Efficient Approximations for Cache-Conscious Data Placement

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  - Otherwise, we have a ***cache miss*** and o has to be copied to the cache (possibly by evicting another item)
- Given a sequence Σ of accesses, our goal is to *minimize cache misses* over Σ

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- We analyze the runtime based on n and N and assume that t and k are small constants

Hardness of CDP

Theorem (Petrank and Rawitz, POPL 2002)

For a cache with more than two lines, CDP is not only NP-hard, but also hard to approximate within any non-trivial factor $O(N^{1-\epsilon})$ *unless P=NP*

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- This is a very strong hardness result and it holds for the simple cache structure in CDP
- All following works are heuristics with no guarantees

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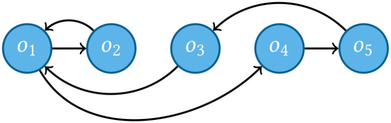
- Experimental Results
  - Not practical
    - Only applicable to small caches with a handful of lines
    - Due to exponential dependence on k
  - On these small caches, our approach beats the heuristics in 84-88% of the cases
  - Not covered in the talk (Please see the paper)

Access Graphs



$$\Sigma = \langle o_1, o_2, o_1, o_4, o_5, o_3, o_3, o_1, o_2 \rangle$$

Higher-order Access Hypergraphs

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Approximation Theorem

Theorem

For any $\epsilon > 0$, by applying the approach above using the sparsified access hypergraph $\tilde{G}_{d_\epsilon}$ of order $d_\epsilon := \lceil t \cdot k + \frac{t \cdot k}{\epsilon} \rceil$, we obtain a $(1+\epsilon)-$approximation of the optimal number of cache misses in a direct-mapped cache, i.e. $\mathit{Misses}_k(\hat{f}, \Sigma) \leq (1+\epsilon) \cdot \mathit{Misses}_k(f^*, \Sigma)$.

Graph Coloring

- Every edge in our hypergraph starts and ends with the same vertex

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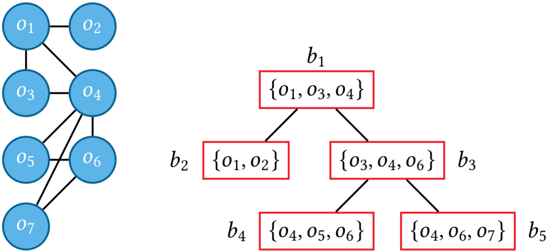
- NP-hard Problem: Find a coloring that minimizes missed edges.

Treewidth-based Dynamic Programming



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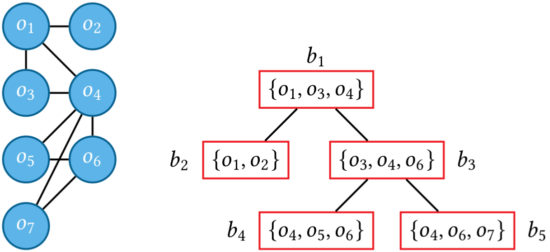

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- Do a linear-time bottom-up dynamic programming as if you are coloring a tree

$$\texttt{dp}[b_i, \text{partial coloring } c] =$$

$$\text{minimum number of missed edges in the subtree of } b_i$$

$$\text{if we color the vertices in } b_i \text{ according to } c$$

Conclusion

- CDP is really hard, even when the sequence of accesses is given apriori
- It is not as hard as previously thought since real-world instances are sparse
- The sparsity (tree-likeness) can be exploited to obtain $(1+\epsilon)$-approximations for any $\epsilon > 0$
- CDP requires both approximation and parameterization (by the treewidth of access graphs) to become tractable
- We provided the first positive theoretical result for CDP but there is still a long way until it becomes practical
- The current algorithm can be used for limit studies and comparison of heuristics