

1 **FASTER SUBPIXEL REGISTRATION FOR FORWARD LOOKING**
2 **SONAR RECONSTRUCTION***

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4 **Abstract.** This paper presents a benchmarking comparison of subpixel image registration in
5 Julia and Matlab as applied to forward looking SONAR reconstruction. The phase-correlation area-
6 based registration algorithm is investigated and a novel Julia modification to the existing algorithm
7 for GPU usage is offered. All implementations achieve the same results with varying execution
8 times. Julia consistently outperforms the Matlab counterparts. Results show a 2.97 times speedup
9 for serialized implementations and a 13.56 times speedup for parallel implementations, using Julia
10 as compared to Matlab. The novel Julia implementation utilizing GPUs shows a 6.52 times speedup
11 compared to the serial Julia code.

12 **Key words.** Julia, Matlab, SONAR, Image Registration, Phase Correlation

13 **1. Introduction.** Recent advances in forward-looking SONAR technologies lend
14 to large collections of acoustic imagery at high frame rates, providing a useful ba-
15 sis for reconstructing underwater environments. SONAR provides unique challenges
16 compared to optical imagery, such as low signal-to-noise ratios and inhomogenous
17 intensity across the image. Reconstruction methods for all imagery rely on image
18 registration, which is the ability to align two images by means of a model and data
19 transformation. To achieve a full reconstruction of a high frame rate dataset, it would
20 be simple to transform nearby frames via image registration and concatenate for a
21 global reconstruction. However, cumulative errors arise so global alignment is needed
22 to employ consistency between consecutive and non-consecutive image pairs. Many
23 methods exist for global image alignment, one such method that has shown great
24 performance on SONAR datasets is pose-graph optimization. [5] [4]

25 The full pose-graph optimization for forward-looking SONAR reconstruction is
26 described in [4], and is outside of this project’s scope. A short summary suffices to
27 motivate the work.

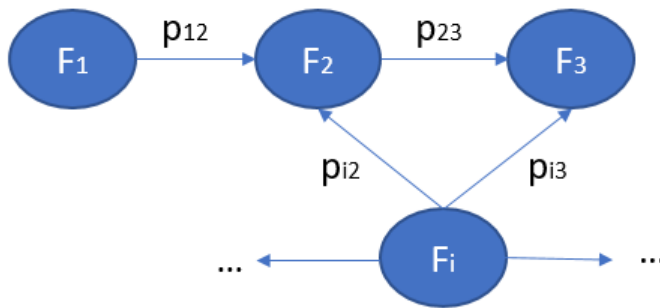


FIG. 1. Pose-graph structure for forward looking SONAR reconstruction algorithms.

28 Suppose you have a dataset of SONAR frames $\mathbf{F} = [F_1, F_2, F_3, \dots, F_n]$ and con-

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29 straints based off neighbor pairwise image registrations $\mathbf{p} = [p_{12}, p_{23}, \dots, p_{1n}, p_{2n}, \dots]$.
 30 The resulting graph structure is shown in Fig. (1). Each vertex of the graph repre-
 31 sents the position of the SONAR image on the reference frame. Image registration
 32 constraints make up the edges linking corresponding vertices. [5] [4]

33 The graph is initialized using the simple concatenation of transformations for
 34 neighbor vertices. Then, the graph is optimized using a select list of candidate frames
 35 for further image registration. One option for selecting candidate frames is to register
 36 all frames with the full rest of the set. However, this brings a huge computational
 37 burden and becomes untenable very quickly for sequential image registration. The
 38 standard approach, set by [5], is to compute registrations of each frame with several
 39 neighbor frames using a mixed window. The window size is estimated according
 40 operational parameters of the SONAR, such as range and mean velocity. [5] [4]

41 In practice, this results in a small loop of image registrations for one vertex with
 42 its selected neighbors, computed for all vertices. This is still very computationally
 43 intensive. For example, one case of image registration using a dataset of 700 frames
 44 resulted in over 12,600 calls to the image registration function. Depending on your
 45 implementation, this can easily create a bottleneck.

46 The aim of this project is to investigate options to ease this computational burden.
 47 The main contributions of this work are twofold: a series of benchmarks comparing
 48 phase-correlation subpixel registration in Julia and Matlab and a novel modification
 49 to existing phase-correlation subpixel registration in Julia that utilizes GPUs. All
 50 Matlab and Julia code make use of existing implementations [1] [6].

51 The paper is organized as follows. Image registration is described in [section 2](#)
 52 along with a summary of the current implementations in Julia and Matlab. Our
 53 benchmarking results are in [section 3](#) followed by discussion. Our new modification
 54 to the existing Julia subpixel registration is described in [section 5](#) and the conclusions
 55 are described in [section 6](#).

56 **2. Phase-correlation Subpixel Registration Algorithm.** Aligning images
 57 by means of a model and data transformation, or image registration, has wide appli-
 58 cability yet is often computationally and data intensive [2]. Registration techniques
 59 are broadly studied and generally fall into two categories: feature-based approaches
 60 and area-based approaches. To estimate the projection relating one image to another,
 61 feature-based approaches rely on a small set of localized, distinguishable points while
 62 area-based approaches utilize the intensity information from the full image [7],[5].
 63 Area-based methods therefore become advantageous when features are not well local-
 64 ized or have poor resolution, which is typical of forward looking SONAR imagery [7].
 65 Prior work shows sub-pixel accuracy area-based methods outperform similar feature-
 66 based methods for simplified forward-looking SONAR geometries, and are thus the
 67 focus of this work [4].

68 Area-based methods, often referred to as Fourier-based methods, assess a simi-
 69 larity metric between two images in the frequency domain. A common similarity
 70 metric is the cross correlation. The particular area-based method shown to perform
 71 well on SONAR imagery is the phase-correlation registration algorithm, and will be
 72 the subject of analysis for this project. [5]

73 The basis of the phase correlation algorithm is the Fourier shift property, which
 74 says that if you have two images, $f(x, y)$ and $g(x, y)$ related by a translation,

$$75 \quad f(x, y) = g(x - t_x, y - t_y)$$

76 ,

77 their resulting spectrum (obtained via Fourier transform) encodes this information
 78 into the phase

$$79 \quad F(u, v) = G(u, v) \exp -i(u * t_x - v * t_y).$$

80 The basic workflow for pixel level accuracy phase-correlation registration is shown
 81 in figure 2 and provides the basis for the subpixel level accuracy algorithm. The final
 82 translation is achieved through identifying the peak of the cross-power spectrum's
 83 inverse Fourier transform.

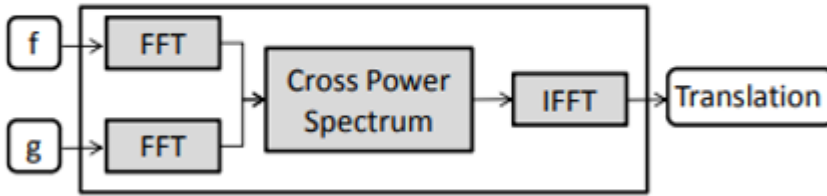


FIG. 2. Workflow for pixel-level accuracy phase-correlation registration, pulled from [5].

84 To achieve subpixel level accuracy in an efficient manor, the following workflow
 85 is implemented, developed in [2]. Define $\kappa = 1/s$ where s is the fraction of a pixel the
 86 images should be registered within (i.e., s is the subpixel precision). Define M, N as
 87 the x and y pixel dimensions, respectively.

- 88 • Obtain initial estimate as starting point.
 - 89 1. Set $\kappa_0 = 2$
 - 90 2. Compute $F(u, v)$ and $G(u, v)$
 - 91 3. Embed the product $F(u, v) * G(u, v)$ in a larger array of zeros with
 92 dimensions $[\kappa_0 M, \kappa_0 N]$
 - 93 4. Compute an inverse FFT to obtain the upsampled cross power spectrum
 - 94 5. Locate the peak
- 95 • Obtain refined estimate using a 1.5×1.5 pixel region about the initial estimate
 96 (in original pixel units).
 - 97 1. Set $\kappa_1 \approx \sqrt{\kappa}$
 - 98 2. Compute $F(u, v)$ and $G(u, v)$
 - 99 3. Embed the product $F(u, v) * G(u, v)$ in a larger array of zeros with
 100 dimensions $[\kappa_1 M, \kappa_1 N]$
 - 101 4. Compute an inverse FFT to obtain the upsampled cross power spectrum
 - 102 5. Locate the peak
- 103 • Obtain refined estimate using a $\frac{3}{\kappa_1} \times \frac{3}{\kappa_1}$ region about the new estimate (in
 104 original pixel units).
 - 105 1. Set $\kappa = \frac{1}{s}$
 - 106 2. Compute $F(u, v)$ and $G(u, v)$
 - 107 3. Embed the product $F(u, v) * G(u, v)$ in a larger array of zeros with
 108 dimensions $[\kappa M, \kappa N]$

- 109 4. Compute an inverse FFT to obtain the upsampled cross power spectrum
 110 5. Locate the peak

111 Typically this phase-correlation registration method is advertised as computationally efficient and less memory-intensive. However, many implementations do not utilize modern computing features such as parallelism or graphical processing units (GPUs) [4],[1]. A high-performance solution in Julia has been developed and assessed for feature-based registration algorithms as applied to medical imagery, where they often perform very well [3]. Performance of area-based algorithms is less documented, and no studies seek a high-performance solution for SONAR data to the author’s awareness.

119 **3. Benchmarking results.** Here we state the first results, a comparison of the above phase-correlation subpixel registration implemented in Matlab and Julia [1], [6]. Corresponding to the motivation use case, phase-correlation registrations were performed on real-world SONAR data collected by an unmanned underwater vehicle for mapping purposes.

124 Both implementations computed the same results within a tolerance of $\epsilon = 0.001$, deemed acceptable for this use case. Each frame has dimensions [661X484]. A subset of 9 frames were selected for registration within a loop, and execution time is evaluated for the entire loop. This directly applies to the pose-graph optimization for SONAR reconstruction application, we seek to optimize for.

129 Figure 3 shows the results for a serialized loop using the existing Matlab and Julia methods. The loop was performed 10 times to ascertain confidence in the results. Matlab timing is computed using the ‘tic’ and ‘toc’ functions. Julia timing is generated using the ‘@btime’ function from the BenchmarkTools package.

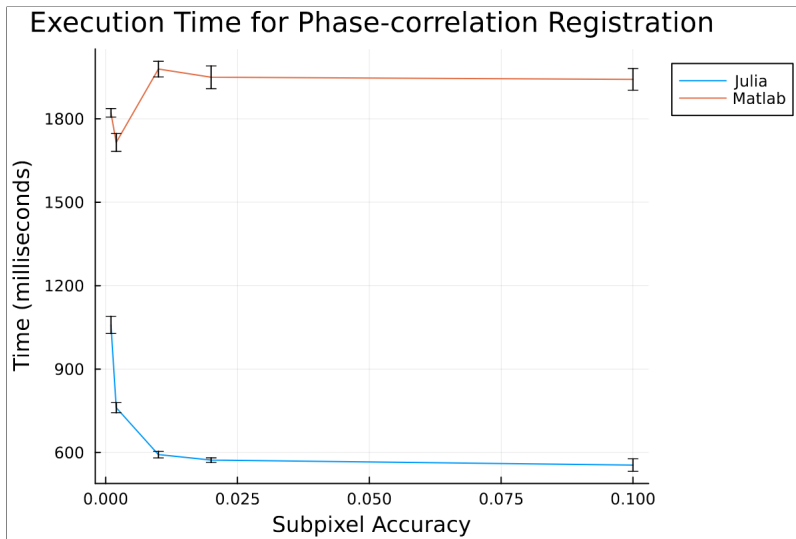


FIG. 3. *Baseline comparisons for phase-correlation registration implementations.*

133 Figure 4 shows the results for a parallelized loop using the existing Matlab and
 134 Julia methods. Parallelism in Matlab is achieved using the ‘parfor’ technique. Parallelism in Julia is achieved using the Threads package. Both were performed on an 8
 135 core machine with Intel Xeon E5 CPUs. The loop was performed 10 times to ascertain confidence in the results. Matlab timing is computed using the ‘tic’ and ‘toc’ func-
 136
 137

138 tions. Julia timing is generated using the '@btime' function from the BenchmarkTools
 139 package.

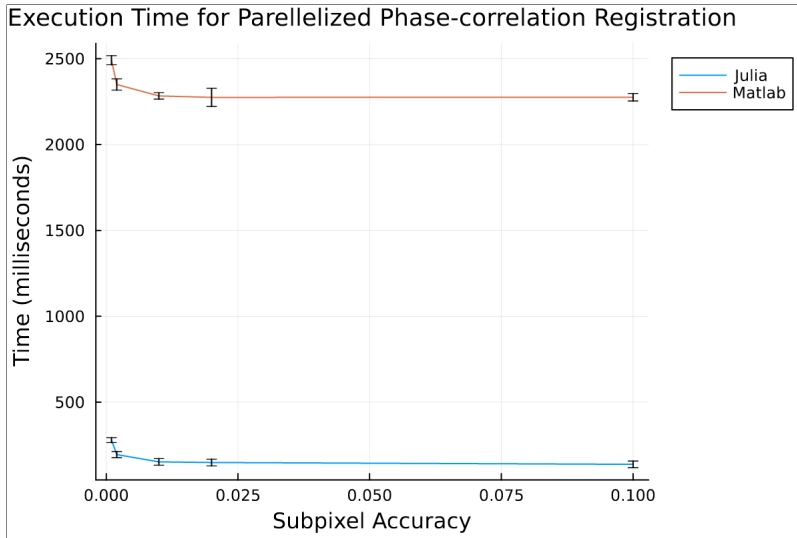


FIG. 4. *Parallelized comparisons for phase-correlation registration implementations.*

140 **4. Discussion of benchmarking results.** Julia implementations give much
 141 faster execution times for both the serial and parallelized versions as compared to
 142 Matlab. This is expected because the Julia code is type stable, meaning its compiled
 143 version is essentially statically-typed after the first call. Even though our function is
 144 called in a loop, Julia is able to compile and optimize the specific registration function
 145 rather than having to optimize for the entire loop in its underlying LLVM code, which
 146 is advantageous. Matlab is known to run faster for vectorized code with pre-allocated
 147 arrays. While the existing implementation does a good job at pre-allocation, it is not
 148 well vectorized, so the execution time is understandably slower.

149 The parallelized Julia code is much faster than both Matlab versions and the se-
 150 rial Julia version. Despite this loop not being an obvious candidate for optimization
 151 due to its small number of iterations, each function call is independent so Julia is able
 152 to execute this on multiple threads with little overhead. Matlab, on the other hand,
 153 experiences a great deal of overhead and is surprisingly much slower than its serial
 154 counterpart. There are no obvious oddities to Matlab's phase-correlation implemen-
 155 tation that would cause such a slow down, such as use of parallelization within one of
 156 the sub-functions. It is generally advised to assess the computational burden of func-
 157 tion calls within a loop before introducing parallelism in Matlab, as less burdensome
 158 functions often have too much overhead for a benefit, and this seems to be the case
 159 with phase-correlation registration.

160 Each method scales similarly with respect to the degree of subpixel accuracy.
 161 Requesting registration within a smaller fraction of a pixel results in longer execution
 162 times. This is due to higher dimension data from upsampling, which is more expensive
 163 for each operation.

164 **5. Algorithmic results.** Here we describe our second result, a novel modifica-
 165 tion to the existing subpixel phase-correlation registration in Julia utilizing CUDA
 166 to execute expensive operations on GPUs. This algorithm is based on the Subpixel-

167 Registration package and has been tested with a NVIDIA Volta GPU. This code is
 168 not particularly optimized, but has shown beneficial performance over the standard
 169 algorithm limited to CPUs.

```

70 function upsampled_dft(data::CuArray{T},region_size,upsample_factor,offsets,) where {T<:Complex}
71     shiftrange = CuArray(collect(1:Int64(region_size)))
72     idxoffset = map(first, axes(data))
73     sample_rate = inv(T(upsample_factor))
74     freqs = CuArray(fftfreq(size(data, 2), sample_rate))
75
76     off2 = CuArray([last(offsets)])
77     ioff2 = CuArray([last(idxoffset)])
78
79     kernel = @. (shiftrange - off2 - ioff2) * freqs'
80     kernel = kernel .* (2*pi)
81     kernel = map(x->cis(x), kernel)
82     _data = kernel * data'
83
84     freqs = fftfreq(size(data, 1), sample_rate)
85     off1 = CuArray([first(offsets)])
86     ioff1 = CuArray([first(idxoffset)])
87
88     kernel = @. (shiftrange - off1 - ioff1) * freqs'
89     kernel = kernel .* (2*pi)
90     kernel = map(cis, kernel)
91     _data = kernel * _data'
92     return _data
93 end

```

FIG. 5. Julia code snippet that utilizes GPUs using CUDA functionality.

170 Only most expensive portion of this algorithm, the upsampled discrete Fourier
 171 transform, is shown for brevity. Full code will be made available on GitHub at [https://](https://github.com/remartell/mit_18337_SubpixelRegistration.git)
 172 github.com/remartell/mit_18337_SubpixelRegistration.git. CuArrays were utilized to
 173 implement GPU calculations. Although this comes at a memory cost for transferring
 174 data between the CPU and GPU, execution time was the metric of interest for this
 175 project.

176 Figure 6 shows a comparison for each Julia implementation for a variety of sub-
 177 pixel accuracy requirements. As expected, the serial version performs worst as there
 178 are no optimizations. The parallelized version performs, on average, 3.90 times faster
 179 than serial. The CUDA version performs, on average, 1.68 times faster than the
 180 parallel version and 6.52 times faster than the serial version.

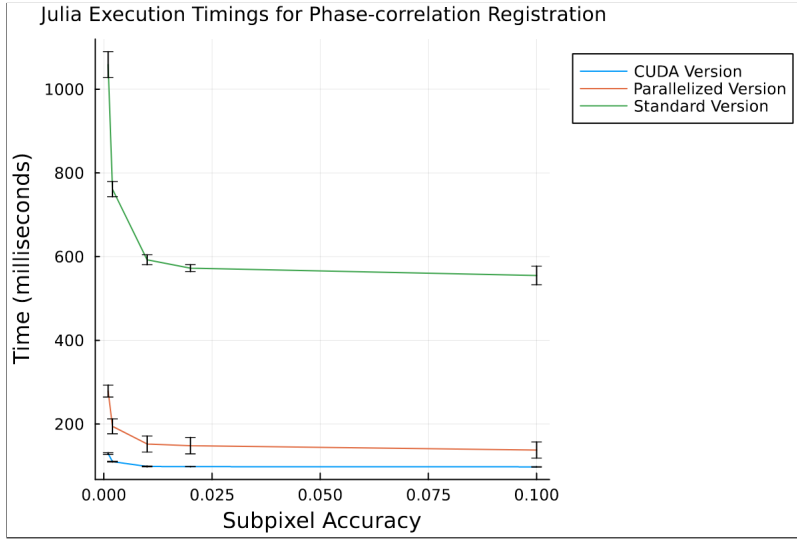


FIG. 6. Julia comparisons for various phase-correlation registration implementations.

181 **6. Conclusions.** In this work, we assessed performance of the phase-correlation
 182 image registration algorithm in Julia and Matlab as used in forward looking SONAR
 183 applications. It is well confirmed that Julia outperforms Matlab with the serial and
 184 parallel implementations. For serialized implementations, Julia executes 2.97 times
 185 faster than its Matlab counterpart. For parallelized implementations, Julia executes
 186 13.56 times faster than its Matlab counterpart. A GPU comparison was not possible
 187 due to limitations in Matlab’s GPU code conversion tool, but Julia clearly showed
 188 benefit for using GPUs with great ease to the user. Compared to the Julia serial
 189 version, the CUDA version runs 6.52 times faster.

190 Following the original motivation of this project, the example of 12,600 function
 191 calls to the phase-correlation registration function would take approximately 27 min-
 192 utes to achieve $\frac{1}{1000}$ subpixel accuracy when utilizing GPUs. Comparatively, Matlab’s
 193 fastest offering, the serial version, would take over 8 hours to complete these compu-
 194 tations. Julia provides a serious benefit with little cost to the user.

195 Future work on benchmarking would compare the memory use for Matlab and
 196 Julia algorithms. Matlab does not have easy command-line tools for assessing the
 197 memory footprint like Julia’s BenchmarkTools and CUDA packages, so this was not
 198 possible for this project. Further work on the novel Julia algorithm would optimize
 199 the CUDA implementation, perhaps utilizing a GPU kernel to avoid any CPU calcu-
 200 lations.

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203

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