Multirobot transport of deformable objects using deformation modes

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***Abstract*— We present a formation controller for transporting deformable objects in 2D space with a team of mobile robots. We assume the deformation of the transported object is determined by the deformation of the robotic formation. The goal is to reach a target configuration consisting of a desired shape, scale, position and orientation, allowing linear and quadratic deformations of the robotic formation relative to the target configuration. The use of this range of deformation modes enables preserving the integrity of the object while making the transport system highly flexible. The controller is tested in simulations and in real experiments with unicycle robots.**

I. Introduction

Transporting a deformable object with a team of robots can be a challenging task that requires high coordination, especially if the object is large, heavy or fragile, or if the trajectory to follow requires the team of robots to execute very specific actions. Formation control allows multirobot systems to manipulate these objects with very accurate movements to prevent damage during the transport.

The manipulation of deformable objects with multiple robots is a field widely covered in prior works [1]. In this context, transport tasks have been considered in different scenarios [2]–[6]. Some related work [7] presents the coordinated motion of the team of robots, modelled by single-integrator dynamics, through a linear combination of translation, shape-preserving transformation and affine transformation of a reference configuration. This combination enables the robots to carry out efficient rotation and resizing maneuvers. Other studies propose a solution exploiting measures of deformation with a linear control law considering single-integrator dynamics [8] or assuming double-integrator dynamics [9] to add inertial effects to the system. The controller in [9] allows driving a deformable object to a desired state by driving the team to a target configuration, defined as a combination of shape, scale, position and orientation in 2D space.

Deformation modes have been used to model deformable objects in computer graphics. Two examples are [10], which exploited the Finite Element Method, and [11], which proposed a geometric approach based on shape matching. This latter approach was used in [12] to estimate deformability in robotics applications. Modal analysis has also been recently exploited for shape control [13]. Linear deformation modes (i.e., stretching and shearing) were considered in [9] to control deformation during the transport, enhancing flexibility.

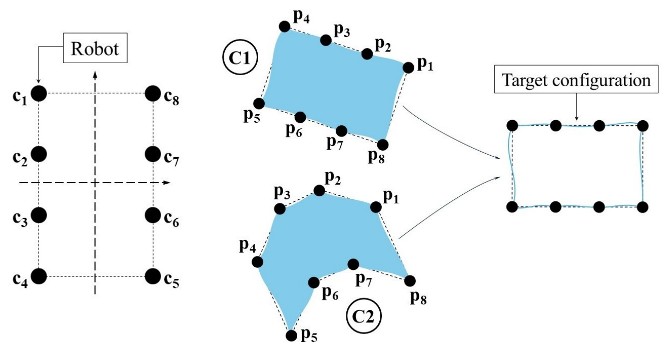

Fig. 1. (Left) Reference configuration of the formation. (Right) Representation of two achievable configurations (C1 and C2) during the transport of a deformable object by a team of mobile robots steered with the proposed controller to a target configuration, defined as a combination of desired shape, scale, position and orientation. If the task does not require that the object's shape is modified, the formation will be kept in a shape-preserving configuration (C1). However, if the formation needs to deform the object, the deformed formation will be constrained to linear and quadratic deformation modes allowing stretching, shearing, bending and twisting while avoiding other unsuitable and unpredictable deformation patterns (C2).

In this paper, we propose a method to transport a deformable object grasped around its contour by multiple mobile robots that allows the agents to perform maneuvers that deform the object only with linear and quadratic deformation modes during the trajectory. Some situations require the transported object to be deformed linearly, stretching or shearing it, as considered in [9]. Others, however, also need to deform it in a quadratic way, especially when the task entails changes in direction, causing the object to bend or twist. Therefore, we introduce quadratic deformation modes, inspired by [11], to increase adaptability relative to [9].

II. Multirobot Control with Deformation Modes

We consider a formation of N robots moving in a 2D space grasping the transported object through rotational joints, as illustrated in Fig. 1. We denote the position of robot $i \in \{1, \dots, N\}$ by $\mathbf{p}_i$. We assume single-integrator dynamics, i.e., $\dot{\mathbf{p}}_i = \mathbf{u}_i$, with $\mathbf{u}_i$ the control input. We group for the full team $\mathbf{p} = [\mathbf{p}_1^\intercal, \dots, \mathbf{p}_N^\intercal]^\intercal \in \mathbb{R}^{2N}$ and $\mathbf{u} = [\mathbf{u}_1^\intercal, \dots, \mathbf{u}_N^\intercal]^\intercal \in \mathbb{R}^{2N}$. The control strategy is based on separately controlling each configuration parameter (shape, scale, position and orientation) and using a linear combination of those controllers.

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This work was supported via project REMAIN - S1/1.1/E0111 (Interreg Sudoe Programme, ERDF), and via projects PID2021-124137OB-I00 and TED2021-130224B-I00 funded by MCIN/AEI/10.13039/501100011033, by ERDF A way of making Europe and by the European Union NextGenerationEU/PRTR. R. Marcos-Saavedra was supported by Cátedra de Transformación Industrial (Universidad de Zaragoza - Gobierno de Aragón). M. Aranda was supported via a María Zambrano Fellowship funded by the Spanish Ministry of Universities and by the European Union-NextGenerationEU.

As in [9], we assume that by suitably controlling the shape and scale of the team of robots we can control the deformation of the object and maintain its integrity. This assumption is valid, e.g., for highly deformable objects whose shape adapts to the shape of the team of robots.

Shape Control. We define $\mathbf{c_i} = [c_{ix}, c_{iy}]^\intercal$ as the position of the robot i in the reference configuration. Note that the target configuration is defined with the same shape as the reference one up to a scale, translation and rotation (Fig. 1). In order to control the shape of the formation during the transport, two types of configurations are considered. The shape-preserving configuration keeps the team in the same shape as the reference configuration. Let us define the following matrix:

$$\mathbf{C}_H = \mathbf{K} \begin{bmatrix} c_{1x} & c_{1y} & \dots & c_{Nx} & c_{Ny} \\ -c_{1y} & c_{1x} & \dots & -c_{Ny} & c_{Nx} \end{bmatrix}^\intercal \in \mathbb{R}^{2N\times 2}, \tag{1}$$

where $\mathbf{K} = (\mathbf{I}_N - (1/N)\mathbf{1}_N\mathbf{1}_N^\intercal) \otimes \mathbf{I}_2 \in \mathbb{R}^{2N\times 2N}$ is a centering matrix which translates the centroid to zero, $\mathbf{I}_N$ is the $N \times N$ identity matrix, $\mathbf{1}_N$ is a column vector of N ones and $\otimes$ denotes the Kronecker product.

Notice that for any $\mathbf{h}_H = [h_{H1}, h_{H2}]^\intercal$ the condition $\mathbf{Kp} - \mathbf{C}_H\mathbf{h}_H = \mathbf{0}$ is equivalent to $\mathbf{p}_i = [[h_{H1}, h_{H2}]^\intercal, [-h_{H2}, h_{H1}]^\intercal]\, \mathbf{c}_i$ for every robot i with the sets of points $\mathbf{p}_i$ and $\mathbf{c}_i$ both having zero centroid. This represents a rotation and uniform scaling of the reference configuration [9]. Note that we use centering for optimality [9], [11]. Therefore, if this condition is satisfied while the robots are moving, the team's shape will be kept. To define $\mathbf{h}_H$ we propose to use a least-squares shape alignment strategy: i.e., we choose $\mathbf{h}_H$ so that $\|\mathbf{Kp} - \mathbf{C}_H\mathbf{h}_H\|$ is minimum, being $\|\cdot\|$ the Euclidean norm. We define henceforth $\mathbf{h}_H = \mathbf{C}_H^+\mathbf{Kp} = \mathbf{C}_H^+\mathbf{p}$, as $\mathbf{C}_H^+\mathbf{K} = \mathbf{C}_H^+$, where $+$ denotes the Moore-Penrose inverse. We can formulate the following cost function associated with shape preservation:

$$\gamma_H = \frac{1}{2}\|\mathbf{Kp} - \mathbf{C}_H\mathbf{h}_H\|^2 = \frac{1}{2}\mathbf{p}^\intercal\mathbf{A}_H\mathbf{p}\,, \tag{2}$$

where $\mathbf{A}_H = \mathbf{K} - \mathbf{C}_H\mathbf{C}_H^+$. Note that $\mathbf{A}_H$ is constant, symmetric, idempotent and positive semidefinite. Then, we propose a controller for preserving the shape of the team following the negative gradient of γ_H:

$$\mathbf{u}_H = -k_H\mathbf{A}_H\mathbf{p}\,, \tag{3}$$

where k_H is a positive control gain.

The second configuration we consider is the deformed configuration which is expressed by deformation modes up to order two, allowing the formation to deform in a controlled way. Inspired by [11], we define this deformation with linear ($\mathbf{L}_i$), quadratic ($\mathbf{Q}_i$) and mixed ($\mathbf{M}_i$) terms. Analogously to $\mathbf{C}_H$ above, we can define the matrix

$$\mathbf{C}_G = \mathbf{K}\begin{bmatrix} \mathbf{L}_1 & \mathbf{Q}_1 & \mathbf{M}_1 \\ \vdots & \vdots & \vdots \\ \mathbf{L}_N & \mathbf{Q}_N & \mathbf{M}_N \end{bmatrix} \in \mathbb{R}^{2N\times 10}, \tag{4}$$

$$\mathbf{L}_i = \begin{bmatrix} c_{ix} & c_{iy} & 0 & 0 \\ 0 & 0 & c_{ix} & c_{iy} \end{bmatrix} \in \mathbb{R}^{2\times 4}, \tag{5}$$

$$\mathbf{Q}_i = \begin{bmatrix} c_{ix}^2 & c_{iy}^2 & 0 & 0 \\ 0 & 0 & c_{ix}^2 & c_{iy}^2 \end{bmatrix} \in \mathbb{R}^{2\times 4}, \tag{6}$$

$$\mathbf{M}_i = \begin{bmatrix} c_{ix}c_{iy} & 0 \\ 0 & c_{ix}c_{iy} \end{bmatrix} \in \mathbb{R}^{2\times 2}, \tag{7}$$

where $\mathbf{L}_i$ terms can only represent shear and stretch, whereas $\mathbf{Q}_i$ and $\mathbf{M}_i$ terms can represent bend and twist.

Similarly to the shape-preserving control, we choose a vector $\mathbf{h}_G \in \mathbb{R}^{10}$ which minimizes $\|\mathbf{Kp} - \mathbf{C}_G\mathbf{h}_G\|$. Then, we propose a deformation controller following the negative gradient of a cost function $\gamma_G = \frac{1}{2}\mathbf{p}^\intercal\mathbf{A}_G\mathbf{p}$:

$$\mathbf{u}_G = -k_G\mathbf{A}_G\mathbf{p}\,, \tag{8}$$

being k_G a positive control gain and $\mathbf{A}_G = \mathbf{K} - \mathbf{C}_G\mathbf{C}_G^+$, which is constant, symmetric, idempotent and positive semidefinite.

Scale Control. To fully control the deformation we also control the scale of the team using the variable $s = \|\mathbf{h}_H\|$. For achieving the desired scale, s_d, we propose the following control term, where k_s is a positive control gain and $s > 0$ can be assumed [9]:

$$\mathbf{u}_s = k_s\,(s_d - s)\,(1/s)\,\mathbf{C}_H\mathbf{h}_H\,. \tag{9}$$

Translation and Rotation Control. Translation and rotation of the formation do not alter the relative positions of the robots, so they do not affect the object's deformation. The translation controller is responsible for driving the team of robots as a whole to achieve a desired absolute position, $\mathbf{g}_d$, of the formation centroid $\mathbf{g} = \frac{1}{N}[\mathbf{p}_1, \dots, \mathbf{p}_N]\,\mathbf{1}_N$. The rotation controller rotates the shape around the formation centroid until the desired angle, θ_d, is reached. The angle can be obtained as $\theta = \mathrm{atan2}\,(h_{H2}, h_{H1})$. Control terms to achieve these transformations are, respectively,

$$\mathbf{u}_c = k_c\,\mathbf{1}_N \otimes (\mathbf{g}_d - \mathbf{g})\,, \tag{10}$$

$$\mathbf{u}_\theta = k_\theta\,(\theta_d - \theta)\,(\mathbf{I}_N \otimes \mathbf{S})\,\mathbf{C}_H\mathbf{h}_H\,, \tag{11}$$

where k_c and k_θ are positive control gains and $\mathbf{S} = [[0, 1]^\intercal, [-1, 0]^\intercal]$. In practice it is possible to take $\theta_d = 0$ for convenience and without loss of generality.

Full Formation Controller. The full control law results from the linear combination of the individual controllers:

$$\mathbf{u} = \mathbf{u}_H + \mathbf{u}_G + \mathbf{u}_s + \mathbf{u}_c + \mathbf{u}_\theta\,. \tag{12}$$

III. Experimental Validation

We validate our controller using the Robotarium [14] with multiple unicycle robots. Note that dynamic model conversion and avoidance of collisions (between agents, and with obstacles) are handled by the Robotarium. We define error variables for position, scale and orientation as $\mathbf{e}_g = \mathbf{g} - \mathbf{g}_d$, $e_s = s - s_d$, $e_\theta = \theta - \theta_d$, respectively.

We first conduct simulations for a formation of twelve robots manipulating a deformable sheet. The object is modelled with the As-Rigid-As-Possible (ARAP) technique [15]. The results are illustrated in Fig. 2. The videos of the simulations can be seen in [16].

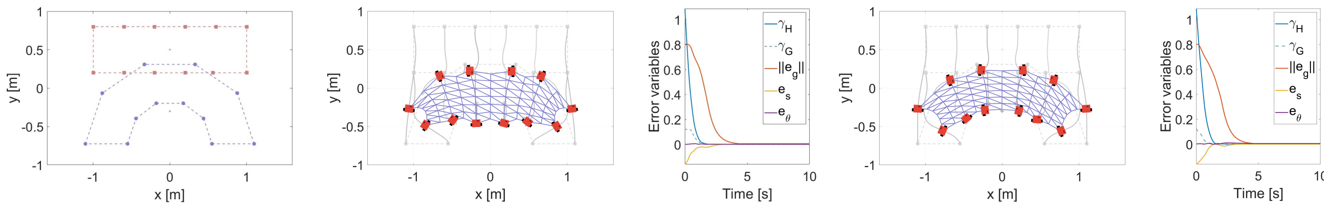

Fig. 2. Simulation results. From left to right, the plots are: 1st, initial configuration (blue circles, bottom) and target configuration (red squares, top); 2nd and 3rd, robot paths and error variables in Case 1; 4th and 5th, robot paths and error variables in Case 2. The configuration of the formation at instant $t = 0.8$ [s] is overlapped on the paths for both cases, showing that only in Case 2 the deformation tends to preserve a quadratic, bending-like pattern. In both cases, control gains are $k_H = 5$, $k_c = 0.5$, $k_s = 2$, and $k_\theta = 0.15$.

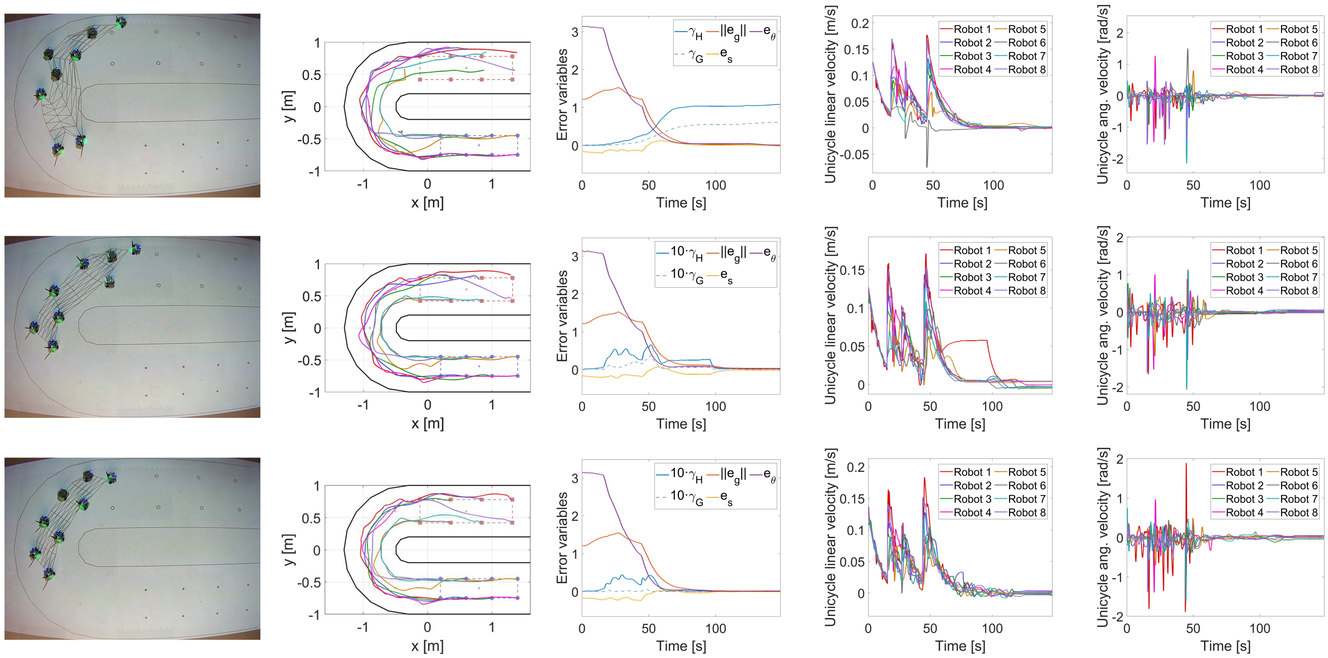

Fig. 3. Experimental results. For each row, the five plots from left to right are: representative top-view snapshots, robot paths, control errors, unicycle linear velocities, and unicycle angular velocities. Each row corresponds to a different shape-control strategy: 1st, the shape is not controlled in any way, $k_H = k_G = 0$, and the shape errors (γ_H, γ_G) do not converge to zero; 2nd, deformation is not controlled, $k_H \neq 0$ and $k_G = 0$; and 3rd, deformation is controlled, $k_H \neq 0$ and $k_G \neq 0$. In the 2nd and 3rd cases, although the shape cannot be preserved, all the errors reach zero values, but only in the 3rd strategy the object deforms in a controlled manner. The improvement of the parameters in the 3rd case with respect to the 2nd one can be seen in the error evolution between 50–100 [s]. The values of the remaining control gains are $k_c = k_s = k_\theta = 0.1$ in the three tests.

We consider a situation where a bent initial configuration has to be driven to a straight target one. We test two cases. For Case 1 we choose $k_G = 0$. The team of robots quickly approaches the same shape as the reference configuration, but deforming the object in an uncontrolled way. For Case 2 we select $k_G = 10$. This allows the formation to maintain a quadratic deformation. The movements are more efficient than in Case 1, producing gradual changes and staying close to a quadratic deformation pattern during the transient period.

Finally, we test the proposed controller in a real scenario with obstacles, where a team of eight robots transports a simulated sheet (also modelled with ARAP) along a curved corridor to a target configuration (Fig. 3). To achieve the experiment's goal we manually define three intermediate waypoints, as a combination of shape, scale, position and orientation, to guide the team during the task. The first strategy we test is to control the formation scale, position and orientation, but not the shape. Therefore, all errors converge to zero except those related to shape. The second strategy consists in applying $k_H = 0.2$ while maintaining $k_G = 0$, so that the formation tends to preserve the shape of the object but the deformations, which appear inevitably due to the constraints of this particular scenario, are uncontrolled. In the third strategy, we use $k_H = 0.1$ and $k_G = 1$. In this case, the object deforms in a controlled way, preserving its integrity. In the last two strategies all error variables converge to zero. These results support the interest of the proposed approach based on linear and quadratic deformation modes. Video results of these experiments are in [16].

Future work may involve the formal analysis of the controller, or the extension to 3D, with a different formulation of the rotation and uniform scaling transformation.

References

[1] R. Herguedas, G. López-Nicolás, R. Aragüés, and C. Sagüés, "Survey on multi-robot manipulation of deformable objects," in *Proc. IEEE Int. Conf. Emerg. Technol. Fact. Autom.*, 2019, pp. 977–984.

[2] J. Alonso-Mora, R. Knepper, R. Siegwart, and D. Rus, "Local motion planning for collaborative multi-robot manipulation of deformable objects," in *Proc. IEEE Int. Conf. Robot. Automat.*, 2015, pp. 5495–5502.

[3] D. Koung, O. Kermorgant, I. Fantoni and L. Belouaer, "Cooperative multi-robot object transportation system based on hierarchical quadratic programming," in *IEEE Robot. Automat. Lett.*, vol. 6, no. 4, pp. 6466–6472, 2021.

[4] R. Herguedas, G. López-Nicolás, and C. Sagüés, "Multirobot transport of deformable objects with collision avoidance," *IEEE Syst. J.*, vol. 17, no. 2, pp. 3224–3234, 2023.

[5] J. Hu, W. Liu, H. Zhang, J. Yi, and Z. Xiong, "Multi-robot object transport motion planning with a deformable sheet," *IEEE Robot. Automat. Lett.*, vol. 7, no. 4, pp. 9350–9357, 2022.

[6] C. Yang, G. N. Sue, Z. Li, L. Yang, H. Shen, Y. Chi, A. Rai, J. Zeng, and K. Sreenath, "Collaborative navigation and manipulation of a cable-towed load by multiple quadrupedal robots," *IEEE Robot. Automat. Lett.*, vol. 7, no. 4, pp. 10041–10048, 2022.

[7] M. Aranda, R. Aragüés and G. López-Nicolás, "Combined leaderless control of translational, shape-preserving, and affine multirobot formations," in *IEEE Robot. Automat. Lett.*, vol. 8, no. 11, pp. 7567–7574, 2023.

[8] M. Aranda, J. A. Corrales and Y. Mezouar, "Deformation-based shape control with a multirobot system," *Proc. IEEE Int. Conf. Robot. Automat.*, 2019, pp. 2174–2180.

[9] R. Herguedas, M. Aranda, G. López-Nicolás, C. Sagüés and Y. Mezouar, "Double-integrator multirobot control with uncoupled dynamics for transport of deformable objects," in *IEEE Robot. Automat. Lett.*, vol. 8, no. 11, pp. 7623–7630, 2023.

[10] A. Pentland and J. Williams, "Good vibrations: modal dynamics for graphics and animation," *SIGGRAPH Comput. Graph.*, vol. 23, no. 3, pp. 207-214, 1989.

[11] M. Müller, B. Heidelberger, M. Teschner and M. Gross, "Meshless deformations based on shape matching," in *ACM Trans. Graph.*, vol. 24, pp. 471–478, 2005.

[12] P. Güler, A. Pieropan, M. Ishikawa, and D. Kragic, "Estimating deformability of objects using meshless shape matching," in *Proc. IEEE/RSJ Int. Conf. Intell. Robots Syst.*, pp. 5941–5948, 2017.

[13] B. Yang, B. Lu, W. Chen, F. Zhong, and Y.-H. Liu, "Model-free 3-D shape control of deformable objects using novel features based on modal analysis," *IEEE Trans. Robot.*, vol. 39, no. 4, pp. 3134–3153, 2023.

[14] S. Wilson, P. Glotfelter, L. Wang, S. Mayya, G. Notomista, M. Mote, and M. Egerstedt, "The Robotarium: Globally impactful opportunities, challenges, and lessons learned in remote-access, distributed control of multirobot systems," *IEEE Control Syst. Mag.*, vol. 40, no. 1, pp. 26–44, 2020.

[15] O. Sorkine and M. Alexa, "As-rigid-as-possible surface modeling," in *Proc. Eurographics Symp. Geometry Process.*, 2007, pp. 109–116.

[16] R. Marcos-Saavedra, M. Aranda and G. López-Nicolás, "Multirobot transport of deformable objects using deformation modes," Accessed: May 6, 2024. [Online]. Available: https://youtube.com/playlist?list=PLjm-KzNA87CoOYVSBtOoxgU3No-d4tw-K&si=d_aZh1NJCUoBdQIr.