Big Data for Public Policy
Causal Inference [Apr 29]

ETH Zürich | 860-0033-00L

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Outline

Prologue: Learning Objectives

1. Implement and evaluate machine learning pipelines.
2. **Implement and evaluate causal inference designs.**
   - Evaluate (find problems in) causal claims.
   - Apply the standard research designs to produce causal evidence for a given empirical setting – or articulate why it is not possible.
   - Implement these research designs using Stata regressions.
3. Limitation of ML: Understand how (not) to use data science tools.

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- Google/Facebook understand this with A/B testing; social scientists want to use this to assist public policy.

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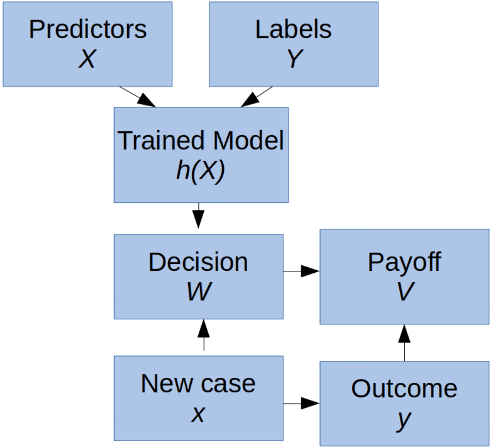
- There isn't a machine learning dataset to train a model on.
  - we cant experimentally force people to wear a mask or not.
- How do we solve that?

- **Glossary for machine learning vs causal inference terms:** `https://bit.ly/ML-Econ-Glossary`.

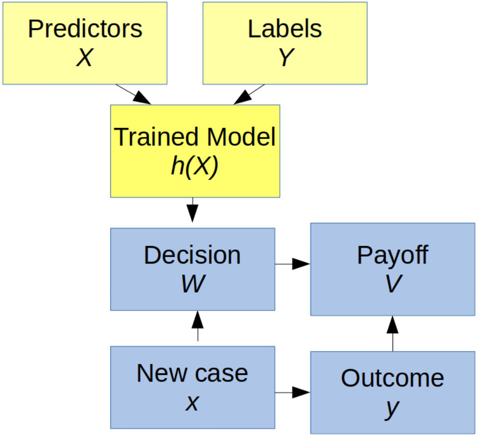
Outline

Decision-Making Schema



- A decision-maker observes facts x and makes decision w, which produces payoff $V = u(y, w)$.
- Decision-maker has access to a history of cases with facts X and labels Y, can learn a machine prediction $\hat{y} = h(x)$.

Previously (Machine Learning)



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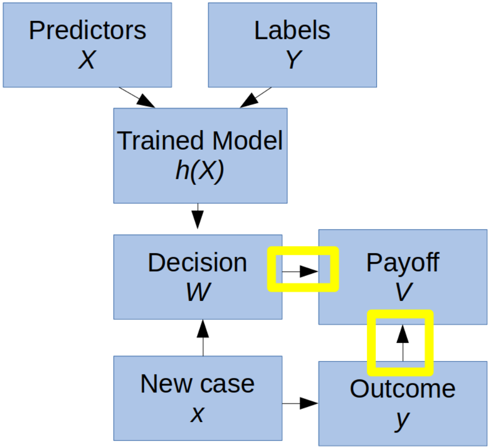
- Good decision-making requires accurate predictions for a relevant outcome (e.g. recidivism) based on observables. We can learn those predictions from data.

Today (Causal Inference)



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- Decision-maker has access to a history of cases with facts X and labels Y, can learn a machine prediction $\hat{y} = h(x)$.
- In addition to having a good prediction $h(\cdot)$, decision-maker wants to know $u(y, w)$.

- Good decision-making requires accurate *counterfactual* predictions for how changes in decisions impact the payoff-relevant outcome.

Outline

Counterfactual predictions $\leftrightarrow$ Causal parameters

- Let's say the payoff function $v = u(y, w; \beta)$ has learnable causal parameters β.
  - e.g., the effect of prison sentence w on crime rates v, given recidivism y.
- How to learn β?
  - what we call *empirical* or *econometric* analysis.
  - requires causal inference.
  - this is the focus in applied economics research

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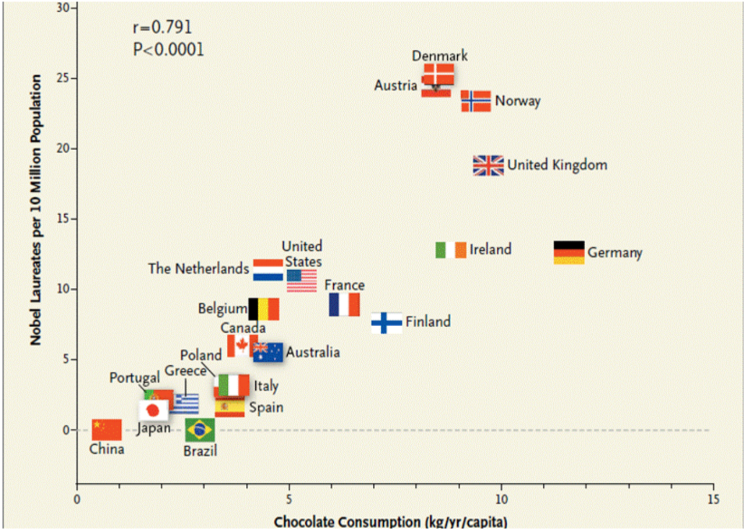
  - etc.

Zoom Chat Activity (2 minutes)

Re-write this "prediction" question as a "what if" question – chat to me privately on Zoom.:

- What is the probability that Ludwig will commit murder if he faces the death penalty?

Correlation does not imply causation



More here: `http://www.tylervigen.com/spurious-correlations`

Outline

Basics

- Represent a **treatment** for row i as a binary random variable $D_i = 0, 1$.
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- The **causal effect** of the medicine (treatment) for individal i is $V_{1i} - V_{0i}$.
  - the difference in the outcome between treatment and control.
- **Problem**: For i, we can observe V_{1i} (individual takes medicine) or V_{0i} (no medicine), **but not both.**

Outline

Illustration

- Let's take some imaginary data where we can time travel and observe participants Leo and Mia both with/without the medicine:

		Leo	Mia
V_{0i}	life expectancy without medicine	3	5
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- In this imaginary data, the medicine would work for Leo, but not for Mia.

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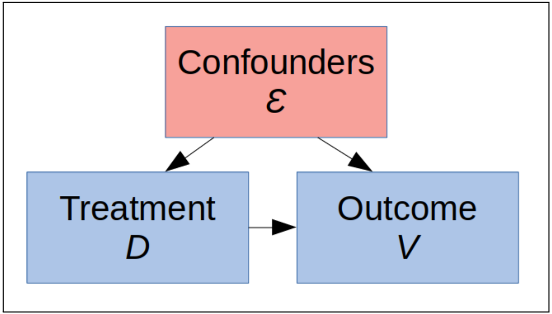
- Note that $V_{\text{Leo}} < V_{\text{Mia}}$:
  - based on these outcomes, one would be led to believe that the medicine actually harms the patient!
  - This is **selection bias** or **confounding**.

Outline

Selection Bias due to Confounders

- Leo has a pre-existing tendency in life expectancy, that is correlated with treatment assignment.
  - this tendency is a **confounder** or **omitted variable**
  - if we could observe this tendency, we could control or adjust for it.
  - but if unobserved, resulting analysis will be biased.



→ Observational studies of medicines don't work well, because relatively sick individuals will be more likely to take the medicine.

Formalizing Selection Bias or Confounding

The difference in observed outcomes between treatment group and control group is:

$$\underbrace{\mathbb{E}[V_{1i}|D_i = 1]}_{\text{avg outcome for treatment}} - \underbrace{\mathbb{E}[V_{0i}|D_i = 0]}_{\text{avg outcome for control}}$$

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- When does the difference in observed outcomes capture the **average treatment effect** (on the treated)?
  - only if there is no selection bias:

$$\mathbb{E}[V_{0i}|D_i = 1] = \mathbb{E}[V_{0i}|D_i = 0]$$

(equivalent to saying their are no confounders).

Questions: Answer by Private Zoom Chat (2 minutes)

- If last name starts with A-M:
  - what are likely confounders for the effect of education on income?
- If last name starts with N-Z:
  - Why is selection bias not a problem in a lab experiment?

Random Assignment

Random assignment $\rightarrow$ D_i independent of potential outcomes:

$$\mathbb{E}[V_{1i}|D_i = 1] = \mathbb{E}[V_{1i}|D_i = 0] = E[V_{1i}]$$
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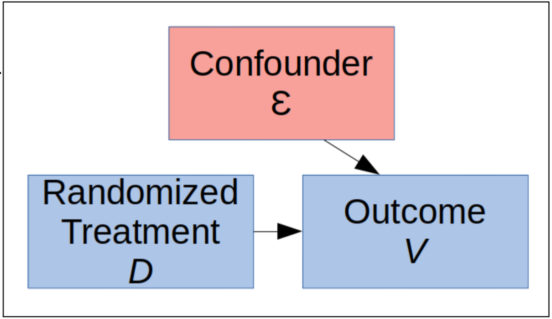

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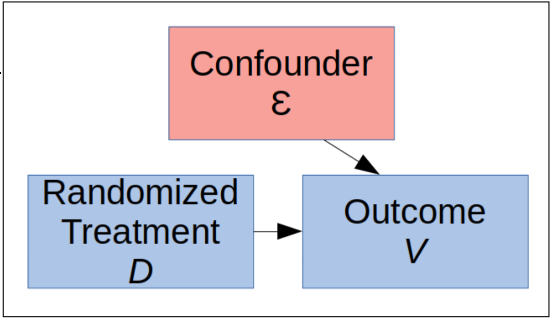

Therefore, the difference in observed outcomes

$$\mathbb{E}[V_{1i}|D_i = 1] - \mathbb{E}[V_{0i}|D_i = 0]$$

captures the average treatment effect:

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and provides a **counterfactual prediction** for effect of taking treatment.

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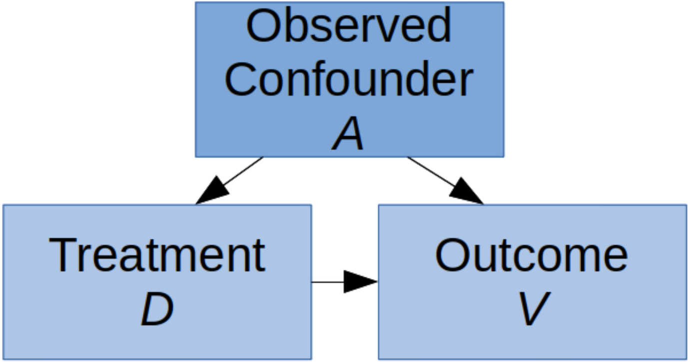
- Today:
  - Adjusting (controlling) for observed confounders
  - Differences-in-differences
- In 2 weeks:
  - Adjusting $\times$ machine learning: Double ML
  - Diffs-in-diffs $\times$ machine learning: Synthetic control

Outline

Adjusting (controlling) for observables



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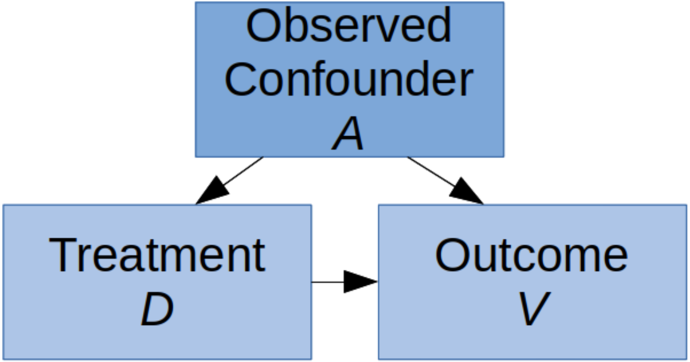

- What if the treated group and the non-treated group differ only by a set of observable characteristics?
- This is the case of observed confounders.
  - also called "selection on observables" or "conditional independence"
  - justifies causal interpretation of regression estimates

Example

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- Conditional Independence holds when

$$\mathbb{E}[V_{0i}|A_i, D_i = 1] = \mathbb{E}[V_{0i}|A_i, D_i = 0]$$

that is, selection bias is zero conditional on observables.

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     - also not a problem
  4. unobserved variables correlated with about treatment and outcome.
     - **this is the problem.**
     - often way to know whether all confounders are observed.

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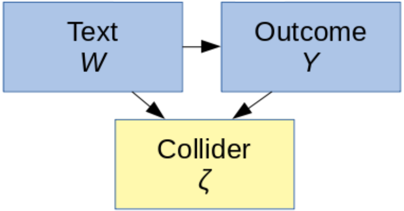

- "Bad controls" (colliders or mediators) are variables that are jointly determined along with the outcome.
  - for example, controlling for occupation in the effect of education on income: education affects both occupation and income.
  - Adjusting for these variables could add bias.

Outline

Introduction to Regression

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- Assume a linear model

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- $\beta =$ the slope parameter summarizing how wages vary with schooling.

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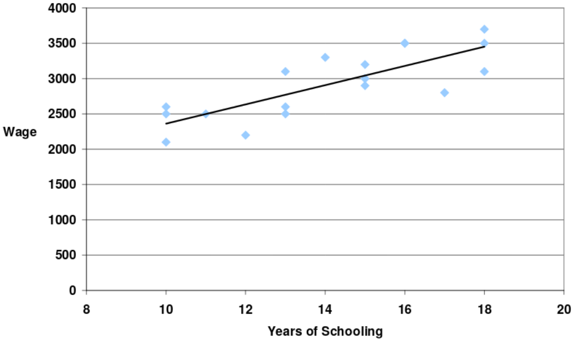
- Assume that s_i is de-meaned. Then the OLS estimator is given by

$$\hat{\beta} = \frac{\sum_{i=1}^{n} s_i V_i}{\sum_{i=1}^{n} s_i^2} = \frac{\text{Cov}[V_i, s_i]}{\text{Var}[s_i]}$$

Interpreting OLS Coefficients



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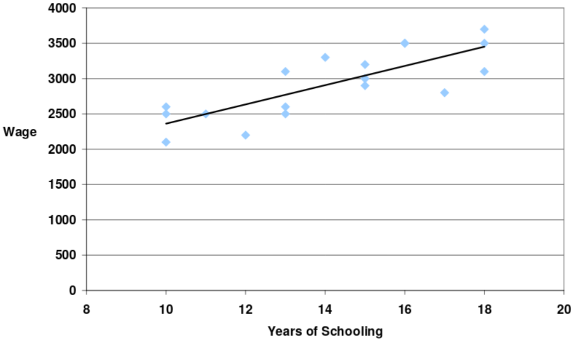

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- Using the estimated constant $\hat{\alpha}$ and estimated slope coefficient $\hat{\beta}$, we obtain a predicted income $\hat{Y}$ for any level of schooling s as

$$\hat{Y}(s) = \hat{\alpha} + \hat{\beta}s$$

Outline

Unbiased Estimates

- The **OLS exogeneity assumption** is $\text{Cov}[s_i, \epsilon_i] = 0$
  - (treatment is uncorrelated with error; equivalent to no confounders).

Unbiased Estimates

- The **OLS exogeneity assumption** is $\text{Cov}[s_i, \epsilon_i] = 0$
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- We have

$$\hat{\beta} = \frac{\sum_{i=1}^{n} s_i V_i}{\sum_{i=1}^{n} s_i^2} = \frac{\sum_{i=1}^{n} s_i(\beta s_i + \epsilon_i)}{\sum_{i=1}^{n} s_i^2}$$
$$= (\frac{\sum_{i=1}^{n} s_i^2}{\sum_{i=1}^{n} s_i^2})\beta + \frac{\sum_{i=1}^{n} s_i(\epsilon_i)}{\sum_{i=1}^{n} s_i^2}$$
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- Taking expectations:

$$\mathbb{E}[\hat{\beta}] = \beta + \mathbb{E}[\frac{\sum_{i=1}^n s_i\epsilon_i}{\sum_{i=1}^n s_i^2}]$$
$$= \beta + \frac{\text{Cov}[s_i, \epsilon_i]}{\text{Var}[s_i]}$$
$$= \beta$$

Endogeneity

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  - That is, an explanatory variable s_i is said to be **endogenous** if it is correlated with unobserved factors (confounders) that are also correlated with the outcome variable.

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  - That is, an explanatory variable s_i is said to be **endogenous** if it is correlated with unobserved factors (confounders) that are also correlated with the outcome variable.
- Since the error term ϵ_i includes all unobserved factors affecting the outcome, we can define **endogeneity** as correlation between an explanatory variable and the error term:

$$\text{Cov}[s_i, \epsilon_i] \neq 0$$

Formalizing omitted variable bias

- Assume that the "true" model is
$$V_i = \beta s_i + \gamma a_i + \eta_i \tag{1}$$
where η_i is random (exogenous), but we cannot measure ability a_i.

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- Taking expectations gives

$$\mathbb{E}[\hat{\beta}] = \beta + \underbrace{\gamma \frac{\text{Cov}[s_i, a_i]}{\text{Var}[s_i]}}_{\text{Omitted variable bias}} + \underbrace{\frac{\text{Cov}[s_i, \eta_i]}{\text{Var}[s_i]}}_{=0 \text{ by assumption}}$$

  - $\rightarrow$if ability is correlated with schooling, $\hat{\beta}$ is a biased estimate for β.

Understanding omitted variable bias

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		Correlation of omitted variable with explanatory variable	
		$\text{Cov}[s,a] > 0$	$\text{Cov}[s,a] < 0$
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- **Poll** 3.2: How does the example of ability/schooling/income fit in this table?

Outline

Statistical Significance

- The value for β provides a prediction for the effect of the explanatory variable on the outcome.
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  - On regression tables, usually reported in parentheses right beneath the point estimate.
- Small *p*-values are often indicated on regression tables with stars to indicate statistical significance.

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Differences-in-Differences

- Example: taxes raised in canton A, but **not** in canton B
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= **price change in treated canton, relative to price change in comparison canton.**

- Identification assumption: **"parallel trends"**
  - Absent tax change, trend in prices would have been the same in cantons A and B.

Outline

Diff-in-Diff Regression

- Can estimate the diff-in-diff effect using

$$Y_{jt} = \alpha + \gamma \text{Treat}_{jt} + \lambda \text{After}_{jt} + \rho \text{Treat*After}_{jt} + \varepsilon_{jt}$$

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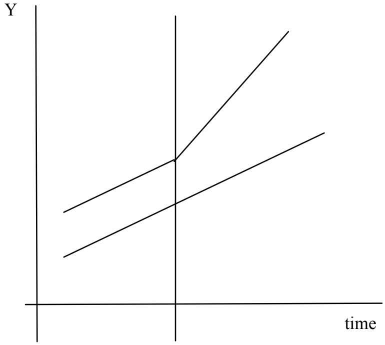
- Interpreting coefficients:
  - α, average in non-treated group, pre-treatment
  - γ, difference between treated and non-treated in pre-treatment period
  - λ, change in the control group after reform
  - ρ, the diff-in-diff treatment effect estimate (change in treatment group, relative to change in control group).

Diff-in-diff: Parallel trends assumption

Good



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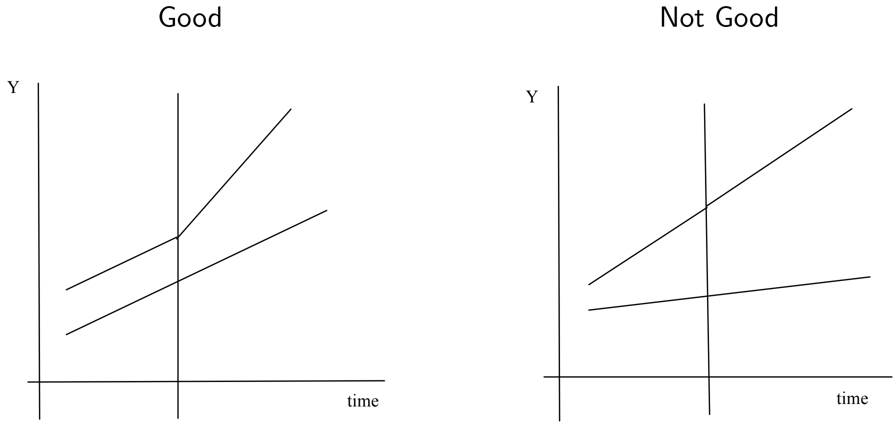

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Fixed-Effects Regression

- **Fixed-effects regression** generalizes diffs-in-diffs to > 2 groups and > 2 periods
  - Requires panel (longitudinal) data
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$$Y_{jt} = \delta_j + \gamma_t + \beta T_{jt} + \varepsilon_{jt}$$

- δ_j = canton fixed effects
  - categorical variables equaling one for canton j's observations, zero otherwise
- γ_t = year fixed effects
  - categorical variables equaling one for year t's observations, zero otherwise

FE regression is an empirical workhorse

- At any given time, taxes and prices across cantons could be correlated for many confounding reasons.
- Diffs-in-diffs holds constant many of the most important confounders:
  - time-invariant canton-level factors
  - nationwide time-varying factors

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- At any given time, taxes and prices across cantons could be correlated for many confounding reasons.
- Diffs-in-diffs holds constant many of the most important confounders:
  - time-invariant canton-level factors
  - nationwide time-varying factors
- Potential confounders must
  - vary over time by canton
  - correlated with outcome variable
  - correlated with the timing of treatment/reforms

Threats to validity for FE regression

- Can check that treatment cantons evolved similarly to comparison cantons before reform.
  - can also add canton-specific trends.

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- Can check that treatment cantons evolved similarly to comparison cantons before reform.
  - can also add canton-specific trends.
- Skeptical questions to ask:
  - Why did the treatment group adopt the policy, and not the control group?
  - Were other policies adopted at the same time that might also affect the outcome?
  - Could the treatment spill over into the comparison cantons?

Activity: Private Zoom Chat (3 minutes)

- Imagine that cantons Zurich and Zug each enact a tax cut and you estimate a negative effect on local employment using fixed effects regression. What are some potential confounding factors that would bias this estimate?
  - chat answers to me privately by zoom.

A note on standard errors

- Consider the regression for cantonal tax cuts and employment. We have 26 cantons.
  - the default standard errors formula for OLS assume that all observations are independent realizations.
- Compare the following analyses:
  - including the 10 years before and after the reform ($N = 260$)
  - including the 20 years before and after ($N = 520$)

A note on standard errors

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  - the default standard errors formula for OLS assume that all observations are independent realizations.
- Compare the following analyses:
  - including the 10 years before and after the reform ($N = 260$)
  - including the 20 years before and after ($N = 520$)
- Using the default SE's, the second analysis would give much more precise estimate, even though the data contain nearly equivalent information.

Solution: Clustering Standard Errors

Cluster standard errors:

- statistically acknowledges how many independent sources of information there are in the data.
- the standard approach is to cluster at the unit where treatment is assigned.
  - in this example, by canton.
- for city-level reforms cluster by city, etc.