

**Review of: Brigitte Le Roux and Henry Rouanet, Geometric Data Analysis, From Correspondence Analysis to Structured Data Analysis, Kluwer, Dordrecht, 2004, xi+475 pp.**

The term “Geometric Data Analysis” is due to Patrick Suppes (Stanford) who writes a Foreword for this encyclopedic view of Correspondence Analysis. The uniqueness of this work lies in the detailed conceptual framework, and in showing how, where and why statistical inference methods come into play.

As a data analysis methodology, Correspondence Analysis is *formal* and *geometric*. The former follows from the fact that mathematical “structures govern procedures”, the mathematics carry the burden of demonstration, and ultimately “this means a fantastic saving in intellectual investment”. The latter is due to the focus on clouds of points in geometric spaces. Unlike the sampling-oriented approach of a good deal of statistical data analysis, including multivariate data analysis, for Correspondence Analysis, “description comes first” since statistics is not reducible to probability.

The word “model” is used, in general, in lots and lots of senses (statistical, mathematical, physical models; mixture model; linear model; noise model; neural network model; sparse decomposition model; even data model). So it is helpful that the methodological models under discussion in this book are termed frame models. As a *geometric frame model*, it is also nice to see due emphasis given to “Euclidean classification” as a “companion method” of Correspondence Analysis. Euclidean classification is most in harmony with Correspondence Analysis when minimum variance (or Ward’s) agglomerative hierarchical clustering is used. The data aggregates are defined in the same Correspondence Analysis metric spaces. The clustering is Euclidean because it is constructed in the Correspondence Analysis factor space.

In social sciences, there are “two distinct conceptions of the role of statistics, namely sustaining a *sociology of variables* versus *constructing a social space*”. The former goes hand in hand with traditional statistical analysis, whereas the latter is part and parcel of Geometric Data Analysis. We will return to the construction of social space below with reference to Bourdieu.

As a “cornerstone” of Correspondence Analysis, one has the “measure versus variable duality”, or the duality of the associated (row, column) vector spaces. When aggregated, the measures are summed, whereas the variables, when they are aggregated, are averaged. A transition notation where subscripts denote measures and superscripts denote variables is handy – much handier than matrix notation – for important aspects of the analysis. Among the latter are: how one’s data departs from the “reference” marginal frequencies; how one’s data is expressed as a “transition” from one to the other of these marginal frequencies; and how the  $\chi^2$  metric is defined on the dual, complementary clouds.

Discussing the suitability of Correspondence Analysis for structured data (that is to say, structured around such variables as age and gender characteristics in a questionnaire), firstly having such structuring factors as supplementary variables allows for their incorporation into the analysis. Later, other analyses such as Analysis of Variance (ANOVA) could be very appropriate. Statistical

inference is a natural complement to descriptive analysis – as Benzécri wrote, “The model should follow the data, not the reverse!” – and both phases together comprise a powerful way to inductively carry out data analysis.

Chapter 2 presents a most accessible and readable introduction to Correspondence Analysis, based on the view of the measure versus variable duality. This leads to Correspondence Analysis being viewed as analysis of a measure over the Cartesian product with strictly positive marginals. All of the essentials of Correspondence Analysis are introduced, going as far as aids to interpretation (e.g. contributions of points to the inertia of axes; quality of representation; data coding; supplementary elements; linkages with matrix expression, Fisher’s linear discriminant analysis, regression, and canonical correlation analysis; links with multidimensional scaling; and how or where CA links up with probabilistic model frames). A range of questions end the chapter, with solutions and with comments.

Chapter 3 dealing with analysis of a Euclidean cloud of points proceeds by way of Huyghens, and spectral decomposition (i.e., eigenvalue, eigenvector decomposition) to concentration (hyper)ellipsoids, and following discussion of the partition of a cloud the reader is led to clustering. Pride of place is accorded agglomerative hierarchical clustering with the minimum variance criterion (Ward’s method; “Euclidean classification” since the input consists of Euclidean projections on factors). The reciprocal nearest neighbors algorithm is described but not the nearest neighbor chain algorithm (which is more manageable from a computational complexity point of view, since it has an  $O(n^2)$  computational bound for most widely used agglomerative criteria). For the nearest neighbor chain and reciprocal nearest neighbor algorithms, which came out of Benzécri’s lab in the early 1980s and are now in R and Clustan among other packages, see Murtagh (2005). The ultrametric is not discussed. For the view of Correspondence Analysis being a “tale of three metrics”,  $\chi^2$ , Euclidean and ultrametric, see Murtagh (2005). Chapter 3 ends with exercises and solutions.

Chapter 4 deals comprehensively with Principal Components Analysis, viewed from the points of view of formal description and also interpretation of results, through the prism of Correspondence Analysis. In terms of number of pages it is more than half the length of Dunteman (1989) (but admittedly shorter than the 500 page length Jolliffe, 2002). The treatment of PCA in Le Roux and Rouanet is unique and in the spirit of Geometrical Data Analysis.

In chapter 5, attention is given to Multiple Correspondence Analysis. A paradigmatic case is questionnaire analysis, where each question has a number of response modalities, and just one among these response modalities is indicated by a subject. Much that is theoretical (e.g., Burt table; viewpoints linked to discriminant analysis, and canonical correlation analysis) and practical (e.g., choice of active questions; data coding for handling infrequent modalities) is covered. Before ending the chapter with questions and solutions, there is a detailed case study, relating to a French Government survey of cultural and leisure activities (3002 respondents, carried out in 1997).

We have already mentioned “structured data” above. Examples of such structuring context, available in addition to the essential data being studied,

include gender, age and education variables in the cultural and leisure questionnaire; or authors and years in textual-based authorship assessment. Due to the overall aim of studying the effect of experimental (independent, under analyst control) variables on the dependent variables (response variables), and also the need often to take nesting and crossing structures of variables into account, there is linkage in chapter 6, “Structured Data Analysis”, with ANOVA, MANOVA and regression. These techniques are discussed within the model frame of Correspondence Analysis: they are “grafted” into the Geometric Data Analysis framework. A case study is provided based on visually-based annotated video of basketball players, with the aim of selecting potential high achievers.

Chapter 7 on stability analysis first touches on alternatives, functional analysis (Krzanowski, Critchley, Tanaka, Pack and Jolliffe, Bénasséni), and bootstrapping (Diaconis and Efron, Daudin et al., Lebart et al.), before returning to the approach of Escofier and Le Roux that perturbs a cloud relative to a reference cloud, and checks what are the effects of doing this. A range of case studies is used to investigate an outcome without a given group being present, or to investigate an outcome following deletion of a single point, or to investigate the discarding of variables or modalities. A range of stability results for eigenvalues is also considered.

Often in applications inference is needed, based on statistical modeling, and the authors encourage a comprehensive approach to “Inductive Data Analysis” in chapter 8. The point is made that descriptive statistics are based on relative frequencies and do not depend on sample size, whereas test statistics (for induction or inference) do combine an erstwhile descriptive statistic with the sample size. There is much to reflect on in this chapter. Traditional significance testing, frequentist and combinatorial inference, Bayesian inference, – all are discussed in a way that is well-grounded geometrically and eminently readable.

Chapter 9, 85 pages in length, deals with three major case studies: a medication treatment study of Parkinson’s patients; French political attitudes before, and voting in, elections in 1997; and part of a large “Education Program for Gifted Youth (EPGY)” study carried out at Stanford University. There is a great wealth of experience etched deeply into these application studies.

An overview of applicable mathematics rounds off the book. We have noted earlier the authors’ eschewing of matrix formalism in favor of transitions. Matrix description is to be found throughout, but the appropriateness and powerfulness of the transition based approach is given pride of place. The transition formalism uses quite lightly the Einstein tensor notation that is used more extensively in Benzécri (1973). Van Rijsbergen (2004) provides another very commendable survey of geometric data analysis in information retrieval, using Dirac notation that is now standard in quantum physics.

The transition formalism is nicely expressed by mapping, or morphism, diagrams (as were used a great deal in Cailliez and Pagès, 1976; and which have been taken further in McCullagh, 2002, who appraises statistical modeling through the prism of category theory). Le Roux and Rouanet are unique in their succinct introduction to homomorphisms and endomorphisms, the application of spectral decomposition in this context, and discussion of extremality

properties.

A major feature of this book, following the example of Benzécri, is the close practical and experienced attention throughout paid to the epistemology of data analysis, i.e. the interface between data and reality. This is nowhere stronger than in the the authors' continuation of Bourdieu's (see e.g. Bourdieu, 1984) approach to mapping out of a social field or space. In chapters 5, 6 and 9, there is appreciable discussion of Bourdieu's use of Geometric Data Analysis.

In conclusion, this book constitutes essential background material on Geometric Data Analysis, and, for the seasoned professional, a most valuable source of reference.

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