

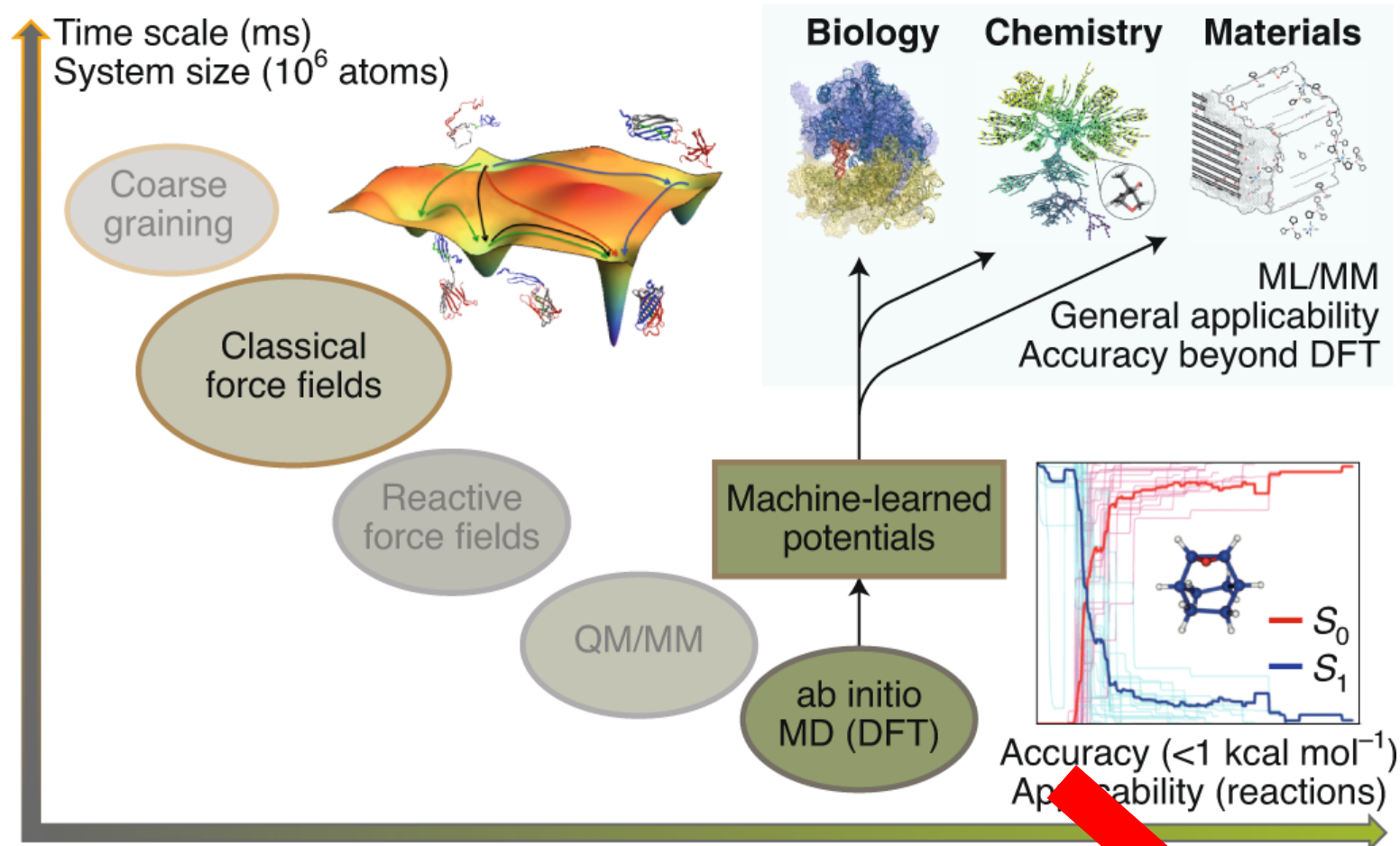
# Feasibility Study of *GNN/ACE* for Chemistry

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# Computational Chemistry

Cost-accuracy tradeoff limits simulation of *large* systems in *long* time scale

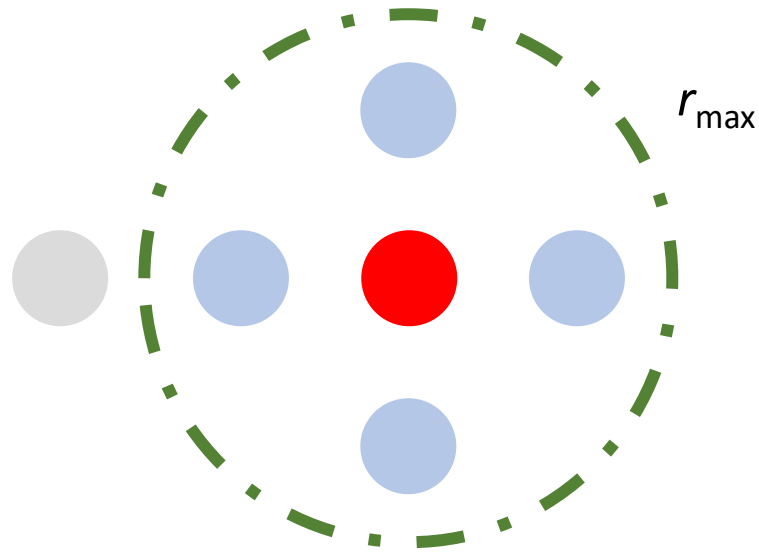


For AIMD,  
*interatomic potentials*  
are the quantities that matter

MD requires updates  
for each time step

# Atomic Cluster Expansion

Expand the energies (and forces) as a many-body expansion of atoms within some cutoff distance



$$E_i = \varepsilon_{(i)}^{(1)} + \frac{1}{2} \sum_j \varepsilon_{(i,j)}^{(2)} + \frac{1}{6} \sum_{j,k} \varepsilon_{(i,j,k)}^{(3)} + \dots$$



Rotational and Translational Invariance

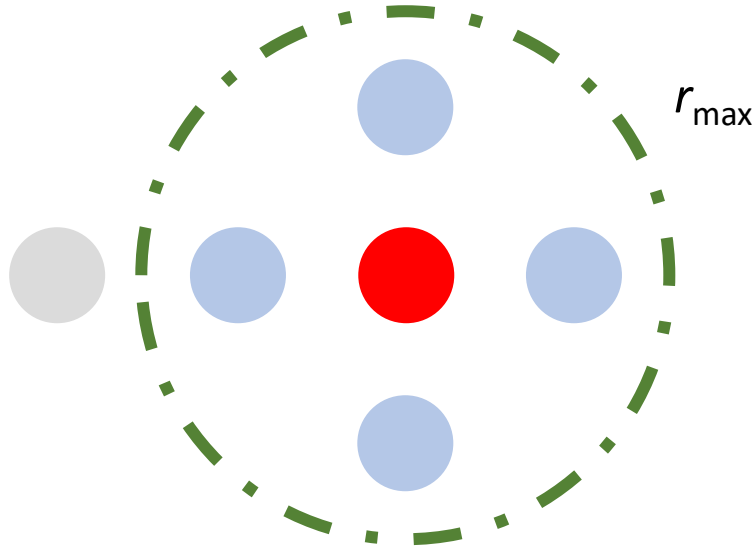
Permutation Invariance

...

through tensor product states of hydrogen-like wavefunctions

# Atomic Cluster Expansion

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Define neighborhood density

$$\rho(i, \mu) = \sum_{j \neq i} \delta_{\mu, \mu_j} \delta(r - r_{ji})$$

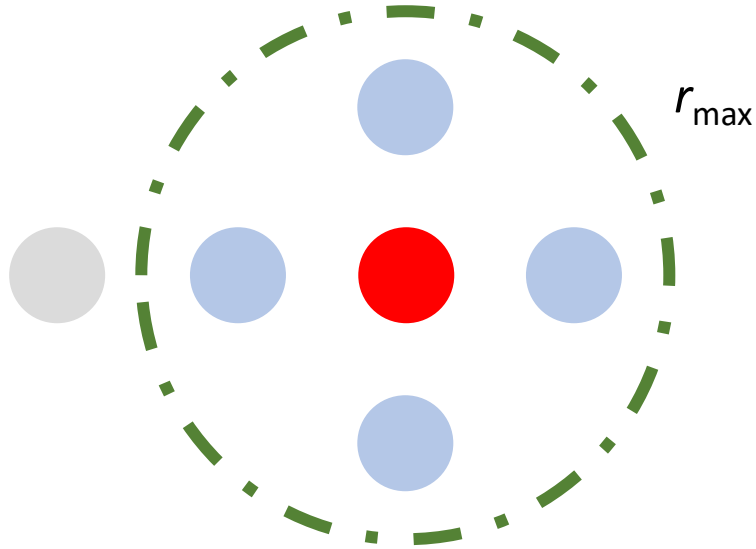
Project neighborhood density into hydrogen-like orbitals

$$\phi_{\mu_i, \mu_j, n, l, m} = R_{n, l}^{\mu_i, \mu_j}(r_{ji}) Y_{l, m}(r_{ji})$$

$$A_{i, (\mu_i, \mu_j, n, l, m)} = \langle \rho(i, \mu) | \phi_{\mu_i, \mu_j, n, l, m} \rangle = \sum_j \phi_{\mu_i, \mu_j, n, l, m}(r_{ji})$$

# Atomic Cluster Expansion

Expand the energies (and forces) as a many-body expansion of atoms within some cutoff distance



$$E_i = \varepsilon_{(i)}^{(1)} + \frac{1}{2} \sum_j \varepsilon_{(i,j)}^{(2)} + \frac{1}{6} \sum_{j,k} \varepsilon_{(i,j,k)}^{(3)} + \dots$$

Take tensor products for n-th order cluster terms

$$A_{i,v} = \prod_{t=1}^v A_{i,(\mu_t, n_t, l_t, m_t)} \quad \leftarrow \text{over all possible } n\text{-body combinations}$$

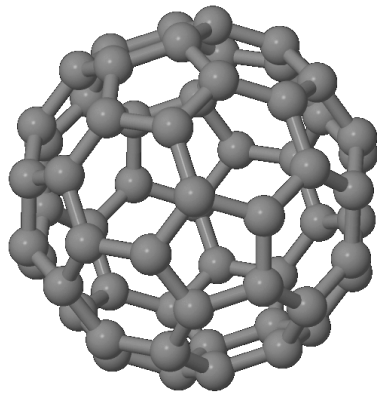
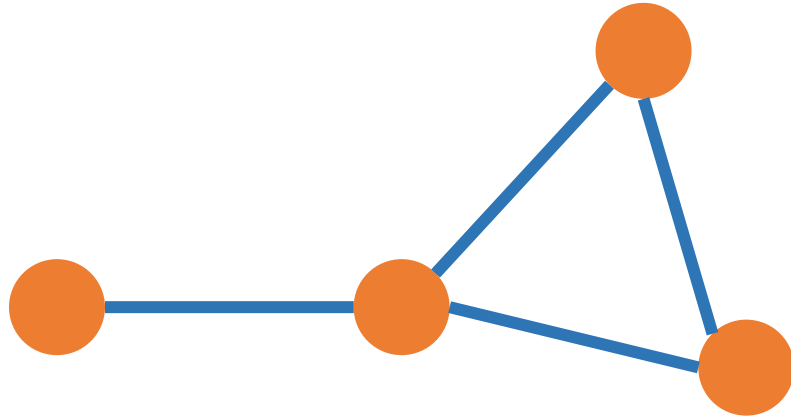
Set up energy estimator based on the basis

$$\psi_i^{(p)} = \sum_v c_v^{(p)} B_{i,v} = \sum_v c_v^{(p)} \sum_{v'} c_{v,v'} A_{i,v}$$

$$E_i = \mathcal{F}(\psi_i^{(0)}, \dots, \psi_i^{(p)}, \dots)$$

# Graph Neural Networks

Input data are graphs...



represented as adjacency matrices

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

each **node**, **edge**, and **graph** can have attributes

**Molecular geometry** as input graphs,  
**ACE local descriptors** as node attributes  
**Energies** and **forces** as graph attributes (output)

Jrnl

# Workflow

Training / Test dataset : 1) molecular geometry  
2) DFT energies

Expressed in ACE basis set, with local descriptors

Build normalized adjacency matrix

Define simple GNN model

Train the neural net

Compare with linear (least squares) solution

PotentialLearning.jl (xyz.jl)

PotentialLearning.jl (ace.jl)

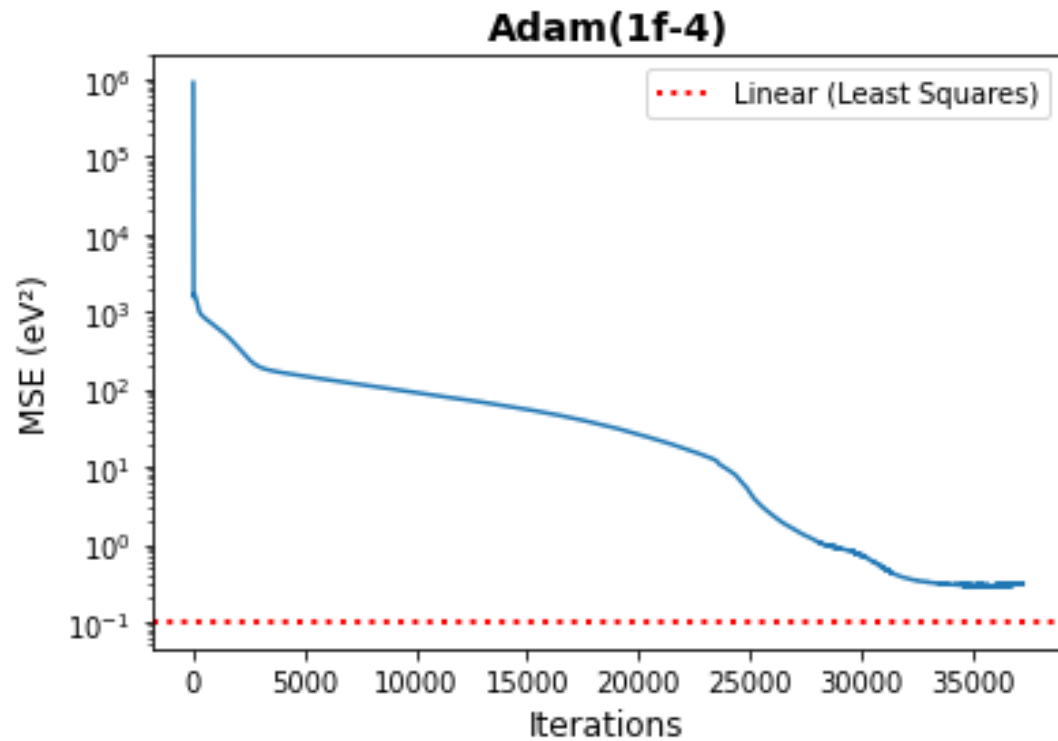
GraphNeuralNetworks.jl

Flux.jl

# Training

a-HfO<sub>2</sub>: 96 atoms (either Hf or O) in periodic cell

Least squares fit of 200 input data, 100 test data: MSE = 0.10 eV (67 sec)



Initial training with small fixed learning rate  
(200 input data, 100 test data)  
~ hrs time scale

To improve convergence, use

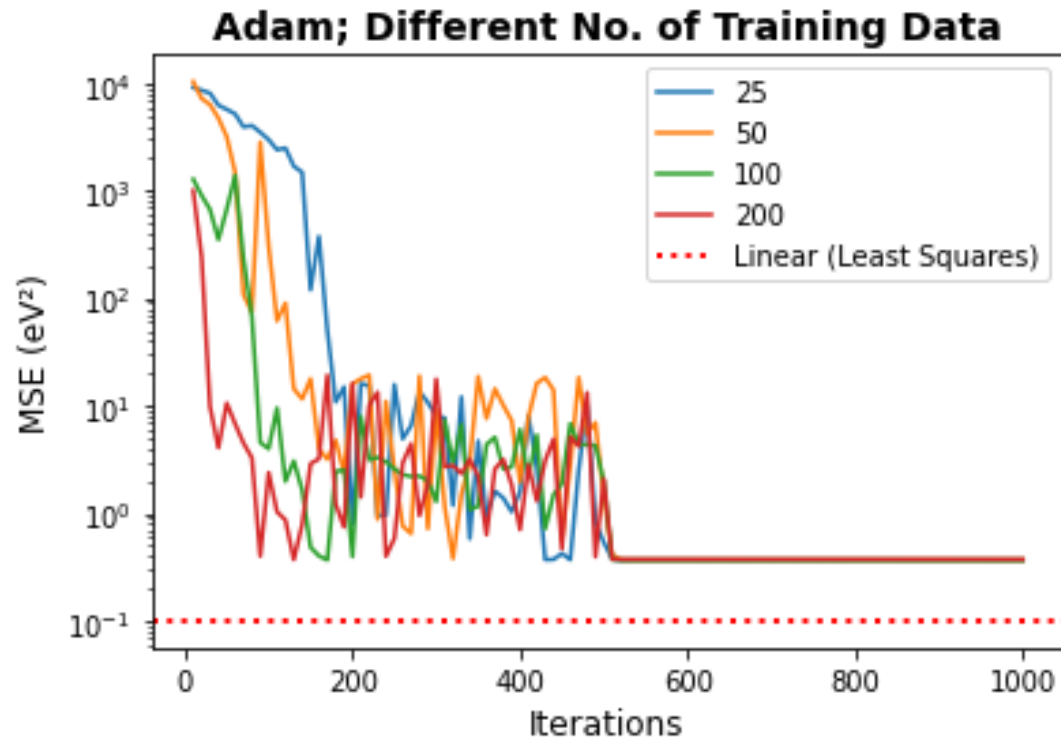
- Normalized adjacency matrix
- Xavier initialization
- Larger learning rate, change momentum



# Training

a-HfO<sub>2</sub>: 96 atoms (either Hf or O) in periodic cell

Least squares fit of 200 input data, 100 test data: MSE = 0.10 eV (67 sec)



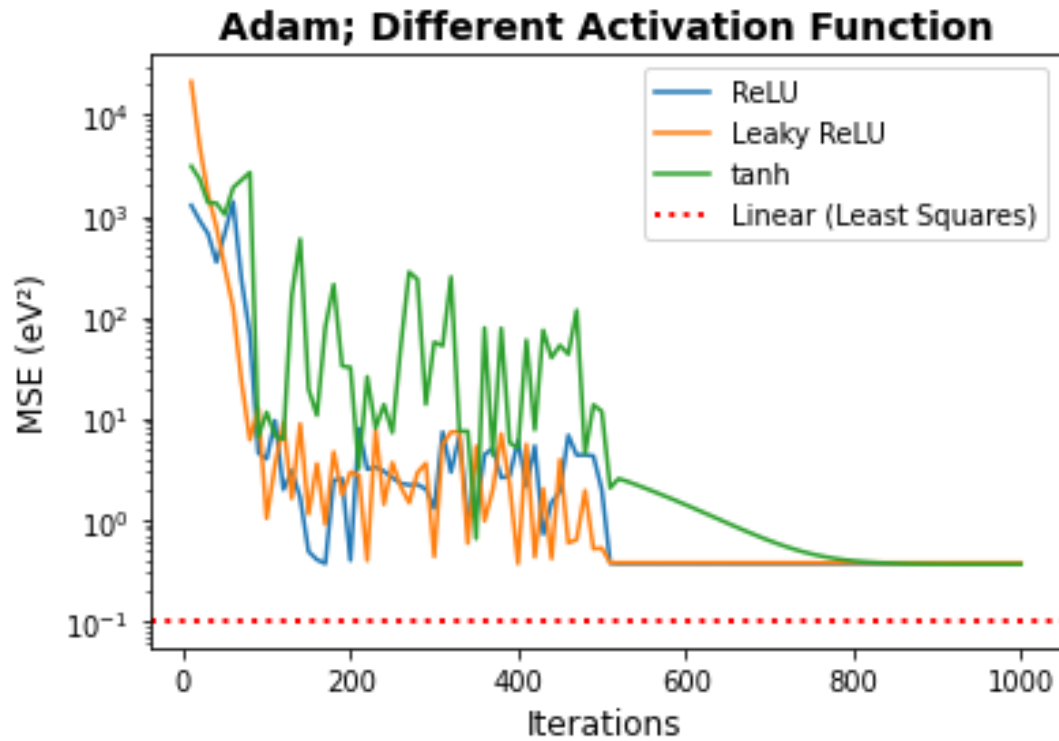
Improved training (~ minutes time scale)

GPU code tested, but difference insignificant  
(compared to 20-threaded CPU runs)

# Training

a-HfO<sub>2</sub>: 96 atoms (either Hf or O) in periodic cell

Least squares fit of 200 input data, 100 test data: MSE = 0.10 eV (67 sec)



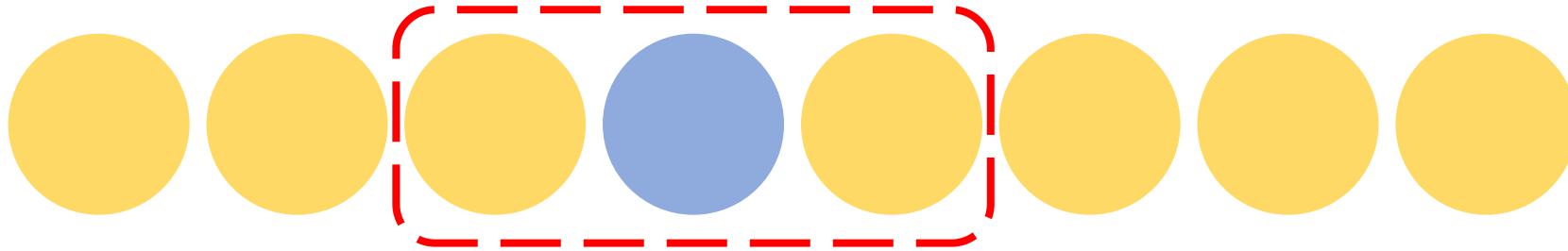
Modifications to the activation function do not cause significant changes

## Next Steps...

Verify with other dataset  
(i.e. FitSNAP)

Connect edge embedding in GNN & cluster expansion terms with quantum embedding methods

- Edge embedding over multiple edges
- Handle incomplete attributes



Experiment larger GNN models with full dataset