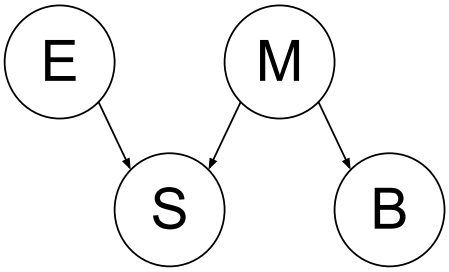
2 . December 21, 2012

A smell of sulphur (S) can be caused either by rotten eggs (E) or as a sign of the doom brought by the Mayan Apocalypse (M). The Mayan Apocalypse also causes the oceans to boil (B). The Bayesian network and corresponding conditional probability tables for this situation are shown below. For each part, you should give either a numerical answer (e.g. 0.81) or an arithmetic expression in terms of numbers from the tables below (e.g. $0.9 \cdot 0.9$).

Note: be careful of doing unnecessary computation here.

$P(E)$	
$+e$	0.4
$-e$	0.6

$P(S\|E,M)$			
$+e$	$+m$	$+s$	1.0
$+e$	$+m$	$-s$	0.0
$+e$	$-m$	$+s$	0.8
$+e$	$-m$	$-s$	0.2
$-e$	$+m$	$+s$	0.3
$-e$	$+m$	$-s$	0.7
$-e$	$-m$	$+s$	0.1
$-e$	$-m$	$-s$	0.9



$P(M)$	
$+m$	0.1
$-m$	0.9

$P(B\|M)$		
$+m$	$+b$	1.0
$+m$	$-b$	0.0
$-m$	$+b$	0.1
$-m$	$-b$	0.9

(a) Compute the following entry from the joint distribution:

$P(-e,-s,-m,-b) =$ $P(-e)P(-m)P(-s|-e,-m)P(-b|-m) = (0.6)(0.9)(0.9)(0.9) = 0.4374$
by expanding the joint according to the chain rule of conditional probability.

(b) What is the probability that the oceans boil?

$P(+b) =$ $P(+b|+m)P(+m) + P(+b|-m)P(-m) = (1.0)(0.1) + (0.1)(0.9) = 0.19$
by marginalizing out m according to the law of total probability.

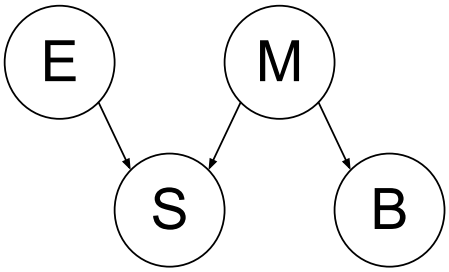
(c) What is the probability that the Mayan Apocalypse is occurring, given that the oceans are boiling?

$P(+m|+b) =$ $\frac{P(+b|+m)P(+m)}{P(+b)} = \frac{(1.0)(0.1)}{0.19} \approx .5263$
by the definition of conditional probability.

The figures and table below are identical to the ones on the previous page and are repeated here for your convenience.

$P(E)$	
$+e$	0.4
$-e$	0.6

$P(S\|E,M)$			
$+e$	$+m$	$+s$	1.0
$+e$	$+m$	$-s$	0.0
$+e$	$-m$	$+s$	0.8
$+e$	$-m$	$-s$	0.2
$-e$	$+m$	$+s$	0.3
$-e$	$+m$	$-s$	0.7
$-e$	$-m$	$+s$	0.1
$-e$	$-m$	$-s$	0.9



$P(M)$	
$+m$	0.1
$-m$	0.9

$P(B\|M)$		
$+m$	$+b$	1.0
$+m$	$-b$	0.0
$-m$	$+b$	0.1
$-m$	$-b$	0.9

(d) What is the probability that the Mayan Apocalypse is occurring, given that there is a smell of sulphur, the oceans are boiling, and there are rotten eggs?

$P(+m|+s,+b,+e) =$

$$\frac{P(+m,+s,+b,+e)}{\sum_m P(m,+s,+b,+e)} = \frac{P(+e)P(+m)P(+s|+e,+m)P(+b|+m))}{\sum_m P(+e)P(m)P(+s|+e,m)P(+b|m)}$$
$$= \frac{(0.4)(0.1)(1.0)(1.0)}{(0.4)(0.1)(1.0)(1.0)+(0.4)(0.9)(0.8)(0.1)}$$
$$= \frac{0.04}{0.04+0.0288} \approx .5814$$

(e) What is the probability that rotten eggs are present, given that the Mayan Apocalypse is occurring?

$P(+e|+m) =$ $P(+e) = 0.4$

The first equality holds true as we have $E \perp\!\!\!\perp M$ (E is independent of M), which can be inferred from the graph of the Bayes' net.