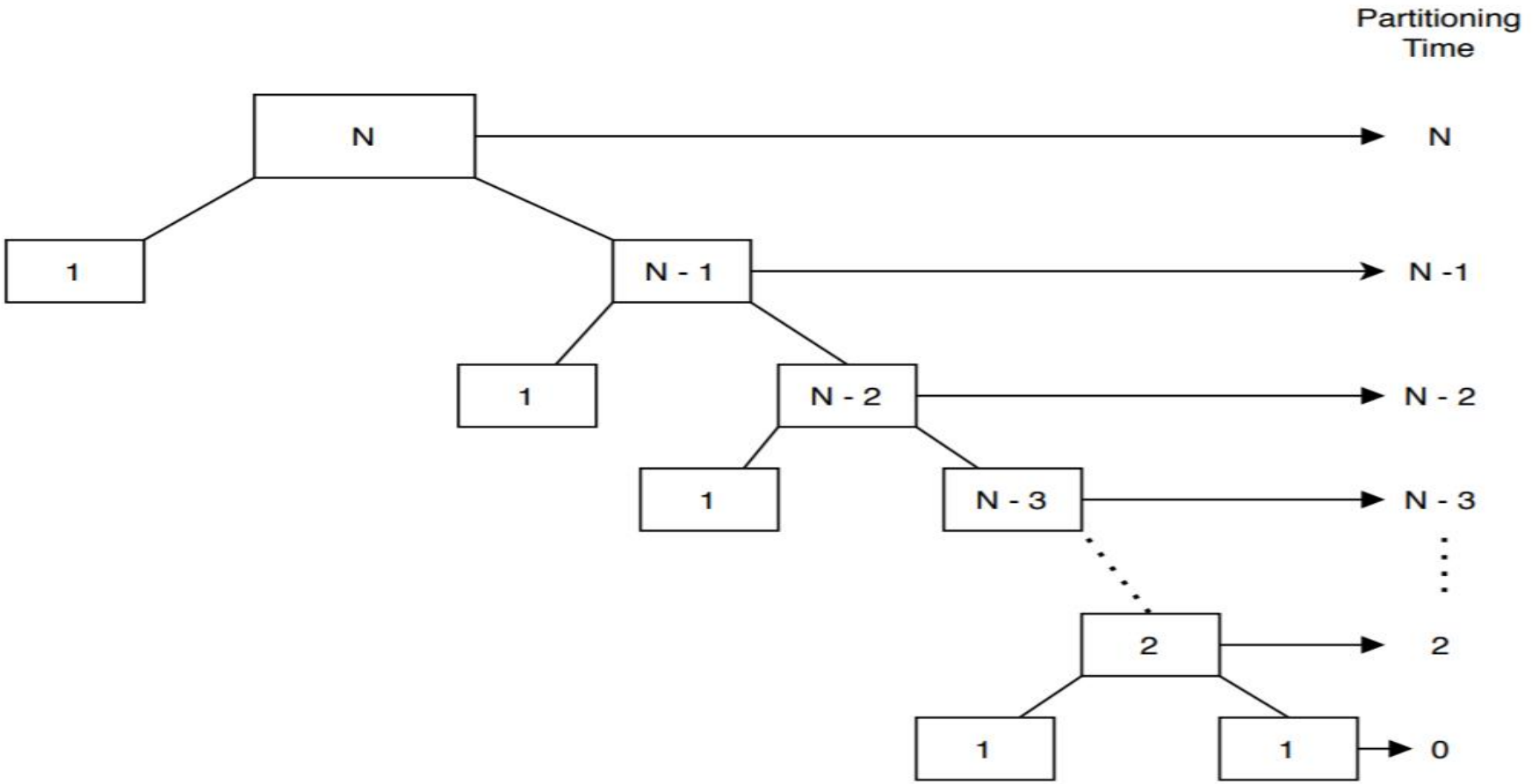
- **Let's assume the input of the Quicksort is a sorted array and we choose the leftmost element as a pivot element.**

- **In this case, we'll have two extremely unbalanced arrays. One array will have one element and the other one will have (N - 1) elements.**

Worst-case running time

- **Similarly, when the given input array is sorted reversely and we choose the rightmost element as the pivot element, the worst case occurs. Again, in this case, the pivot elements will split the input array into two unbalanced arrays.**

Worst Case Time Complexity Analysis



Worst Case Time Complexity Analysis

- **All the numbers in the box denote the size of the arrays or the subarrays.**

- **Let's consider an input array of size N. The first partition call takes N times to perform the partition step on the input array.**

- **Each partition step is invoked recursively from the previous one. Given that, we can take the complexity of each partition call and sum them up to get our total complexity of the Quicksort algorithm**

Therefore, the time complexity of the Quicksort algorithm in worst case is

$$[N + (N-1) + (N-2) + (N-3) + ... + 2] = [\frac{N(N+1)}{2} - 1] = \mathcal{O}(N^2)$$

Or

Alternatively, we can create a recurrence relation for computing it.

- **In the worst case, after the first partition, one array will have 1 element and the other one will have (N-1) elements.**

- **Let's say T(N) denotes the time complexity to sort N elements in the worst case:**

- **T(N) = Time needed to partition N elements + Time needed to sort (N - 1) elements recursively = N + T(N - 1)**

- **Again for the base case when N = 0 and 1, we don't need to sort anything. Hence, the sorting time is 0 and T(0)= T(1) = 0**

Now, we're ready to solve the recurrence relation

T(N) = N + T(N- 1),
T(N - 1) = (N - 1) + T(N- 2),
T(N - 2) = (N - 2) + T(N- 3),
T(N - 3) = (N - 3) + T(N- 4),
……
……
……
T(3) = 3 + T(2),
T(2) = 2 + T(1),
T(1) = 0

As a result, T(N) = N + (N-1) + (N-2) ... + 3 + 2

$$= \left[\frac{N(N+1)}{2} - 1\right] = O(N^2)$$

Advantages of Quick Sort

- **Fast and efficient :** Quick sort is the most favored users' choice to perform sorting functions quickly and efficiently. It allows users to achieve the same result much quicker than other sorting algorithms

- **Cache friendly :**The most talked-about feature of quicksort is its in-place sorting. This means, at the time of sorting, the algorithm doesn't need additional storage space. Thus, the sorted list occupies the same storage as the unsorted list, and the sorting takes place in the given area

Advantages of Quick Sort

- **Good for parallelization:** Quick sort can be parallelized easily, taking advantage of multiple processors or computing resources. By splitting the input array into sub-arrays and sorting them concurrently, it can achieve faster sorting times on systems with parallel processing capabilities.

- **Simple implementation:** Quick sort's algorithmic logic is relatively straightforward, making it easier to understand and implement compared to some other complex sorting algorithms

Advantages of Quick Sort

- **Time complexity:** Time complexity is the time taken by the algorithm to run until its completion. Compared to other sorting algorithms, it can be said that the time complexity of quick sort is the best

- **Space complexity:** Quick sort has a space complexity of O(logn), making it a suitable algorithm when you have restrictions in space availability

Disadvantages of Quick Sort

- **Unstable:** Quick sort is undoubtedly a fast and efficient sorting algorithm, but when it comes to stability, you might want to reconsider your options. This sorting algorithm is regarded unstable as it does not retain the original order of the key-value pair.

- **Worst-case time complexity:** This is actually a drawback of quick sort. It usually occurs when the element selected as the pivot is the largest or smallest element, or when all the elements are the same. These worst cases drastically affect the performance of the quick sort.

Disadvantages of Quick Sort

- **Dependency on Pivot Selection:** The choice of pivot significantly affects the performance of quick sort. If a poorly chosen pivot divides the array into highly imbalanced partitions, the algorithm's efficiency may degrade. This issue is particularly prominent when dealing with already sorted or nearly sorted arrays.

- **Recursive Overhead:** Quick sort heavily relies on recursion to divide the array into subarrays. Recursive function calls consume additional memory and can lead to stack overflow errors when dealing with large data sets. This makes quick sort less suitable for sorting extremely large arrays.

Disadvantages of Quick Sort

- **Inefficient for Small Data Sets:** Quick sort has additional overhead in comparison to some simpler sorting algorithms, especially when dealing with small data sets. The recursive nature of quick sort and the constant splitting of the array can be inefficient for sorting small arrays or arrays with a small number of unique elements.

- **Not Adaptive:** Quick sort's performance is not adaptive to the initial order of the elements. It does not take advantage of any pre-existing order or partially sorted input. This means that even if a portion of the array is already sorted, the quick sort will still perform partitioning operations on the entire array.

Applications of Quick Sort

- **Commercial Computing is used in various government and private organizations for the purpose of sorting various data like sorting files by name/date/price, sorting of students by their roll no., sorting of account profile by given id, etc.**

- **The sorting algorithm is used for information searching and as Quicksort is the fastest algorithm so it is widely used as a better way of searching.**

- **It is used everywhere where a stable sort is not needed.**

Applications of Quick Sort

- **Quicksort is a cache-friendly algorithm as it has a good locality of reference when used for arrays.**
- **It is tail -recursive and hence all the call optimization can be done.**
- **It is used to implement primitive type methods.**
- **It is used in operational research and event-driven simulation.**

Conclusion:

Overall, quick sort is a powerful data structure algorithm with advantages such as efficiency and versatility. It is an essential tool for developers to have in their belt when dealing with large amounts of data. Quicksort is an efficient, unstable sorting algorithm with time complexity of O(n log n) in the best and average case and $O(n^2)$ in the worst case.


Thank you!