Firm Investment with Shareholder Inequality*

Michael Carter†

June 12, 2026

Abstract

When households hold equity instead of capital, incomplete markets make it unclear how corporate firms should maximize shareholder value. I resolve this longstanding issue by modeling an economy where firms maximize their net market value in a setting with heterogeneous households and endogenous equity prices. I find that changes to the household income distribution from 1970 to 2010 generate a 25% increase in the price to cash flow ratio and a 26% decrease in dividend yields, explaining approximately half of the observed change in dividend yields and a third of the observed change in price to cash flow. I also use the model to examine the role of wealth inequality through unanticipated redistribution shocks. Redistributing wealth equally across all households significantly depresses investment, leading to an output contraction lasting decades longer than a typical productivity shock.

JEL Codes: D52, E13, G12, L21

Keywords: Incomplete markets, Asset prices, Firm objectives

*I thank Aubhik Khan, Julia Thomas, Kyle Dempsey, and Gianluca Violante for valuable comments and suggestions, as well as conference and seminar participants at the Ohio State macroeconomics colloquium, the Liberal Arts Macroeconomics Workshop, the U.S. Naval Academy, Bowdoin College, Elon University, James Madison University, Lafayette College, Colgate University, the University of Mississippi, and the Econometric Society World Congress.

†Department of Economics, Lafayette College. Email: cartermj@lafayette.edu

1 Introduction

When markets are incomplete, households facing individual and aggregate risk engage in precautionary savings. When households directly hold capital and rent it to firms, this precautionary savings motive increases the capital stock.[1] In the data, however, households typically save by holding equity in corporate firms, and those firms invest on behalf of their shareholders.[2] To understand investment over the business cycle, we need to understand how corporate firms make dynamic investment decisions on behalf of their heterogeneous shareholders. Firms generally maximize shareholder value, but shareholders have different valuations across aggregate states when markets are incomplete. An investment choice that maximizes value for one shareholder may not maximize value for another.

The primary theoretical contribution of this paper is a mechanism that disciplines dynamic firm choices when shareholders have different valuations of future states. Households save in a mutual fund, which then owns the production firms. This mutual fund finds the market stochastic discount factor (SDF) as a share-weighted average of interior shareholders' expected marginal rates of substitution over aggregate states. On its own, the stochastic discount factor only *prices* an asset; it does not necessarily pin down firm behavior.[3] To discipline the production firm's choices, I introduce the threat of a private equity firm that takes over producers who are not maximizing their cum-dividend share price (or net market value). Together, these financial intermediaries solve the technical issue of firm decision-making with shareholder heterogeneity.

I use this model to quantitatively study two topics. I first examine how idiosyncratic household risk shapes firm investment over the business cycle. I find that the increase in household earnings risk seen in the data from 1970 to 2010 increases the price-cash flow ratio and decreases dividend yields. I then examine a set of unanticipated wealth redistribution shocks to study the role of wealth inequality in determining firm dynamics. Higher wealth inequality increases output and wages, though poor households are made worse off with a lower rate of return on savings.

Under incomplete markets, payoffs in each state are not uniquely priced. This uncertainty about the value of payoffs causes two problems. First, it becomes more difficult to price assets. An asset can generally be priced as $P = \mathbb{E}[M'X]$ where X is a vector of payoffs and M is the stochastic discount factor (SDF). With complete markets, this SDF is the vector of probability-weighted state-contingent claim prices. When markets are incomplete, however, there are infinitely many potential discount factors M that satisfy $P = \mathbb{E}[M'X]$ if there is more than one aggregate state.[4]The

[1] Aiyagari (1994) documents that households accumulate more capital in response to uninsurable individual risk. Krusell and Smith (1998) find that this precautionary savings further increases in a setting with aggregate uninsurable risk.

[2] In 2019, aggregate investment was composed of corporate investment (64%), household investment (20%, primarily housing), sole proprietorships and partnerships (10%), and nonprofits or other tax-exempt institutions (6%), according to the U.S. Bureau of Economic Analysis (2025) Fixed Asset Investment data.

[3] As in the finance literature, an SDF can price a bond or a Lucas tree without influencing those future payouts. In macroeconomic models with dynamic firms and a representative household (e.g. Khan and Thomas (2008)), the SDF determines firm behavior because all households value payments equally.

[4] Following the textbook explanation from Campbell (2018), suppose there are N assets and S aggregate states, with N<S. Then the matrix P is 1×N, the vector of payoffs X is S×N, and the discount factor M' is 1×S. The vector of payoffs

second problem is that firms no longer have a well-defined objective in incomplete markets. Firms generally want to maximize shareholder value. Under incomplete markets, each shareholder might have different valuations of payoffs in each future state. This difference in valuation means that they might each suggest different capital plans if they were solely in charge of the firm.[5]

To resolve the joint problems of asset pricing and firm discounting, I introduce a pair of financial agents that find the stochastic discount factor (or pricing kernel) and discipline the production sector's investment and dividend choices. Households save in equity through the mutual fund, which holds shares of production firms.[6] Each household's savings choice depends on the aggregate price, future payoffs, and current idiosyncratic state. The mutual fund holds a portfolio of all production firms in the economy, which plays the dual role of simplifying the household's problem to a single continuous choice variable and preventing production firms from having an incentive to become financial innovators.[7] The mutual fund measures a pricing kernel as the post-trade share-weighted marginal rates of substitution of all interior shareholders.[8] This pricing kernel determines the price that the mutual fund is willing to pay for *any* asset, given its future returns. The mutual fund resolves the asset pricing issue but does not resolve the question of firm behavior.

Production firms own capital, invest, pay dividends, and produce using a decreasing returns technology subject to aggregate productivity risk.[9] A private equity sector looks for opportunities to take over production firms and make them more valuable, similar to the outside manager proposed by Grossman and Hart (1979). If the private equity firm can find a profitable investment deviation, it will take over the production firm for a single period; otherwise the private equity firm is inactive. Equilibrium requires production firms avoid challenges by the private equity sector. To prevent the private equity firm from finding a deviation, the production firms will (in equilibrium) value future payoffs with the pricing kernel found by the mutual fund. Discounting future payoffs with the same pricing kernel as the mutual fund results in firms that maximize their net market value (or cum-dividend share price), consistent with the Fisher separation theorem introduced in Fisher (1930).[10]

---

has a maximum rank of N, so there are infinitely many M' that satisfy the pricing equation.

[5] A further discussion of disagreement about capital plans and Pareto Optimality is presented in Section 3.4.

[6] This mutual fund is similar to capital mutual funds in Carlstrom and Fuerst (1997), though the mutual fund in this paper holds shares of production firms instead of capital.

[7] Financial innovation can happen when a firm creates a new set of payoffs that were not spanned by the previous set of possible investment choices. If a firm promises a tiny deviation in one future aggregate state, households could trade this firm purely as a financial asset, even if it does not meaningfully change output.

[8] Weighing marginal rates by post-trade shareholding is a key difference from the literature. Grossman and Hart (1979) use pre-trade weights, which I will show results in firms with lower value at the same capital level. Constantinides and Duffie (1996) and Constantinides and Ghosh (2017) use post-trade weights but require permanent income shocks, which eliminates differences in intertemporal marginal rates of substitution across households.

[9] The assumption of decreasing returns means that firms earn real economic profits, which the firm can either pay out as dividends or reinvest into the business. Carceles-Poveda and Coen-Pirani (2010) show that the firm's value equals its capital stock with constant returns to scale, even with heterogeneous shareholders.

[10] While firms maximize their value, the equilibrium of this model will generally not be constrained efficient. Production firms are atomistic, so they do not consider their investment's impact on wages and other prices. A social planner will be able to find an improvement by changing aggregate capital and shifting wages.

I use this model to examine the relationship between household risk and macroeconomic aggregates by modeling the observed increase in earnings variance from 1970 to 2010 in the United States.[11] This higher level of risk results in a lower expected rate of return on capital, higher aggregate investment, and lower dividend yields. As individual earnings risk increases, aggregate demand for savings increases. Households want to hold more insurance against negative labor productivity shocks. Firms see household demand for savings and increase their investment. This leads to a higher capital stock, which smooths out aggregate productivity shocks during recessions. However, the increased capital stock lowers the marginal product of capital, which translates to a lower dividend yield. Changes to the earnings process explain 51.0 percent of the observed fall in dividend yield and 34.0 percent of the rise in the price-cash flow ratio for U.S. public entities over the same dates.

I also show that the wealth distribution directly influences firm behavior. The most extreme case I consider transfers all wealth to 5 percent of households as an unanticipated shock.[12] Firms in this setting are entirely owned by rich households who have a low marginal propensity to consume and therefore a low valuation of current dividends, driving firms to increase their investment. At the date of the wealth transfer, investment increases by over ten percent, which is funded by a decrease in aggregate consumption. Increased investment eventually increases aggregate output and wages. Aggregate consumption remains below the baseline for 12 years before the increased level of output finally offsets the increase in investment. Despite higher output and wages, most households are much worse off in this scenario. As firms accumulate capital, the rate of return falls, making saving less appealing for low-wealth households.[13] They remain stuck near the borrowing constraint until they eventually get a higher idiosyncratic productivity shock and start slowly saving away from the borrowing constraint. To my knowledge, this is the first paper that examines the effect of wealth redistribution shocks in a setting with dynamic, shareholder-owned firms.

The discounting approach developed in this paper nests many common special cases, including representative households, exogenously imposed zero trade restrictions as in Krusell, Mukoyama and Smith (2011), and endogenous zero trade as in Constantinides and Duffie (1996) or Constantinides and Ghosh (2017).

[11] Heathcote, Storesletten and Violante (2010) document the increase in persistent earnings risk over this time period and find that it leads to an increase in consumption. However, they focus on labor market results and set interest rate outside the model, while I focus on capital investment and have an endogenous rate of return on savings.

[12] The top 5% of households held 85% of equity in 2016, as documented in Table B7 of the online appendix of Saez and Zucman (2016). The experiment is too extreme in its redistribution, but it cleanly highlights the role of wealth inequality.

[13] Greenwald et al. (2021) find that low interest rates increase inequality by making low-wealth households worse off and high-wealth households better off. My results are consistent with their finding while also suggesting that wealth inequality can itself cause these lower rates of return.

Related Literature

An extensive literature studies the role of household risk in incomplete markets where households own capital. Aiyagari (1994) develops this in a setting without aggregate risk, which Krusell and Smith (1998) extends with aggregate risk. Challe and Ragot (2016) documents the role of household precautionary savings with unemployment risk over the business cycle. I contribute to this literature by including a production sector that operates a decreasing returns to scale production technology. When this is the case, shareholder-owned firms earn real economic profits and must decide how much of these profits to pay out as dividends or reinvest in the business through investment. Hubmer, Krusell and Smith (2021) model income inequality alongside increasing returns to savings, changes to tax structure, and preference heterogeneity to generate the observed increases to the wealth to output ratio, with savings via capital.

Household risk and firm behavior are frequently modeled in the entrepreneurship literature. Cagetti and De Nardi (2006) and Boar, Gorea and Midrigan (2022) describe settings where household wealth generates a distribution of entrepreneur firms. In their setting, small businesses take their discounting directly from their entrepreneurial owners. My approach focuses on larger corporate firms who are responsible for nearly two thirds of total investment rather than the small businesses, which account for 10 percent of investment.

My paper is most closely related to the firm discounting and price perception literature. Early work by Drèze (1974) describes the problem of uncertainty in the firm's valuation of future payoffs. Grossman and Hart (1979) proposed aggregating discount factors weighted by current shareholding to try to discipline the production sector's problem. This model features two shortcomings. First, the compensations to shareholders rely on off-equilibrium perceptions of price changes. Each household has a different belief about how prices will change after a deviation in investment. Second, the discount factor used by firms cannot be used to find the value of shares of the firm, except in the special case of constant returns.[14] In contrast, I construct a model where price perceptions are common across households and consistent with equilibrium. Additionally, the stochastic discount factor in my model both disciplines the firm's choices and prices assets. Firms that weigh the future by methods proposed by Grossman and Hart (1979) will have opportunities for Pareto improvements among interior shareholders. However, differences between aggregate outcomes generated by different discounting regimes are relatively minor.

Instead of using price perceptions, Krusell, Mukoyama and Smith (2011) exogenously imposes zero trade, which results in a setting where a single household pins down the aggregate discount factor. My model nests their approach. Empirically, Gormsen and Huber (2025) documents that corporate firms' discount rates for capital projects are higher than the safe rate, ruling out the use of the safe rate as a valid discounting mechanism. My model replicates this apparent risk aversion by firms through curvature in the aggregate market pricing kernel.

Bejan (2020) finds that financial innovation can break the standard link between firm value and

[14] Carceles-Poveda and Coen-Pirani (2010) prove that the firm's value is equal to its capital stock with constant returns to scale and shareholder heterogeneity.

discounted returns. Her model focuses on a control group of stakeholders whose preferences govern firm behavior. This setting would be particularly useful when studying the problem of a firm with a block of a few, distinct shareholders. My setting is more general and assumes that control of a firm is pinned down by the external threat of a private equity challenge. Moreover, the pricing kernel described in this paper can be used to value any asset, including those governed by a control group.

The asset pricing literature also relates closely to my work. Constantinides and Duffie (1996), Braun and Nakajima (2012), and Constantinides and Ghosh (2017) combine household marginal rates of substitution to create an aggregate stochastic discount factor. The discount factors calculated in their settings differ from the standard result from representative household models, which partly explains the equity premium. However, these papers construct an income process that results in zero trade while my model allows for shareholders to change over time. Marcet and Singleton (1999) finds asset prices are higher with higher income risk, but they only focus on the price and not the discount factor required to find that price. Krueger and Lustig (2010) documents that a lack of insurance for idiosyncratic risk only shifts the price of aggregate risk if household risk is correlated with aggregate risk. Paron (2021) documents a similar phenomenon in continuous time models. Idiosyncratic risk is uninsurable in this paper, which suggests that their results would hold in this setting.

The paper proceeds as follows. Section 2 describes the model environment in detail, with particular focus given to the problem of the private equity firm. Section 3 describes the conditions required for equilibrium and shows how I derive the aggregate stochastic discount factor. Section 4 discusses business cycle moments and impulse responses with varying levels of realistic idiosyncratic risk. Section 5 describes a counterfactual wealth redistribution experiment. Section 6 concludes.

2 Model Environment

This model features three dynamic agents and one static off-equilibrium agent. Production firms own capital and make nontrivial intertemporal decisions on behalf of their shareholders.[15] Households face idiosyncratic labor productivity risk and can only save in equity. Households save in equity through a mutual fund which bundles shares of production firms. Finally, a private equity firm tries to take over a production firm with the support of the mutual fund.

I begin the description of this economy with details about the maximization problem facing each household, the production firms, and the mutual fund. Once the dynamic agents are introduced, I describe the static off-equilibrium private equity firm.

[15] The firm's decisions are nontrivial in the sense that their choices are not the result of aggregation of investors' choices.

2.1 Households

There is a single production good which is used for both consumption and investment, which is the numeraire. There are a unit measure of households in this economy, identified by their start of period assets a and idiosyncratic labor productivity η. Each household has identical, time-separable, concave, strictly increasing preferences over consumption $u(c)$. Each supplies labor inelastically and saves in equity a. I assume η follows a Markov chain; $\eta \in \mathbf{N} \equiv \{\eta_1, \dots, \eta_{N_\eta}\}$, where $\Pr(\eta' = \eta_j | \eta = \eta_i) = \pi_{ij} \geq 0$ and $\sum_{j=1}^{N_\eta} \pi_{ij} = 1$ for each $i = 1, \dots, N_\eta$. For simplicity and without loss of generality, I assume higher indexed values of η denote higher productivity levels: $\eta_1 < \eta_2 < \cdots < \eta_{N_\eta}$.

The household's asset holding in the mutual fund is given by $a \in \mathbf{A} \subset \mathbf{R}$. The set $\mathbf{A}$ is bounded above by $\bar{A}$ and below by $\underline{a}$. The upper bound $\bar{A}$ is set outside the model and is chosen at a high enough level such that no households choose it in equilibrium. The lower bound $\underline{a}$ is a parameter in the model. This lower bound $\underline{a}$ must fall in the range $[\underline{A}, 1]$, where $\underline{A}$ is the natural borrowing limit. I derive an expression for the natural borrowing limit in Appendix B. If $\underline{a} = 0$, this constraint would prohibit short sales. If $\underline{a} = 1$, the economy is in exogenously-imposed no-trade equilibrium, similar to the setting in Krusell, Mukoyama and Smith (2011).

I summarize the distribution of households over (a, η) using the probability measure μ_H defined on the Borel algebra $\mathcal{S}$ generated by the open subsets of the product space, $\mathbf{S} = \mathbf{A} \times \mathbf{N}$.

I require two more components to fully define the aggregate state. The first is aggregate exogenous TFP z. I assume z follows a Markov chain; $z \in \mathbf{Z} \equiv \{z_1, \dots, z_{N_z}\}$, where $\Pr(z' = z_n | z = z_m) = \pi_{mn} \geq 0$ and $\sum_{n=1}^{N_z} \pi_{mn} = 1$ for each $m = 1, \dots, N_z$. As with labor productivity, higher indexed levels of z are more productive: $z_1 < z_2 < \cdots < z_{N_z}$.

The final component of the aggregate state is the distribution of firms over their start of period capital, $k \in \mathcal{K} \subset \mathbf{R}_{++}$. Similar to households, I summarize the distribution of firms over k using the probability measure μ_F.[16] The production firms are described in detail in Section 2.3. The aggregate state of the economy is then $\mathbb{Z} \equiv (z, \mu_H, \mu_F)$.

The per-productivity-unit wage $w(\mathbb{Z})$ is taken as given by the household. In each state, equity pays dividends $D(\mathbb{Z})$ and is traded at the ex-dividend price $P(\mathbb{Z})$, which households take as given.

To describe the recursive problem of each household, let $V(a, \eta_i; \mathbb{Z})$ be the start of period value of a household with assets a, individual productivity η_i, and the aggregate state given by $\mathbb{Z} \equiv$

[16] In this paper, all production firms make the same investment choice and the μ_F distribution can be replaced by the aggregate capital stock K.

(z_m, μ_H, μ_F). The dynamic problem of each household is given by:

$$V(a, \eta_i; \mathbb{Z}) = \max_{c,a'} u(c) + \beta \sum_{j=1}^{N_\eta} \pi_{ij} \sum_{n=1}^{N_z} \pi_{mn} V(a', \eta_j; z_n, \mu'_H, \mu'_F) \tag{1}$$

$$\text{s.t. } c + P(\mathbb{Z})a' \leq (P(\mathbb{Z}) + D(\mathbb{Z}))\, a + w(\mathbb{Z})\eta_i \tag{2}$$

$$\underline{a} \leq a' \tag{3}$$

$$\mu'_F = \Gamma_F(\mathbb{Z}), \qquad \mu'_H = \Gamma_H(\mathbb{Z})$$

where β is the common subjective discount factor. Equation 3 is the borrowing limit if $\underline{a} < 0$, a ban on short sales if $\underline{a} = 0$, or a minimum savings rule if $\underline{a} > 0$. The distribution of households over individual productivity and shareholding evolves over time according to a mapping Γ_H which depends on the current aggregate state. That is, $\mu'_H = \Gamma_H(z, \mu_H, \mu_F)$. This evolution depends on the asset choices of households in the previous period and the realization of idiosyncratic shocks. The distribution of firms over capital is similar, with $\mu'_F = \Gamma_F(z, \mu_H, \mu_F)$. The household takes both of these laws of motion as given when making its shareholding choice.

Let $c(a, \eta; \mathbb{Z})$ and $a(a, \eta; \mathbb{Z})$ be the decision rules for consumption and future shareholding of a household with current state (a, η) and aggregate state $\mathbb{Z}$.

2.2 Equity Mutual Fund

A risk-neutral, perfectly competitive mutual fund sector bundles shares of the production firms and sells the bundle to households as equity. Each period, the intermediary collects dividends from production firms, chooses how many shares of each production firm it wants to hold for the next period, and pays out aggregate dividends to households. Aggregate dividends are the dividends collected from production firms plus the net revenue from changing its shareholding of production firms.

The mutual fund chooses aggregate dividends D (with decision rule $D(\mathbb{Z})$) and its portfolio of future shareholding in production firms $\{s'_k\}$ to maximize its net market value. It buys shares $\{s'_k\}$ in each firm indexed by their capital level k at price $p(k; \mathbb{Z})$ and collects dividends $d(k; \mathbb{Z})$. The goal of the intermediary is to maximize net market value, with payoffs in future states valued by the price vector χ, which the intermediary takes as given.[17] $\chi(z_n|\mathbb{Z})$ is the pricing kernel which assigns value to payoffs for each future uncertain aggregate state conditional on the current aggregate state $\mathbb{Z}$. This pricing kernel is measured from households via a Walrasian auctioneer and is described in more detail in Section 3.3.

[17] As in Makowski (1983), maximizing net market value expands the budget constraint of all households who held positive shares at the start of the period.

The mutual fund's recursive problem is written as:

$$J(\{s_k\};\mathbb{Z}) = \max_{\{s'_k\}} D + \sum_{n=1}^{N_z} \chi(z_n|\mathbb{Z}) J(\{s'_k\}; z_n, \mu'_H, \mu'_F) \tag{4}$$

$$\text{s.t.} \quad D \leq \int_{\mathcal{K}} \left((p_k + d_k)s_k - p_k s'_k\right) \mu(dk) \tag{5}$$

$$\mu'_F = \Gamma_F(\mathbb{Z}), \qquad \mu'_H = \Gamma_H(\mathbb{Z})$$

The mutual fund plays three key roles in this economy. It prevents production firms from becoming financial innovators, it simplifies the problem of price discovery, and it makes this problem tractable.

First and most importantly, it prevents atomistic production firms from becoming financial innovators. DeAngelo (1981), Makowski (1983), and Krouse (1985) argue that shareholders will be unanimous in supporting the firm's decision to maximize net market value if firms are sufficiently small. That is, shareholders have to believe that a firm's deviation will not change the set of available prices or future outcomes. By imposing a mutual fund between the household and the production sector, a production firm will not be able to change the available choice set for households when it produces differently than its peers.

Second, it simplifies the potential problem of price discovery. It could be difficult for every production firm to ask its shareholders how they value payoffs across time, aggregate those answers, and predict future shareholding. A mutual fund sector could much more realistically study markets and make prices available to production firms.[18] The information channel is not strictly necessary due to the full information setting, but it may be a potential avenue for future research.

Finally, the mutual fund makes this model reasonably tractable. If shareholders were allowed to own individual production firms, shareholding would become a portfolio choice and the distribution of households would increase by the number of firms in the economy.[19] Tractability has important implications for equilibrium. Imagine if two production firms start the period identically, but attract different types of shareholders due to animal spirits. This would change capital plans, which would then change which shareholders flock to that firm. It is unclear if a fixed point even exists for a problem like this. Assuming a mutual fund holds equity is consistent with recent empirical findings from Backus, Conlon and Sinkinson (2021), which find that 80% of shares in the S&P500 are held by asset managers.

[18] An example of this behavior comes from Investor Relations departments at large firms. These groups frequently interact with institutional investors, who might provide feedback about how proposed capital investment plans would change share prices.

[19] Clearly, households do differentiate their portfolios for a variety of reasons. There is also evidence from Brockman et al. (2022) that firm risk earns a return premium over the market rate. This would not be the case with a single mutual fund holding all production firms.

2.3 Production Firms

A unit measure of production firms produce a homogeneous output using productivity-adjusted labor ℓ and their start of period capital stock k. They produce using a strictly increasing and concave production function $y = zF(k, \ell)$. The variable z is the common exogenous stochastic TFP level which was described in the household section. The firm hires each unit of labor productivity at a price $w(\mathbb{Z})$, which the firm takes as given.

A firm enters each period with its predetermined stock of capital, $k \in \mathcal{K} \subset \mathbf{R}_{++}$. The goal of each production firm is to maximize dividends plus discounted future value, with payoffs in future states valued by the price vector $\tilde{\chi}$. Each firm chooses labor to maximize period profits, then selects future capital and current dividends. A portion of the firm's capital stock δ depreciates each period. The firm pays a convex adjustment cost $I(k', k)$ that depends on both its current and future capital levels. This adjustment cost ensures that firms issue procyclical dividends, as is the case in the data.

The mutual fund values future payoffs at χ while the production firms value future payoffs at $\tilde{\chi}$. Each production firm takes the price vector $\tilde{\chi}$ as given. One important distinction here is that the discount factor used by each production firm $\tilde{\chi}$ is not assumed to be the same as the mutual fund's discount factor χ. These will be the same in equilibrium, but not because it was imposed as a modeling assumption.

As before, I use a shorthand for the aggregate state $\mathbb{Z} \equiv (z_m, \mu_H, \mu_F)$. Each production firm's problem can be written recursively as:

$$G(k; z_m, \mu_H, \mu_F) = \max_{\ell, k'} d + \sum_{n=1}^{N_z} \tilde{\chi}(z_n|\mathbb{Z}) G(k'; z_n, \mu'_H, \mu'_F) \tag{6}$$

$$\text{s.t.} \quad d + k' + I(k', k) \leq z_m F(k, \ell) - w(\mathbb{Z})\ell + (1 - \delta)k \tag{7}$$

$$\mu'_F = \Gamma_F(\mathbb{Z}), \qquad \mu'_H = \Gamma_H(\mathbb{Z})$$

where $\tilde{\chi}(z_n|\mathbb{Z})$ is the valuation of dividends plus firm price in each future state $\{z_n, \mu'_H, \mu'_F\}$, conditional on the current aggregate state. This discount factor is taken as given by each production firm and is described in detail in section 3.3.

Let $g(k; \mathbb{Z})$ and $d(k; \mathbb{Z})$ be the decision rules for future capital and dividends of a firm with current capital k and aggregate state $\mathbb{Z}$. Its choice of labor to hire is given by $\ell(k; \mathbb{Z})$.

2.4 Private Equity

The private equity sector disciplines the choices of the production sector. The private equity firm is modeled from observed shareholder proxy battles. Fos (2017) documents that a majority of shareholder challenges state their goal as increasing market value and that shareholder challenges targeting market value are more likely to succeed.

After production firms propose a choice for their future capital stock k', private equity firms attempt to find an arbitrage opportunity. Each period, a unit measure of deep-pocketed risk-neutral private equity firms are randomly matched with the unit measure of production firms.[20] Each private equity firm attempts to find an arbitrage opportunity by taking over a single production firm for a single period.[21] To do so, it purchases 100 percent of shares of its matched production firm from the mutual fund before dividends are paid. The mutual fund willingly sells shares to the private equity firm at the price $p(k';\mathbb{Z}) + d(k';\mathbb{Z})$, which leaves its budget constraint entirely unchanged.

The private equity firm then implements a new investment plan $\hat{k}'$, keeps the dividends, and sells the production firm's shares back to the mutual fund. The private equity firm keeps the new dividends $d(\hat{k}';\mathbb{Z})$. After the private equity firm implements a new plan, the mutual fund is willing to buy back the shares at the new price $p(\hat{k}';\mathbb{Z})$ as long as that price satisfies the mutual fund's first order condition in Equation 10.

The private equity firm only acts if it can earn a strictly positive profit from taking over the production firm, which only happens when the production firm's original choice of capital differs from the private equity firm's choice of capital ($k' \neq \hat{k}'$).

The private equity firm's problem is written as:

$$\max_{\hat{k}'} \left(p(k;\mathbb{Z};\hat{k}') + d(k;\mathbb{Z};\hat{k}') \right) - \left(p(k;\mathbb{Z};g(k;\mathbb{Z})) + d(k;\mathbb{Z};g(k;\mathbb{Z})) \right) \tag{8}$$

where hat variables denote the alternate investment plan. The price $p(k;\mathbb{Z};\hat{k}')$ (and dividend $d(k;\mathbb{Z};\hat{k}')$) describes the price (dividend) of a firm that started with capital level k in aggregate state $\mathbb{Z}$ which pursues capital investment plan $\hat{k}'$. In contrast, the price $p(k;\mathbb{Z};g(k;\mathbb{Z}))$ (and dividend $d(k;\mathbb{Z};g(k;\mathbb{Z}))$) reflects the price (dividend) of a production firm that follows its originally planned investment strategy $g(k;\mathbb{Z})$.

This private equity firm ultimately pins down the objective of the production firm. In Section 4.4, I discuss the quantitative difference between the economies governed by this discount factor compared to other common approaches in the literature.

[20] The deep-pocketed assumption abstracts away from questions about how a buyout of the production firm might be financed. In observed private equity challenges, activist challengers typically only acquire 5-10% of the shares of a firm to exert significant influence on the production firm's managers. However, activist shareholders who buy a fraction of the firm to implement a new management plan would result in identical behavior as the deep pocketed private equity firms modeled in this paper. Because these firms never act in equilibrium, the deep pocketed assumption never influences the real economy.

[21] The unit measure of private equity firms being randomly matched with production firms prevents private equity firms from being able to exert market power. If there were a single private equity firm with deep pockets, it might generate profitable deviations by changing the production decisions of a large group of production firms and thereby shifting aggregate prices. This pooled ownership distortion is consistent with the Common Ownership hypothesis described in Azar, Schmalz and Tecu (2018).

3 Equilibrium Definition and Discount Factor Properties

This section describes the determination of the equilibrium market pricing kernel χ and the production firm's discount factor $\tilde{\chi}$. The section proceeds in three steps. First, I derive a candidate pricing kernel, χ^*, by aggregating household Euler equations using post-trade share weights. Next, I show that χ^* (and a class of equivalent transformations) satisfies first-order Pareto efficiency for interior shareholders under a set of admissible side-payments. Finally, I show that production firms discounting with χ^* implement cum-dividend value maximization and therefore preempt private equity challenges.

3.1 Recursive Competitive Stock Market Equilibrium

A recursive competitive stock market equilibrium is a set of functions,

$$\{w, G, \chi, \tilde{\chi}, d, \ell, g, p, J, P, D, V, s, c, a\}$$

that jointly solve the household, firm, and mutual fund's problems, and clear the markets for goods, labor, production firm shares, and mutual fund shares, as described by the following:

i. V solves Eq. 1 with policy functions $\{c, a\}$.

ii. J solves Eq. 4 with policy functions $\{D, s\}$.

iii. G solves Eq. 6 with policy functions $\{d, \ell, g\}$.

iv. The market for equity clears, which pins down aggregate equity price P:

$$\int_{\mathbf{S}} a(a, \eta; \mathbb{Z}) \mu(d[a \times \eta]) = 1.$$

v. The intermediary holds all shares of each production firm, which pins down production firm price p:

$$s'_k = 1 \; \forall \; k$$

vi. The labor market clears, which pins down wage:

$$\int_{\mathcal{K}} \ell(k; \mathbb{Z}) \mu(dk) = \int_{\mathbf{S}} \eta \mu(d[a \times \eta]).$$

vii. The goods market clears:

$$\int_{\mathcal{K}} \left(g(k; \mathbb{Z}) - (1-\delta)k + I(g(k; \mathbb{Z}), k)\right) \mu(dk) + \int_{\mathbf{S}} c(a, \eta; \mathbb{Z}) \mu(d[a \times \eta])$$
$$= \int_{\mathcal{K}} zF(k, \ell(k; \mathbb{Z})) \mu(dk).$$

viii. Γ_H is defined by:

$$\mu'(A, \eta_j) = \int_{\{a, \eta_i | (a(a, \eta_i; \mathbb{Z})) \in A\}} \pi_{ij} \mu(d[a \times \eta_i]) \quad \forall\, (A, \eta_j) \in \mathcal{S}.$$

ix. Γ_F is defined by:

$$\mu'(k) = \int_{\{k | (g(k; \mathbb{Z})) \in K\}} \mu(dk) \quad \forall\, k \in \mathcal{K}.$$

x. Each private equity firm cannot find a profitable deviation, which is satisfied when the maximum of Equation 8 is zero. This pins down the optimal capital choice and therefore a class of discount factors $\tilde{\chi}$ that can generate that capital choice.[22]

The mutual fund's discount factor χ is determined by market clearing for equity (equilibrium condition iv). The production firm's discount factor $\tilde{\chi}$ is determined by equilibrium condition x and will be derived in Section 3.3. The novel feature of this equilibrium is item x. By assumption, managers of production firms do not want to be ousted by the private equity managers.

3.2 Optimal Choices and the Equilibrium Stochastic Discount Factor

Each household's optimal choice of a' satisfies

$$Pu'(c) = \beta \mathbb{E}_{\eta', z'} (P' + D') u'(c') + \lambda_a. \tag{9}$$

where λ_a is the multiplier on the borrowing constraint $a' \geq \underline{a}$ and equals zero when the household's choice is interior. Households for whom $\lambda_a = 0$ are referred to as *interior shareholders*. Constrained households, in contrast, are those who would prefer to hold less equity than permitted by the borrowing constraint. Interior shareholders are important because their Euler equations price aggregate equity.[23] If market returns changed, these households would change their holdings, which in turn would influence the equilibrium market price. Constrained households, by contrast, cannot change their shareholding in response to minor changes in future payments. To denote the set of interior shareholders, I use the indicator function

$$\mathbb{I} = \mathbf{1}\{\lambda_a = 0\}.$$

Given the aggregate stochastic discount factor χ, the mutual fund chooses shareholding of each

[22] Appendix A describes the class of $\tilde{\chi}$ that can preempt a private equity challenge when $N_z \geq 2$. Any alternate discount factors that satisfy the criteria in Appendix A result in the same prices and quantities, so these alternate formulations are not economically meaningful.

[23] Traders with negative equity (short sellers) for whom λ_a is equal to zero are still considered interior. Their choices still discipline the equilibrium market price.

production firm with the optimality condition,

$$p(k;\mathbb{Z}) = \sum_{n=1}^{N_z} \chi(z_n|\mathbb{Z})[p'(g(k;\mathbb{Z});z_n,\mu'_H,\mu'_F) + d'(g(k;\mathbb{Z});z_n,\mu'_H,\mu'_F)]. \tag{10}$$

For an arbitrary discount factor $\tilde{\chi}$, each production firm's optimal choices are given by

$$1 + \frac{\partial I(k',k)}{\partial k'} = \sum_{n=1}^{N_z} \tilde{\chi}(z_n|\mathbb{Z}) D_1 G(k';z_n,\mu'_H,\mu'_F). \tag{11}$$

The private equity firm's optimal deviation, evaluated using the aggregate market willingness to pay χ, satisfies

$$-\frac{\partial d}{\partial \hat{k}'} = \sum_{n=1}^{N_z} \chi(z_n|\mathbb{Z}) \left(\frac{\partial p'}{\partial \hat{k}'} + \frac{\partial d'}{\partial \hat{k}'} \right). \tag{12}$$

3.3 Market Pricing Kernel

To derive the pricing kernel, I aggregate household Euler equations among interior shareholders. Suppressing the aggregate state $\mathbb{Z}$ and writing $m_i(n) \equiv \frac{\sum_{j=1}^{N_\eta} \pi_{ij} u'(c'_{jn})}{u'(c)}$ as the expected marginal rate of substitution for household $i = (a, \eta)$, Equation 9 can be rewritten in terms of equity price as

$$P = \beta \sum_{n=1}^{N_z} \pi_{mn} (P'_n + D'_n) m_i(n) + \frac{\lambda_a}{u'(c)}.$$

Multiplying through by the household's post-trade shareholding[24] a' and integrating over households yields

$$\int_{\mathbf{S}} P a' \mu(d[a \times \eta]) = \int_{\mathbf{S}} \beta \sum_{n=1}^{N_z} \pi_{mn} (P'_n + D'_n) m_i(n) a' \mu(d[a \times \eta]) + \int_{\mathbf{S}} \frac{\lambda_a}{u'(c)} a' \mu(d[a \times \eta]). \tag{13}$$

From the equity market clearing condition, the left hand side equals P. To eliminate the residual wedge term, I split the group among interior shareholders $\mathbb{I} = 1$ and constrained households $\mathbb{I} = 0$. For constrained shareholders, their Euler equation rearranges to

$$\frac{\lambda_a}{u'(c)} = P - \beta \sum_{n=1}^{N_z} (P'_n + D'_n) m_i(n) > 0.$$

[24] Using post-trade shareholding is a nontrivial choice. See section 3.4 for a detailed explanation.

Substituting this expression into Equation 13 and grouping terms yields

$$P = \sum_{n=1}^{N_z} (P'_n + D'_n) \frac{\beta \pi_{mn} \int_{\mathbf{S}} \mathbb{I} \sum_{j=1}^{N_\eta} \pi_{ij} \frac{u'(c'_{jn})}{u'(c)} a' \mu(d[a \times \eta])}{\int_{\mathbf{S}} \mathbb{I} a' \mu(d[a \times \eta])}. \tag{14}$$

The value of equity is determined by discounted future payoffs, where discounting comes from interior (or marginal) shareholders' expected marginal rates of substitution across aggregate states, weighted by their end of period (post-trade) shareholding. This generates the candidate pricing kernel

$$\chi^*(z_n|\mathbb{Z}) \equiv \beta \pi_{mn} \frac{\int_{\mathbf{S}} \mathbb{I} \sum_{j=1}^{N_\eta} \pi_{ij} \frac{u'(c'_{jn})}{u'(c)} a' \mu(d[a \times \eta])}{\int_{\mathbf{S}} \mathbb{I} a' \mu(d[a \times \eta])}. \tag{15}$$

Assumption 1 (Finite present value of aggregate dividends)**.** *The value of the aggregate dividend payment stream under the pricing kernel χ^* is finite.*

This assumption is reasonable under the criteria outlined in Santos and Woodford (1997). The asset is in positive net supply, is productive, pays weakly positive dividends, and each household receives a positive stream of payments for labor in each period. Therefore, no household will ever have marginal utility of consumption that is zero, infinite, or undefined. Defining χ^*_s as the state contingent discount factor for each future state and given Assumption 1, the ex-dividend price of a production firm can be written as the typical discounted stream of dividends,

$$p(k;\mathbb{Z}) = \sum_{t=1}^{\infty} \mathbb{E}\left[\left(\prod_{s=0}^{t-1} \chi^*_s \right) d_t \,\middle|\, k_0 = k,\ k_{t+1} = g(k_t;\mathbb{Z}_t) \right]. \tag{16}$$

3.4 Pareto Efficiency

Trivially, *any* aggregation of household optimality conditions (Equation 9) will deliver the equilibrium price of an asset. However, firms and markets need to know how to value alternate, off-equilibrium capital plans. To demonstrate that post-trade share weighting among interior shareholders prices deviations in capital, I show that χ^* implements the unique capital plan that is first-order Pareto efficient for interior shareholders[25] under a set of admissible side payments.

Consider a hypothetical ε deviation in the firm's future capital, moving from k' to $(k' + \varepsilon)$ after the equity market has cleared. I evaluate the first-order welfare consequences of this deviation for each interior shareholder and characterize the conditions under which redistributive transfers can deliver a Pareto improvement.

[25] Only interior shareholders are compensated because they are the set of households who can make marginal adjustments to their shareholding if they are unhappy with a firm's future plans. A further discussion of this point is presented in Section 3.6 below.

3.4.1 Side payments and first-order welfare effects

For interior shareholder i indexed by $i = (a, \eta)$ for whom $\lambda_a = 0$, the capital deviation changes lifetime utility through three channels: a change in the current cum-dividend value of their pre-trade position, a change in the price at which they settle their post-trade position, and a change in the expected future payoff stream from their post-trade position. Following Grossman and Hart (1979), the first-order change in household i's lifetime utility from the deviation, valued in current consumption units, is

$$b_i = a_i \left(\frac{\partial p}{\partial k'} + \frac{\partial d}{\partial k'} \right) - a_i' \frac{\partial p}{\partial k'} + a_i' \beta \sum_{n=1}^{N_z} \pi_{mn} m_i(n) \left(\frac{\partial p'}{\partial k'} + \frac{\partial d'}{\partial k'} \right). \tag{17}$$

Let $\mathbf{U}$ be the set of interior shareholders. A transfer scheme $\{t_i\}$ is admissible if it satisfies $\int_{\mathbf{U}} t_i \mu(di) = 0$ (mean zero lump sum transfers among interior shareholders) and does not push any household across the borrowing constraint. Under admissible transfers, the deviation generates a Pareto improvement if and only if there exist transfers such that every household's first-order welfare change is weakly nonnegative and a positive measure is strictly positive.

Assumption 2 (Interior shareholders)**.** *Every household with $\lambda_a = 0$ has sufficient slack in their budget constraint that an admissible transfer of magnitude $O(\varepsilon)$ does not push them to the borrowing constraint.*

Lemma 1 (Pareto improvements via side payments)**.** *Let Assumption 2 hold. A first-order admissible Pareto improvement exists at the candidate plan k' if and only if $\int_{\mathbf{U}} b_i \mu(di) \neq 0$.*

Proof. **Sufficiency** Suppose without loss of generality that $\int_{\mathbf{U}} b_i \mu(di) > 0$. Define the transfer scheme $t_i = -b_i \varepsilon + \delta_i$, where $\delta_i \geq 0$ is any non-negative measurable function satisfying $\int_{\mathbf{U}} \delta_i \mu(di) = \varepsilon \int_{\mathbf{U}} b_i \mu(di)$ and $\delta_i > 0$ on a set of positive measure. Such a δi exists because the right hand side is positive by hypothesis. Each household's net first order welfare change is $b_i \varepsilon + t_i = \delta_i \geq 0$, with strict inequality on a set of positive measure. The deviation combined with $\{t_i\}$ Pareto-dominates the original plan.

Necessity Suppose $\int_{\mathbf{U}} b_i \mu(di) = 0$. Any admissible transfer scheme $\{t_i\}$ with $\int_{\mathbf{U}} t_i \mu(di) = 0$ yields aggregate welfare change $\int_{\mathbf{U}} (b_i \varepsilon + t_i) \mu(di) = 0$. Therefore the net welfare change cannot be weakly nonnegative with strict positivity on a set of positive measure. No first-order admissible Pareto improvement exists. □

The next step links the side payments integral $\int_{\mathbf{U}} b_i \mu(di)$ to the candidate kernel χ^* and the equilibrium objects of the firm's problem. For convenience, define the total amount of pre- and post-trade shares held by interior shareholders as

$$S_I \equiv \int_{\mathcal{S}} \mathbb{I} a_i \, \mu(di) \qquad \text{and} \qquad S_I' \equiv \int_{\mathcal{S}} \mathbb{I} a_i' \, \mu(di).$$

Integrating Equation 17 over interior shareholders and rewriting in terms of the candidate discount factor in Equation 15 simplifies to

$$\int_{\mathbf{S}} \mathbb{I}b_i \mu(di) = \left(\frac{\partial p}{\partial k'} + \frac{\partial d}{\partial k'}\right) S_I - \frac{\partial p}{\partial k'} S_I' + S_I' \sum_{n=1}^{N_z} \chi^*(z_n|\mathbb{Z}) \left(\frac{\partial p'}{\partial k'} + \frac{\partial d'}{\partial k'}\right). \tag{18}$$

Lemma 2 (Interior Shareholder Pareto efficiency and net market value maximization)**.** *Let Assumption 2 hold, let $S_I > 0$ and $S_I' > 0$, and suppose the mutual fund prices deviations consistently with its demand for shares, so that*

$$\frac{\partial p}{\partial k'} = \sum_{n=1}^{N_z} \chi(z_n|\mathbb{Z}) \left(\frac{\partial p'}{\partial k'} + \frac{\partial d'}{\partial k'}\right). \tag{19}$$

Then, for any feasible candidate plan k':

(i) When $\chi = \chi^*$, $\int_{\mathbf{S}} \mathbb{I}b_i\, \mu(di) = S_I\, \partial(p+d)/\partial k'$. k' admits no first-order admissible Pareto improvement among interior shareholders if and only if it maximizes net market value.

(ii) Conversely, if k' maximizes net market value and admits no first-order admissible Pareto improvement, then $\sum_n \left[\chi(z_n|\mathbb{Z}) - \chi^*(z_n|\mathbb{Z})\right] \partial(p'+d')/\partial k' = 0$: the pricing rule is identified up to the equivalence class of χ^* characterized in Appendix A.

Proof. (i) With $\chi = \chi^*$ in (19), the second and third terms of Equation 18 cancel exactly, leaving $\int \mathbb{I}b_i\, \mu(di) = S_I\big(\partial p/\partial k' + \partial d/\partial k'\big)$. Since $S_I > 0$, the integral vanishes if and only if $\partial(p+d)/\partial k' = 0$, and Lemma 1 converts the vanishing of the integral into the absence of a first-order admissible Pareto improvement.

(ii) Setting $\int \mathbb{I}b_i\, \mu(di) = 0$ and $\partial(p+d)/\partial k' = 0$ in Equation 18 and dividing by $S_I' > 0$ yields $\partial p/\partial k' = \sum_n \chi^*(z_n|\mathbb{Z})\, \partial(p'+d')/\partial k'$. The third term of 18 carries χ^* regardless of the fund's pricing rule, since it aggregates the households' own Euler equations. Subtracting (19) gives the stated restriction, the single linear condition defining the class in Appendix A at the net market value maximizing plan. □

3.5 Production Firm Optimality

I now turn to the production firm's discount factor $\tilde{\chi}$ and its relationship to the firm's market valuation. Define the value of a firm with capital k in aggregate state $\mathbb{Z}$ as the present value of the dividend stream generated by the best feasible capital plan:

$$G(k;\mathbb{Z}) = \sup_{\{k_{t+1}\}} \sum_{t=0}^{\infty} \mathbb{E}\left[\left(\prod_{s=0}^{t-1} \tilde{\chi}_s\right) d_t \,\middle|\, k_0 = k\right], \tag{20}$$

where the supremum is taken over plans satisfying the firm's budget constraint (Equation 7) in every date and state, dividends d_t are those implied by the plan, and labor is chosen optimally period by period.

Lemma 2 establishes that the net market value maximizing capital plan is achieved with discounting χ^*, and that this plan is first order Pareto efficient among interior shareholders. The proposition below extends that first order relationship to the firm's stock market valuation. A firm discounting by χ^* has a value equal to its cum-dividend share price and vice-versa.

Proposition 1 (χ^* discounting and value maximization)**.** *Let Assumption 1 hold, and suppose the firm's period problem is strictly concave in k' (as is the case under the functional forms of Section 4). Then:*

(i) If $\tilde{\chi} = \chi^*$, the firm's value coincides with its cum-dividend share price on $\mathcal{K} \times \mathbf{Z}$:

$$G(k;\mathbb{Z}) = p(k;\mathbb{Z}) + d(k;\mathbb{Z}).$$

(ii) Conversely, if $G(k;\mathbb{Z}) = p(k;\mathbb{Z}) + d(k;\mathbb{Z})$, the firm's capital policy coincides with the net-market-value-maximizing plan k^*, and $\tilde{\chi}$ belongs to the class of discount factors characterized in Appendix A, of which χ^* is a member.

Proof. **(i)** Suppose $\tilde{\chi} = \chi^*$. The firm's capital plan g satisfies Equation 11 in every date and state. Capital is bounded due to decreasing returns and depreciation, and the term $1 + \partial I(k', k)/\partial k'$ is bounded along the plan, so Assumption 1 implies the transversality condition

$$\lim_{T\to\infty} \mathbb{E}\left[\left(\prod_{s=0}^{T-1} \chi_s^*\right)\left(1 + \frac{\partial I(k_{T+1}, k_T)}{\partial k_{T+1}}\right) k_{T+1}\right] = 0.$$

Because the period objective is concave in (k, k'), the Euler equation and the transversality condition are jointly sufficient for optimality. $G(k;\mathbb{Z})$ equals the present value of dividends discounted by χ^*. By Equation 16, that present value is $p(k;\mathbb{Z}) + d(k;\mathbb{Z})$.

(ii) Suppose $G(k;\mathbb{Z}) = p(k;\mathbb{Z}) + d(k;\mathbb{Z})$ pointwise. Differentiating with respect to current capital and using the firm's budget constraint,

$$\frac{\partial G}{\partial k} = \frac{\partial p}{\partial k} + \frac{\partial d}{\partial k} \qquad \text{and} \qquad \frac{\partial d}{\partial k'} = -\left(1 + \frac{\partial I(k', k)}{\partial k'}\right).$$

Substituting each of these into Equation 11 gives the same condition as Equation 12 with $\chi = \tilde{\chi}$. By equilibrium condition (x), the maximum of Equation 8 is zero, so the incumbent plan attains the private equity optimum. That problem is strictly concave, its maximizer $\hat{k}' = k^*$ is unique, and the firm's policy satisfies $g(k;\mathbb{Z}) = k^*$. Evaluating the displayed condition at $k' = k^*$, $\tilde{\chi}$ satisfies the single linear restriction defining the equivalence class in Appendix A, of which χ^* is a member.

□

Corollary 1 (No private equity challenge in equilibrium)**.** *In any equilibrium in which the production firm discounts with $\tilde{\chi} = \chi^*$, the private equity sector cannot find a profitable deviation.*

Proof. By Proposition 1(i), $G = p + d$, so the production firm's optimality condition coincides with the private equity firm's (Equations 11 and 12). Both problems are strictly concave with the same unique maximizer, so $k' = \hat{k}'$ and the maximum of Equation 8 is zero. □

3.6 Constrained Shareholders

The Pareto efficiency argument focuses solely on interior shareholders. Households for whom $\lambda_a > 0$ prefer to hold less equity and are not included in the side-payment regime. Infra-marginal households do not discipline equity markets. By definition, they value a firm's future returns less than the market price and would not change their shareholding choice in response to minor investment changes. Because they cannot express their preferences by trading in equity markets, firms place no weight on those unobserved preferences.

3.7 Analysis

I will simplify notation in the remainder of this paper and use χ for both the market and the firm discount factors. In the sections below, I describe properties of the discount factor, including how it compares to discount factors proposed in the literature and its uniqueness and unanimity properties.

3.7.1 Properties of the Discount Factor

The discount factor described in Equation 15 features a number of useful properties. I discuss below how it nests a number of standard models, including the representative household case, the exogenous no-trade case, constant returns to production environments, and the Makowski (1983) Criterion.

First, it neatly nests the representative household discount factor. This discount factor is standard in the literature featuring dynamic firms, such as in Khan and Thomas (2013). With a representative household, the discount factor can be written as:

$$\chi_{\text{rep}}(z_n|\mathbb{Z}) = \pi_{mn}\beta\frac{u'(c'_n)}{u'(c)}.$$

This is a special case of the discount factor I derive in Equation 15.

Another useful feature is that this discount factor nests the exogenous no-trade approach proposed by Krusell, Mukoyama and Smith (2011). In their setting, a minimum savings rule of $\underline{a} = 1$ means that only the single highest income shareholder holds the asset on the margin and therefore uniquely prices the asset. This is generated exactly in my model in Equation 15 when $\underline{a} = 1$.

The proposed discount factor also meets a criterion as set out by Makowski (1983). That criterion requires the discount factor used by the firm satisfies $P = \max_{a,\eta_i}[\sum_n SDF_{a,\eta_i}(P'_n + D'_n)]$, where SDF_{a,η_i} is the stochastic discount factor across aggregate states for the household indexed by $\{a, \eta_i\}$. Every household that chooses $a' > \underline{a}$ will satisfy this condition as shown by the optimality conditions in Equation 9.[26]

[26]Households who choose $a' = \underline{a}$ will have $\lambda_a > 0$, which means only their SDF will not satisfy the Makowski criterion.

3.7.2 Comparison to Alternatives

General alternate approaches to discounting are typically written as,

$$\chi_{\text{alt}}(z_n|\mathbb{Z}) = \pi_{mn}\beta \int_{\mathbf{S}} \frac{\sum_{j=1}^{N_\eta} \pi_{ij} u'(c'_{jn})}{u'(c)} \mathbf{W}_{a,\eta} \mu(d[a \times \eta]), \tag{21}$$

where $\mathbf{W}_{a,\eta}$ is a model-specific weighting for shareholder indexed by $\{a, \eta\}$. Grossman and Hart (1979) considers both pre-trade and date-zero shareholding as possible weights, and Drèze (1974) uses pre-trade shareholding. I discuss the quantitative difference between my model and the discounting approach proposed by Grossman and Hart (1979) in Section 4.4.

3.7.3 Uniqueness and Unanimity

A common question in this literature is around the concept of unanimity. The traditional view is that all shareholders should agree about the firm's capital plan. Carceles-Poveda and Coen-Pirani (2009) shows that models with a constant returns to scale production function, no adjustment costs, and no short selling can deliver this style of unanimity. DeAngelo (1981) and Makowski (1983) instead propose that unanimity is better understood as the case where ex-ante shareholders unanimously prefer for the firm to engage in net value (or cum-dividend price) maximization. The best a firm can do for its shareholders is to maximally expand their budget sets, which is done by maximizing the firm's net value.

In my model, households are unanimous in their preference for firm value maximization, as is the case in DeAngelo (1981) or Makowski (1983). Households are not unanimous, however, in the firm's exact choice of future capital. That is, some households would run the firm differently if they were the firm's sole owner.[27]

A lack of unanimity about specific capital plans is not a shortcoming of the model. Empirically, unanimity is unimportant in capital choice as long as firms maximize their net market value. Shareholder votes about capital choices (or firm management in general) are rare. Fos (2017) documents that the most frequent cause for shareholder challenges are poor stock performance. Additionally, he finds that proxy challenges that target capital structure tend to be less successful.

3.8 Algorithm

The solution algorithm uses a modified version of the backward induction process developed by Reiter (2010), a detailed description of which is included in the supplemental online appendix. I discretize the aggregate state z into 5 states using the Rouwenhorst algorithm developed in Rouwenhorst (1995). I discretize the idiosyncratic productivity process across $N_\eta = 13$ levels following the process in Hubmer, Krusell and Smith (2021), with the bottom 90% of households

[27] In contrast, Boar, Gorea and Midrigan (2022) describes a model where entrepreneurs run individual firms. A story about entrepreneurs is helpful in understanding the persistence of wealth inequality, but it can't explain the investing behavior of the (much larger) corporate sector.

following a lognormal AR(1) process and the top 10% of households following a Pareto distribution. To proxy the (degenerate) distribution of firms μ_F, I use a 11 log spaced grid points of aggregate capital, which proxies aggregate total wealth.[28] Although there is a representative sector of production firms, each firm still needs to understand its off-equilibrium value in order to know that it is making the optimal capital investment choice for itself. A coarse grid is used to solve household and firm value functions, which has 69 and 59 knots, respectively.[29] A fine grid is used for the proxy distribution and simulation, which has 699 grid points for shareholding. Den Haan (2010) errors for share price (future capital stock) average 4.87E-05 (1.18E-05) with a maximum of 3.12E-04 (7.32E-05).[30] The algorithm runs in approximately 65 hours on ten cores of an Intel Xeon Gold 6230R CPU.

4 Business Cycle Moments and Impulse Responses

I now apply my method to a simple real business cycle model to see how aggregate behavior varies with idiosyncratic risk. To begin, I specify explicit forms for the utility and production functions. Households value consumption with CRRA utility of the form:

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

where σ is the relative risk aversion parameter. I assume a Cobb-Douglas production function: $y = zF(k, \ell) = zk^{\alpha}\ell^{\gamma}$ with $\alpha + \gamma \in (0, 1]$. I further assume a quadratic adjustment cost function: $I(k', k) = \frac{\psi}{2k}(k' - k)^2$. If $\psi = 0$, this nests a model without adjustment costs.

4.1 Earnings Process

Empirically, the labor earnings distribution has a thick right tail. Therefore, I follow Hubmer, Krusell and Smith (2021) in drawing labor productivity for the 10% of highest income households from a Pareto distribution. Their productivity η follows $F^{-1}_{Pareto(\kappa)}\left(\frac{F(\eta)-0.9}{1-0.9}\right)$ where $F(\cdot)$ and $F^{-1}(\cdot)$ are the CDF and inverse CDF of a Pareto distribution with lower bound $F^{-1}(0.9)$ and shape κ.[31] The shape coefficient κ is calibrated to match the top 1% of labor income earnings in 1970 and 2010 documented in Piketty and Saez (2003). The top 1% of income shares are 6.4% in 1970 and 11.0% in 2010, leading to shape coefficients (κ) of 2.48 and 1.92, respectively.

[28] Because the distribution of firms is degenerate, aggregate capital K' directly follows from the representative firm's choice of capital k'.

[29] In practice, Den Haan errors for equity price average below 1E-4 once there are at least 39 points on the shareholding grid. Relative to a coarse grid with 199 points for shareholding, 69 knots are accurate to within 0.2 percent for common great ratios like consumption to GDP, price to cash flow, capital to output, and dividend yield.

[30] This level of accuracy arrives at the fourth outer loop, after which there is no notable improvement.

[31] Unlike Hubmer, Krusell and Smith (2021), I do not include a zero-earnings state. Adding unemployment risk would increase the precautionary savings motive, but it is not clear if the highest income households should face the same unemployment risk as low income households. I also do not include a transitory risk component, which would also increase the precautionary savings motive.

For the lowest 90% of households, the log income process follows $\eta_t = \rho_\eta \eta_{t-1} + \epsilon$. The innovation to the income process is drawn from a mean-zero normal distribution with standard deviation σ_η. Each household's labor productivity is then given by e^η. The persistence of the labor productivity shock is held constant at $\rho_\eta = 0.973$, consistent with estimates from Heathcote, Storesletten and Violante (2010). The earnings risk σ_η in 1970 (2010) is 0.087 (0.146), also matching estimates from Heathcote, Storesletten and Violante (2010).

4.2 Numerical Strategy

Table 1 summarizes the model parameters. The period of the model is annual.

External Idiosyncratic Labor Parameters					
Parameter		Value		Source	
Labor productivity risk	σ_η	Varies		Heathcote, Storesletten and Violante (2010)	
Labor productivity persistence	ρ_η	0.973		″	
Labor Pareto Coefficient	κ	Varies		Piketty and Saez (2003)	
Set Economic Parameters					
Parameter		Value		Source	
Risk Aversion	σ	1.40		Literature	
Labor's Share	γ	0.63		Private labor share	
TFP volatility	σ_z	0.012043		Solow Residual	
TFP persistence	ρ_z	0.789767		Solow Residual	
Depreciation	δ	0.068		Average I/K	
Calibrated Parameters					
Parameter		Value	Target	Data	Model
Discounting	β	0.95049	Average Real Return	3.93%	3.9163%
Capital's Share	α	0.246807	K/Y	2.30	2.30153
Adjustment Cost	ψ	1.624977	Corr(D,Y)	0.2745	0.263335
Maximum borrowing	$\underline{a}$	-0.00874	Average wealth of poorest 25%	-0.286	-0.2239

Table 1: The coefficient of risk aversion σ is set within the standard range used in the business cycle literature. Labor's share of output γ is set to match the average payments to employees relative to output in the corporate sector. The parameters governing the TFP process σ_z and ρ_z are estimated from the Solow residual of annual US productivity with a smoothing parameter of 100. The labor productivity process for the bottom 90% of households is taken from Heathcote, Storesletten and Violante (2010) for both 1970 and 2010. The earnings of the top 10% are determined by the relative earnings of the top 1% documented in Hubmer, Krusell and Smith (2021). Depreciation δ matches the average of US investment divided by the capital stock measured in the fixed asset tables. Discounting β is chosen to approximate the average real rate of return documented in Gomme, Ravikumar and Rupert (2011). Capital's share α matches the average private capital to output ratio. The adjustment cost ψ is chosen to match the correlation between dividends and output. The borrowing constraint $\underline{a}$ is chosen such that the poorest quartile of households have the same relative wealth as measured by the Survey of Consumer Finances in 1989.

Most parameters are taken from outside the model or can be set to directly match observed moments in the data. The coefficient of relative risk aversion is set at 1.4, which is well within the range of risk aversion parameters standard in the literature.[32] Additionally, the value chosen matches Evans (2005), which estimates average risk aversion from 20 OECD countries from tax filings. Total factor productivity follows an AR(1) process estimated to match the HP-filtered productivity process measured in the US. Depreciation is the average ratio of investment to the capital stock, measured in the BEA fixed asset tables.

The remaining parameters are jointly estimated within the model to match target moments. Discounting β primarily targets the average real rate of return. The target in the data comes from Gomme, Ravikumar and Rupert (2011) and is measured in the model as the average dividend yield. Capital's share of output α primarily targets the average capital to output ratio, which is approximately 2.3 in the data. The adjustment cost ψ primarily targets the correlation between HP-filtered innovations to GDP and dividends. The borrowing limit $\underline{a}$ targets the average wealth of the poorest quartile of households, measured in the 1989 vintage of the Survey of Consumer Finance. In the data, the average household in the poorest quartile has wealth that is -0.29% of the average household wealth in the whole economy.[33] Practically, this means that there is some borrowing, but not a significant amount. The calibration slightly misses this moment.

4.3 Business Cycle Analysis

I conduct three sets of comparison studies to understand how varying household risk and firm structure jointly influence business cycle dynamics. The first study examines how the economy might change as individual risk increases from the 1970 level to the 2010 level. The second study examines how an economy would differ with a representative household. The final study examines what an economy would look like when households directly hold capital instead of investing in firms (approximating Krusell and Smith (1998)). The model moments for all four models are summarized in Table 2.

4.3.1 Increasing Idiosyncratic Risk

An increase in income risk causes production firms to save in capital on behalf of their shareholders. Households use this higher average market value to smooth consumption across riskier idiosyncratic states.

However, as idiosyncratic risk increases, the average return on equity falls from 3.9% to 2.9%. This reflects two driving factors. First, the demand for shares rises as idiosyncratic risk increases. High income households want to insure against a low idiosyncratic productivity shock, so they save in equity, which drives up the price of the asset and lowers average returns. This precautionary savings motive holds even in the case of the highest income households and would hold

[32] Changing risk aversion primarily influences the correlation between dividend payments and output, which is separately targeted by the adjustment cost parameter.

[33] To put this borrowing limit in dollar terms: Average household wealth was $436,600 in 1989. This borrowing limit imposes a maximum negative net worth of $3,816.

	1970 earnings process (Calibrated)					
	Y	C	I	D	P	r^e
Average	0.524	0.442	0.082	0.112	2.855	3.93%
σ/μ	0.021	0.016	0.047	0.010	0.020	0.269
SD(X)/SD(GDP)	1.000	0.788	2.280	0.481	0.979	13.015
Corr(X,GDP)	1.000	0.989	0.961	0.263	0.979	0.322
AutoCorr	0.806	0.849	0.737	0.944	0.864	-0.083
	2010 earnings process					
	Y	C	I	D	P	r^e
Average	0.541	0.447	0.093	0.107	3.677	2.91%
σ/μ	0.021	0.016	0.046	0.011	0.020	0.355
SD(X)/SD(GDP)	1.000	0.768	2.244	0.522	0.963	17.194
Corr(X,GDP)	1.000	0.987	0.964	-0.032	0.979	0.329
AutoCorr	0.805	0.851	0.740	0.873	0.864	-0.083
	Representative Household Economy					
	Y	C	I	D	P	r^e
Average	0.505	0.434	0.071	0.116	2.230	5.22%
σ/μ	0.021	0.017	0.049	0.010	0.021	0.214
SD(X)/SD(GDP)	1.000	0.801	2.375	0.477	1.027	10.299
Corr(X,GDP)	1.000	0.990	0.957	0.477	0.980	0.310
AutoCorr	0.808	0.848	0.736	0.978	0.866	-0.081
	CRS Economy (1970 Earnings)					
	Y	C	I	D	P	r^e
Average	0.707	0.534	0.173	0.088	2.551	3.46%
σ/μ	0.022	0.015	0.052	0.048	0.019	0.054
SD(X)/SD(GDP)	1.000	0.679	2.350	2.170	0.867	2.435
Corr(X,GDP)	1.000	0.908	0.928	-0.607	0.751	0.562
AutoCorr	0.837	0.949	0.737	0.702	0.984	0.715

Table 2: Columns are output, consumption, investment, dividends, equity price, and realized return on equity, respectively. σ/μ is the standard deviation divided by the average. SDX/SDY is the relative standard deviation of the variable divided by the relative standard deviation of output. CORR(X,Y) describes the variable X's correlation with output. AutoCorr is the variable's correlation with itself over time.

even without aggregate risk.[34] The presence of aggregate risk amplifies the risk associated with the lower productivity draws. Firms will invest more, which further lowers the rate of return on capital. These results are consistent with Huggett (1993), Aiyagari (1994), and Krusell and Smith (1998), all of which find that households demand additional savings as risk increases. The increase in individual earnings risk also slightly decreases the relative volatility of investment. Firm dividends become smaller, more volatile, and countercyclical.

If there is more individual earnings risk, why aren't firms helping to smooth consumption by issuing more countercyclical dividends? As the right tail of households becomes more productive, these households hold larger shares of wealth and therefore exert more influence on the aggregate pricing kernel. Wealthy households can naturally smooth consumption across aggregate states by changing their equity holdings. Because these wealthy households are not much worse off during a recession than they are during an expansion, firms that pay dividends in a recession are not as valuable as firms that retain value across the business cycle.

Model Fit of Untargeted Financial Moments One key feature of this model is that it can generate meaningful changes in financial moments through changes in household composition. These changes are outlined in Table 3. Under the 1970 level of earnings risk, the model overstates the initial price to cash flow ratio.

With a simple shift in earnings risk, the model is able to replicate secular trends in equity markets. The first is a trend toward lower dividend yields. In the 1970's, dividends were approximately 4.0% of equity price, while they are now closer to 2%. My model features dividend yields of 3.9% on average with a 1970's level of risk, which falls to 2.9% with the 2010's level of idiosyncratic risk. The increase in earnings risk explains 51.0% of the observed drop in dividend yields. As idiosyncratic risk increases, households want more savings for low aggregate productivity states. This leads to an increase in average investment, which increases capital stock and asset prices. In this setting, dividends fall as more investment is needed to maintain the capital stock. Together, the increase in price and decrease in dividends lead to declining dividend yields.

	Data		Model	
Year	Dividend Yield	P/CF	Dividend Yield	P/CF
1970-1979	4.0%	8.1	3.9%	14.7
2010-2019	2.0%	14.0	2.9%	18.4
Change	-50.7%	72.8%	-25.9%	24.8%

Table 3: Dividend yields and Price / (Cash Flow) are evaluated across a 10-year average. Dividend yields are Shiller (2016) and Cash Flows are from French (2026). Model dividend yield is evaluated as average dividend divided by average price. Model Price to Cash Flow ratio is price divided by the sum of investment and dividends.

Additionally, the model also partly replicates the observed rise in the price to cash flow ratio documented in French (2026). An increase in household income risk explains 34.0% of the rise in

[34] In the model, the highest earning workers know that they will eventually return to a low idiosyncratic productivity draw. The real-world proxy is that high income workers know they will eventually retire.

the price to cash flow ratio. This cash flow metric is included to provide an alternate proxy for firm valuation that should be immune to secular shifts in the importance of share repurchases.

Household Distributions Each setting creates an endogenous distribution of wealth over shareholding, described below. Table 4 shows how the wealth distribution changes with increasing levels of household earnings risk.

	Top 0.01%	0.01%-0.1%	0.1%-1%	1%-5%	5%-10%	Bottom 90%
Model - 1970	1.3%	6.1%	19.4%	25.0%	14.5%	33.8%
Model - 2010	3.1%	11.0%	20.3%	21.9%	13.0%	30.7%
Data - 1970	3.1%	6.2%	17.4%	27.2%	14.8%	31.3%
Data - 2010	10.2%	9.4%	17.8%	22.5%	12.8%	27.2%

Table 4: Distribution of wealth in each scenario compared to the data. The modeled wealth distribution is evaluated at the median level of aggregate productivity and at the aggregate level of capital closest to the simulated average. The 1970 and 2010 data are from Table B1 in the online appendix of Saez and Zucman (2016).

As earnings risk increases, the wealthiest households accumulate more wealth. This finding is standard in the literature, so this result primarily demonstrates that this model generates expected results. While the distribution of wealth is untargeted, the model describes the data quite well. As income inequality increases, the top one percent of households become significantly more wealthy and the bottom 95% of households hold a lower proportion of wealth. However, most households become wealthier on average.

Wealth Percentile	Top 0.01	0.01-0.1	0.1-1	1-5	5-10	10-25	25-50	50-75	Bottom 25
Δ Wealth Share	1.82	4.98	0.91	-3.12	-1.43	-2.15	-1.17	0.12	0.04
Δ Wealth	215.6%	134.8%	34.9%	12.7%	16.0%	15.8%	15.3%	39.4%	-3.4%

Table 5: Changes in wealth share and wealth after an increase in idiosyncratic risk. The first column shows the percentage point change in wealth held by each subset of households. The second column shows the absolute change in wealth held by each subset, measured in consumption-equivalent terms.

Table 5 documents how wealth levels change as idiosyncratic risk increases from the 1970 level to the 2010 level. Even the groups that lose wealth share tend to gain in terms of absolute wealth.

Shareholder Valuation Differences Across States Shareholder disagreement appears most clearly when examining the distribution of households by their intertemporal marginal rates of substitution across aggregate states. Panel A of Figure 1 shows the marginal rates of substitution across future aggregate states, weighted by post-trade shareholding. In a setting with complete markets, we would expect each of these distributions to be degenerate. Under incomplete markets, most shares tend to be concentrated near the aggregate market valuation, though meaningful difference exists between shareholders.

The differences across households are more apparent when looking at marginal rates of substitution across states weighted by household population. Panel C of Figure 1 illustrates the broad

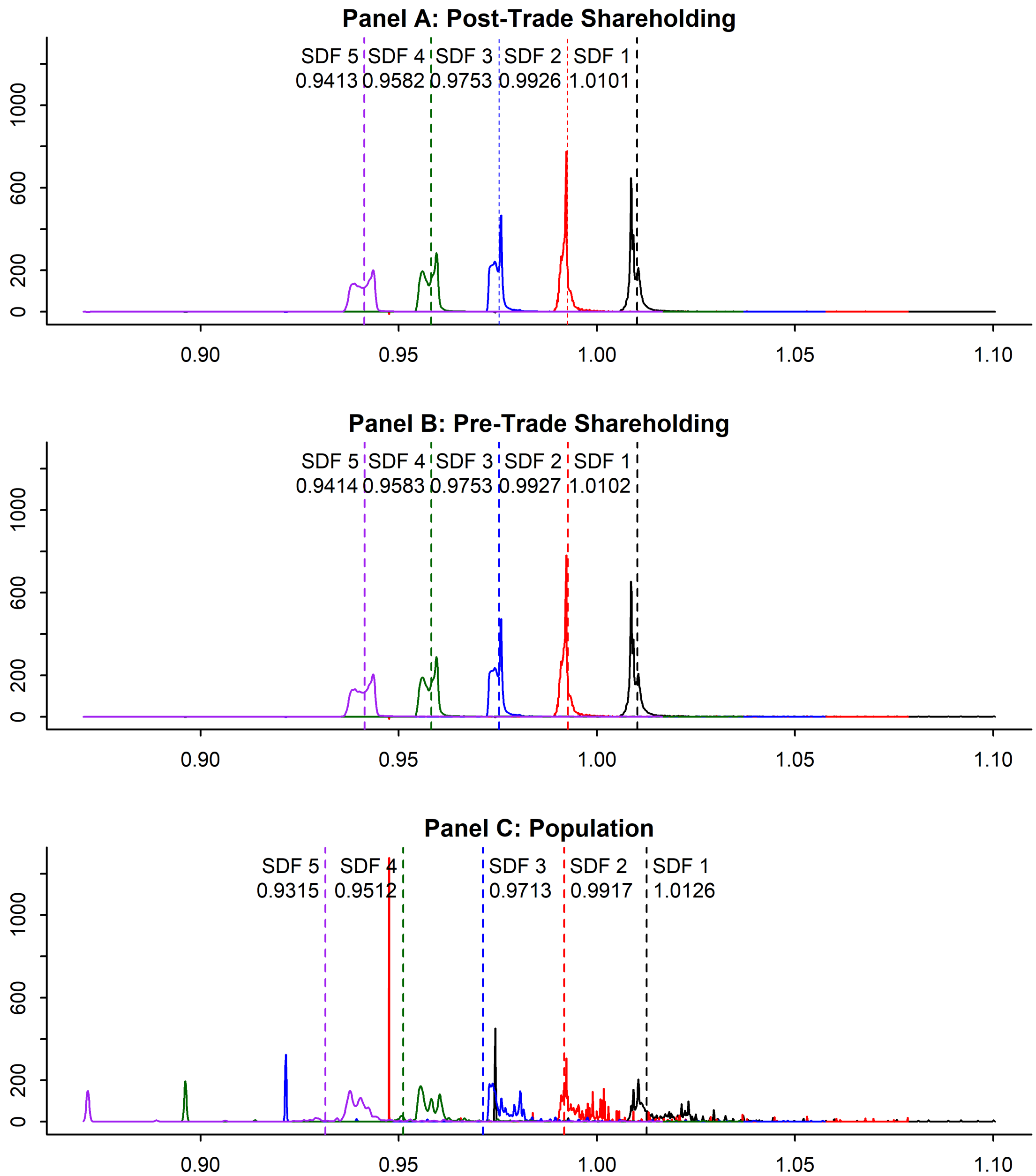

Figure 1: Distribution of households over their intertemporal marginal rate of substitution (IMRS) $\beta \frac{\mathbb{E}_{\eta'} u'(c')}{u'(c)}$ for each future aggregate state, conditional on starting in the median TFP state, the modal capital knot on the solution grid, and the 2010's level of earnings risk. Panel A shows the IMRS weighted by the post-trade shareholding of households who are not at the borrowing limit. Panel B shows the IMRS weighted by the pre-trade shareholding, similar to the process proposed in Grossman and Hart (1979). Panel C shows the IMRS weighted by population. Vertical dashed lines indicate the implied aggregate IMRS using each weighting scheme.

range of marginal rates of substitution. For each future state, there is a long, thin right tail of households that value the future up to 11 percent more than the aggregate valuation. There also exists a mass of households that are fully against their borrowing limit. These households value the future by an average of 7 percent less than the aggregate. However, the households in these tails are relatively unimportant to shaping aggregate discount factors. For example, 98.0% of shares are held by the 60.4% of households whose valuation of the median future aggregate state is within 50 basis points of the aggregate share-weighted IMRS. So while nearly 40% of households differ from the aggregate valuation by more than 50 basis points, these households only hold 2.0% of shares.

The difference between pre-trade and post-trade shareholding is minor, as seen in comparing panels A and B in Figure 1. Once a household has self-insured to a sufficient degree, it can buy or sell equity to smooth consumption in response to a negative idiosyncratic productivity shock. Using pre-trade weights slightly alters the value of future payouts.

Further, no individual investor has an intertemporal marginal rate of substitution that exactly matches the aggregate pricing kernel. In the simulated economy, the closest match as measured by absolute distance is a household that is 47 times richer than average with a productivity draw 22 times higher than average. However, this household does not have any special properties. It is not the richest household, the most productive household, or any sort of "median" household. In this preference aggregation, there are no dictators. Viewed another way, this discounting satisfies the Makowski criterion, but it would not be generated by the process described in Makowski (1983).

4.3.2 Comparison to a Representative Household Economy

The first and third sub-panels of Table 2 compare the baseline model to one with a representative household. Without idiosyncratic risk, households do not need firms to accumulate as much capital for precautionary savings. Aggregate demand for savings falls, which drives up the real interest rate. Firms invest less and pay out larger average dividends.

4.3.3 Returns to Scale and Adjustment Costs

The first and last sub-panels of Table 2 compare the baseline model to one with the same level of idiosyncratic risk, but with constant returns to scale technology and no adjustment costs. In this alternate setting, equilibrium firm value is equal to the capital stock. Therefore, this alternate formulation can be viewed as one where households directly hold capital.

In the setting where the firm is trivial, the most significant change is in the sign of the correlation of dividends to output.[35] Without an adjustment cost, households dissave their capital stock to smooth consumption over the business cycle. Consumption is higher and slightly less volatile, and has a much higher autocorrelation. Investment becomes more volatile and less strongly correlated with GDP.

[35] In the spirit of comparing this setting to one where households directly hold capital, dividends are here best thought of as the sum of rental returns less the net change in capital.

The last notable difference between these settings is that the real rate of return falls by about 0.5 percentage points. Because capital can be sold off freely during a recession, savings have a higher insurance value over the business cycle. Demand increases, which lowers the rate of return on equity.

4.4 Business Cycles with Alternate Discounting

The literature frequently uses two alternate approaches to model firm behavior in incomplete markets with shareholder heterogeneity. The approach proposed by Grossman and Hart (1979) proposes using pre-trade shareholder weights to calculate the firm's discount factor. The second common approach is for the firm to discount future dividends using some other market safe rate, such as the rate on bonds (as is the case in Kaplan, Moll and Violante (2018)). How different does an economy look when using these alternate approaches?

Table 6 shows how model economies change under alternate discounting schemes. The pre-trade stochastic discount factor is calculated as in Grossman and Hart (1979). The "safe rate" discount factor is calculated using the implied safe rate from interior shareholders, spread across probability states: $\chi_{\text{safe}}(z_x|\mathbb{Z}) = \pi_{mx}\sum_{n=1}^{N_z}\chi(z_n|\mathbb{Z})$. "Include Constrained" aggregates MRS across all shareholders, including those who choose $a' = \underline{a}$.

Measure	Baseline	Pre-trade	Safe Rate	Include Constrained
Capital/Output Ratio	2.537	2.536	2.540	2.539
Dividend Yield	2.903%	2.904%	2.902%	2.903%
Price/Cash Flow	18.38	18.38	18.37	18.37
Price/Capital Ratio	2.680	2.682	2.676	2.677
Corr(D,Y)	-0.032	-0.030	-0.034	-0.032

Table 6: Comparison of select average model moments under alternate discounting regimes. The baseline setting matches the standard calibration with a 2010's level of income risk. The pre-trade approach mirrors the approach used by Grossman and Hart (1979). The Safe Rate approach uses an implied safe rate, calculated as $\chi_{\text{safe}}(z_x|\mathbb{Z}) = \pi_{mx}\sum_{n=1}^{N_z}\chi(z_n|\mathbb{Z})$. The Baseline (including constrained) model includes households who are at the borrowing limit in its calculation of the SDF.

While alternate discounting regimes permit Pareto improvements, the aggregate difference is quite small. To see why, consider the expression for the difference between expressions for the stochastic discount factor when $\underline{a} = 0$,

$$\chi_{\text{error}}(z_n|\mathbb{Z}) = \pi_{mn}\beta\int_{\mathbf{S}}\frac{\sum_{j=1}^{N_\eta}\pi_{ij}u'(c'_{jn})}{u'(c)}(a'-a)\mu(d[a\times\eta]). \tag{22}$$

Practitioners should consider Equation 22 when thinking about the importance of using the correct discount factor. There will only be a large difference in outcomes if two conditions are met. First, there must be a sufficiently large mass of shareholders who make large shareholding changes. Second, those households who make large purchases (or sales) must have large differences in their relative valuation of aggregate states. In the calibrated model, the largest change in shareholding comes when low income and low wealth households draw the highest level from the productivity distribution. These households save significantly so they can preserve their risky income into the future. However, these only represent approximately 0.03% of the households in the economy. The remainder of households make very small changes to their shareholding.[36]

4.5 Impulse Responses

I next use the model to predict how idiosyncratic risk and capital structure influence recoveries from aggregate productivity shocks. I start by simulating the economy for 900 periods with TFP at the median level. This produces a steady state distribution of households and a constant level of capital. The economy then experiences a shock that is one standard deviation in size, which decays naturally at the rate of ρ_z. In each simulated date, I solve the equity price and law of motion using the process described in Section 3.8.

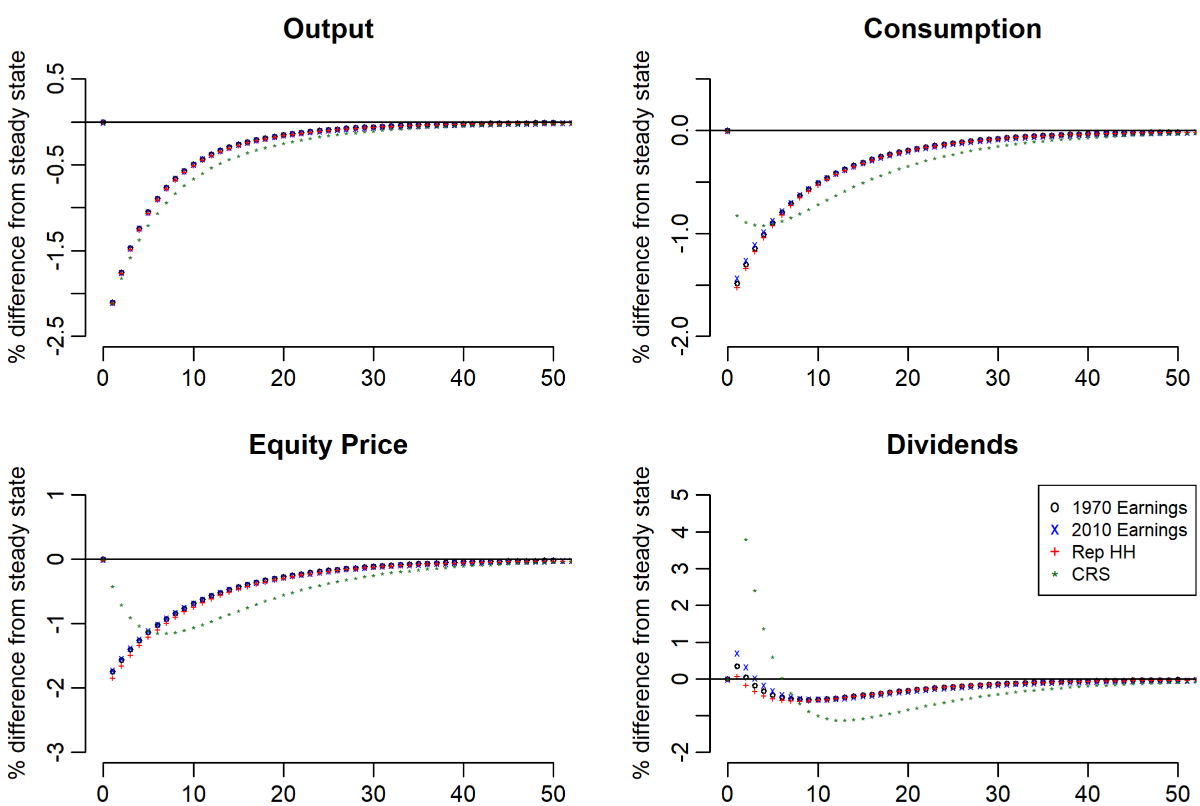

Figure 2: Impulse responses to a standard deviation negative aggregate productivity shock. Each impulse response is run through four settings. The first two use the levels of idiosyncratic risk in 1970 and 2010. The third setting shuts down the idiosyncratic risk channel. The final setting is run with constant returns and no adjustment cost, which represents a setting where households directly hold capital.

Figure 2 shows impulse responses under the same four scenarios as described in Section 4.3. With more income inequality, the economy recovers to the steady state slightly faster than economies

[36] A larger difference would appear if there were other large, uninsurable shocks to households. Idiosyncratic shocks to the discounting value β, for example, would generate enormous differences between models.

with less risk. With more inequality, firms are owned by wealthier households. These wealthy households are patient and can afford to wait out a recession by slightly dissaving in equity. Because these households are patient, the firm is not pressured to issue dividends and it hews closely to its existing investment strategy.

The setting without adjustment costs and with constant returns to scale (which proxies households directly holding capital) demonstrates strikingly different dynamics. Because capital stock is equal to share price in this setting, equity price reaches its minimum long after the peak of the recession. Households smooth consumption at the date of the shock by consuming their capital stock. Naturally, this capital dissaving prolongs the recession relative to the other settings.

5 Wealth Redistribution Experiments

I next use unanticipated wealth redistribution shocks to examine the role of wealth inequality in shaping outcomes. In these experiments, I first simulate the economy for 900 periods without aggregate productivity shocks to find a steady state level for the economy. The end of this 900 year simulation is defined as date 0. In date 1, I shock the economy with an unanticipated wealth redistribution shock. At the beginning of the period of the shock, I redistribute assets to match a chosen target distribution. I then simulate the economy as described in Section 3.8. These shocks use the baseline 1970's level of earnings risk.

	Top 1%	1%-5%	5%-10%	Bottom 90%
Endogenous Wealth Distribution	26.7%	25.0%	14.5%	33.8%
Equality Shock	1.0%	4.0%	5.0%	90.0%
Inequality Shock	20.0%	80.0%	0.0%	0.0%

Table 7: Distribution of wealth in each shock setting. The baseline case with a TFP shock uses the wealth distribution from the 1970 baseline model and is consistent with the first line in Table 4. The "equality" case distributes all wealth evenly among all households, while the "inequality" case distributes all wealth equally among 5 percent of households. The first line replicates the simulated wealth distribution in Table 4.

I run two wealth redistribution experiments. The first shocks the economy to equality and the second gives all wealth to 5 percent of households, both distributed evenly across idiosyncratic productivity levels. For comparison, I also include a single standard deviation negative TFP shock. Table 7 describes the distribution of wealth in each shock scenario.

Figure 3 documents the results of these experiments. When increasing inequality, investment and equity price immediately increase. Wealthy households are more patient than other households, so firms increase their investment, which leads to output growth. Aggregate consumption falls while the economy focuses on building its capital stock, though it eventually becomes higher than the baseline 12 years after the initial shock. The kink in investment during the first three years following the shock is driven by reallocation of equity among households. Equity was redistributed independently of idiosyncratic productivity, so some high-income households entered date 1 without savings. During the first few periods, these high-income households bought

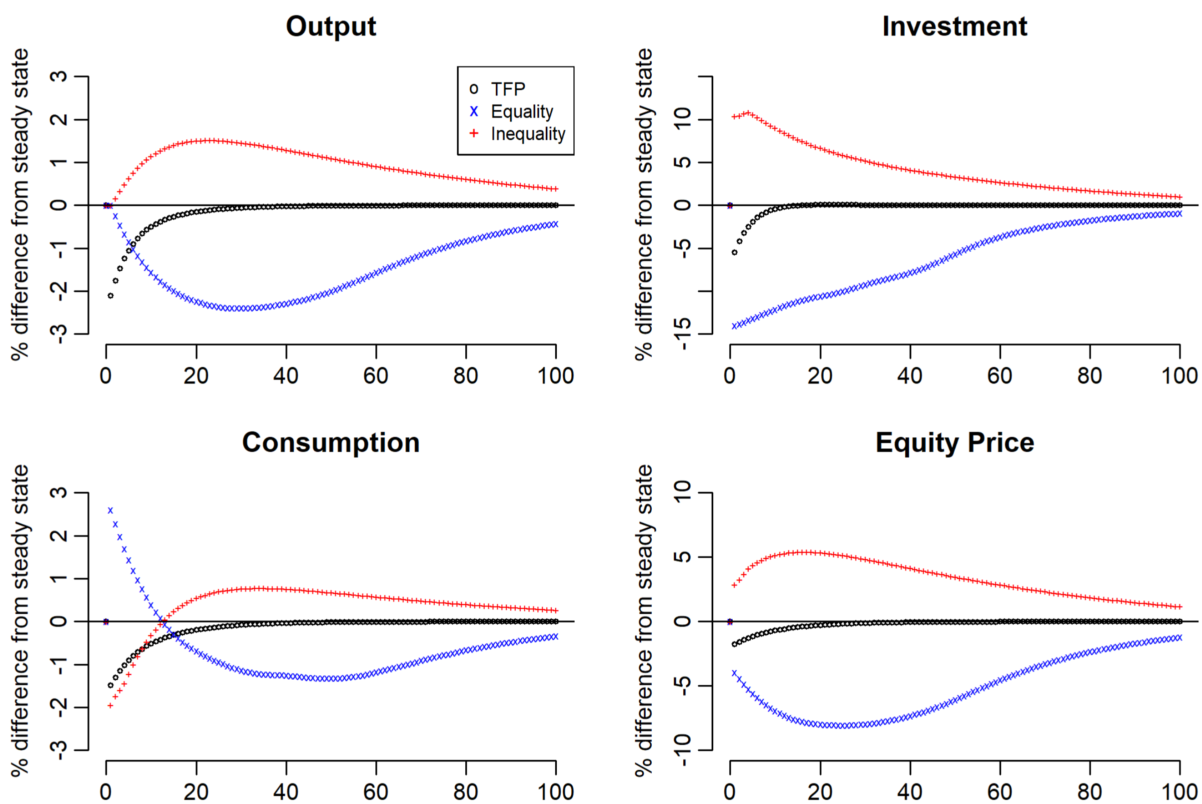

Figure 3: Impulse responses to unanticipated wealth redistribution shocks or standard TFP shock. Each of these starts from a steady state with zero TFP innovations for 900 years, then are shocked at date 1. “Equality” reallocates all wealth equally. “Inequality” gives 5 percent of households all wealth. “TFP” shows the effect of a single standard deviation negative TFP shock for comparison.

up shares from the low-income wealthy households. Once these households stopped buying up equity, investment begins to follow a more stable path.

In the full equality case, no households are near their borrowing limit and none are very rich. Households do not need much precautionary savings because they have large buffer stocks already, so investment and equity price falls significantly (14.0% and 8.0%) and remains below the steady state by at least 1% for 98 years. Aggregate consumption increases in the first 12 years as firms dissave and pay out excess dividends to shareholders.

Relative to a TFP shock, wealth redistribution shocks are incredibly persistent. Output returns to within half a percent of the steady state within 10 years of an aggregate productivity shock. In comparison, it takes 97 (92) years for output to return to within half a percent of its steady state level after an equality (inequality) redistribution shock.

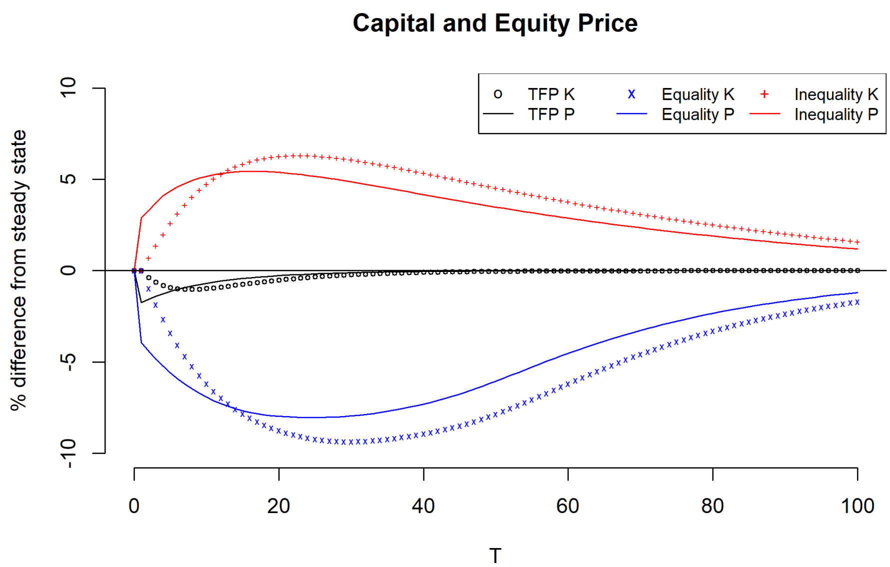

Figure 4: Percentage change in equity price and capital following a wealth or TFP shock.

Figure 4 demonstrates that wealth shocks persistently influence the aggregate economy via capital accumulation. When shocked toward high inequality, households that earn high income but with low wealth demand shares to insure themselves against future low idiosyncratic productivity draws. Wealthy households know they can smooth consumption across risky aggregate states by selling shares, so they have relatively little curvature in their stochastic discount factors. We also see that the equity price hits its peak (trough) years before the capital stock hits its peak (trough).

6 Conclusion

This model resolves the open question regarding how corporate firms should value future payouts when their shareholders have differing valuations of uncertain aggregate states. The equilibrium discounting mechanism drives firms to maximize their net market value, consistent with observed shareholder-firm interactions. An increase in idiosyncratic household productivity risk increases capital investment and lowers equity rate of return. Additionally, the increase in idiosyncratic earnings risk observed from 1970 to 2010 explained approximately half of the observed decline in dividend yields and increase in the price to cash flow ratio.

This discounting method can be immediately extended to a setting with firm heterogeneity with only a small change to the solution algorithm. There is evidence that household earnings are becoming more dispersed over time (Hubmer, Krusell and Smith, 2021) alongside evidence that firm dynamism is falling (Akcigit and Ates, 2023). These phenomena have not been studied in tandem in part because there is not a consensus method for linking heterogeneous shareholders to firm choices.

This model is also well-suited to studying tax policy in a real business cycle setting with idiosyncratic risk. To properly model the importance of wealth, dividend, corporate, and capital gains taxes, production firms need to make nontrivial decisions about investment and dividend payments. The model also allows researchers to study firms where shareholding is significantly constrained, as may be the case with 401k plans, pension systems, or Social Security funds invested in private firms.

A Alternate Equivalent Admissible Discount Factors

From Corollary 1, the condition required to preempt a shareholder challenge can be written as

$$\sum_{n=1}^{N_z} \chi^*(z_n|\mathbb{Z}) \frac{\partial p' + \partial d'}{\partial \hat{k}'} = \sum_{n=1}^{N_z} \tilde{\chi}(z_n|\mathbb{Z}) \frac{\partial p' + \partial d'}{\partial k'}. \tag{23}$$

This equation shows that there are a continuum of possible discount factors that can lead to the same capital choice k^*.To illustrate this fact, consider a simplified version of the firm's problem where the adjustment cost is set to zero, $\psi = 0$. Define χ_0 as the discount factor vector calculated by equation 15, which is size $[N_z\ 1]$. From the firm's first order condition, the choice of k' solves

$$1 = \sum_{n=1}^{N_z} \chi_n \frac{\partial \pi_z'}{\partial k'}$$

which is a simplified version of Equation 11 absent adjustment costs. Then, define Π as the value of $\frac{\partial \pi_z'}{\partial k'}$ evaluated at $k' = k^*$. Π is also a vector of size $[N_z\ 1]$. A discount factor that induces the optimal investment level is given by any χ that solves:

$$1 = \chi' \Pi \tag{24}$$

As long as there are two or more exogenous states that occur with positive probability, there are a continuum of χ that satisfy the equation above. For example, a firm could state a single cost of capital, as documented empirically in Gormsen and Huber (2025).

B Endogenous Borrowing Limit

The natural borrowing limit in any state $\underline{a}(\mathbb{Z})$ is recursively defined as the maximum amount a household can borrow in any aggregate state such that they will always be able to afford at least the natural borrowing limit in any future aggregate state. Formally, each borrowing limit is defined recursively by,

$$\underline{a}(\mathbb{Z}) = \max_{\mathbb{Z}'|\mathbb{Z}} \left(\frac{P(\mathbb{Z}')\underline{a}(\mathbb{Z}') - w(\mathbb{Z}')\underline{\eta}}{P(\mathbb{Z}') + D(\mathbb{Z}')} \right) \tag{25}$$

where $\underline{\eta}$ is the lowest labor productivity state. Due to the Inada condition, no household will ever choose to borrow up to the natural borrowing limit. A scalar interpretation of this natural borrowing limit would be $\underline{\mathrm{A}} = \max_{\mathbb{Z}} \underline{\mathrm{a}}(\mathbb{Z})$.

C Algorithm Details

I discretize the aggregate state z into 5 states using the Rouwenhorst algorithm developed in Rouwenhorst (1995). I discretize the idiosyncratic productivity process following the process in Hubmer, Krusell and Smith (2021), with the bottom 90% of households following a lognormal AR(1) process and the top 10% of households following a Pareto distribution. The idiosyncratic productivity distribution has $N_\eta = 13$. To proxy the (degenerate) distribution of firms μ_F, I use a log spaced grid of aggregate capital, which proxies aggregate total wealth.[37] I linearly space log capital ($M = \ln(K)$) into 11 grid points. Finally, I discretize the choice values for a' and k'. Although there is a representative sector of production firms, each firm still needs to understand its off-equilibrium value in order to know that it is making the optimal capital investment choice for itself. A coarse grid is used to solve household and firm value functions, which has 69 and 59 knots, respectively.[38] A fine grid is used for the proxy distribution and simulation, which has 699 grid points for shareholding. A fine grid over capital is not needed due to the degenerate distribution of firms.

I start with a naïve guess of the distribution of households by assuming that all households start with shareholding $a = 1$ and individual productivity is distributed at the stationary level. I guess that $K' = K$, which also pins down aggregate dividends. Wage is determined by the first order conditions from the representative firm's choice of labor. The guess of share price is trivial, but I start by assuming $P = \beta D/(1-\beta)$. This guess of the price is consistent with an asset priced

[37] Because the distribution of firms is degenerate, aggregate capital K' directly follows from the representative firm's choice of capital k'.

[38] In practice, Den Haan errors for equity price average below 1E-4 once there are at least 39 points on the shareholding grid. Relative to a coarse grid with 199 points for shareholding, 69 knots are accurate to within 0.2 percent for common great ratios like consumption to GDP, price to cash flow, capital to output, and dividend yield.

in a riskless Lucas (1978) economy. Finally, I also guess starting levels for the firm's value G, the household's value V, and the household's period marginal utility of consumption (MUC). For a starting guess, I set MUC^0 as the marginal utility of consuming the entire labor wage endowment at the equilibrium wage.

In the algorithm below, I search for equity price via bisection. Conditional on a guess of the future aggregate capital stock, demand for shares is weakly decreasing in price, so there is a single price that clears the equity market. I solve the household's problem with the endogenous grid method, and I solve the firm's problem via golden section search.

Before running the looping algorithm, I solve the household's problem without aggregate risk and with exogenously chosen prices and dividends.

In each iteration o, the algorithm proceeds as follows:

1 **Outer Loop:** In each aggregate state indexed by (z, M),

  (a) **Solve LOM:** Guess a future aggregate state M'_g, which implies aggregate dividends D_g

    i. **Clear the Equity Market via bisection:** Guess a price for equity P.

      A. Solve each household's optimal choice of a' given the previous V^o, proposed law of motion M'_g, dividends D_g, and future prices and dividends P^o & D^o, which are consistent the proposed law of motion.

      B. Using the proxy distribution, measure total shareholding $A(P) = \int_{\mathbf{S}} a' \mu(d[a \times \eta])$.

      C. If $A(P) - 1 >$ precision (1E-6), there is excess demand for shares, so the price needs to rise. Return to step 1(a)i

      D. If $1 - A(P) >$ precision, there is insufficient demand for shares, so the price is too high and needs to fall. Return to step 1(a)i.

    ii. Measure the implied stochastic discount factor χ (Equation 15 of the main text) using the previous iteration's marginal utility of consumption MUC^o array to approximate the value of future marginal utility.

    iii. **Consistency with aggregates:** Solve the representative firm's problem given the discount factor χ and firm values G^o, which yields k'.

      A. If $k' - K' >$ precision, the guess of M'_g was too low. Guess a higher K' and return to step 1a

      B. If $K' - k' >$ precision, the guess of M'_g was too high. Guess a lower K' and return to step 1a

  (b) **Update prices:** Once the aggregate capital law of motion is consistent with the representative firm's choices under the household aggregate discount factor, update the guess of $P^{o+1}, D^{o+1}, K'^{o+1}$.

2 **Update Value Functions:** With a consistent guess of aggregates, solve the household's and the firm's problem conditional on the updated guesses of $P^{o+1}, D^{o+1}, K'^{o+1}$

   (a) $V^{o+1} = u(c) + \beta \mathbb{E} V^o$

   (b) $MUC^{o+1} = u'(c)$

   (c) $G^{o+1} = d + \chi G^o$

   (d) If the norm of the value function is sufficiently small $|V^{o+1} - V^o| <$ precision, proceed to the simulation. Otherwise, return to step 1.

3 **Simulation:** Once value functions have converged, simulate the economy for 1,750 periods. Throw out the first 750 periods, and use the remainder to calculate the new proxy distribution for each proxy aggregate state.

I run a total of six iterations of the outer loop and simulation process. Den Haan (2010) errors for share price (future capital stock) average 4.87E-05 (1.18E-05) with a maximum of 3.12E-04 (7.32E-05).[39] The algorithm runs in approximately 65 hours on ten cores of an Intel Xeon Gold 6230R CPU.

D Decomposing Earnings Risk

The modeled change from 1970 to 2010 featured two changes to the earnings process. The parameter σ_η increased earnings risk for the bottom 90% of households, while the parameter κ increased earnings risk for those in the top 10% of the income distribution. How do these different changes contribute to the overall change in the economy?

I decompose the changes in wealth to GDP and capital to GDP along both of these axes. Table 9 presents the results. The increase to the permanent risk component of the lognormal earnings process drives the largest share of the change to both wealth and capital. Increasing the income risk for the top 10% of households creates a change that is about half the size of the change to most households' earnings.

This decomposition shows that simpler income processes generally understate the importance of income risk. The tails of the income distribution play a critical role in understanding how the economy responds to changes in the income distribution. Simply increasing the income share of the top 10% households generates half as much change as changing the bottom 90% of households' income risk.

[39] This level of accuracy arrives at the fourth outer loop, after which there is no notable improvement.

Table 8: Changes to Economic Ratios - Decomposition by Labor Risk Type

	% Δ Wealth/GDP	% Δ Capital/GDP	Δ Dividend Yield (p.p)
Total Change	24.8%	10.2%	-1.01
σ_η	15.8%	6.9%	-0.71
κ	7.8%	3.7%	-0.38
Remainder	1.2%	-0.4%	0.07

Table 9: Decomposition of observed changes in ratios by labor shock change. The percentage change variables are the same variables discussed in Table III of the main text. The first two columns show percentage changes in their respective variables, while the last column shows percentage point changes to dividend yields. Total Change shows the total modeled change from 1970's risk to 2010's risk. σ_η shows how much each variable changed when the model was run with only a change to the innovation risk to the lognormal component of the earnings process. κ shows how much each variable changed when the Pareto component alone was changed. The remainder term shows how much the joint change contributes to the total modeled change.

References

Aiyagari, S. R. 1994. "Uninsured Idiosyncratic Risk and Aggregate Saving." *The Quarterly Journal of Economics*, 109(3): 659–684.

Akcigit, Ufuk, and Sina T. Ates. 2023. "What Happened to US Business Dynamism?" *Journal of Political Economy*, 131(8): 2059–2124.

Azar, José, Martin C. Schmalz, and Isabel Tecu. 2018. "Anticompetitive Effects of Common Ownership." *Journal of Finance*, 73(4): 1513–1565.

Backus, Matthew, Christopher Conlon, and Michael Sinkinson. 2021. "Common Ownership in America: 1980–2017." *American Economic Journal: Microeconomics*, 13(3): 273–308.

Bejan, Camelia. 2020. "Investment and financing in incomplete markets." *Economic Theory*, 69(1): 149–182.

Boar, Corina, Denis Gorea, and Virgiliu Midrigan. 2022. "Why Are Returns to Private Business Wealth So Dispersed?" National Bureau of Economic Research Working Paper 29705.

Braun, R. Anton, and Tomoyuki Nakajima. 2012. "Uninsured Countercyclical Risk: An Aggregation Result and Application to Optimal Monetary Policy." *Journal of the European Economic Association*, 10(6): 1450–1474.

Brockman, Paul, Tao Guo, Maria Gabriela Vivero, and Wayne Yu. 2022. "Is idiosyncratic risk priced? The international evidence." *Journal of Empirical Finance*, 66: 121–136.

Cagetti, Marco, and Mariacristina De Nardi. 2006. "Entrepreneurship, Frictions, and Wealth." *Journal of Political Economy*, 114(5): 835–870.

Campbell, John Y. 2018. *Financial decisions and markets: a course in asset pricing.* Princeton ; Oxford:Princeton University Press.

Carceles-Poveda, Eva, and Daniele Coen-Pirani. 2009. "Shareholders' unanimity with incomplete markets." *International Economic Review*, 50(2): 577–606.

Carceles-Poveda, Eva, and Daniele Coen-Pirani. 2010. "Owning capital or being shareholders: An equivalence result with incomplete markets." *Review of Economic Dynamics*, 13(3): 537–558.

Carlstrom, Charles T., and Timothy S. Fuerst. 1997. "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis." *The American Economic Review*, 87(5): 893–910.

Challe, Edouard, and Xavier Ragot. 2016. "Precautionary Saving Over the Business Cycle." *The Economic Journal*, 126(590): 135–164.

Constantinides, George M., and Anisha Ghosh. 2017. "Asset Pricing with Countercyclical Household Consumption Risk." *The Journal of Finance*, 72(1): 415–460.

Constantinides, George M., and Darrell Duffie. 1996. "Asset pricing with heterogeneous consumers." *Journal of Political Economy*, 104(2): 219–240.

DeAngelo, Harry. 1981. "Competition and Unanimity." *The American Economic Review*, 71(1): 18–27.

Den Haan, Wouter J. 2010. "Assessing the accuracy of the aggregate law of motion in models with heterogeneous agents." *Journal of Economic Dynamics and Control*, 34(1): 79–99.

Drèze, Jacques H. 1974. "Investment Under Private Ownership: Optimality, Equilibrium and Stability." In *Allocation under Uncertainty: Equilibrium and Optimality.* , ed. Jacques H Drèze, 129–166. London:Palgrave Macmillan UK.

Evans, David J. 2005. "The Elasticity of Marginal Utility of Consumption: Estimates for 20 OECD Countries." *Fiscal Studies*, 26(2): 197–224.

Fisher, Irving. 1930. *The Theory of Interest : As Determined by Impatience to Spend Income and Opportunity to Invest It.* New York:MacMillan.

Fos, Vyacheslav. 2017. "The Disciplinary Effects of Proxy Contests." *Management Science*, 63(3): 655–671.

French, Kenneth R. 2026. "Kenneth R. French - Data Library." Accessed 05/27/2026.

Gomme, Paul, B. Ravikumar, and Peter Rupert. 2011. "The return to capital and the business cycle." *Review of Economic Dynamics*, 14(2): 262–278.

Gormsen, Niels Joachim, and Kilian Huber. 2025. "Corporate Discount Rates." *American Economic Review*, 115(6): 2001–2049.

Greenwald, Daniel L, Matteo Leombroni, Hanno Lustig, and Stijn Van Nieuwerburgh. 2021. "Financial and Total Wealth Inequality with Declining Interest Rates." National Bureau of Economic Research Working Paper 28613.

Grossman, Sanford J., and Oliver D. Hart. 1979. "A Theory of Competitive Equilibrium in Stock Market Economies." *Econometrica*, 47(2): 293.

Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante. 2010. "The Macroeconomic Implications of Rising Wage Inequality in the United States." *Journal of Political Economy*, 118(4): 681–722.

Hubmer, Joachim, Per Krusell, and Anthony A. Smith. 2021. "Sources of US Wealth Inequality: Past, Present, and Future." *NBER Macroeconomics Annual*, 35: 391–455.

Huggett, Mark. 1993. "The risk-free rate in heterogeneous-agent incomplete-insurance economies." *Journal of Economic Dynamics and Control*, 17(5-6): 953–969.

Kaplan, Greg, Benjamin Moll, and Giovanni L. Violante. 2018. "Monetary policy according to HANK." *American Economic Review*, 108(3): 697–743.

Khan, Aubhik, and Julia K. Thomas. 2008. "Idiosyncratic shocks and the role of nonconvexities in plant and aggregate investment dynamics." *Econometrica*, 76(2): 395–436.

Khan, Aubhik, and Julia K. Thomas. 2013. "Credit shocks and aggregate fluctuations in an economy with production heterogeneity." *Journal of Political Economy*, 121(6): 1055–1107.

Krouse, Clement G. 1985. "Competition and Unanimity Revisited, Again." *The American Economic Review*, 75(5): 1109–1114.

Krueger, Dirk, and Hanno Lustig. 2010. "When is market incompleteness irrelevant for the price of aggregate risk (and when is it not)?" *Journal of Economic Theory*, 145(1): 1–41.

Krusell, Per, and Anthony A. Smith. 1998. "Income and wealth heterogeneity in the macroeconomy." *Journal of Political Economy*, 106(5): 867–896.

Krusell, Per, Toshihiko Mukoyama, and Anthony A. Smith. 2011. "Asset prices in a Huggett economy." *Journal of Economic Theory*, 146(3): 812–844.

Lucas, Robert E. 1978. "Asset Prices in an Exchange Economy." *Econometrica*, 46(6): 1429.

Makowski, Louis. 1983. "Competition and Unanimity Revisited." *The American Economic Review*, 73(3): 329–339.

Marcet, Albert, and Kenneth J. Singleton. 1999. "Equilibrium Asset Prices and Savings of the Heterogeneous Agents in the Presence of Incomplete Markets and Portfolio Constraints." *Macroeconomic Dynamics*, 3(2): 243–277.

Paron, James. 2021. "Heterogeneous-agent asset pricing." *SSRN Electronic Journal*.

Piketty, T., and E. Saez. 2003. "Income Inequality in the United States, 1913-1998." *The Quarterly Journal of Economics*, 118(1): 1–41.

Reiter, Michael. 2010. "Solving the incomplete markets model with aggregate uncertainty by backward induction." *Journal of Economic Dynamics and Control*, 34(1): 28–35.

Rouwenhorst, K. Geert. 1995. "10 Asset Pricing Implications of Equilibrium Business Cycle Models." In *Frontiers of Business Cycle Research.* , ed. Thomas F. Cooley, 294–330. Princeton University Press.

Saez, Emmanuel, and Gabriel Zucman. 2016. "Wealth Inequality in the United States since 1913: Evidence from Capitalized Income Tax Data." *The Quarterly Journal of Economics*, 131(2): 519–578.

Santos, Manuel S., and Michael Woodford. 1997. "Rational Asset Pricing Bubbles." *Econometrica*, 65(1): 19.

Shiller, Robert J. 2016. *Irrational exuberance.* . Revised and expanded third edition, [fourth printing, and first paperback printing] ed., Princeton Oxford:Princeton University Press.

U.S. Bureau of Economic Analysis. 2025. "Table 6.7. Investment in Private Fixed Assets by Industry Group and Legal Form of Organization." Accessed 05/22/2026.