# Fast Statistical Inference for Complex Generative Models

Michael Gutmann

http://homepages.inf.ed.ac.uk/mgutmann

Institute for Adaptive and Neural Computation
School of Informatics, University of Edinburgh

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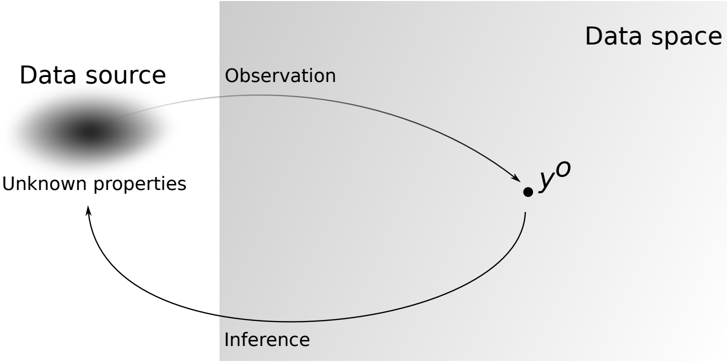
# Reference

M.U. Gutmann and J. Corander
Bayesian optimization for likelihood-free inference of simulator-based statistical models
*Journal of Machine Learning Research*, 17(125), 2016

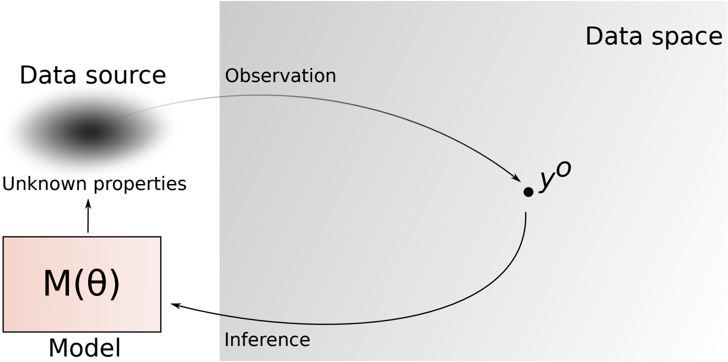
# Overall goal

- Inference: Given data $y^o$, learn about properties of its source
- Enables decision making, predictions, ...



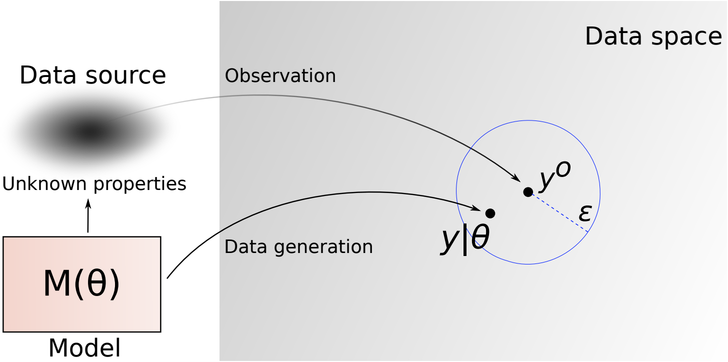
# Approach

- Set up a model with potential properties $\theta$ (hypotheses)
- See which $\theta$ are in line with the observed data $y^o$



# The likelihood function $L(\theta)$

- Measures agreement between $\theta$ and the observed data $y^o$
- Probability to generate data like $y^o$ if hypothesis $\theta$ holds



# Performing statistical inference

- If $L(\theta)$ is known, theory tells us what to do
- Maximum likelihood estimation

$$\hat{\theta} = \mathrm{argmax}_\theta \, L(\theta)$$

- Bayesian inference

$$p(\theta|y) \propto p(\theta) \times L(\theta)$$

$$\text{posterior} \propto \text{prior} \times \text{likelihood}$$

Allows us to learn from data by updating probabilities

# Problem statement

Likelihood-free inference:

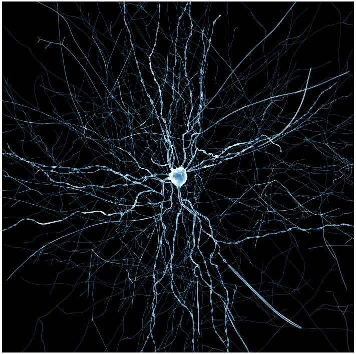
Perform statistical inference for models where

1. the likelihood function is too costly to evaluate
2. sampling – simulating data – from the model is possible

# Importance of likelihood-free inference

One reason: Such models occur widely

- Evolutionary biology:
  Simulating the evolution of life
- Neuroscience:
  Simulating neural circuits
- Astrophysics:
  Simulating the formation of galaxies, stars, or planets
- Computer vision:
  Simulating naturalistic scenes
- Health science:
  Simulating the spread of an infectious disease
- ...



Simulated neural activity in rat somatosensory cortex
(Figure from `https://bbp.epfl.ch/nmc-portal`)

# Simulator-based models

- We call such models "simulator-based models"
- Different communities use different names:
  - Stochastic simulation models
  - Generative models
  - Implicit models
  - Probabilistic programs
  - ...
- Allow us to perform experiments in silico
- Allow us to propagate uncertainty

# Flavors of likelihood-free inference

- There are several flavors of likelihood-free inference. In Bayesian setting e.g.
  - Approximate Bayesian computation (ABC)
  - Synthetic likelihood (Wood, Nature, 2010)
- General idea: Identify the values of the parameters of interest $\theta$ for which simulated data resemble the observed data
- Simulated data resemble the observed data if some distance measure $d \geq 0$ is small.

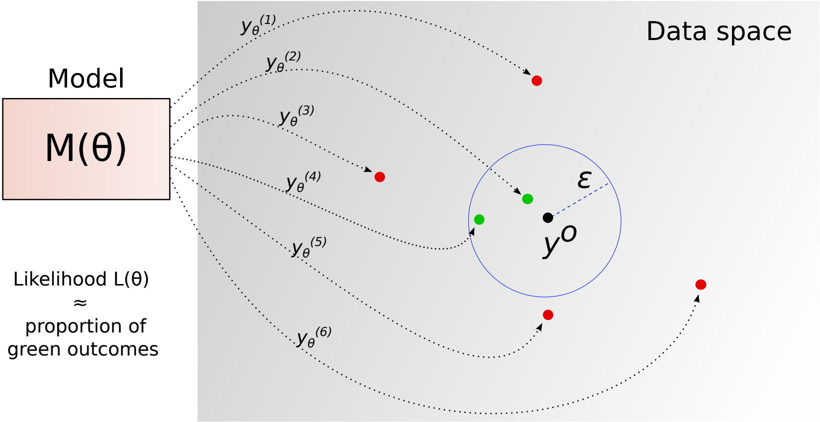
*Here: Focus on ABC, see reference paper for synthetic likelihood*

# Meta ABC algorithm

- Let $y^o$ be the observed data.
- Iterate many times:
  1. Sample $\theta$ from a proposal distribution $q(\theta)$
  2. Sample $y|\theta$ according to the model
  3. Compute distance $d(y, y^o)$ between simulated and observed data
  4. Retain $\theta$ if $d(y, y^o) \leq \epsilon$
- Different choices for $q(\theta)$ give different algorithms
- Produces samples from the (approximate) posterior when $\epsilon$ is small

# Implicit likelihood approximation

Likelihood: Probability to generate data like $y^o$ if hypothesis $\theta$ holds

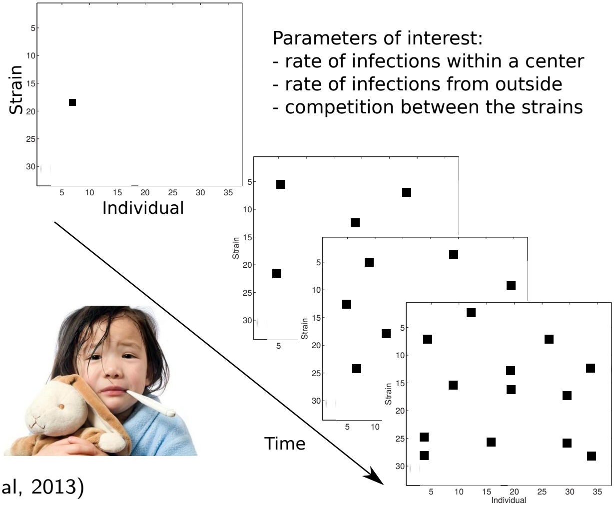



$$L(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \mathbb{1} \left( d(y_\theta^{(i)}, y^o) \leq \epsilon \right)$$

# Example: Bacterial infections in child care centers

- Likelihood intractable for cross-sectional data
- But generating data from the model is possible

Parameters of interest:
- rate of infections within a center
- rate of infections from outside
- competition between the strains



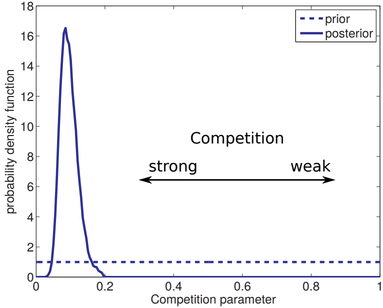

(Numminen et al, 2013)

# Example: Bacterial infections in child care centers

- Data: *Streptococcus pneumoniae* colonization for 29 centers
- Inference with Population Monte Carlo ABC
- Reveals strong competition between different bacterial strains

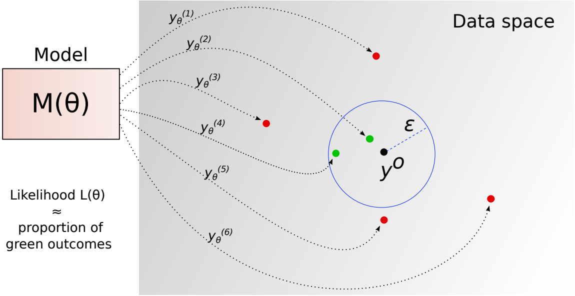
Expensive:

- 4.5 days on a cluster with 200 cores
- More than one million simulated data sets



# Why is the ABC algorithm so expensive?

1. It rejects most samples when $\epsilon$ is small
2. It does not make assumptions about the shape of $L(\boldsymbol{\theta})$
3. It does not use all information available
4. It aims at equal accuracy for all parameters




$$L(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \mathbb{1}\left(d(y_\theta^{(i)}, y^o) \leq \epsilon\right)$$

# Proposed solution

(Gutmann and Corander, 2016)

1. It rejects most samples when $\epsilon$ is small
   $\Rightarrow$ Don't reject samples – learn from them
2. It does not make assumptions about the shape of $L(\theta)$
   $\Rightarrow$ Model the distances, assume average distance is smooth
3. It does not use all information available
   $\Rightarrow$ Use Bayes' theorem to update the model
4. It aims at equal accuracy for all parameters
   $\Rightarrow$ Prioritize parameter regions with small distances

*equivalent strategy applies to*
*inference with synthetic likelihood*

# Modeling (points 1 & 2)

- Data are tuples $(\theta_i, d_i)$, where $d_i = d(y_\theta^{(i)}, y^o)$
- Model the conditional distribution of $d$ given $\theta$
- Estimated model yields approximation $\hat{L}(\theta)$ for any choice of $\epsilon$

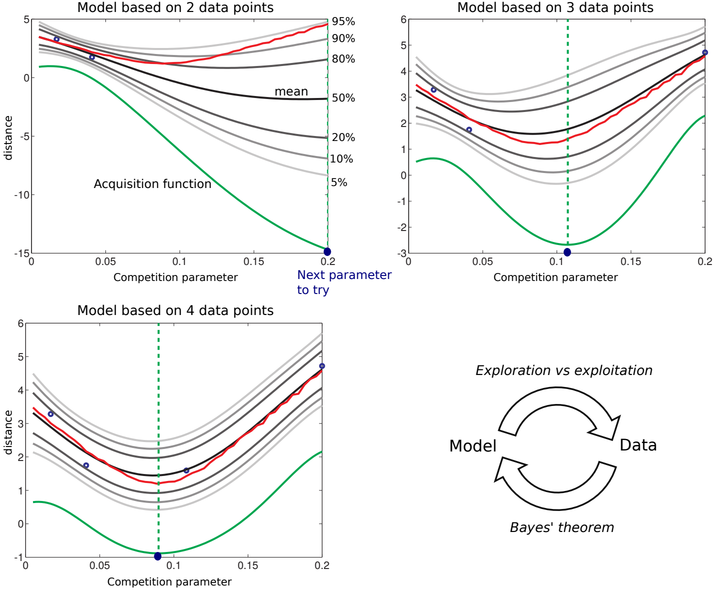
$$\hat{L}(\theta) \propto \widehat{\Pr}\left(d \leq \epsilon \mid \theta\right)$$

  $\widehat{\Pr}$ is probability under the estimated model.
- Here: Use (log) Gaussian process as model (with squared exponential covariance function)
- Approach not restricted to Gaussian processes.

# Data acquisition (points 3 & 4)

- Samples of $\theta$ could be obtained by sampling from the prior or some adaptively constructed proposal distribution
- Give priority to regions in the parameter space where distance $d$ tends to be small.
- Use Bayesian optimization to find such regions

# Bayesian optimization for likelihood-free inference



# Example: Bacterial infections in child care centers

- Comparison of the proposed approach with a standard population Monte Carlo ABC approach.
- Roughly equal results using 1000 times fewer simulations.

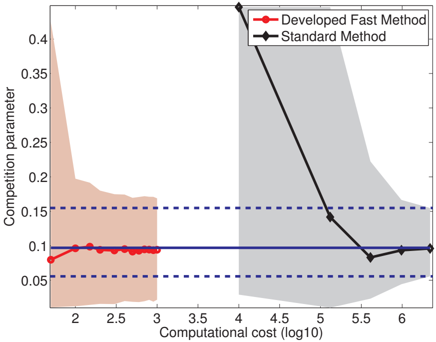
4.5 days with 200 cores

↓

90 minutes with seven cores

Posterior means: solid lines,

credibility intervals: shaded areas or dashed lines.




(Gutmann and Corander, 2016)

# Example: Bacterial infections in child care centers

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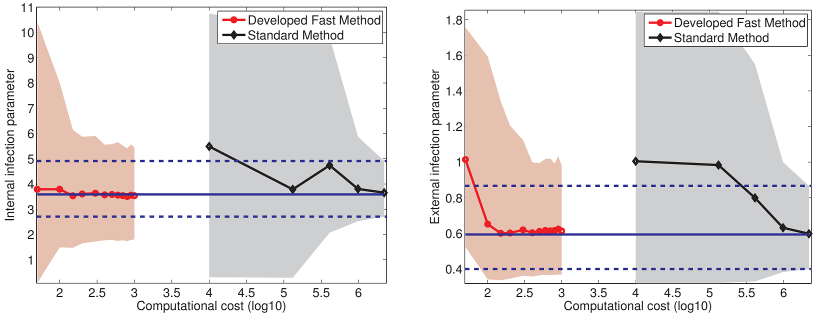


Posterior means are shown as solid lines, credibility intervals as shaded areas or dashed lines.

# Further benefits

- The proposed method makes the inference more efficient.
  - Allowed us to perform far more comprehensive data analysis than with standard approach (Numminen et al, 2016)
- Enables inference for models which were out of reach till now
  - model of evolution where simulating a single data set took us 12-24 hours (Marttinen et al, 2015)
- Enables easier assessment of parameter identifiability for complex models
  - model about transmission dynamics of tuberculosis (Lintusaari et al, 2016)

# Open questions

- Model: How to best model the distance between simulated and observed data?
- Acquisition function: Can we find strategies which are optimal for parameter inference?
- Efficient high-dimensional inference: Can we use the approach to infer the joint distribution of 1000 variables?

  *see reference paper for a discussion*

  *for first answers:* `http://homepages.inf.ed.ac.uk/mgutmann`

# Summary

- Topic: Inference for models where the likelihood is intractable but sampling is possible
- Inference principle: Find parameter values for which the distance between simulated and observed data is small
- Problem considered: Computational cost
- Proposed approach: Combine statistical modeling of the distance with decision making under uncertainty (Bayesian optimization)
- Outcome: Approach increases the efficiency of the inference by several orders of magnitude

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