

Lean cheat sheet

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March 24, 2026

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1 Logic

Notation

For a complete list of notation, click on the $\forall$ icon in “Lean Infoview”, select “Documentation...” and then “Docs: Show Unicode Input Abbreviations”.

Implication	$\rightarrow$	<code>\r, \to, \imp</code>
Left arrow	$\leftarrow$	<code>\l, \gets</code>
Equivalence	$\leftrightarrow$	<code>\iff, \leftrightharrow, \lr, \<-></code>
Logical AND	$\wedge$	<code>\and, \wedge</code>
Logical OR	$\vee$	<code>\or, \v, \vee</code>
Negation	$\neg$	<code>\n, \not</code>
Universal quantifier	$\forall$	<code>\all, \forall</code>
Existential quantifier	$\exists$	<code>\ex, \exists</code>
Not equal	$\neq$	<code>\neq, \ne</code>
Turnstile	$\vdash$	<code>\vdash, \ -, \goal</code>

Conventions

- $A \rightarrow B \rightarrow C$ means $A \rightarrow (B \rightarrow C)$
- $\neg A$ means $A \rightarrow \text{False}$
- $x \neq y$ means $(x = y) \rightarrow \text{False}$
- $\forall x < 42, P\ x$ is a shorthand for $\forall x, x < 42 \rightarrow P\ x$

Accessors

- **Modus ponens:** `h.mp` and `h.mpr` turn a hypothesis $h : p \leftrightarrow q$ into $p \rightarrow q$ and $q \rightarrow p$, respectively.
- **Modus tollens:** `h.mt` turns a hypothesis $h : p \rightarrow q$ into $\neg q \rightarrow \neg p$.
- Left and right injection into **Or**:
 - `Or.inl` hl proves $hl \vee hr$ using hl ,
 - `Or.inr` hr proves $hl \vee hr$ using hr .

Tactics

left

- Reduces $\vdash a \vee b$ to $\vdash a$.

right

- Reduces $\vdash a \vee b$ to $\vdash b$.

tauto

- **tauto** concludes the proof by automatically establishing logical statements.

push_neg

- **push_neg** rewrites the goal by pushing negations deeper. For example, $\vdash \neg \forall x, \exists y, x \leq y$ transforms into $\vdash \exists x, \forall y, y < x$.

contrapose

- **contrapose** turns a goal $P \rightarrow Q$ into $\neg Q \rightarrow \neg P$
- **contrapose** h changes the goal to $\neg h$ and replaces h by the negation of the original goal

contrapose!

- **contrapose!** h first does **contrapose** and then applies **push_neg** to both the goal and h

Contradiction tactics

exfalse

- **exfalse** changes the goal to **False**. Use it to start a proof by contradiction.

by_contra

- **by_contra** h replaces the goal g with **False** and introduces a new hypothesis $h : \neg g$. It can be used to prove the goal by contradiction.

by_contra!

- **by_contra!** is the same as **by_contra** except that it also automatically normalizes negations by using **push_neg**.

contradiction

- **contradiction** closes the goal if its hypotheses are “trivially contradictory”. For example, if there is a hypothesis $h : \mathbf{False}$, or two contradictory hypotheses $h : P$ and $h' : \neg P$, or a decidable false hypothesis such as $h : 2 + 2 = 3$.

2 Sets and functions

Notation

Element	$\in$	<code>\in</code>
Not element	$\notin$	<code>\notin</code>
Subset	$\subseteq$	<code>\ss</code> , <code>\sub</code> , <code>\subset</code>
Intersection	$\cap$	<code>\i</code> , <code>\inter</code> , <code>\cap</code>
Union	$\cup$	<code>\union</code> , <code>\un</code> , <code>\cup</code>
Set difference	$\setminus$	<code>\</code>
Empty set	$\emptyset$	<code>\emptyset</code> , <code>\empty</code> , <code>\varnothing</code>

Some further notation and conventions:

- Universal set: **Set.univ** is the set of all elements of a given type
- $\{y \mid P\ y\}$ unfolds to $(\text{fun } y \mapsto P\ y)$, so $x \in \{y \mid P\ y\}$ reduces to $P\ x$.
- The preimage $f^{-1}\ s$ is defined as $\{x \mid f\ x \in s\}$, so $x \in f^{-1}\ s$ reduces to $f\ x \in s$.
- **Injective** f is defined as $\forall x_1\ x_2, f\ x_1 = f\ x_2 \rightarrow x_1 = x_2$.

We write $P\ a$ to mean that proposition P holds for $a : \alpha$, so under the hood a property or predicate on a type α is just a function $P : \alpha \rightarrow \mathbf{Prop}$. In *Mathlib*, **Set** α is defined to be $\alpha \rightarrow \mathbf{Prop}$ and $x \in S$ is defined to be $S\ x$.

Set-builder notation is used to define

- $s \cap t$ as $\{x \mid x \in s \wedge x \in t\}$,
- $s \cup t$ as $\{x \mid x \in s \vee x \in t\}$,
- $\emptyset$ as $\{x \mid \text{False}\}$,
- univ as $\{x \mid \text{True}\}$.

Tactics

`tauto_set`

- Proves any relation between sets by first reformulating it as a logical statement and then applying `tauto`.

3 Core tactics

Tactics

`intro`

- If the goal is $\vdash A \rightarrow B$ then `intro` h replaces it with $\vdash B$ and introduces a new hypothesis $h : A$. Omitting h or replacing it by `_` leaves the hypothesis unnamed.
- If the goal is $\vdash A \rightarrow B \rightarrow C$ then `intro` $ha\ hb$ replaces it with C and introduces two new hypotheses, $ha : A$ and $hb : B$.
- If the goal is $\vdash \neg A$ then `intro` h replaces it with $\vdash \text{False}$ and introduces a hypothesis $h : A$.
- If the goal is $\vdash A \wedge B \rightarrow C$ then `intro` $\langle ha, hb \rangle$ introduces hypotheses $ha : A$ and $hb : B$, and replaces the goal with C .
- If the goal is $\vdash \forall (a : \mathbb{N}), a + a = 2 * a$ then `intro` x introduces a hypothesis $x : \mathbb{N}$ and changes the goal to $\vdash x + x = 2 * x$.

`revert`

- `revert` x is the inverse of `intro` x .

`exact`

- `exact` h concludes the proof if the goal is $\vdash A$ and there is a hypothesis $h : A$.
- `exact` $\langle hp, hq \rangle$ closes the goal $\vdash P \wedge Q$ using hypotheses $hp : P$ and $hq : Q$.
- `exact` $h1\ h2$ is a shorthand for `apply` $h1$; `exact` $h2$.
- `exact` $h\ a$ closes the goal by using hypothesis $h : (\forall y : Y), P(y)$ with $y = a$. It seems that `apply` h has the same effect.
- `exact?` performs library search.

`assumption`

- `assumption` closes the goal if there is a matching hypothesis.

refine

- **refine** $\{?_, h, ?_\}$ uses hypothesis $h : Q$ to replace the goal $\vdash P \wedge Q \wedge R$ with two new goals: $\vdash P$ and $\vdash R$.
- **refine** $h ?_$ is equivalent to **apply** h since it uses hypothesis $h : P \rightarrow Q$ to change the goal from $\vdash Q$ to $\vdash P$.
- **refine** $\{2, ?_\}$ is equivalent to **use** 2 since it changes an existential goal $\vdash \exists (n : \mathbb{N}), n^4 = 16$ to $\vdash 2^4 = 16$.
- The cases can be given names: **refine** $h ?a ?b$; **case** $a \Rightarrow \dots$; **case** $b \Rightarrow \dots$.

refine'

apply

- If the goal is $\vdash B$ and $h : A \rightarrow B$ is a hypothesis, then **apply** h replaces the goal by $\vdash A$. This is *backwards reasoning* from the goal.
- If $h1 : A \rightarrow B$ and $h2 : A$ are hypotheses then **apply** $h1$ **at** $h2$ transforms the second hypothesis into $h2 : B$. This is *forwards reasoning*.
- If $h : A \rightarrow B \rightarrow C$ is a hypothesis and the goal is $\vdash C$ then **apply** h creates two cases with goals $\vdash A$ and $\vdash B$. Use $\backslash.$ to address the two cases. Can use $<;>$ if both cases can be address in the same way.
- If $h : \neg P$ is a hypothesis and the goal is $\vdash \text{False}$ then **apply** h changes the goal to P since $\neg P$ is definitionally equivalent to $P \rightarrow \text{False}$.
- If $h : \forall a, \varphi a = \psi a$ is a hypothesis and the goal is $\vdash \varphi x = \psi x$ then **apply** h will conclude the proof.
- **apply** h **at** $h1$ $\vdash$ applies hypothesis h at both $h1$ and the goal.
- **apply?** performs library search.

change

- **change** g will change the current goal to g if they are definitionally equivalent.
- **change** t' **at** h will change the hypothesis $h : t$ to $h : t'$, assuming t and t' are definitionally equivalent.
- **change** a **with** b will change occurrences of a with b in the goal, assuming a and b are definitionally equivalent.
- **change** a **with** b **at** h does the same for hypothesis h .

show

- **show** g selects which of the goals to prove and unifies it with g .

rename_i ???????? rename inaccessible variable (one with dagger)

clear

- **clear** h removes the hypothesis h .

triv

- **triv** concludes the proof if the goal is **True**.

rfl

- `rfl` tries to close the goal by using reflexivity, e.g., when the goal is $\vdash A \leftrightarrow B$.

rw

- `rw [h]` rewrites the goal using hypothesis h .
- `rw [← h]` rewrites the goal using hypothesis h in the backwards direction.
- `rw [h] at h1` rewrites the hypothesis $h1$.
- `rw?` searches for possible rewrites through the library.
- `rw? at h` does the same for hypothesis h .

erw

- Like `rw` but tries a little harder.

nth_rw

- `nth_rw i j [h]` rewrites i -th to j -th occurrences of hypothesis h .

rwa

- `rwa` is the same as `rw` but afterwards closes any remaining goals using `assumption`.

unfold

- `unfold def` unfolds the definition def in the goal. Its action is similar to `rw [def]`.
- `unfold?` followed by shift-clicking an expression in the tactic state gives a list of rewrite suggestions for the selected expression. Clicking on a suggestion replaces `unfold?` by a tactic that performs this rewrite.

unfold_let

use

- `use 42` chooses the variable under $\exists$ in the goal to have value 42.
- `use 42, h1, h2` chooses the value of the variable and uses hypotheses $h1$ and $h2$ to close the goal.
- `use ha` turns $ha : a, \vdash a \wedge b$ into $ha : a, \vdash b$.
- $\langle 42, h1, h2 \rangle$ does the same but using Lean's "anonymous constructor" notation.
- `example :  $\exists x : \mathbb{R}, 41 < x \wedge x < 43 := \langle 42, \text{by norm\_num} \rangle$ .`

delta

`decide` ?????????? brute force trial for finite types

`fin_cases` ?????????? consider finite number of cases

done

- `done` succeeds if and only if there are no remaining goals.

sorry

admit

4 Splitting into cases

Notation

- Tactic combinator: `<;>` applies the subsequent tactic to all subgoals generated by **cases**. For example, `cases h; repeat assumption` tries to close all cases by using existing hypotheses. This can be abbreviated as `cases h <;> assumption`.

Tactics

by_cases

- **by_cases** $h : P$ splits the goal into two cases, assuming $h : P$ in the first branch and $h : \neg P$ in the second.

if ...then ...else ...

- `if h : t then tac1 else tac2` is equivalent to `by_cases h : t · tac1 ; · tac2`.

cases'

- **cases'** h **with** hp hq breaks up the hypothesis
 - $h : P \wedge Q$ into two new hypotheses: $hp : P$ and $hq : Q$,
 - $h : P \vee Q$ into two goals: one with hypothesis $hp : P$ and another with $hq : Q$.
- **cases'** h **with** z hf replaces hypothesis $h : \exists x : X, Px$ with $hf : Pz$ and introduces a new hypothesis $z : X$.
- **cases'** h z **with** x hx uses hypothesis $h : \forall b : Z, \exists a : X, fa = b$ to create two new hypotheses: $x : X$ and $hx : fx = z$.
- If $n : \mathbb{N}$ then **cases'** n **with** m gives two goals: one where n is replaced by 0 and another where it is replaced by **succ** m .

cases

- **cases** is the structured version of **cases'** that uses `cases h with` followed by `| constructor_name => tactic` branches.
- `cases h with | inl hp => ... | inr hq => ...` on $h : P \vee Q$ creates two goals:
 - `| inl hp =>` with hypothesis $hp : P$,
 - `| inr hq =>` with hypothesis $hq : Q$.
- `cases n with | zero => ... | succ m => ...` on $n : \mathbb{N}$ creates two goals:
 - `| zero =>` where n is replaced by 0,
 - `| succ m =>` where n is replaced by **succ** m .
- Unlike **cases'**, the named branches can appear in any order, making proofs more robust to upstream changes.

rcases

- **rcases** h with $\langle g, \langle fg, gf \rangle \rangle$ breaks up hypothesis $h : \exists g, f \circ g = id \wedge g \circ f = id$ into three hypotheses: $g : Y \rightarrow X$, $fg : f \circ g = id$, $gf : g \circ f = id$.
- **rcases** h with $\langle a, rfl \rangle$ unpacks the hypothesis h into a and another unnamed hypothesis that is immediately used to rewrite the goal.
- **rcases** h with $hl \mid hr$ breaks the hypothesis $h = (\dots) \vee (\dots)$ into two cases called hl and hr .

rintro

- **rintro** is a combination of **intro** and **rcases**.
- **rintro** $hP \langle hQ, hR \rangle$ is equivalent to **intro** hP followed by **intro** h and **cases'** h with $hQ \ hR$.
- **rintro** $x \ y \ h$ replaces goal $\vdash \forall (x \ y : \mathbb{R}), x = y \rightarrow y = x$ by $\vdash y = x$ and introduces hypotheses $x \ y : \mathbb{R}$ and $h : x = y$.
- **rintro** $x \ y \ rfl$ is equivalent to **rintro** $x \ y \ h$; **subst** h , which speeds up the previous example by immediately changing the goal to $\vdash x = x$ with hypothesis $x : \mathbb{R}$.

interval_cases

- **interval_cases** n searches for upper and lower bounds on a variable n , and if bounds are found, splits into separate cases for each possible value of n .

constructor

- **constructor** breaks the goal $P \wedge Q$ into two cases: P and Q . Each of them can then be addressed using $\cdot$ (entered as $\backslash \cdot$). If both cases can be addressed the same way, use **constructor** $<;>$, followed by $\cdot$ and commands that address both cases simultaneously.
- **constructor** breaks the goal $A \leftrightarrow B$ into two cases: $A \rightarrow B$ and $B \rightarrow A$.

5 Reals

Notation

- Reals: $\mathbb{R}$ is $\backslash R$

Tactics

norm_num

- **norm_num** normalizes the numerical expression in the goal.

subst

- **subst** h removes hypothesis $h : x = y$ and replaces all occurrences of y by x throughout.
- **subst** y has the same effect, while **subst** x substitutes all occurrences of x by y .
- **subst** $h1 \ h2$ makes two substitutions simultaneously based on hypotheses $h1$ and $h2$.

subst_vars

- **subst_vars** performs all possible substitutions with **subst**.

specialize

- `specialize h 42` replaces the variable under $\forall$ in hypothesis h with the value 42.

generalize ?????????? look for an expression and replace it by a variable

clean_value ??????????

6 Functions

Notation

- Function composition: $\circ$ is `\o`
- Map: $\mapsto$ is `\mapsto`
- Identity function: `id` (`id x` is definitionally equal to x)
- Subscripts: x_1 is `x\1` or `x\_1`
- Equivalence: $\simeq$ is `\equiv` or `\simeq`

Conventions

- $X \simeq Y$ is the type of functions from X to Y with a two-sided inverse.

Commands

fun

- `fun x y => x + y` creates an anonymous function (lambda expression).
- λ is a shorthand for `fun`.

def

- `def f :  $\mathbb{N} \rightarrow \mathbb{R}$  := fun n  $\mapsto n^2 + 3$` defines a function.

Tactics

apply_fun

- `apply_fun f` turns goal $\vdash x = y$ into $\vdash f(x) = f(y)$.
- `apply_fun f at h` turns hypothesis $h : x = y$ into $h : f(x) = f(y)$.

choose

- `choose g hg using sur` creates a new hypothesis $hg : \forall (y : Y), f(g y) = y$ which asserts that $g : Y \rightarrow X$ is a one-sided inverse of function $f : X \rightarrow Y$ whose surjectivity is guaranteed by hypothesis `sur : Function.Surjective f`.

ext

- `ext x` changes the goal from $f = g$ to $f x = g x$ and introduces a new hypothesis $x : X$ where X is the domain of both functions f and g .

7 Creating new hypotheses

have

- **have** $h1 := h$ creates a copy of hypothesis h called $h1$.
- **have** $h : \langle \text{statement} \rangle := \text{by } \langle \text{proof} \rangle$ introduces a new hypothesis h defined as $\langle \text{statement} \rangle$ and justified by $\langle \text{proof} \rangle$.

suffices ????????? similar to have

obtain

- **obtain** is a combination of **have** and **rcases**.
- **obtain** $\langle \text{patt} \rangle : \text{type} := \text{proof}$ is equivalent to **have** $h : \text{type} := \text{proof}$; **rcases** h with patt .
- **obtain** $\langle w, hw \rangle := h$ destructs a hypothesis $h : \exists x, P x$ into a witness x and proof $hx : P x$
- **obtain** $\langle hP, hQ \rangle := P \wedge Q$ creates two new hypotheses, $hP : P$ and $hQ : Q$.

let

set

- **set** $a := t$ **with** h is a variant of **let** $a := t$. It adds the hypothesis $h : a = t$ to the local context and replaces t with a everywhere it can.

8 Structures

8.1 Groups

Notation

- Inverse: $^{-1}$ is `\-` or `\inv`
- Quotient: $/$ is `\quot` or `\/` (not the same as the “/” character)
- Group isomorphisms: $\simeq$ where $\simeq$ is `\simeq` or `\equiv` (short for `MulEquiv`)
- Ring isomorphisms: $\simeq + *$ (short for `RingEquiv`)

Conventions

- $a * b * c$ means $(a * b) * c$

Tactics

group

- Automatically proves any group identity that does not require the hypotheses.

abel

- Similar but for abelian groups.

8.2 Lattices

Notation

- Less than or equal: $\leq$ is `\le`
- Greater than or equal: $\geq$ is `\ge`
- Greatest lower bound, infimum, or meet: $\sqcap$ is `\inf`, `\meet`, `\glb`, or `\sqcap`
- Least upper bound, supremum, or join: $\sqcup$ is `\sup`, `\join`, `\lub`, or `\sqcup`
- The bottom element: $\perp$ is `\bot`
- The top element: $\top$ is `\top`

8.3 Linear algebra

Notation

- Linear maps: $V \rightarrow_l [k] W$ where $\rightarrow_l$ is `\tol`, `\rl`, `\to_l`, `\r_l`, or `\linearmap`
- Vector (neither row nor column): $![1, 2, 3]$
- Vector dot product: $![1, 2] \cdot_v ![3, 4]$ where $\cdot_v$ is `\cdot_v`
- Matrix: $!![1, 2; 3, 4]$
- Matrix multiplication: $!![1, 2; 3, 4] * !![3, 4; 5, 6]$
- Matrix-vector multiplication: $!![1, 2; 3, 4] *_v ![1, 1]$
- Vector-matrix multiplication: $![1, 1] *_v !![1, 2; 3, 4]$
- Transpose: $!![1, 2; 3, 4]^T$ where ^{T} is `\^T`
- Inverse: $!![1, 2; 3, 4]^{-1}$ where $^{-1}$ is `\-`, `\inv`, `\-1`, or `\^-1`

9 Tactic combinators

;

- $t_1 ; t_2$ means tactic t_1 followed by t_2 .

<;>

- `cases <;> assumption` splits the proof into cases and uses `assumption` in each case.

►

- `►` is entered as `\t` or `\blacktriangleright`.
- Given $h : a = b$ and $e : p\ a$, the term $h \blacktriangleright e$ has type $p\ b$. You can also view $h \blacktriangleright e$ as a “type casting” operation where you change the type of e by using h .
- `example {f : ℝ → ℝ} (h : Surjective f) : ∃ x, f x2 = 4 :=
let ⟨x, hx⟩ := h 2
⟨x, hx ► by norm_num`

<|

try

- **try assumption** tries to close the goal by using some hypothesis. If no hypothesis applies then no progress is made.

repeat

- **repeat** t keeps applying tactic t until it fails.

repeat'

- Repeat the given tactic as many times as possible. Subgoals where it fails are left over.

all_goals

- **all_goals** t applies the tactic t to every goal, and succeeds if all applications succeed.

any_goals

- **any_goals** t applies the tactic t to every goal. If t fails on all of the goals, the entire tactic fails.

first

- **first** $| t_1 | t_2 | \dots | t_n$ tries each tactic t_i in order until one of them succeeds, or else fails.

@

- **@x** disables automatic insertion of implicit parameters of the constant x . **@e** for any term e also disables the insertion of implicit lambdas at this position.

10 Arithmetic

ring

- **ring** evaluates the goal expression in commutative (semi)ring.

ring_nf

- **ring_nf** rewrites the ring expression in the goal into a normal form.

noncomm_ring

- Similar but for non-commutative rings.

linarith

- **linarith** uses linear arithmetic to prove the goal.

nlinarith

- **nlinarith** is extension of **linarith** with some preprocessing to allow solving nonlinear arithmetic problems.

linear_combination

polyrith

- **polyrith** finds the right coefficients for **linear_combination**.

calc

```
example {a b : C} (h1 : a - 5 * b = 4) (h2 : b + 2 = 3) : a = 9 :=
  calc
    a = a - 5 * b + 5 * b := by ring
    _ = 4 + 5 * b := by rw [h1]
    _ = -6 + 5 * (b + 2) := by ring
    _ = -6 + 5 * 3 := by rw [h2]
    _ = 9 := by ring
```

field_simp

- Reduce an expression in a field to an expression of the form n/d where neither n nor d contains any division symbol.

11 Simplification

dsimp

- **dsimp** stands for “definitional simplifier” since it simplifies the goal using only theorems that hold by reflexivity. For example, it can be used to substitute arguments into a function: **(fun** $n \mapsto n^2 + 3$) 37 simplifies into $37^2 + 3$.
- **dsimp only [...]** simplifies using only the given set of rules.
- **dsimp only** is equivalent to **dsimp only []**, meaning that it does not use any additional rules for simplification.

simp

- **simp** simplifies the goal using lemmas (tagged with **[simp]**) and hypotheses.
- **simp at h** simplifies the hypothesis h .
- **simp only [...]** simplifies using only the given set of rules.

simp_all

simp

- **simp** [rules, ...] using e
- **simp** [rules, ...]
- [...] asdf

simp_rw

aesop

12 Other

congr

trans

- **trans** y changes the goal from $x \leq z$ to two new goals: $x \leq y$ and $y \leq z$. It can be used whenever the goal contains a transitive relation.

Tactics for general-purpose or domain-specific automation:

- **suggest** searches the library for an applicable lemma.
- **simp** is a general-purpose simplifier.
- **abel**, **ring**, **linarith**, **omega**, continuity: domain-specific automation.
- **tidy**, **finish**, **solve_by_elim**: general purpose automation.

13 Metacommands

- **#eval**
- **#check**
- **#print**
- **#print axioms theorem_name** – prints all the axioms used in the proof of **theorem_name**
- **#reduce**
- **#synth**
- **#version** – prints the Lean version

Search commands:

- **#leansearch**
- **#loogle**
- **#moogle**
- **#statesearch**

14 Rules of thumb

Goal

- $\vdash \exists a, \dots$ – use **use**
- $\vdash \forall a, \dots$ – use **intro**
- $\vdash a \rightarrow b$ – use **intro**
- $\vdash a \leftrightarrow b$ – use **constructor**
- $\vdash a \wedge b$ – use **constructor**
- $\vdash a \vee b$ – use **left** or **right**
- $\vdash s \subseteq t$ – **intro** $x \ xs$

Hypothesis

- $h : x \in f' 's$ – use **rcases** h with $\langle x, \ xs, \ fxeq \rangle$

Table

$\vdash \exists x, f\ x$	<code>use 42</code>	$\vdash f\ 42$
$\vdash \forall x, f\ x$	<code>intro x</code>	$x : \alpha$ $\vdash f\ x$
$\vdash a \leftrightarrow b$	<code>constructor</code>	$\vdash a \rightarrow b$ $\vdash b \rightarrow a$
$\vdash a \rightarrow b$	<code>intro h</code>	$h : a$ $\vdash b$
$\vdash a \wedge b$	<code>constructor</code>	$\vdash a$ $\vdash b$
$\vdash a \vee b$	<code>left</code> <code>right</code>	$\vdash a$ $\vdash b$
$\vdash s \subseteq t$	<code>intro x xs</code>	$x : \alpha$ $xs : x \in s$ $\vdash x \in t$
$\vdash s = t$	<code>ext x</code>	$x : \alpha$ $\vdash x \in s \leftrightarrow x \in t$
$\vdash x \in s \cup t \rightarrow p$	<code>rintro (xs xt)</code>	$xs : x \in s$ $\vdash p$ $xt : x \in t$ $\vdash p$
$\vdash y \in f''s$	<code>use x</code>	$\vdash x \in s \wedge f\ x = y$
$\vdash y \in f''s \rightarrow p$	<code>rintro (x, xs, fx)</code>	$x : \alpha$ $xs : x \in s$ $fx : f\ x = y$ $\vdash p$
$h : y \in f''s$	<code>rcases h with (x, xs, fx)</code>	$x : \alpha$ $xs : x \in \alpha$ $fx : f\ x = y$