# Statistics — Moments of a distribution

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> *Moments in statistics are popularly used to describe the characteristic of a distribution.*

1 Moment: Measure of central location

2 Moment: Measure of dispersion

3 Moment: Measure of asymmetry

4 Moment: Measure of peakedness

> ***First moment- Mean***

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Measure the location of the central point.
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$$\overline{x} = \frac{\sum_{i=1}^{N} x_i}{N}$$

> ***Second moment- Standard Deviation (SD, σ(Sigma)):***

Measure the spread of values in the distribution OR how far from the normal.

$$\sigma^2 = \frac{\sum_{i=1}^{N}(X_i - \bar{X})^2}{N}$$

σ = (Variance)^.5

**Small SD** : Numbers are close to mean
**High SD** : Numbers are spread out

**For normal distribution:**
Within 1 SD: 68.27% values lie
Within 2 SD: 95.45% values lie
Within 3 SD: 99.73% values lie

**Advantages over Mean Absolute Deviation(MAD):**
1. Mathematical properties- Continuous, differentiable.
2. SD of a sample is more consistent estimate for a population- When drawing repeated samples from a normally distributed population, the standard deviations of samples are less spread out as compare to mean absolute deviations.

## *Third moment- Skewness*

Measure the symmetry in the distribution.

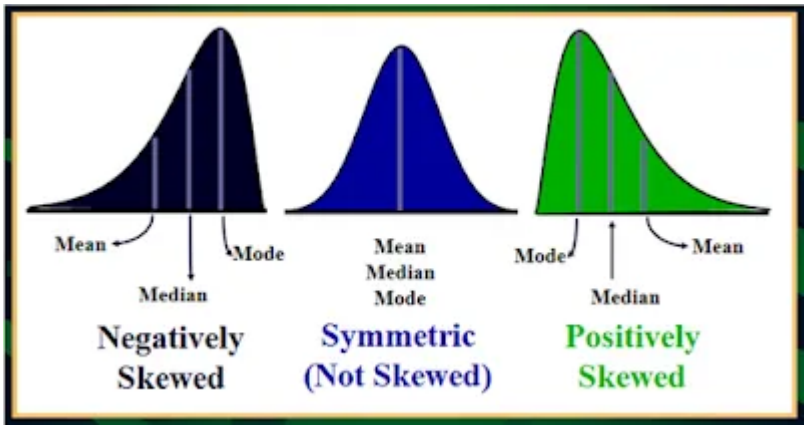
$$Skew = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{(X_i - \bar{X})}{\sigma}\right]^3$$

Skewness=0 **[Normal Distribution, Symmetric]**

**Other Formulas:**
1. Skewness = (Mean-Mode)/SD
2. Skewness = 3*(Mean-Median)/SD
(Mode = 3*Median-2*Mean)

**Transformations** (to make the distribution normal)**:**
a. Positively skewed (right): Square root, log, inverse

b. Negatively skewed (left) : Reflect and square[sqrt(constant-x)], reflect and log, reflect and inverse




## *Fourth moment- Kurtosis:*

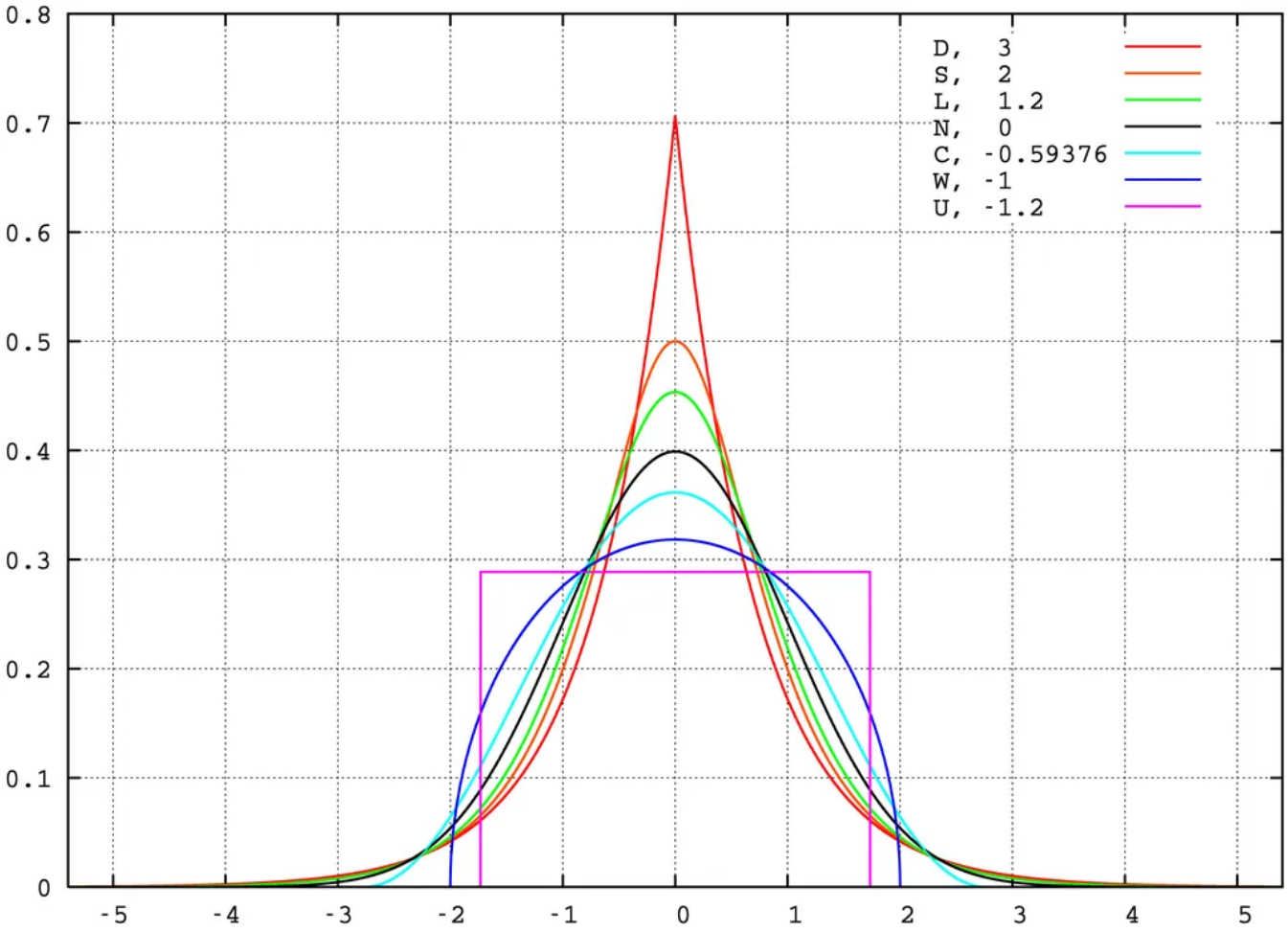
Measure the amount in the tails.

$$Kurt = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{(X_i - \bar{X})}{\sigma}\right]^4$$

Kurtosis=3 **[Normal Distribution]**
Kurtosis<3 [Lighter tails]
Kurtosis>3 [Heavier tails]

**Other Formulas:**
*Excess Kurtosis = Kurtosis - 3*

**Understanding:**
Kurtosis is the average of the standardized data raised to fourth power. Any standardized values less than |1| (i.e. data within one standard deviation of the mean) will contribute petty to kurtosis. The standardized values that will contribute immensely are the outliers.
High Kurtosis alerts about attendance of outliers.




**Excess Kurtosis** for Distributions [Laplace (D)ouble exponential; Hyperbolic (S)ecant; (L)ogistic; (N)ormal; ©osine; (W)igner semicircle; (U)niform]

**References:**


*SD and variance:* *https://www.mathsisfun.com/data/standard-deviation.html*

*Advantages of the mean deviation:*
*http://www.leeds.ac.uk/educol/documents/00003759.htm*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