Astroseismology and Tidal Physics with GYRE

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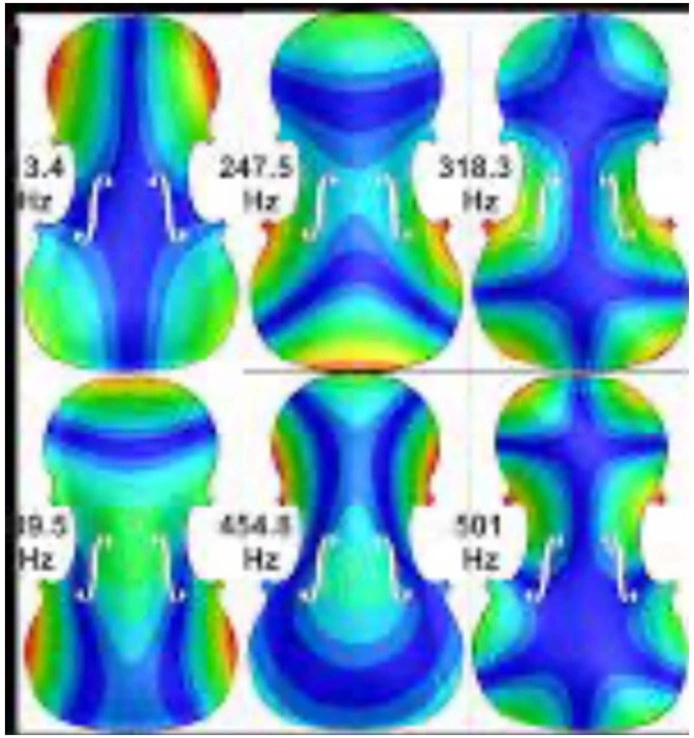
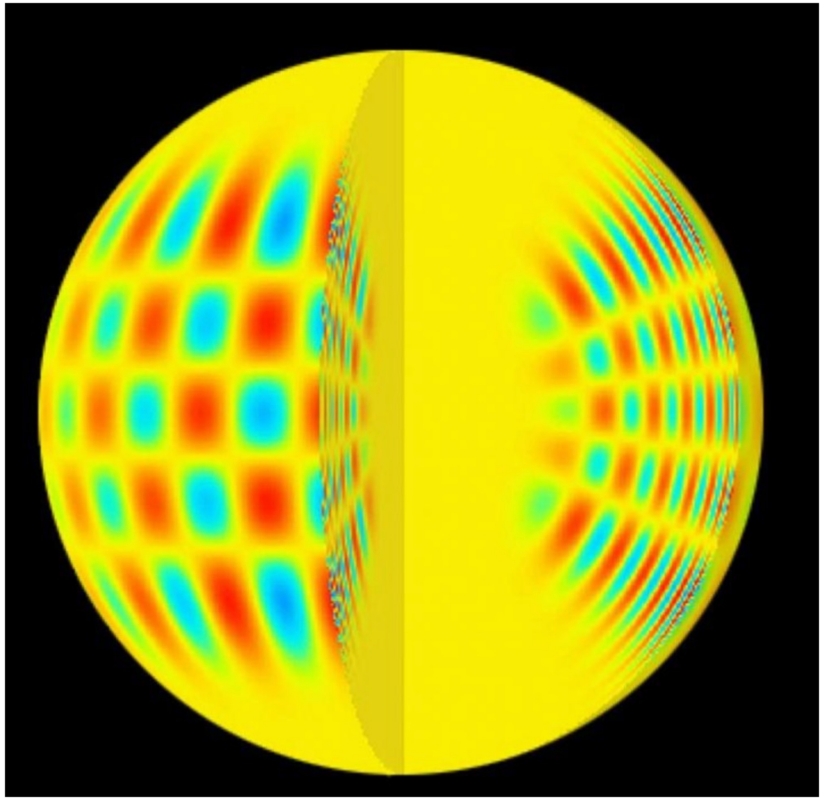
Astroseismic Analysis 101



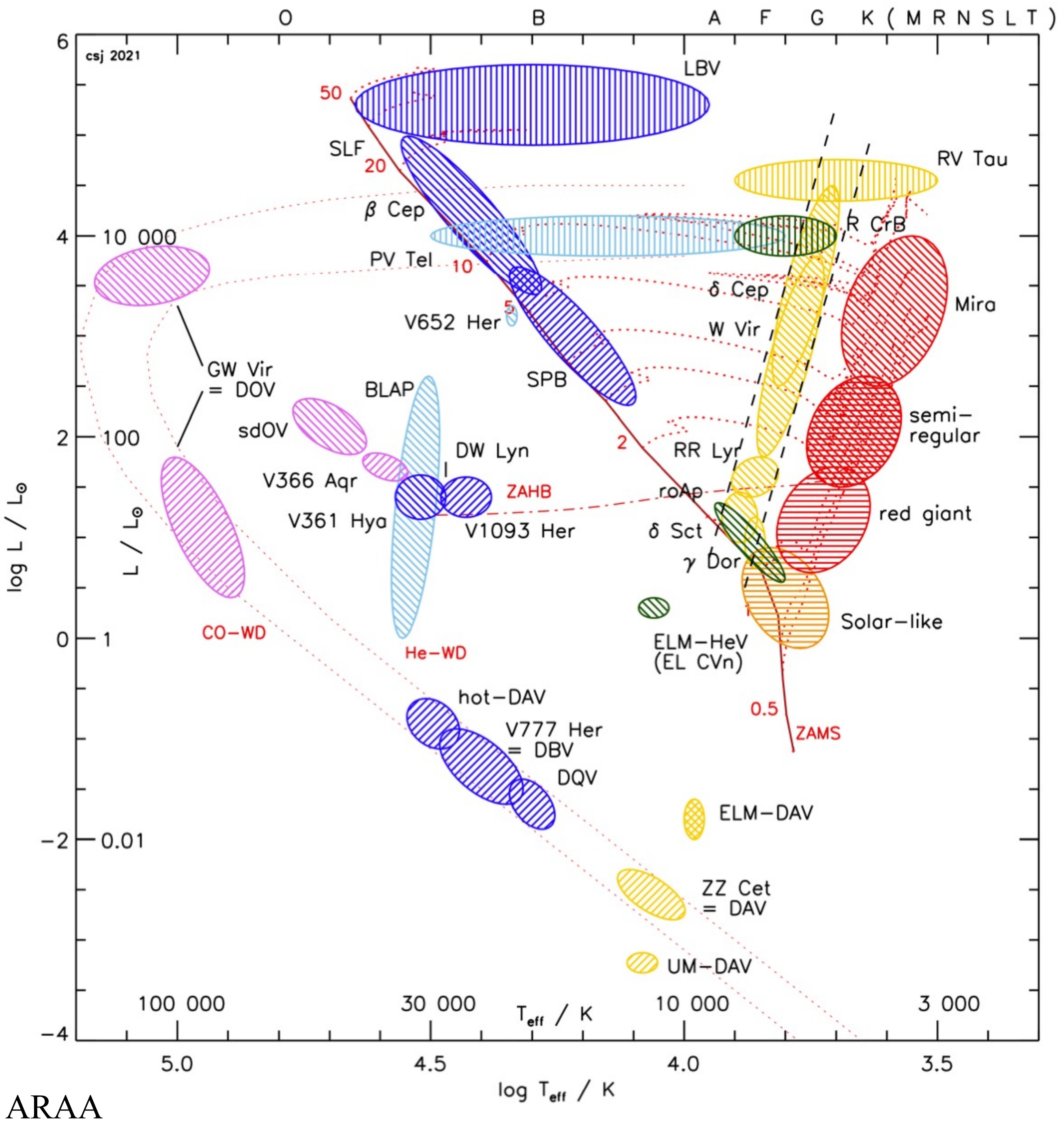


- Stars are pulsating;
- Different inner structures with different oscillation frequencies in stars are analogous to different sounds from different musical instruments;
- Stellar oscillations can be treated as the heat engine mechanism: a region gains heat by compression + driving oscillation successfully → thermal to mechanical energy

Astroseismic Analysis

- Given the measured oscillation mode periods, one can compare these periods with those predicted by models of different mass, temperature, etc., to see which parameters agree best with the data.

- A ton of real astroseismology experts in the teaching team

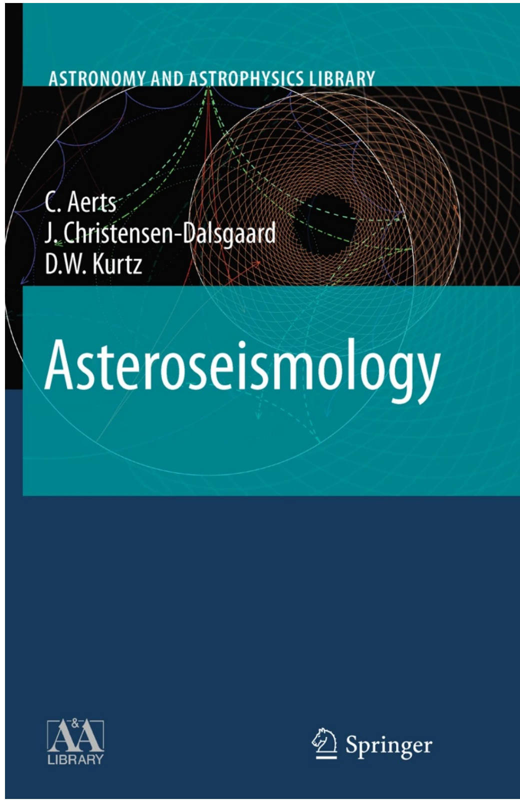


Kurtz, 2022, ARAA

Physics of Stellar Oscillations

- Jørgen Christensen-Dalsgaard lecture notes “Stellar Oscillations”
- Unno, W.’s book (2nd version): Nonradial oscillations of stars, 1989

- Linearized equations of mass, motion, energy and Poisson Equation (from fluid dynamics and thermal physics)
- Then the perturbed fluid equations are:



- Continuity $$\rho' + \mathrm{div}\,(\rho_0 \boldsymbol{\delta r}) = 0$$
- Equation of motion $$\rho_0 \frac{\partial^2 \boldsymbol{\delta r}}{\partial t^2} = \rho_0 \frac{\partial \boldsymbol{v}}{\partial t} = -\nabla p' + \rho_0 \boldsymbol{g}' + \rho' \boldsymbol{g}_0$$
  pressure **buoyancy** **gravity**
- Poisson $$\nabla^2 \Phi' = 4\pi G \rho'$$
- Energy equation $$p' + \boldsymbol{\delta r} \cdot \nabla p_0 = \frac{\Gamma_{1,0} p_0}{\rho_0} (\rho' + \boldsymbol{\delta r} \cdot \nabla \rho_0)$$

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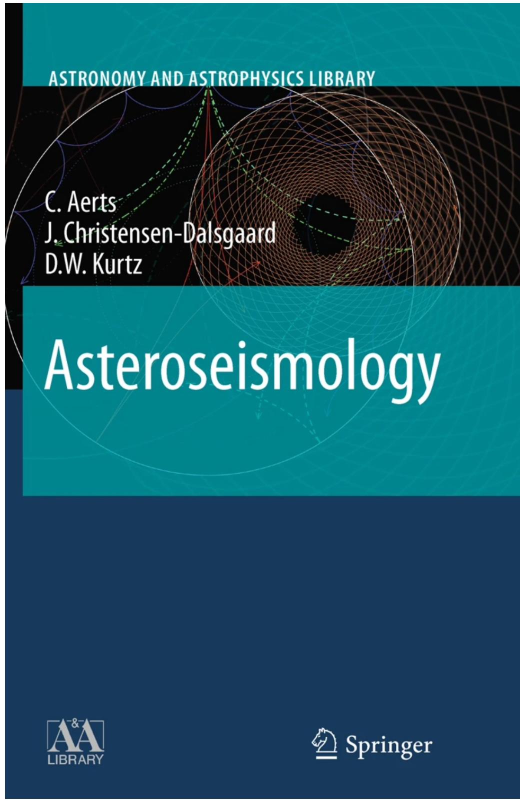

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Physics of Stellar Oscillations

MESA

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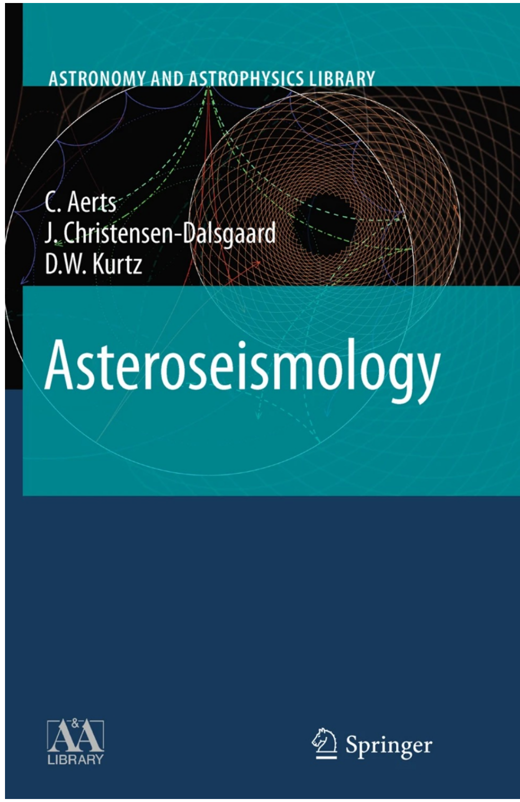

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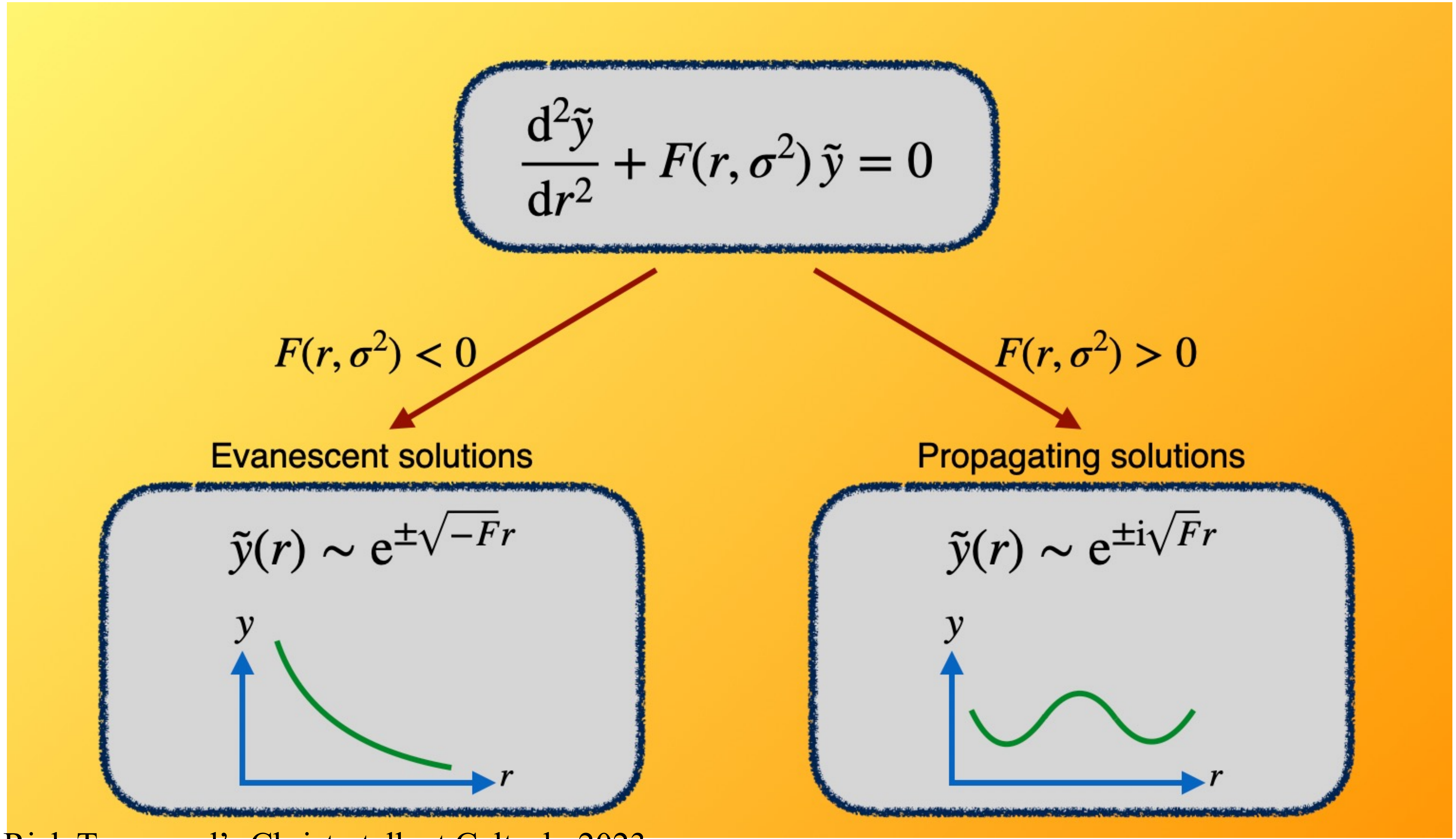
Next Step…

- The solution to those differential equations are assumed to have periodic time dependence $\rho', P', \ldots \propto \mathrm{e}^{\mathrm{i}\sigma t}$
- Separate variables in spherical coordinates $y'(\mathbf{r}) = \tilde{y}(r)\, Y_\ell^m(\vartheta, \varphi)$
- Combining the continuity equation, equation of motion, and energy equation, the equation for the mode radial displacement can be rewritten as:

$$\frac{\mathrm{d}^2\tilde{y}}{\mathrm{d}r^2} + F(r, \sigma^2)\, \tilde{y} = 0$$

Evanescent and Propagating Solutions



Slide credit to: Rich Townsend's Christy talk at Caltech, 2023

Mode Classification

p-mode: $\sigma^2 > N^2$ and $\sigma^2 > S_\ell^2$

g-mode: $\sigma^2 < N^2$ and $\sigma^2 < S_\ell^2$

$$F(r,\sigma^2) = \frac{1}{\sigma^2 c^2}\left(\sigma^2 - S_\ell^2\right)\left(\sigma^2 - N^2\right)$$

square of Lamb frequency
sound wave frequency

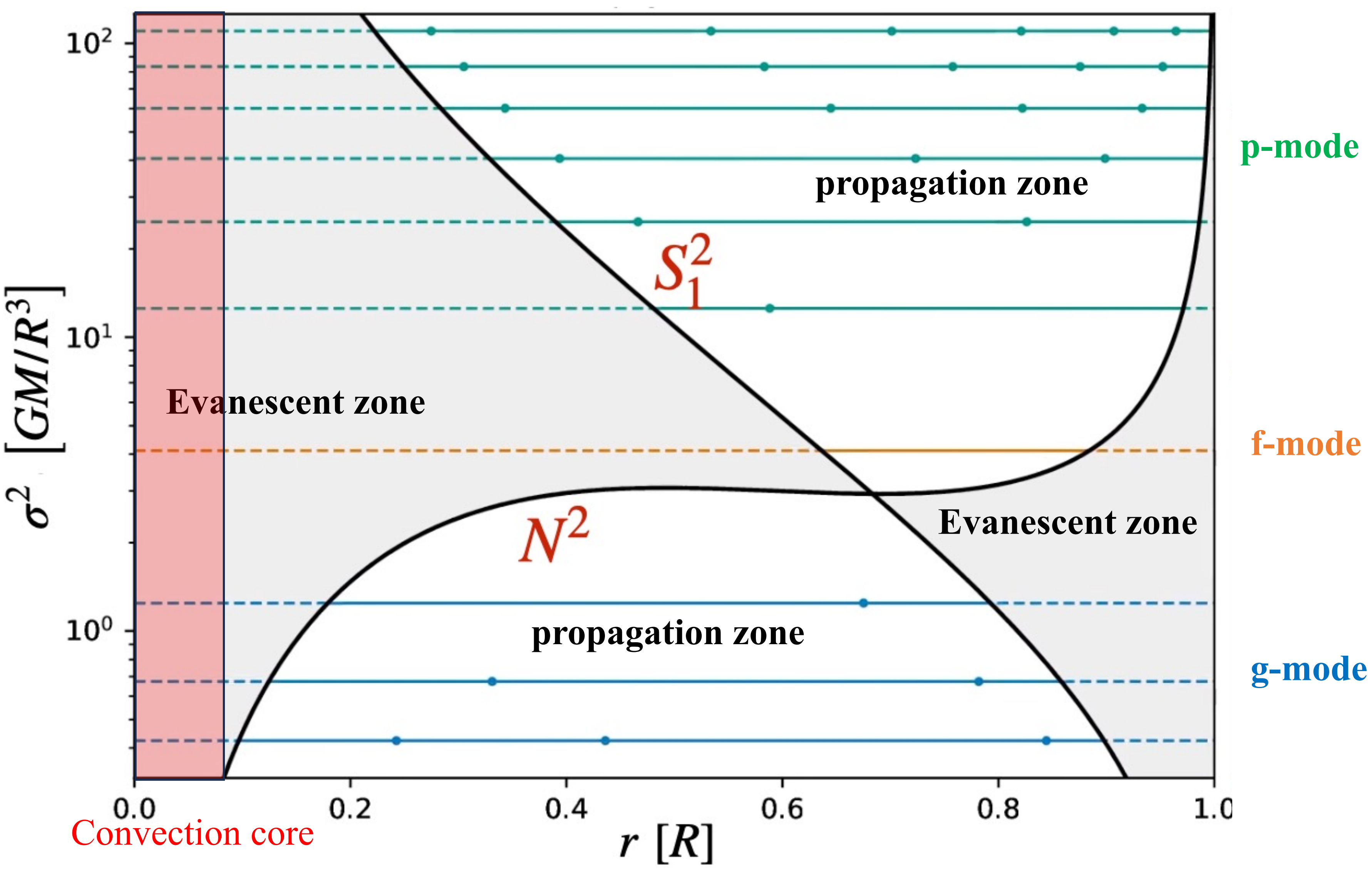
$$S_\ell^2 \equiv \frac{\ell(\ell+1)c^2}{r^2}$$

square of Brunt-Väisälä frequency
buoyancy frequency

$$N^2 \equiv \frac{1}{\gamma}\frac{\mathrm{d}\ln P_0}{\mathrm{d}r} - \frac{\mathrm{d}\ln \rho_0}{\mathrm{d}r}$$

Slide credit to: Rich Townsend's Christy talk at Caltech, 2023

Propagation Diagram of a Given Massive Star



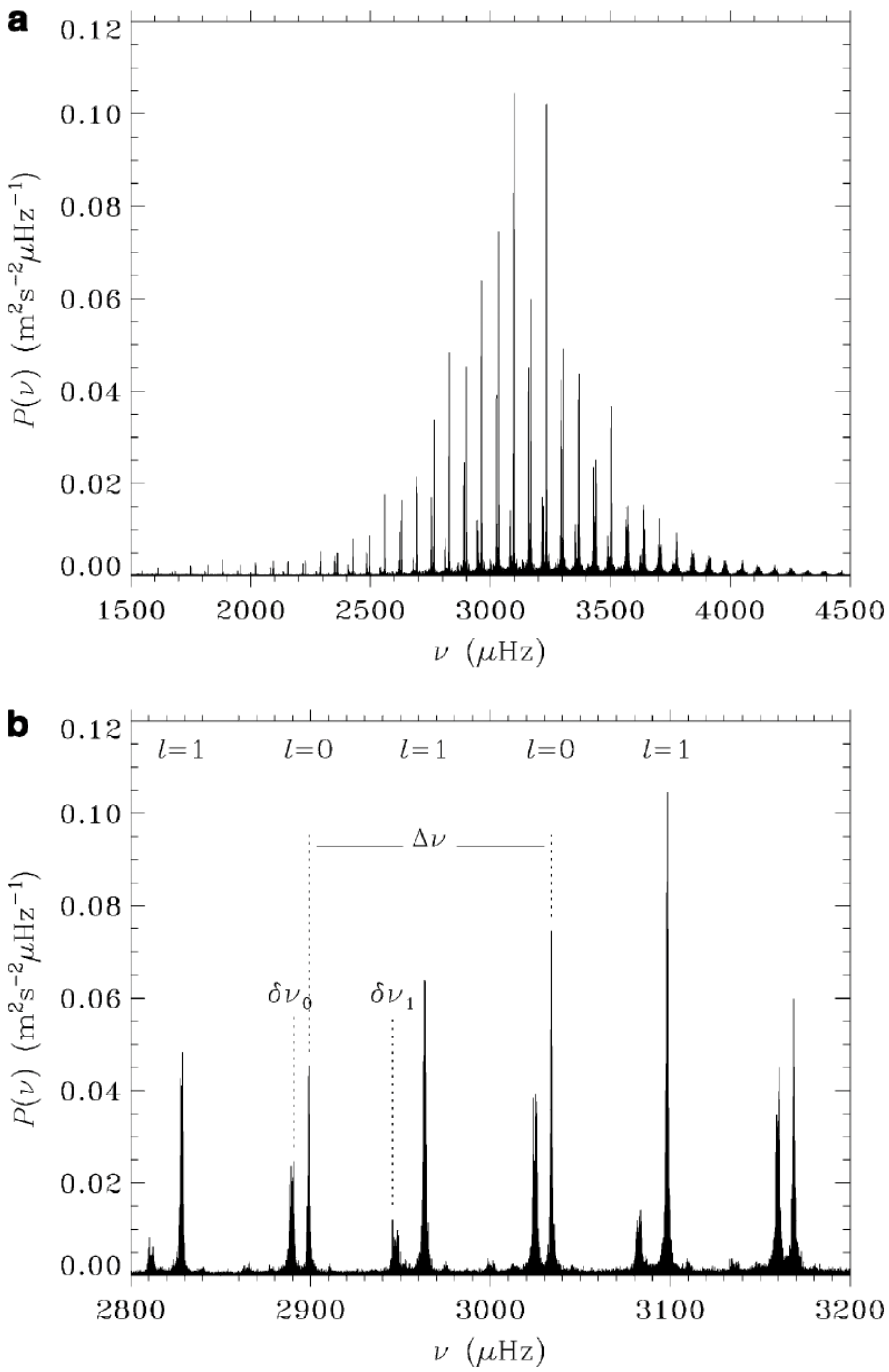
Base plot credit to: Rich Townsend's Christy talk at Caltech, 2023

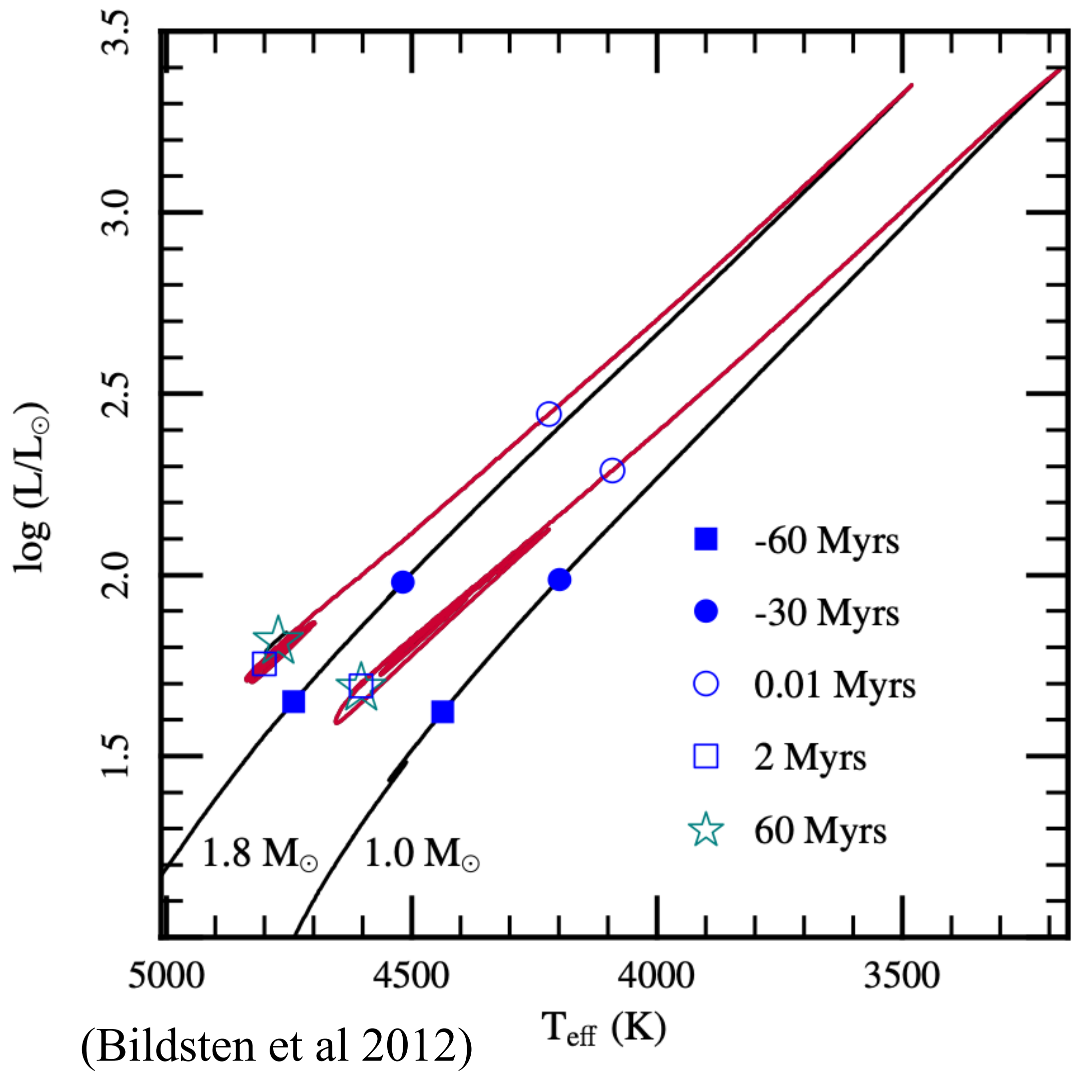
Application 1: Testing if Stellar Evolution Code is Correct

- Surprisingly, the very classical process, helium flash had been successfully modeled in recent 15 years, with MESA (Bildsten et al 2012)

- Three ways to test if code doing physics correctly:
  - Result is converged
  - Compare with analytical solutions (e.g. polytrope models)
  - Compare with observations

Application 1: Testing Helium Flash Model





(Bildsten et al 2012)

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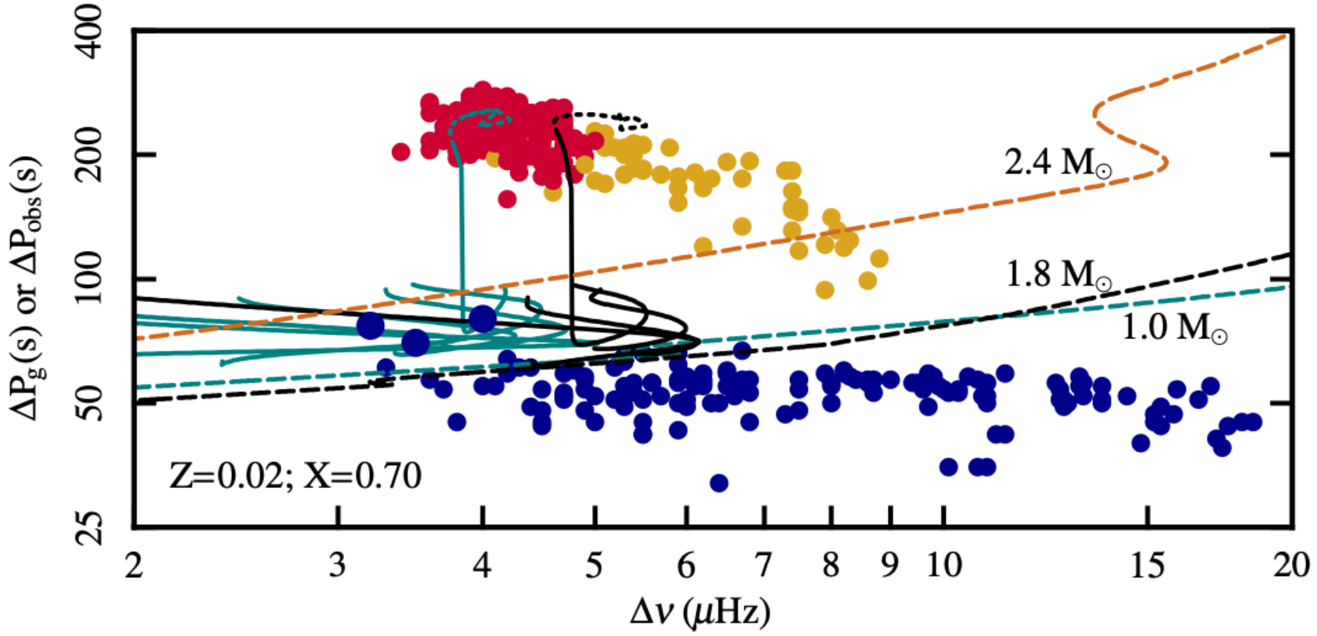
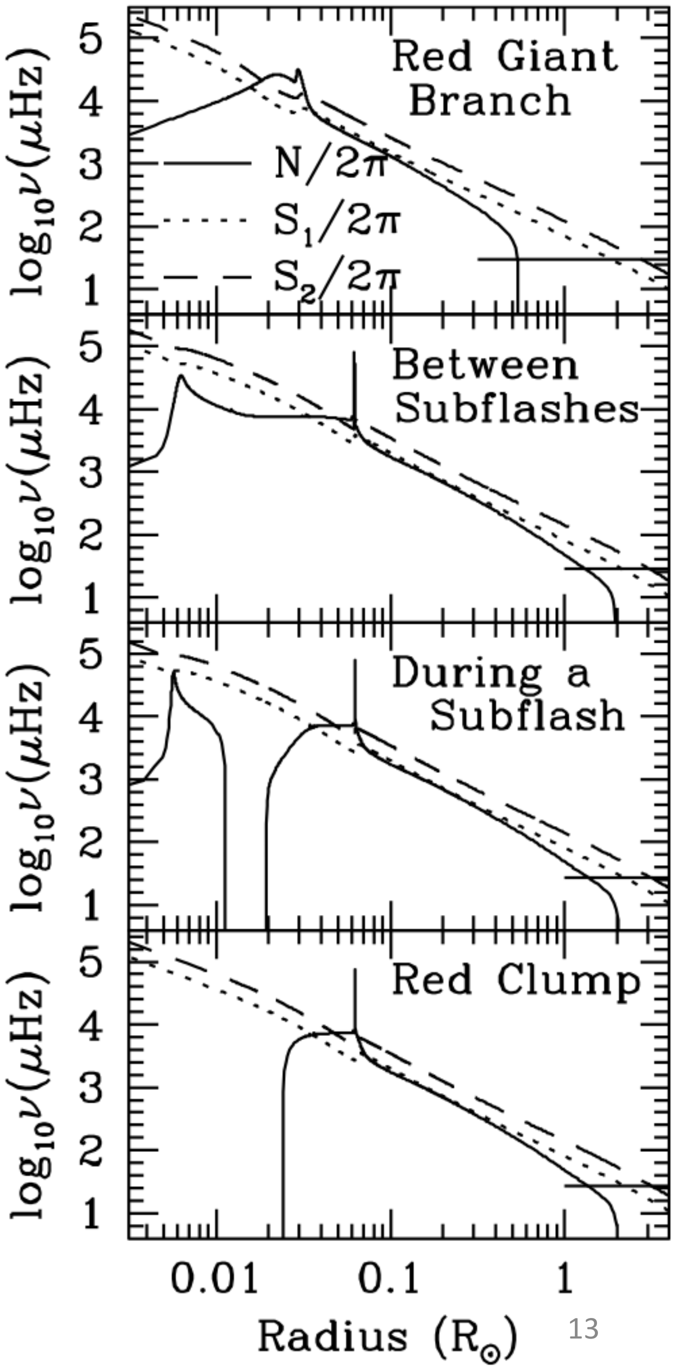


- Dashed lines: RGB stars, burning H into He in the shell;
- Solid lines: flashes;
- Dotted lines: helium burning red clump stars.

- Red clump stars, experiences through helium flashes, has longer period spacing

Model: Bildsten et al, 2012; Data: Bedding et al, 2011



Minilab 1 Make a model of KOI-54 star

- Why we used three inlists for running the model?
- Should we call this model as massive or low-mass star? Reason?

Application 2: Constrain the Inner Structure of Stars

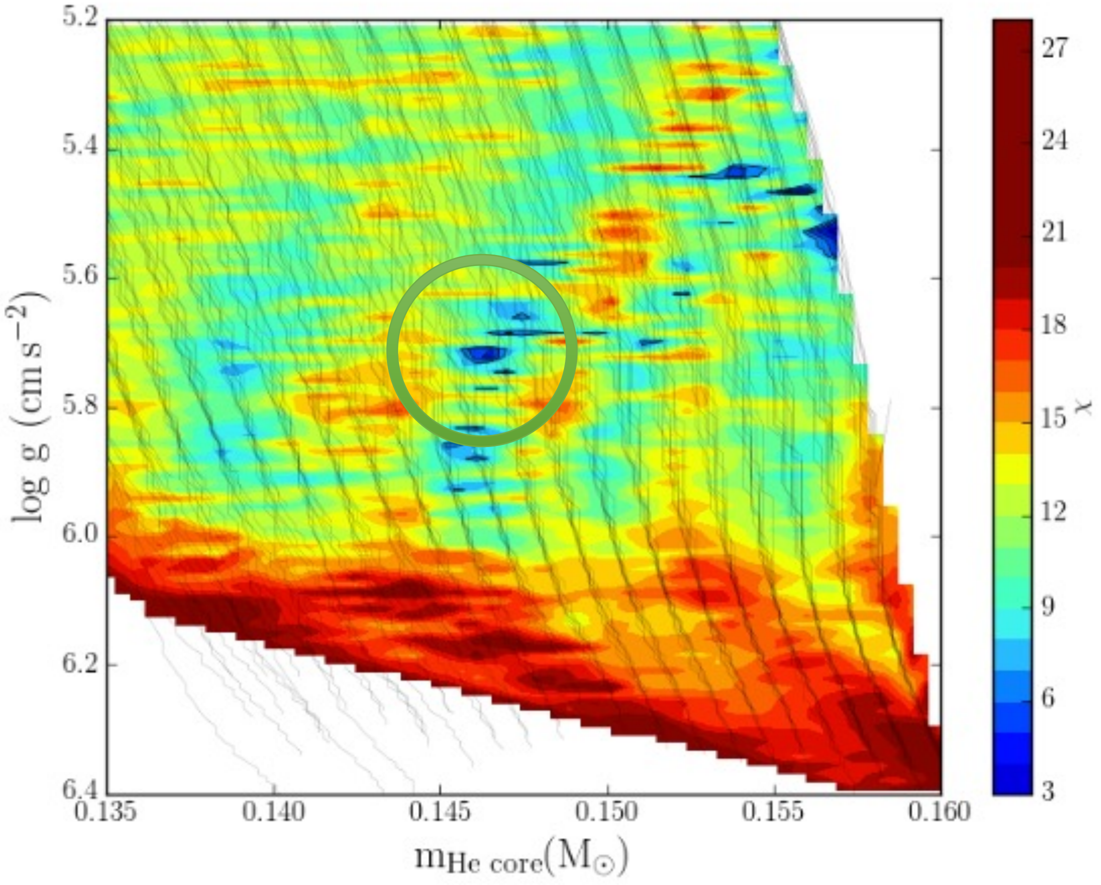
Observation found 7 pulsating periods of this extremely low-mass white dwarfs in the J1112 system, two of them are possible p-modes!

Table 2
Frequency Solution for SDSS J111215.82+111745.0

ID	Period (s)	Frequency (μHz)	Amplitude (mma)	S/N
f_1	2258.528 ± 0.003	442.7662 ± 0.0007	7.49 ± 0.08	26.5
f_2	2539.695 ± 0.005	393.7480 ± 0.0007	6.77 ± 0.09	23.0
f_3	1884.599 ± 0.004	530.6170 ± 0.0011	4.73 ± 0.08	16.9
f_4	2855.728 ± 0.010	350.1734 ± 0.0013	3.63 ± 0.09	11.5
f_5	1792.905 ± 0.005	557.7542 ± 0.0017	3.31 ± 0.08	11.9
f_6	134.275 ± 0.001	7447.388 ± 0.010	0.44 ± 0.08	*4.4*
f_7	107.56 ± 0.04	9297.4 ± 3.6	0.38 ± 0.14	*4.1*

Note. 1 mma = 0.1% relative amplitude.

(Hermes et al 2013)



Color: rms difference between the 7 measured and the model periods (my unpublished plot)

- Spectroscopic data (all surface information):

log g = 5.9 cm/s^2
T_{eff} = 9400 K

- Best-fit model:

$m_{He} = 0.146\ M_\odot$
$m_H = 0.014\ M_\odot$
(10% hydrogen!)
$m_{tot} = 0.160\ M_\odot$

$R = 0.08\ R_\odot$ (big radius!)

(Sun & Arras, 2018)

Tidal Physics

Post-Main Sequence Evolution of Binaries: 3 Effects

1. On the Red Giant and Asymptotic Giant Branches, Stars Expand to >1AU radius
   - All close-in planets and stars are engulfed by the expanding star
2. Orbital Decay Due to **Tides** Brings the Companion inward to Meet the star, Rather than Waiting for the Star to Expand to the Orbit
3. Due to the Strong Wind of the Stars, the Orbits Expand
   - All close-in planets and stars are survived from merger

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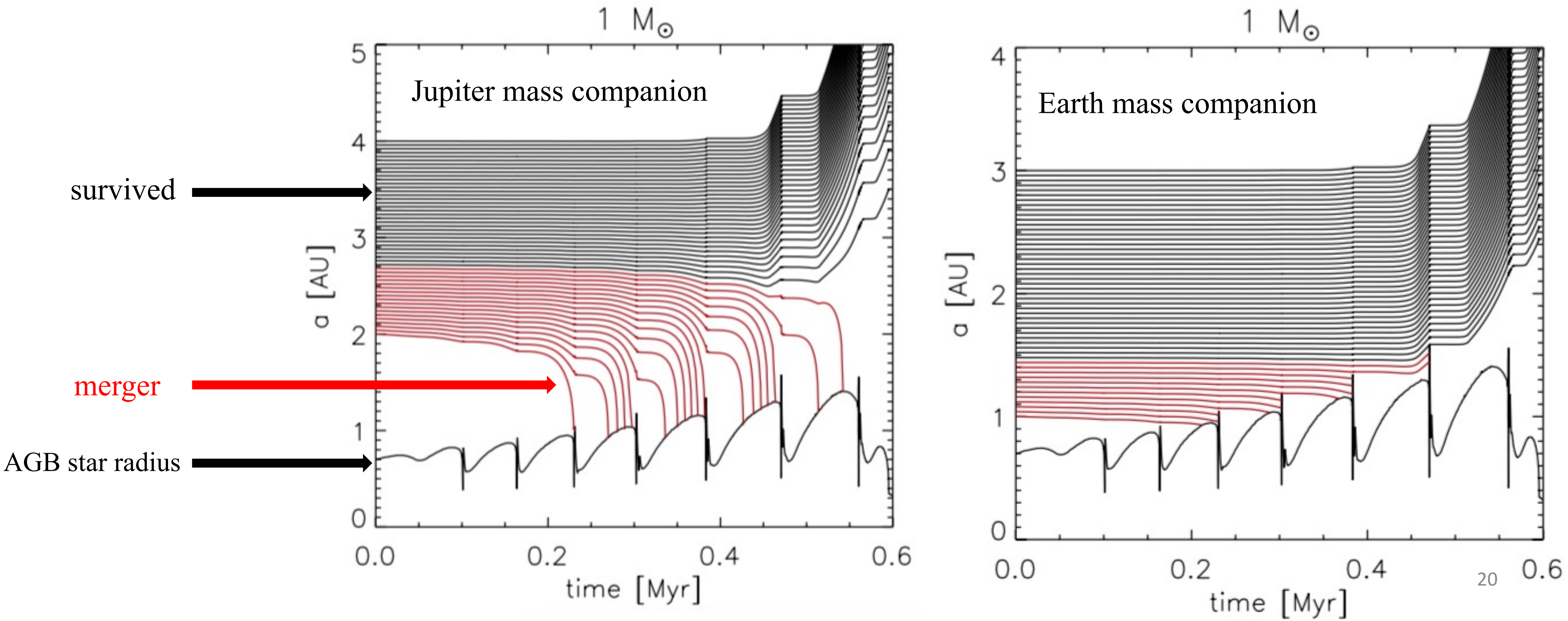
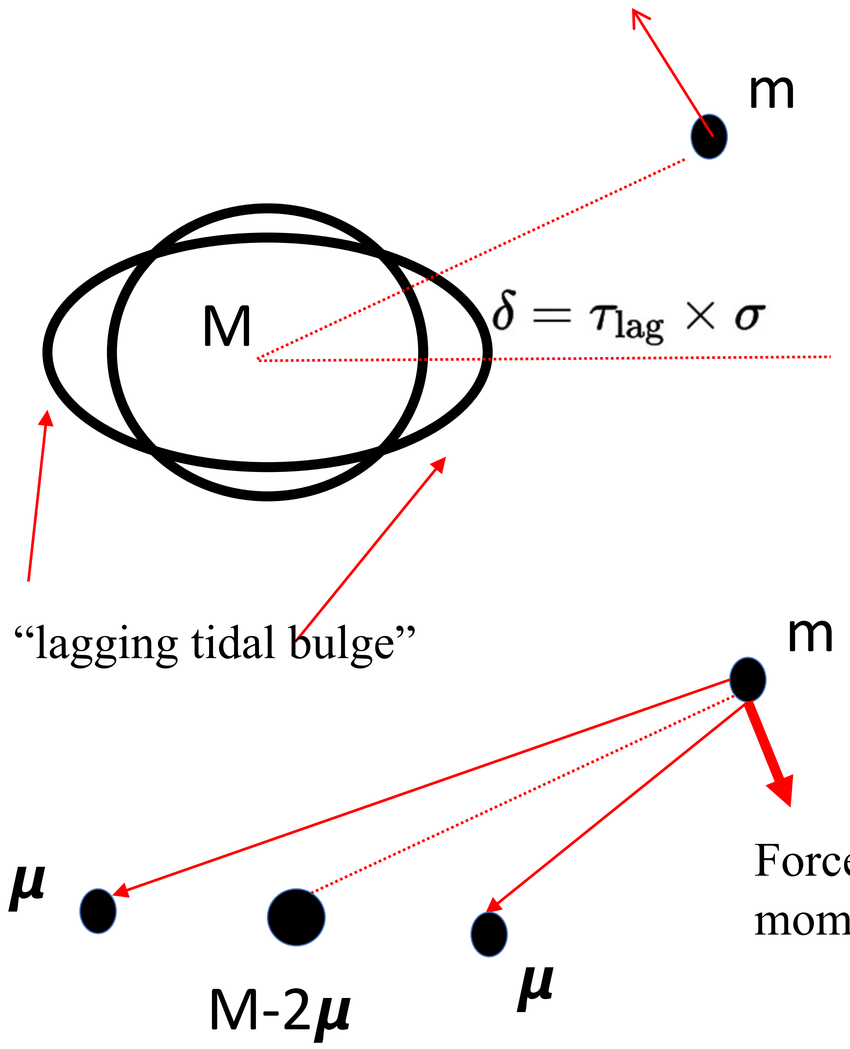
Previous Work on Engulfment of the Secondary by Evolved Stars

Mustill, A. J., & Villaver, E. 2012, ApJ, 761, 121



Basic Idea of How Tides Change the Orbit

G. Darwin's theory of tides: Friction causes the tidal bulge to lag behind the companion.



Force decreases the orbital angular momentum

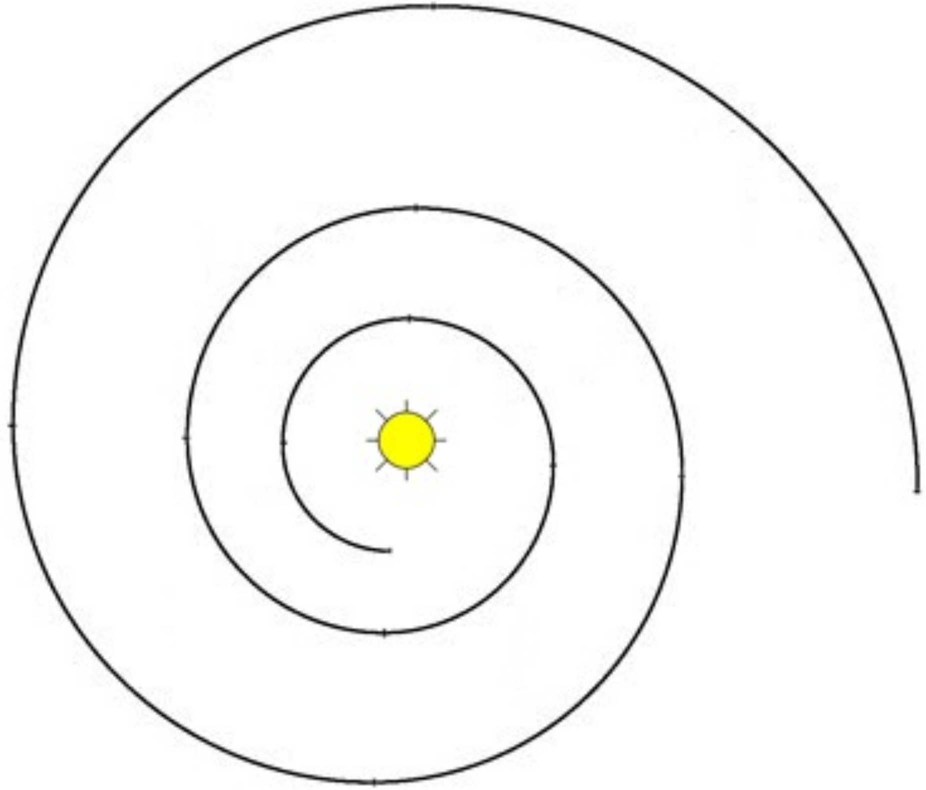
- The spin angular momentum of the star increases and the orbital angular momentum decreases
- The orbit will shrink
- The lag time is a free parameter

$$\dot{a} = -\frac{a}{\tau_{\text{lag}}}\left(\frac{m}{M}\right)\left(1+\frac{m}{M}\right)\left(\frac{R}{a}\right)^8\left(1-\frac{\Omega}{n}\right)$$

Hut P. (1981), A&A, 99, 126

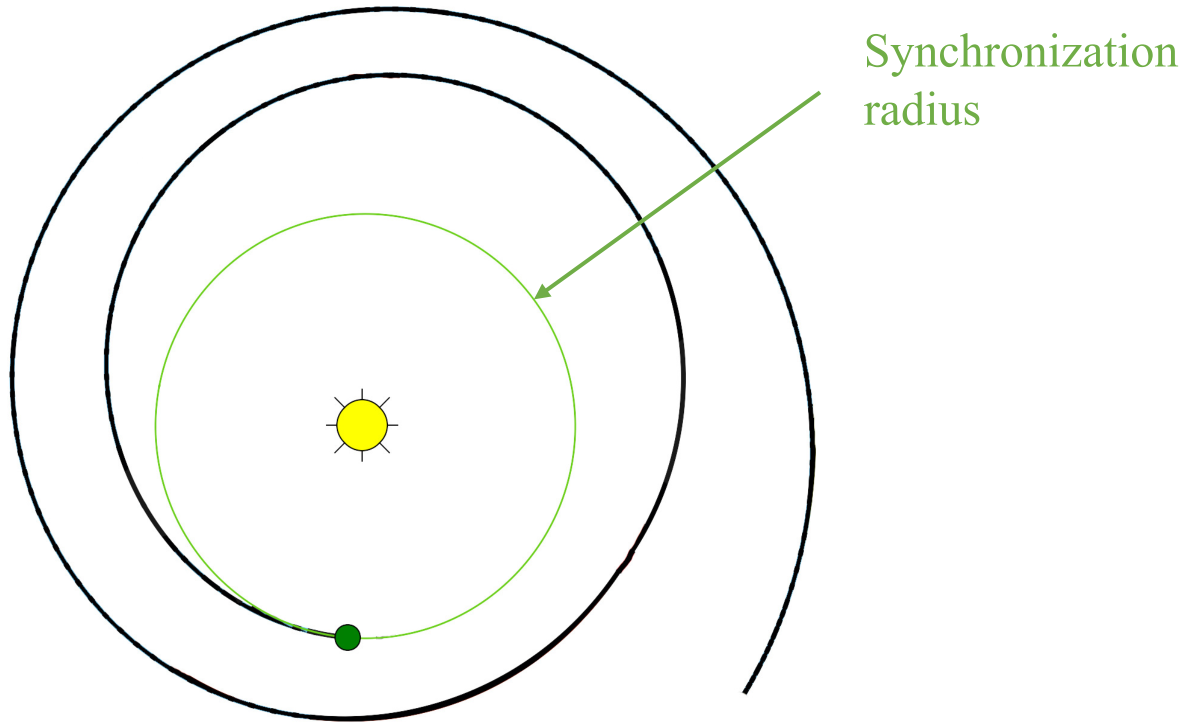
Two Possible Regimes of the Tidal Evolution

Low mass companion



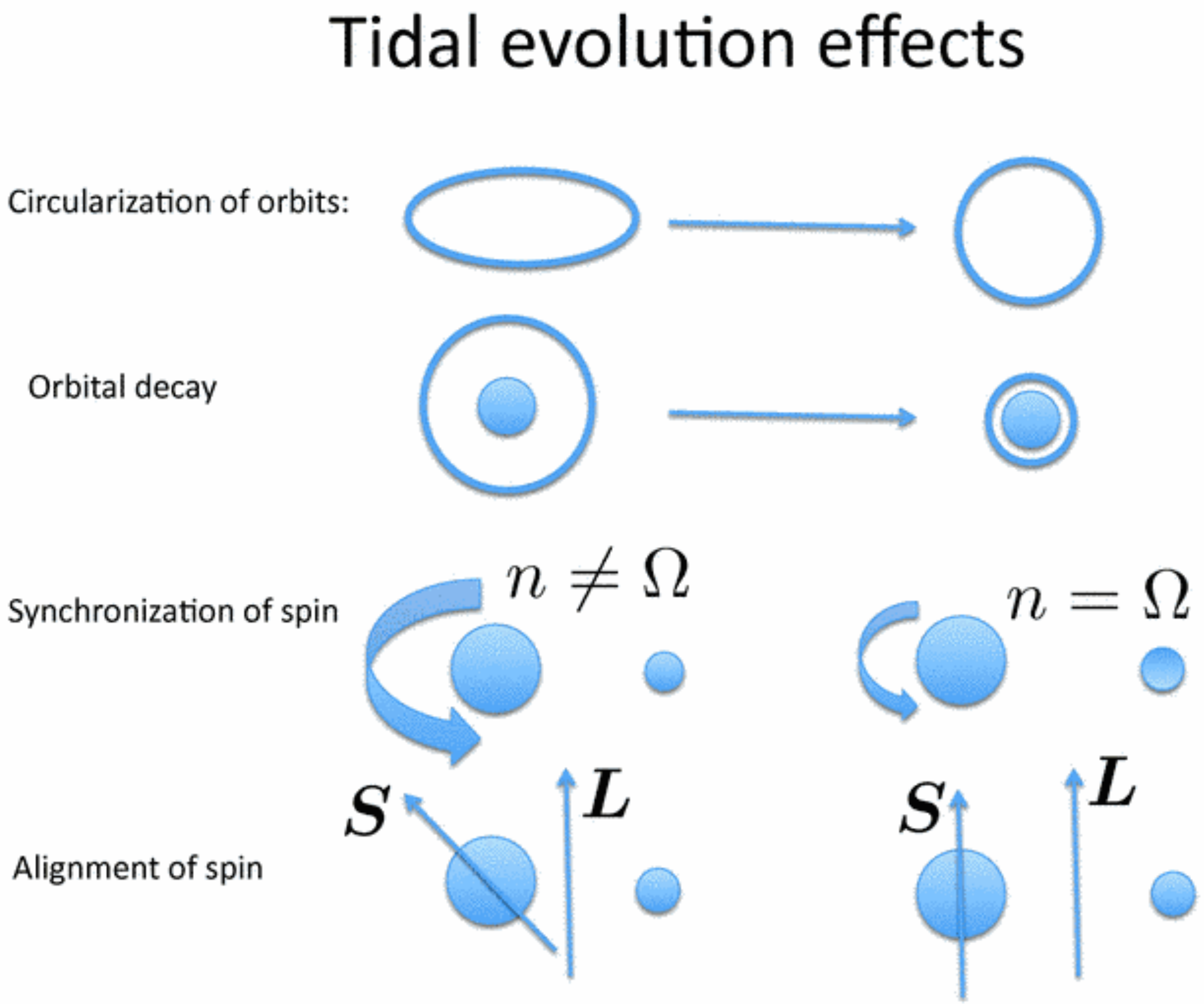
- Can’t synchronize the stellar spin
- Orbit decay
- Merger

High mass companion



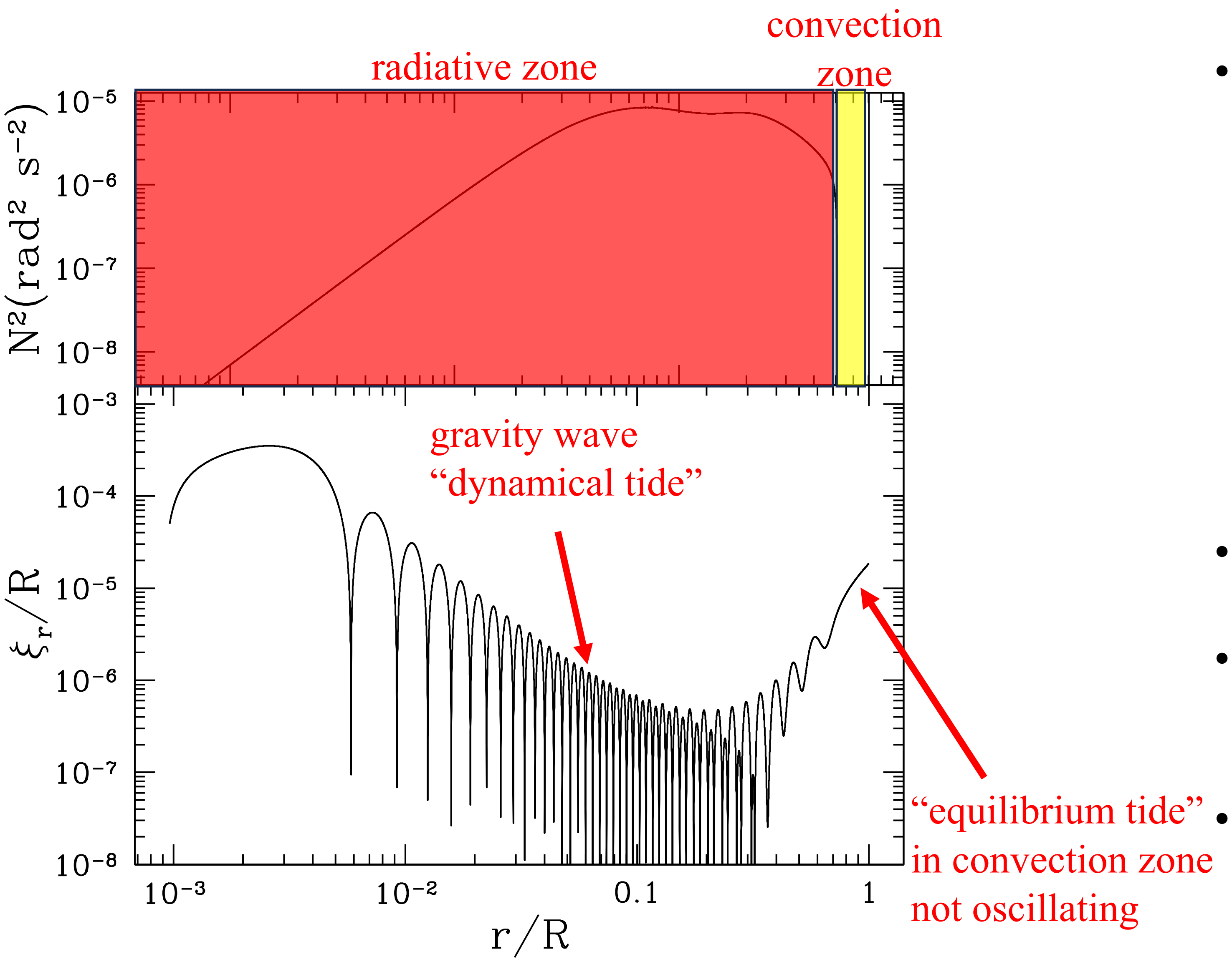
- Can synchronize the stellar spin
- Orbital decay slows down to the nuclear evolution timescale of the primary

Summary of How tides change the orbit of a two body system



Slide Credit: Phil Arras (KITP talk: Tides and Nonlinear Waves in Solar Like Stars, 2011/10/28)

Physics of Tidal Flow



- Add in the tidal acceleration in the momentum equation for nonadiabatic oscillations

$$\sigma_{m,k}^2 \tilde{\xi}_{r;} = \frac{1}{\rho}\frac{d\tilde{P}'}{dr} - \frac{\tilde{\rho}'}{\rho^2}\frac{dP}{dr} + \frac{d\tilde{\Psi}'}{dr},$$

$$\sigma_{m,k}^2 \tilde{\xi}_{h} = \frac{1}{r}\left(\frac{\tilde{P}'}{\rho} + \tilde{\Psi}'\right),$$

- Compute linear response to the tidal force
- Equilibrium tide approximation: set ω=0 to get non-resonant response due to tides.
- Dynamical tide approximation: resonant excitation of internal gravity waves.

Numerical solution is to deal with the lag time angle

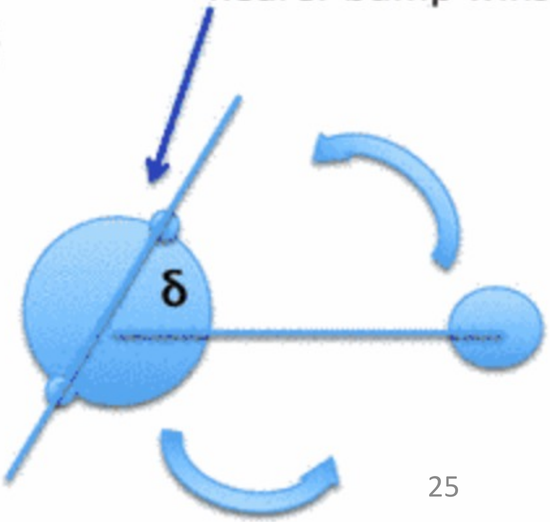
Origin of Secular Tidal Evolution

tidal potential due to companion: $U \sim \epsilon\ e^{-i\omega t}$ $\quad$ $\epsilon =$ strength of tide

density perturbation response to tide: $\delta\rho \sim \epsilon\ e^{i(\delta - \omega t)}$ $\quad$ lag time $= \delta/\omega$ due to dissipation

external potential from this density perturbation acts back on companion: $\delta\phi \sim \epsilon\ e^{i(\delta - \omega t)}$

out-of-phase force on companion leading to secular evolution: $f \sim \epsilon^2 \sin(\delta)$



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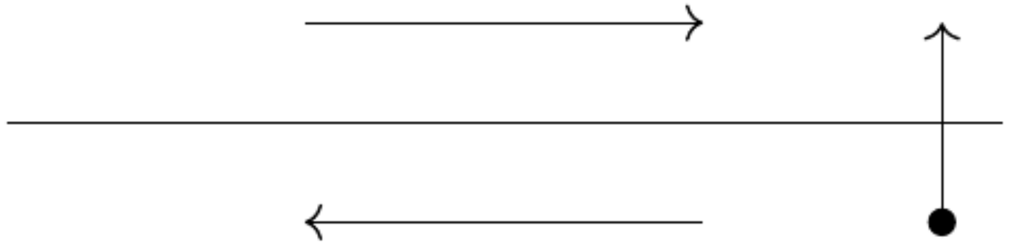
Turbulent Viscosity Damping in Convective Zones

Test particle case:

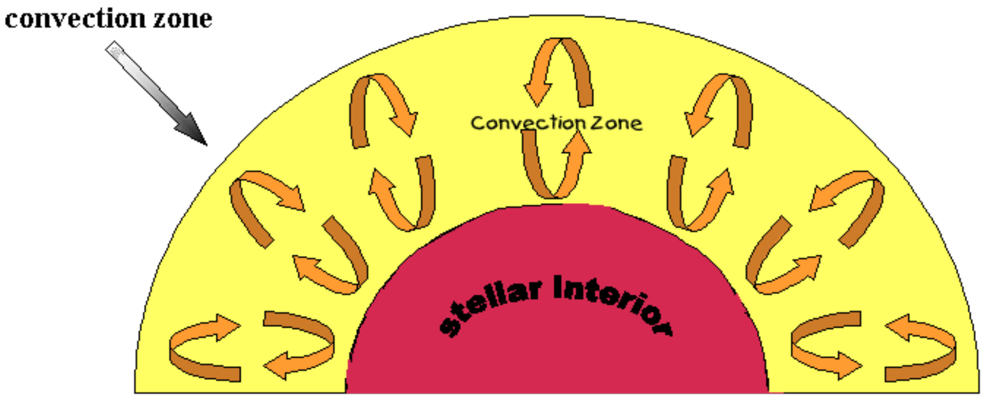
Molecular viscosity



tiny!

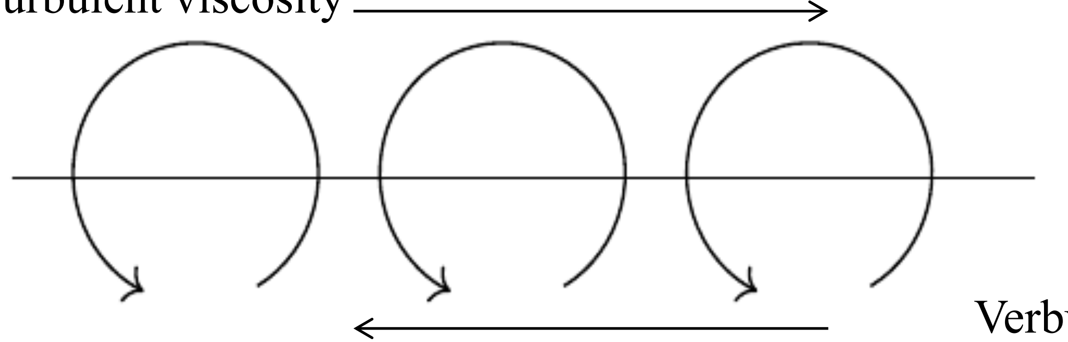
$$\vec{F}_{\text{shear}} = \rho \nu_{\text{mol}} \nabla^2 \vec{v}_{\text{tide}}$$

During this process, a shear force is generated, and this corresponds to an Energy Dissipation Rate



Eddies in stellar convection zone:

Turbulent viscosity



$$\nu_{\text{turb}} = \ell_{\text{eddy}} v_{\text{eddy}}$$

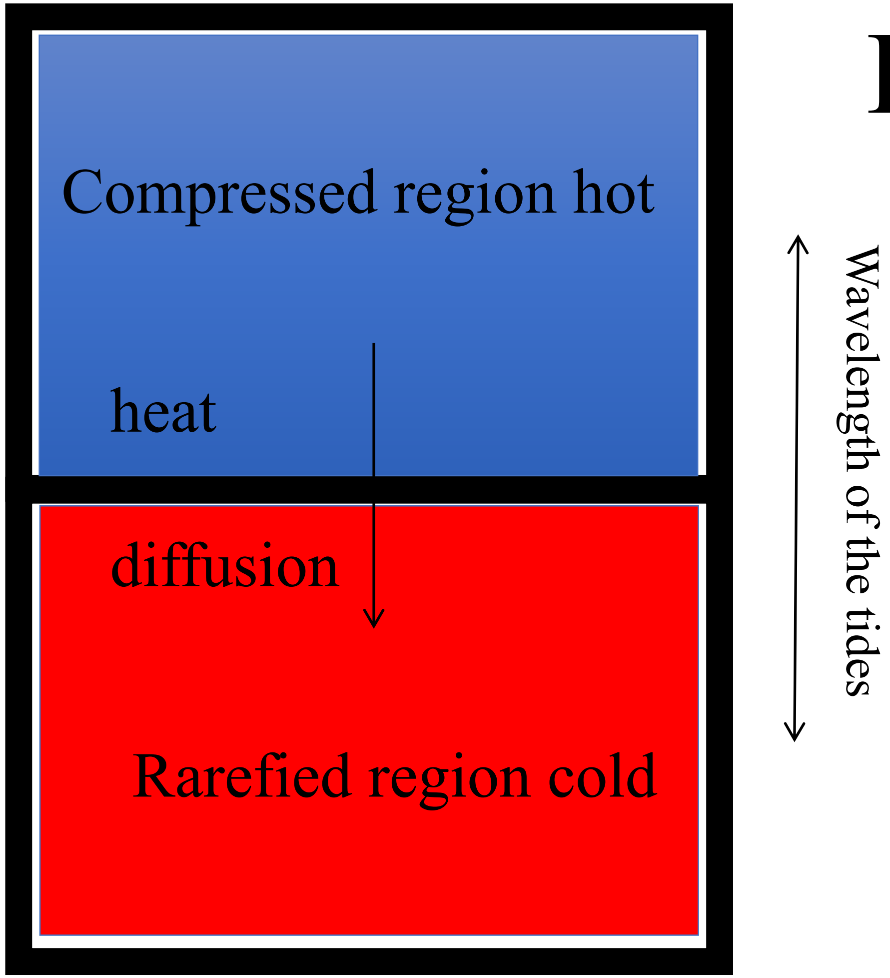
$$\dot{E} \approx M \nu_{\text{turb}} |\nabla \vec{v}_{\text{tide}}|^2$$

Fluid dynamics

The rate of change in orbital parameters (observable) are from this Energy Dissipation Rate

Verbunt, F., Phinney, E. S. 1995, A&A, 296, 709

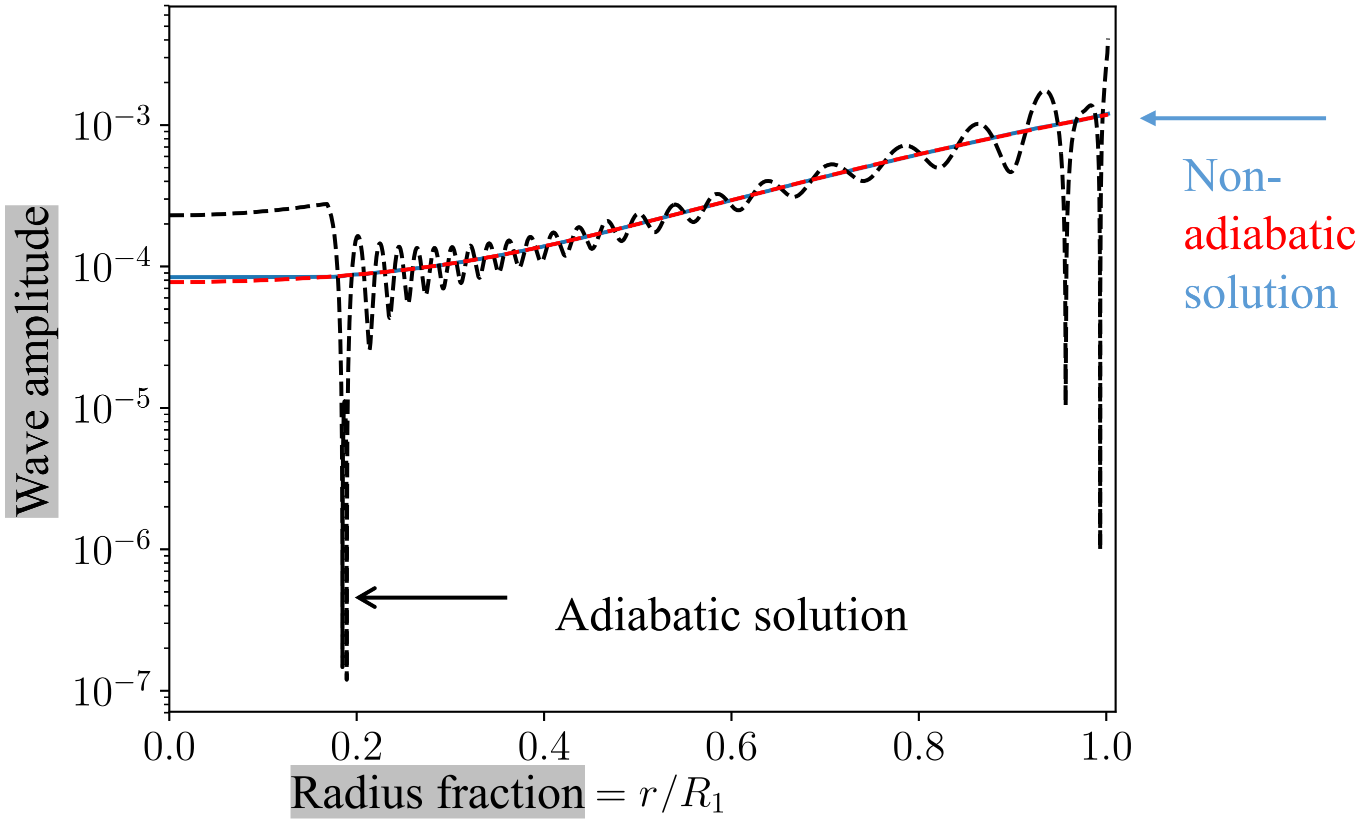
Radiative Diffusion Damping



- χ = heat diffusion coefficient
- P = 2π/ω = wave period
- Heat diffused distance in time P is d ≈ $(\chi P)^{1/2}$

- If diffusion distance d ≥ wavelength

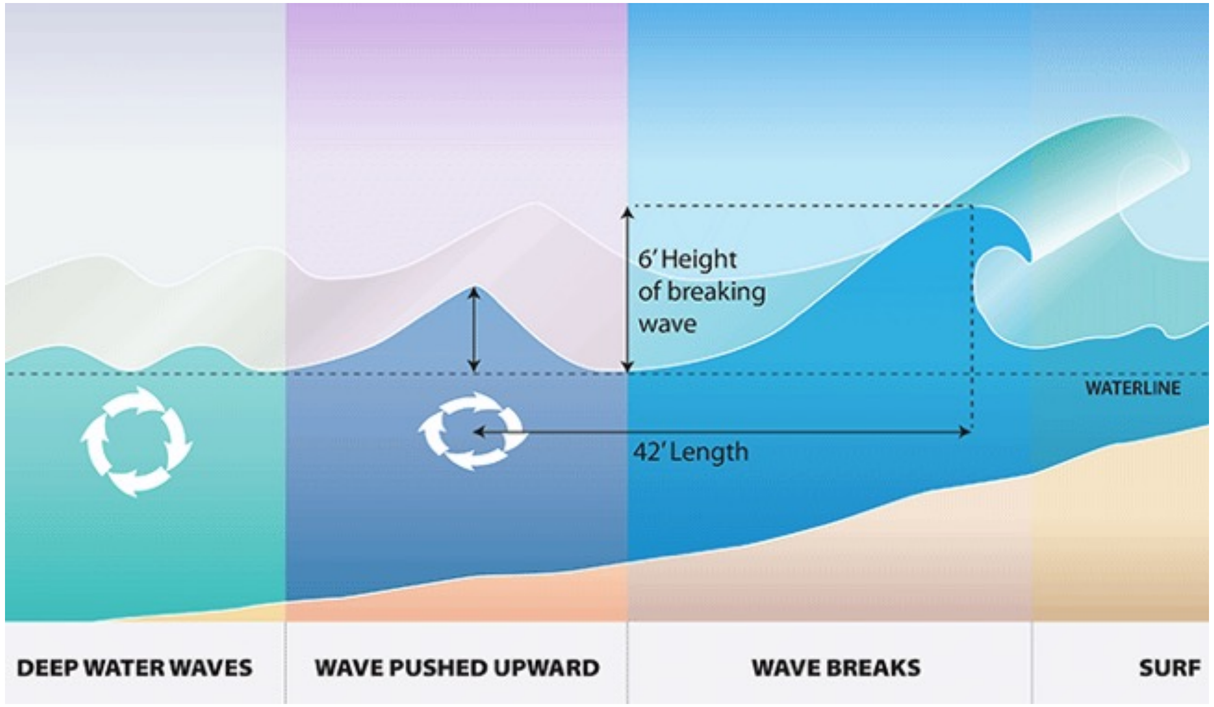
=> the wave will be strongly damped by radiative damping



A high order internal gravity wave mode with strong radiative damping. The nonadiabatic solution is close to the equilibrium tide solution (no oscillatory feature).

Nonlinear Damping in Water Waves

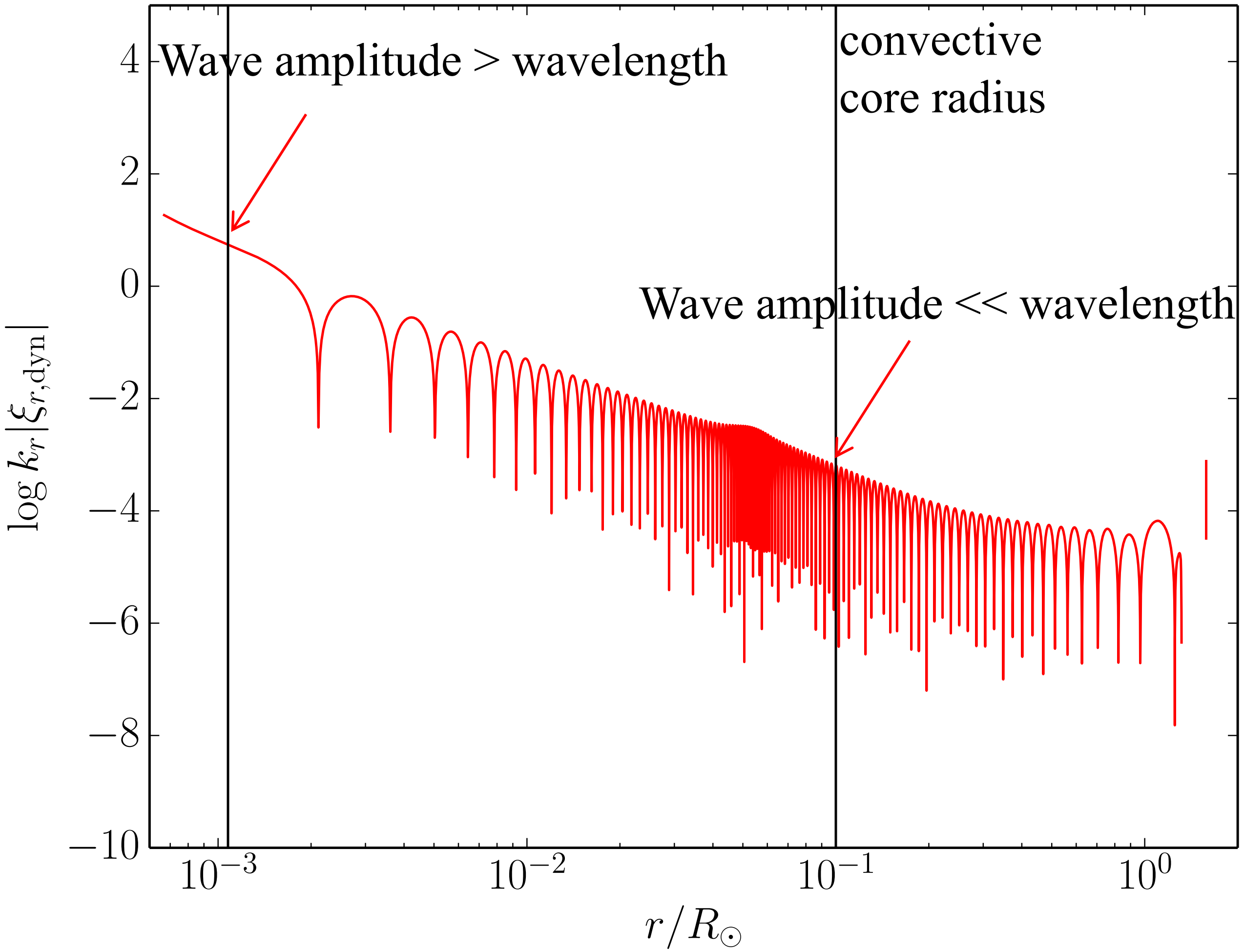
- Ocean is shallower near the shore
- The wave height increases near the shore
- The criterion for wave braking is wave height > wavelength



Nonlinear Damping for the Dynamical Tides

- Gravity wave steepen towards the center of the star, $\xi_r \sim 1/r^2$
- Wave braking when the wave amplitude > wavelength
- After the wave breaking, the wave energy deposits as heat

- The radiative core allows the dynamical tide to grow to large amplitude and break
- Gravity waves are evanescent in a convective core, giving small amplitude and no wave breaking



Minilab 2: Take Home Points

Table 2: Astroseismic Mode Properties

Name	Analogous to	Restoring Force	Dispersion Relation	Relation between# nodes and Wave Frequency	Amplitude Property
p-mode	sound/acoustic wave	pressure ($\nabla p'$)	$\sigma^2 = c^2(k_r^2 + k_h^2)$	increase	$\xi_r > \xi_h$
f-mode	surface gravity/deep water wave	gravity	$\sigma^2 = \frac{g_0}{R}\frac{2\ell(\ell-1)}{2\ell+1}$		–
g-mode	internal gravity wave	buoyancy ($\rho' g_0$)	$\sigma^2 = \frac{N^2}{1 + k_r^2/k_h^2}$	decrease	$\xi_r < \xi_h$

With WKB analysis:

- High order p-mode has constant frequency spacing
- High order g-mode has constant period spacing

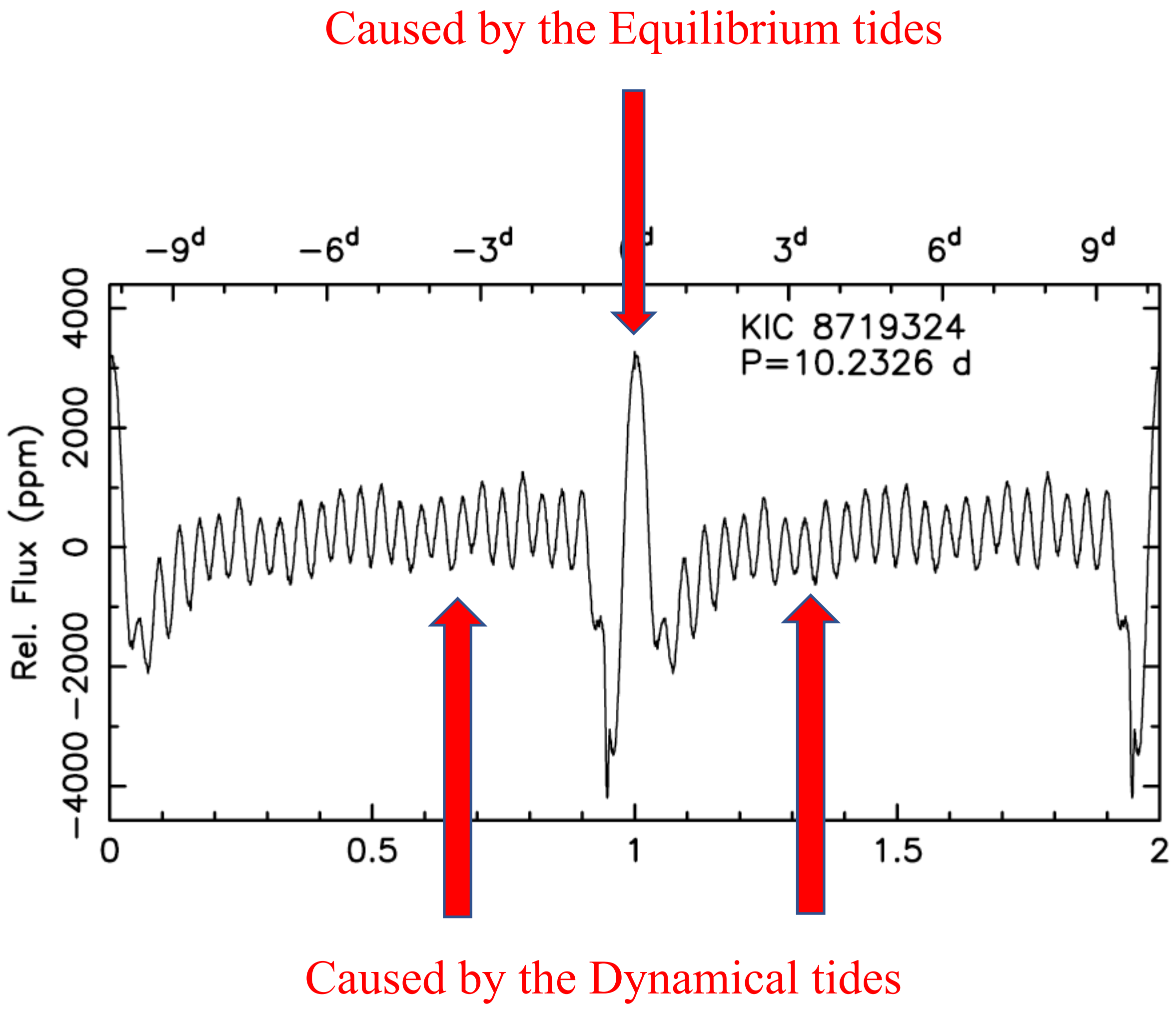
Introducing GYRE-TIDES: a New Open-Source Code to Model Stellar/Binary Tides

Sun, M., Townsend R. H. D., Guo Z. 2023, ApJ, 945, 43

GYRE

- Regular GYRE: free oscillation (natural mode of the star); GYRE-tides: forced oscillation;
- Traditional way of calculating tidal dissipation rate is incorrect;
- GYRE-tides: no approximation, fully numerical;
- Wide applications: from massive binaries (tidal induced oscillations) to planetary systems (orbital decay).
- Code is available at https://gyre.readthedocs.io/en/stable/user-guide/frontends/gyre_tides.html

Tides in Massive Stars – the Heartbeat Phenomenon



Thompson et al. (2012), ApJ, 753, 86

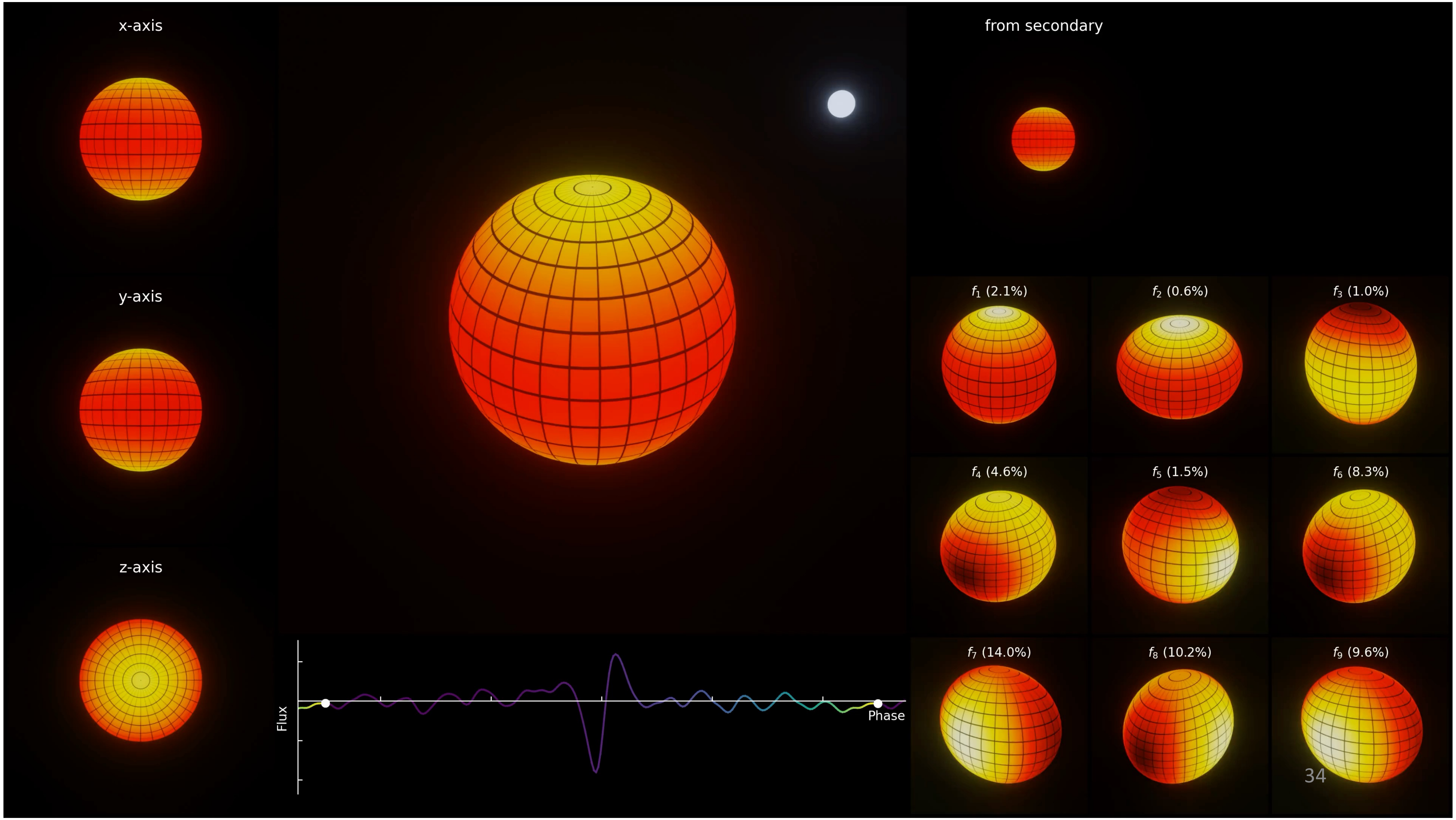
The Heartbeat features are usually observed in:

- high eccentricities binaries ($0.3 < e < 0.9$);
- intermediate-mass stars ($1.2\ M_\odot < M < 2.5\ M_\odot$);
- have also recently been reported in high-mass stars (up to $30\ M_\odot$).

The Kepler mission found 173 heartbeat stars (Kirk et al 2016), almost all of these stars are A- and F-type main sequence stars;

Other space and ground-base observations found heartbeat phenomenon for O and B type stars.

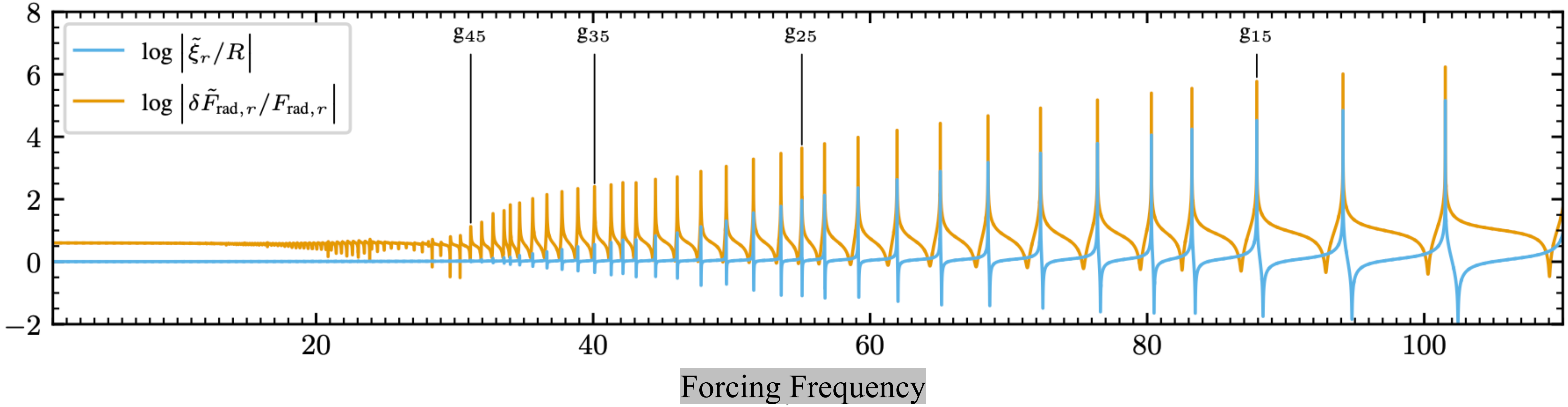
The Heartbeat Stars from the Eccentric Binaries



Movie credit:
Rich Townsend

Modeling the Heartbeat Star KOI-54

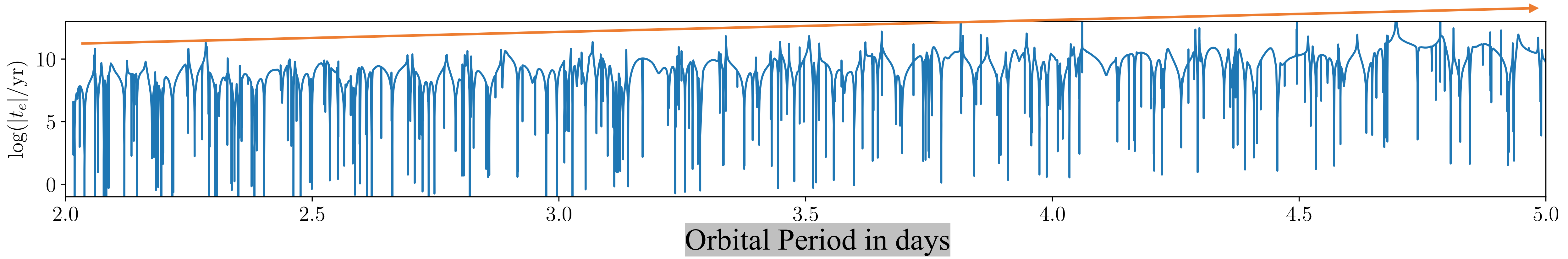
Sun, M., Townsend R. H. D., Guo Z. 2023, ApJ, 945, 43



↑ Surface displacement and flux versus forcing frequency for KOI-54 system, forced by a fixed-strength potential.

- The two stars in KOI-54 have similar mass, 2.32 $M_\odot$ and 2.38 $M_\odot$ with an orbital period of 42 days. The system is highly eccentric with e=0.83.
- At low forcing frequency (left of the figure), the solution is dominated by the equilibrium tides; At high forcing frequency, the spikes correspond to the excitation of the internal gravity waves (also known as the dynamical tides).

Tides in Changing the Orbital Elements of the Binary Systems

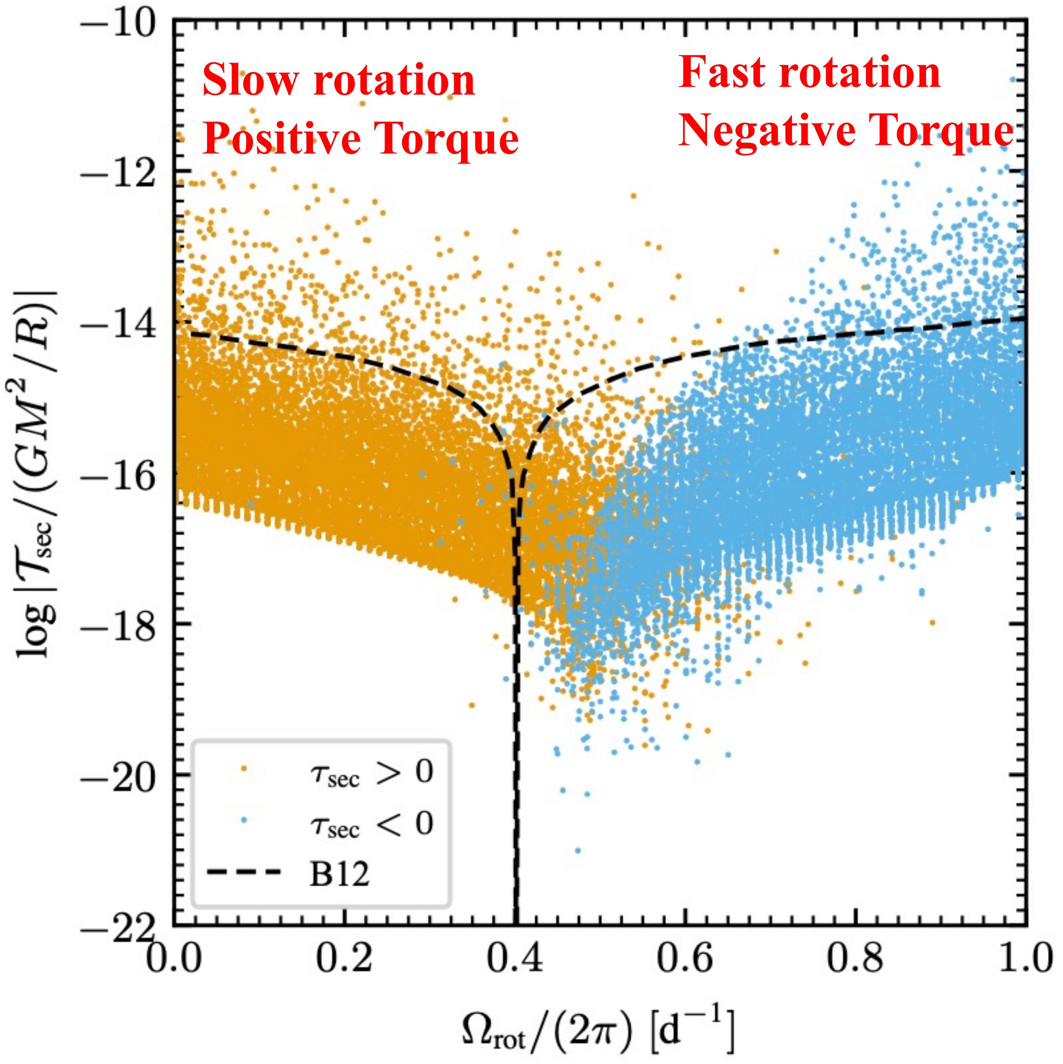


→ the rate-of-change in eccentricity as a function of orbital period, predicted by GYRE-tide for an eccentric 1.4 solar mass neutron star raising tides on a 5 solar mass main-sequence primary.

- Generally, these timescales are smaller for short orbital periods, and larger for long periods (the strong dependence of tidal strength on orbital separation).

- The spikes can be seen where the timescales become very short (caused by dynamical tides).

GYRE Stellar Torque on KOI-54 Primary Star



← tidal torque on the primary star as a function of the star's rotation rate; to calculate the torque on the KOI-54 primary star, we use gyre_tides to evaluate the response of the KOI-54 primary star model in the tidal potential for 25,000 rotation frequencies in the interval 0 ≤ rotation frequency ≤ 1 day^-1.

- we are significantly sampling the dense forest of resonances;
- a torque that's generally a positive torque at small rotation frequencies, and negative at high frequencies;

Tidal Physics Summary

- Traditional theory of tides: rely on parameterized equations
- Numerical solution of tidal response: equilibrium tides + dynamical tides
- Damping mechanism: convective damping, radiative damping
- New tool for understanding tides: open-source code GYRE-tides, no more parameterized equations, applicable to massive binaries

Minilab 3: Take-aways of the Theory of Tides

Table 1. Properties of the equilibrium and the dynamical tides

Properties	Equilibrium tide	Dynamical tide (internal gravity wave)
Mode Excitation	Time varying gravitational potential of the secondary object	
Numerical Solution is	nonwave-like	wave-like
Mode Damping (non-adiabatic)	Convective damping	Radiative and nonlinear damping
Radial Amplitude Strong at	Surface convective zone	Inner radiative zone
Tidal Dissipation Rate Depends on $P_{\rm orb}$	$P_{\rm orb}^{-4}$ - $P_{\rm orb}^{-6}$	$P_{\rm orb}^{-7.67}$ - $P_{\rm orb}^{-10}$