Coordinate-free formation stabilization based on relative position measurements ⋆

Miguel Aranda [a], Gonzalo López-Nicolás [a], Carlos Sagüés [a], Michael M. Zavlanos [b]

[a] Inst. de Investigación en Ingeniería de Aragón - Universidad de Zaragoza, C/ María de Luna 1, E-50018 Zaragoza, Spain

[b] Dept. of Mechanical Engineering and Materials Science, Duke University, Durham, NC 27708, United States

Abstract

This paper presents a method to stabilize a group of agents moving in a two-dimensional space to a desired rigid geometric configuration. A common approach is to use information of relative interagent position vectors to carry out this specific control task. However, existing works in this vein either require the agents to express their measurements in a global coordinate reference, or generally fail to provide global stability guarantees. Our contribution is a globally convergent method that uses relative position information expressed in each agent's local reference frame, and can be implemented in a distributed networked fashion. The proposed control strategy, which is shown to have exponential convergence properties, makes each agent move so as to minimize a cost function that encompasses all the agents in the team and captures the collective control objective. The coordinate-free nature of the method emerges through the introduction of a rotation matrix, computed by each agent, in the cost function. We consider that the agents form a nearest-neighbor communications network, and they obtain the required relative position information via multi-hop propagation, which is inherently affected by time-delays. We support the feasibility of such distributed networked implementation by obtaining global stability guarantees for the formation controller when these time-delays are incorporated in the analysis. The performance of our approach is illustrated with simulations.

Key words: Multiagent systems; Formation stabilization; Autonomous mobile robots; Networked control systems; Time-delay systems

1 Introduction

We address in this paper the problem of controlling a multiagent group, which has many interesting applications, e.g. autonomous multivehicle control, cooperative sensing, surveillance, or search and rescue missions. In particular, we are interested in the stabilization of a team of mobile agents to a desired geometric configuration. Diverse methods have been proposed to carry out this task. Approaches typically rely on defining the multiagent formation in terms of absolute positions the agents must reach (Ren & Atkins, 2007; Zavlanos & Pappas, 2007; Dong & Farrell, 2008; Sabattini, Secchi, & Fantuzzi, 2011) or in terms of relative quantities such as position vectors or distances between the agents (Mesbahi & Egerstedt, 2010). In particular, relative position-based formation stabilization (Lin, Francis, & Maggiore, 2005; Olfati-Saber, Fax, & Murray, 2007; Ji & Egerstedt, 2007; Dimarogonas & Kyriakopoulos, 2008; Cortés, 2009; Kan, Dani, Shea, & Dixon, 2012; Coogan & Arcak, 2012; Oh & Ahn, 2014) specifies the formation in terms of relative distances and bearings between agents. This leads to a unique desired shape and, by using linear consensus-based control laws, permits global stabilization when the formation graph (i.e. the graph encapsulating the interactions between agents) is connected. However, methods based on position information (either absolute or relative) require the agents' measurements used for the control to be expressed in a global reference frame. In particular, in relative position-based approaches, the agents must share a common sense of orientation. For flexibility (i.e. in GPS-denied environments), simplicity and autonomy of the agents, a scenario where they can rely on their independent onboard sensors, i.e. use only locally referred information, is interesting.

⋆ This work was supported by Ministerio de Economía y Competitividad/European Union (project DPI2012-32100), Ministerio de Educación (FPU grant AP2009-3430), and NSF (CNS # 1261828 and CNS # 1302284). Corresponding author M. Aranda. Tel. +34 876555075.

Email addresses: marandac@unizar.es (Miguel Aranda), gonlopez@unizar.es (Gonzalo López-Nicolás), csagues@unizar.es (Carlos Sagüés), michael.zavlanos@duke.edu (Michael M. Zavlanos).

Distance-based formation control (Olfati-Saber & Murray, 2002; Hendrickx, Anderson, Delvenne, & Blondel, 2007; Krick, Broucke, & Francis, 2008; Dimarogonas & Johansson, 2009; Oh & Ahn, 2011) addresses this scenario. Since the relative bearings between the agents cannot be expressed in a common reference, this methodology resorts to specifying the formation in terms of interagent distances only, and requires the formation graph to be rigid (Anderson, 2008). Still, when generic numbers of agents are considered, no leader agents are used, and the desired task is for the team to acquire a rigid shape, these schemes provide only local stability guarantees, even with a complete formation graph. Global stabilization to a rigid formation is not achievable using negative-gradient, distance-based formation control (Dimarogonas & Johansson, 2009; Anderson, 2011). In addition, using distances to specify the formation implies that the target shape is always defined up to a reflection of the pattern, i.e. it is not unique. Even if only distances are used in the specification, knowledge of the directions to the neighboring agents is required in these methods to compute the control inputs.

We present here an approach that requires the same knowledge as distance-based controllers (i.e. locally expressed relative positions), but achieves global stability. The final positions of the agents form a specified rigid shape, defined up to translation and rotation. The control task is accomplished via minimization of a cost function defined for each agent in terms of global information, i.e. the relative positions of all other agents. We address the misalignment between orientation references by introducing in the cost function a local rotation matrix acting on the relative interagent vectors. These rotations, on which the agents implicitly agree, capture the method's independence of any global reference.

Our multiagent team constitutes a networked system, in which the agents interact via communications. If they have up-to-date relative position information, we show that our controller has exponential convergence properties. Realistically, the mobile agents are subject to strict limitations in terms of power consumption, computational resources and communication ranges, which require the use of a distributed, nearest-neighbor network scheme. With such setups, system scalability is greatly enhanced, but multi-hop communication causes the information used by the agents to be affected by significant time-delays. This effect needs to be introduced in the model of our system, which then becomes a nonlinear time-delay system (Gu, Kharitonov, & Chen, 2003; Richard, 2003) that is, in addition, interconnected (Hua & Guan, 2008; Papachristodoulou, Jadbabaie, & Munz, 2010; Nedic & Ozdaglar, 2010). As illustrated by this relevant literature, obtaining stability results for such a system in general conditions (e.g. varying delays, or switching network topologies) is a complex problem. We present a Lyapunov-based study of our system's convergence, considering that the network's communication pattern, in terms of active links and time-delay values, is fixed. Constant point-to-point delays are a usual assumption in the study of nonlinear time-delay systems (Richard, 2003; Papachristodoulou, Jadbabaie, & Munz, 2010), while the consideration of a fixed interaction/communication graph topology is ordinary in multiagent formation stabilization methods (Dimarogonas & Kyriakopoulos, 2008; Cortés, 2009; Dimarogonas & Johansson, 2009; Guo, Lin, Cao, & Yan, 2010; Oh & Ahn, 2011). Then, by assuming the agents' motions satisfy certain bounds regarding maximum accelerations and minimum interagent separations, we establish an upper bound for the system's worst-case point-to-point time-delay such that global stability is ensured.

Other relevant work addressing coordinate-free multiagent motion using not merely distances includes López-Nicolás, Aranda, Mezouar, and Sagüés (2012), where a formation is stabilized using a central coordinator which employs visual sensing. In Zhang (2010), each agent computes its motion using global information and both relative position and velocity information are employed, while here we use only positions. Distributed schemes have been presented addressing behaviors different from rigid-shape stabilization, e.g. rendezvous (Cortés, Martínez, & Bullo, 2006; Yu, LaValle, & Liberzon, 2012), flocking (Jadbabaie, Lin, & Morse, 2003) and other coordinated motion patterns (Moshtagh, Michael, Jadbabaie, & Daniilidis, 2009), or assuming the agents agree on (and then maintain) a common orientation reference before or during the control execution (Cortés, 2009; Oh & Ahn, 2014). Although formation schemes based on leader agents (Desai, Ostrowski, & Kumar, 2001; Guo, Lin, Cao, & Yan, 2010) have also been very popular, leaderless approaches, such as the one we propose, provide greater robustness and flexibility.

To summarize, our contribution is an approach implementable in a distributed networked fashion that globally stabilizes a multiagent group to an arbitrary unique rigid shape in the absence of a global coordinate system, and not relying on central coordinators or leader agents.

The rest of this paper is organized as follows: In Section 2 we define the formation control problem and discuss the communication patterns in the underlying networked system. In Section 3, we describe the proposed coordinate-free control method. Section 4 presents the stability analysis of our approach, addressing both a scenario where the agents have instantaneous global information, and the case where they form a distributed networked system in which information propagation is affected by time-delays. Simulation results are presented in Section 5. Finally, a brief discussion and the conclusion of the paper are given in Section 6.

2 Problem formulation

Consider a group of N agents in $\mathbb{R}^2$ having single integrator kinematics, i.e. satisfying:

$$\dot{\mathbf{q}}_\mathbf{i} = \mathbf{u}_\mathbf{i}, \tag{1}$$

where $\mathbf{q_i} \in \mathbb{R}^2$ denotes the position vector of agent i and $\mathbf{u_i} \in \mathbb{R}^2$ is its control input. We define a desired configuration, or formation shape, by a certain, fixed, reference layout of the N agents in their configuration space. The way in which we encode the desired configuration is through a set of interagent relative position vectors. To capture the existence or absence of an interaction in our control method between every pair of agents, we define an undirected formation graph, $\mathcal{G}_f = (\mathcal{V}, \mathcal{E}_f)$, where $\mathcal{V}$ is a set of N vertices, each one associated with an agent, and $\mathcal{E}_f$ is a set of links, each one expressing the connection between a pair of agents. Then, for every neighbor j of agent i in $\mathcal{G}_f$, we denote as $\mathbf{c_{ji}} \in \mathbb{R}^2$ the vector from i to j in the reference layout of the agents that defines the desired configuration. The agents are not interchangeable, i.e. each of them has a fixed place in the target formation. We then consider that the agents are in the desired configuration if the reference layout has been achieved, up to a rotation and translation.

The problem that we set out to solve in this paper is specified as follows:

Problem 1 *Given an initial configuration in which the agents are in arbitrary positions, the control objective is to stabilize them in a set of final positions such that the group is in the desired configuration.*

We also define a graph $\mathcal{G}_c = (\mathcal{V}, \mathcal{E}_c)$ to capture the communications in our distributed networked system. The edges $\mathcal{E}_c$ express the presence or absence of a direct communication link between every pair of nodes (associated with agents) in $\mathcal{V}$. Each agent in the system is assumed to be able to obtain an estimation of the relative positions of its set of neighbors in $\mathcal{G}_f$. This is the only information used by the proposed control strategy, which is described in the following section.

3 Coordinate-free control strategy

We pose the formation control problem as the minimization of the following cost function for each agent i:

$$\gamma_i = \frac{1}{4} \sum_{j \in N_i} \sum_{k \in N_i} ||\mathbf{q_{jk}} - \mathbf{R_i c_{jk}}||^2, \tag{2}$$

where the set N_i includes the neighboring agents of i in the formation graph and agent i as well. In addition, $\mathbf{q_{jk}} = \mathbf{q_j} - \mathbf{q_k}$, with the position vectors expressed in an arbitrary global coordinate frame, and $\mathbf{R_i} \in SO(2)$ is a rotation matrix:

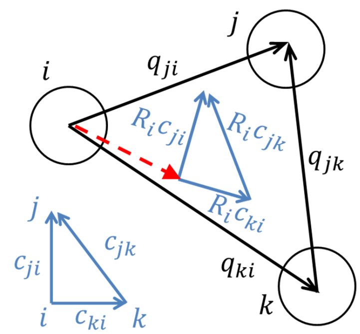

Fig. 1. Illustration of the cost function γ_i for agent i. Three agents i, j and k are considered. γ_i is defined by the sum of squared norms of the differences between the current relative vectors (i.e. $\mathbf{q_{ji}}, \mathbf{q_{jk}}, \mathbf{q_{ki}}$ in the figure) and the vectors that encode the desired pattern, (i.e. $\mathbf{c_{ji}}, \mathbf{c_{jk}}, \mathbf{c_{ki}}$ in the figure), rotated by the matrix $\mathbf{R_i}$. To minimize γ_i, agent i moves in the direction of the vector shown with a dashed line. The control strategy for agents j and k is analogous.

$$\mathbf{R_i} = \begin{bmatrix} \cos\alpha_i & -\sin\alpha_i \\ \sin\alpha_i & \cos\alpha_i \end{bmatrix}, \tag{3}$$

where the value of α_i is discussed below. The cost function is a sum of squared distances that expresses how separated the set of agents is from the desired configuration. It can be observed that γ_i accounts for agent i's distance to its neighbors, but also for the distances between its neighbors. Also, the local rotation matrix $\mathbf{R_i}$ in (2) is the key component of our method that allows it to be coordinate-free. The geometric meaning of the cost function is illustrated in Fig. 1. We compute the matrix $\mathbf{R_i}$ so as to minimize γ_i. For this, we express the function in terms of α_i and the components of the vectors, $\mathbf{q_{jk}} = [q^x_{jk}, q^y_{jk}]^T$, $\mathbf{c_{jk}} = [c^x_{jk}, c^y_{jk}]^T$:

$$\gamma_i = \frac{1}{4} \sum_{j \in N_i} \sum_{k \in N_i} \Big[(q^x_{jk} - c^x_{jk} \cos\alpha_i + c^y_{jk} \sin\alpha_i)^2 + (q^y_{jk} - c^x_{jk} \sin\alpha_i - c^y_{jk} \cos\alpha_i)^2 \Big]. \tag{4}$$

To minimize γ_i with respect to α_i, we solve $\frac{\partial \gamma_i}{\partial \alpha_i} = 0$. After manipulation, this leads to:

$$\frac{\partial \gamma_i}{\partial \alpha_i} = \frac{1}{2} \Big[\sin\alpha_i \sum_{j \in N_i} \sum_{k \in N_i} (q^x_{jk} c^x_{jk} + q^y_{jk} c^y_{jk}) - \cos\alpha_i \sum_{j \in N_i} \sum_{k \in N_i} (-q^x_{jk} c^y_{jk} + q^y_{jk} c^x_{jk}) \Big] = 0. \tag{5}$$

Solving (5) with respect to the rotation angle α_i we get:

$$\alpha_i = \arctan \frac{\sum_{j \in N_i} \sum_{k \in N_i} \mathbf{q_{jk}^T c_{jk}^\perp}}{\sum_{j \in N_i} \sum_{k \in N_i} \mathbf{q_{jk}^T c_{jk}}}. \tag{6}$$

where $\mathbf{c_{jk}^\perp} = [(0,1)^T, (-1,0)^T]\mathbf{c_{jk}}$. Let us define the variable $T_i \in \mathbb{R}$ which will be useful for the analysis of the controller throughout the paper:

$$T_i = \tan \alpha_i = \frac{\sum_{j \in N_i} \sum_{k \in N_i} \mathbf{q_{jk}^T c_{jk}^\perp}}{\sum_{j \in N_i} \sum_{k \in N_i} \mathbf{q_{jk}^T c_{jk}}}. \tag{7}$$

Observe from (6) that there are two possible solutions for α_i, separated by π radians. In order to select the correct one, we compute the second order derivative from (5):

$$\frac{\partial^2 \gamma_i}{\partial \alpha_i^2} = \frac{1}{2}\Big[\cos \alpha_i \sum_{j \in N_i} \sum_{k \in N_i} (q_{jk}^x c_{jk}^x + q_{jk}^y c_{jk}^y) + \sin \alpha_i \sum_{j \in N_i} \sum_{k \in N_i} (-q_{jk}^x c_{jk}^y + q_{jk}^y c_{jk}^x)\Big]. \tag{8}$$

By considering together (5) and (8), it can be readily seen that one of the solutions of (6) minimizes γ_i, while the other maximizes the function. The solution that is a minimum satisfies the condition $\frac{\partial^2 \gamma_i}{\partial \alpha_i^2} > 0$. If we isolate the term $\sin \alpha_i$ in (5) and then substitute it in (8), we get that this condition holds when $\sin(\alpha_i)/\sum_{j \in N_i} \sum_{k \in N_i} \mathbf{q_{jk}^T c_{jk}^\perp} > 0$. From this, it is straightforward to deduce that the value of α_i that minimizes γ_i, i.e. the value used in our controller, is given by:

$$\alpha_i = atan2(\sum_{j \in N_i} \sum_{k \in N_i} \mathbf{q_{jk}^T c_{jk}^\perp}, \sum_{j \in N_i} \sum_{k \in N_i} \mathbf{q_{jk}^T c_{jk}}), \tag{9}$$

where the $atan2$ function gives the solution of (6) for which α_i is in the quadrant that corresponds to the signs of the two input arguments. Note that the case $atan2(0,0)$, for which α_i is not defined, is theoretically possible for degenerate configurations of the agents where γ_i is constant for all α_i, see (5). These degeneracies are linked to the desired geometry (i.e. are not related with our control strategy). They are measure zero, i.e. they will never occur in practice and, therefore, we do not consider them in our analysis.

The control law for agent i is then obtained as the negative gradient of the cost function with respect to $\mathbf{q_i}$. In particular, by separating the terms in γ_i depending directly on $\mathbf{q_i}$ from the part of the function depending on $\mathbf{R_i}$, we can express the controller as follows:

$$\mathbf{\dot{q}_i} = K_c \left[-\frac{\partial \gamma_i}{\partial \mathbf{q_i}} - \frac{\partial \gamma_i}{\partial \alpha_i}\frac{\partial \alpha_i}{\partial \mathbf{q_i}}\right] = K_c \left[\sum_{j \in N_i} \mathbf{q_{ji}} - \mathbf{R_i} \sum_{j \in N_i} \mathbf{c_{ji}}\right], \tag{10}$$

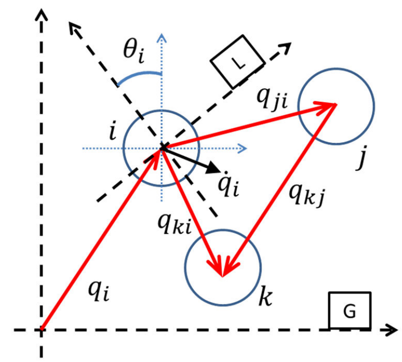

Fig. 2. Representation of the quantities used in the control law for agent i, expressed in an arbitrary global frame G. Three agents i, j and k are depicted, and the local frame L of agent i is shown.

where we have used that $\frac{\partial \gamma_i}{\partial \alpha_i} = 0$, and K_c is a positive control gain.

Let us show that each agent can compute its control input in the absence of any global coordinate reference. For this, we denote as θ_i the rotation angle between the arbitrary global frame and the local frame in which i operates, and by $\mathbf{P_i}(\theta_i) \in SO(2)$ the corresponding rotation matrix. Let us now write down the local control law computed by i, using a superscript L to denote that the quantities are expressed in the local frame:

$$\mathbf{\dot{q}_i^L} = K_c \left[\sum_{j \in N_i} \mathbf{q_{ji}^L} - \mathbf{R_i^L} \sum_{j \in N_i} \mathbf{c_{ji}^L}\right]. \tag{11}$$

Observe that $\mathbf{\dot{q}_i^L} = \mathbf{P_i \dot{q}_i}$. Each agent minimizes γ_i^L, i.e. the cost function expressed in its local frame. Given that $\mathbf{q_{jk}^L} = \mathbf{P_i q_{jk}}$ for all j, k, we can write, through simple manipulation:

$$\gamma_i^L = \frac{1}{4} \sum_{j \in N_i} \sum_{k \in N_i} ||\mathbf{q_{jk}^L} - \mathbf{R_i^L c_{jk}}||^2 = \frac{1}{4} \sum_{j \in N_i} \sum_{k \in N_i} ||\mathbf{q_{jk}} - \mathbf{P_i^{-1} R_i^L c_{jk}}||^2. \tag{12}$$

Since $\mathbf{R_i}$ minimizes γ_i and $\mathbf{R_i^L}$ minimizes γ_i^L, and the minimum is unique (as shown earlier in the section), we clearly have from (2) and (12) that $\gamma_i = \gamma_i^L$, and thus $\mathbf{R_i} = \mathbf{P_i^{-1} R_i^L}$, i.e. $\mathbf{R_i^L} = \mathbf{P_i R_i}$. Therefore, we can readily see that (11) has the form (10) when expressed in the global frame. Figure 2 provides illustration of the variables used by the controller with respect to the global and local frames.

4 Stability analysis

We note the similarity of the expression (10) to consensus-based formation control laws in the literature (Olfati-Saber, Fax, & Murray, 2007; Dimarogonas & Kyriakopoulos, 2008; Mesbahi & Egerstedt, 2010). The difference is given by the rotation matrix that we introduce. In a scenario where the use of information is distributed (i.e. $\mathcal{G}_f$ is not complete), the proposed control method requires the agents to reach consensus in this rotation matrix for the system to be stable. Whether this consensus will occur or not depends on the formation graph and the geometry of the desired formation and is not trivial to determine. In this paper, we focus our attention on the case where $\mathcal{G}_f$ is complete, i.e. $N_i = N_t \ \forall i$, with N_t being the set that contains all the agents. We first assume that up-to-date global information is always available to the agents, i.e., the communication graph $\mathcal{G}_c$ is also complete. Then, we consider a distributed scenario where the agents receive the information in $\mathcal{G}_f$ through nearest neighbor communications that are subject to time-delays. In this latter case, $\mathcal{G}_c$ can be any connected graph and the introduction of time-delays captures the effect of multi-hop information propagation in the networked system.

4.1 Stability with global information

Let us analyze the proposed control method considering that all the agents have instantaneous access to complete information of the system. We obtain the following result:

Theorem 2 *Assume that both $\mathcal{G}_f$ and $\mathcal{G}_c$ are complete graphs. Then the multiagent system evolving according to (10) converges exponentially to the desired configuration.*

PROOF. Notice first from equation (7) that when all the N agents have global information, the value of T_i will be the same for all of them at any given time instant. Let us denote by $T(t)$ this common variable, i.e. $T(t) = T_i(t) \ \forall i$ and by T_0 its initial value. Accordingly, we denote as $\mathbf{R}(\alpha, t)$ the common rotation matrix that the agents compute, and as $\mathbf{R_0}(\alpha_0)$ the initial one, with α and α_0 being the respective rotation angles.

We will compute the time derivative of T, which can be obtained as:

$$\dot{T} = \dot{T}_i = \sum_{j \in N_i} \left(\frac{\partial T_i}{\partial \mathbf{q_j}} \right)^T \dot{\mathbf{q}}_\mathbf{j}. \qquad (13)$$

Unless otherwise stated, all the sums that follow in the proof are carried out over the elements of N_t. We use the nomenclature $P = \sum_i \sum_j \mathbf{q_{ij}^T c_{ij}}$, $P_\perp = \sum_i \sum_j \mathbf{q_{ij}^T c_{ij}^\perp}$. Thus, $T = P_\perp / P$. From (7), we have the following expression:

$$\dot{T} = \sum_j \left[\frac{P \sum_k \mathbf{c_{jk}^\perp}^T - P_\perp \sum_k \mathbf{c_{jk}}^T}{P^2} K_c \sum_k (\mathbf{q_{kj}} - \mathbf{R c_{kj}}) \right]. \qquad (14)$$

Now, the following identity can be readily shown to be true by expressing all the vectors with subindex jk in terms of their two separate components j, k and then regrouping them:

$$\sum_j \left[\sum_k \mathbf{c_{jk}}^T \sum_k \mathbf{q_{jk}} \right] = \frac{N}{2} \sum_j \sum_k \mathbf{q_{jk}}^T \mathbf{c_{jk}}. \qquad (15)$$

By applying the above identity on all four addends in the numerator of equation (14), we get:

$$\dot{T} = K_c \frac{P \sum_j \sum_k \mathbf{c_{jk}^\perp}^T (\mathbf{R c_{jk}}) - P_\perp \sum_j \sum_k \mathbf{c_{jk}}^T (\mathbf{R c_{jk}})}{2P^2/N}. \qquad (16)$$

Given that $\mathbf{c_{jk}}^T (\mathbf{R c_{jk}}) = \cos(\alpha) ||\mathbf{c_{jk}}||^2$, $\mathbf{c_{jk}^\perp}^T (\mathbf{R c_{jk}}) = \sin(\alpha) ||\mathbf{c_{jk}}||^2$, and $T = \tan \alpha = P_\perp / P$, it is straightforward to see that $\dot{T} = 0$. Furthermore, we observe that the angle α, obtained as $atan2(P_\perp, P)$ (9), must always stay in the same quadrant and is, therefore, constant. A change of quadrant would imply a jump in the value of the angle, which is not possible given that the evolution of the system is continuous. Therefore, α is constant as the system evolves, and the rotation matrix computed by the agents remains constant for all time, i.e. $\mathbf{R} = \mathbf{R_0}$.

Let us now write down the dynamics of the relative position vector between any pair of agents i and j:

$$\dot{\mathbf{q}}_\mathbf{ij} = \dot{\mathbf{q}}_\mathbf{i} - \dot{\mathbf{q}}_\mathbf{j} = K_c \left[\sum_k (\mathbf{q_{ki}} - \mathbf{q_{kj}}) - \mathbf{R_0} \sum_k (\mathbf{c_{ki}} - \mathbf{c_{kj}}) \right] = - K_c N \cdot (\mathbf{q_{ij}} - \mathbf{R_0 c_{ij}}). \qquad (17)$$

Thus, we conclude that the multiagent system converges exponentially to the desired configuration. $\square$

Remark 3 *The proposed controller allows to predict collisions. To illustrate this, observe that the predicted evolution of the interagent vector $\mathbf{q_{ij}}$ at a given initial instant t_0 has, from (17), the following form:*

$$\mathbf{q_{ij}}(t) = \mathbf{q_{ij}}(t_0) e^{-K_c N (t - t_0)} + \mathbf{R_0 c_{ij}} \left[1 - e^{-K_c N (t - t_0)} \right]. \qquad (18)$$

Thus, $\mathbf{q_{ij}}$ will e.g. become null at time $t = t_0 + \ln(2)/(K_c N)$ if it is satisfied that $\mathbf{q_{ij}}(t_0)$ and $\mathbf{R_0 c_{ij}}$ are parallel, have equal length and lie on opposite sides of the coordinate origin. A general collision risk occurs when the following conditions hold: $\mathbf{q_{ij}}^T \mathbf{R_0 c_{ij}^\perp} = 0$ and

$\mathbf{q_{ij}}^T \mathbf{R_0} \mathbf{c_{ij}} < 0$. *Every agent can evaluate these two conditions for all other agents already at the beginning of the control execution. This interesting prediction property can facilitate the actual avoidance of collisions. Although we do not address here formal guarantees in this respect, a possible strategy that may be explored is for the agents that predict a future collision to modify their control gains temporarily. The idea is that, then, the resulting gain imbalance among the agents would slightly modify the rotation matrix computed by the group, changing the agents' trajectories and preventing the collision.*

4.2 Stability in a networked distributed scenario

Next, we analyze the behavior of the networked distributed implementation of the system we propose. The information used by the agents is subject to time-delays due to its propagation across the multiagent group, modeled by the communications graph $\mathcal{G}_c$. Clearly, the maximum number of communication hops needed for an agent to get the relative position information of another agent is $N-1$. Assuming that the delay associated with each of these hops is bounded by a certain time lapse Δt, the worst-case delay is given by:

$$D = (N-1) \cdot \Delta t. \tag{19}$$

The effect of time-delays on the stability of nonlinear control systems, such as the one we propose, is well known to be complex (Richard, 2003) and it is often difficult to find bounds for the maximum worst-case time-delay that makes the system stable. In our case, the interconnected nonlinear dynamics and the presence of multiple different delays complicate things further (Gu, Kharitonov, & Chen, 2003). We present next a Lyapunov-based stability study of our control method.

Let us define $\mathbf{d_i} = \sum_{j \in N_t} \mathbf{q_{ij}} - \mathbf{R} \mathbf{c_{ij}}$, where $\mathbf{R} = \mathbf{R_i} \ \forall i$ can be parameterized by $T = T_i \ \forall i$, computed as in (7). We denote as $\tau_{ji} < D$ the delay in the transmission of information from agent j to agent i, not necessarily a direct neighbor of j, across the network. We model the effect of this delay in our system as an error in the position vector measured from i to j, which we call $\epsilon_{\mathbf{ji}}$, expressed as follows:

$$\epsilon_{\mathbf{ji}}(t) = \mathbf{q_{ji}}(t - \tau_{ji}) - \mathbf{q_{ji}}(t), \ \ \forall i, j \in N_t. \tag{20}$$

We also define $\mathbf{e_i}$ as the error vector, caused by delays, affecting each agent's control input (10), such that the actual motion vectors can be expressed as follows:

$$\dot{\mathbf{q}}_{\mathbf{i}} = -K_c \cdot (\mathbf{d_i} - \mathbf{e_i}), \tag{21}$$

and we denote as $||\mathbf{e}|| = \sum_{i \in N_t} ||\mathbf{e_i}||$ the aggregate error for the complete system. Let us formulate the following assumptions regarding the characteristics of the system, which will be used in our analysis:

A1: $\mathcal{G}_c$ is a fixed connected graph and τ_{ji} is constant $\forall t \ \forall i, j$. In other words, the network's communication pattern, defined by the graph's nodes, edges and their associated time-delays, is fixed over time. Notice that this implies that the evolution of the state of the system is continuous. In general, the delays between different agents are not equal, i.e. $\tau_{ji} \neq \tau_{kl} \ \ \forall i, j, k, l \in N_t$ if $j \neq k$ or $i \neq l$.

A2: The magnitude of the total disturbance created by the time-delays, which is expressed by $||\mathbf{e}||$, is bounded by the system's state, i.e. $||\mathbf{e}|| < B \cdot \sum_i ||\mathbf{d_i}||$ for some constant $B > 1$.

A3: For any given time t, if $\sum_{j \in N_t} \sum_{k \in N_t} \mathbf{q_{jk}}^T \mathbf{c_{jk}}^\perp = 0$ in a given reference frame at t, then there exists a value M such that $|\sum_{j \in N_t} \sum_{k \in N_t} \mathbf{q_{jk}}^T \mathbf{c_{jk}}| > M$ in the same frame. This assumption means that initially not all the agents are close together and that they will remain sufficiently separated throughout the control execution. Also, consistently with A2, we assume that the magnitudes of the errors generated by the delays are bounded by the distances between agents in such a way that $|\sum_{j \in N_t} \sum_{k \in N_t} \epsilon_{\mathbf{jk}}^{\mathbf{T}} \mathbf{c_{jk}}| < M/p$ and $|\sum_{j \in N_t} \sum_{k \in N_t} \epsilon_{\mathbf{jk}}^{\mathbf{T}} \mathbf{c_{jk}}^\perp| < M/p$ for some $p > 2$.

A4: The magnitude of the acceleration of any given agent is bounded by a finite value $A_{max} > 0$ for all $t > t_0$, where t_0 is the system's initial time.

We enunciate next a Lemma that will be used in the subsequent proof of our global stability result.

Lemma 4 *Consider the multiagent system evolving according to the strategy (10), with $\mathcal{G}_f$ being a complete graph, and subject to a worst-case point-to-point time-delay D. If A1-A4 hold, the magnitude of the error due to the delays satisfies:*

$$||\mathbf{e}|| \leq 2(N-1) K_e \sum_j ||\dot{\mathbf{q}}_{\mathbf{j}}(t - D)|| (D + A_{max} \cdot D^2/2),$$

with $K_e = 1 + (4(N-1) \cdot \max_{j,k} ||\mathbf{c_{jk}}|| \cdot \max_i || \sum_j \mathbf{c_{ji}}||)/M$.

PROOF. We will look for a bound on the global magnitude of the error caused by the delays, considering separately for each agent the errors affecting its computation of the rotation matrix and those perturbing its measured relative position vectors. As in the previous proof, note that all the sums that follow are carried out over all the elements in N_t. In addition, we omit the dependence of variables on time except when necessary, for clarity reasons. From (10) and (20), the input for every agent can

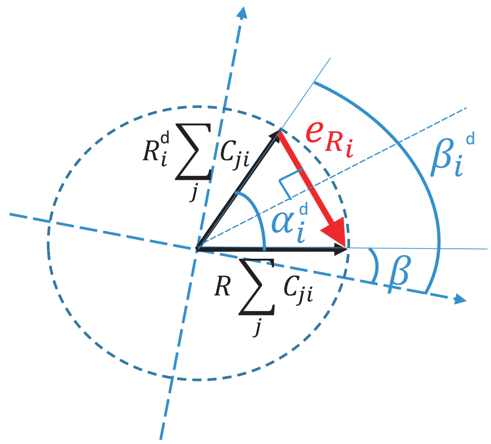

Fig. 3. Representation of the error vector $\mathbf{e_{Ri}}$ due to delays. The variables are expressed in an arbitrary global frame.

be expressed as follows:

$$\dot{\mathbf{q}}_\mathbf{i}(t) = K_c \cdot \left[\sum_j (\mathbf{q_{ji}}(t) + \epsilon_\mathbf{ji}(t)) - \mathbf{R_i^d}(T_i^d) \sum_j \mathbf{c_{ji}} \right]. \tag{22}$$

where $T_i^d = \frac{\sum_j \sum_k (\mathbf{q_{jk}^T}(t) + \epsilon_\mathbf{jk}^\mathbf{T}(t)) \mathbf{c_{jk}^\perp}}{\sum_j \sum_k (\mathbf{q_{jk}^T}(t) + \epsilon_\mathbf{jk}^\mathbf{T}(t)) \mathbf{c_{jk}}}$. Now, we define an error vector due to the error in the rotation matrix computed by agent i as:

$$\mathbf{e_{Ri}} = \mathbf{R} \sum_j \mathbf{c_{ji}} - \mathbf{R_i^d} \sum_j \mathbf{c_{ji}}. \tag{23}$$

$\mathbf{R}(T)$ is the rotation matrix at time t unaffected by delays, i.e. $T = \frac{\sum_j \sum_k \mathbf{q_{jk}^T}(t) \mathbf{c_{jk}^\perp}}{\sum_j \sum_k \mathbf{q_{jk}^T}(t) \mathbf{c_{jk}}}$. On the other hand, the errors in the relative position measurements result in the following error vector:

$$\mathbf{e_{qi}} = \sum_j \epsilon_\mathbf{ji}, \tag{24}$$

in such a way that, from (22) and (21), the total error for agent i is:

$$\mathbf{e_i} = \mathbf{e_{qi}} + \mathbf{e_{Ri}}. \tag{25}$$

We will now bound the magnitude of $\mathbf{e_{Ri}}$, in such a way that the bound is defined in terms of $\mathbf{e_{qi}}$. Figure 3 illustrates the geometry behind the error vector $\mathbf{e_{Ri}}$. Using trigonometry, it can be seen that:

$$||\mathbf{e_{Ri}}|| = 2|\sin(\alpha_i^d/2)| \cdot ||\sum_j \mathbf{c_{ji}}||, \tag{26}$$

where α_i^d is the difference between the rotation angles encapsulated by $\mathbf{R}$ (denoted as β) and $\mathbf{R_i^d}$ (denoted as β_i^d). These two angles are expressed in a common global frame. Let us choose, without loss of generality, a frame where $\mathbf{R} = \mathbf{I_2}$, i.e. $\beta = 0$. Then, $\beta_i^d = \alpha_i^d$ and, considering (7), one can see that:

$$T_i^d = \tan \beta_i^d = \tan \alpha_i^d = \frac{\sum_j \sum_k \epsilon_\mathbf{jk}^\mathbf{T} \mathbf{c_{jk}^\perp}}{\sum_j \sum_k (\mathbf{q_{jk}^T} + \epsilon_\mathbf{jk}^\mathbf{T}) \mathbf{c_{jk}}} \leq \frac{2|\sum_j \sum_k \epsilon_\mathbf{jk}^\mathbf{T} \mathbf{c_{jk}^\perp}|}{M}, \tag{27}$$

where the inequality is due to A3, choosing $p = 2$. Now observe that:

$$|\sum_j \sum_k \epsilon_\mathbf{jk}^\mathbf{T} \mathbf{c_{jk}^\perp}| \leq \max_{j,k} ||\mathbf{c_{jk}}|| \sum_j \sum_k ||\epsilon_\mathbf{jk}||, \tag{28}$$

and that we can also write:

$$\sum_j \sum_k ||\epsilon_\mathbf{jk}|| \leq \sum_j \sum_k ||\epsilon_\mathbf{ji}|| + ||\epsilon_\mathbf{ki}|| = 2(N-1) \sum_j ||\epsilon_\mathbf{ji}||, \tag{29}$$

where a constant $(N-1)$ appears instead of N due to the fact that $||\epsilon_\mathbf{ii}|| = 0 \ \forall i$. Thus, substituting (28) and (29) in (27), we reach the following bound for T_i^d:

$$T_i^d \leq \frac{4(N-1) \max_{j,k} ||\mathbf{c_{jk}}|| \sum_j ||\epsilon_\mathbf{ji}||}{M}. \tag{30}$$

Now, we can write:

$$|\sin(\alpha_i^d/2)| \leq |\alpha_i^d|/2 = |\arctan(T_i^d)|/2 \leq |T_i^d|/2, \tag{31}$$

and therefore, substituting (31) and (30) in (26), we get:

$$\begin{aligned} ||\mathbf{e_{Ri}}|| &\leq |T_i^d| \cdot ||\sum_j \mathbf{c_{ji}}|| \\ &\leq \frac{4(N-1) \max_{j,k} ||\mathbf{c_{jk}}|| \max_i ||\sum_j \mathbf{c_{ji}}||}{M} \sum_j ||\epsilon_\mathbf{ji}||, \end{aligned} \tag{32}$$

which holds $\forall i$. Finally, we obtain the following bound for the total magnitude of the error vector associated with agent i:

$$||\mathbf{e_i}(t)|| \leq ||\mathbf{e_{qi}}(t)|| + ||\mathbf{e_{Ri}}(t)|| \leq K_e \sum_j ||\epsilon_\mathbf{ji}(t)||, \tag{33}$$

with $K_e = 1 + (4(N-1) \cdot \max_{j,k} ||\mathbf{c_{jk}}|| \cdot \max_i ||\sum_j \mathbf{c_{ji}}||)/M$. The two terms including a maximum in this expression depend on the specific geometry of the desired configuration. In particular, they are the maximum interagent desired distance and the largest sum of the desired vectors from one agent to all the others. These terms are always greater than zero. We then have that the magnitude of the system's global error is:

$$||\mathbf{e}|| \leq K_e \sum_i \sum_j ||\epsilon_\mathbf{ji}||. \tag{34}$$

From A1, $||\epsilon_{\mathbf{ji}}||$ is a continuous function. In addition, each relative-position error $\epsilon_{\mathbf{ji}}$ emerges from the motion that j and i have performed in the delay interval (20). The distance traveled by every robot is limited by A_{max} by assumption A4, which gives the bound:

$$\sum_i \sum_j ||\epsilon_{\mathbf{ji}}(t)|| \leq \sum_i \sum_j \int_{t-D}^{t} ||\dot{\mathbf{q}}_{\mathbf{ji}}(\tau)|| d\tau$$

$$\leq \sum_i \sum_j \int_{t-D}^{t} (||\dot{\mathbf{q}}_{\mathbf{j}}(\tau)|| + ||\dot{\mathbf{q}}_{\mathbf{i}}(\tau)||) d\tau$$

$$\leq 2(N-1) \sum_j \int_{t-D}^{t} ||\dot{\mathbf{q}}_{\mathbf{j}}(\tau)|| d\tau$$

$$\leq 2(N-1) \sum_j ||\dot{\mathbf{q}}_{\mathbf{j}}(t-D)|| (D + A_{max} \cdot D^2/2), \quad (35)$$

where a constant $(N-1)$, instead of N, appears because $||\dot{\mathbf{q}}_{\mathbf{ii}}(\tau)|| = 0 \ \forall i$. (34) and (35) result in the statement of the Lemma. □

Theorem 5 *Consider the multiagent system evolving according to (10), with $\mathcal{G}_f$ being a complete graph, and subject to a worst-case point-to-point time-delay D. If A1-A4 are satisfied and D satisfies the following expression, which is a function of parameters of the system:*

$$D < \left(\sqrt{1 + \frac{A_{max}}{K_c(1+B)K_e(N-1)N\sqrt{N}}} - 1 \right) / A_{max}, \quad (36)$$

then the system converges asymptotically to the desired configuration.

PROOF. We will propose a Lyapunov function and, from the expression of its dynamics, state a stability condition that relates the error caused by delays, $||\mathbf{e}||$, with the non-delayed vector norms $||\mathbf{d_i}||$, see (21). Then, using the bound provided in Lemma 4 and exploiting a constraint inspired by a theorem for stability of time-delay systems, we will obtain an upper bound for D that ensures the stability condition holds.

We define the following candidate Lyapunov function for the system:

$$V = \frac{1}{2N} \sum_i ||\mathbf{d_i}||^2. \quad (37)$$

Notice that V is globally positive definite and radially unbounded. In addition, $V = 0 \Leftrightarrow \mathbf{q_{ij}} = \mathbf{Rc_{ij}} \ \forall i, j \in N_t$ (i.e. the agents are in the desired configuration). We can express the function's time derivative, considering separately the terms depending on T, as follows:

$$\dot{V} = \sum_i \left(\frac{\partial V}{\partial \mathbf{q_i}} \right)^T \dot{\mathbf{q}}_{\mathbf{i}} + \frac{\partial V}{\partial T} \dot{T} = \sum_i \mathbf{d_i}^T \dot{\mathbf{q}}_{\mathbf{i}}, \quad (38)$$

where we have used the fact that $\frac{\partial V}{\partial T} = 0$, which can be readily verified (see Section 3). Substituting (21) in (38), we have:

$$\dot{V} = K_c \sum_i \left[-||\mathbf{d_i}||^2 + \mathbf{d_i}^T \mathbf{e_i} \right]. \quad (39)$$

From (39), clearly, $\dot{V} < 0$ if and only if:

$$\sum_i \mathbf{d_i}^T \mathbf{e_i} < \sum_i ||\mathbf{d_i}||^2. \quad (40)$$

Using that $\sum_i \mathbf{d_i}^T \mathbf{e_i} \leq \sum_i ||\mathbf{d_i}|| \cdot ||\mathbf{e}||$, the condition (40) is satisfied if:

$$\sum_i ||\mathbf{d_i}|| \cdot ||\mathbf{e}|| < \sum_i ||\mathbf{d_i}||^2, \quad (41)$$

and since $\sum_i ||\mathbf{d_i}||^2 / (\sum_i ||\mathbf{d_i}||)^2 \geq 1/N$, we get that the following is a sufficient condition to ensure $\dot{V} < 0$:

$$||\mathbf{e}|| < \frac{\sum_i ||\mathbf{d_i}||}{N}. \quad (42)$$

In what follows, we provide guarantees under which (42) holds true. Due to assumption A2, we have from (21) that at time $t - D$:

$$\sum_i ||\dot{\mathbf{q}}_{\mathbf{i}}(t-D)|| \leq K_c (\sum_i ||\mathbf{d_i}(t-D)|| + ||\mathbf{e_i}(t-D)||)$$

$$\leq K_c(1+B) \sum_i ||\mathbf{d_i}(t-D)||, \quad (43)$$

and by substituting (43) in the expression stated in Lemma 4, we obtain:

$$||\mathbf{e}|| \leq$$

$$K_c 2(N-1)(1+B)K_e(D + \frac{A_{max}D^2}{2}) \sum_i ||\mathbf{d_i}(t-D)||. \quad (44)$$

Let us call $\overline{V}(t) = \max_{\tau \in [t-D,t]} V(\tau)$. Observe now that if $V(t) < \overline{V}(t)$, the fact that V grows at t will not compromise the stability of the system, since it does not make $\overline{V}(t)$ grow. It is thus only required that $\dot{V} < 0$ whenever $V(t) = \overline{V}(t)$ (Gu, Kharitonov, & Chen, 2003). This argument is the main idea behind Razumikhin's theorem for stability of time-delay systems. The condition $V(t) = \overline{V}(t)$ would be hard for us to use, since it requires knowledge of the evolution of the system in the interval $[t-D, t]$. Let us introduce a different condition. In particular, it is clear that whenever $V(t) = \overline{V}(t)$, it also holds that $V(t) \geq V(t-D)$. Then, we can simply consider this latter case so as to prove stability, since the case $V(t) = \overline{V}(t)$ is included in it. Therefore, we will use the condition that stability is ensured if $\dot{V}(t) < 0$ when it holds that $V(t) \geq V(t-D)$.

To enforce this stability condition, we first use that if $V(t) \geq V(t-D)$, then $\sum_i ||\mathbf{d_i}(t)|| \geq \sum_i ||\mathbf{d_i}(t-D)||/\sqrt{N}$. This can be readily seen from (37). Thus, by substituting the latter expression in (44), we get:

$$\frac{||\mathbf{e}||}{\sum_i ||\mathbf{d_i}||} \leq K_c 2(N-1)\sqrt{N}(1+B)K_e(D+\frac{A_{max}D^2}{2}). \tag{45}$$

Then, we enforce (42) in order to guarantee $\dot{V} < 0$. From (45), it can be readily seen that (42) is satisfied and therefore, the system is stable, if the parameters of the system satisfy the following condition:

$$K_c(1+B)K_e(D+A_{max}\cdot D^2/2)2(N-1)N\sqrt{N} < 1. \tag{46}$$

Thus, by solving (46) with respect to D, we obtain that the system is asymptotically stable if the worst-case time-delay satisfies:

$$D < \left(\sqrt{1+\frac{A_{max}}{K_c(1+B)K_e(N-1)N\sqrt{N}}}-1\right)/A_{max}, \tag{47}$$

which is the statement of the theorem. □

Remark 6 *The bound (47) is conservative. There are various reasons for this: for instance, the delays (and the errors they generate) are assumed to be maximum and to affect the system in the worst possible manner, and the Lyapunov-based stability conditions used are only sufficient. Thus, we are overestimating the effects of delay, and it is clear that the system will be able to accommodate worst-case delays considerably larger than the bound we obtained.*

Let us provide a more realistic bound for the worst-case delay in a typical practical scenario. We focus on the case where the delays are small compared to the time required for the system to converge with zero delays, which is, from (17), inversely proportional to K_cN. We assume the agents do not increase their speed, or do it negligibly, after t_0. Thus, we can consider $A_{max} = 0$. We can also use in practice the less restrictive stability condition $||\mathbf{e}|| < \sum_{i\in N_t} ||\mathbf{d_i}||$ instead of the theoretical one (42). We get an expression analogous to (46), which results in the bound:

$$D_r < 1/(C_rK_c(N-1)\sqrt{N}), \tag{48}$$

where $C_r = 2(1+B)K_e$. This new bound for the delay is still conservative, but it captures better the relationship between the maximum admissible delay and the time required for the system with zero delays to converge. Figure 4 illustrates the behavior of the bounds D and D_r as a function of K_c and N (with the other parameters kept constant). We see that these bounds decrease as K_c and N grow.

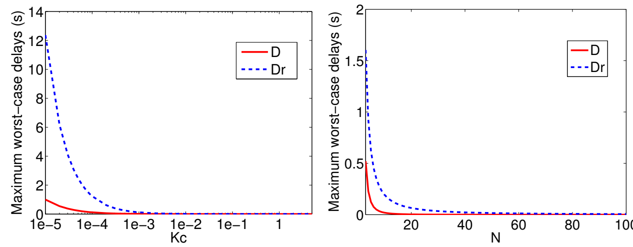

Fig. 4. Maximum delays D, from (47), and D_r, from (48), as a function of K_c -shown in logarithmic scale- (left), and N (right).

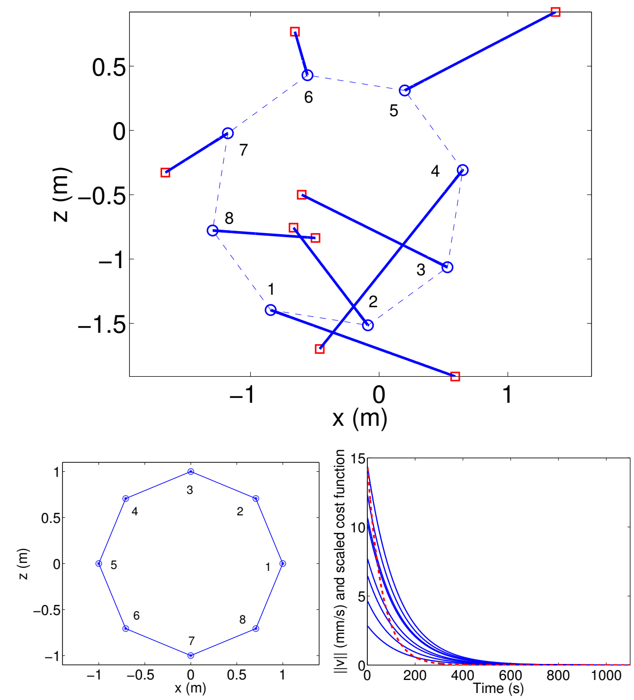

Fig. 5. Simulation results for the eight-agent circular formation. Top: Paths followed by the agents. The final positions are joined by dashed lines. The agents are labeled with numbers. Bottom-Left: Desired geometric configuration. Bottom-Right: Evolution of the norm of the velocity vectors for the eight agents (full lines) and the sum of their cost functions scaled by a constant factor (dashed line).

Remark 7 *Since $||\mathbf{e}||$ is generated by the motion of the agents, it is reasonable to assume, as we do in A2, that this magnitude will be bounded by that motion, governed by the vectors $\mathbf{d_i}$. Note that, thanks to allowing $B > 1$, A2 implies that $||\mathbf{e}||$ can be larger (but not arbitrarily larger) than the sum of norms of $\mathbf{d_i}$. In essence, A2 states that when all $\mathbf{d_i}$ vanish, $||\mathbf{e}||$ will vanish too. Observe that this ensures that the agents remain static once they reach the desired configuration (since $V = 0$ implies all $\mathbf{d_i} = \mathbf{0}$, i.e. $||\mathbf{e}|| = \mathbf{0}$, and in consequence the motion vectors*

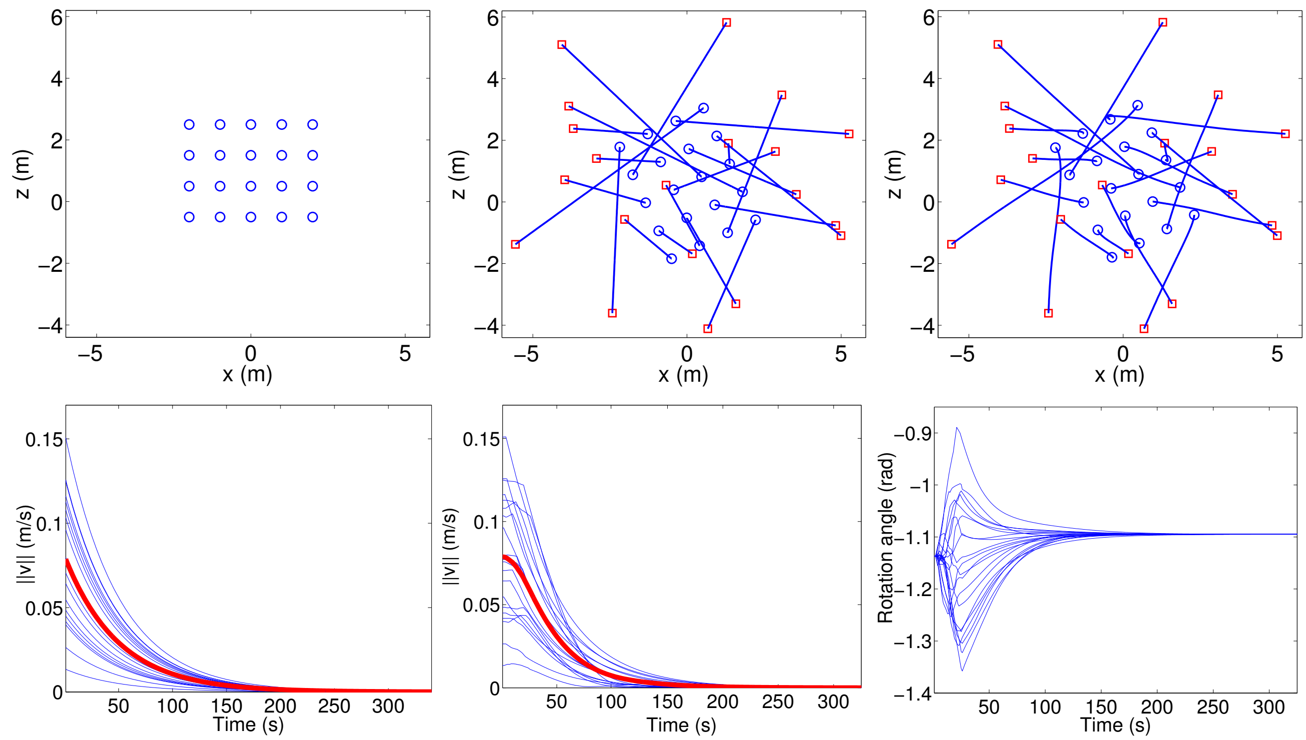

Fig. 6. Simulation results for a twenty-agent rectangular grid formation, considering a worst-case time-delay $D = 25s$. Top: desired configuration (left); agents' paths with zero delay (center) and with delays (right). The starting positions are marked with squares, while the final positions are marked with circles. Bottom: time evolution of the magnitudes of the agents' velocity vectors (thin lines) and their mean value (thick line), with zero delay (left) and with delays (center). Evolution of the formation rotation angles (expressed in a fixed arbitrary global reference frame) estimated by the agents, with delays (right).

perturbed by delays in (21) are null). It is immediate to see that the final formation is static when global up-to-date information is available (Section 4.1), looking at (10).

Remark 8 *Notice that it is ensured that A3 will be satisfied if two given agents remain separated (or, equivalently, if the agents do not all rendezvous), which is reasonable to assume. We analyze next the bounds M and p defined in this assumption. For small delays, the agents will reach the desired configuration moving along nearly straight lines. This steady convergence behavior means that typically, M will be equal to the smallest of two values: the initial sum $|\sum_j \sum_k \mathbf{q_{jk}}^T(t_0)\mathbf{c_{jk}}|$, and the value of the same sum when the desired formation is reached (i.e. when $\mathbf{q_{jk}} = \mathbf{c_{jk}} \;\; \forall j, k$), which is $\sum_j \sum_k ||\mathbf{c_{jk}}||^2$. From assumption A3, given that $|\sum_j \sum_k \mathbf{q_{jk}}^T \mathbf{c_{jk}^{\perp}}| = 0$, we have that the sum $\sum_j \sum_k \mathbf{q_{jk}}^T \mathbf{c_{jk}}$ will be nonzero. Concerning p, it depends on how the errors caused by delays $||\epsilon_{\mathbf{jk}}||$ compare with the interagent distances $||\mathbf{q_{jk}}||$. p will be large for small delays. We can safely choose it as equal to 2 for simplicity.*

Remark 9 *We can ensure that the acceleration bound A_{max} (A4) holds by reducing the control gain K_c below its nominal value, if necessary. Notice that this gain reduction would imply an increase of the admissible worst-case delay. Thus, the bound (47) always holds. The minimum gain achievable will always be positive. To see this, we reason that as the gain tends to zero, the agents would stop moving. Therefore, they would be well below the limit A_{max}, and K_c would not need to decrease further. It is worth highlighting that for any given worst-case delay of finite value, the system can be ensured to be stable by choosing the control gain K_c appropriately, according to (47) or (48).*

5 Simulations

This section presents simulation results to illustrate the performance of the proposed formation control method. In the first example, we consider a circular formation composed of eight agents. As discussed in Section 4, a complete formation graph is employed. A gain $K_c = 1e-3$ is selected. The results are displayed in Fig. 5. The paths followed by the agents from arbitrary initial positions using our control strategy are rectilinear and show exponential convergence to the desired configuration, as theoretically expected. The desired geometric formation is also shown in the same figure. Observe that both the location and the orientation of the final agent configuration are arbitrary. In addition, we represent the norms of the agents' velocity vectors, which vanish when convergence to the target formation is reached, and the sum of the cost functions computed by all the agents.

The second simulation example illustrates the behavior of the system when the information used by the agents to compute their control inputs is affected by time-delays. This case is interesting from a practical perspective, since it corresponds to a scenario where the agents obtain directly only *local* information, and acquire the rest of the information through multi-hop communications. It gives rise to a truly distributed framework. The total delay that a message experiences until it reaches its intended destination is equal to the sum of the delays incurred as the message travels through the individual links in the network on its way to its destination. We consider in the simulations that the transmission of information between every pair of agents has a different total delay, and that all these delays are constant and contained in the interval $(0, D)$. We also assume the agents have synchronized clocks, a common practical requirement in networked multiagent systems (Cortés, Martínez, & Bullo, 2006; Mesbahi & Egerstedt, 2010; Schwager, Julian, Angermann, & Rus, 2011). We consider that every relative position measurement in the network has an associated time stamp. These time stamps are used by each agent to integrate consistently its own measurements with those received from its direct neighbors when computing its estimates of the relative positions of non-direct neighbors.

Figure 6 depicts the results for an example where the desired formation shape is a rectangular grid composed of twenty agents. We compare the performance of the system with no delays and with a worst-case delay of $D = 25s$. The same initial configuration is considered in both cases. The control gain is chosen as $K_c = 1e-3$. When introducing the time-delays, we must specify the initial conditions, i.e. the state of the system for $t \in (-D, 0)$. We choose the agents to be static during that interval. The effects of the delays on the trajectories can be observed in the plots showing the agents' paths and velocities. In this case, the delays are small and the disturbance they generate is not very significant. Observe that in the presence of delays, the agents eventually reach an agreement in the rotation angles they compute (expressed in a common frame). The bounds described in section 4.2 are: $D = 0.01s.$, obtained from (47) considering $B = 2$ and neglecting A_{max}, and $D_r = 0.2s.$, computed from (48) using $C_r = 60$. As stated previously, the system is stable for worst-case delays larger than these bounds.

Figure 7 illustrates how the system's performance degrades as the worst-case time-delay increases. A wedge-shaped ten-agent desired formation is used in this case. The gain is $K_c = 3e-3$. It can be seen that when D increases, the Lyapunov function (37) eventually becomes non monotonic. In addition, for large delays (larger than $50s.$ in this case) the system is not stable. The performance of the system remains close to that of the zero-

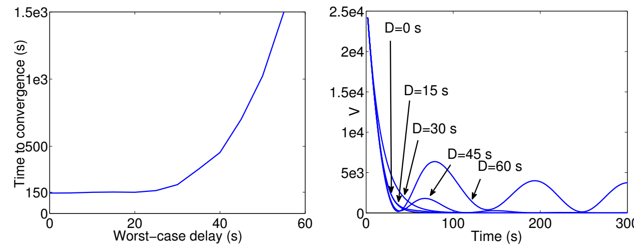

Fig. 7. Simulation results for a ten-agent wedge-shaped formation. Left: Time to convergence vs. the worst-case delay D. Right: Evolution of the Lyapunov function V for D equal to 0, 15, 30, 45 and $60s$.

delay case as long as D remains small compared to the time to convergence without delays. The figure also illustrates how the degradation of the convergence speed gets worse as D becomes larger. The time to convergence shown is estimated as the one at which the remaining control error is less than 1% of the initial one. For this example, the bounds described in section 4.2, computed as in the previous simulations, have the following values: $D = 0.02s.$ (47) and $D_r = 0.19s.$ (48).

6 Discussion and Conclusion

We have presented a control method through which a group of mobile agents can be stabilized to a desired geometric configuration. The approach employs the relative interagent positions for this task, and its relevance lies in the fact that it does not require a global common coordinate system or orientation agreement among the agents. In this paper, we focused in particular on a scenario in which all the agents have complete information of the system, obtained through multi-hop communications. This is practically feasible for a number of agents in the hundreds with state-of-the-art technology in wireless communications and mobile computing. Observe that, even if complete information is employed, the fact that it is expressed locally represents a significant difficulty if a coordinated behavior is desired, as discussed in the introduction of the paper. The use of full system information means that our method provides performance advantages, e.g. in terms of speed of convergence, when compared with locally-optimal, partial information-based approaches. For the sake of scalability and flexibility, it would be interesting to make the use of information distributed in our scheme, i.e. to require each agent to know the relative positions of only a subset of the other agents. We have observed in extensive simulations that the presented method can be successfully extended, in some cases, to scenarios with distributed information (i.e. incomplete formation graphs). In particular, the stability of the controller is observed to depend on both the characteristics of the formation graph and the geometry of the desired configuration. In light of (10), stabilization to the desired formation in this case may be posed as a nonlinear consensus problem where agreement on the rotation matrices $\mathbf{R_i}$ is required.

Our future work will consider the case in which the agents use only partial information of the group, corresponding to a subset of neighbors. The final objective is to characterize the classes of formation graphs and desired geometric configurations for which the proposed method can be effective. In addition, another relevant issue to explore is the consideration of nonholonomic kinematic agents.

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